Title: Metastring theory and Modular spacetime

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Abstract: In this talk I will review a recent reformulation of string theory which does not rely on an a priori space-time interpretation or a pre-assumption of locality and include form the onset stringy symmetries such as T-duality.

I will explain how this resulting theory, called metastring, leads to formulation where the string is chiral and the target is phase space instead of space-time. I will discuss metastring theory on a flat background and summarize a variety of technical and interpretational ideas. These include a discussion of moduli space of Lorentzian worldsheets, a generalization of the world sheet renormalisation group, a description of the geometry of phase space, a study of the symplectic structure and of closed and open boundary conditions, and the string spectrum and operator algebra.

What emerges from these studies is a new quantum notion of space-time that we call modular space-time. This new geometrical concept is fundamental quantum and modular. It is closely linked with T-duality and implements in a precise way a notion of relative locality

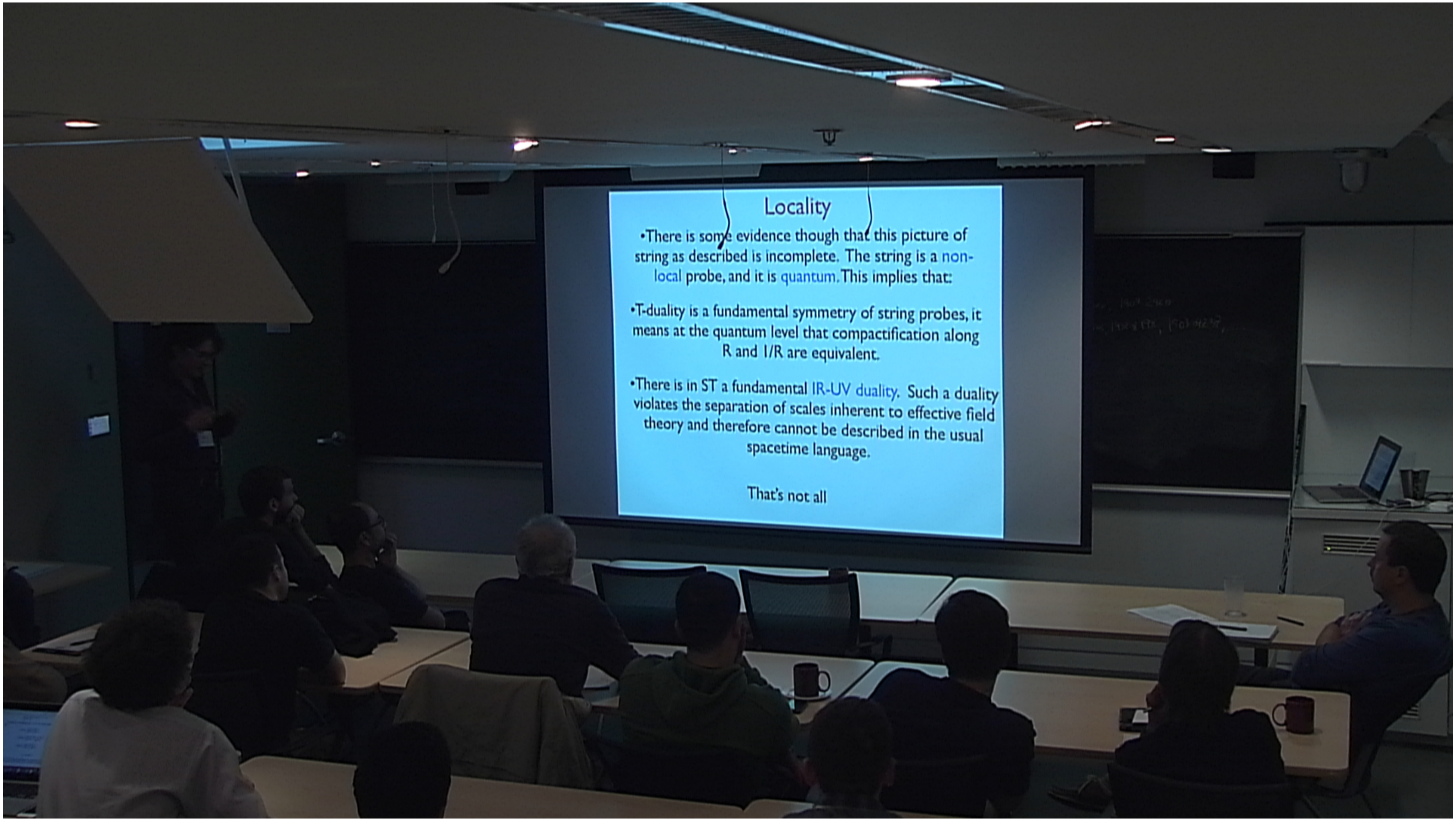
String theory magic

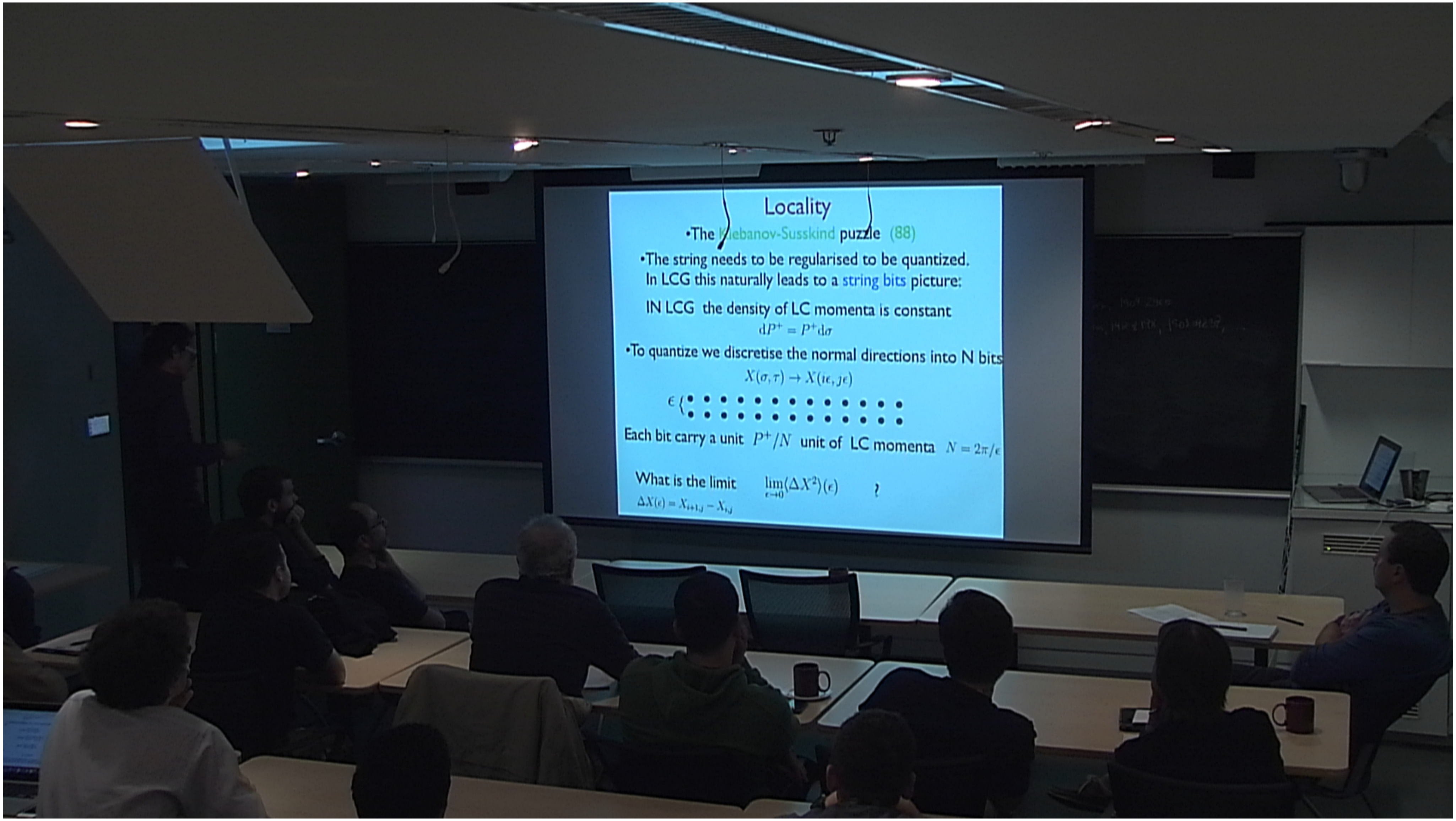
The usual interpretation of String theory is that it can be understood as describing an infinite number of particle states following Regge trajectories $M^2 \propto S$

string modes	⟷	particle states
correlation function	⟷	S-matrix
higher genus	⟷	Feynman loops

