Title: 1PI Effective Action for Superstring Perturbation Theory

Date: Apr 23, 2015 09:00 AM

URL: http://pirsa.org/15040171

Abstract:

1PI Effective Action for Superstring Perturbation Theory

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Waterloo, April 2015

PLAN

1. Conventional approach and its shortcomings

2. Off-shell amplitudes

3. 1PI effective string field theory

We shall follow RNS formalism with picture changing operators.

But our considerations are quite general and might extend to other approaches as well.

References

Zwiebach, hep-th/9206084

Pius, Rudra, A.S., , arXiv:1311.1257, 1401.7014, 1404.6254

A.S., arXiv:1408.0571, 1411.7478, 1501.00988, work in progress

A.S., Witten, arXiv:1504.00609

Conventional approach to computing g-loop S-matrix elements in superstring theory

1. Represent physical states by BRST invariant vertex operators in the world-sheet superconformal field theory of matter and ghost system.

2. Compute correlation functions of vertex operators inserted at ‘punctures’ and additional ghost and ‘picture changing operators’ on a genus g Riemann surface.

3. Integrate the result over the moduli space of the Riemann surface with punctures.

However this approach is insufficient for addressing many issues even within the perturbation theory.

1. Mass renormalization

2. Vacuum shift

LSZ formula for S-matrix elements in QFT

$$\lim_{k_i^2 \to -m_{i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{Z_i^{-1/2} \times (k_i^2 + m_{i,p}^2)\}$$

$G^{(n)}$: n-point Green's function

$a_1, \cdots a_n$: quantum numbers, $k_1, \ldots k_n$: momenta

$m_{i,p}$: physical mass of the i-th external state

– given by the locations of the poles of two point function in the $-k^2$ plane.

Z_i: wave-function renormalization factors, given by the residues at the poles.

In contrast, string amplitudes compute 'truncated Greens function on classical mass-shell'

$$\lim_{k_i^2 \to -m_i^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2) \, .$$

m_i: tree level mass of the i-th external state.

$k_i^2 \to -m_i^2$ condition is needed to make the vertex operators BRST invariant.

String amplitudes:

$$\lim_{k_i^2 \to -m_i^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2)\,,$$

The S-matrix elements:

$$\lim_{k_i^2 \to -m_{i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{Z_i^{-1/2} \times (k_i^2 + m_{i,p}^2)\}$$

The effect of Z_i can be taken care of. Witten

The effect of mass renormalization is more subtle.

⇒ String amplitudes compute S-matrix elements directly if $m_{i,p}^2 = m_i^2$ but not otherwise.

– Includes BPS states, massless gauge particles and all amplitudes at tree level.

Problem with vacuum shift

Example: In many compactifications of SO(32) heterotic string theory on Calabi-Yau 3-folds, one loop correction generates a Fayet-Ilioupoulos term.

Effect: Generate a potential of a charged scalar ϕ of the form

$$c(\phi^*\phi - K\, g_s^{\,2})^2$$

c, K: positive constants, g_s: string coupling

Dine, Seiberg, Witten; Atick, Dixon, A.S.; Dine, Ichinose, Seiberg
Atick, A.S.; Witten; D'Hoker, Phong; Berkovits, Witten

Correct vacuum: $|\phi| = g_s\sqrt{K}$

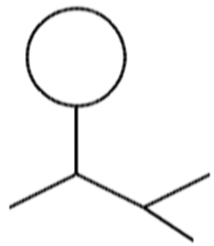
– not described by a world-sheet CFT

– conventional perturbation theory fails.

Even in absence of mass renormalization and vacuum shift we have to deal with infrared divergences in the integration over moduli space at intermediate stages.

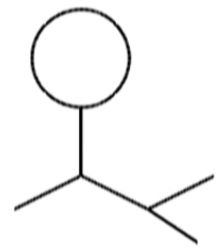
Witten

Consider a tadpole diagram in a QFT:



This diverges if a massless state propagates along the vertical propagator.

Often the result vanishes after loop integration due to SUSY.



In string theory, this translates to a specific regularization procedure for integration over moduli spaces of Riemann surfaces.

1. Put an upper cut-off L on certain modulus corresponding to the Schwinger parameter of the vertical propagator.

2. Do integration over the other moduli first.

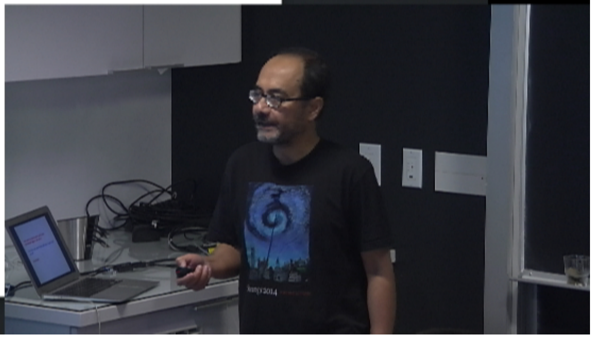
3. Then let L go to infinity.

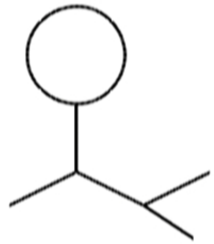
Witten

This works but requires an IR cut-off at the intermediate stages of calculation.

How do we circumvent these difficulties / need for IR cut-off?

Go off-shell.





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Off-shell amplitudes

· · · ; Nelson; Zwiebach; Pius, Rudra, A.S.

1. Relax the constraint of conformal and BRST invariance on the vertex operators

– result will depend on the world-sheet metric around the punctures where the vertex operators are inserted.

2. Choose a local coordinate system w_i around the i-th puncture for each i and take the metric around the puncture $w_i = 0$ to be $|dw_i|^2$.

A different choice of the local coordinate system e.g. $y_i = f(w_i) \Rightarrow$ different metric $|dy_i|^2 = |f'(w_i)|^2 \, |dw_i|^2$

$\Rightarrow$ different off-shell amplitudes for the same external states.

