Title: Magnetized relativistic plasma as a Weyl metal

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Abstract: It has been recently established that a magnetized relativistic plasma yields an interesting example of a Weyl metal. I discuss the properties of magnetized relativistic plasma and its possible role in some astrophysics phenomena.

Magnetized relativistic plasma as a Weyl metal

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Chiral separation effect

- Axial current induced by fermion chemical potential

$$\left\langle j_5^3 \right\rangle_{\text{free}} = -\frac{eB}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Exact result (is it?), which follows from chiral anomaly relation

- No radiative correction expected…

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The chiral anomaly and CSE

$$\partial_\mu j_5^\mu = \frac{e^2}{8\pi^2} F_{\lambda\sigma} \widetilde{F}^{\lambda\sigma} = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

Ambjorn, Greensite, Peterson (1983): Only LLL generates the chiral anomaly.

Axial current induced in CSE: $\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$

In a free theory, $\langle \vec{j}_5 \rangle$ is generated only in LLL.

The connection between $\partial_\mu j_5^\mu$ and $\langle \vec{j}_5 \rangle$:

$$\mathcal{L} = \cdots \ \mu\psi^\dagger\psi + eA_0\psi^\dagger\psi$$

$$\mu \ \rightarrow \ eA_0^{\text{ext}} \ \Rightarrow \ \langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu = -\frac{e^2\vec{B}}{2\pi^2}A_0^{\text{ext}}$$

Then, $\partial_\mu \langle j_5^\mu \rangle = \partial_i \langle j_5^i \rangle = -\frac{e^2\vec{B}}{2\pi^2}\vec{\nabla} A_0^{\text{ext}}$

$$\xrightarrow{\vec{E}=-\vec{\nabla}A_0} \ \partial_\mu \langle j_5^\mu \rangle = \frac{e^2\vec{B}}{2\pi^2} \cdot \vec{E} \quad \text{(anomalous relation!)}$$

Is the relation $\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$ exact?

Landau spectrum at B≠0

- Dirac equation with massless fermions (e<0)

$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla}+ie\vec{A}\right)\right]\Psi = 0$$

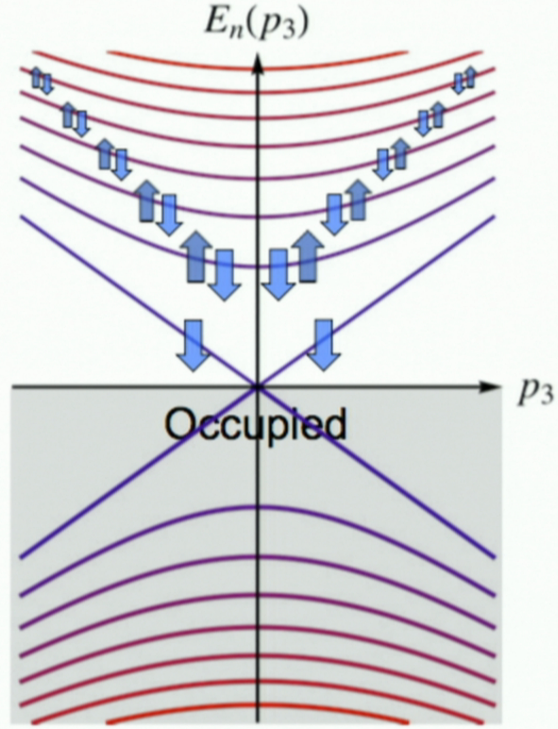
- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

$s = \pm 1/2$ (spin)

where $n = s + k + \frac{1}{2}$

$k = 0,1,2,\ldots$(orbital)



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Effect of Interactions (Motivation from Graphene)

[E. Gorbar, V. M., I. Shovkovy, Phys. Rev. C **80**, 032801(R) (2009)].

