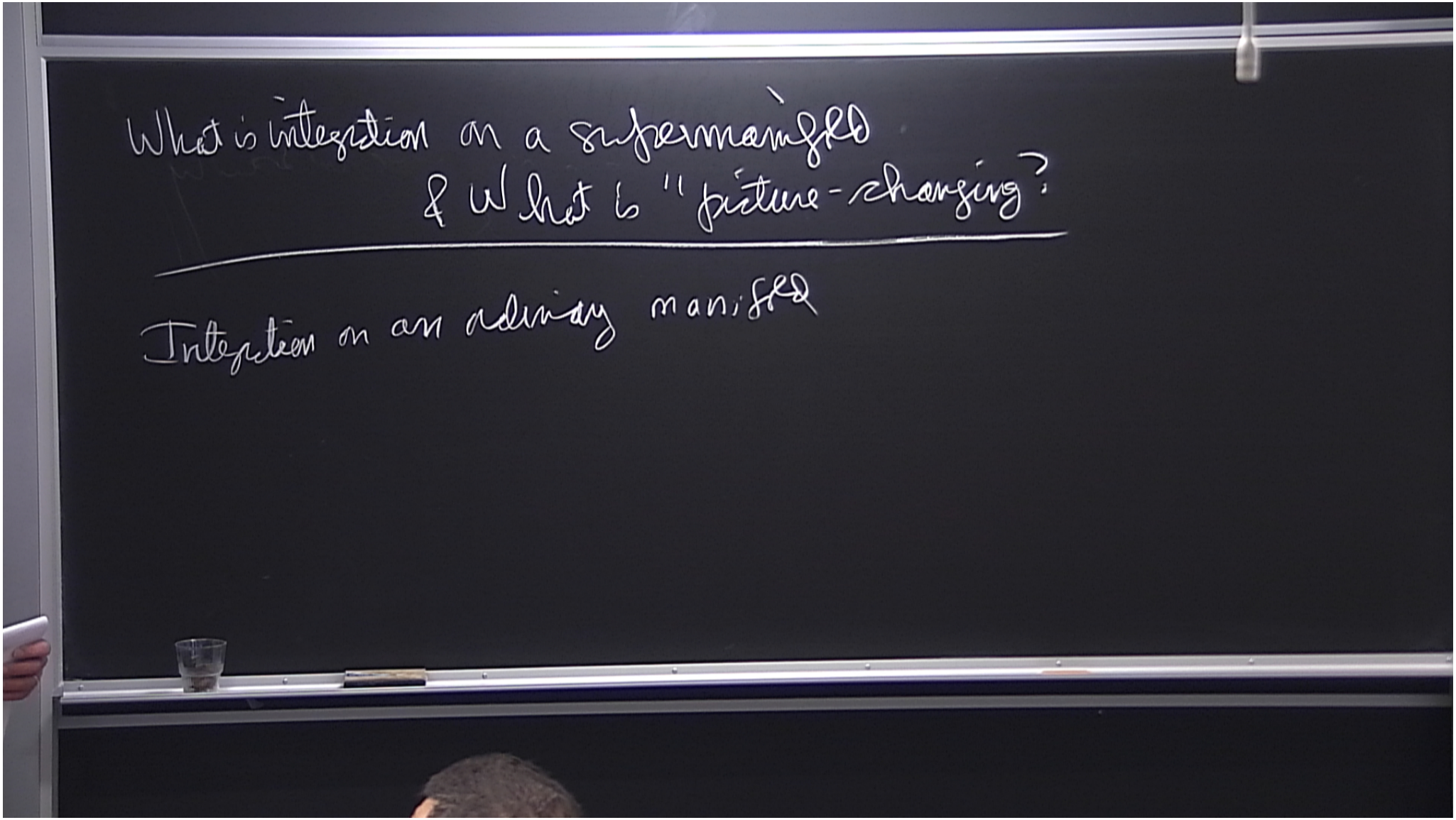
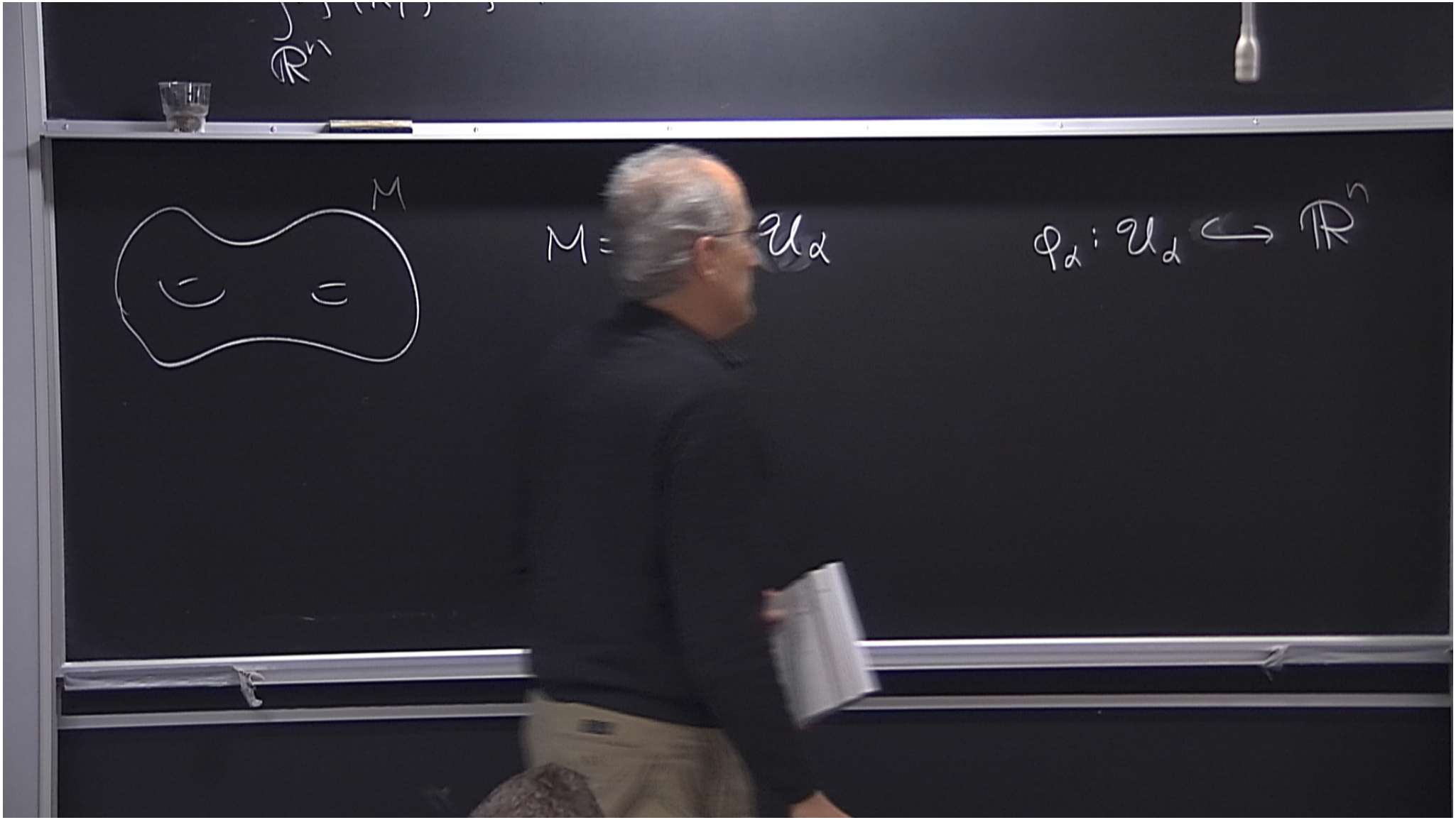
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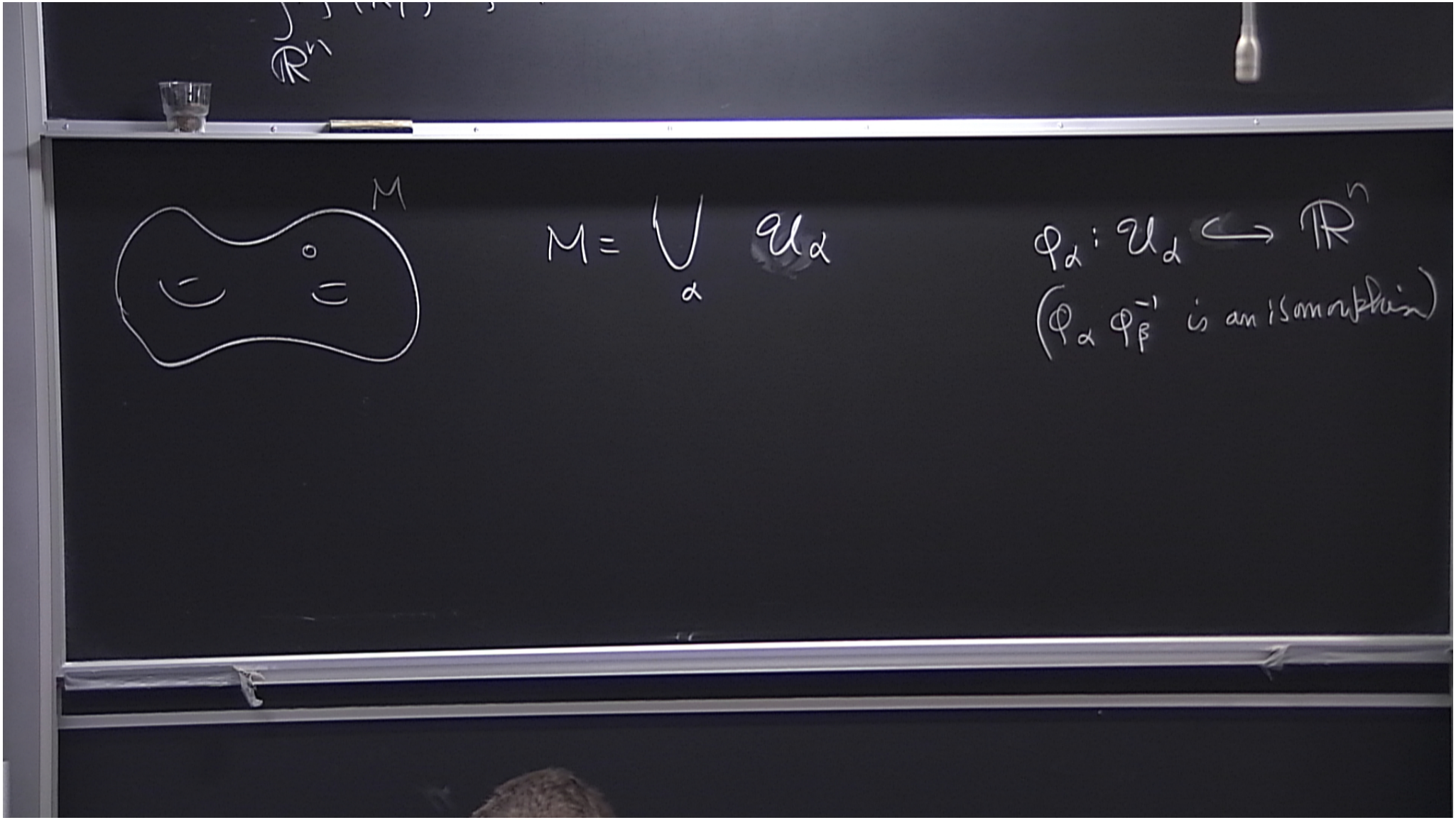
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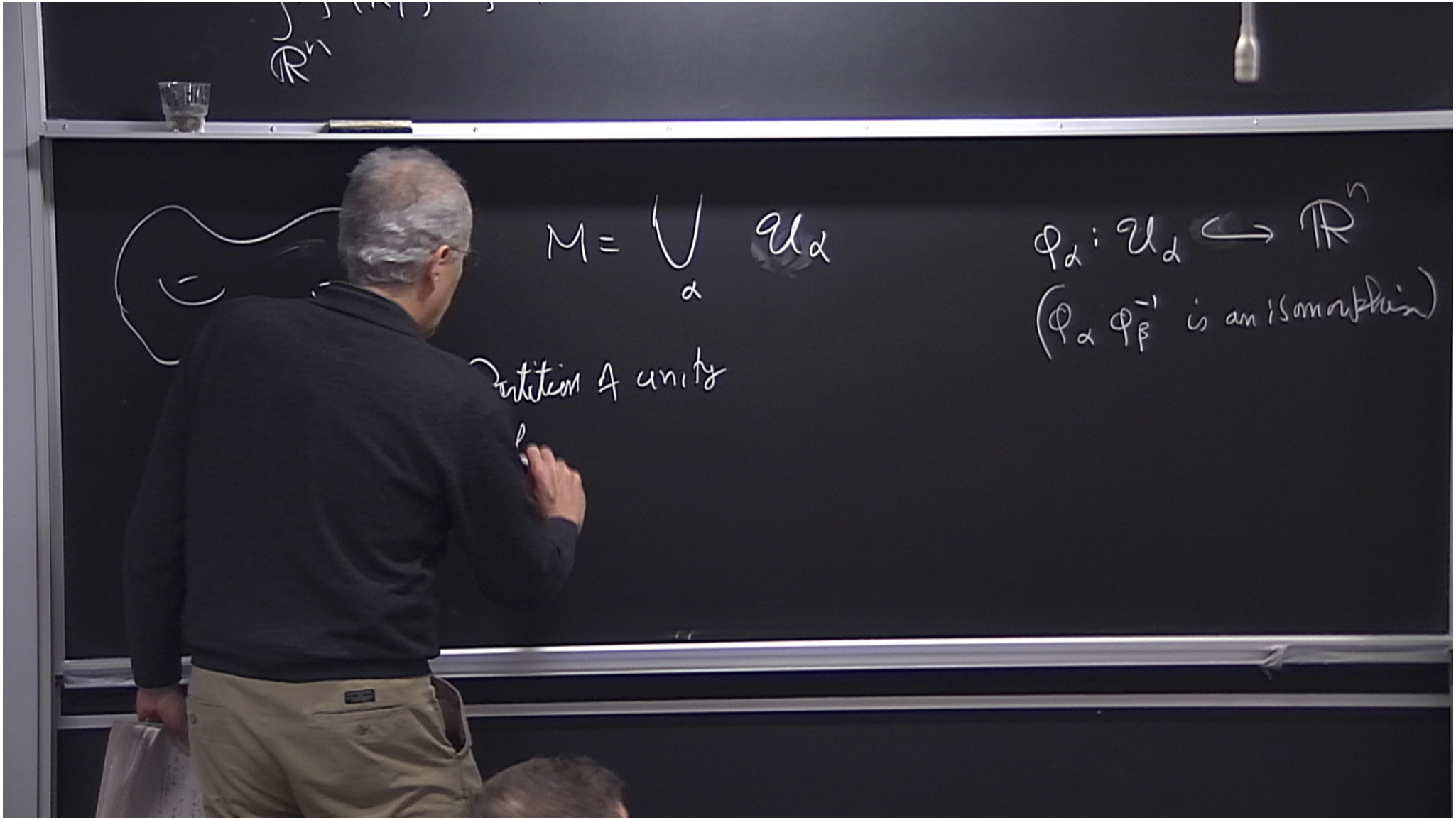
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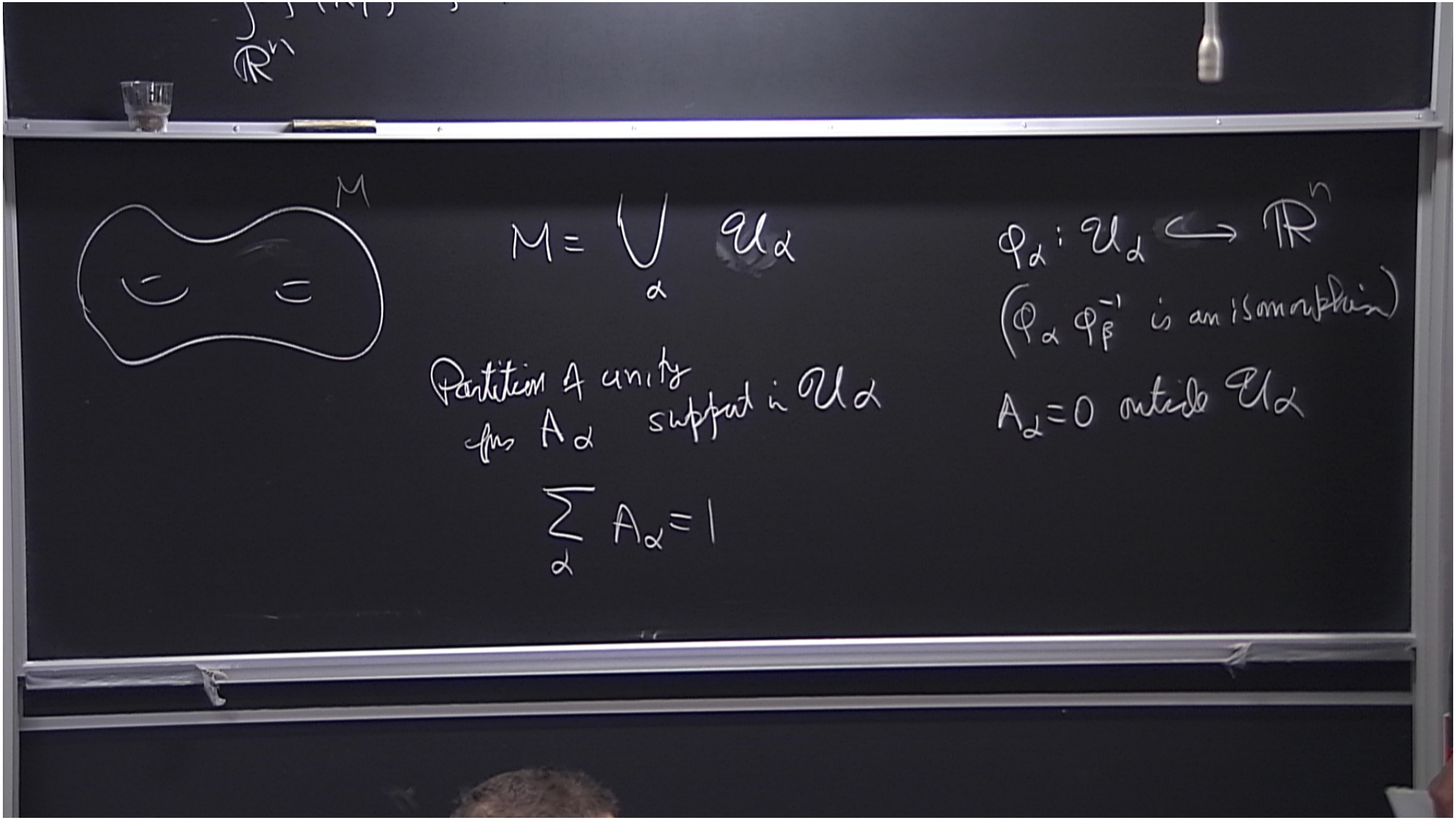
Abstract:


What is integration on a supermanifold
& What is "picture-changing"?
Integration on an ordinary manifold


M
$M = \mathcal{U}_\alpha$
$\varphi_\alpha : \mathcal{U}_\alpha \hookrightarrow \mathbb{R}^n$


$\mathbb{R}^n$
M
$M = \bigcup_\alpha \mathcal{U}_\alpha$
$\varphi_\alpha : \mathcal{U}_\alpha \hookrightarrow \mathbb{R}^n$
($\varphi_\alpha \varphi_\beta^{-1}$ is an isomorphism)

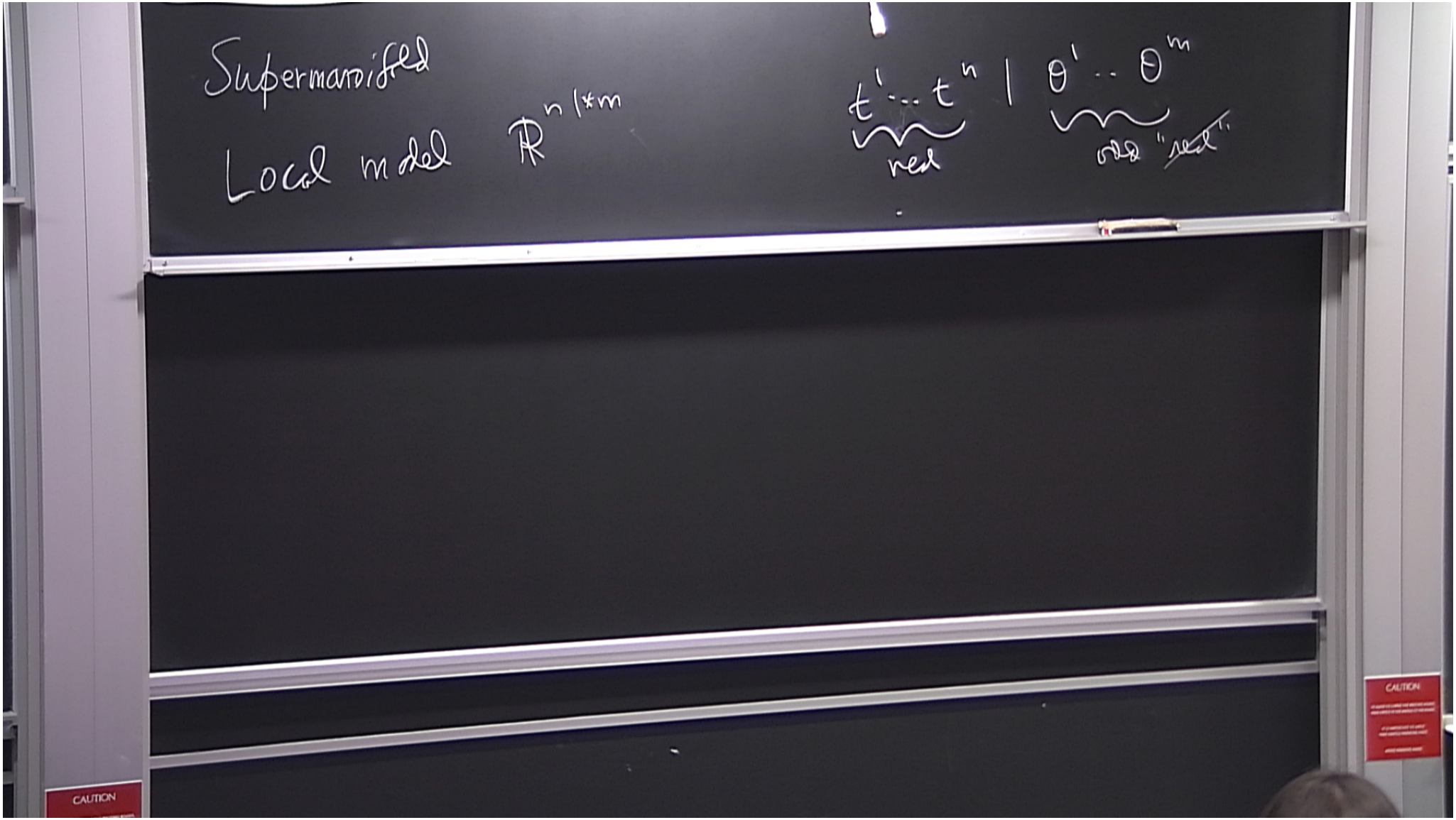

$\mathbb{R}^n$
$M = \bigcup_\alpha \mathcal{U}_\alpha$
$\varphi_\alpha : \mathcal{U}_\alpha \hookrightarrow \mathbb{R}^n$
$(\varphi_\alpha \varphi_\beta^{-1}$ is an isomorphism)
Partition of unity

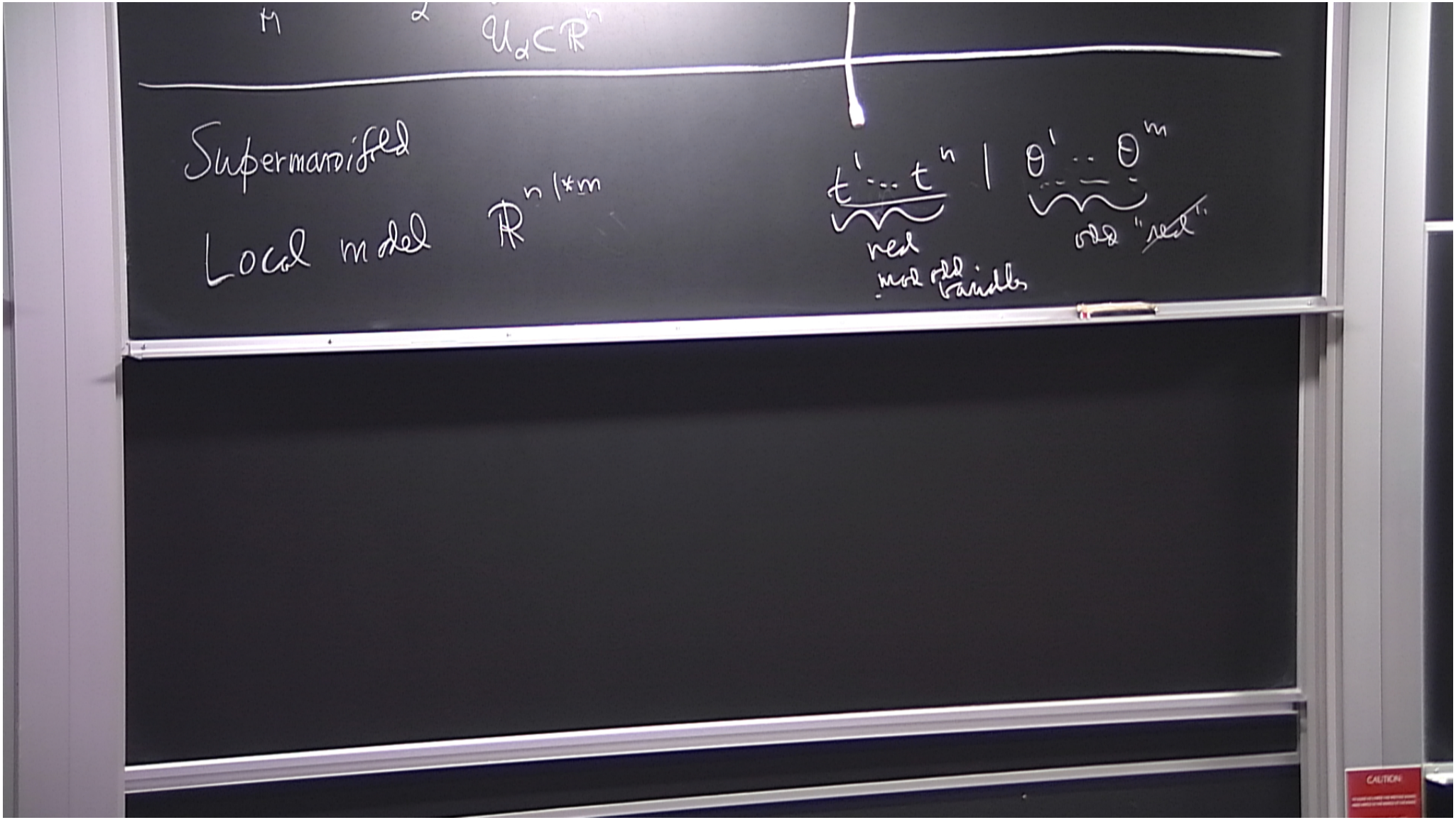

M
$M = \bigcup_\alpha \mathcal{U}_\alpha$
$\varphi_\alpha : \mathcal{U}_\alpha \hookrightarrow \mathbb{R}^n$
$(\varphi_\alpha \varphi_\beta^{-1}$ is an isomorphism)
Partition of unity
A_α supported in $\mathcal{U}_\alpha$
$A_\alpha = 0$ outside $\mathcal{U}_\alpha$
$\sum_\alpha A_\alpha = 1$

