

Title: Explicit multiloop amplitude computations with the pure spinor formalism

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Abstract: In this talk I will show how to use the pure spinor formalism to compute multiloop string amplitudes, including their precise overall coefficients. I will also discuss the relevance of the recent four-point 3-loop and five-point 2-loop amplitudes for verifying some S-duality predictions.

MEASURES

$$[d] T_{\alpha_1 \dots \alpha_5} = c_1 \epsilon_{\alpha_1 \dots \alpha_6} d\tau^{\alpha_6} \dots d\tau^{\alpha_6}$$

$$[dw^i] = c_w T_{\alpha_1 \dots \alpha_5} \epsilon^{\tau_1 \dots \tau_6} dw_{\alpha_6}^{\tau_6} \dots dw_{\alpha_6}^{\tau_6}$$

$$c_1 = \left(\frac{\pi}{2}\right)^{-2} \frac{1}{11!} \left(\frac{A_g}{4\pi^2}\right)^{11/2}$$

$$T_{\tau_1 \dots \tau_5} = (p^m)_{\alpha_1} (p^u)_{\alpha_2} (p^p)_{\alpha_3} (p^{map})_{\alpha_4 \alpha_5}$$

THE VARIABLES

FERMIONIC

s^α	11	11g
r_α	11	11
d_α	16	16g
σ^α	16	16

BOSONIC

λ^α	11	11
$\bar{\lambda}_\alpha$	11	11
w_τ	11	11g
$\bar{w}^\alpha$	11	11g
x^m	10	10

NON
ZERO MODES

ZERO
MODES

EXPLICIT MULTILoop AMPLITUDE COMPUTATIONS WITH PS FORMALISM

BASED ON .

1504.02759	M. GOMEZ, CM, SCHLOTTERER
1308.6567	M. GOMEZ, CM
1003.0678	M. GOMEZ, CM

BASED ON 1001.06734 M. GOMEZ, CM, SCHLOTTERER (SPT @ (LOOPS))
 1308.6567 M. GOMEZ, CM (4PT 3 LOOPS)
 1003.0678 M. GOMEZ, CM (4PT, TREE, 1, 2 LOOPS)

IN THIS TALK:
 1 LOOP $\rightarrow$ 4PT, SPT
 2 LOOP $\rightarrow$ 4PT SPT
 3 LOOP $\frac{4PT}{\downarrow}$
 Low ENERGY LIMIT

Loop 1  $\rightarrow$ $(\gamma_{mpqr}) (\gamma_{s_{12}}) (1_{34} + (12 \leftrightarrow 34))$

1308.6567

1003.0678

M. GOMEZ, CM

(4PT, 3 LOOPS)

M. GOMEZ, CM

(4PT, TREE, 1, 2 LOOPS)

IN THIS TALK:

1 LOOP $\rightarrow$ 4PT, 5PT

2 LOOP $\rightarrow$ 4PT SPT

3 LOOP $\rightarrow$ 4PT

LOW ENERGY LIMIT

2005

$$M_n^{(g)} = S_g e^{2g-2} \int \prod_{i=1}^{3g-3} d^2 z_i \int \sum K N(b_{m_i}) U_1 \dots U_n \rangle^2$$

$\uparrow$ SYMMETRY $\uparrow$ TEICHMULLER $\uparrow$ REGULATOR $\nwarrow$ BELTRAMI DIFF

$S_1 = \frac{1}{2}, S_2 = \frac{1}{2}, S_3 = 1$

loop  $\leftrightarrow (\lambda \gamma_{mnpqr} \lambda) (\lambda \gamma_s W_{12}^{mnpq}) F_{34}^{rs} + (12 \leftrightarrow 34)$