String magic

CFT consistency = Conformal symmetry Mutual locality, Modularity	⟷	Spacetime consistency = Locality, Unitarity causality.


Locality
•There is some evidence though that this picture of string as described is incomplete. The string is a non-local probe, and it is quantum. This implies that:
•T-duality is a fundamental symmetry of string probes, it means at the quantum level that compactification along R and 1/R are equivalent.
•There is in ST a fundamental IR-UV duality. Such a duality violates the separation of scales inherent to effective field theory and therefore cannot be described in the usual spacetime language.
That's not all


Locality
•The Klebanov-Susskind puzzle (88)
•The string needs to be regularised to be quantized.
In LCG this naturally leads to a string bits picture:
IN LCG the density of LC momenta is constant
$\mathrm{d}P^+ = P^+ \mathrm{d}\sigma$
•To quantize we discretise the normal directions into N bits
$X(\sigma,\tau) \to X(i\epsilon, j\epsilon)$
ϵ
Each bit carry a unit P^+/N unit of LC momenta $N = 2\pi/\epsilon$
What is the limit $\lim_{\epsilon \to 0} (\Delta X^2)(\epsilon)$?
$\Delta X(\epsilon) = X_{i+1,j} - X_{i,j}$

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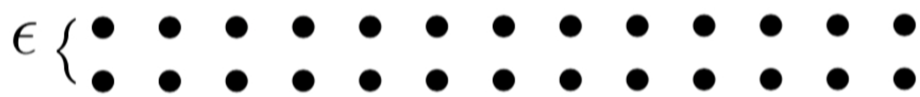

Each bit carry a unit P^+/N unit of LC momenta $N = 2\pi/\epsilon$

What is the limit $\lim_{\epsilon \rightarrow 0} \langle \Delta X^2 \rangle (\epsilon)$?

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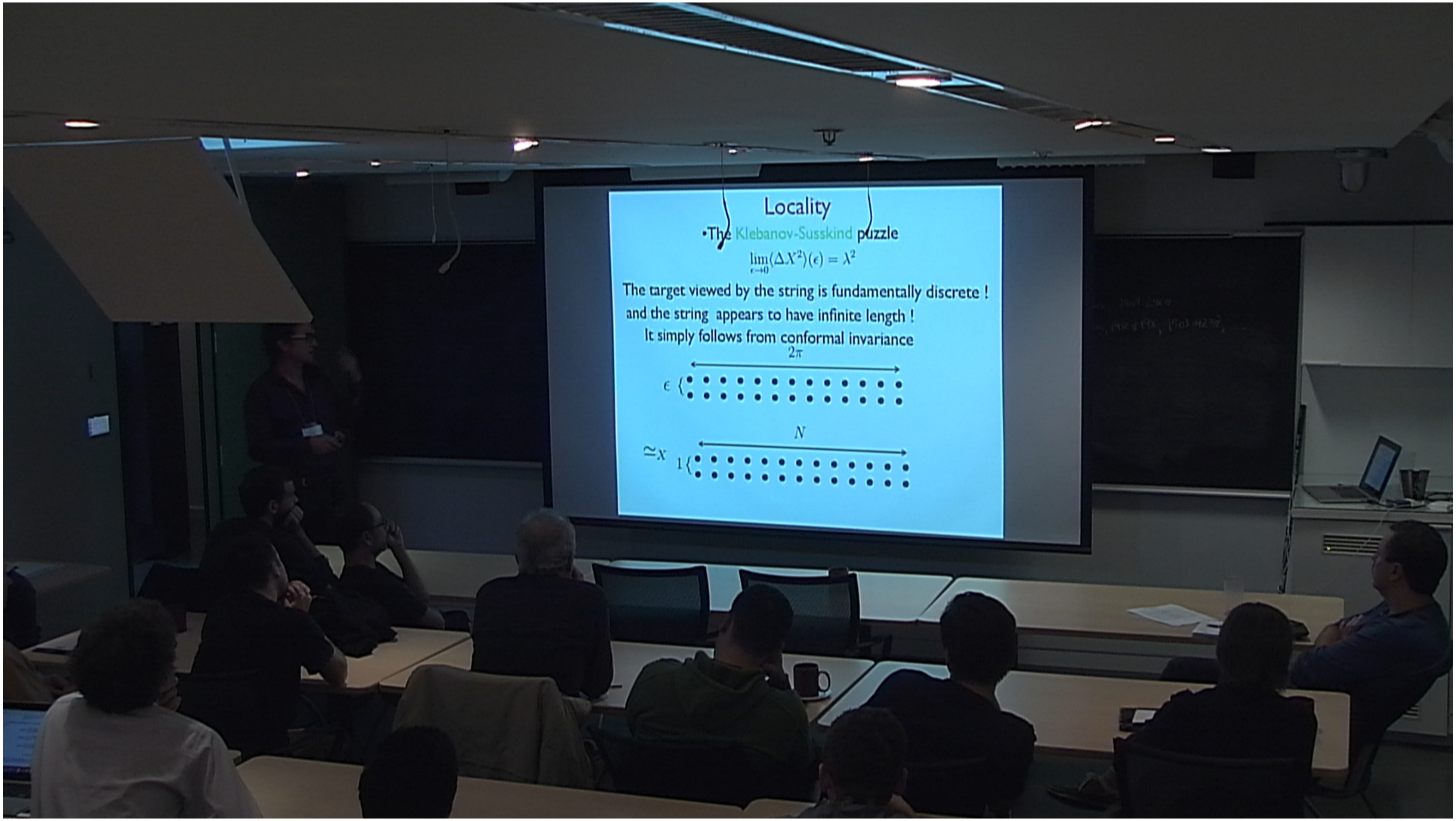
Locality
•The Klebanov-Susskind puzzle
$\lim_{\epsilon\to 0}\langle \Delta X^2\rangle(\epsilon) = \lambda^2$
The target viewed by the string is fundamentally discrete !
and the string appears to have infinite length !
It simply follows from conformal invariance
2π
ϵ
N
$\simeq x$
1

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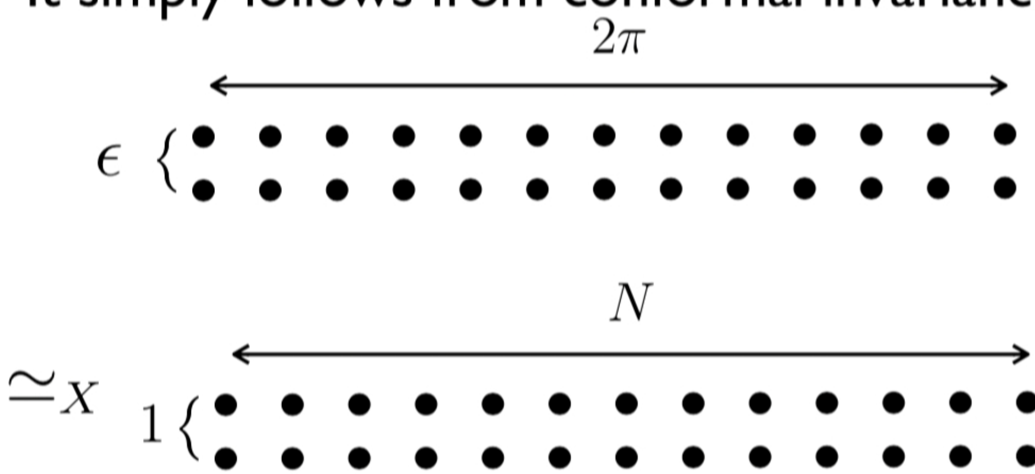

What is String theory?