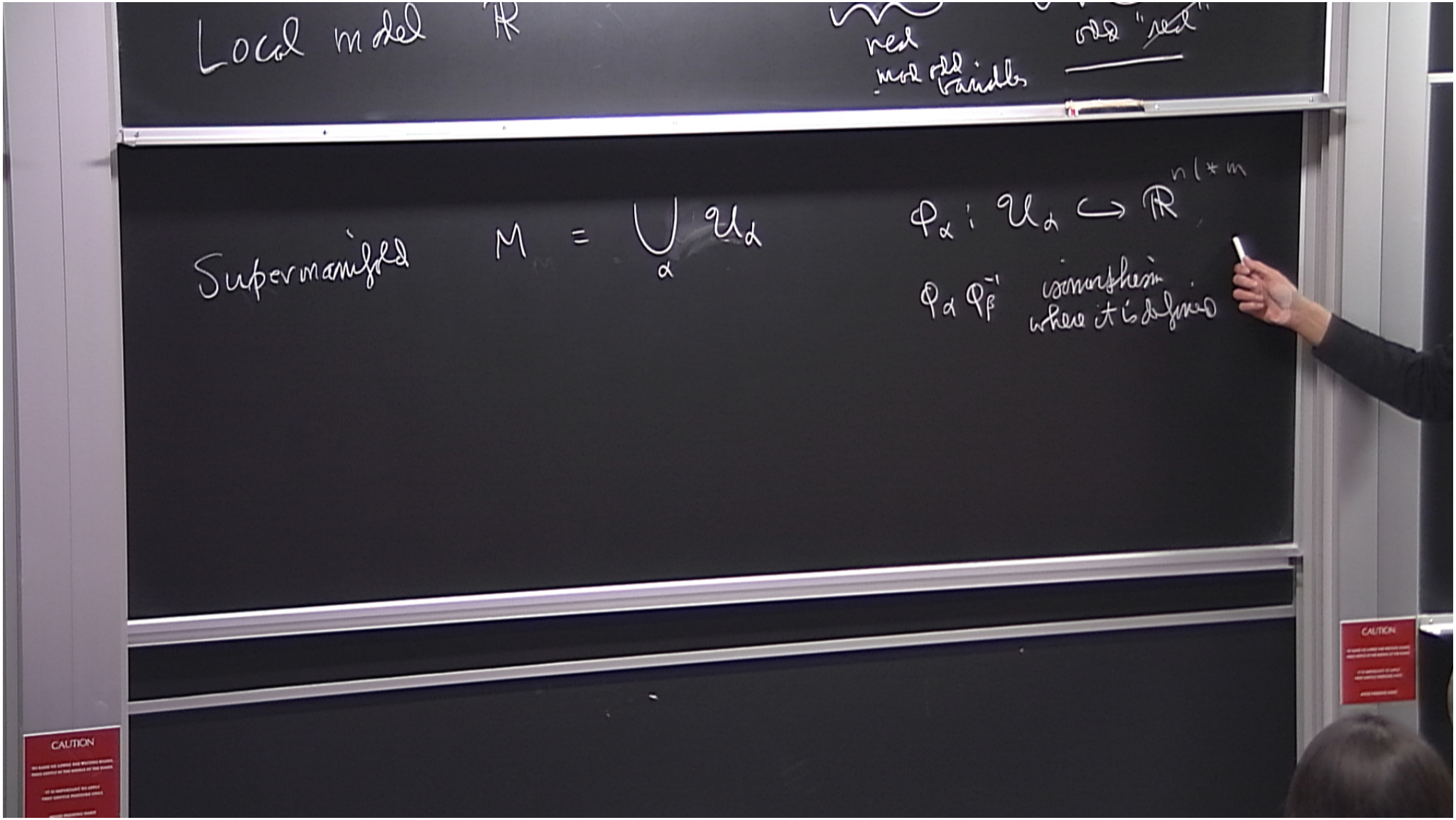
ω = n-form on M

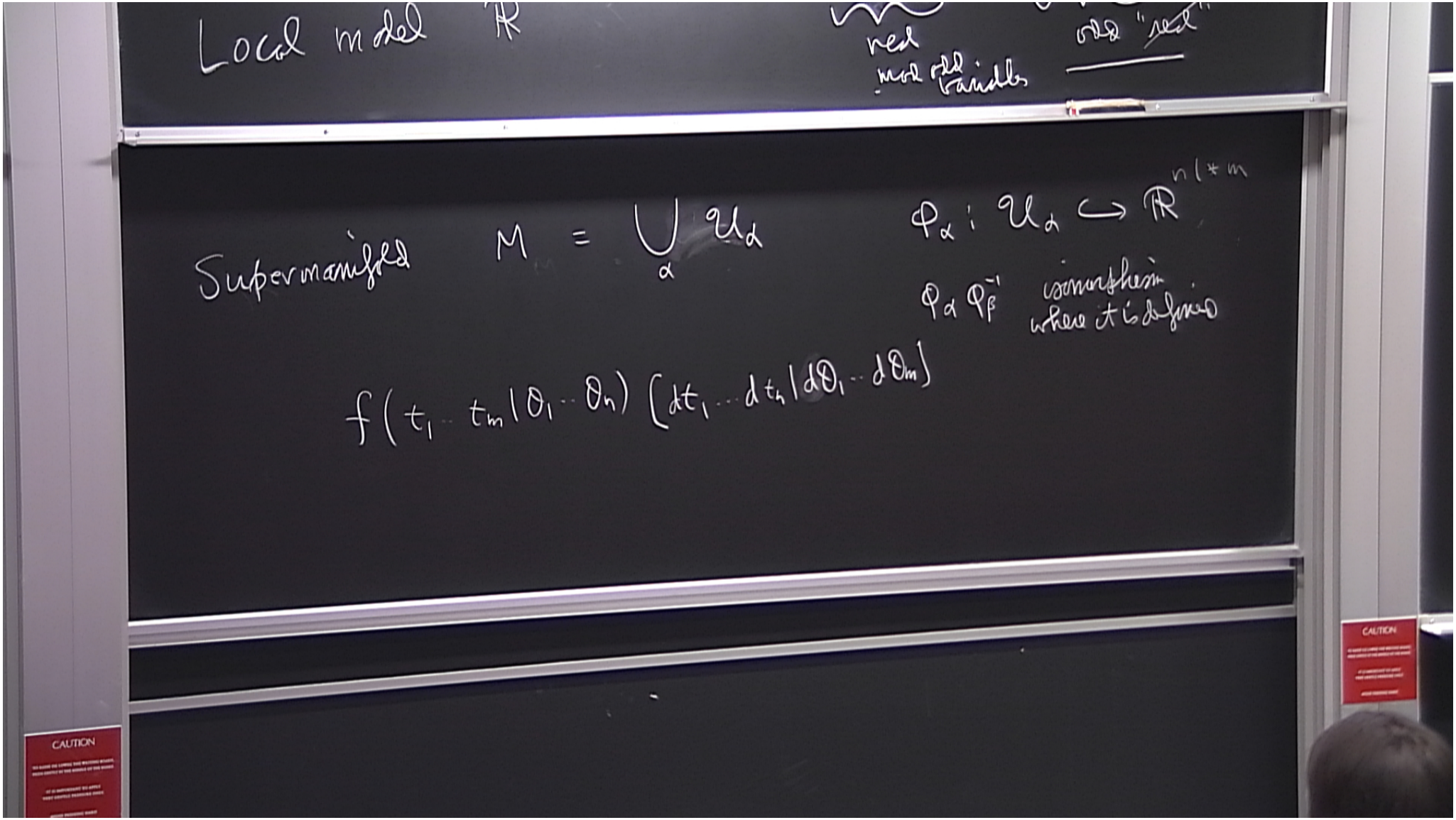
Define

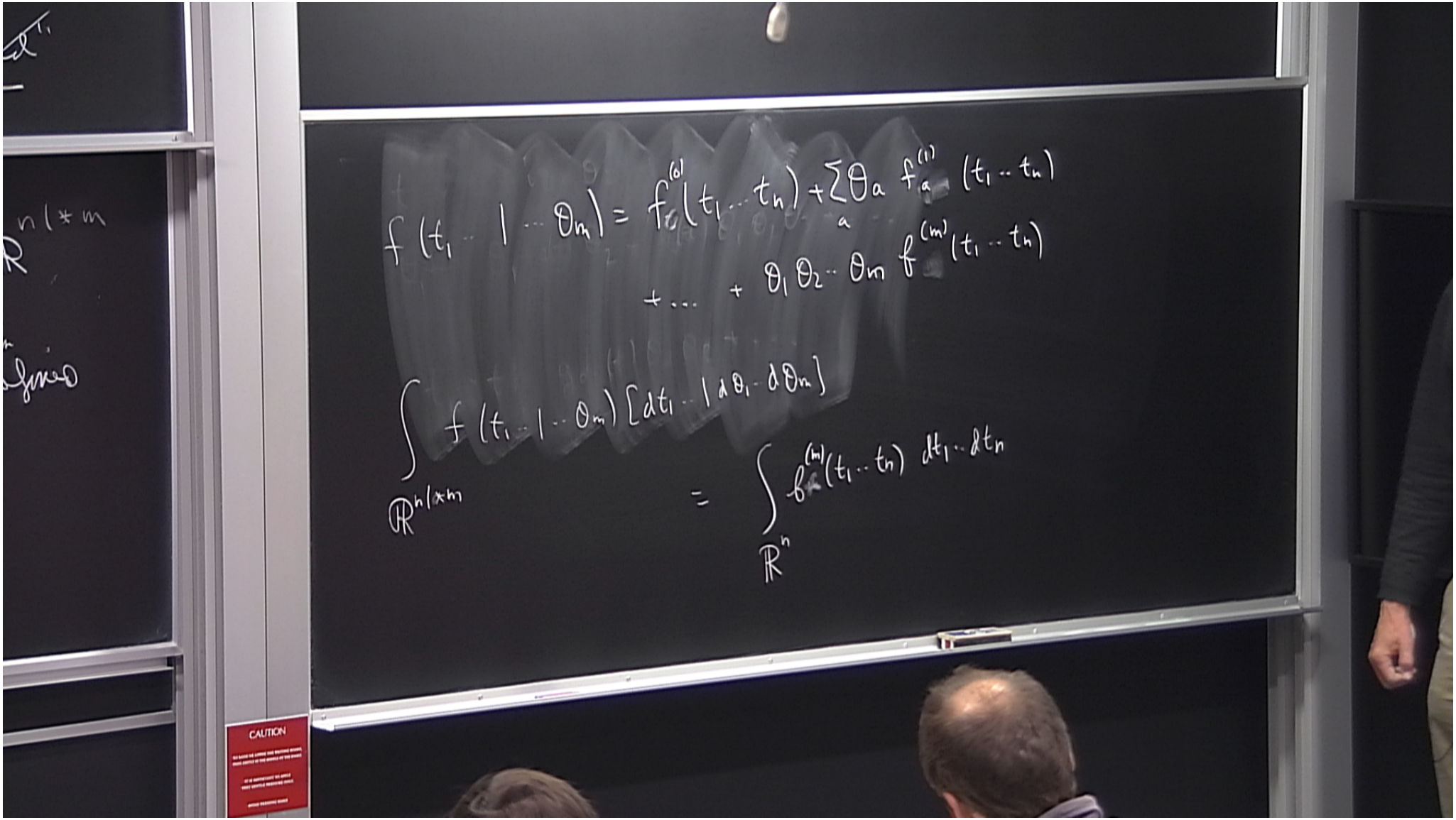
$$\int_M \omega = \sum_\alpha \int_{\mathcal{U}_\alpha \subset \mathbb{R}^n} A_\alpha \omega$$

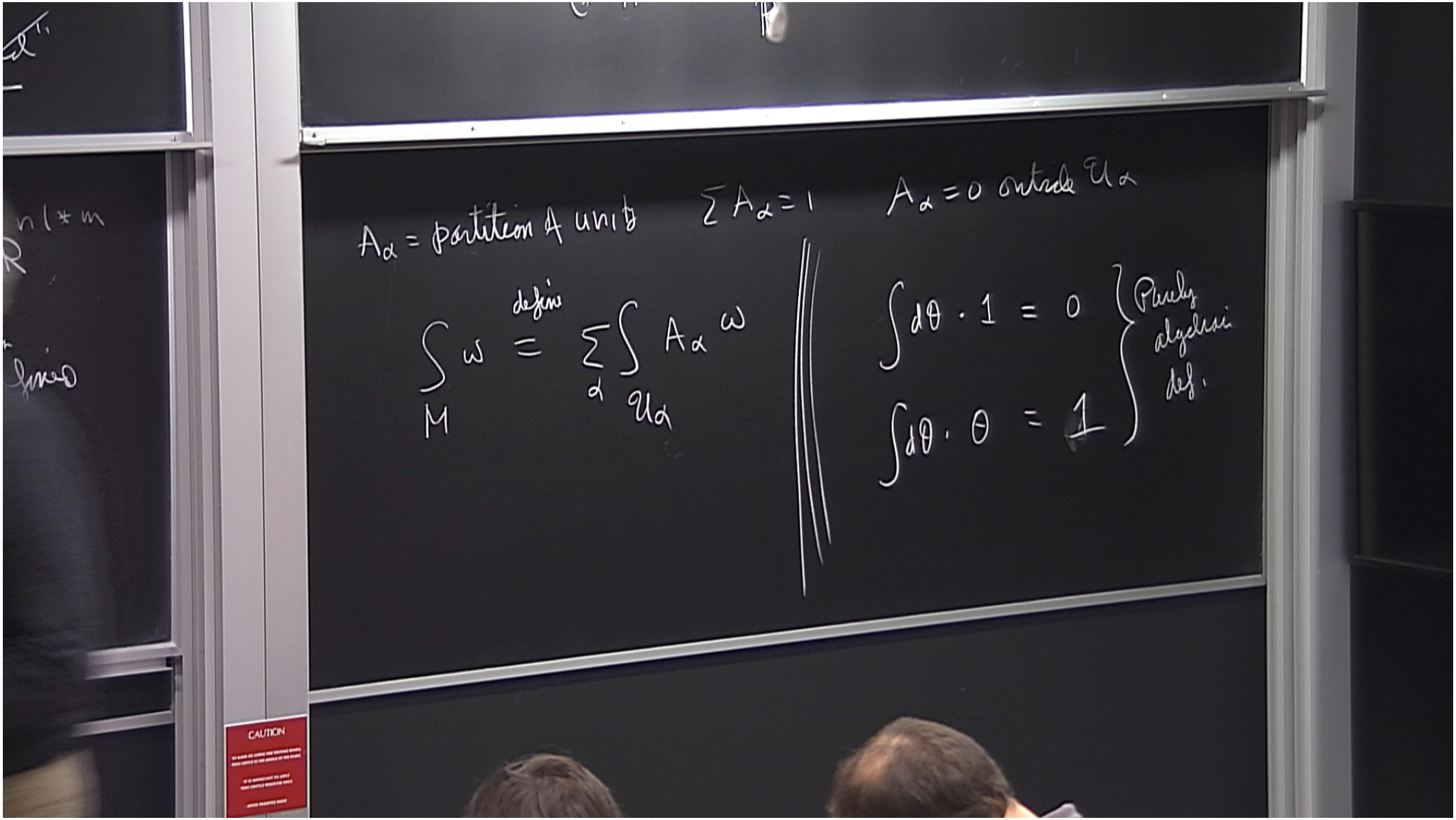

Supermanifold
Local model $\mathbb{R}^{n|*m}$
$t^1 \dots t^n \mid \theta^1 \dots \theta^m$
real
odd "real"

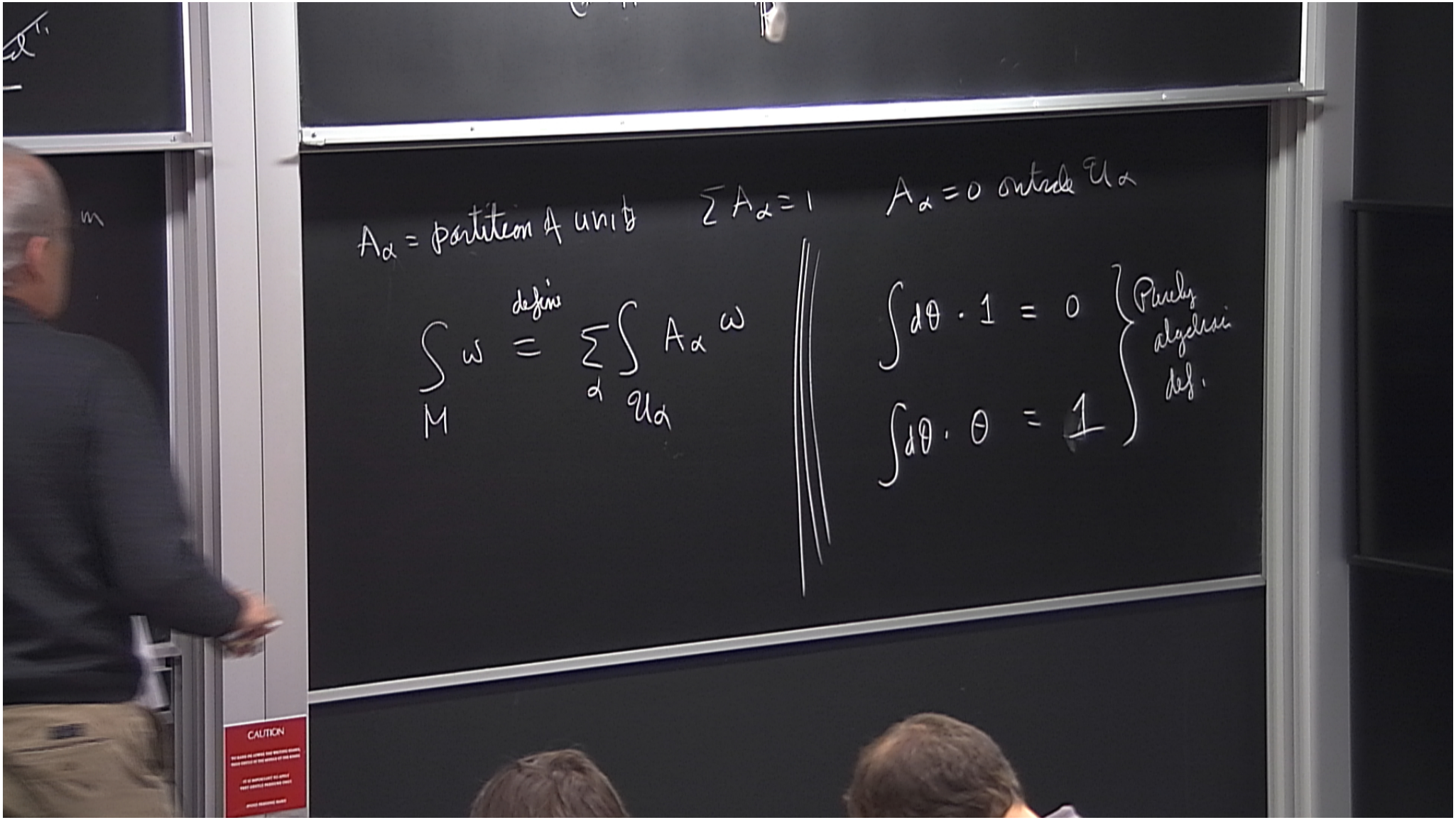

M
$U_\alpha \subset \mathbb{R}^n$
Supermanifold
Local model $\mathbb{R}^{n|*m}$
$t^1 \dots t^n \mid \theta^1 \dots \theta^m$
real
real
CAUTION

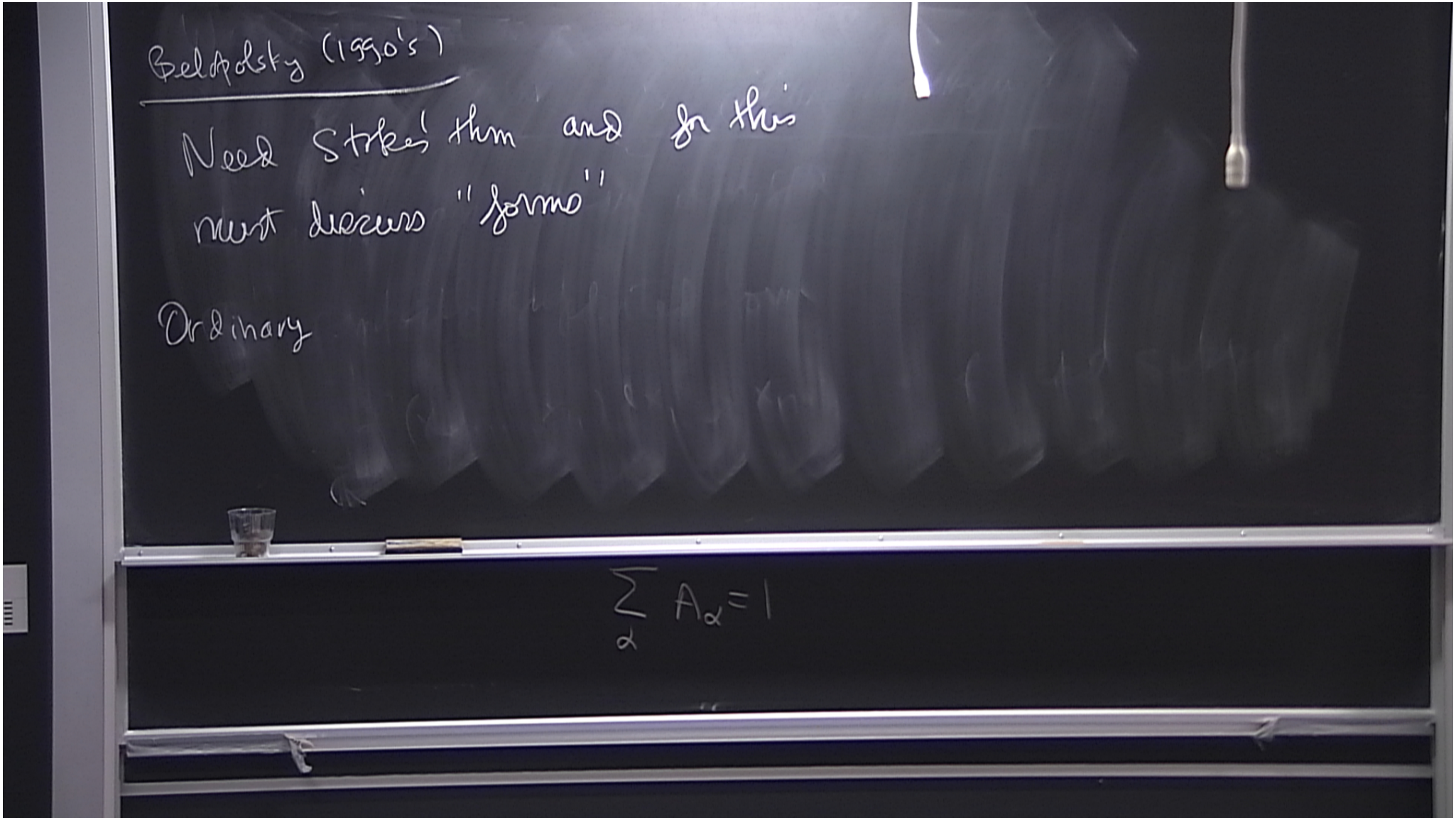

Local model
Supermanifold
$M = \bigcup_\alpha \mathcal{U}_\alpha$
$\varphi_\alpha : \mathcal{U}_\alpha \hookrightarrow \mathbb{R}^{n|m}$
$\varphi_\alpha \varphi_\beta^{-1}$ isomorphism
CAUTION

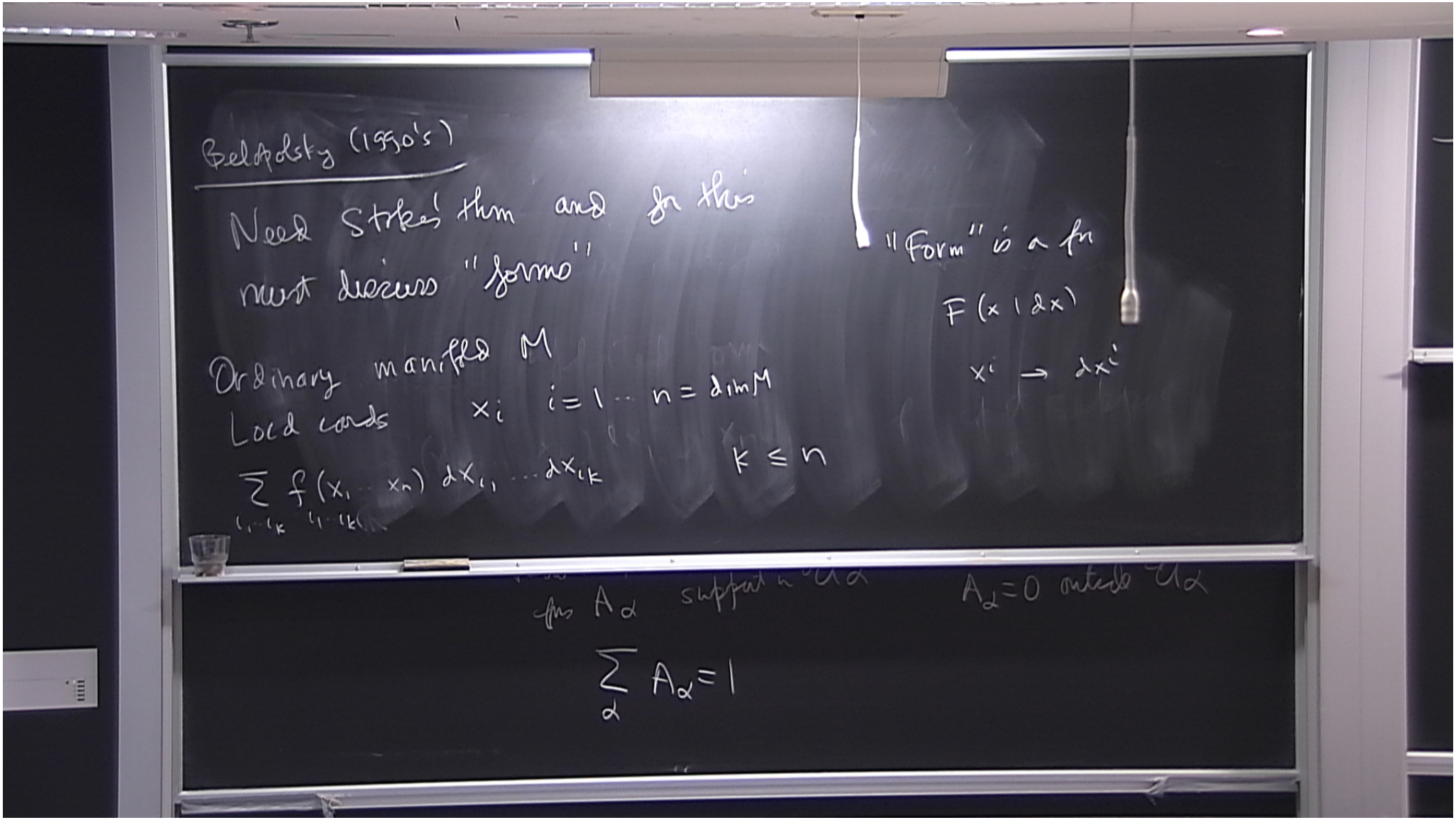

Local model
Supermanifold
$M = \bigcup_\alpha \mathcal{U}_\alpha$
$\varphi_\alpha : \mathcal{U}_\alpha \hookrightarrow \mathbb{R}^{n|m}$
$\varphi_\alpha \varphi_\beta^{-1}$ isomorphism where it is defined
$f(t_1 \dots t_m | \theta_1 \dots \theta_n)\,[dt_1 \dots dt_n | d\theta_1 \dots d\theta_m]$
CAUTION
CAUTION