MEASURES

$$\cdot [d\lambda] T_{\alpha_1 \dots \alpha_5} = c_1 \varepsilon_{\alpha_1 \dots \alpha_5} d\lambda^{\alpha_1} \dots d\lambda^{\alpha_5}$$

$$\cdot [d\omega^I] = c_w T_{\alpha_1 \dots \alpha_5} \varepsilon^{I_1 \dots I_5} d\omega_{\alpha_1}^{I_1} \dots d\omega_{\alpha_5}^{I_5}$$

$$\cdot c_1 = \left(\frac{\pi}{2}\right)^{-2} \frac{1}{\pi!} \left(\frac{A_g}{4\pi^2}\right)^{11/2}$$

$$\cdot T_{\alpha_1 \dots \alpha_5} = \binom{11}{\alpha_1} \binom{10}{\alpha_2} \binom{9}{\alpha_3} \binom{8}{\alpha_4} \binom{7}{\alpha_5}$$

THE VARIABLES

FERMIONIC

S^α	11	11g
$R_{\alpha\gamma}$	11	11
$d_{\alpha\gamma}$	16	16g
σ^α	16	16

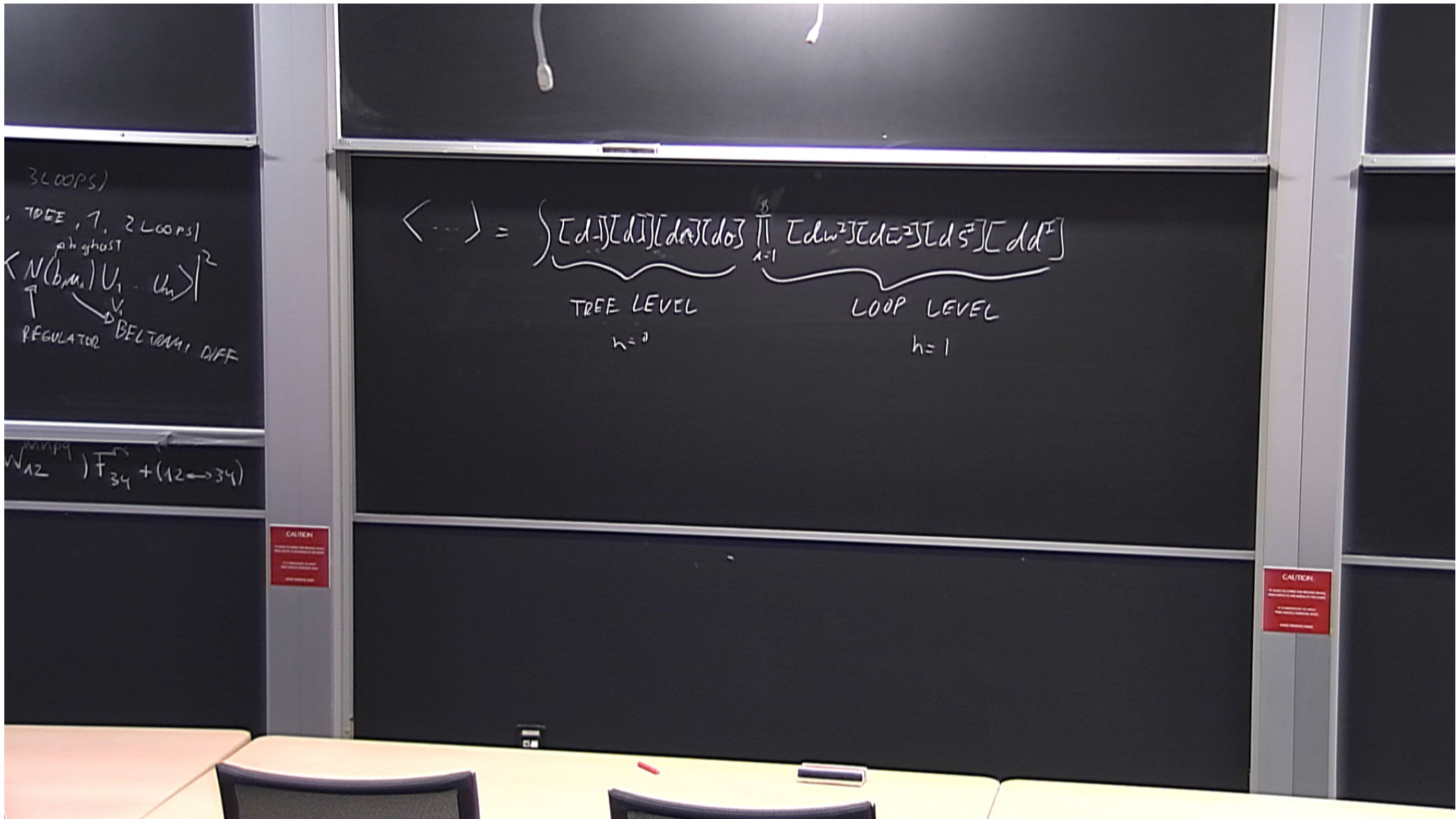
BOSONIC

λ^α	11	11
$\bar{\lambda}_\alpha$	11	11
w_α	11	11g
$\bar{w}^\alpha$	11	11g
x^m	10	10

NON ZERO MODES

2 ZERO MODES

$(1^2 0^5)$



$$\langle \dots \rangle = \underbrace{\int [d\psi][d\bar{\psi}][d\phi][d\phi^\dagger]}_{\text{TREE LEVEL } h=0} \prod_{i=1}^B \underbrace{[d\omega^i][d\bar{\omega}^i][d\zeta^i][d\bar{\zeta}^i]}_{\text{LOOP LEVEL } h=1}$$

3 LOOPS)
TREE, 1, 2 LOOPS)
ghost
 $\langle N(b_{\mu\nu}) U_1 \dots U_n \rangle^2$
↑
REGULATOR
↓
BELTRAMI DIFF

$\omega_{12} \rightarrow F_{34} + (12 \rightarrow 34)$