- Any additional consequences of the CSE relation?

$$\left\langle j_5^3 \right\rangle_{\text{free}} = -\frac{eB}{2\pi^2}\mu \quad \text{(free theory!)}$$

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- Any dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \Delta\overline{\psi}\gamma^3\gamma^5\psi$$

- Note: Δ=0 is not protected by any symmetry (Δ is 3-dim analog of Haldane mass in graphene)

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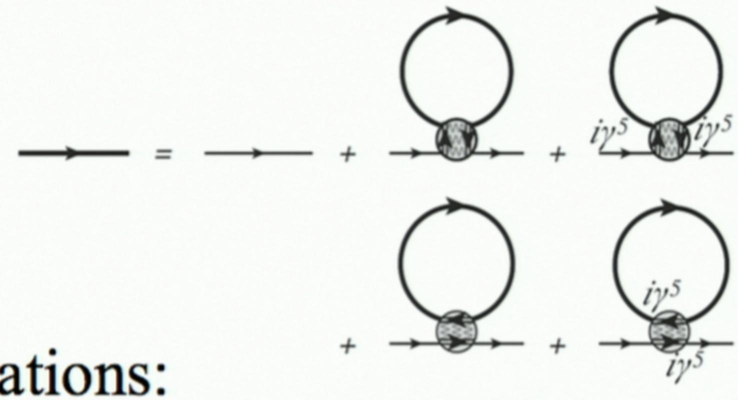
Chiral shift in NJL model

[Gorbar, V.M., Shovkovy, Phys. Rev. C **80**, 032801(R) (2009)]

- NJL model (local interaction)



- "Gap" equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \left\langle j^0 \right\rangle$$ ("effective" chemical potential)

$$m = m_0 - G_{\text{int}} \langle \overline{\psi} \psi \rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\text{int}} \left\langle j_5^3 \right\rangle$$ (chiral shift parameter)

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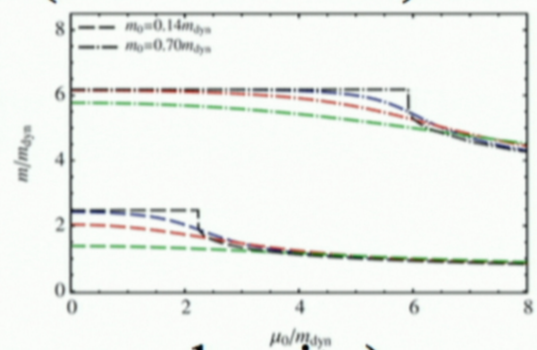
Solutions

- Magnetic catalysis solution (vacuum state):

$$m^2_{\text{dyn}} \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\text{int}}|eB|}\right)$$

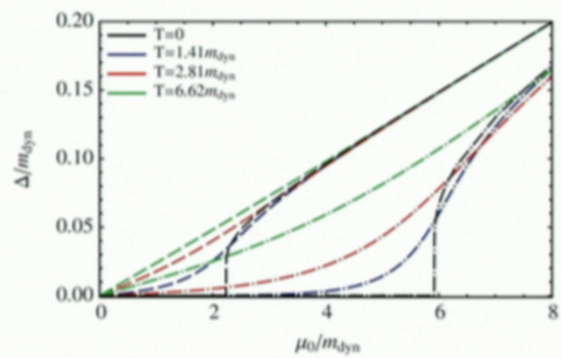


$$\Delta = 0 \quad \& \quad \mu = \mu_0$$

- State with a chiral shift (nonzero density):

$$m_{\text{dyn}} = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2}$$



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Chiral shift @ Fermi surface

- Chirality is $\approx$ well defined at Fermi surface ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