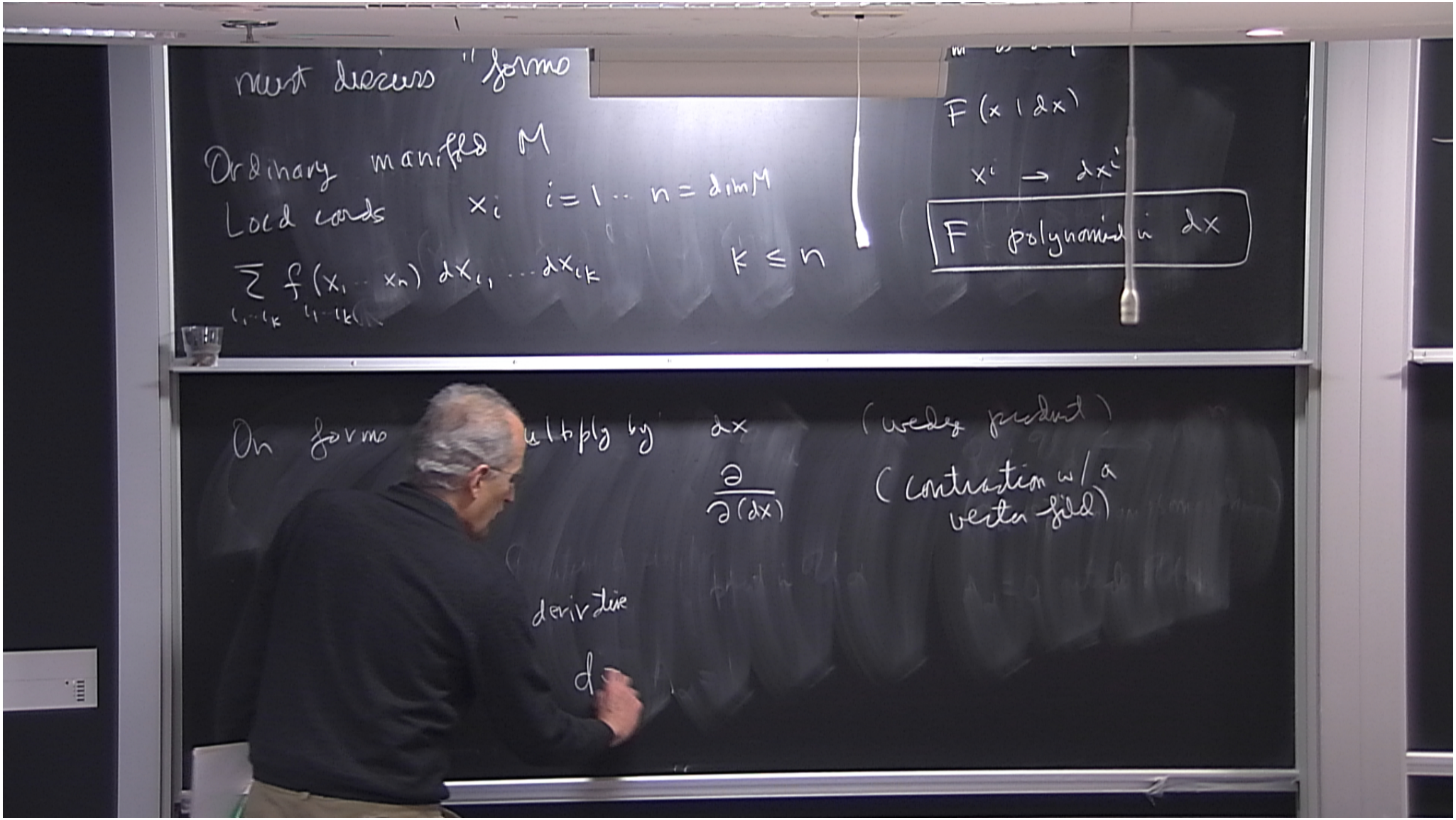

$f(t_1 \cdots | \cdots \theta_m) = f_0^{(0)}(t_1 \cdots t_n) + \sum_a \theta_a f_a^{(1)}(t_1 \cdots t_n)$
$+ \cdots + \theta_1 \theta_2 \cdots \theta_m f^{(m)}(t_1 \cdots t_n)$
$\int_{\mathbb{R}^{n|*m}} f(t_1 \cdots | \cdots \theta_m) [dt_1 \cdots | d\theta_1 \cdots d\theta_m]$
$= \int_{\mathbb{R}^n} f^{(m)}(t_1 \cdots t_n)\, dt_1 \cdots dt_n$
CAUTION

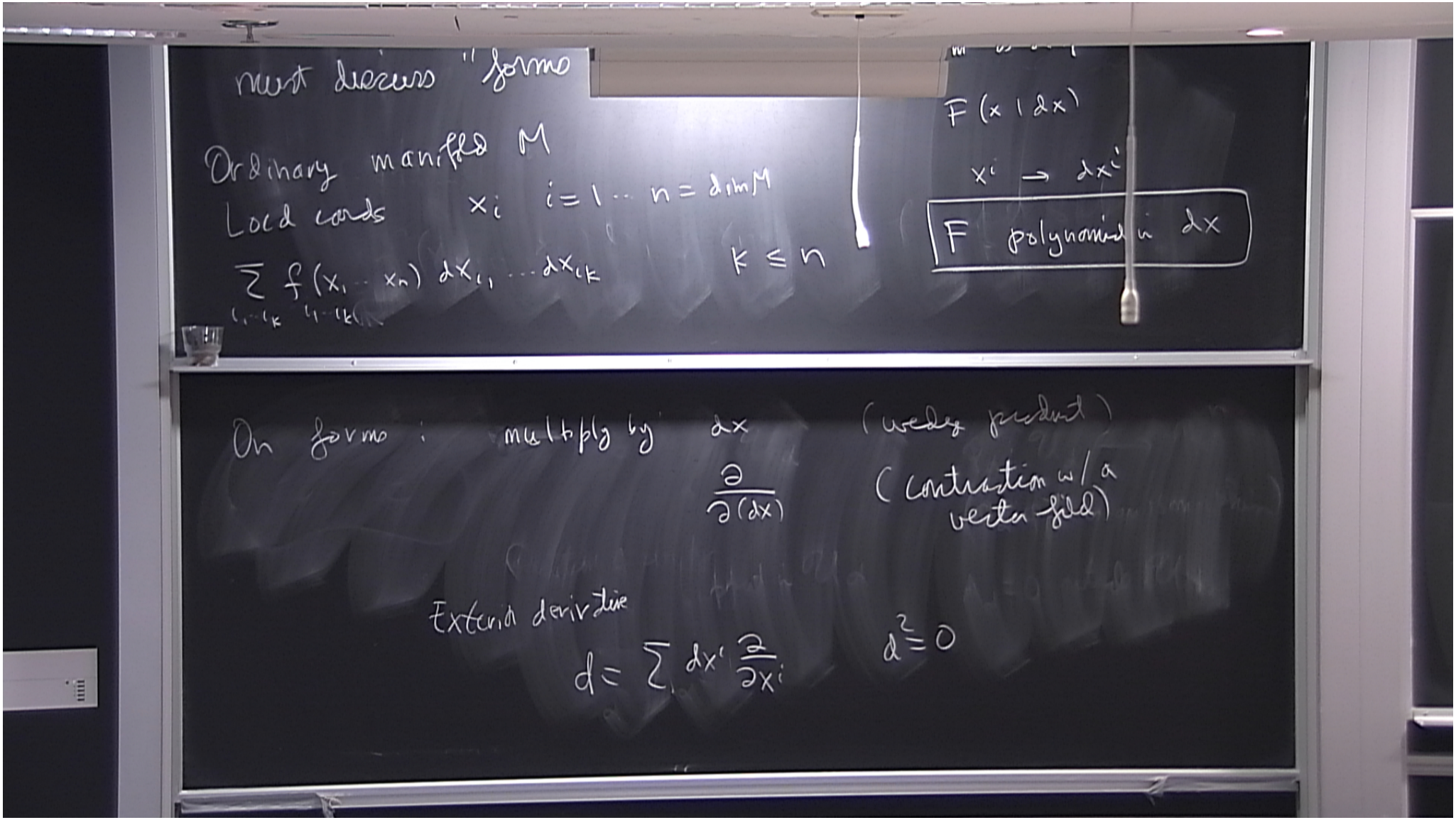

A_α = partition of unity
$\sum A_\alpha = 1$
$A_\alpha = 0$ outside U_α
$\int_M \omega \overset{\text{defin}}{=} \sum_\alpha \int_{U_\alpha} A_\alpha \omega$
$\int d\theta \cdot 1 = 0$
$\int d\theta \cdot \theta = 1$
Purely algebraic def.
CAUTION

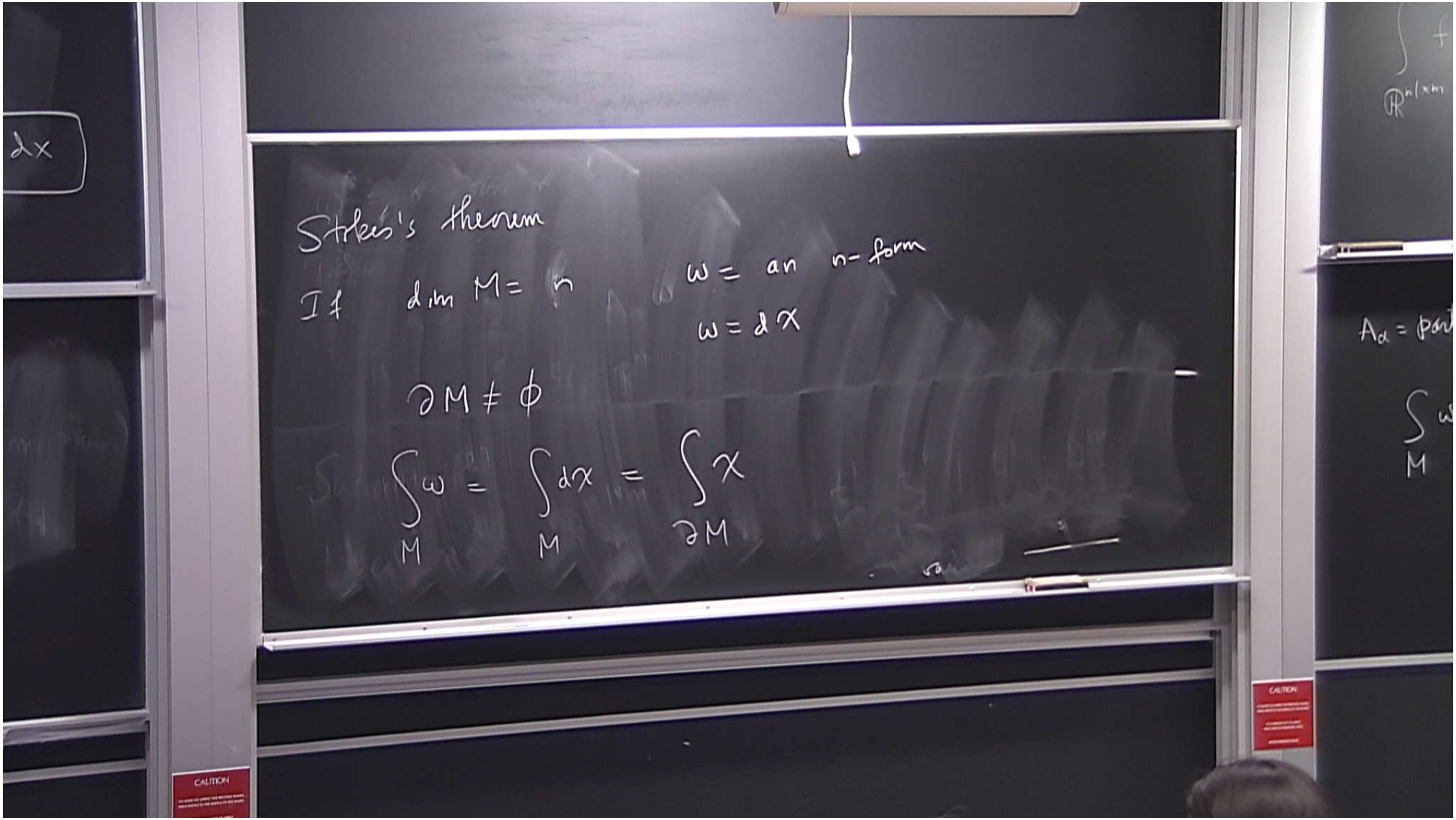

A_α = partition of unity
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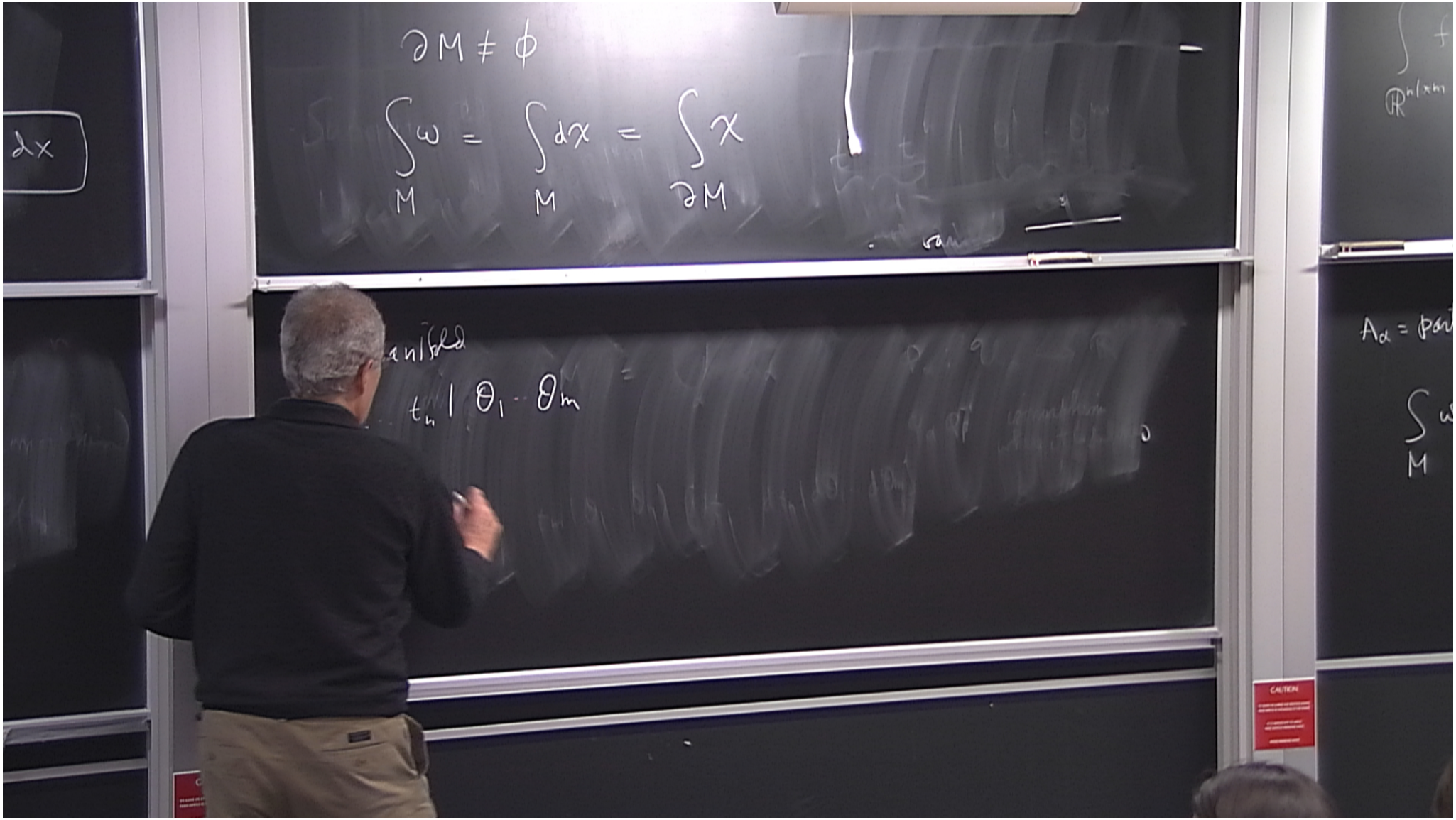

Beldadsky (1990's)
Need Stokes' thm and so this
must discuss "forms"
Ordinary
$\sum_\alpha A_\alpha = 1$

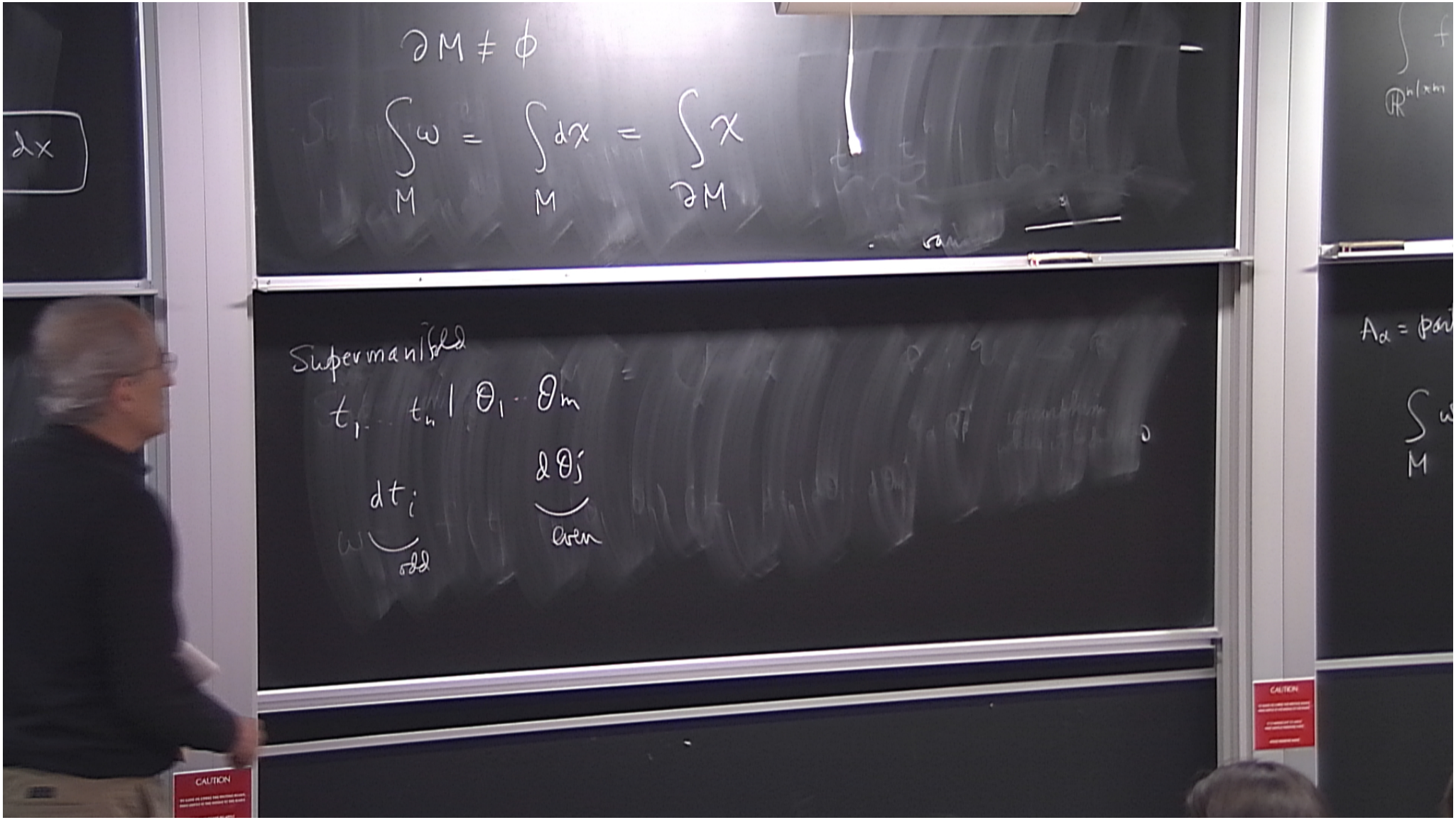

Beldpdsky (1950's)
Need Stokes thm and so this
must discuss "forms"
Ordinary manifold M
Local coords x_i $i = 1 \cdots n = \dim M$
$\sum_{i_1 \cdots i_k} f(x_1 \cdots x_n)\, dx_{i_1} \cdots dx_{i_k}$ $k \leq n$
"Form" is a fn
$F(x \mid dx)$
$x^i \to dx^i$
A_α
$A_\alpha = 0$ outside $\mathcal{U}_\alpha$
$\sum_\alpha A_\alpha = 1$

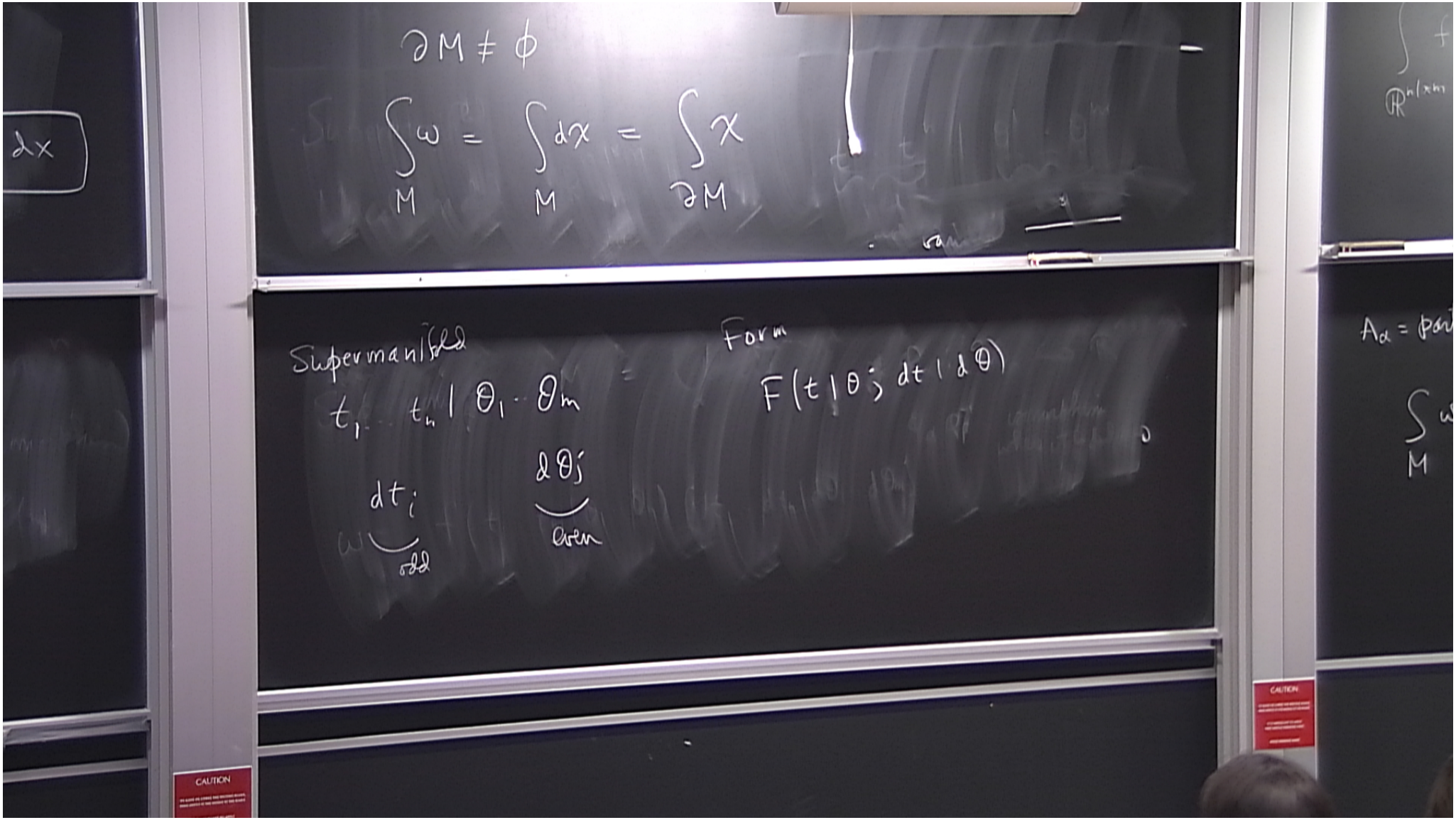

must discuss "forms"
Ordinary manifold M
Local coords x_i $i = 1 \cdots n = \dim M$
$\sum_{i_1 \cdots i_k} f(x_1 \cdots x_n)\, dx_{i_1} \cdots dx_{i_k}$ $k \le n$
$F(x \mid dx)$
$x^i \to dx^i$
F polynomial in dx
On forms multiply by dx (wedge product)
$\frac{\partial}{\partial (dx)}$ (contraction w/ a vector field)
derivative
d

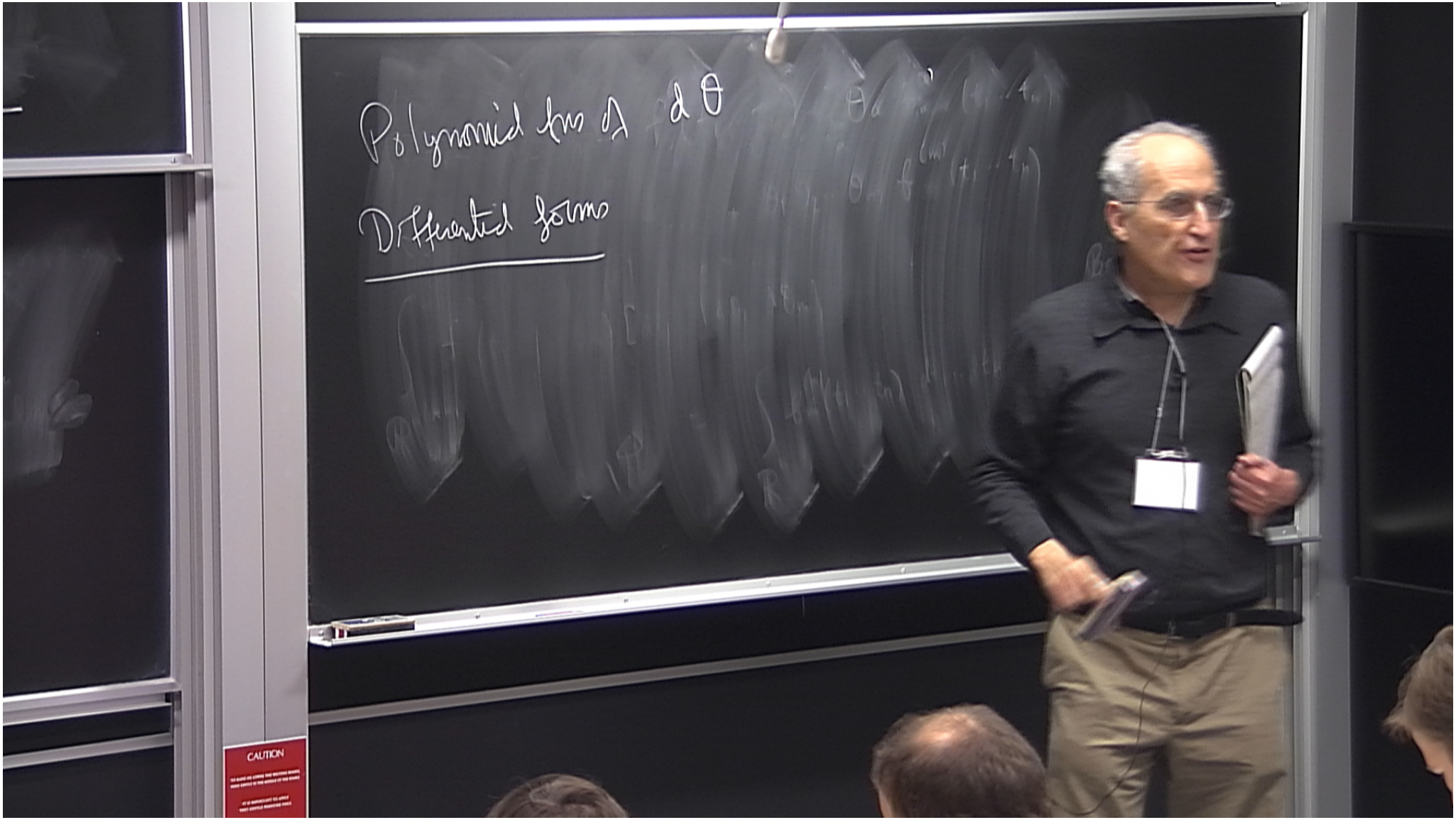

must discuss "forms"
Ordinary manifold M
Local coords x_i $i = 1 \cdots n = \dim M$
$\sum_{i_1 \cdots i_k} f(x_1 \cdots x_n)\, dx_{i_1} \cdots dx_{i_k}$ $k \leq n$
$F(x \mid dx)$
$x^i \to dx^i$
F polynomial in dx
On forms: multiply by dx (wedge product)
$\frac{\partial}{\partial (dx)}$ (contraction w/ a vector field)
Exterior derivative
$d = \sum_i dx^i \frac{\partial}{\partial x^i}$ $d^2 = 0$