$$\langle \dots \rangle = \underbrace{\int [d\lambda][d\bar{\lambda}][d\psi][d\bar{\psi}]}_{\text{TREE LEVEL}} \prod_{i=1}^{\beta} \underbrace{[d\omega^i][d\bar{\omega}^i][d\psi^i][d\bar{\psi}^i]}_{\text{LOOP LEVEL}}$$

$$\langle M \rangle_{(p,g)} = \frac{\int [_{\text{TREE LEVEL}}]^{h=0} M(\lambda, \bar{\lambda}, \psi, \bar{\psi}) e^{-\lambda \bar{\lambda} + \eta \psi}}{(\lambda \bar{\lambda})^{3-p}} \quad h=1$$

$$\int_{\theta^5}^{2600} [_{\theta^5}]_{(p,g)} = N_{(p,g)} = \# \frac{1}{A_g^{5/2}}$$

$$(\bar{\lambda} n d^2 d^0) = (\bar{\lambda} r^{m+p} r) (d^2 r^{m+p} d^0)$$

$$1\text{Loop } b^{(1)} = \frac{(\bar{\lambda} r d^1 d^1)}{(i\lambda)^2} \omega_1(y) \omega_2(y) + \frac{\pi_1^m (\bar{\lambda} r^m d^1)}{(i\lambda)} \omega_1(y) \omega_2(y)$$

$$\int d^2 z |b(y)|^2 = \frac{1}{2\pi} \int d^2 y |b(y)|^2 = \int d^2 \Omega \left| \frac{d\Omega_1}{dz_1} \right|^2 d^2 z = |\bar{\lambda} n d^1|^2 \int d^2 \Omega$$

$$\int d^2 y \omega_1(y) \omega_2(y) \mu_1 = \frac{d\Omega_2}{dz_2} \quad (\text{RAUCH VARIATIONAL FORMULA})$$

$$\int d^2 z_1 d^2 z_2 d^2 z_3 \left| (b, u_1)(b, u_2)(b, u_3) \right|^L = \left| \varepsilon^{i_1 i_2 i_3} \frac{d\Omega_{i_1}}{dz_1} \frac{d\Omega_{i_2}}{dz_2} \frac{d\Omega_{i_3}}{dz_3} \right|^L$$