For superstring theories we need insertion of picture changing operators (PCO) on the Riemann surface.

Off-shell amplitudes depend not only on the choice of local coordinates at the punctures but also on the locations of the PCO's.

Are the physical quantities computed from off-shell amplitudes independent of the choice of local coordinates and PCO locations?

For simplicity we shall discuss heterotic string theory but the analysis for type II is similar.

Some notations (ignoring subtleties):

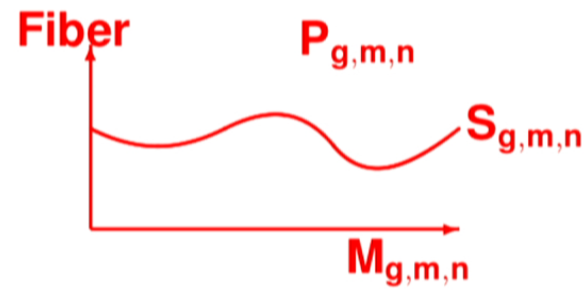
Q_B: BRST charge

$\mathcal{X}(z)$: PCO

$M_{g,m,n}$: (6g-6+2m+2n) dimensional moduli space of genus g Riemann surfaces with (m,n) punctures of (NS,R) type in $(-1, -1/2)$ picture number.

$P_{g,m,n}$: A fiber bundle with $M_{g,m,n}$ as the base and possible choices of local coordinates at punctures and PCO locations as fibers.

Number of PCO's: $2g - 2 + m + n/2$



A choice of local coordinate system and PCO locations corresponds to a section $S_{g,m,n}$ of this fiber bundle.

Dimension of $S_{g,m,n} = 6g - 6 + 2m + 2n$.

Note: We could also choose formal weighted average of multiple sections.

Procedure for constructing an off-shell amplitude

1. For a given set of external off-shell states collectively called ϕ, construct p-forms $\omega_p(\phi)$ on $P_{g,m,n}$ satisfying

$$\omega_p(\sum_i Q_B^{(i)}\phi) = (-1)^p d\omega_{p-1}(|\phi\rangle)$$

$Q_B^{(i)}$: BRST charge acting on i-th state

ω_p is constructed from appropriate correlation functions of off-shell vertex operators and ghost and PCO insertions on the Riemann surface (details later).

2. Genus g, (m+n)-point amplitude

$$\int_{S_{g,m,n}} \omega_{6g-6+2m+2n}(|\phi\rangle)$$

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Goal: Prove that all physical quantities computed from the off-shell amplitudes are independent of the choice of the section $S_{g,m,n}$ even though the amplitudes themselves are not.

For this we work within a specific class of local coordinates

– gluing compatible local coordinate system.

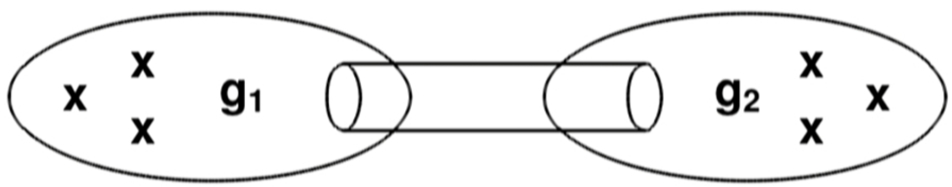
Consider a genus g_1, $(m_1 + n_1)$-punctured Riemann surface and a genus g_2, $(m_2 + n_2)$-punctured Riemann surface.

Take one puncture from each of them, and let w_1, w_2 be the local coordinates around the punctures at $w_1 = 0$ and $w_2 = 0$.

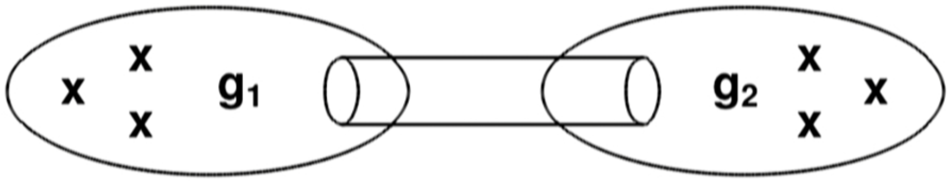
Glue them via the identification (plumbing fixture)

$$w_1 w_2 = e^{-s+i\theta}, \quad 0 \leq s < \infty, \quad 0 \leq \theta < 2\pi$$

– gives a family of new Riemann surfaces of genus $g_1 + g_2$ with 2 fewer punctures.



Gluing compatibility: Choice of local coordinates at the punctures and the PCO locations on the genus $g_1 + g_2$ Riemann surface must agree with the one induced from the local coordinates at the punctures and PCO locations on the original Riemann surfaces.



For gluing at NS puncture the no of PCO's on the final surface is the sum of the number of PCO's on the individual surfaces.

$$(2g_1 - 2 + m_1 + n_1/2) + (2g_2 - 2 + m_2 + n_2/2)$$

$$= 2(g_1 + g_2) - 2 + (m_1 + m_2 - 2) + (n_1 + n_2)/2$$

For gluing at R-puncture the sum of the number of PCO's on the individual surfaces is <u>one less</u> than the required number.

$$(2g_1 - 2 + m_1 + n_1/2) + (2g_2 - 2 + m_2 + n_2/2)$$

$$= 2(g_1 + g_2) - 2 + (m_1 + m_2) + (n_1 + n_2 - 2)/2 - 1$$

A consistent prescription: Insert

$$\mathcal{X}_0 \equiv \oint \frac{dw_1}{w_1} \mathcal{X}(w_1) = \oint \frac{dw_2}{w_2} \mathcal{X}(w_2)$$

around either puncture.

$\mathcal{X}_0$ has been used earlier for other purposes.