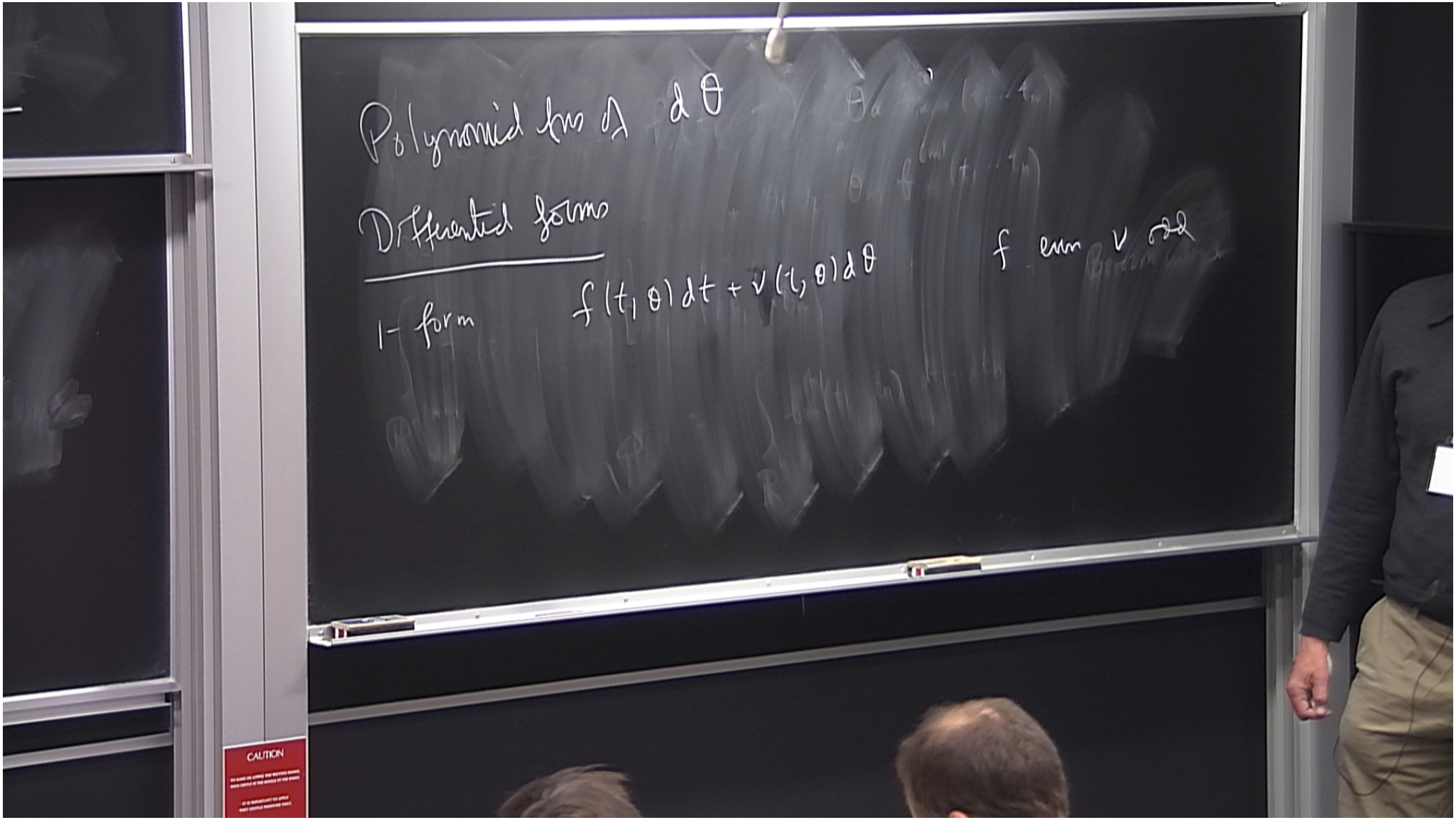

Stokes's theorem
If dim M = n
ω = an n-form
ω = dχ
∂M ≠ ϕ
∫_M ω = ∫_M dχ = ∫_∂M χ

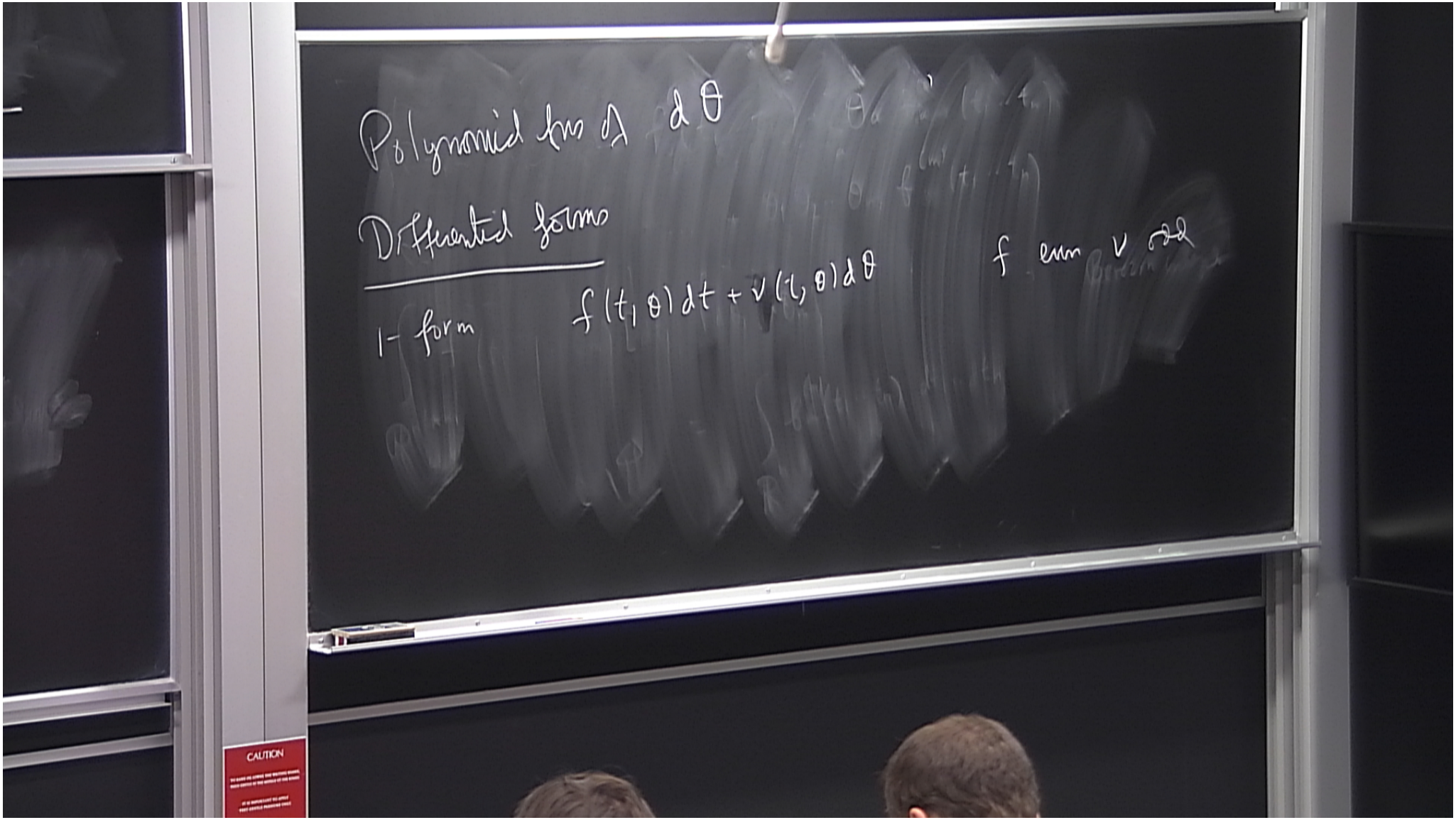

$\partial M \neq \phi$
$\int_M \omega = \int_M dx = \int_{\partial M} x$
manifold
$t_n \mid \theta_1 \cdots \theta_m$

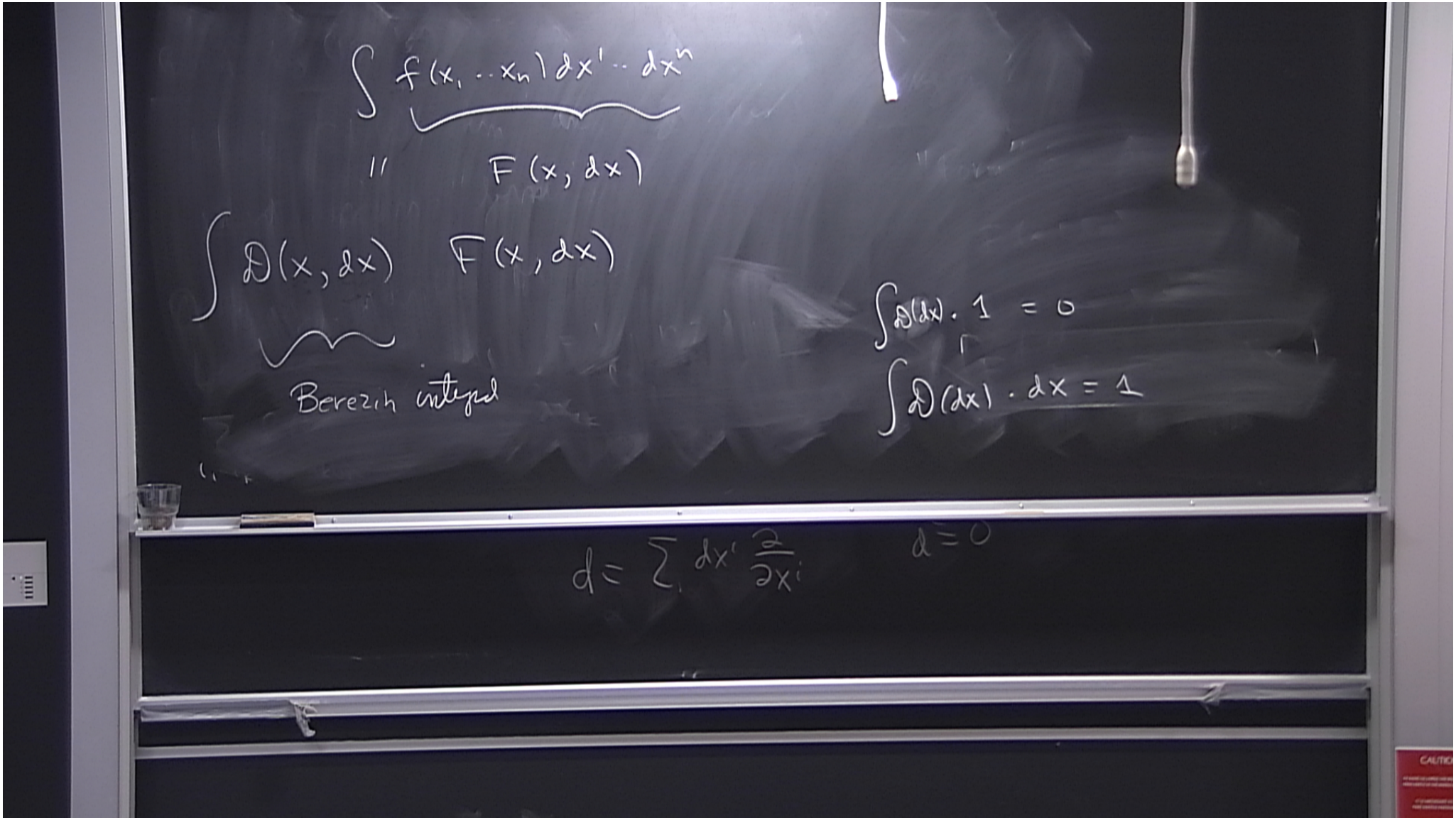

$\partial M \neq \phi$
$\int_M \omega = \int_M dx = \int_{\partial M} x$
Supermanifold
$t_1 \dots t_n \mid \theta_1 \dots \theta_m$
dt_i
odd
$d\theta_j$
even
CAUTION

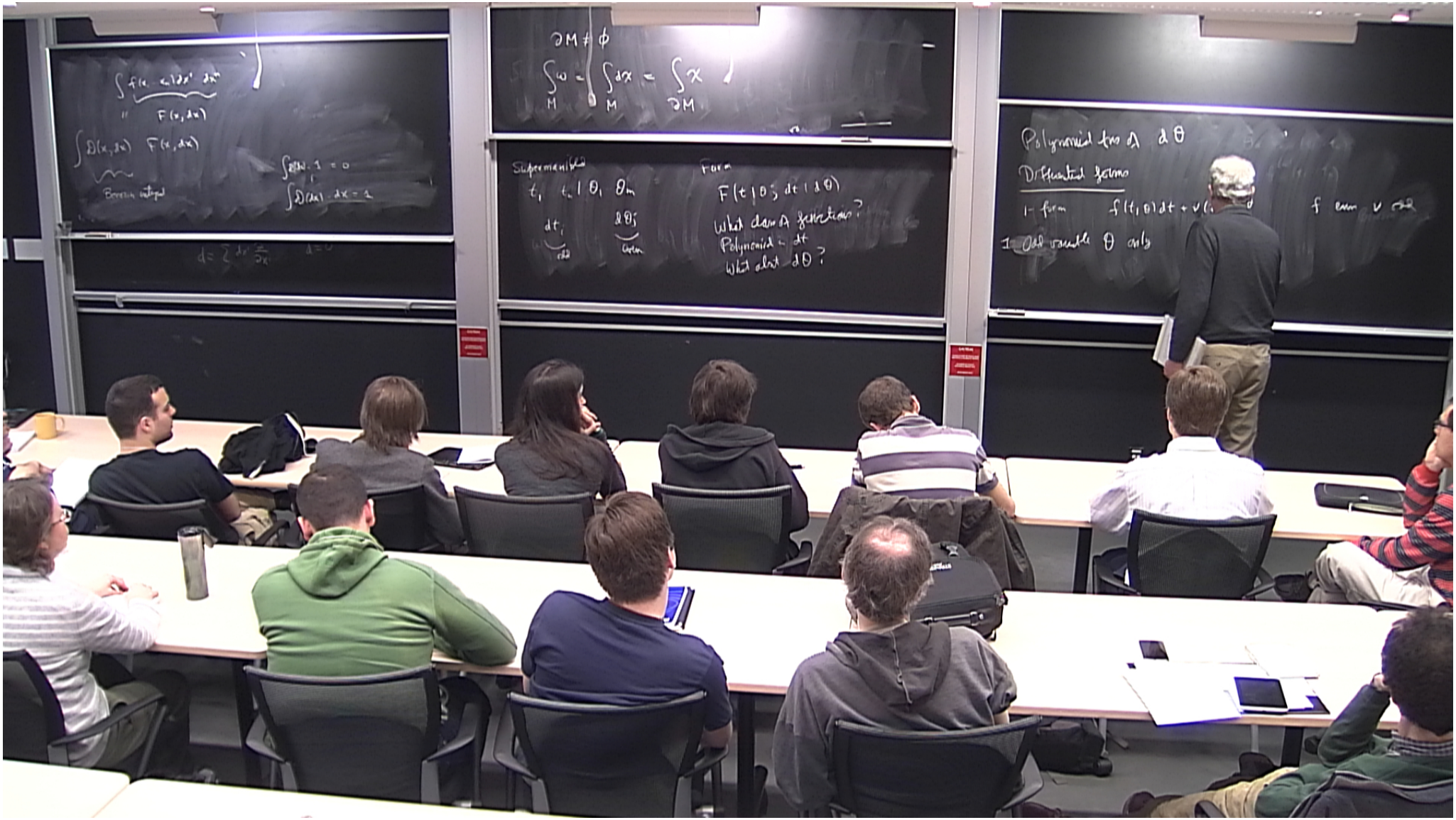

$\partial M \neq \phi$
$\int_M \omega = \int_M dx = \int_{\partial M} x$
Supermanifold
$t_1 \dots t_n \mid \theta_1 \dots \theta_m$
dt_i
$d\theta_j$
Even
Form
$F(t \mid \theta; dt \mid d\theta)$
CAUTION
CAUTION

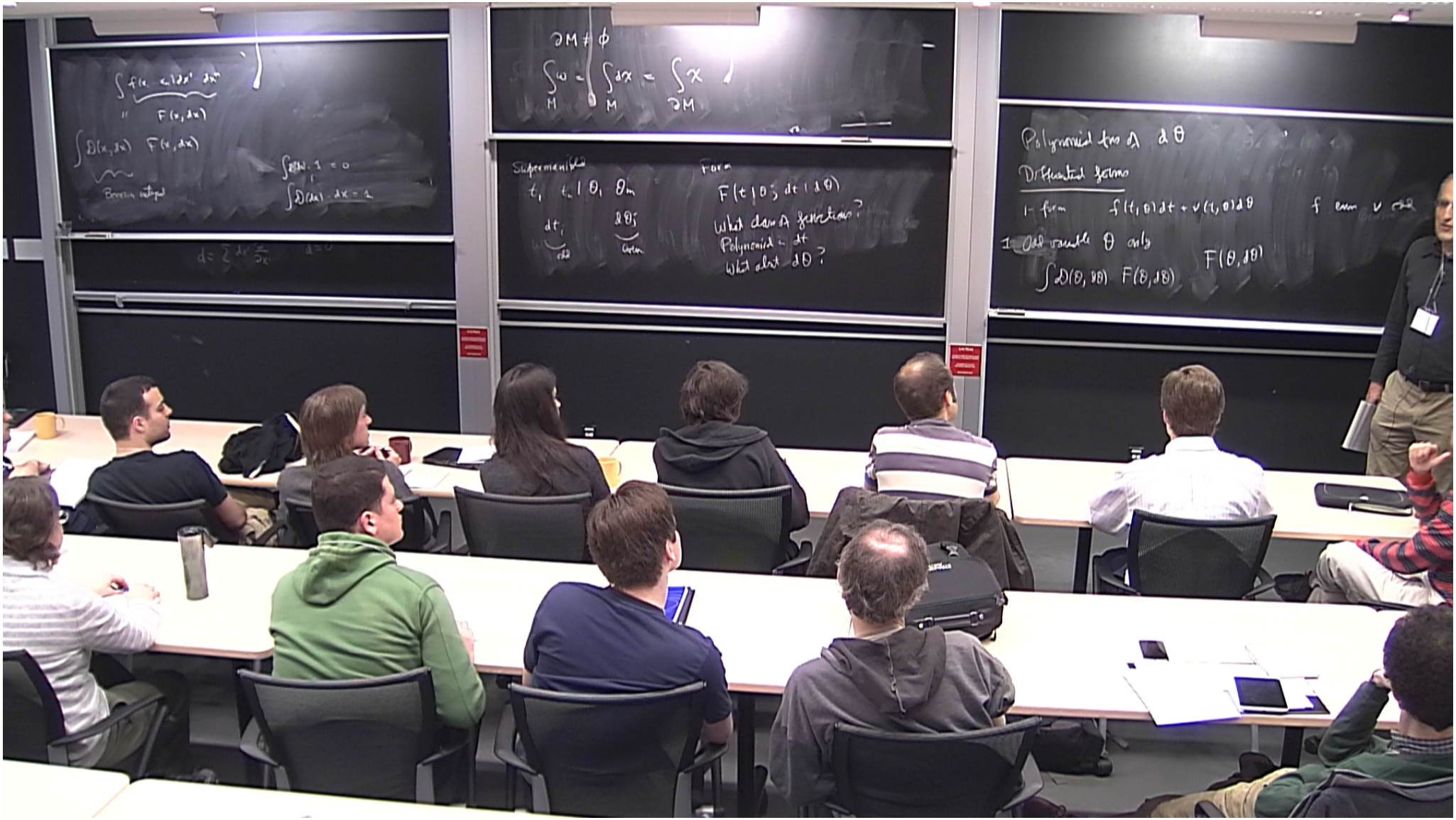

Polynomial ths of λ dθ
Differential forms
CAUTION

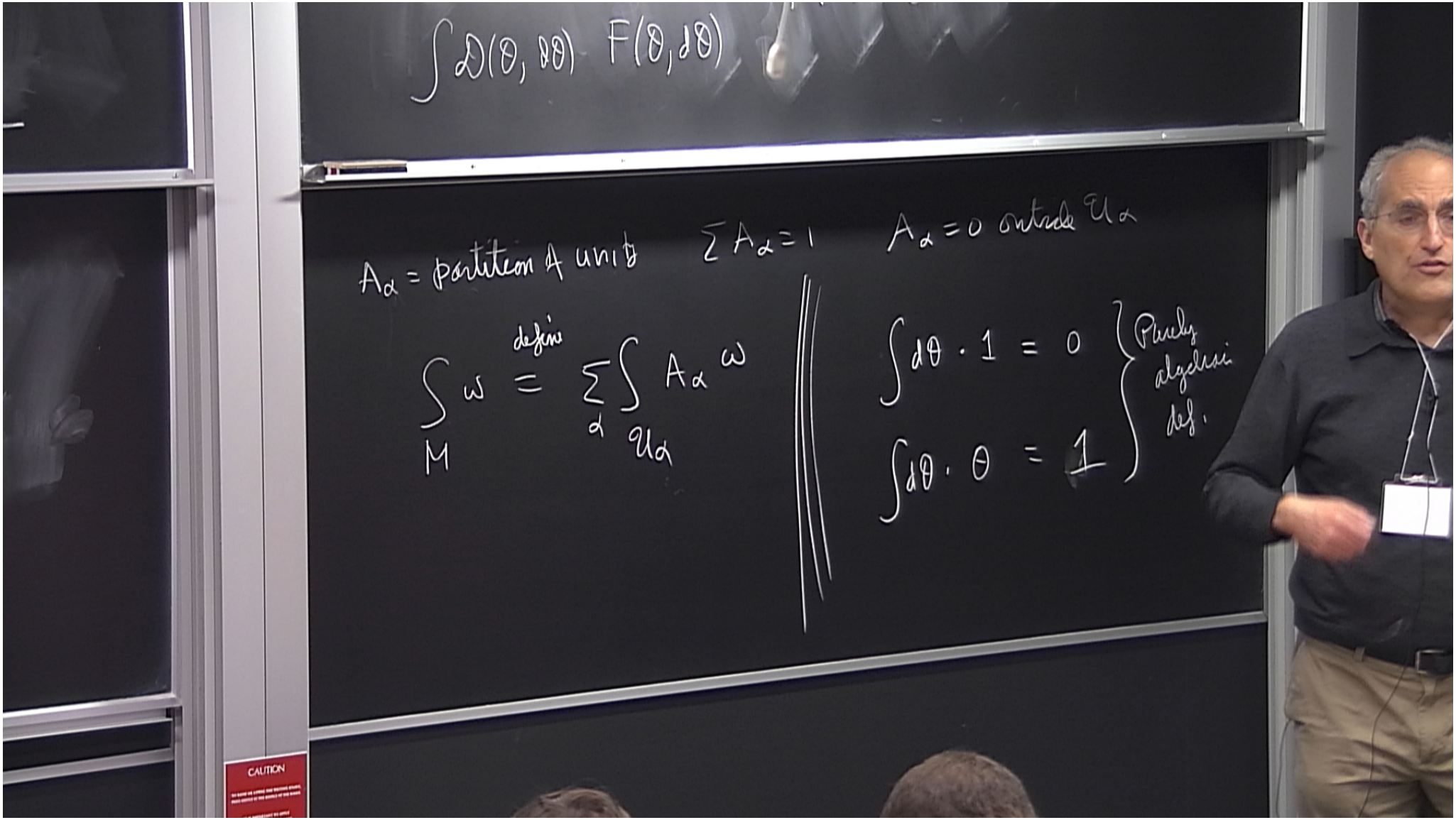

Polynomial terms of λ $d\theta$
Differential forms
1- form $f(t,\theta)dt + v(t,\theta)d\theta$
f and v