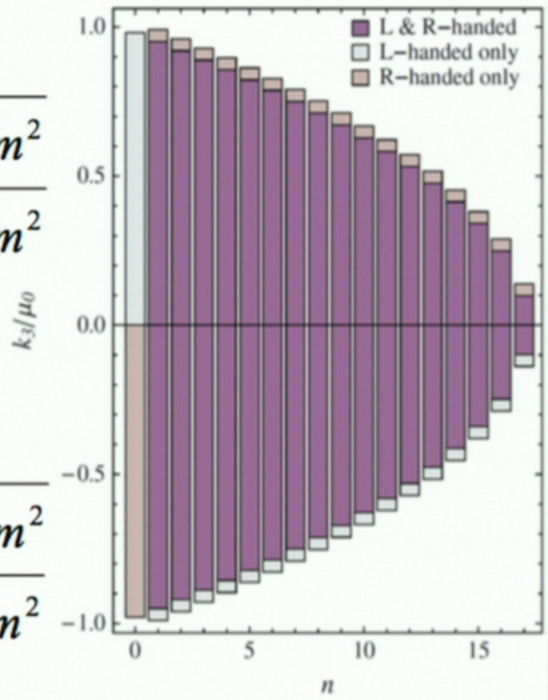
$$k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta)^2 - m^2}$$

$$k^3 = +\sqrt{(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta)^2 - m^2}$$



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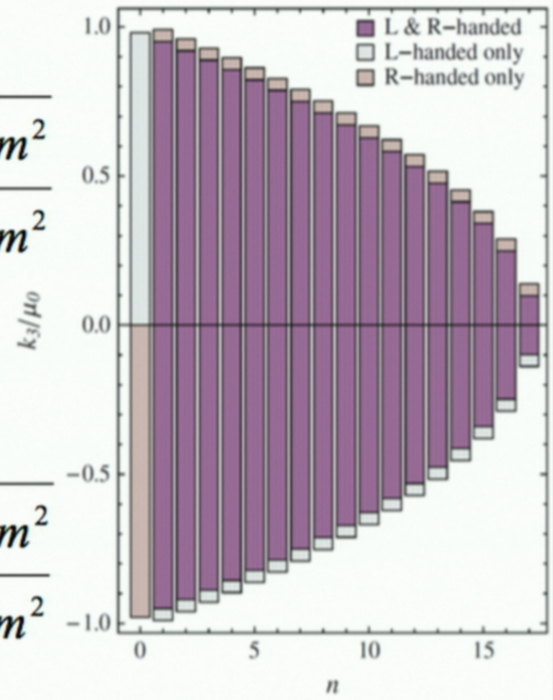


QED in weak field (B→0)

- The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta(p) + \gamma^0\gamma^5\mu_5(p)$$

Near Fermi surface ($p_0 \to 0$, $|\mathbf{p}| \to p_F$)

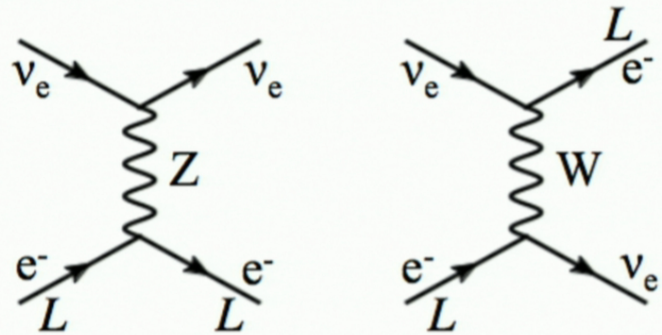
$$\Delta(p) \approx \frac{\alpha e B \mu}{\pi m^2}\left(\ln\frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1\right)$$

$$\mu_5(p) \approx -\frac{\alpha e B \mu}{\pi m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu(|\mathbf{p}| - p_F)} - 1\right)$$

[Gorbar, Miransky, Shovkovy, Wang, Phys. Rev. D **88**, 025043 (2013)]