•A collection of massive particles propagating in spacetime

ST= non local object in spacetime

•A fundamentally discrete theory made out of string bits

ST= non local in a discrete spacetime

•A theory which exhibit at the fundamental level a UV-IR duality.

ST doesn't live in spacetime

In order to make progress we have to let go of the concept of spacetime as we know it.

What spacetime does ST theory live in?

Our strategy: reanalyse ST without assuming locality

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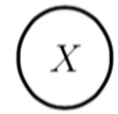
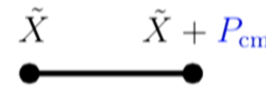
T-duality

String solutions are chiral combination

$$X(\tau,\sigma) = X_L(\tau-\sigma) + X_R(\tau+\sigma)$$



T-Duality $\tilde{X}(\tau,\sigma) = [X_L(\tau-\sigma) - X_R(\tau+\sigma)]/\alpha'$



T-duality exchanges cm Momenta with Monodromy

$$P_{\rm cm} = \frac{1}{\alpha'}\int_0^{2\pi} \partial_\tau X(\tau,\sigma)\mathrm{d}\sigma = \tilde{X}(\tau,\sigma)\Big|_0^{2\pi} = \Delta$$

T-duality exchanges Momentum density with Position density

$$P := \partial_\tau X/\alpha' \quad Q := \partial_\sigma X \qquad (P,Q) \to (\tilde{Q},\tilde{P})$$

- A central theme of our analysis is that the dual coordinate is a momentum coordinate and α' is a conversion factor.

$$\alpha' = \frac{\lambda}{\epsilon}$$

T-duality= Fourier Transform

Consider a string state $|\Psi_\Sigma\rangle$represented by the Path integral

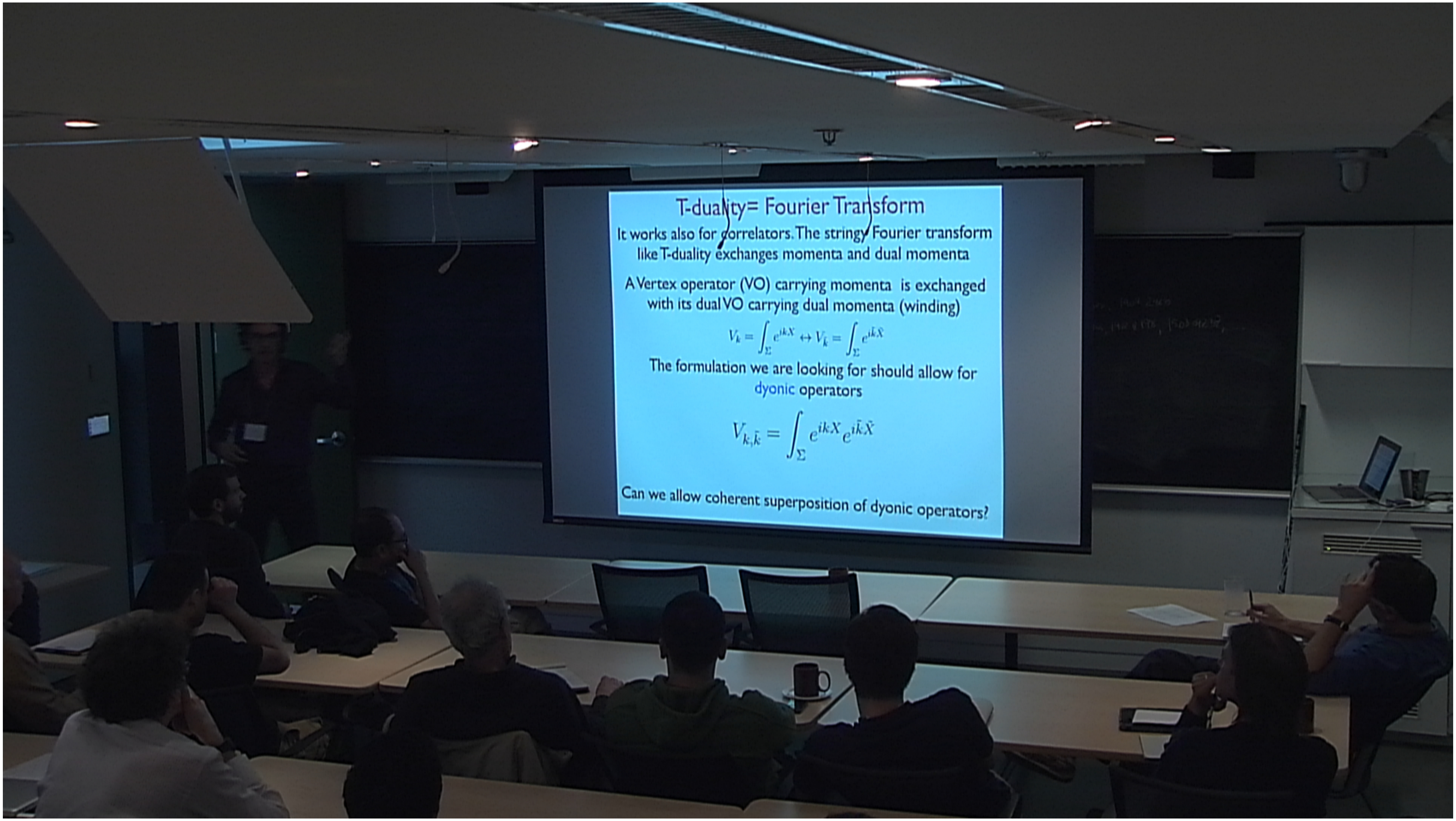
$$\Psi_\Sigma(x^a(\sigma)) = \int_{X|_{\partial\Sigma}=x} DX \int_{Met(\Sigma)} \mathcal{D}\gamma \ \exp\left(\frac{i}{4\pi\lambda^2} S_P(X)\right)$$

Take its Stringy Fourier transform

$$\tilde{\Psi}_\Sigma(\tilde{x}_a(\sigma)) := \int Dx \ \exp\left(\frac{1}{2\pi i\hbar}\int_{\partial\Sigma} x^a \mathrm{d}\tilde{x}_a\right)\Psi(x^a(\sigma))$$

It is given by the dual Polyakov integral in momentum space

$$\tilde{\Psi}_\Sigma(\tilde{x}_a(\sigma)) = \int_{\tilde{X}|_{\partial\Sigma}=x} DX \int_{Met(\Sigma)} \mathcal{D}\gamma \ \exp\left(\frac{i}{4\pi\epsilon^2} S_P(\tilde{X})\right)$$


T-duality= Fourier Transform
It works also for correlators. The stringy Fourier transform like T-duality exchanges momenta and dual momenta
A Vertex operator (VO) carrying momenta is exchanged with its dual VO carrying dual momenta (winding)
$V_k = \int_\Sigma e^{ikX} \leftrightarrow V_{\tilde{k}} = \int_\Sigma e^{i\tilde{k}\tilde{X}}$
The formulation we are looking for should allow for dyonic operators
$V_{k,\tilde{k}} = \int_\Sigma e^{ikX} e^{i\tilde{k}\tilde{X}}$
Can we allow coherent superposition of dyonic operators?