Berkovits, Zwiebach; Erler, Konopka, Sachs

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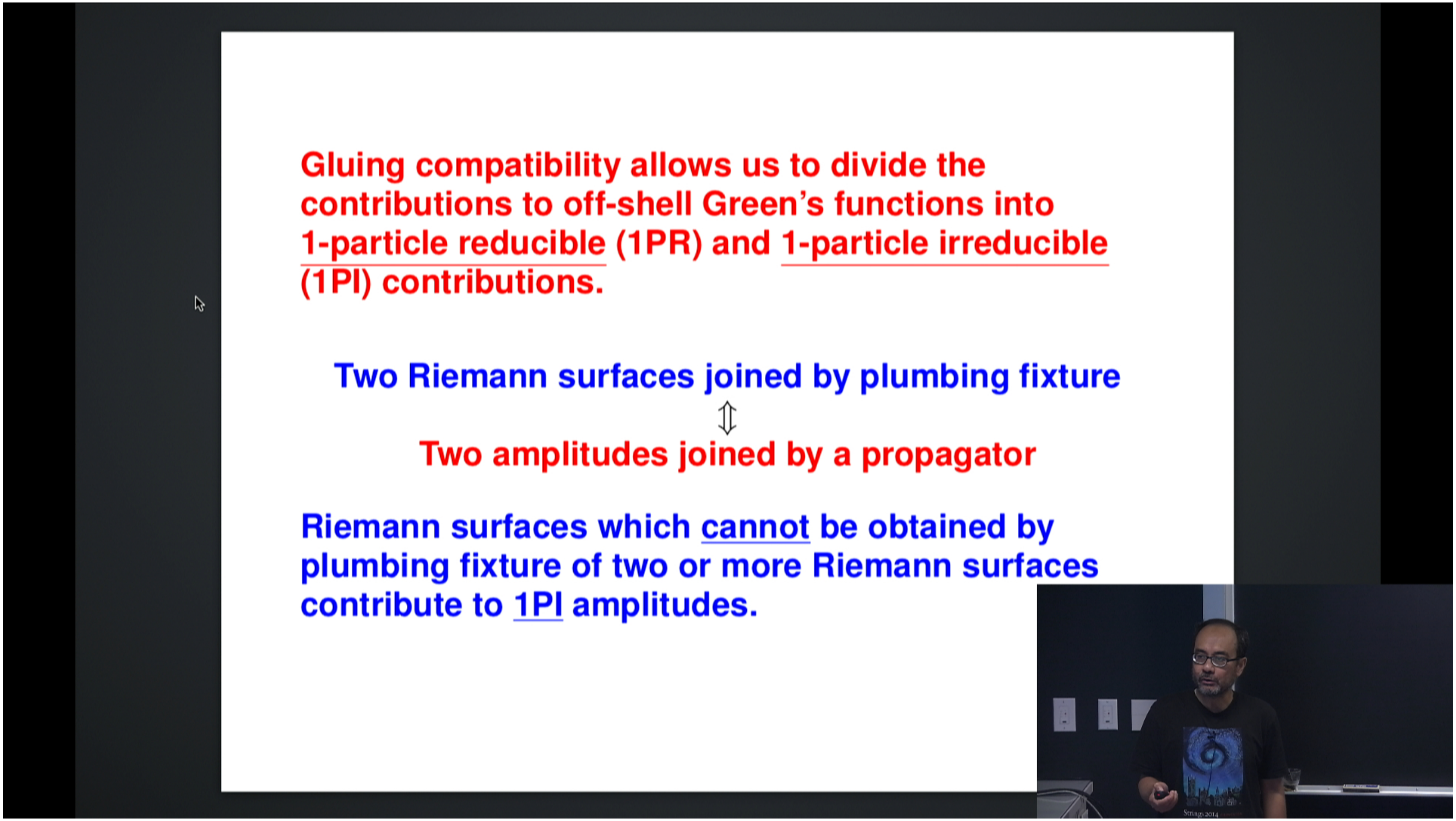
Gluing compatibility allows us to divide the contributions to off-shell Green's functions into 1-particle reducible (1PR) and 1-particle irreducible (1PI) contributions.

Two Riemann surfaces joined by plumbing fixture

$\Updownarrow$

Two amplitudes joined by a propagator

Riemann surfaces which cannot be obtained by plumbing fixture of two or more Riemann surfaces contribute to 1PI amplitudes.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⇕
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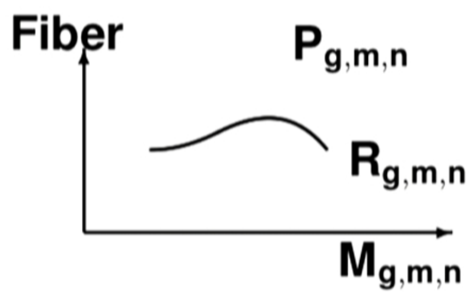
Put another way, for a gluing compatible choice of sections, we can identify a subspace $R_{g,m,n}$ of the full section $S_{g,m,n}$ which we can call the 1PI subspace.



All the Riemann surfaces corresponding to the full section $S_{g,m,n}$ are given by the Riemann surfaces in $R_{g,m,n}$ and their plumbing fixture in all possible ways.

Systematic construction of 1PI regions

1. Begin with 3-punctured sphere and one punctured torus.

The first one has 0-dimensional moduli space and the second one has two dimensional moduli space.

Declare them to be 1PI.

2. Choose local coordiates at the punctures and PCO locations arbitrarily consistent with symmetries

– exchange of punctures on the 3-punctured sphere

– modular transformation for the 1-punctured torus.

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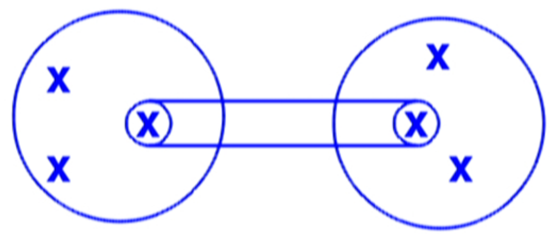
3. Now take two 3-punctured spheres and glue them using plumbing fixture.



$$w_1 w_2 = q, \quad q \equiv e^{-s+i\theta}, \quad 0 \leq s < \infty, \quad 0 \leq \theta < 2\pi$$

Declare these to be 1PR 4-punctured spheres and choose the local coordinates and PCO locations to be those induced from 3-punctured spheres.