Neutrino asymmetry

- Neutrinos equilibrate with the "flow" of L-handed fermions via



- An asymmetric L-handed Fermi surface with

$$\delta p_3 \sim \alpha \, |eB|/\mu$$

should scatter ν_e's more preferably in the direction of the field

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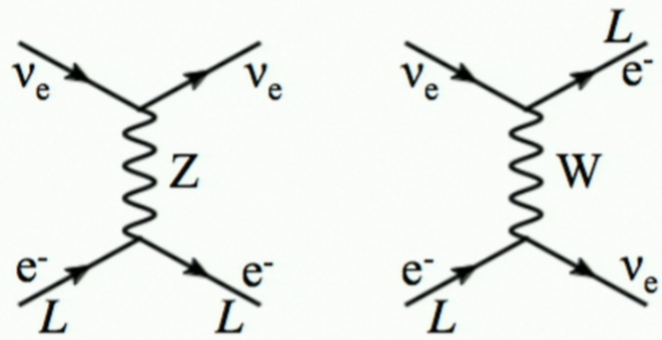

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should scatter ν_e's more preferably in the direction of the field

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Neutrinos from protoneutron star

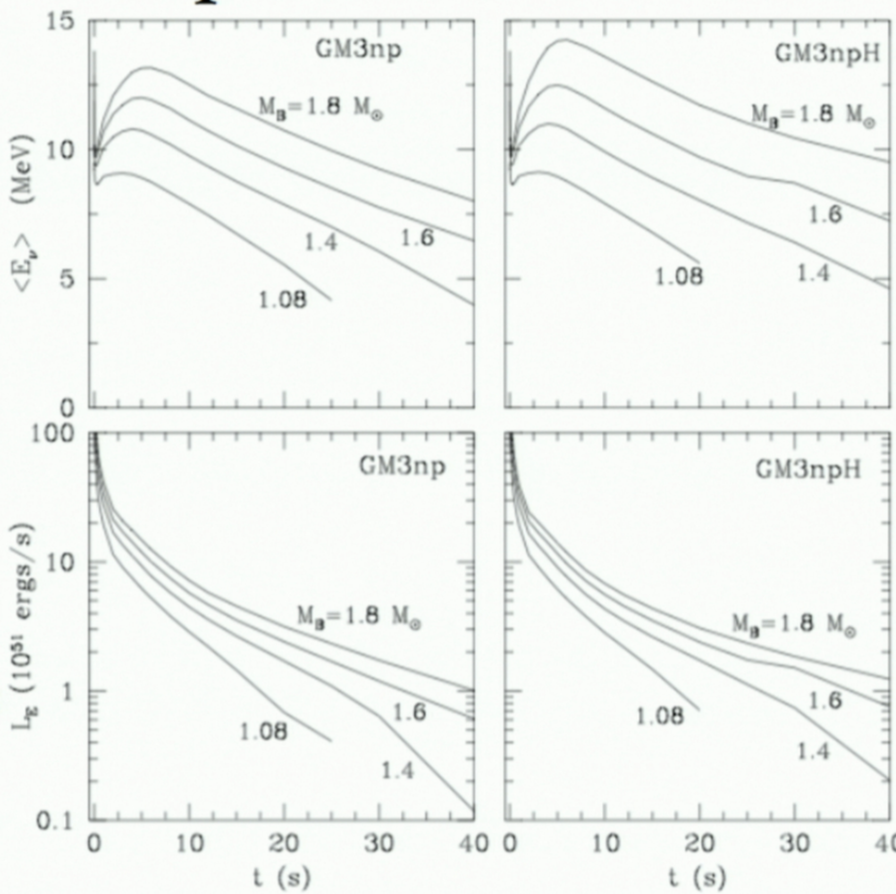
$\langle E_\nu \rangle \approx 5 \text{ to } 10 \text{ MeV}$

$L_E \approx 2\times10^{51}$ erg/s

$\approx 10^{57}$ MeV/s

$$N_{\text{tot}} \approx \frac{10^{57}\text{ MeV/s}}{5\text{ MeV}} 20\text{ s}$$

$$\approx 4\times10^{57}$$



[Pons, Reddy, Prakash, Lattimer, Miralles, Astrophys.J. 513 (1999) 780]

