

Title: Explorations in String Theory -15

Date: Apr 24, 2015 11:30 AM

URL: <http://pirsa.org/15040154>

Abstract:

Harmonic Oscillator

Q

SC

AdS<sub>5</sub> × S<sup>5</sup> strings

decoupling

Symmetries

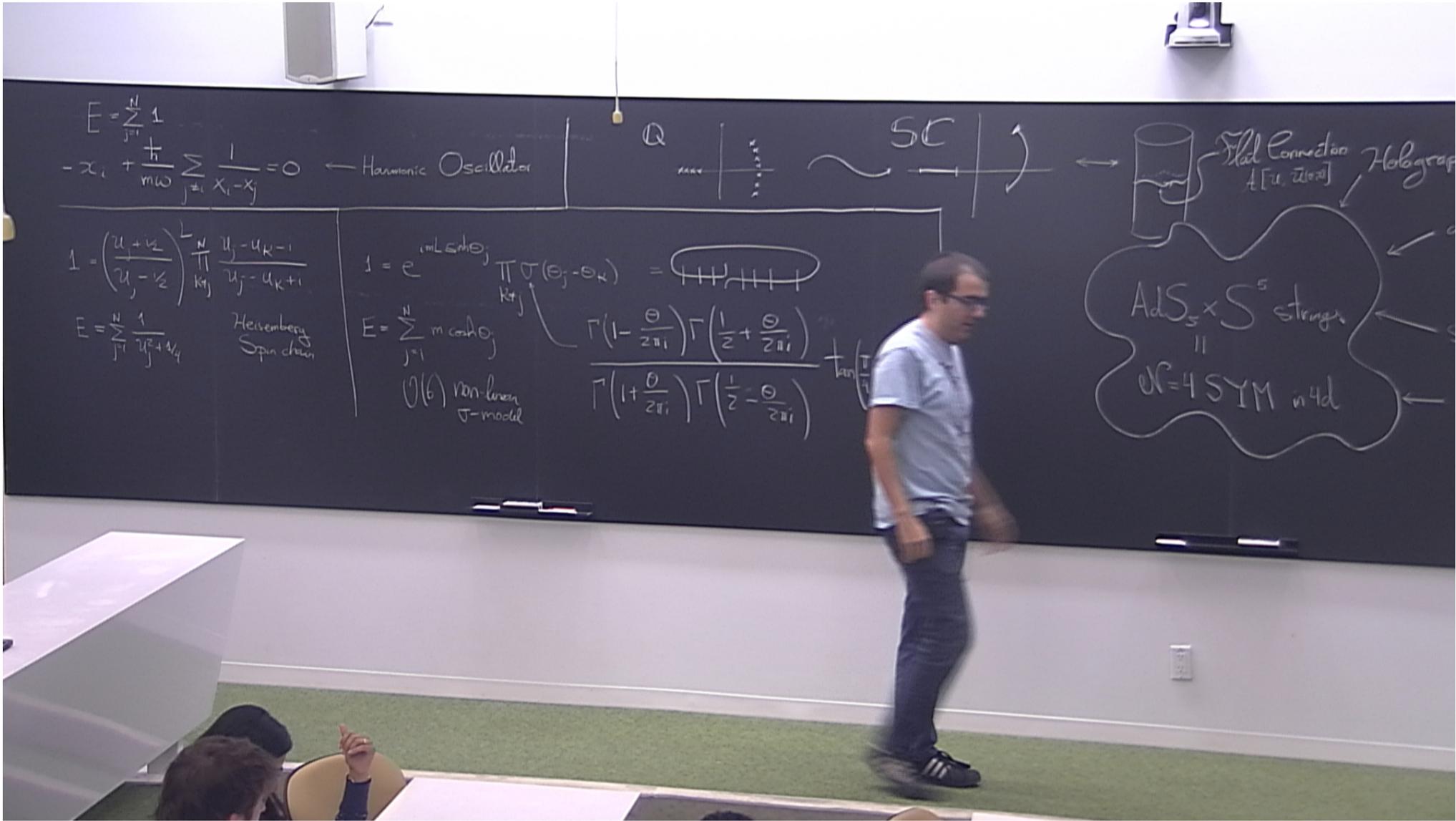
experimental evidence

Holographic principle

Spacetime Connection [U, U(1)]