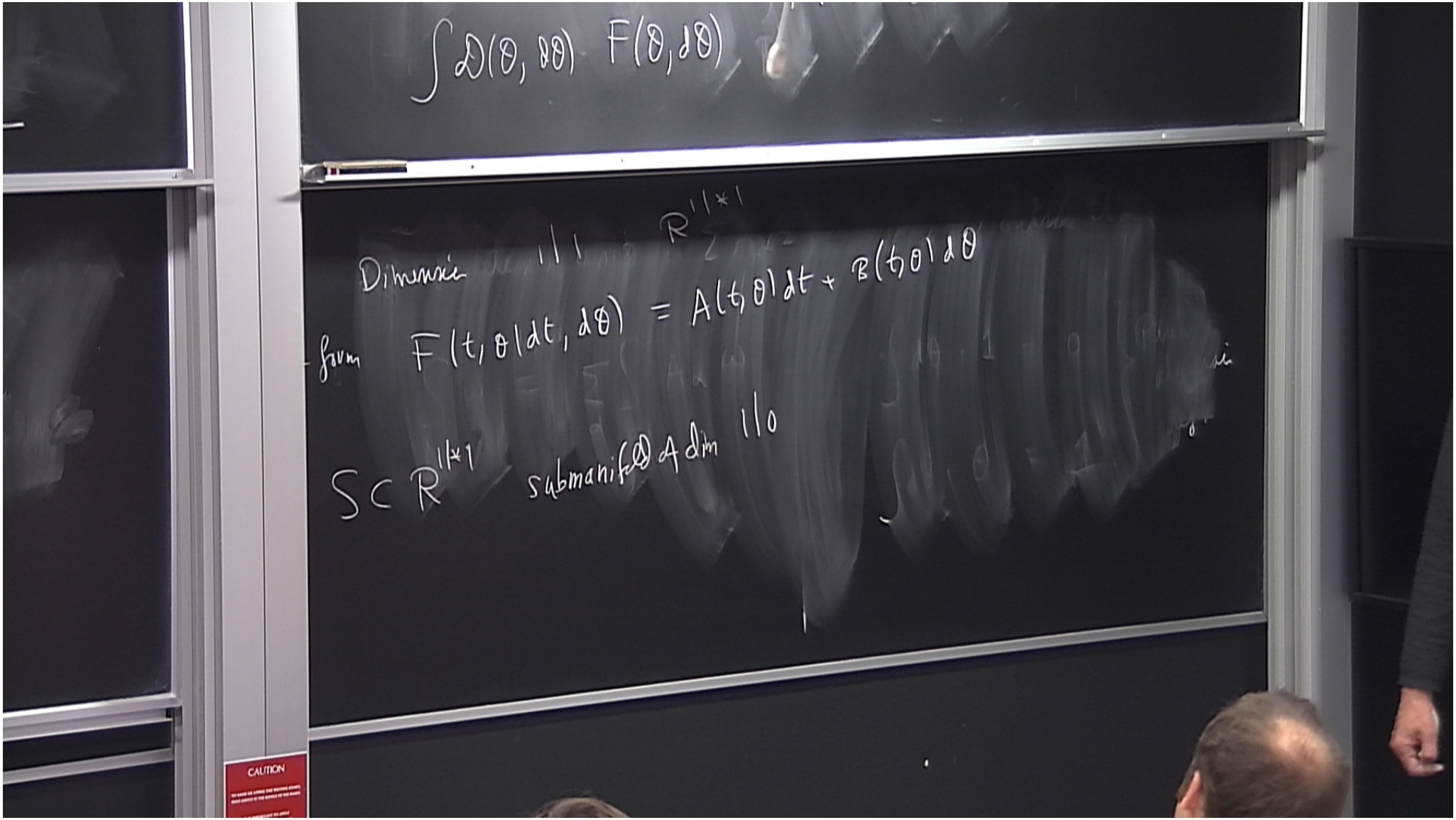

Polynomial forms
Differential forms
1-form
f(t,θ)dt + v(t,θ)dθ
f and v
CAUTION

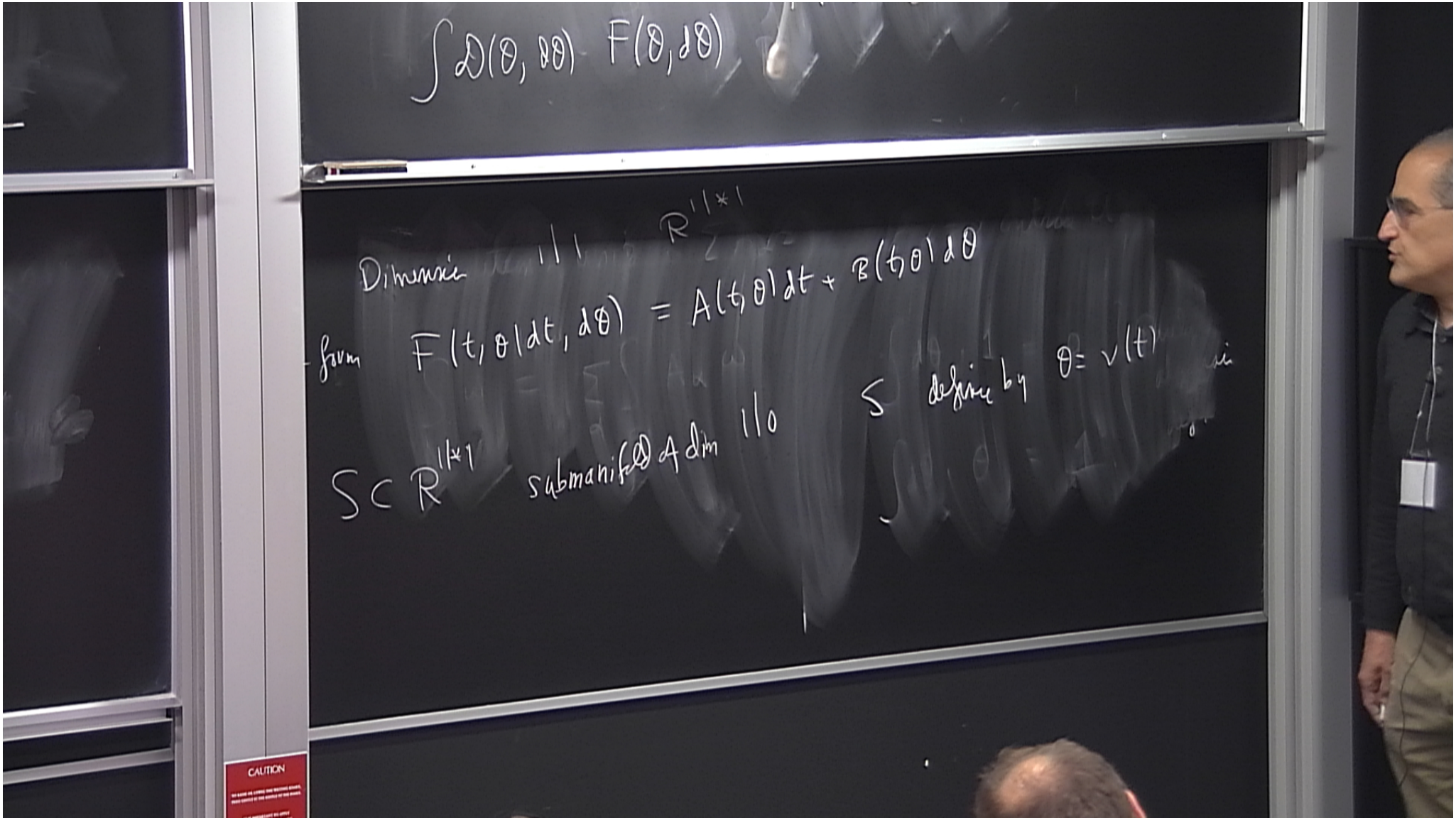

$\int f(x_1 \dots x_n)\, dx^1 \dots dx^n$
$=$ $F(x, dx)$
$\int \mathcal{D}(x, dx)\ F(x, dx)$
Berezin integral
$\int \mathcal{D}(dx) \cdot 1 = 0$
$\int \mathcal{D}(dx) \cdot dx = 1$
$d = \sum_i dx^i \frac{\partial}{\partial x^i}$
$d = 0$

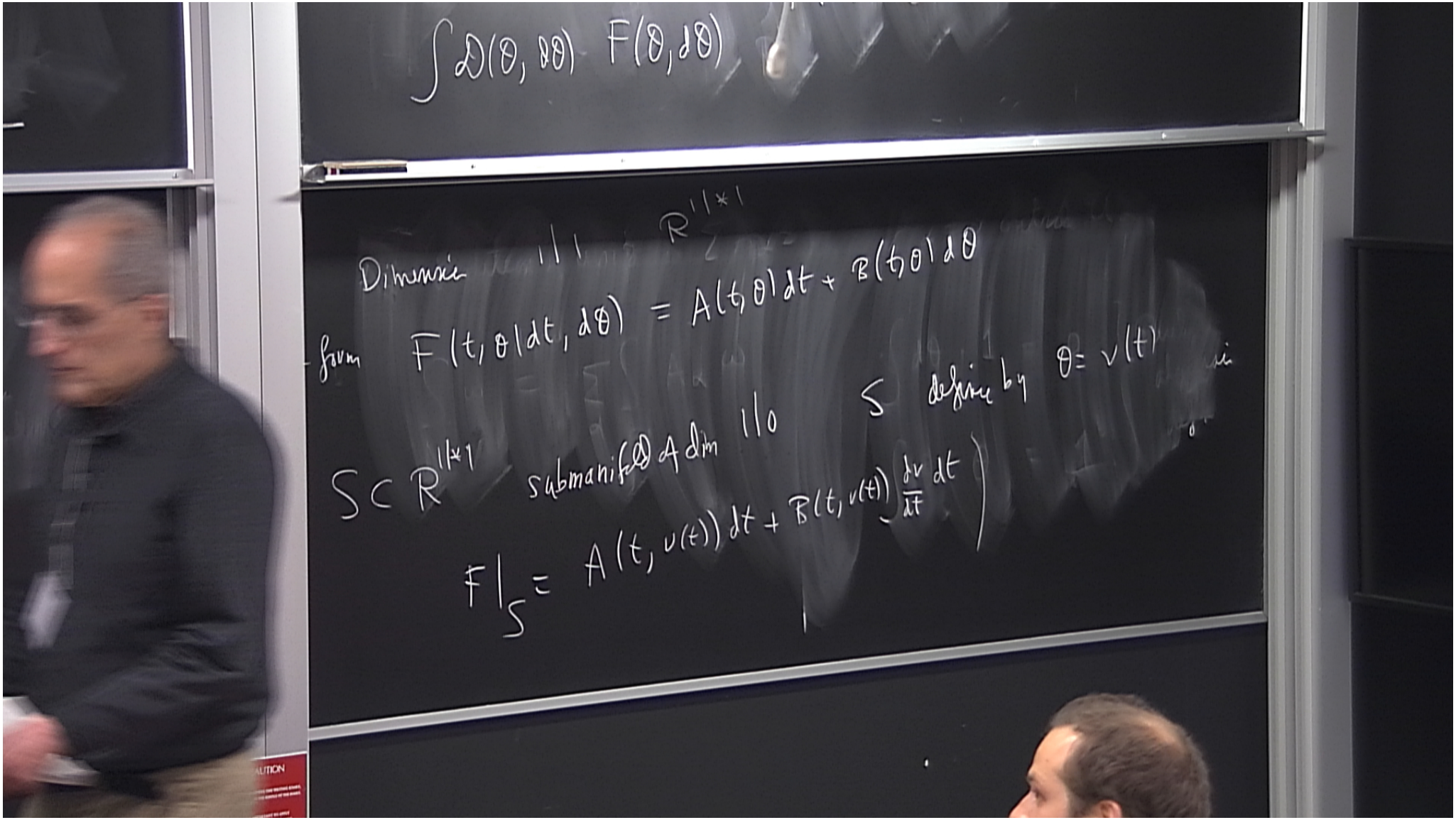

$\partial M \neq \phi$
$\int_M \omega = \int_M d\alpha = \int_{\partial M} \alpha$
Supermanifold
Form
Polynomial in $d\theta$
Differential forms
What class of functions?
Polynomial in dt
What about $d\theta$?

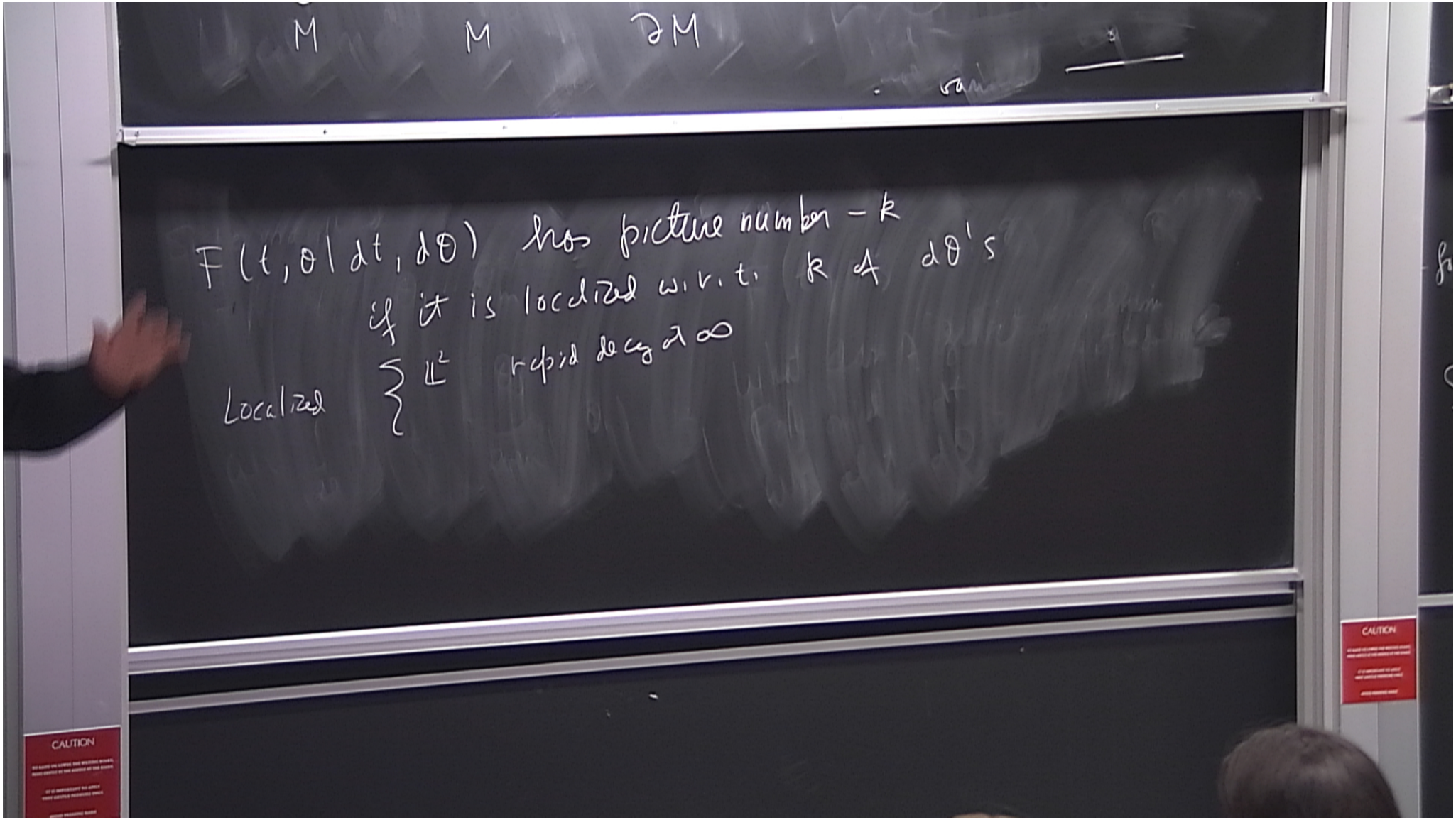

Supermanifold
Form
What class of functions?
Polynomial in dt
What about dθ?
Polynomial in dθ
Differential forms
1-form
Only variable θ only


$\int \mathcal{D}(\theta, d\theta)\ F(\theta, d\theta)$
A_α = partition of unity
$\sum A_\alpha = 1$
$A_\alpha = 0$ outside $\mathcal{U}_\alpha$
$\int_M \omega \overset{\text{defin}}{=} \sum_\alpha \int_{\mathcal{U}_\alpha} A_\alpha\, \omega$
$\int d\theta \cdot 1 = 0$
$\int d\theta \cdot \theta = 1$
Purely algebraic def.
CAUTION


$\int \mathcal{D}(\theta, d\theta)\ F(\theta, d\theta)$
Dimensions $1|1$ $R^{1|*1}$
form $F(t,\theta|dt, d\theta) = A(t,\theta|dt + B(t,\theta|d\theta$
$S \subset R^{1|*1}$ submanifold of dim $1|0$
CAUTION


$\int \mathcal{D}(\theta, d\theta)\ F(\theta, d\theta)$
Dimensions $1|1$ $R^{1|*1}$
form $F(t,\theta|dt, d\theta) = A(t,\theta|dt + B(t,\theta|d\theta$
$S \subset R^{1|*1}$ submanifold dim $1|0$ S defined by $\theta = \nu(t)$
CAUTION


$\int \mathcal{D}(\theta, d\theta)\ F(\theta, d\theta)$
Dimension 1|1 $R^{1|*1}$
form $F(t,\theta|dt, d\theta) = A(t,\theta)dt + B(t,\theta|d\theta$
$S \subset R^{1|*1}$ submanifold of dim 1|0 S defined by $\theta = \nu(t)$
$F|_S = A(t,\nu(t))dt + B(t,\nu(t))\frac{d\nu}{dt}dt$


M
M
∂M
F(t,θ|dt,dθ) has picture number −k
if it is localized w.r.t. k of dθ's
Localized
L²
rapid decay at ∞
CAUTION
CAUTION

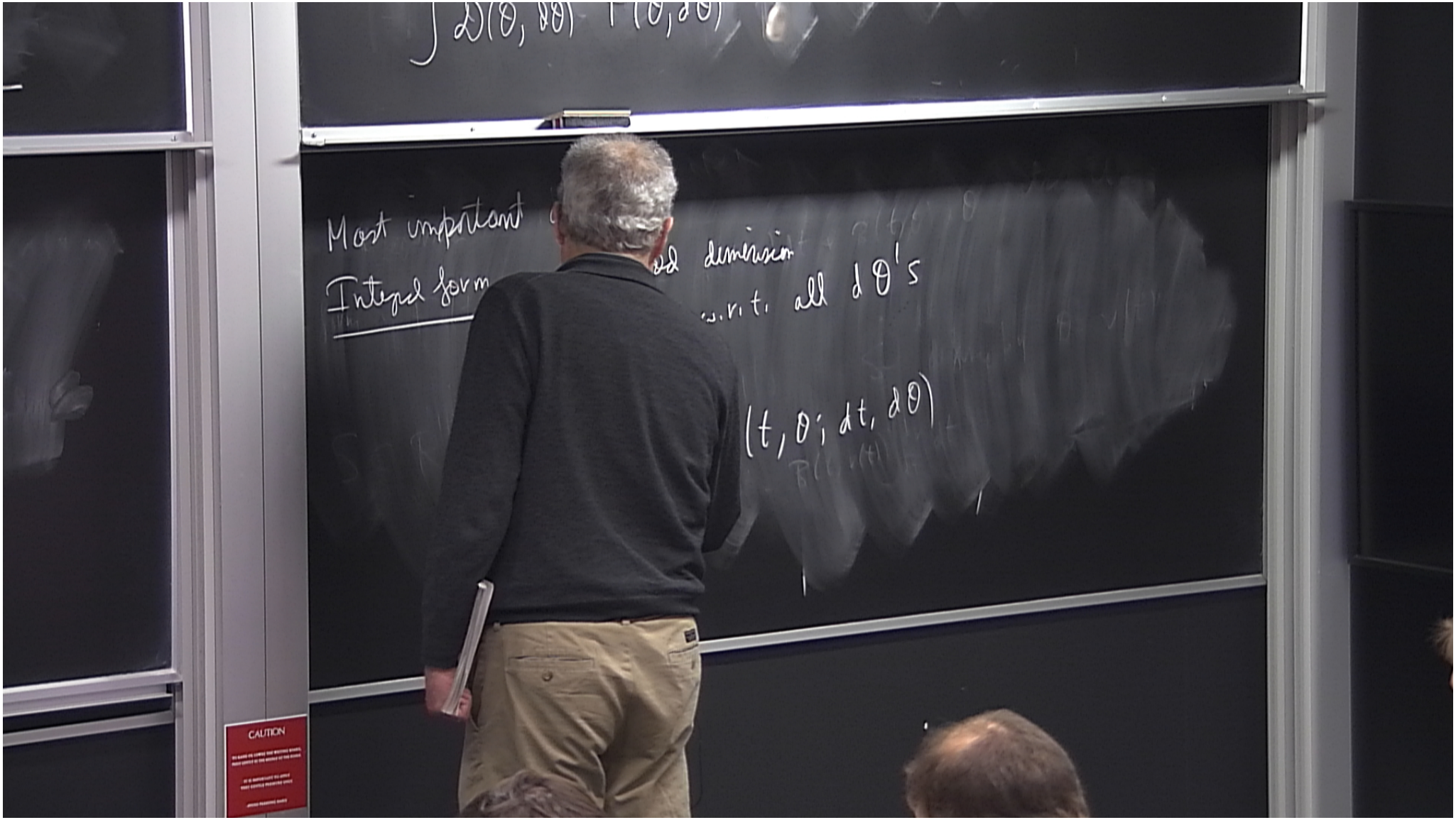
M M ∂M

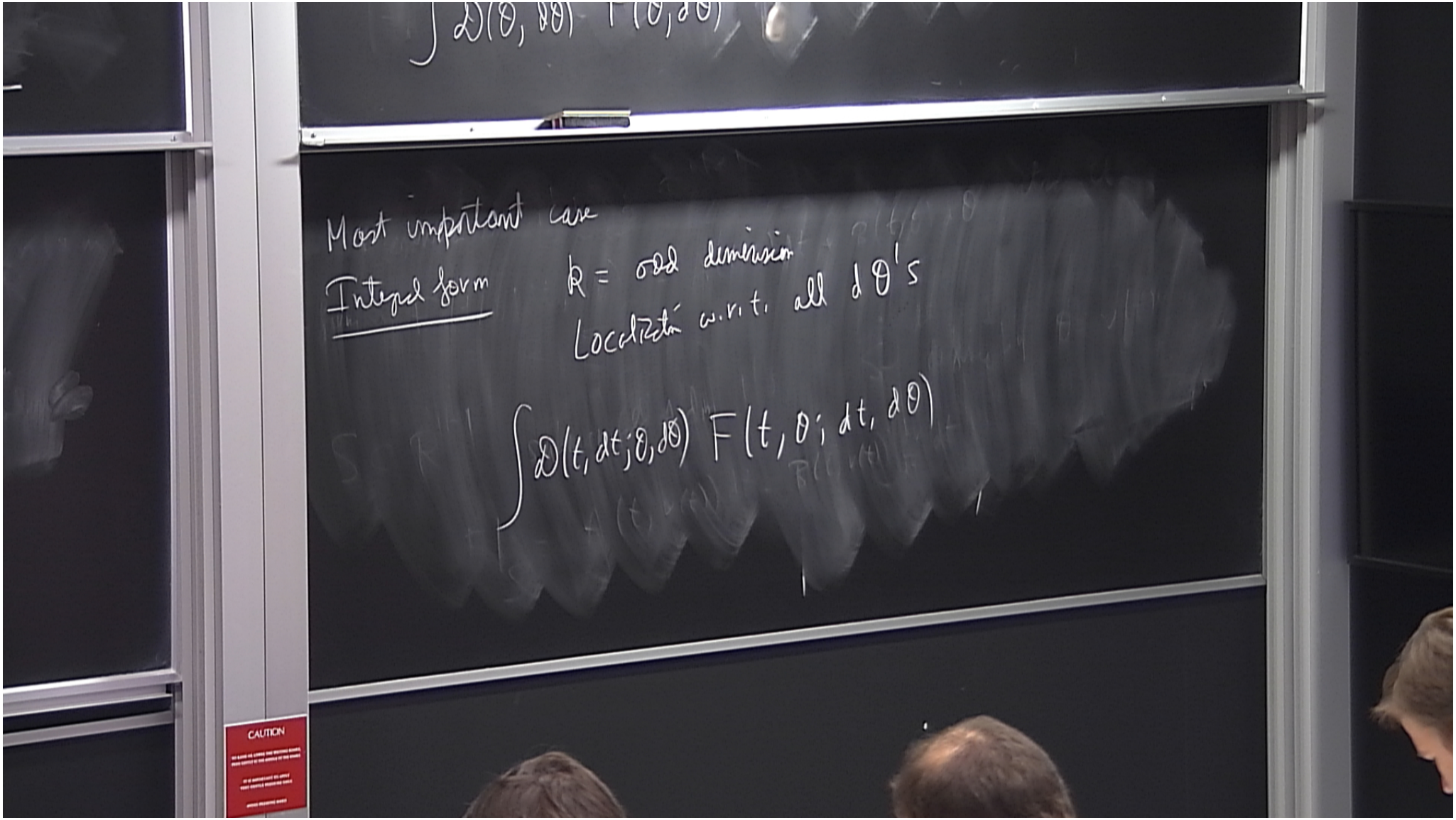
$F(t,\theta|dt,d\theta)$ has picture number $-k$

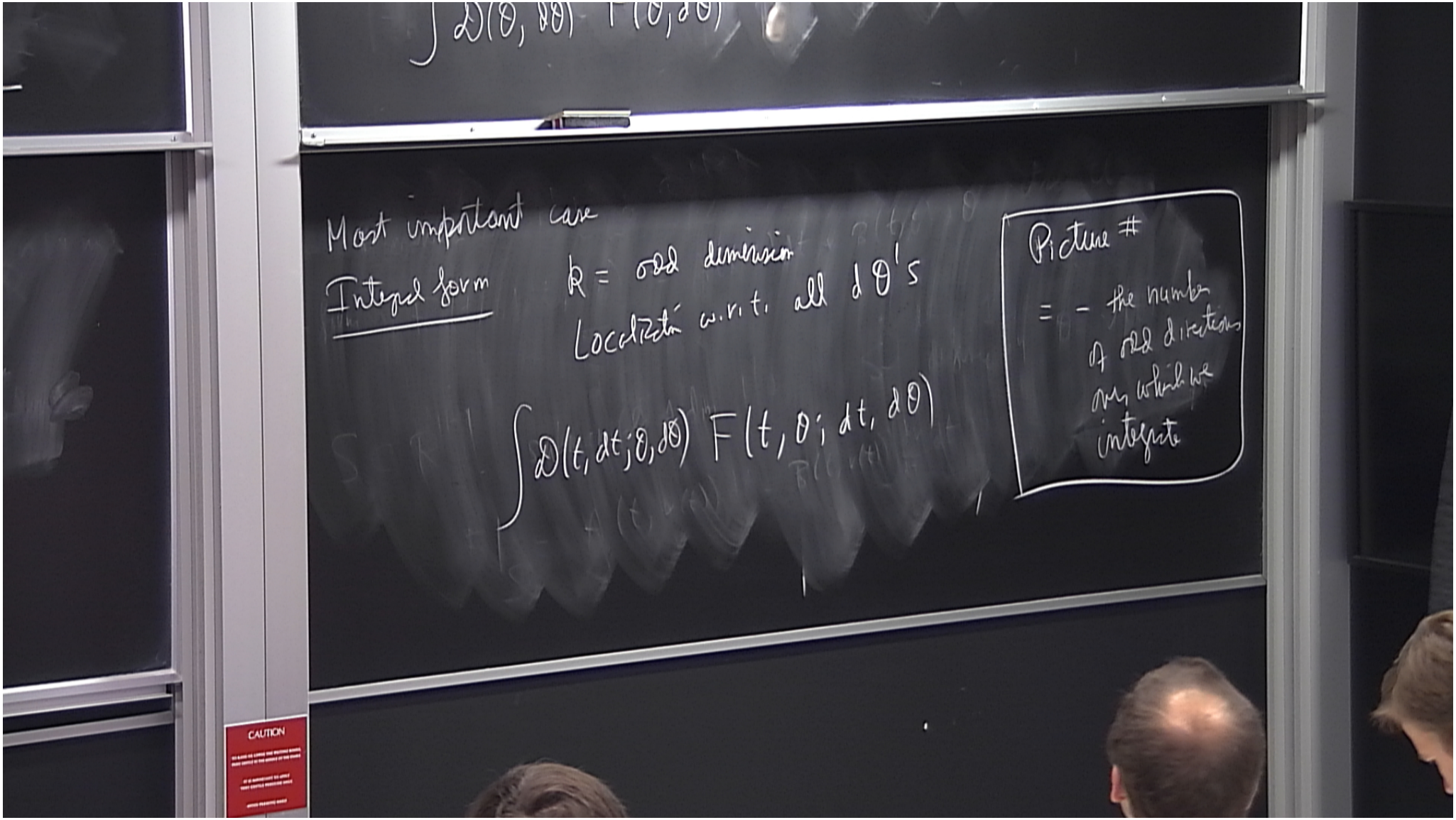
if it is localized w.r.t. k $d\theta$'s

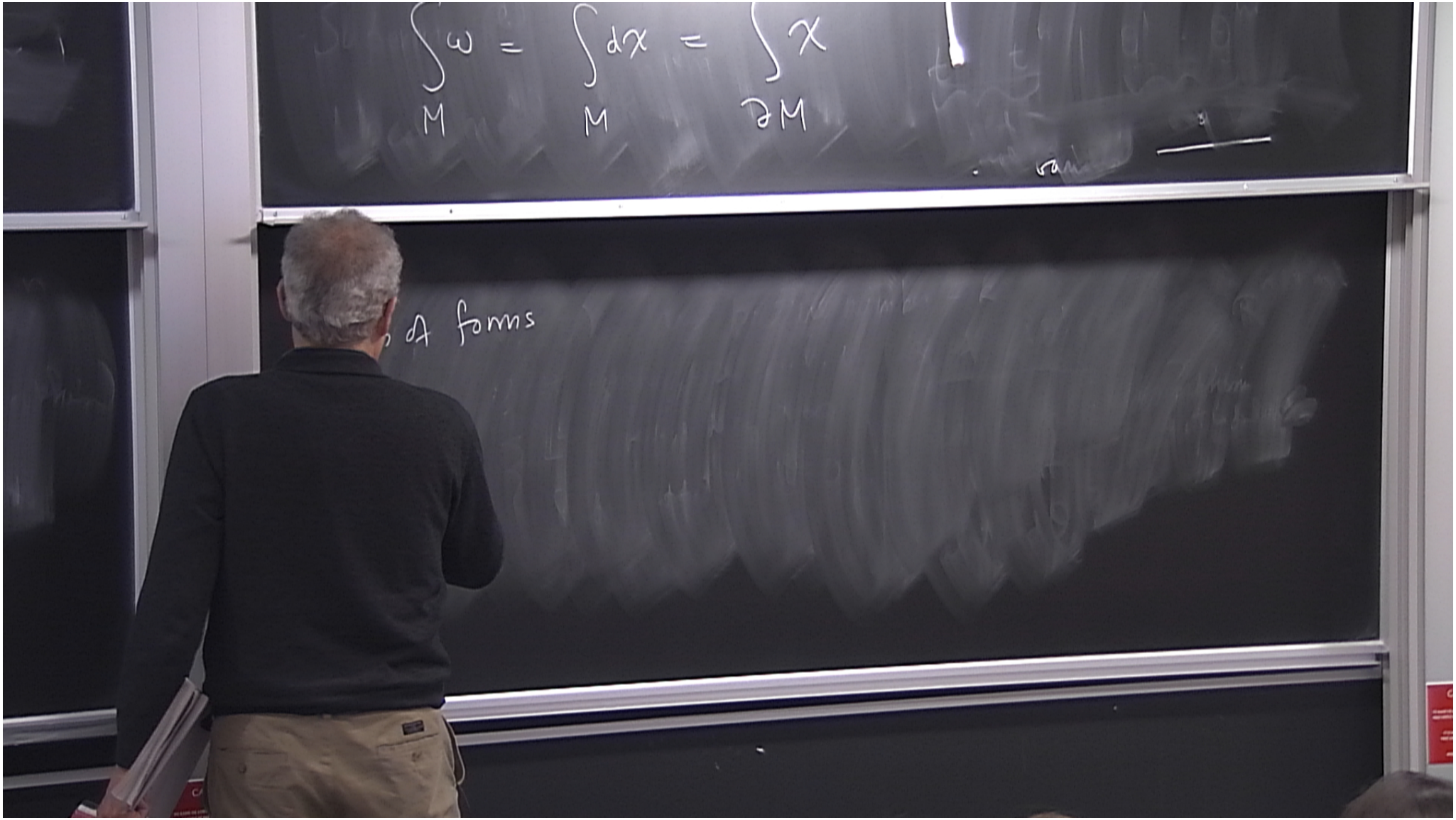
Localized { L^2 rapid decay $\to\infty$

δ-fn support $\neq d\theta=0$


Most important
Integral form
dimension
w.r.t. all dθ's
(t, θ; dt, dθ)
CAUTION


Most important case
Integral form
k = odd dimension
Localized w.r.t. all dθ's
∫ D(t, dt; θ, dθ) F(t, θ; dt, dθ)
CAUTION