T-duality= Born Duality Veneziano 86

Dimensionless Momentum and Position density

$$P := \partial_\tau X/\lambda \quad Q := \partial_\sigma X/\lambda$$

The dynamics of free string is characterized by the Hamiltonian and diffeomorphism constraints

$$H = P^2 + Q^2 = 0$$

$$D = P \cdot Q = 0$$

This is symmetric under the exchange $P \leftrightarrow Q$

M. Born (38) Born duality principle:

Physics should be formulated in a way that incorporate in a democratic form position and momenta

Classical Metastring

We are looking for a formulation of string that generalises the Polyakov spacetime formulation

$$S_P = \int_\Sigma (G_{\mu\nu} + B_{\mu\nu})(X)(\mathrm{d}X^\mu) \wedge (*\mathrm{d}X^\nu)$$

and can include deformations by general dyonic operators

A formulation that liberates the right movers from the left

This formulation is a relativistic phase space formulation whose target fields are coordinates on P given by

$$\mathbb{X}^A = \begin{pmatrix} X^a/\lambda \\ \tilde{X}_a/\epsilon \end{pmatrix} \qquad \alpha' = \frac{\lambda}{\epsilon} \qquad \hbar = \lambda\epsilon$$

P=Phase space?

Classically It carries a symplectic and a bilagrangian structure.

QM: X and $\tilde{X}$ do not commute.

Geometry of Phase space

Usual string background geometry (M, G_{ab}, B_{ab})

What is the meaning of the metastring background as a geometry on relativistic phase space?

$$(P, \omega_{AB}, \eta_{AB}, H_{AB})$$

Geometry of Quantization

Remarkably, in the non relativistic case the same structure appears in the geometry of quantization!

Phase space geometry= geometry of quantisation

$$(P, \omega_{AB}, \eta_{AB}, H_{AB})$$

Phase space

symplectic structure

Symplectic connection

complex structure

Fedosov Q

Geometric Q

Star product.

Hilbert space.

Geometry of Phase space

In Darboux coordinates $\omega = \mathrm{d}p \wedge \mathrm{d}q$

Geometrical Quantisation:

Given a compatible complex structure (ω, I)

We can construct a complex line bundle L
with curvature and define $\mathcal{H} = L^2(\Gamma_{\mathrm{Hol}}, \omega)$

The metric is given by $H = \omega I$

In Darboux coordinate $\mathrm{d}s_H^2 = \mathrm{d}p^2 + \mathrm{d}q^2$

In pedestrian terms H is the metric induced by the Born metric on coherent states.

Bilagrangian and Fedosov

In 1992 Fedosov proved an absolutely beautiful and foundational result about quantization. He showed that given a torsionless symplectic connection $\nabla\omega = 0$ there exists a non-commutative star product.

$$\nabla \to f * g \qquad\qquad f * g - g * f = \frac{\hbar}{i}\{f, g\} + \cdots$$

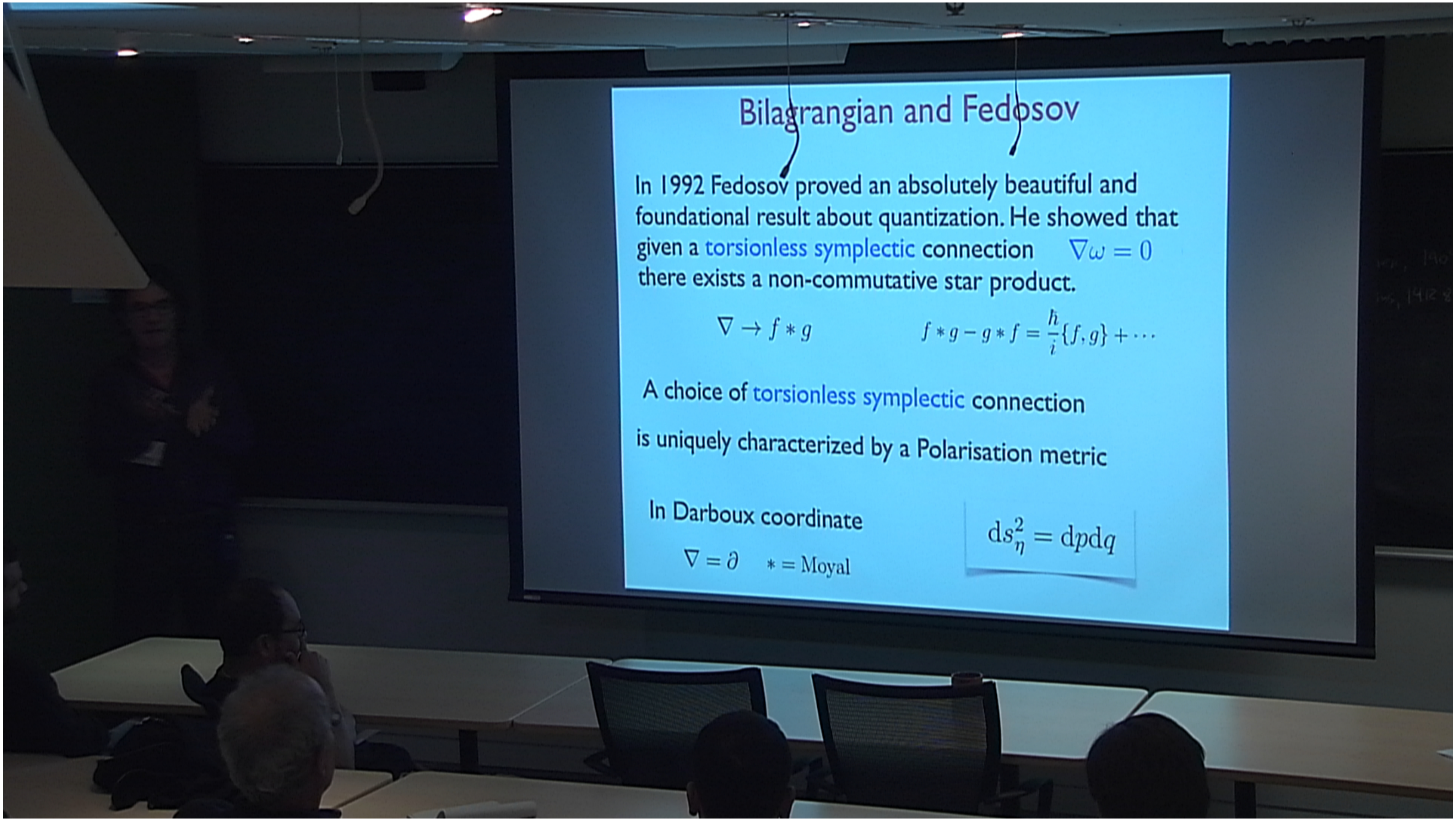
A choice of torsionless symplectic connection

is uniquely characterized by a Polarisation metric

In Darboux coordinate

$$\mathrm{d}s^2_\eta = \mathrm{d}p\mathrm{d}q$$

$$\nabla = \partial \qquad * = \text{Moyal}$$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Solving the dynamics

Eom: $$\partial_\tau \mathbb{X}^A - (J\partial_\sigma \mathbb{X})^A = 0$$

relation momenta-monodromy $P = \dfrac{J(\Delta)}{2\pi}$

Soldering between world sheet null coordinate and chiral structure on target. $\sigma^\pm = \sigma \pm \tau$

$$\partial_\pm \mathbb{X}^A = P_\pm \partial_\sigma \mathbb{X}$$

Chiral projectors $P_\pm = \dfrac{(1 \pm J)}{2}$

Allow to liberate the left geometry from the right.