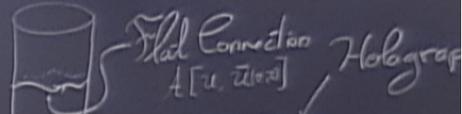
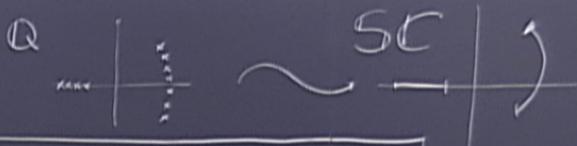
$$1 = e^{i \sum_{j=1}^n m_j \theta_j} \prod_{k < j} \sigma(\theta_j - \theta_k) = \frac{\Gamma(1 - \frac{\theta}{2\pi i}) \Gamma(\frac{1}{2} + \frac{\theta}{2\pi i})}{\Gamma(1 + \frac{\theta}{2\pi i}) \Gamma(\frac{1}{2} - \frac{\theta}{2\pi i})} \tan\left(\frac{\pi}{4} - \frac{\theta}{2i}\right)$$

0(6) non-linear J-model



$$E = \sum_{j=1}^N \frac{1}{j^2}$$

$$-x_i + \frac{1}{m\omega} \sum_{j \neq i} \frac{1}{x_i - x_j} = 0 \quad \leftarrow \text{Harmonic Oscillator}$$



$$1 = \left( \frac{u_j + u_k}{u_j - u_k} \right)^L \prod_{k_j} \frac{u_j - u_{k-1}}{u_j - u_{k+1}}$$

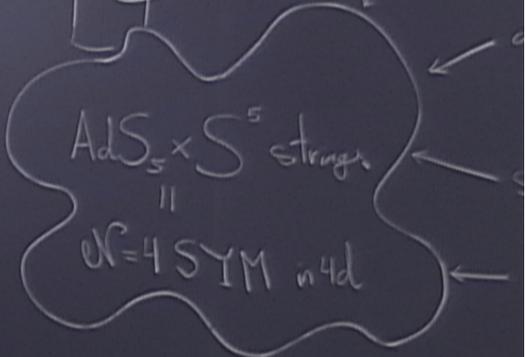
$$E = \sum_{j=1}^N \frac{1}{u_j^2 + u_j^4} \quad \text{Heisenberg Spin chain}$$

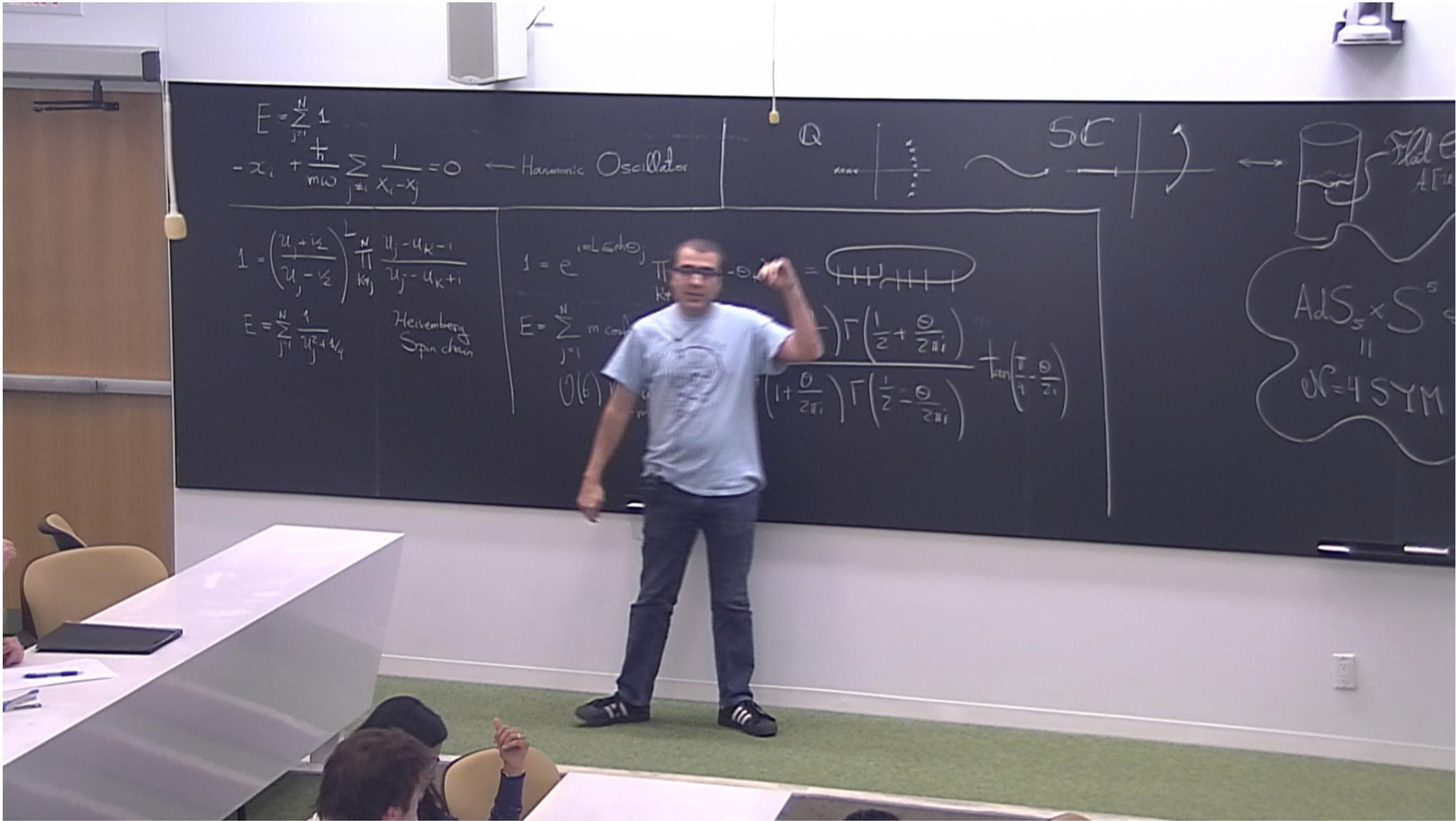
$$1 = e^{imL\sum \theta_j} \prod_{k_j} \sigma(\theta_j - \theta_k) = \text{Diagram}$$