$g=1,2,3$

$g=4$

SCHOTTKY PROBLEM

$d\Omega$

$$d^2 z_1 \dots d^2 z_3 = \int d^2 \Omega_{i_1} d^2 \Omega_{i_2} d^2 \Omega_{i_3}$$

$$(d^2 z_i)^g \rightarrow (d^2 \Omega)^{10}$$

g: 4 SCHOTTKY PROBLEM

$$\text{Im} [B_{(p_1, \dots, p_j)} + \pi_m B_{(p_1, \dots, p_j)} + \dots] = \int \pi |(b, m)|^2$$

$$B_{(2)} = \frac{\bar{\lambda} n d' d'}{(1\bar{\lambda})^2} \quad B_{(3,3)} = \frac{1}{(1\bar{\lambda})^6} (\bar{\lambda} n d' d') (\bar{\lambda} n d' d'^2) (\bar{\lambda} n d^2 d^2)$$

$$N_{(p,q)}^2 \left\langle \prod_{i=1}^N e^{i k x_i} \right\rangle = (2\pi)^{p+q} S(k) \frac{1}{2^8 3^4 5^2} \left(\frac{(7+p)!}{7!} \right) \exp(s_{11}, G_{11})$$

1. INTEGRATE POSITIONS B. CHOS

2. OPE / ZERO MODE TWT.

$$1\text{-LOOP} \quad D_{(1,3)} V_1 V_2 V_3 = \underbrace{\{ \bar{c} d d \} (\varepsilon \cdot \gamma \cdot d) (\bar{\lambda} n d d) V_1 (d' w_{23} (d' L_4) (d' w_{51}) \partial G_{23}}}_{\text{...}}$$

$$T_{112,3,4} = (\bar{\lambda} \gamma^{\mu \alpha \rho \lambda} V_1) (\bar{\lambda} \gamma^{\mu \alpha \rho \lambda} V_2) (\bar{\lambda} \gamma^{\mu \alpha \rho \lambda} V_3) (\bar{\lambda} \gamma^{\mu \alpha \rho \lambda} V_4) \partial G_{23}$$

$$\{ \bar{c} d d \} \varepsilon \cdot \gamma \cdot d (d_1)^5 = G_d T_{\alpha_1 \dots \alpha_5}$$

$$\int d^2 y w_L(x) w_R(y) M_{11} = \frac{d S_{22}}{d z_1} \quad \text{(RAUCH VARIATIONAL FORMULA)}$$

FORMALISM

PT @ 2 LOOPS

ghost

$|U_1 \dots U_n\rangle^2$

BELTRAMI DIFF

$\prod_{i=1}^g \left(\frac{\epsilon T d^2}{715!} \right)^2$

$(11, 11, \dots, 11)$

$g d^2$

$U_1 \dots U_n$

$\frac{d^2 z_i}{dz_i} \frac{d^2 z_j}{dz_j} \frac{d^2 z_k}{dz_k}$

$g=1,2,3$

$g=4$ SCHOTTKY PROBLEM

$(d^2 z_i)^9 \rightarrow (d^2 \Omega)^{10}$

$B_{(2)} = \frac{\bar{\lambda} n d' d'}{(1\pi)^2}$

$B_{(3,3)} = \frac{1}{(2\lambda)^4} (\bar{\lambda} n d' d') (\bar{\lambda} n d' d') (\bar{\lambda} n d' d')$

4PT 2 LOOP

$\int D_{(11,11)} U_1 U_2 U_3 U_4 = D_{(11,11)} (d' w_1) (d' w_2) (d^2 w_3) (d^2 w_4)$

4PT $\neq T_{ABCD} \Delta_{AC} \Delta_{BD}$

$\epsilon^{12} w_1 w_2 + w_1(1) w_1(2) w_2(3) w_2(4) + \dots$

CAUTION

CAUTION