Commutation

A careful analysis of the metastring actions shows that its symplectic form is

$$\Omega = \frac{1}{4\pi}\oint \delta\mathbb{X}^A \eta_{AB} \nabla_\sigma \delta\mathbb{X}^B$$

Generalized Fedosov connection $\nabla\eta = 0, \quad T = \nabla\omega$

The polarisation metric controls the phase space commutation

When constant:

$$\left\{\mathbb{X}^A(\sigma), \mathbb{X}^B(\sigma')\right\} = \eta^{AB}\theta(\sigma - \sigma').$$

Staircase distribution

At the quantum and interacting level the metastring target appears non-commutative ! $X\ \tilde{X}$ are conjugate variables

Spectrum

Vertex operators $V_K = \epsilon_K e^{iK\mathbb{X}}, \quad V_K^A = \epsilon_K e^{iK\mathbb{X}}(\partial_\sigma \mathbb{X})^A$

OPE $K, K' \in \Gamma \longrightarrow K \pm K' \in \Gamma$

The spectrum is a lattice.

Hamiltonian $\frac{1}{2}K^A H_{AB} K^B = (2 - N_+ - N_-)$

Diffeo $\frac{1}{2}K^A \eta_{AB} K^B = (N_- - N_+)$

$N_\pm \in \mathbb{N}$

New fields: $\eta_{AB} \quad A_A$

Moore (93)

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Constraints $$K_\pm^2 = 2(1 - N_\pm)$$ $N_\pm \in \mathbb{N}$, $K_\pm = P_\pm(K)$

The spectra is a double lorentzian integral lattice

Modular invariance → The lattice is self dual

There exist such lattices only in D= 2 mod(8) and they are unique

Criticality $$\Gamma = II_{1,25} \times II_{1,25}$$

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Mutual locality

The vertex operators then commute provided one chooses the phase factors to be $\epsilon_K = e^{\frac{i}{2}K(\eta+\omega)P_{\rm cm}}$

Cocycle

symplectic form

The lattice

How does one recover the usual string?

The monodromy for the usual string are $\Delta^A = (0, \tilde{\delta}_\mu)$

The momenta for the usual string are $K_A = (\tilde{\delta}_\mu / 2\pi, 0)$

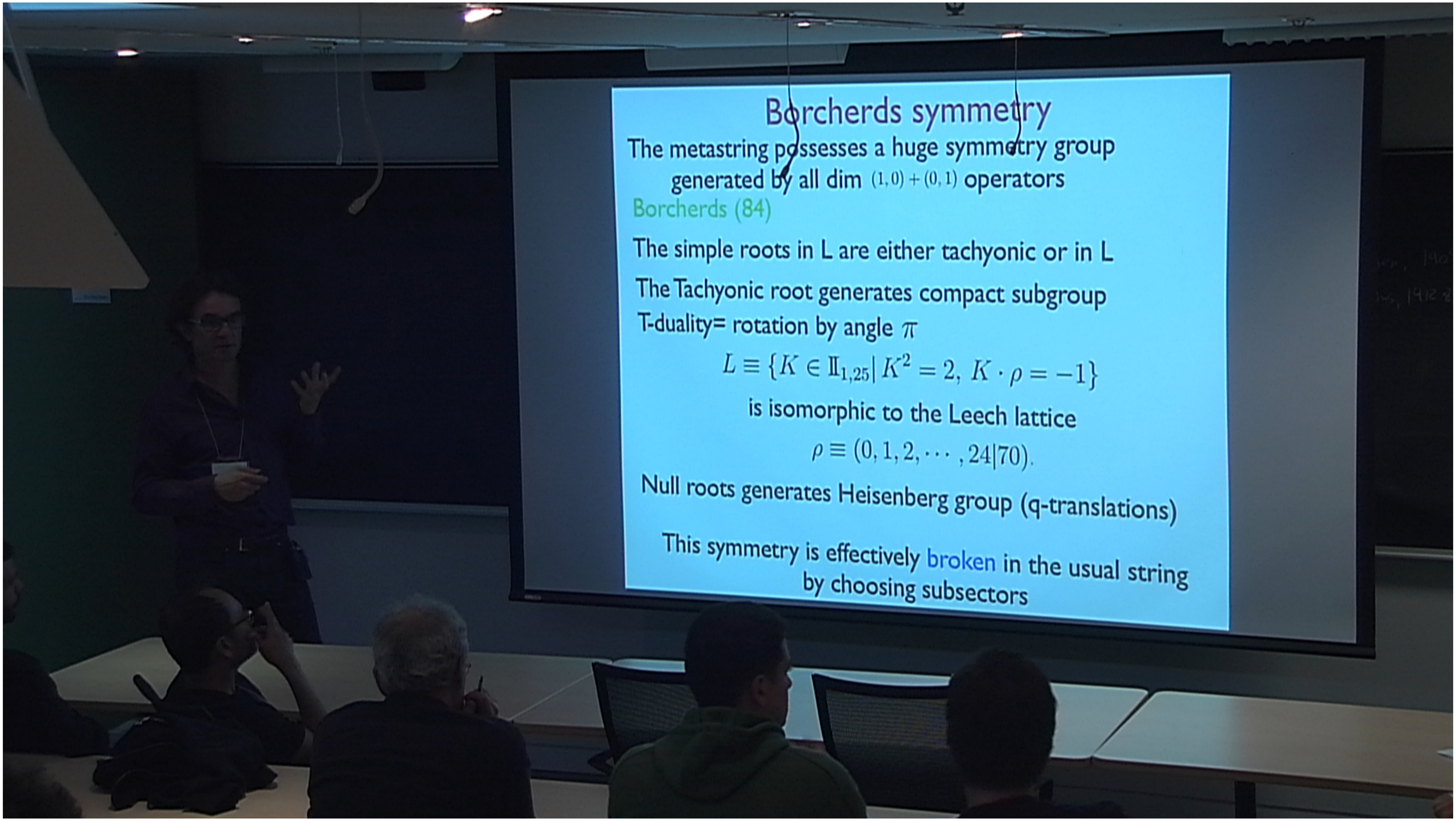
$K_A = J(\Delta)/2\pi$

For the metastring we have that $K_A = (k_\mu, \tilde{k}^\mu)$

The usual string spectrum is recover if we truncate the metastring spectra to operators such that

$$|k| >> |\tilde{k}|$$

The usual locality limit is a large quantum number limit = a classical limit


Borcherds symmetry
The metastring possesses a huge symmetry group generated by all dim $(1,0)+(0,1)$ operators
Borcherds (84)
The simple roots in L are either tachyonic or in L
The Tachyonic root generates compact subgroup
T-duality= rotation by angle π
$L \equiv \{K \in \mathbb{II}_{1,25} | K^2 = 2,\ K\cdot\rho = -1\}$
is isomorphic to the Leech lattice
$\rho \equiv (0,1,2,\cdots,24|70).$
Null roots generates Heisenberg group (q-translations)
This symmetry is effectively broken in the usual string by choosing subsectors

Modular spacetime

Classically we have seen that the metastring forces us to think about spacetime M as a Lagrangian sub-manifold of P

$$\omega|_L = \eta|_L = 0$$

Absolute Locality = Flatness of the Polarization metric $R(\eta) = 0$

What is spacetime for the quantum string?
What are the effective fields?