Most important case
Integral form
k = odd dimension
Localized w.r.t. all dθ's
∫ D(t, dt; θ, dθ) F(t, θ; dt, dθ)
Picture #
= - the number of odd directions over which we integrate
CAUTION


$\int_M \omega = \int_M d\chi = \int_{\partial M} \chi$
of forms

$$\int_M \omega = \int_M d\chi = \int_{\partial M} \chi$$

Class of forms should be closed under

mult by $d\theta$

diff $\frac{\partial}{\partial(d\theta)}$

$\therefore\ d = \sum d\theta^i \frac{\partial}{\partial \theta^i}$

Scaling

$dx \to t\,dx$

$d\theta \to t\,d\theta$

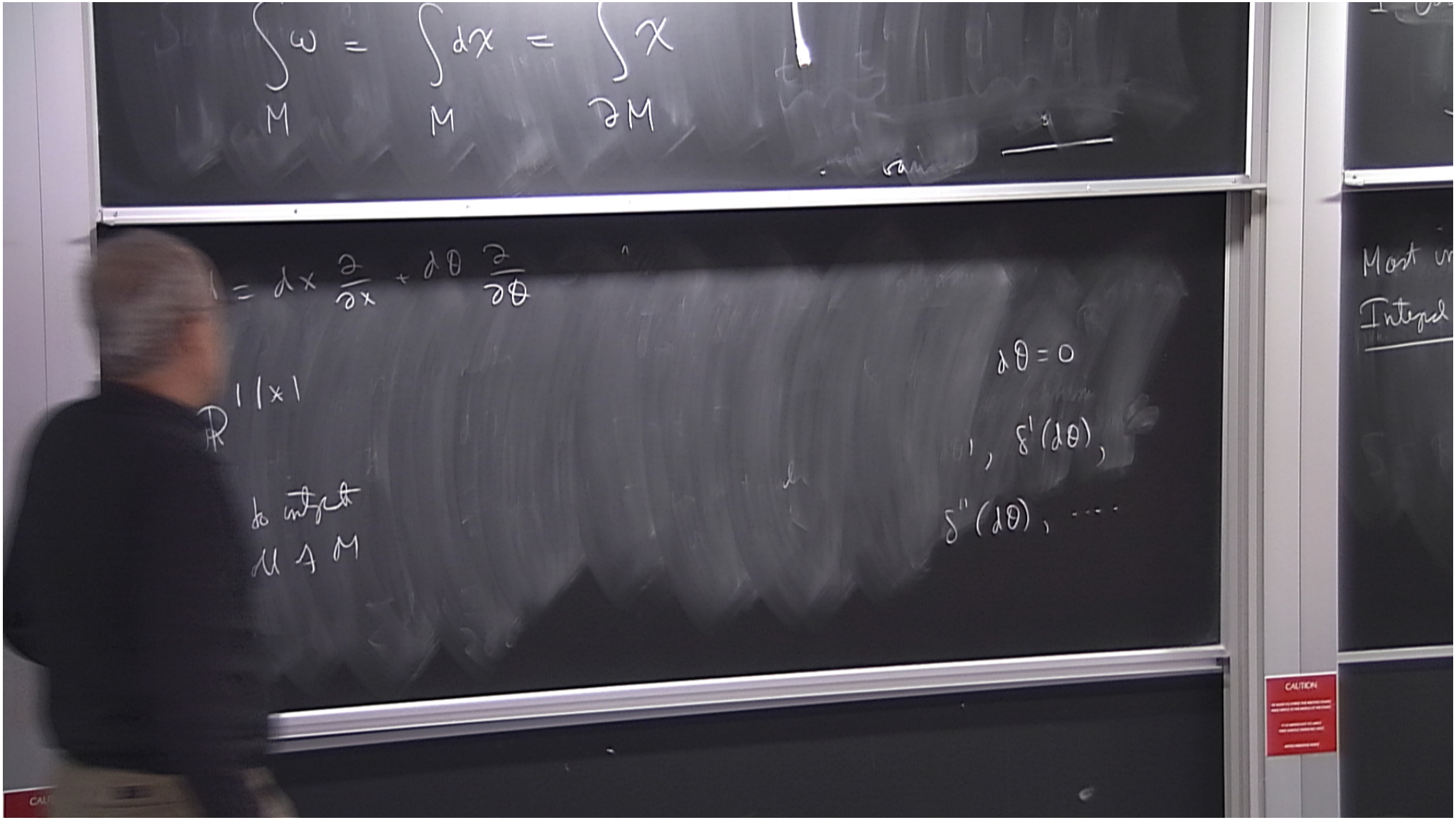
$F \to t^k F$

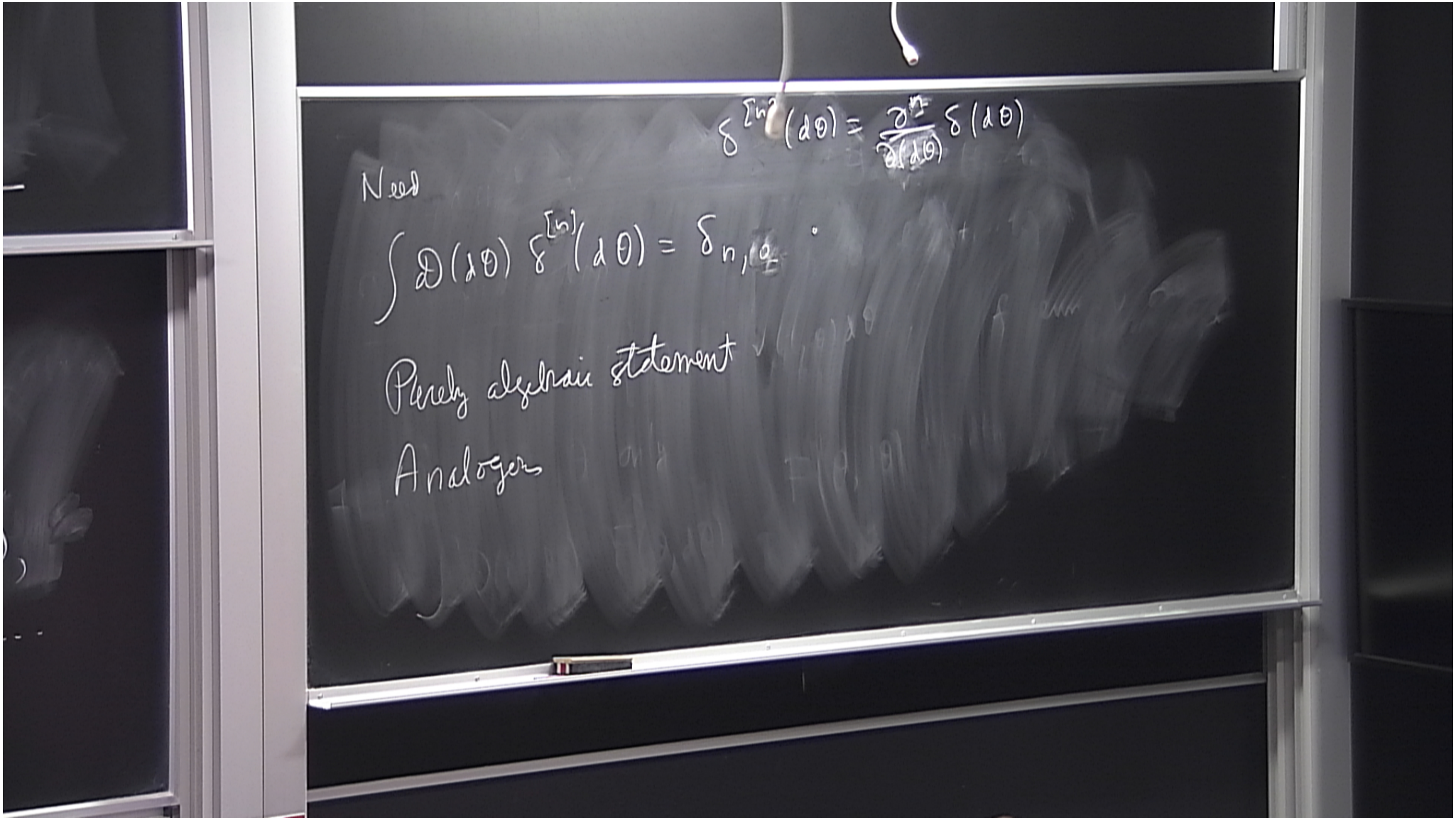
F has ghost number k

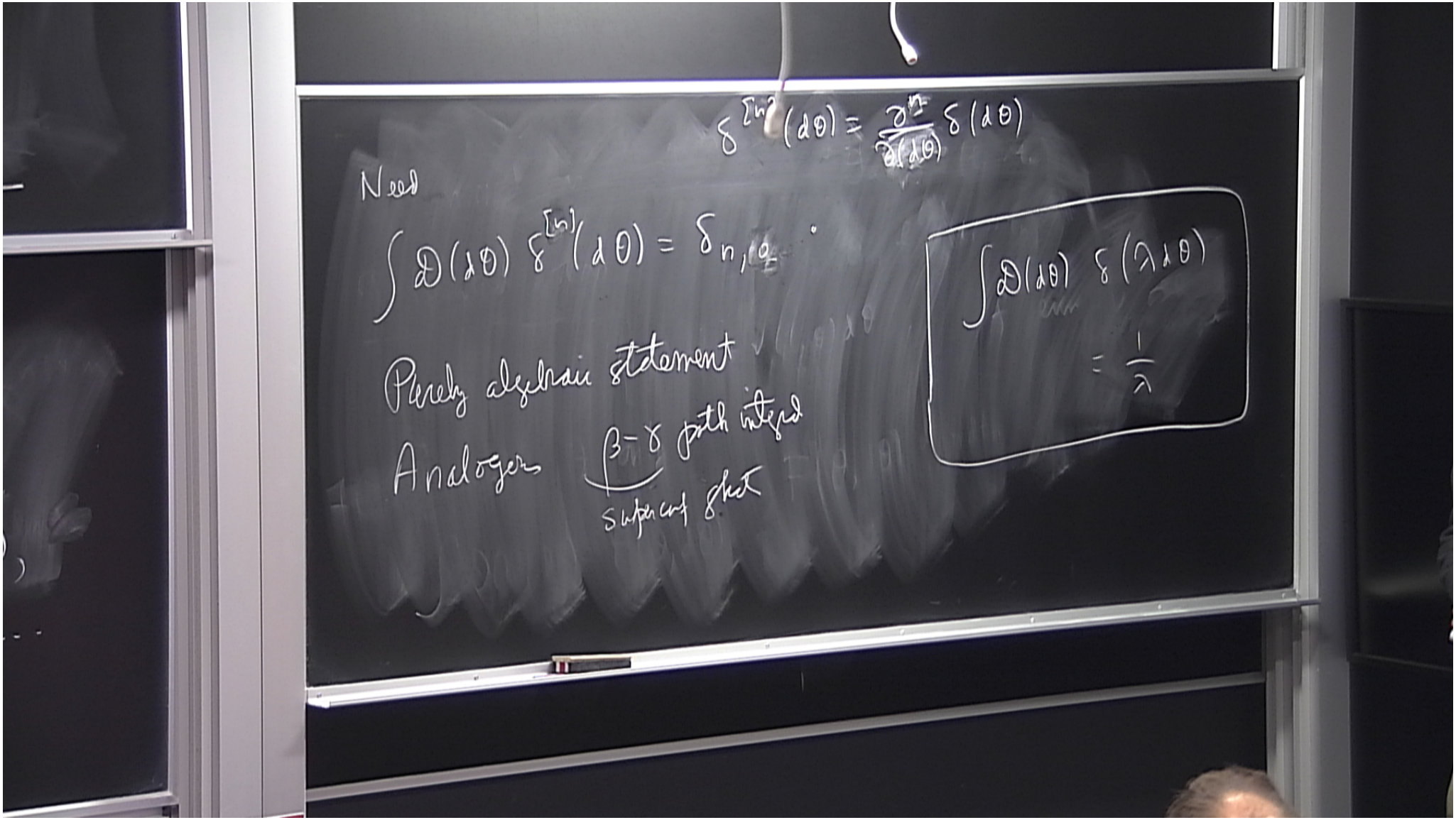
Polynomials

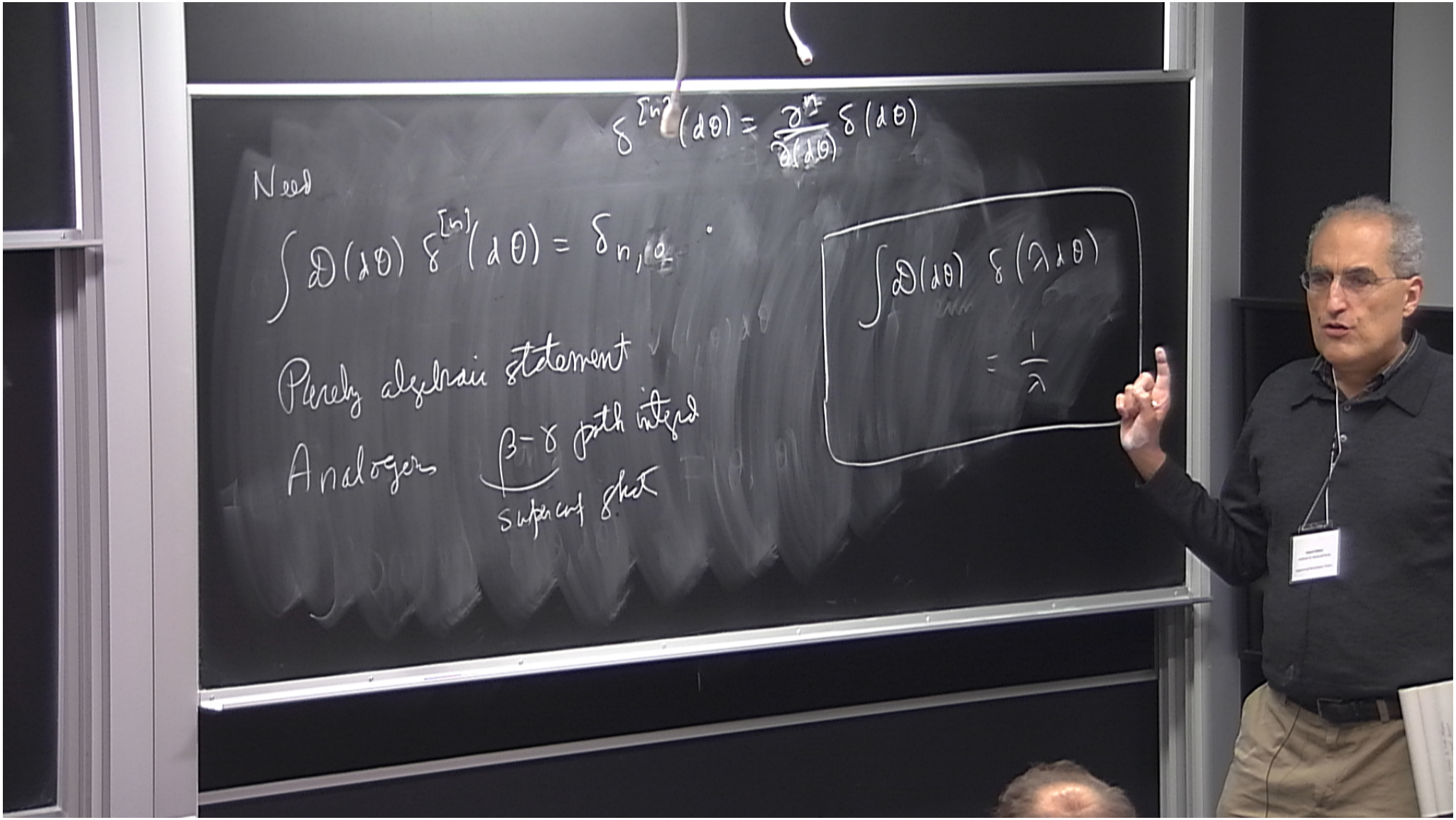
δ-fns at $d\theta = 0$

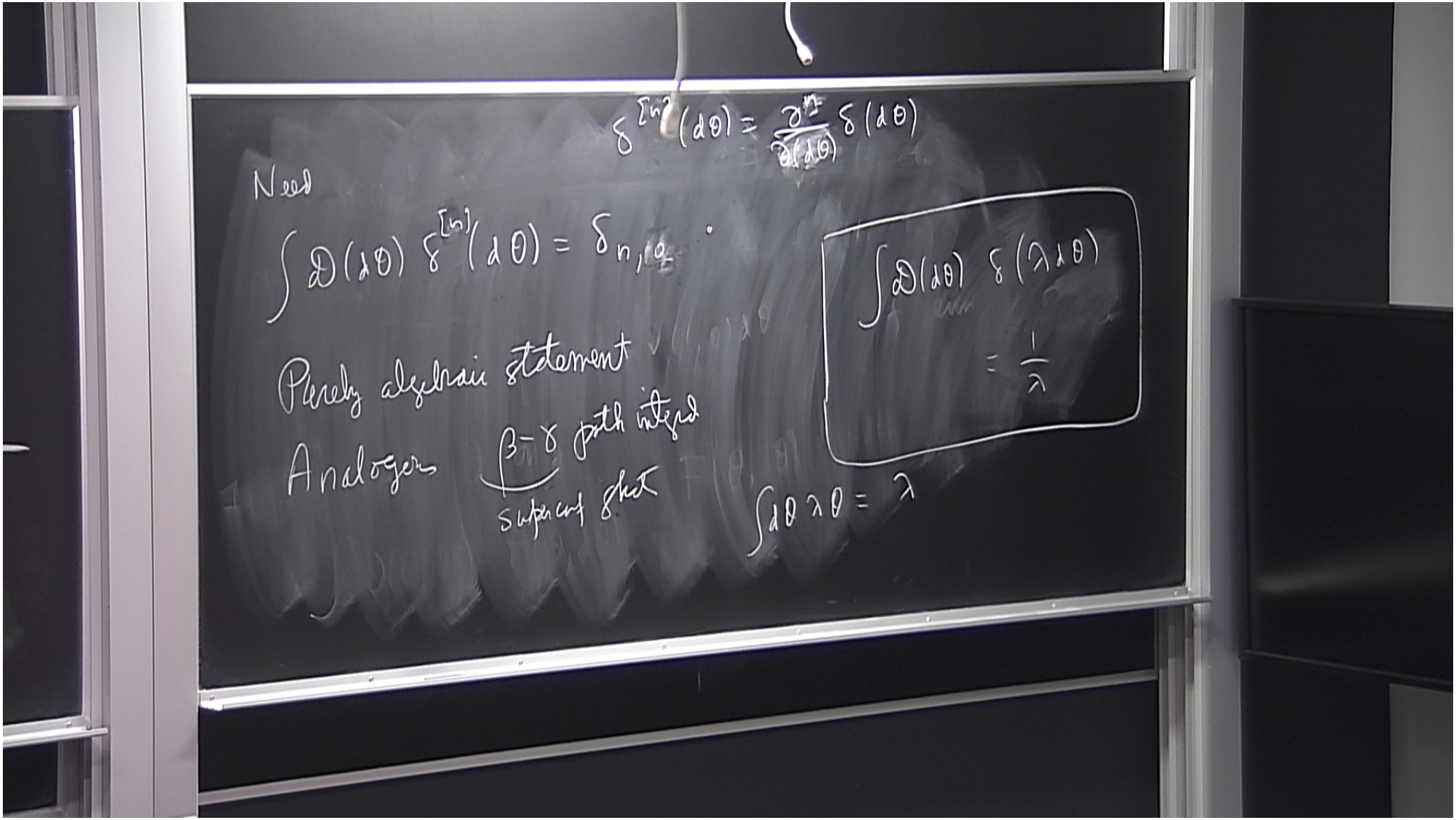
$\{\delta(d\theta),\ \delta'(d\theta),\ \delta''(d\theta),\ \ldots\}$

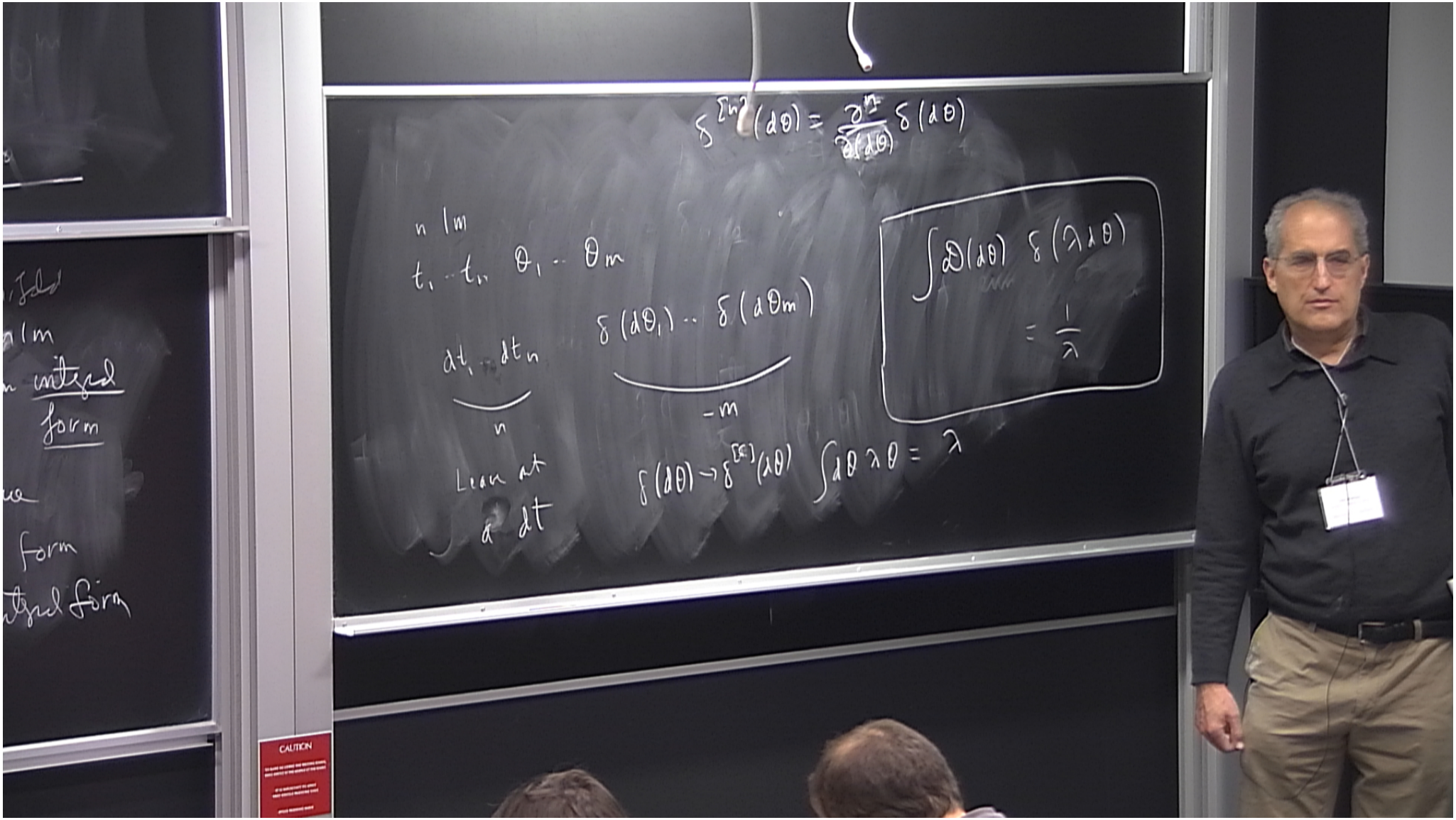

$\int_M \omega = \int_M dx = \int_{\partial M} x$
$= dx \frac{\partial}{\partial x} + d\theta \frac{\partial}{\partial \theta}$
$d\theta = 0$
$\delta'(d\theta)$,
$\delta''(d\theta)$, ...
Most
Integ


$\delta^{[n]}(d\theta) = \frac{\partial^n}{\partial(d\theta)} \delta(d\theta)$
Need
$\int \mathcal{D}(d\theta)\, \delta^{[n]}(d\theta) = \delta_{n,0}$.
Purely algebraic statement
Analogues


$\delta^{[n]}(\lambda\theta) = \frac{\partial^n}{\partial(\lambda\theta)}\delta(\lambda\theta)$
Need
$\int \mathcal{D}(\lambda\theta)\,\delta^{[n]}(\lambda\theta) = \delta_{n,0}$.
$\int \mathcal{D}(\lambda\theta)\,\delta(\lambda\lambda\theta) = \frac{1}{\lambda}$
Purely algebraic statement
Analogous β-γ path integral
Superconf. ghost


$\delta^{[n]}(\lambda\theta) = \frac{\partial^n}{\partial(\lambda\theta)}\delta(\lambda\theta)$
Need
$\int \mathcal{D}(\lambda\theta)\,\delta^{[n]}(\lambda\theta) = \delta_{n,0}$
$\int \mathcal{D}(\lambda\theta)\,\delta(\gamma\lambda\theta) = \frac{1}{\gamma}$
Purely algebraic statement
Analogous β-γ path integral
superconf ghost