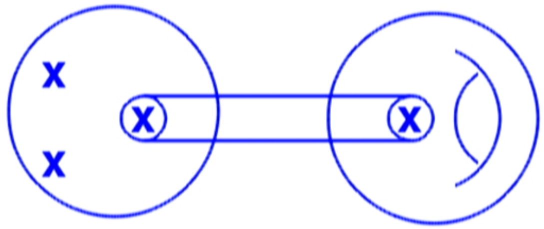
Repeat this for inequivalent permutations of the four punctures i.e. 'sum over s, t and u-channel diagrams'.

The 1PR 4-punctured spheres will typically cover part of the moduli space of 4-punctured spheres.

Declare the rest of the 4-punctured spheres to be 1PI 4-punctured spheres.

On them choose local coordinates and PCO locations arbitrarily consistent with symmetries and continuity.

5. Similarly gluing 3-puntured spheres with 1-punctured tori we get a set of 2-punctured tori.



Declare them to be 1PR and choose local coordinates on them to be those induced from the constituents

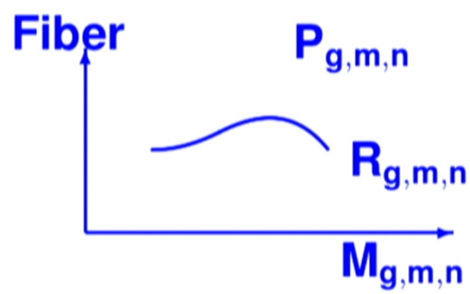
– covers part of the moduli space of 2-punctured tori.

6. Declare the rest of the 2-punctured tori to be 1PI and choose local coordinates and PCO locations on them arbitrarily maintaining symmetries and continuity.

Proceeding this way, for all $P_{g,m,n}$ we can choose

1. Gluing compatible sections $S_{g,m,n}$

2. Identify part of the section $S_{g,m,n}$ as 1PI subspace $R_{g,m,n}$ of $P_{g,m,n}$



Once this division has been made, we can define the 1PI amplitudes as

$$\int_{R_{g,m,n}} \omega_{6g-6+2m+2n}$$

Generating function of these amplitudes is 1PI effective action (almost).

Tree amplitudes computed from 1PI action

= full off-shell string amplitude including loop corrections, given by integrals over the whole section $S_{g,m,n}$

We can now apply standard field theory methods to compute renormalized masses and S-matrix from the 1PI action, as well as to compute amplitudes in a shifted vacuum.

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All separating type degenerations come from 1PR amplitudes

$\Rightarrow$ the 1PI effective action is free from all IR divergences associated with tadpoles, and mass or wave-function renormalization.

Definitions and identities

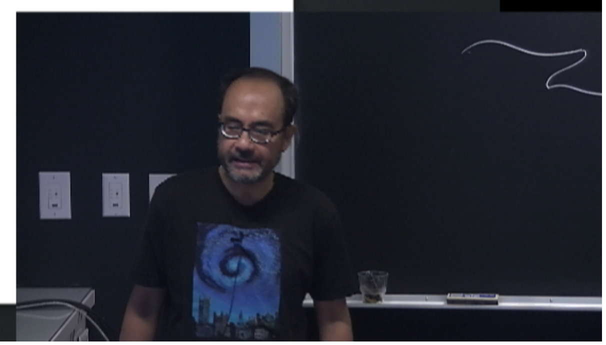
H: Hilbert space of GSO even states of the matter-ghost CFT annihilated by

$$b_0 - \bar{b}_0, \qquad L_0 - \bar{L}_0$$

$\langle A|B\rangle$: BPZ inner product between CFT states

NS-sector vertex operators are grassman even for even ghost number and grassmann odd for odd ghost number

For R-sector it is opposite.



More definitions (following Zwiebach)

Given N states $|A_1\rangle, \cdots |A_N\rangle \in H$, of which m are NS states and $n = N - m$ are R-states we define a multi-linear function

$$\{A_1 \cdots A_N\} = \sum_{g=0}^{\infty} {g_s}^{2g} \int_{R_{g,m,n}} \omega_{6g-6+2(m+n)}(|A_1\rangle, \cdots |A_N\rangle)$$

We also define $[A_1 \cdots A_N] \in H$ via

$$\langle A_0 | c_0^- | [A_1 \cdots A_N] \rangle = \{A_0 A_1 \cdots A_N\}, \quad c_0^- \equiv (c_0 - \bar{c}_0)/2$$

$$b_0^\pm = b_0 \pm \bar{b}_0, \quad L_0^\pm \equiv L_0 \pm \bar{L}_0, \quad c_0^\pm \equiv (c_0 \pm \bar{c}_0)/2$$

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1. Under exchange of A_i and A_j, $\{A_1 \cdots A_N\}$ pick up a sign

$$(-1)^{\gamma_i \gamma_j}$$

γ_i: grassmannality of A_i i.e. 0 if A_i is grassmann even and 1 if A_i is grassmann odd.

2.
$$\sum_{i=1}^{N} (-1)^{\gamma_1 + \cdots \gamma_{i-1}} \{A_1 \cdots A_{i-1} (Q_B A_i) A_{i+1} \cdots A_N\}$$
$$= -\frac{1}{2} \sum_{\substack{\ell, k \geq 0 \\ \ell + k = N}} \sum_{\substack{\{i_a; a=1,\cdots \ell\}, \{j_b; b=1,\cdots k\} \\ \{i_a\} \cup \{j_b\} = \{1,\cdots N\}}} \sigma(\{i_a\}, \{j_b\})$$
$$\{A_{i_1} \cdots A_{i_\ell} (G[A_{j_1} \cdots A_{j_k}])\}$$

$\sigma(\{i_a\}, \{j_b\})$: the sign that one picks up while rearranging $b_0^-, A_1, \cdots A_N$ to $A_{i_1}, \cdots A_{i_\ell}, b_0^-, A_{j_1}, \cdots A_{j_k}$

$$G|s\rangle \equiv \begin{cases} |s\rangle & \text{if } |s\rangle \in H_{NS} \\ \mathcal{X}_0 |s\rangle & \text{if } |s\rangle \in H_R \end{cases},$$

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NS sector string field: An arbitrary state $|\psi_{NS}\rangle \in H$ carrying ghost number 2 and picture number -1.