Modular spacetime

At the quantum level we have seen that the zero-mode algebra is non-commutative

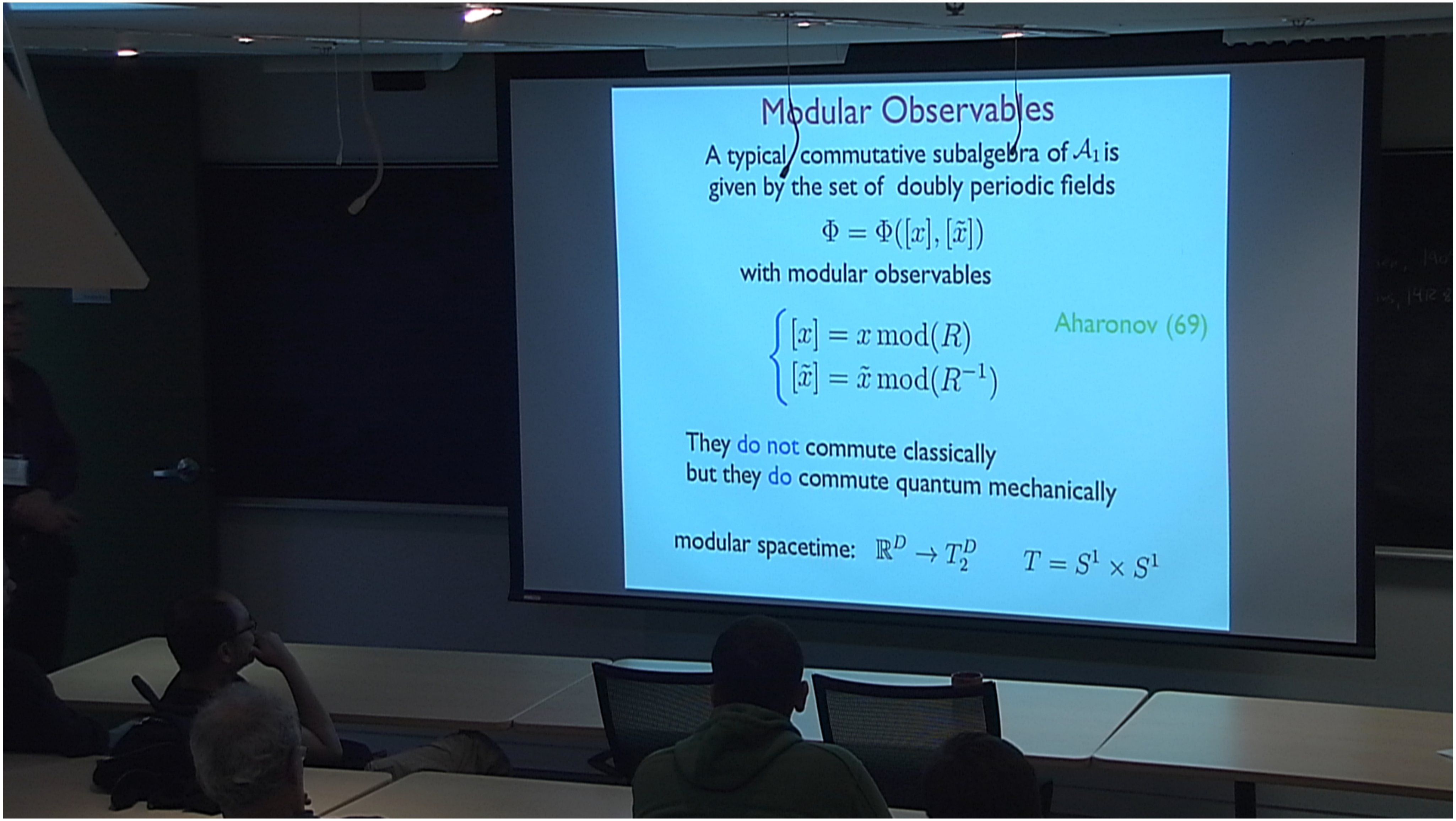
We have to take the Non-Commutative point of view: To talk about a space we have to look at the dual space. The space of functions on it. In the non commutative case it is an algebra.

Here the algebra $\mathcal{A}_D$ is the algebra of bounded function of $(X, \tilde{X})$

$$[X^a, \tilde{X}_b] = 2i\pi\hbar\delta^a_b$$

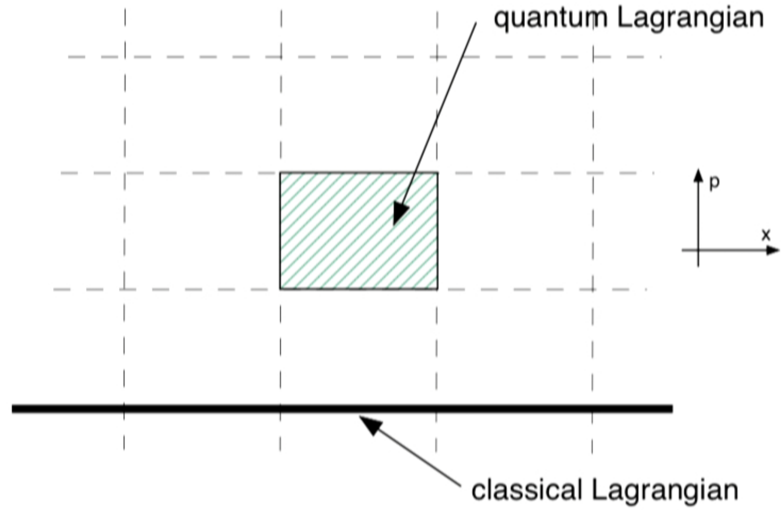
The space of function on modular spacetime is defined as a commutative subalgebra of $\mathcal{A}_D$

modular spacetime = quantum Lagrangian expression of string mutual locality


Modular Observables
A typical commutative subalgebra of $\mathcal{A}_1$ is given by the set of doubly periodic fields
$\Phi = \Phi([x],[\tilde{x}])$
with modular observables
$[x] = x \bmod (R)$
$[\tilde{x}] = \tilde{x} \bmod (R^{-1})$
Aharonov (69)
They do not commute classically
but they do commute quantum mechanically
modular spacetime: $\mathbb{R}^D \to T_2^D$ $T = S^1 \times S^1$

Modular spacetime

modular spacetime: is a cell in phase space



The usual spacetimes are obtained as a semi-classical limit squashing the phase space cell. A coarse graining procedure which concatenates the cells.

Usual moduli reappear as a parametrisation of this limit.

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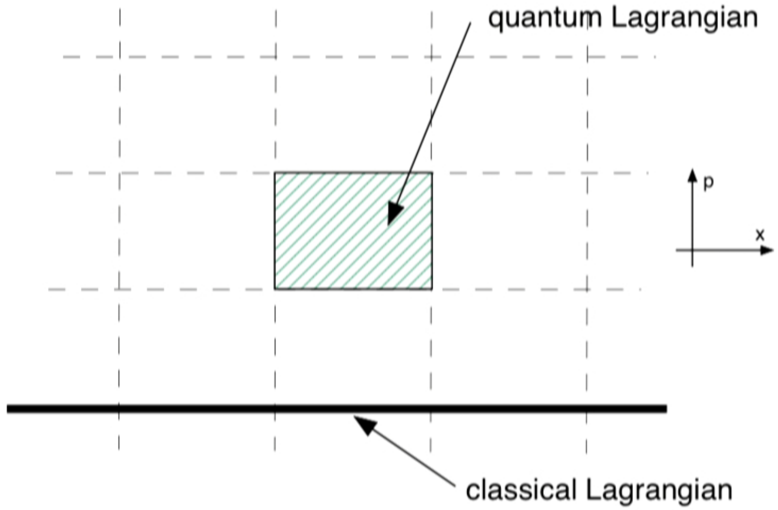