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Total momentum estimate

- Total momentum carrier by L-handed fermions
  - QED (B=10^{18} G and μ=100 MeV):

$$P \sim N\delta p \sim 10^{57} \frac{\alpha|eB|}{\mu} \sim (70\,\text{km/s}) \times 1.4 M_{\text{Sun}}$$

  - QCD (B=10^{18} G and μ=400 MeV):

$$P \sim N\delta p \sim 10^{57} \frac{\alpha_s|eB|}{\mu} \sim (1700\,\text{km/s}) \times 1.4 M_{\text{Sun}}$$

- Pulsar kicks? Possible, but questions remain…

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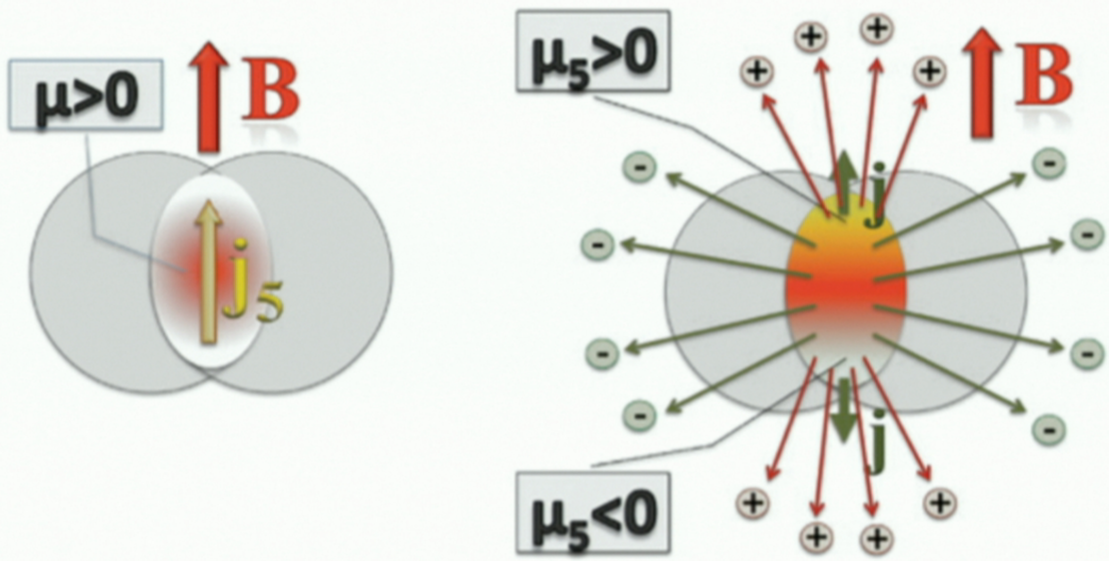
Quadrupole CME

- Start from a small baryon density and B≠0



- Produce back-to-back electric currents

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Experiment

One of the observable implications of the quadrupole CME is the splitting of the elliptic flows of positive and negative pions, i.e., $v_2^{\pi^-} - v_2^{\pi^+} = r_e A$, where A is the net charge asymmetry $A = (\bar{N}^+ - \bar{N}^-)/(\bar{N}^+ + \bar{N}^-)$ and $r_e > 0$ is the slope parameter.

Such a splitting was observed by the STAR collaboration and appears to be in qualitative agreement with the theoretical predictions.

[L. Adamczyk, Phys. Rev. C **89**, 044908 (2014)]
[G. Wang, Nucl. Phys. A **904-905**, 248c-255c (2013)]
[H. Ke, J. Phys. Conf. Ser. **389**, 012035 (2012)]

Summary

- LLL chiral asymmetry plus **interactions** generate chiral shift/asymmetry in higher LLs
- Chiral asymmetry shifts the L-handed and R-handed **Fermi surfaces** along **B**-field direction
- Chiral asymmetry can produce asymmetric **neutrino emission** and generate **pulsar kicks**
- The mechanism is more promising for quark stars, but may even affect all compact stars
- Chiral asymmetry seems to be an important ingredient in dynamics in heavy ion collisions

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