R sector string field: An arbitrary state $|\psi_R\rangle \in H$ carrying ghost number 2 and picture number $-1/2$.

$$|\psi\rangle \equiv |\psi_{NS}\rangle + |\psi_R\rangle$$

If $|\phi_r\rangle$ is a basis in $H_{-1} + H_{-1/2}$, then we can expand

$$|\psi\rangle = \sum_r a_r |\phi_r\rangle$$

The coefficients a_r are the dynamical variables labelling the string field (in momentum space).

Coefficients of NS sector basis states are grassmann even and the coefficients of R-sector basis states are grassmann odd.

We shall first describe the 1PI effective action for NS sector fields.

$$S(|\psi_{NS}\rangle) = g_s^{-2}\left[\frac{1}{2}\langle\psi_{NS}|c_0^- Q_B|\psi_{NS}\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\psi_{NS}{}^{n}\}\right]$$

$\{\psi_{NS}{}^{n}\}$: $\{\psi_{NS}\psi_{NS}\cdots\psi_{NS}\}$ with n copies of ψ_{NS} inside $\{\ \}$.

Invariant under infinitesimal gauge transformation

$$\delta|\psi_{NS}\rangle = Q_B|\lambda_{NS}\rangle + \sum_{n=0}^{\infty}\frac{1}{n!}[\psi_{NS}{}^{n}\lambda_{NS}]$$

$|\lambda_{NS}\rangle$: is an element of H with ghost number 1, picture number -1.

Gauge invariance of $S(|\psi_{NS}\rangle)$ can be proved using the identities involving $\{\cdots\}$ and $[\cdots]$.

We shall first describe the 1PI effective action for NS sector fields.

$$S(|\psi_{NS}\rangle) = g_s^{-2}\left[\frac{1}{2}\langle\psi_{NS}|c_0^- Q_B|\psi_{NS}\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\psi_{NS}{}^n\}\right]$$

$\{\psi_{NS}{}^n\}$: $\{\psi_{NS}\psi_{NS}\cdots\psi_{NS}\}$ with n copies of ψ_{NS} inside $\{\ \}$.

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Equations of motion:

$$Q_B|\psi_{NS}\rangle + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}[\psi_{NS}{}^{n-1}] = 0$$

Note: $\{\psi_{NS}\}$ and [ ] are non-zero from genus 1 onwards

$|\psi_{NS}\rangle = 0$ is not a solution to equations of motions.

We have to first solve the equations of motion and then expand the 1PI action around the solution.

Special importance: Vacuum solution carrying zero momentum

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Special importance: Vacuum solution carrying zero momentum

Iterative construction of the vacuum solution:

Suppose $|\psi_k\rangle$ is the solution to order ${g_s}^k$. $(|\psi_0\rangle = 0)$

P: projection operator to $L_0^+ \equiv L_0 + \bar{L}_0 = 0$ states.

Then

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} (1-P)[\psi_k^{n-1}] + |\phi_{k+1}\rangle ,$$

$|\phi_{k+1}\rangle$ is an $L_0^+ = 0$ state satisfying

$$Q_B|\phi_{k+1}\rangle = -\sum_{n=1}^{\infty} \frac{1}{(n-1)!} P[\psi_k^{n-1}] + \mathcal{O}({g_s}^{k+2}) .$$

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Possible obstruction / ambiguity to solving these arise from the last equation.

rhs could contain a component along a non-trivial element of BRST cohomology.

– reflects the existence of zero momentum massless tadpoles in perturbation theory.

Unless this equation can be solved we have to declare the vacuum inconsistent.

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This also allows us to deal with the cases involving vacuum shift, e.g. when a scalar field χ in low energy theory has potential

$$c(\chi^2 - K\, g_s{}^2)^2 .$$

At order g_s we have three solutions $\chi = 0, \pm g_s\sqrt{K}$.

In 1PI effective feld theory this will be reflected in the existence of multiple solutions for $|\phi_1\rangle$.

The solution corresponding to $\chi = 0$ will have non-zero dilaton one point function at higher order

$\Rightarrow$ an obstruction to extending the corresponding 1PI effective field theory solution to higher order.