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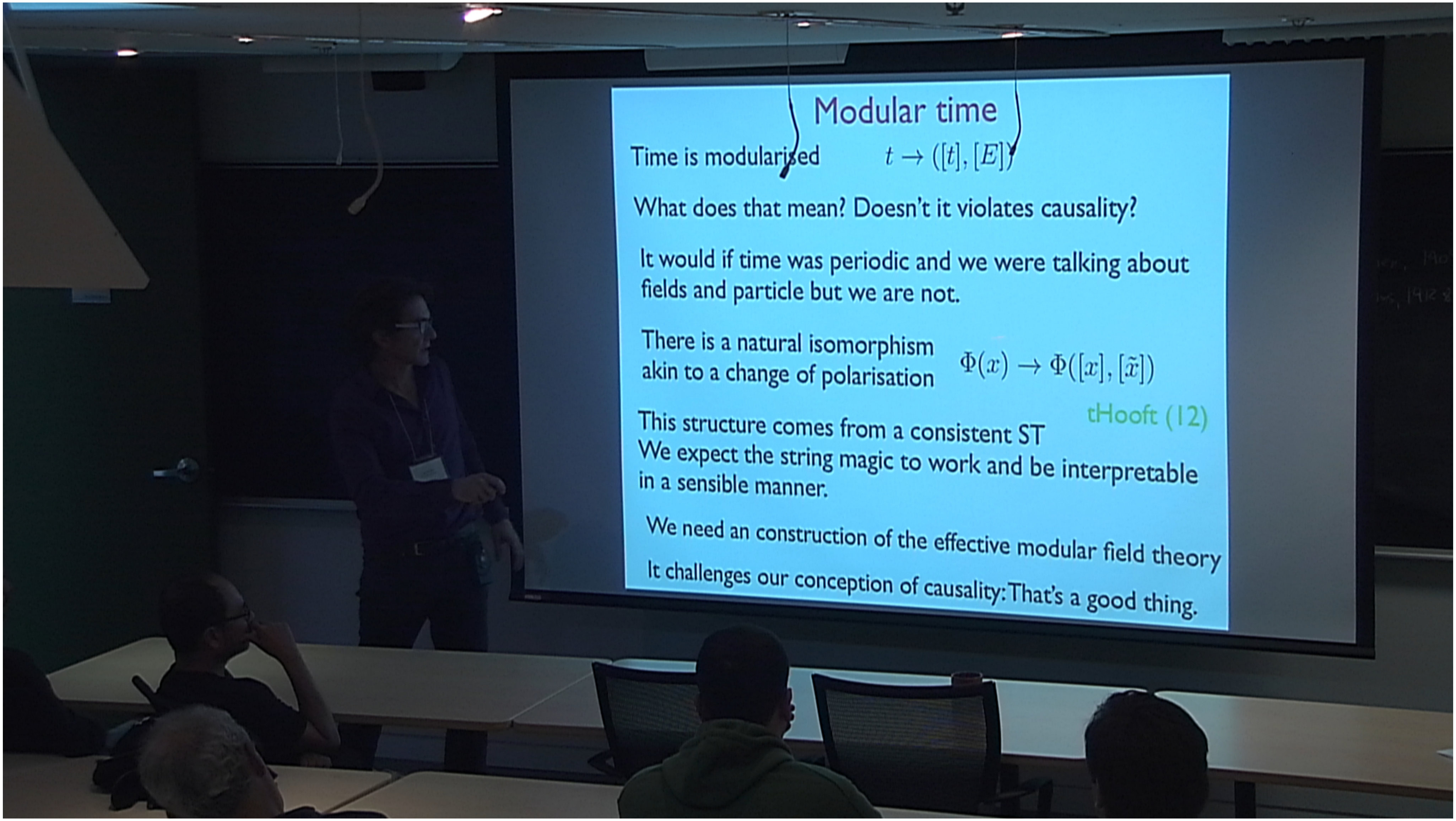
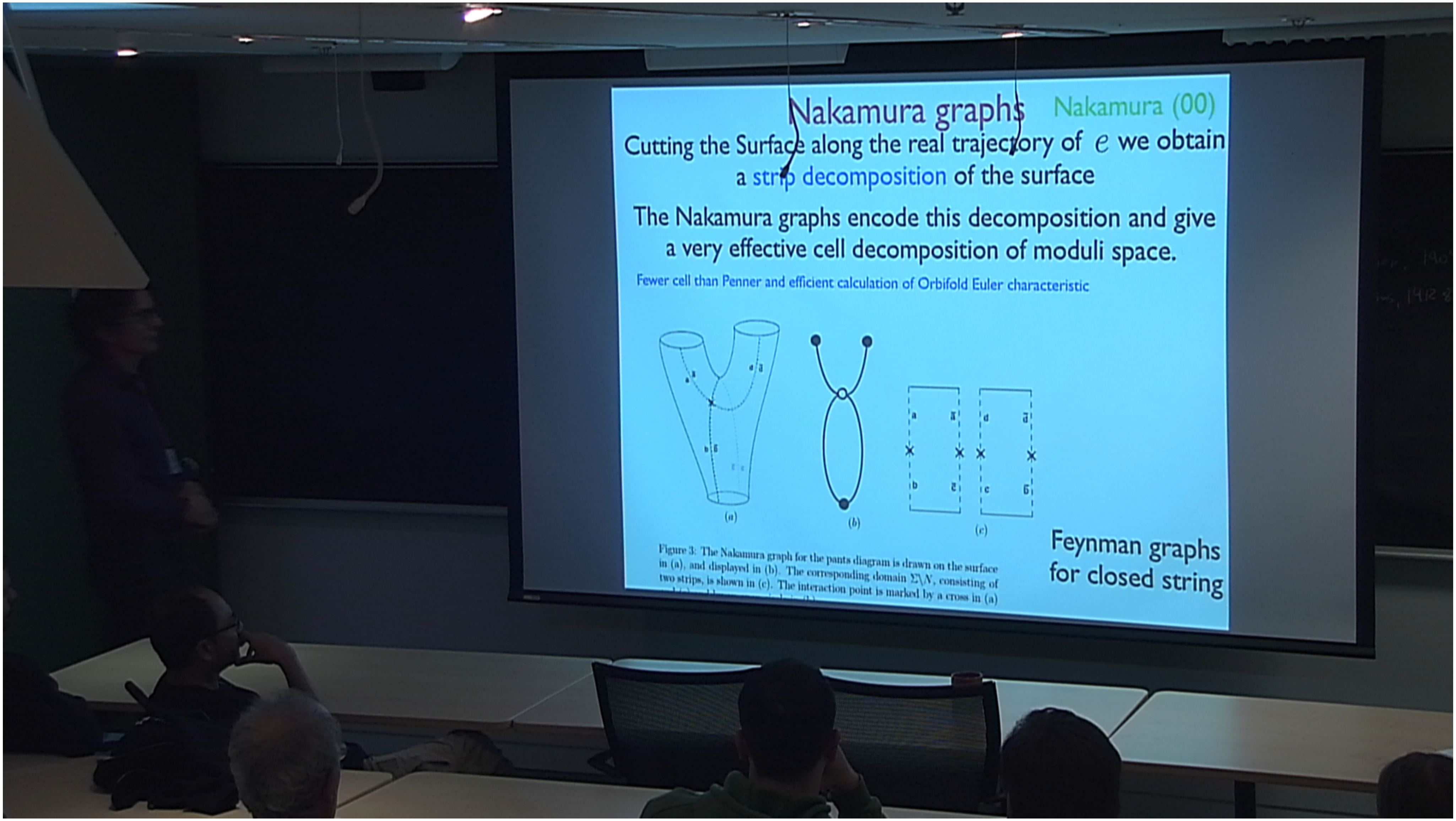

Modular time
Time is modularised $t \to ([t],[E])$
What does that mean? Doesn't it violates causality?
It would if time was periodic and we were talking about fields and particle but we are not.
There is a natural isomorphism akin to a change of polarisation $\Phi(x) \to \Phi([x],[\tilde{x}])$
tHooft (12)
This structure comes from a consistent ST
We expect the string magic to work and be interpretable in a sensible manner.
We need an construction of the effective modular field theory
It challenges our conception of causality: That's a good thing.