$\delta^{[n]}(\lambda\theta) = \frac{\partial^n}{\partial(\lambda\theta)}\delta(\lambda\theta)$
Need
$\int \mathcal{D}(\lambda\theta)\, \delta^{[n]}(\lambda\theta) = \delta_{n,0}$
Purely algebraic statement
Analogous β-γ path integral
superconformal ghost
$\int \mathcal{D}(\lambda\theta)\, \delta(\gamma\lambda\theta) = \frac{1}{\gamma}$
$\int d\theta\, \gamma\theta = \gamma$


$\delta^{[m]}(d\theta) = \frac{\partial^m}{\partial(d\theta)} \delta(d\theta)$
n | m
$t_1 \cdots t_n \quad \theta_1 \cdots \theta_m$
$dt_1 \cdots dt_n$
n
$\delta(d\theta_1) \cdots \delta(d\theta_m)$
m
Learn at
a dt
$\delta(d\theta) \to \delta^{[n]}(d\theta) \quad \int d\theta\, \lambda\theta = \lambda$
$\int \mathcal{D}(d\theta)\, \delta(\lambda\, d\theta) = \frac{1}{\lambda}$
integral
form
form
form
CAUTION

$$\int_M \omega = \int_M d\chi = \int_{\partial M} \chi$$

Codimension 1

Codim 0 $\quad f(\theta)\,\delta(d\theta)$

$\mathbb{R}^{0|*1}$ Codim 1 integral form

$$\chi = f(\theta)\,\delta'(d\theta)$$

Let's verify

$$\int_{\mathbb{R}^{0|1}} d\chi = 0$$

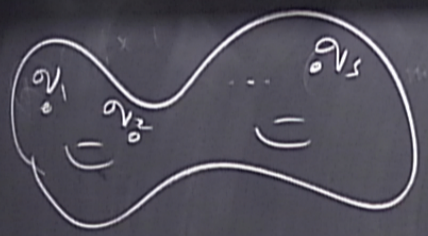
$$d\chi = d\theta \frac{\partial}{\partial\theta} f(\theta)\,\delta'(d\theta)$$

$$= \frac{\partial f}{\partial \theta} \underbrace{d\theta\,\delta'(d\theta)}_{-\delta(d\theta)}$$

CAUTION

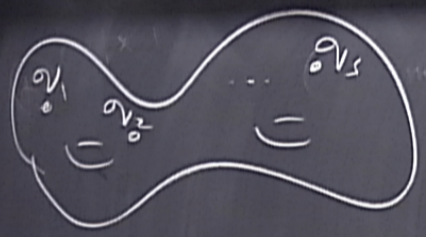
M M ∂M

Superstring Post theory



$\mathcal{M}$ $\overline{\mathcal{M}}$ $\partial\mathcal{M}$

Superstring Perturbation theory



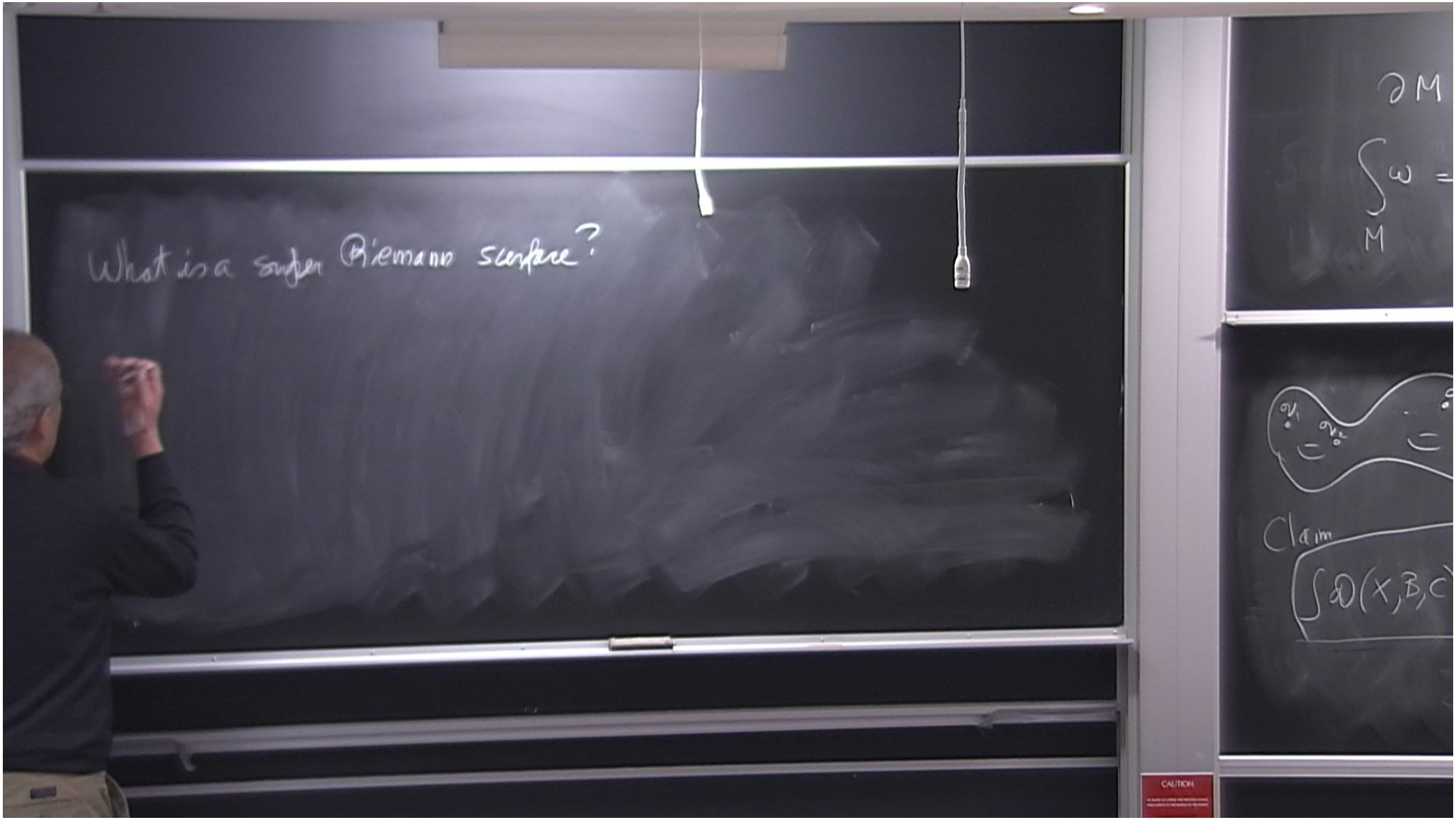
v_i = superconformal vertex operators

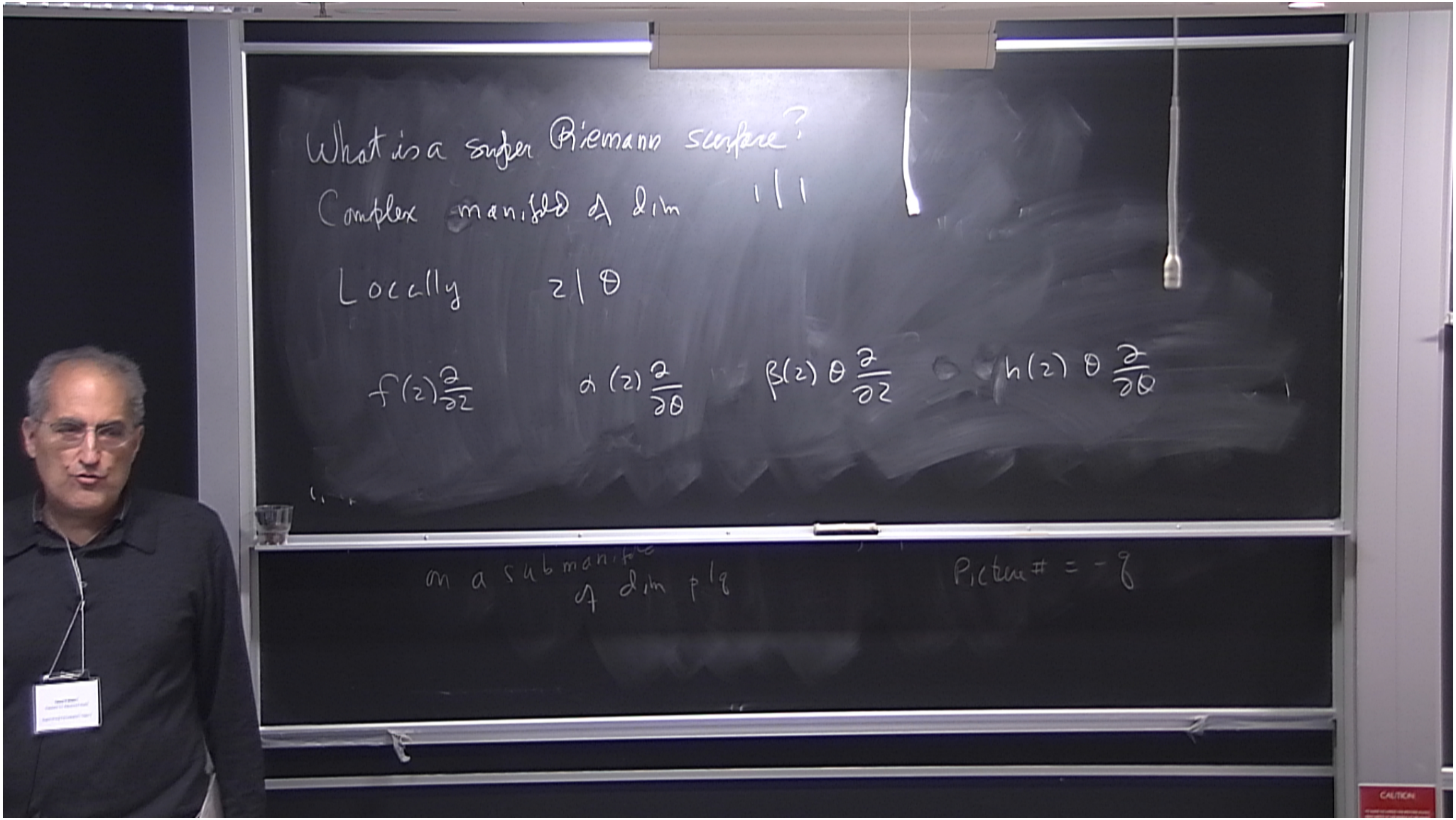
Claim

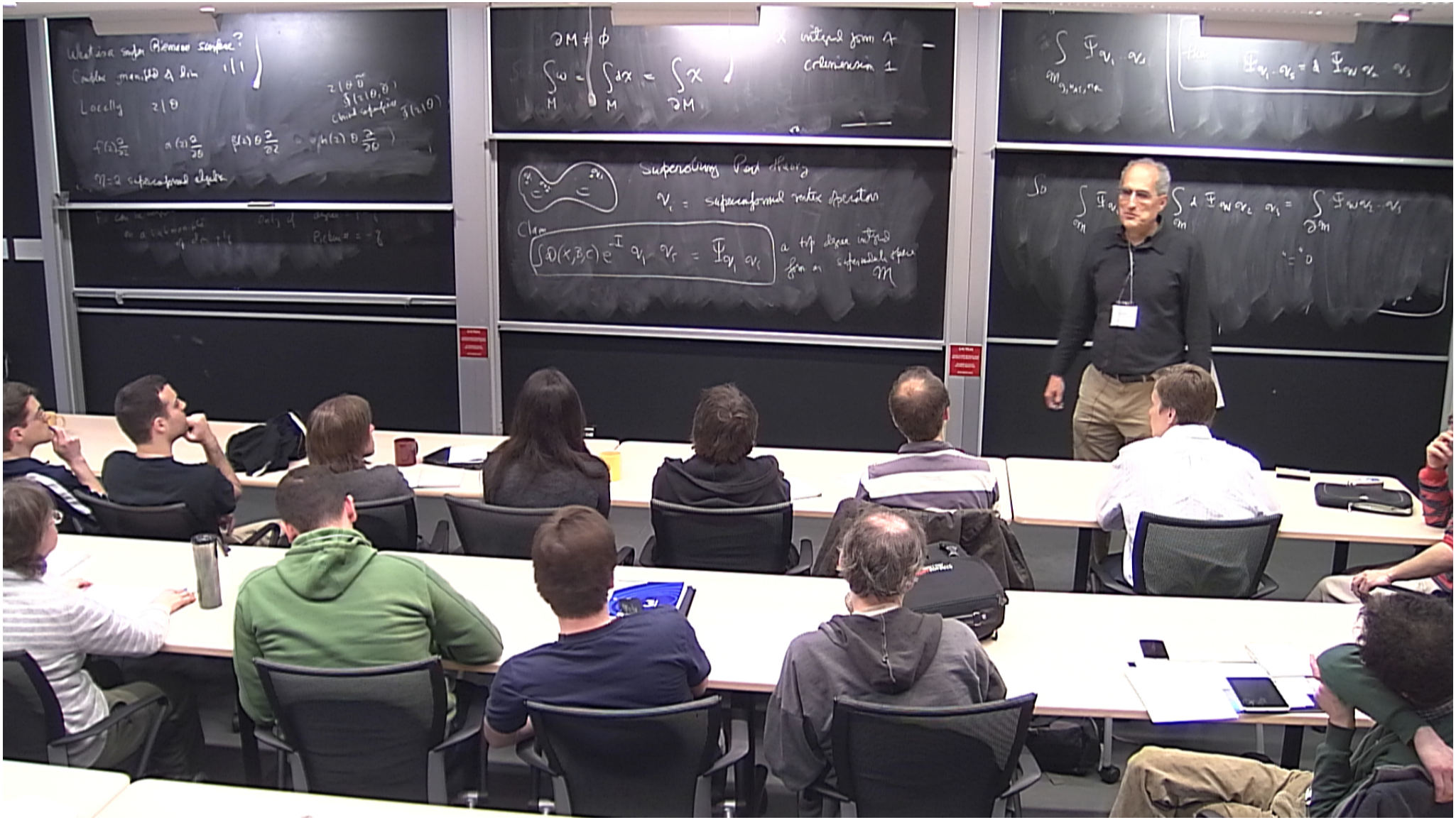
$$\int \mathcal{D}(X,B,C)\, e^{-I}\, v_1 \cdots v_3 = \Psi_{v_1 \cdots v_3}$$

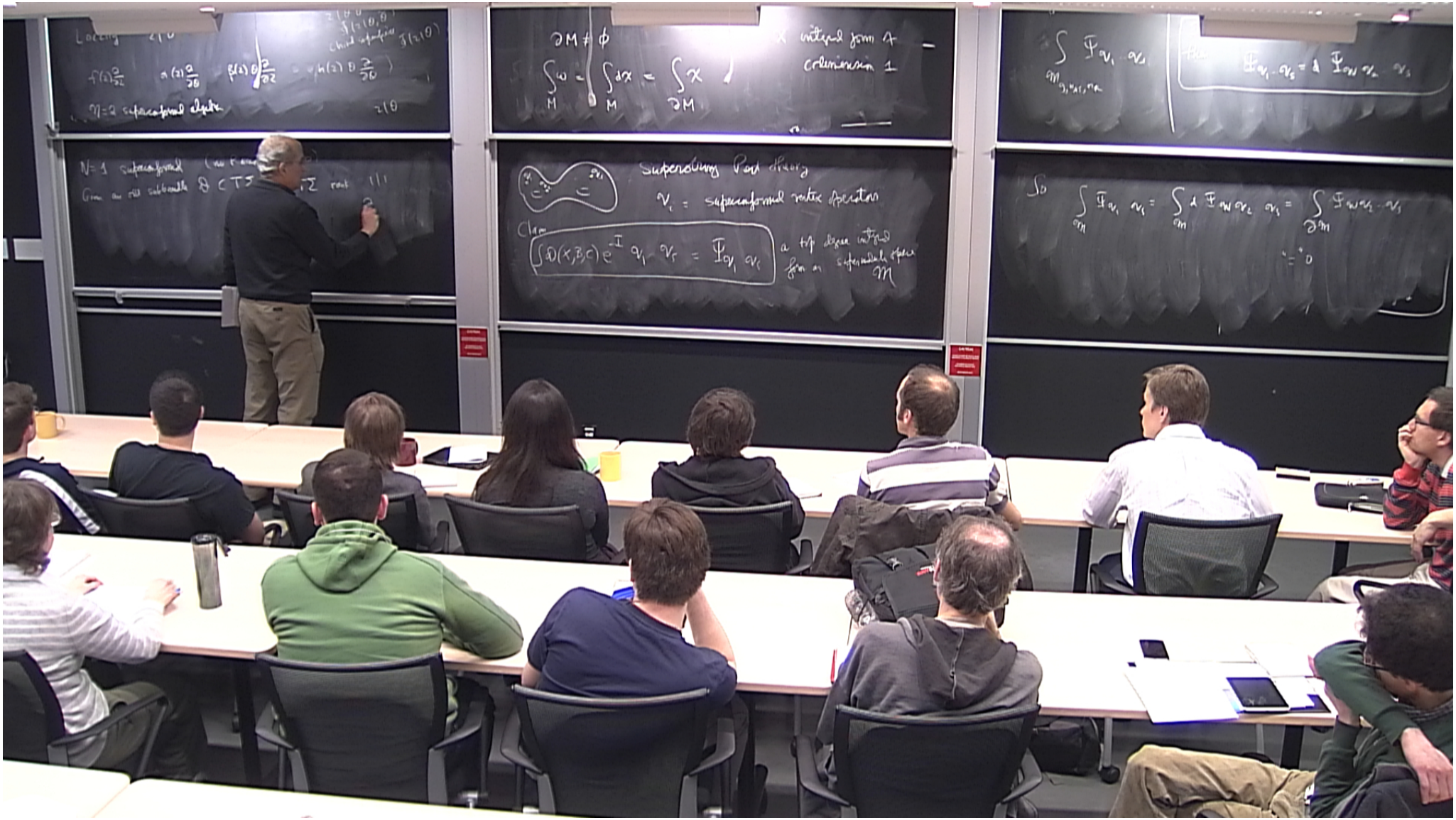
a top degree integral form on a supermoduli space $\mathcal{M}$

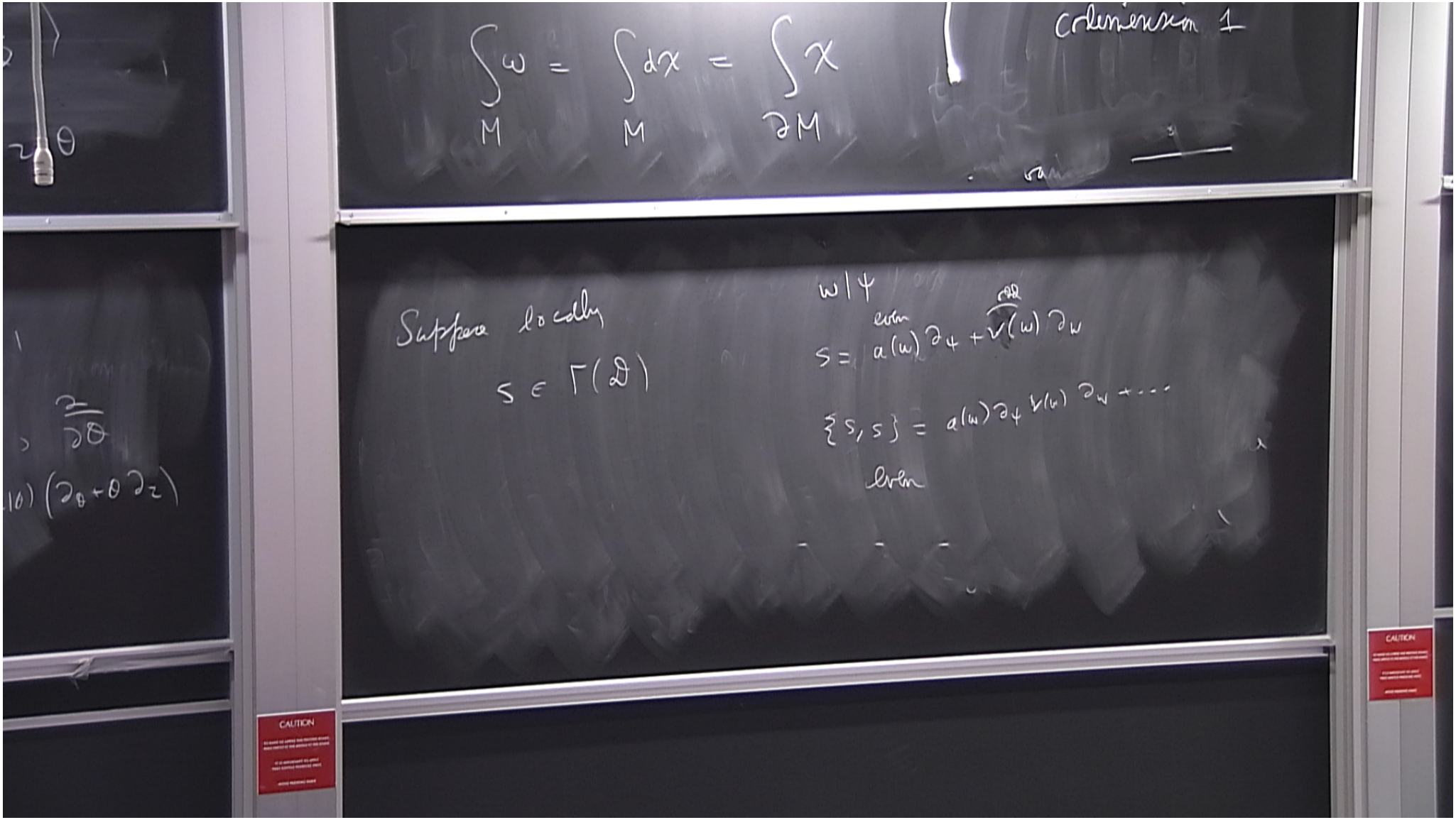
CAUTION

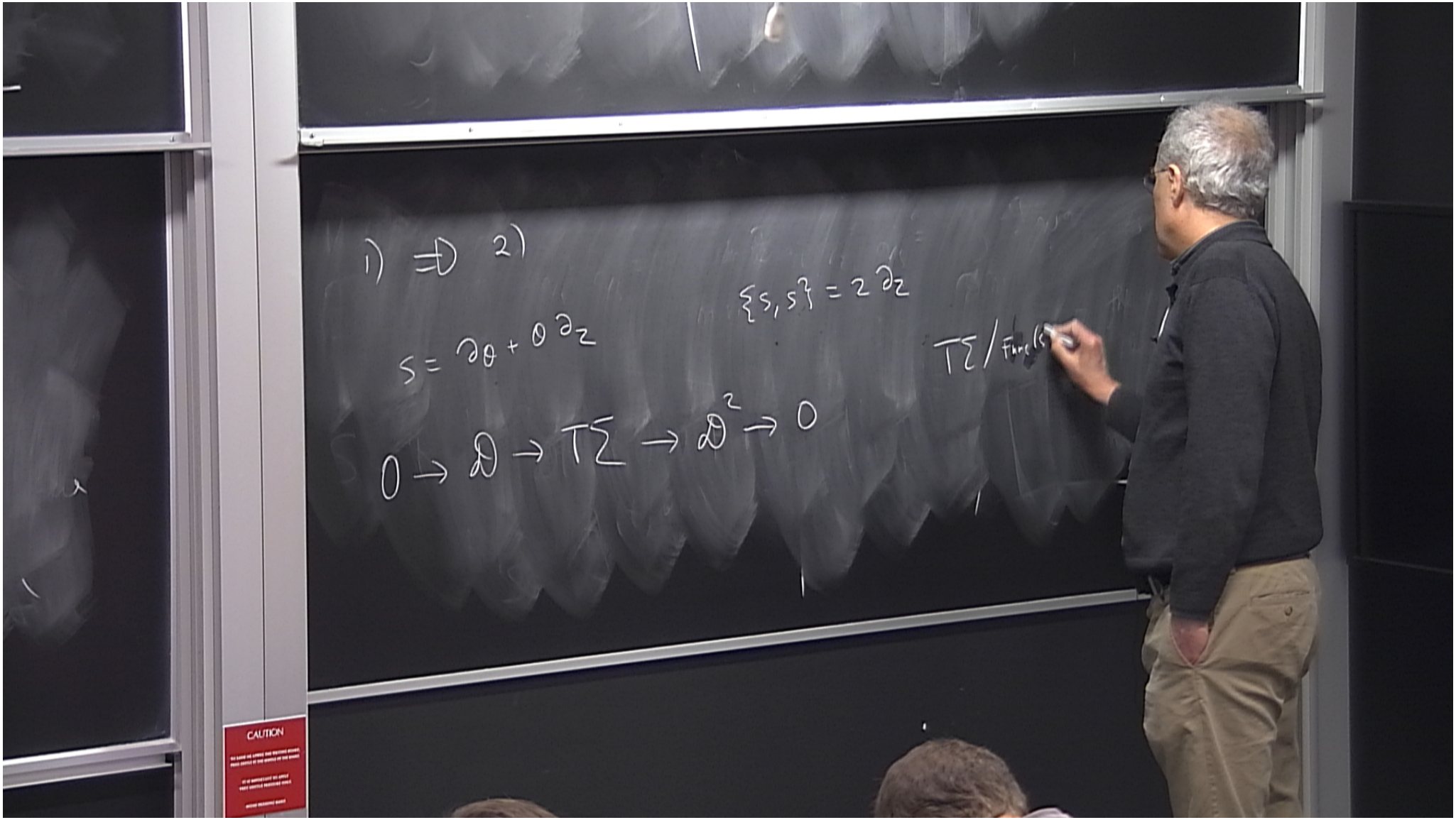

What is a surface Riemann surface?
∂M
Claim

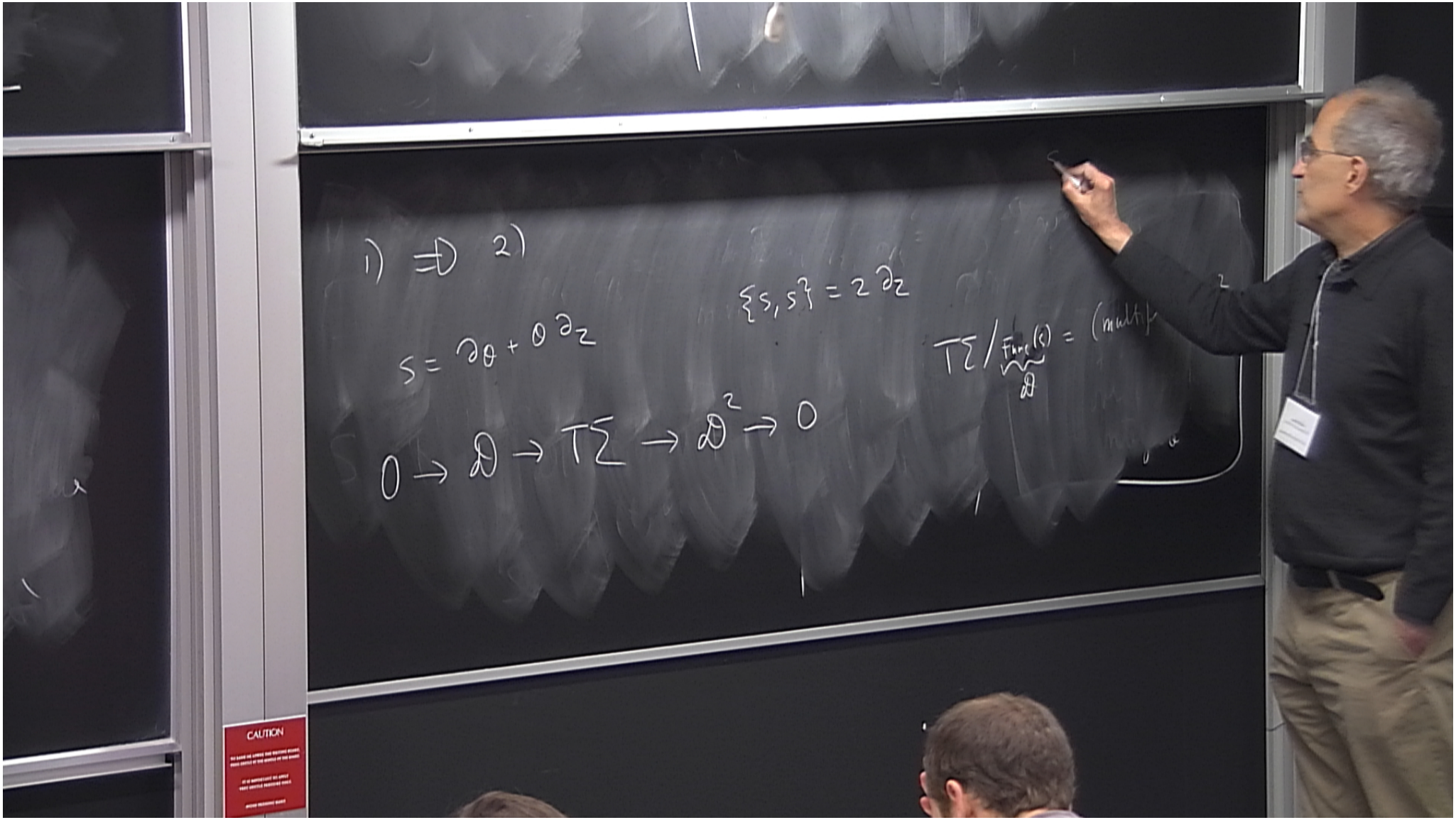

What is a super Riemann surface?
Complex manifold of dim 1|1
Locally z|θ
$f(z)\frac{\partial}{\partial z}$
$\alpha(z)\frac{\partial}{\partial \theta}$
$\beta(z)\theta\frac{\partial}{\partial z}$
$h(z)\theta\frac{\partial}{\partial \theta}$
Picture # = -g