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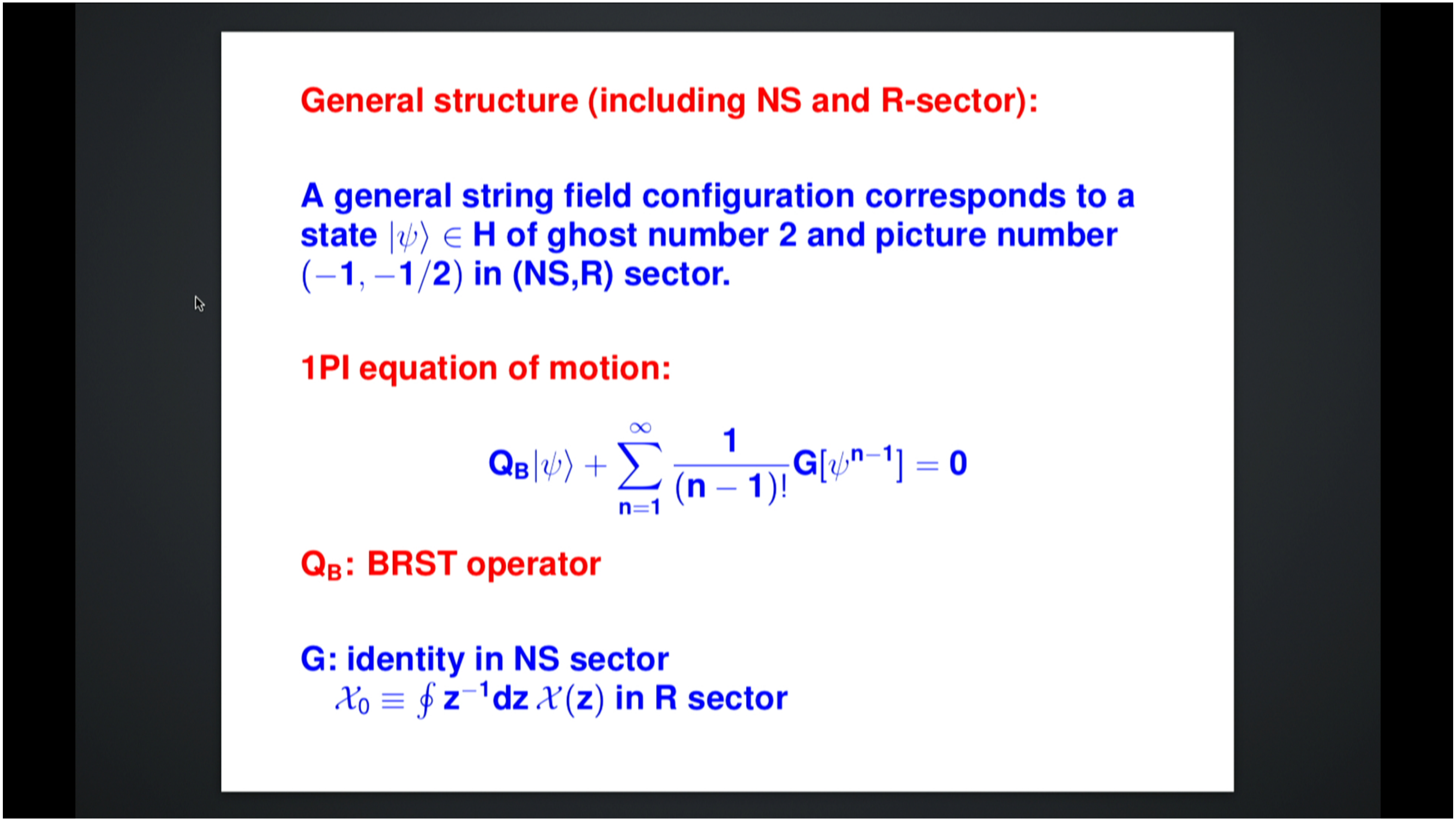
For Ramond sector states it is not possible to write down an action with local kinetic term.

We can only write down the equation of motion.

– related to the fact that Ramond sector states carry picture number $-1/2$ and the inner product between two such states vanish by picture number conservation.

For a string field theory this would be problematic since we would not know how to quantize the theory.

However for 1PI theory this is not a problem since we only need to work at the tree level.

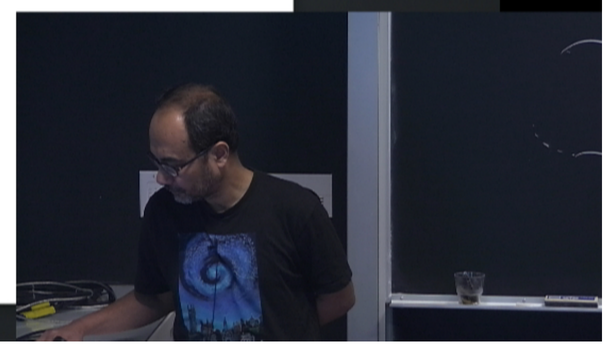

General structure (including NS and R-sector):
A general string field configuration corresponds to a state $|\psi\rangle \in$ H of ghost number 2 and picture number $(-1, -1/2)$ in (NS,R) sector.
1PI equation of motion:
$$Q_B|\psi\rangle + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} G[\psi^{n-1}] = 0$$
Q_B: BRST operator
G: identity in NS sector
$\mathcal{X}_0 \equiv \oint z^{-1} dz\, \mathcal{X}(z)$ in R sector

Gauge transformations

The infinitesimal gauge transformation parameters correspond to states $|\lambda\rangle$ of ghost number 1 and picture number $(-1, -1/2)$ in (NS,R) sector.

Gauge transformation law

$$\delta|\psi\rangle = \mathbf{Q_B}|\lambda\rangle + \sum_{\mathbf{n}=0}^{\infty} \frac{\mathbf{1}}{\mathbf{n}!} \mathbf{G}[\psi^{\mathbf{n}}\,\lambda]$$



Once we have a vacuum solution $|\psi_v\rangle$ we can expand the equations of motion around $|\psi_v\rangle$.

Define: $|\chi\rangle \equiv |\psi\rangle - |\psi_v\rangle$

$$\widehat{Q}_B|A\rangle \equiv Q_B|A\rangle + \sum_{k=0}^{\infty} \frac{1}{k!} G[\psi_v{}^k A] .$$

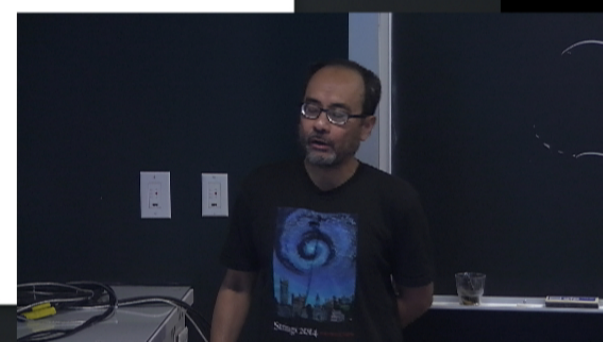
$\widehat{Q}_B^2 = 0$ as a consequence of $|\psi_v\rangle$ satisfying equations of motion.