Nakamura graphs
Nakamura (00)
Cutting the Surface along the real trajectory of e we obtain a strip decomposition of the surface
The Nakamura graphs encode this decomposition and give a very effective cell decomposition of moduli space.
Fewer cell than Penner and efficient calculation of Orbifold Euler characteristic
(a)
(b)
(c)
Figure 3: The Nakamura graph for the pants diagram is drawn on the surface in (a), and displayed in (b). The corresponding domain Σ\V, consisting of two strips, is shown in (c). The interaction point is marked by a cross in (a)
Feynman graphs for closed string

Nakamura graphs   Nakamura (00)

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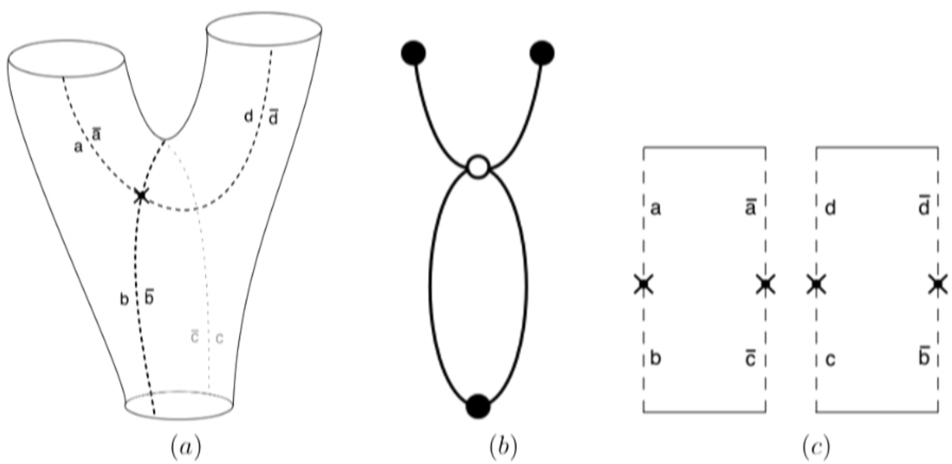


Figure 3: The Nakamura graph for the pants diagram is drawn on the surface in (a), and displayed in (b). The corresponding domain $\Sigma \backslash N$, consisting of two strips, is shown in (c). The interaction point is marked by a cross in (a)

Feynman graphs for closed string

Double RG flow

Our chiral theory cannot be Wick rotated

In a theory where time and space appear non symetrically we need two cut-offs

$$E < \Lambda \quad |p| < \Lambda'$$

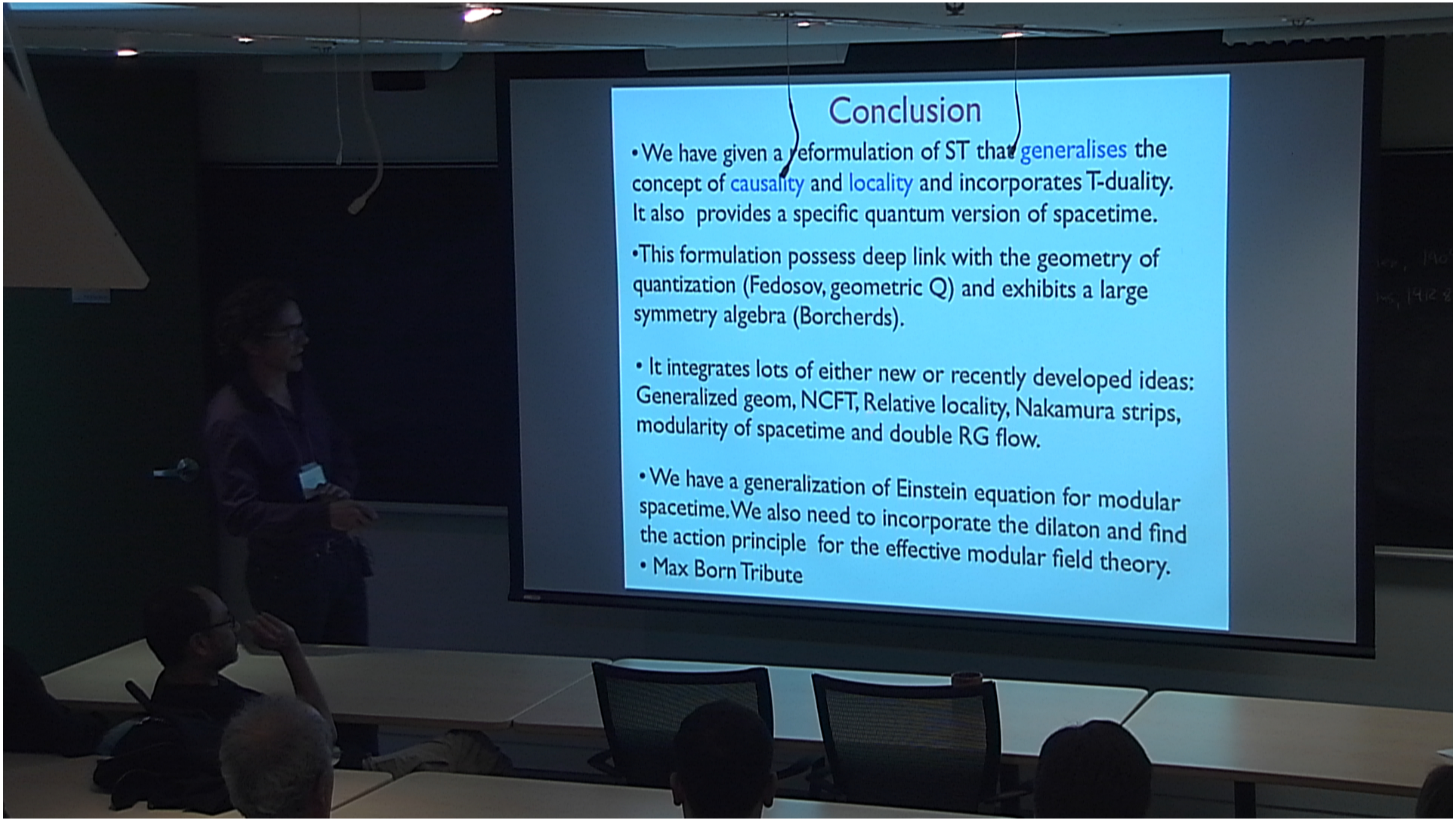
In 2d conformal and Lorentz transformations are on the same footing

$$\mathrm{d}s^2 = e_+ e_- \qquad\qquad e_\pm \to e^{\phi\pm\theta} e_\pm$$

At the fixed point we restore both conformal and Lorentz symmetry

$$\partial_\phi Z = 0 \to (\partial^2 + \tilde{\partial}^2)\delta\eta = 0$$

$$\partial_\theta Z = 0 \to (\partial \cdot \tilde{\partial})\delta\eta + 2(J\delta\eta) = 0$$


Conclusion
• We have given a reformulation of ST that generalises the concept of causality and locality and incorporates T-duality. It also provides a specific quantum version of spacetime.
•This formulation possess deep link with the geometry of quantization (Fedosov, geometric Q) and exhibits a large symmetry algebra (Borcherds).
• It integrates lots of either new or recently developed ideas: Generalized geom, NCFT, Relative locality, Nakamura strips, modularity of spacetime and double RG flow.
• We have a generalization of Einstein equation for modular spacetime. We also need to incorporate the dilaton and find the action principle for the effective modular field theory.
• Max Born Tribute