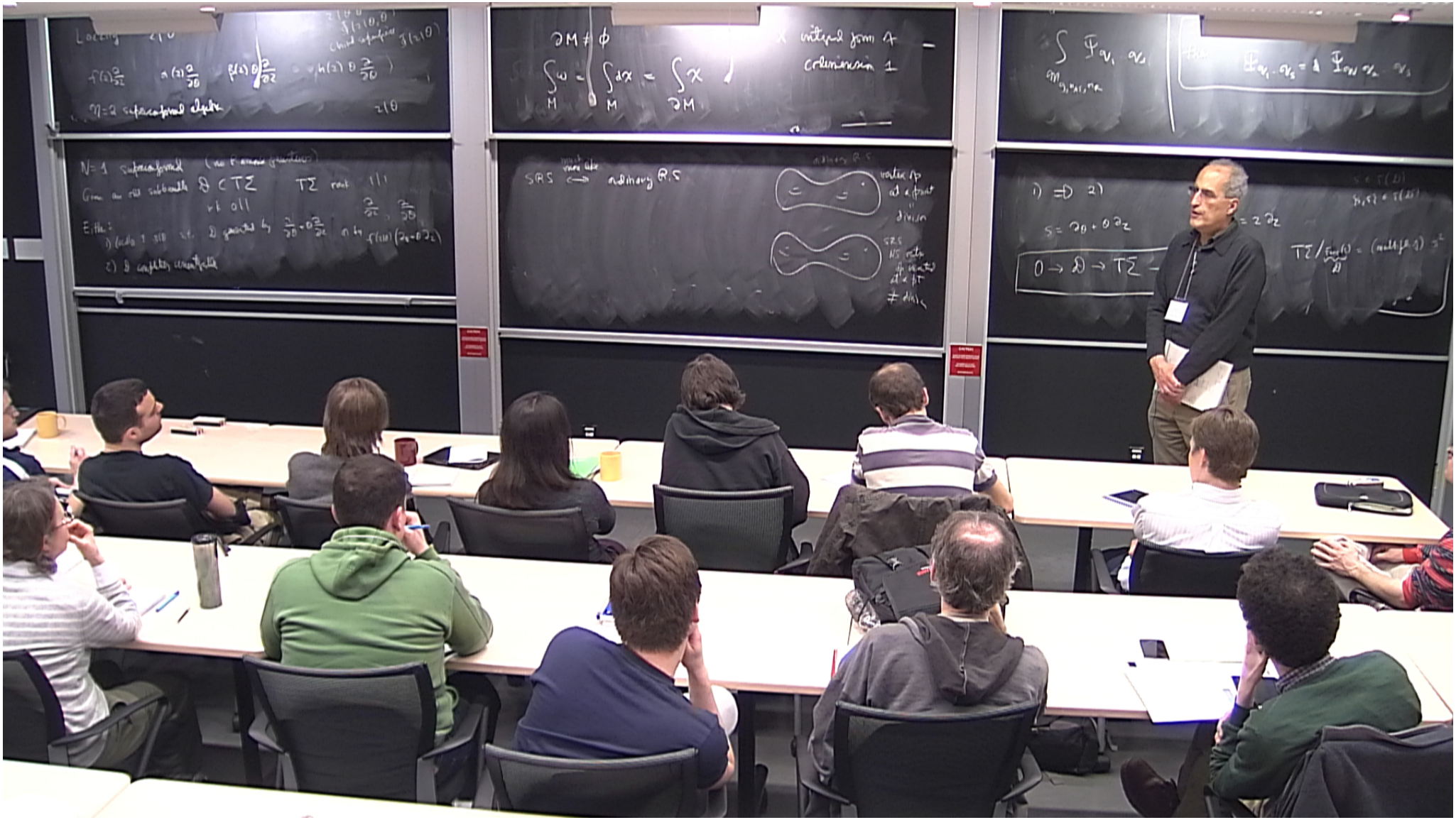

Locally
Superstring Field theory
Claim

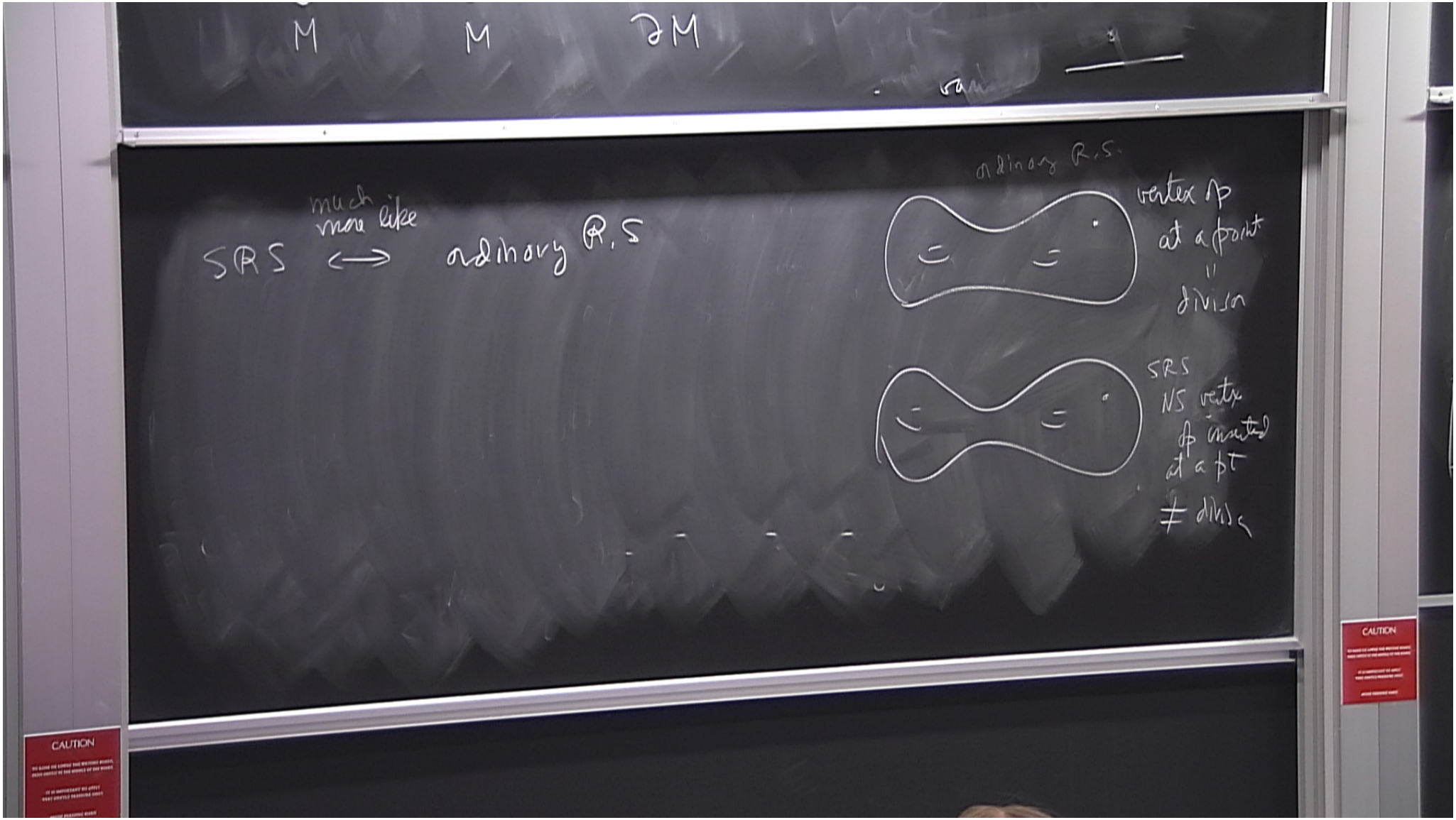

$\partial M \neq \phi$
$\int_M \omega = \int_M d\alpha = \int_{\partial M} \alpha$
integral form 4
codimension 1
Supersolving Ford theory
v_i = superconformal vector operators
Claim
$\int \mathcal{D}(\alpha,\beta,c)\, e^{-I}\, v_1 \cdots v_r = \Phi_{v_1 \cdots v_r}$
a top degree integral form on supermanifold object $\mathcal{M}$
$\int_{\partial m} \Phi_{v_1 \cdots v_r} = \int_m d\, \Phi_{dv_1 v_2 \cdots v_r} = \int_{\partial m} \Phi_{v_1 v_2 \cdots v_r}$
$= 0$

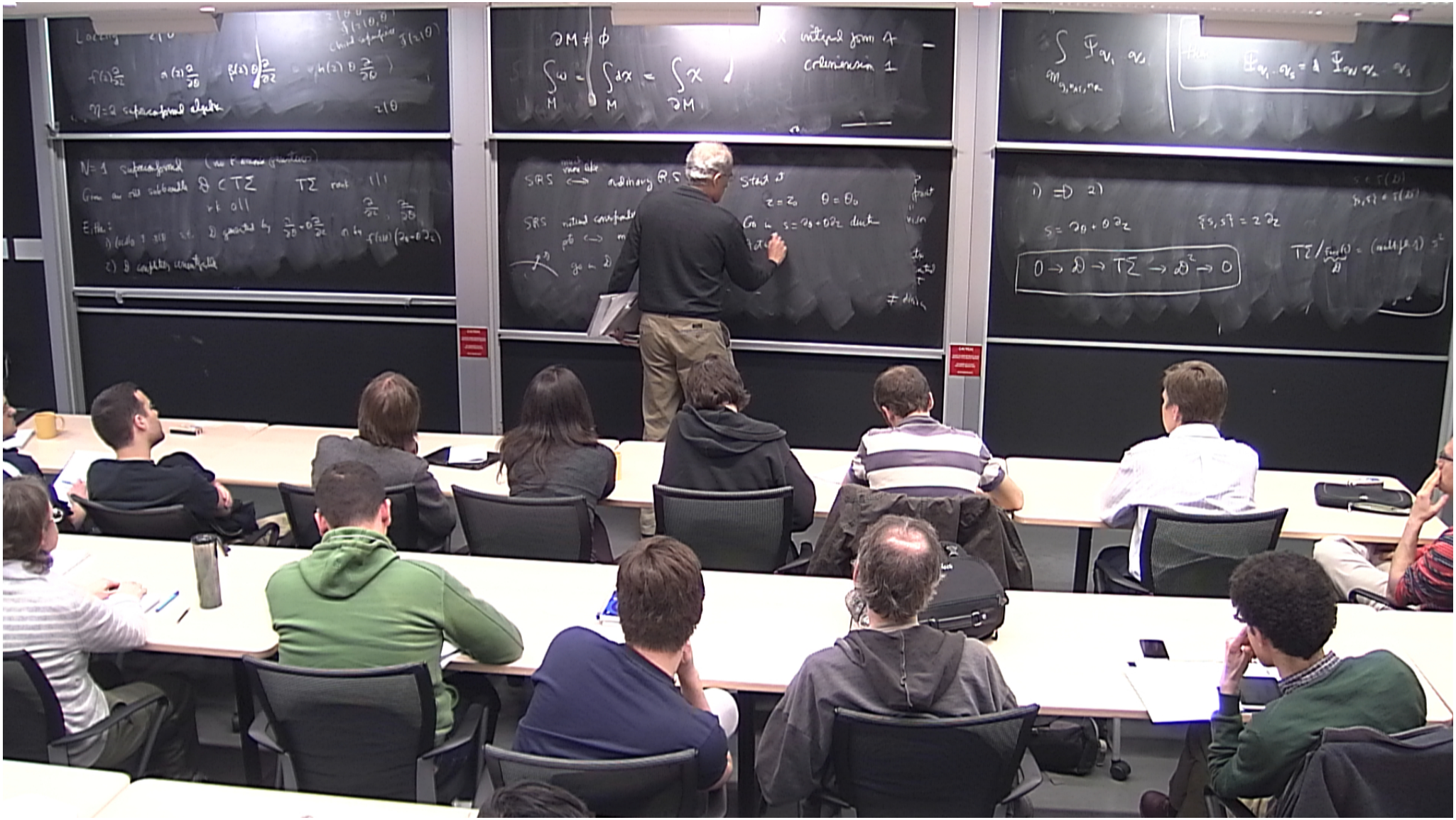

$\int_M \omega = \int_M dx = \int_{\partial M} x$
codimension 1
Surface locally
$s \in \Gamma(\mathcal{D})$
w/ψ
even
$s = a(w)\partial_\psi + \nu(w)\partial_w$
$\{s, s\} = a(w)\partial_\psi \nu(w)\partial_w + \cdots$
even
$\frac{\partial}{\partial\theta}$
$(\partial_\theta + \theta\partial_z)$
CAUTION
CAUTION

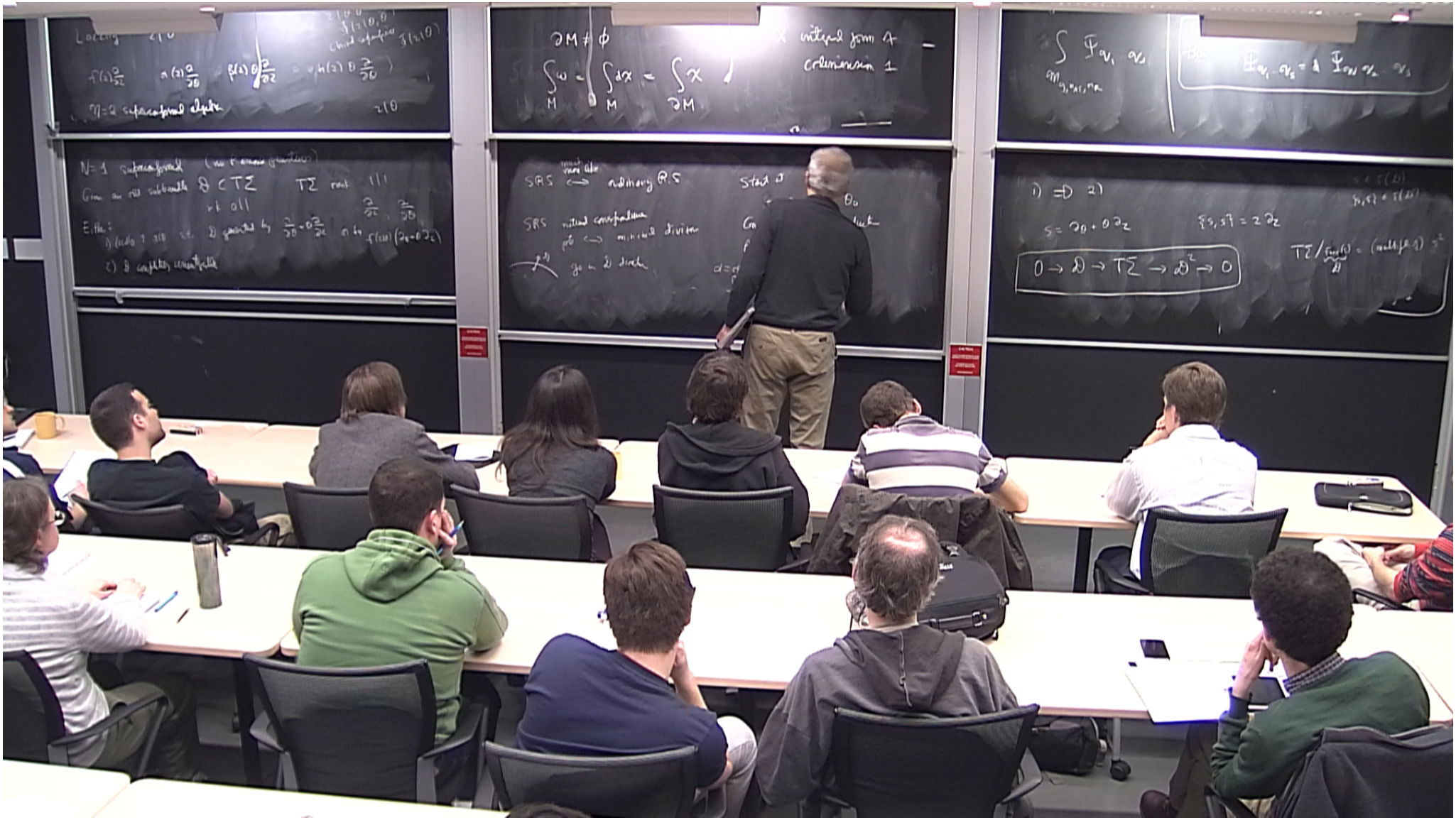

1) ⇒ 2)
$S = \partial_\theta + \theta\partial_z$
$\{S, S\} = 2\partial_z$
$0 \to \mathcal{D} \to T\Sigma \to \mathcal{D}^2 \to 0$
$T\Sigma$/
CAUTION

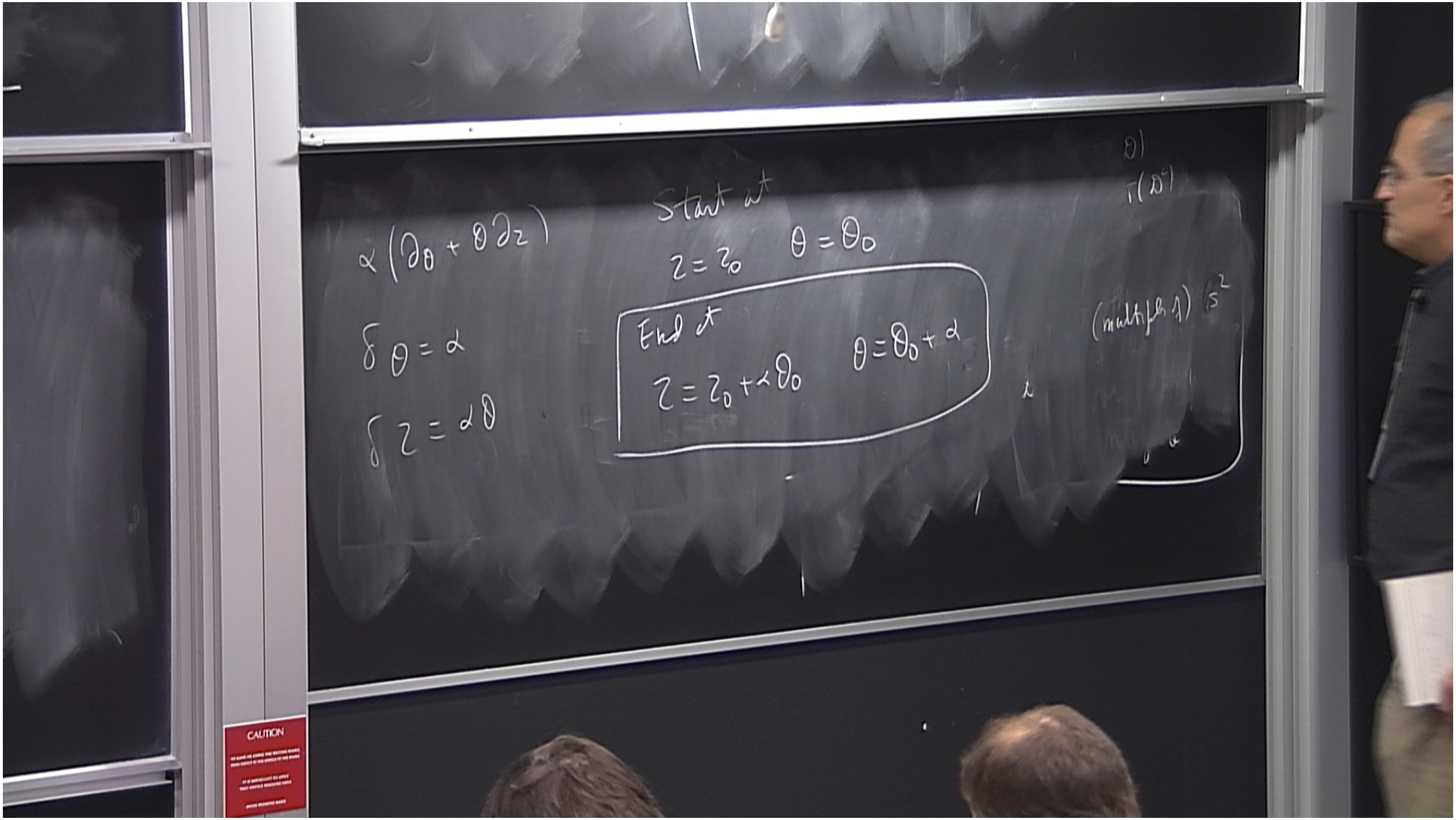

1) ⇒ 2)
$S = \partial_\theta + \theta\partial_z$
$\{S, S\} = 2\partial_z$
$0 \to \mathcal{D} \to T\Sigma \to \mathcal{D}^2 \to 0$
$T\Sigma/\mathcal{D} =$
CAUTION

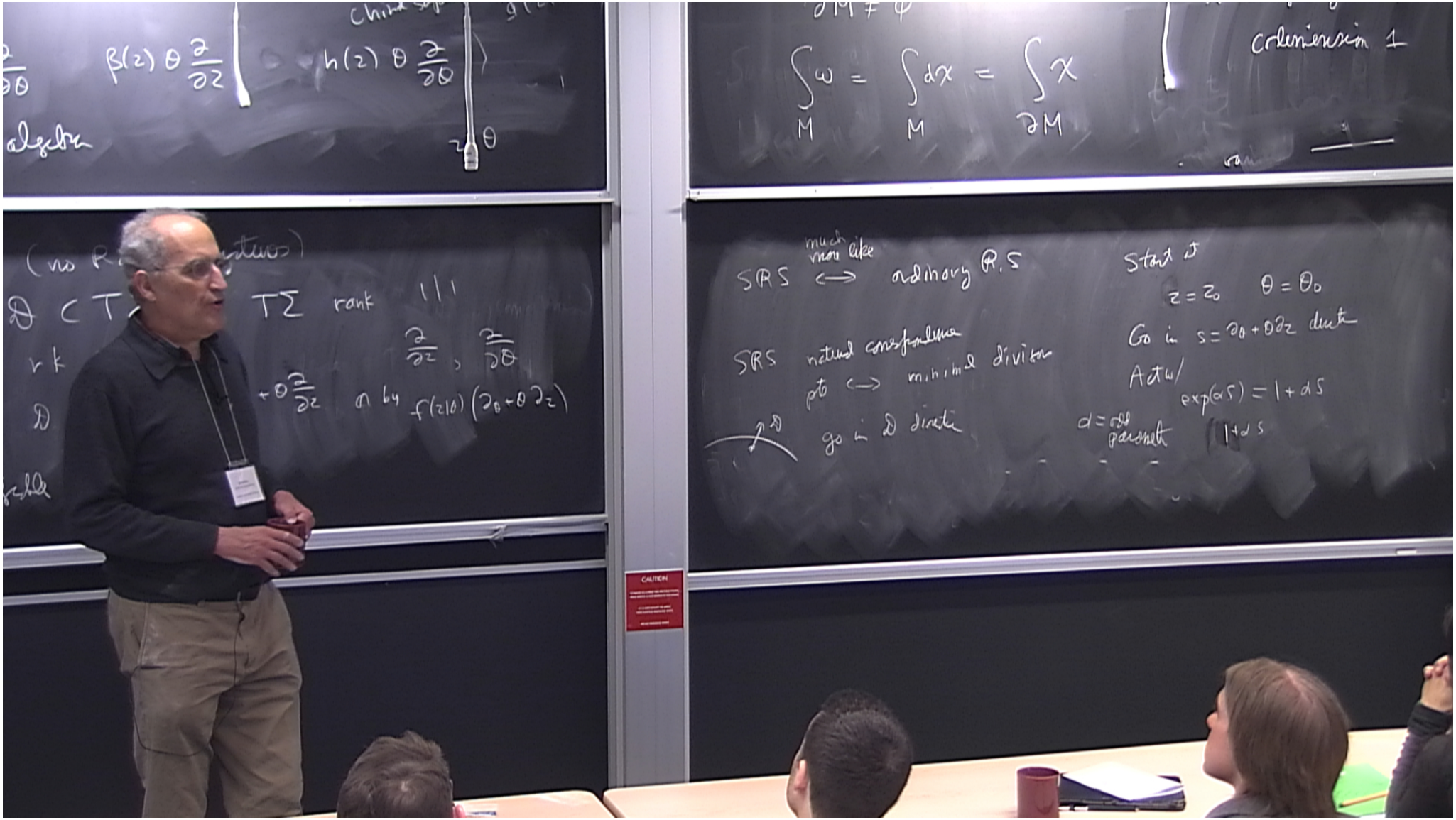

$\partial M \neq \phi$
$\int_M \omega = \int_M d\alpha = \int_{\partial M} \alpha$
$N=1$ superconformal
$\mathcal{D} \subset T\Sigma$
$T\Sigma$
SRS
ordinary R.S
$S = \partial_\theta + \theta \partial_z$
$0 \to \mathcal{D} \to T\Sigma \to$


M
M
∂M
much more like
SRS ⟷ ordinary R.S
ordinary R.S.
vertex ↑ at a point
=
divisor
SRS
NS vertex ↑ inserted at a pt
≠ divisor


$\partial M \neq \phi$
$\int_M \omega = \int_M d\alpha = \int_{\partial M} \alpha$
$0 \to \mathcal{D} \to T\Sigma \to \mathcal{D}^2 \to 0$


$\partial M \neq \phi$
$\int_M \omega = \int_M d\alpha = \int_{\partial M} \alpha$
$0 \to \mathcal{D} \to T\Sigma \to \mathcal{D}^2 \to 0$


$\alpha(\partial_\theta + \theta\partial_z)$
$\delta\theta = \alpha$
$\delta z = \alpha\theta$
Start at
$z = z_0$ $\theta = \theta_0$
End at
$z = z_0 + \alpha\theta_0$ $\theta = \theta_0 + \alpha$
CAUTION


$\beta(z)\,\theta\,\frac{\partial}{\partial z}$
$h(z)\,\theta\,\frac{\partial}{\partial \theta}$
$\int_M \omega = \int_M d\alpha = \int_{\partial M} \alpha$
codimension 1
much more like
SRS ordinary R.S
Start
$z = z_0 \quad \theta = \theta_0$
Go in $s = z_0 + \theta z$ direction
Actw/
$\exp(\alpha s) = 1 + \alpha s$
$1 + \alpha s$
α = odd parameter
SRS natural conformal
pts minimal divisor
go in 2 directi
CAUTION