New ‘shifted’ linearized equations of motion

$$\widehat{Q}_B|\chi\rangle = 0$$

New shifted linearized gauge transformations

$$\delta|\chi\rangle = \widehat{Q}_B|\lambda\rangle$$



Linearized equations of motion around $|\psi_v\rangle$:

$$\widehat{Q}_B|\chi\rangle = 0$$

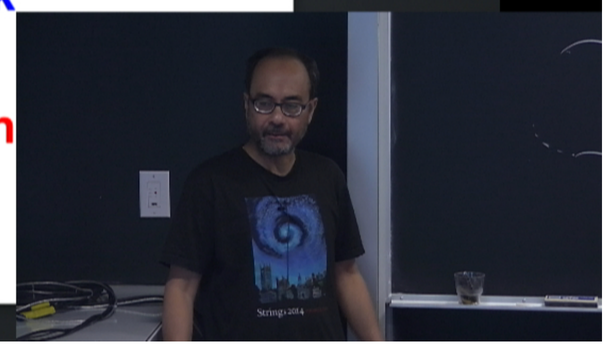
– has two kinds of solution:

1. Solutions which exist for all momentum k of $|\chi\rangle$

– have the form $\widehat{Q}_B|\lambda\rangle$ for some $|\lambda\rangle$ and are pure gauge.

2. Solutions which exist for special values of k^2

– represent physical states with the correspon
values of $-k^2$ giving renormalized mass^2.



This abstract definition can be developed into a fully systematic perturbative scheme.

A similar procedure can be given for the S-matrix elements starting from the LSZ formalism.

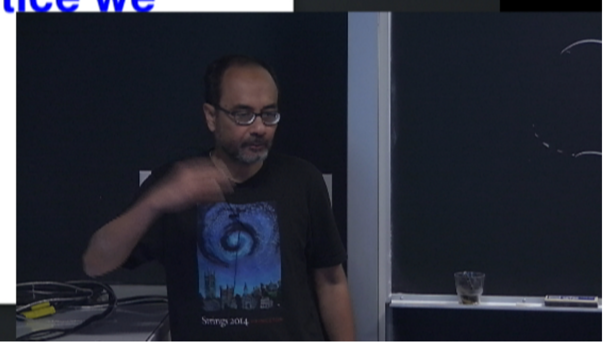
Pius, Rudra, A.S.

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{n=1}^{\infty}\frac{1}{(n-1)!}(1-P)[\psi_k^{n-1}] + |\phi_{k+1}\rangle\,,$$

$$Q_B|\phi_{k+1}\rangle = -\sum_{n=1}^{\infty}\frac{1}{(n-1)!}P[\psi_k^{n-1}] + \mathcal{O}({g_s}^{k+2})\,.$$

Once these equations have been solved, we do not encounter any further tadpole divergence in perturbation theory.

Note: The full solution $|\psi_v\rangle$ is $|\psi_\infty\rangle$, but in practice we shall stop at some fixed order in g_s.



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Pius, Rudra, A.S.

Gauge transformation laws around the shifted vacuum

$$\delta|\chi\rangle = \widehat{\mathbf{Q}}_{\mathbf{B}}|\lambda\rangle + \mathcal{O}(\chi)$$

Global symmetries are generated by those $|\lambda\rangle$ for which

$$\widehat{\mathbf{Q}}_{\mathbf{B}}|\lambda\rangle = \mathbf{0}$$

For such global symmetries we can derive Ward identities.

e.g. unbroken global SUSY $\Rightarrow$ equality of the renormalized masses of bosons and fermions at all mass levels.

also $\Rightarrow$ absence of obstruction to finding vacuum solution.

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{n=1}^{\infty}\frac{1}{(n-1)!}(1-P)[\psi_k^{n-1}] + |\phi_{k+1}\rangle\,,$$

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Section dependence

The definition of $\{A_1 \cdots A_N\}$ and all subsequent analysis depends on the choice of 1PI subspace $R_{g,m,n}$.

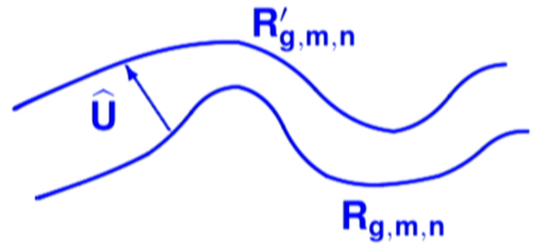
A different choice of gluing compatible 'sections'

$\Rightarrow$ a different choice of $R_{g,m,n}$

$\Rightarrow$ a different set of equations of motion.

Do the renormalized masses and S-matrix elements depend on this choice?

We shall consider the case of infinitesimal deformations from $R_{g,m,n}$ to $R'_{g,m,n}$ labelled by some tangent vector $\widehat{U}$ of $P_{g,m,n}$ at every point of $R_{g,m,n}$.



Result: The change in the equation of motion can be compensated by a field redefinition $|\psi\rangle \to |\psi\rangle + \delta|\psi\rangle$ where

$$\langle\phi|c_0^-|\delta\psi\rangle = -\sum_{g=0}^{\infty} {g_s}^{2g} \sum_{m,n=0}^{\infty} \frac{1}{m!n!}$$

$$\Big[\int_{R_{g,m+1,n}} \omega_{6g-5+2m+2n+2}[\widehat{U}](G|\phi_{NS}\rangle\rangle, |\psi_{NS}\rangle^{\otimes m}, |\psi_R\rangle^{\otimes n})$$

$$+\int_{R_{g,m,n+1}} \omega_{6g-5+2m+2n+2}[\widehat{U}](|\psi_{NS}\rangle^{\otimes m}, G|\phi_R\rangle, |\psi_R\rangle^{\otimes n})\Big]$$

Thus renormalized masses and S-matrix elements remain unchanged.

Construction of $\omega_0(|\phi\rangle)$

$$\omega_0(|\phi\rangle) = (2\pi i)^{-3g+3-n} \left\langle \prod_{j=1}^{2g-2+m+n/2} \mathcal{X}(y_j) \quad \phi \right\rangle_S$$

$\langle\ \rangle_S$: correlation function on the Riemann surface corresponding to a given point in $P_{g,m,n}$

ϕ: product of all external vertex operators inserted using the local coordinates appropriate for the given point in $P_{g,m,n}$.

Construction of $\omega_p(|\phi\rangle)$

– specify contractions of ω_p with tangent vectors of $P_{g,m,n}$

We have two kinds of tangent vectors

1. Tangent vectors associated with deformations of the moduli of the punctured Riemann surface or local coordinates at the punctures.

2. $\partial/\partial y_j$ describing deformation of PCO locations y_j.

1. Contraction of $\omega_p(|\phi\rangle)$ with the tangent vectors of the first kind is given by

$$\omega_p(|\phi\rangle)[V_1, \cdots V_p]$$

$$\equiv (2\pi i)^{-3g+3-n} \langle B[V_1] \cdots B[V_p] \prod_{j=1}^{2g-2+m+n/2} \mathcal{X}(y_j)\,\phi\rangle_S$$

$$B[V] = \int d^2z \, \left(\eta_V(z,\bar{z}) b(z) + \bar{\eta}_V(z,\bar{z}) \bar{b}(\bar{z})\right)$$

η_V: Beltrami differential

2. Contraction of $\omega_p(|\phi\rangle)$ with $\partial/\partial y_k$ has the effect of replacing the $\mathcal{X}(y_k)$ factor by $-\partial\xi(y_k)$. Verlinde, Verlinde

The p-form $\omega_p(|\phi\rangle)$ on $P_{g,m,n}$ defined this way satisfies all the required identities.

A technical issue: Spurious poles

The correlation function used for defining ω_p diverges when PCO locatins collide and also at points where no vertex operators or PCO's coincide. Verlinde, Verlinde

$$f(\{y_i\}, \{w_j\}, \{m_k\}) = 0$$

y_i: location of PCO's

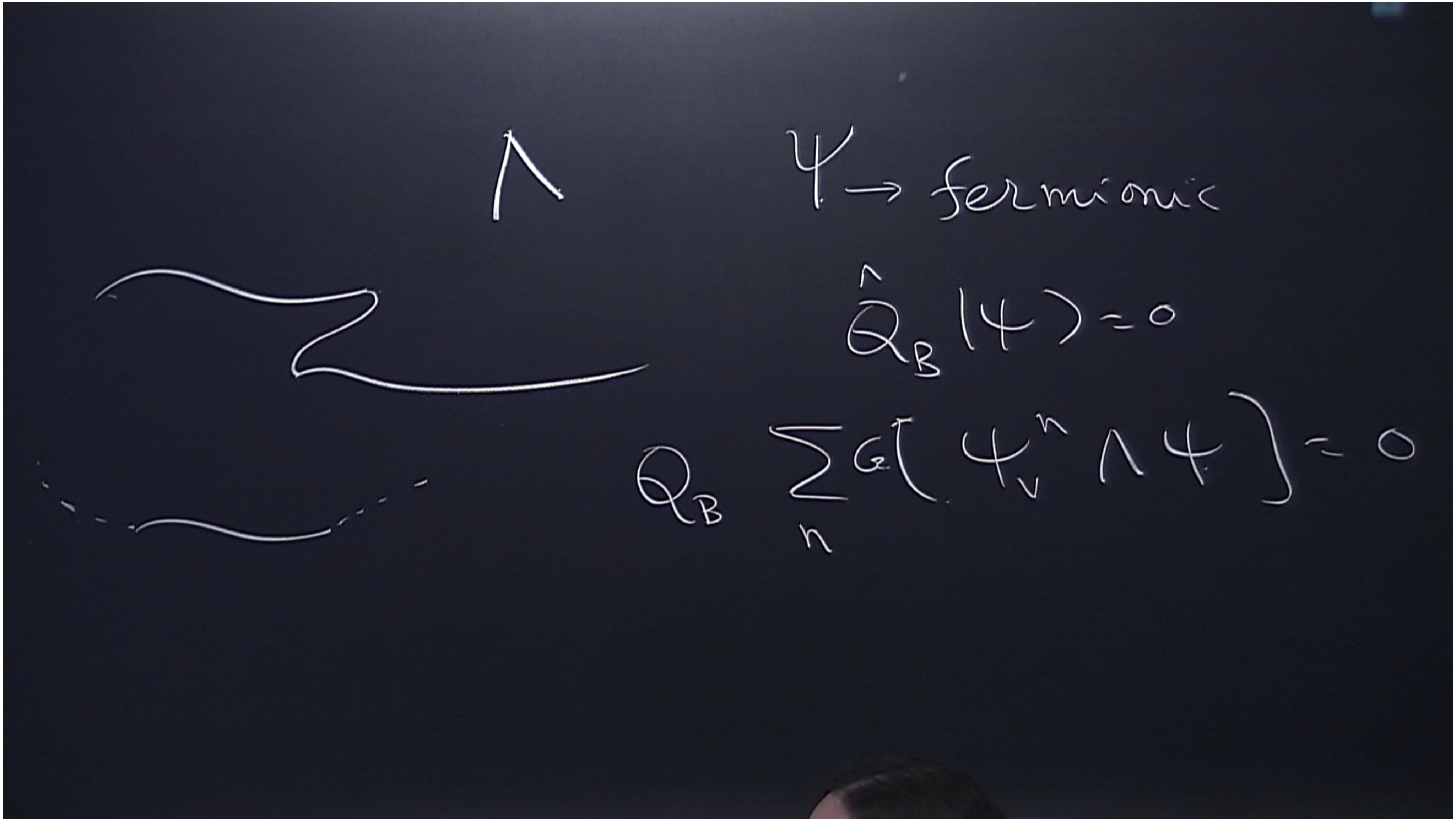
$\{m_k\}$: moduli

w_j: locations of vertex operators

– a real codimension two subspace on the section

– appears even for on-shell amplitudes

– related to the fact that the gauge choice for the world-sheet gravitino breaks down at these points.


Λ
Ψ → fermionic
Q̂_B |Ψ⟩ = 0
Q_B Σ_n G[Ψ_v^n ∧ Ψ] = 0