$$E = \sum_{j=1}^N m \cos \theta_j$$

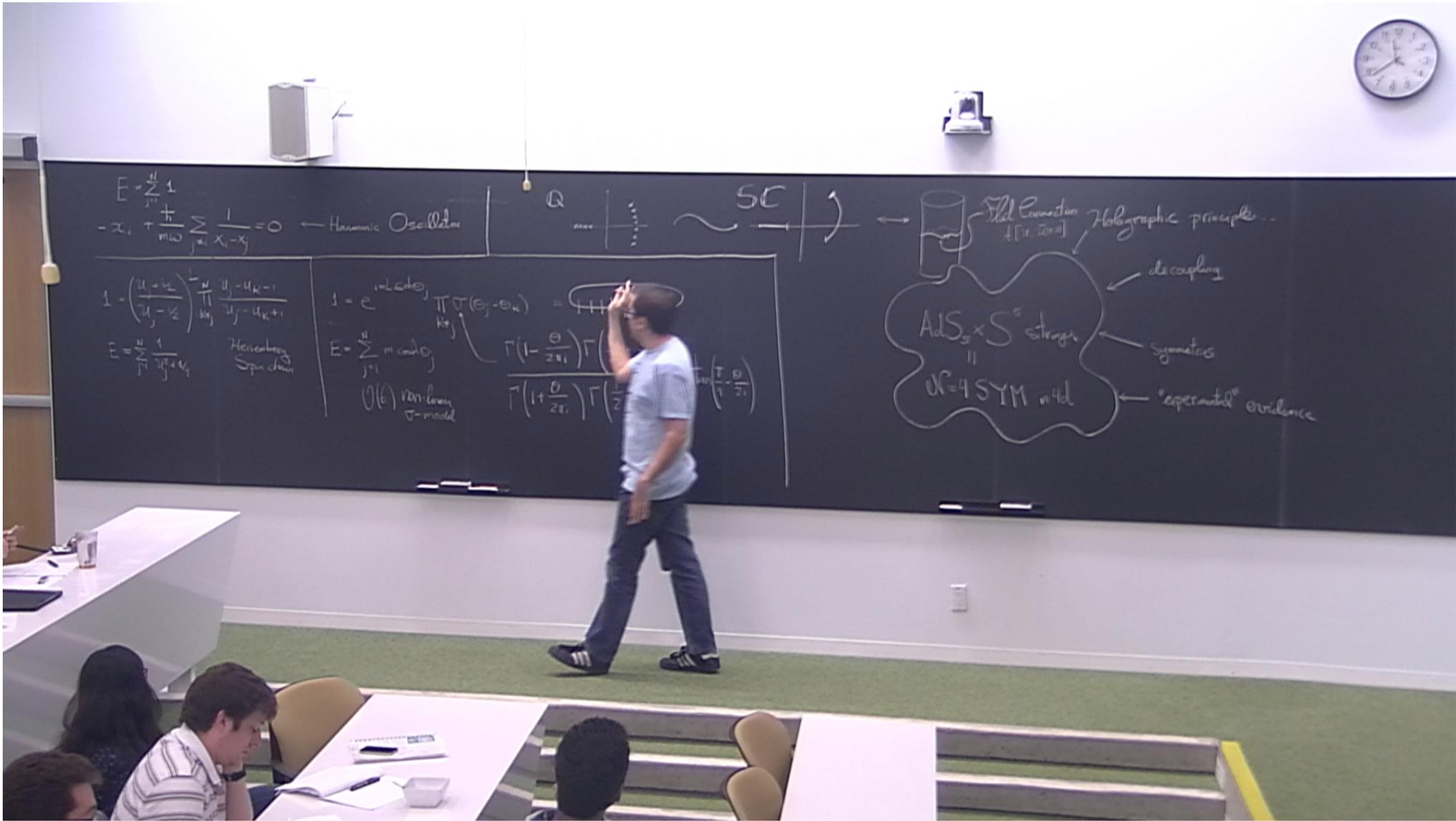
(6) non-linear T-model

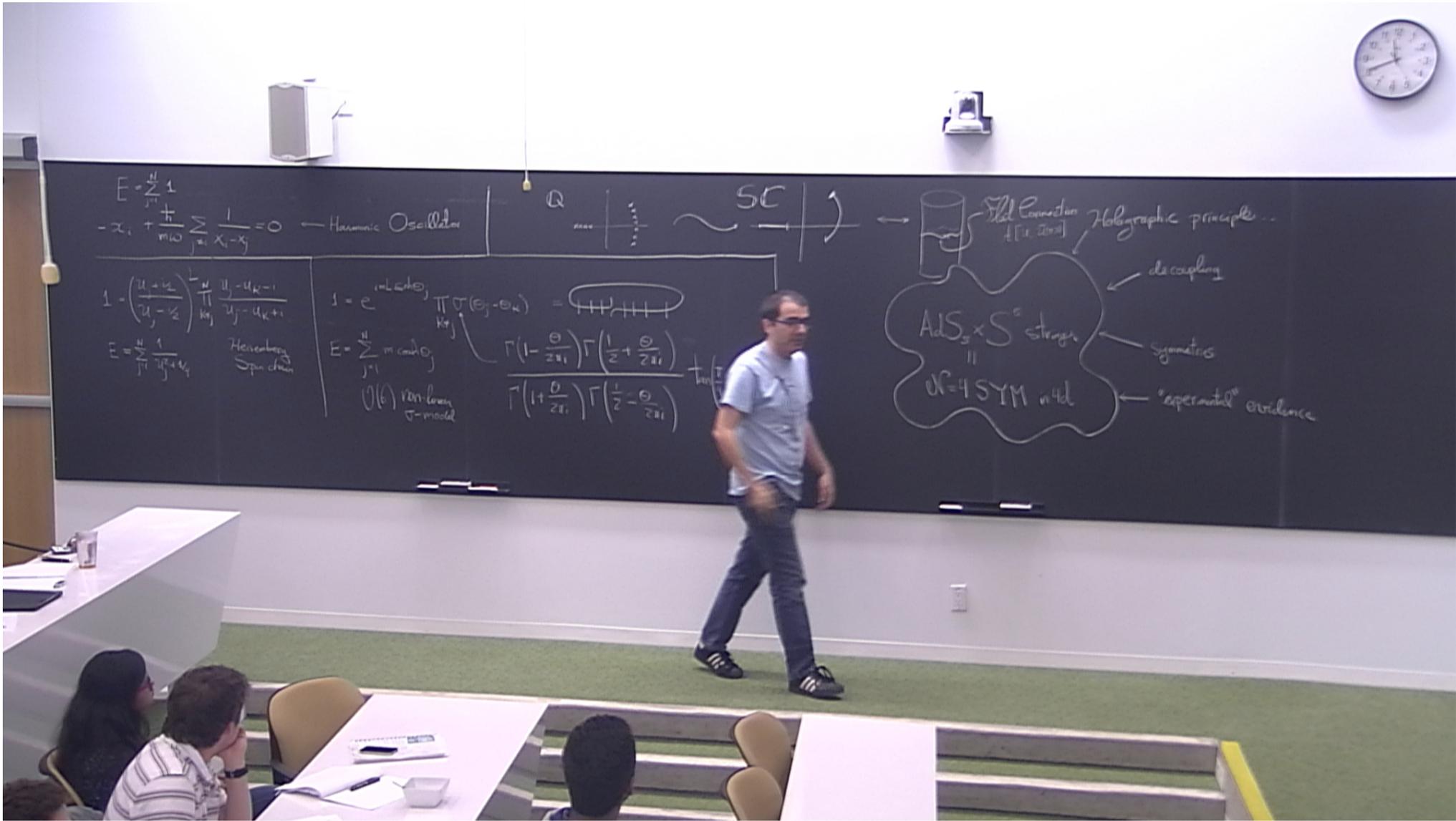
$$\frac{\Gamma(1 - \frac{\theta}{2\pi i}) \Gamma(\frac{1}{2} + \frac{\theta}{2\pi i})}{\Gamma(1 + \frac{\theta}{2\pi i}) \Gamma(\frac{1}{2} - \frac{\theta}{2\pi i})} \frac{1}{\tan(\frac{\theta}{4})}$$

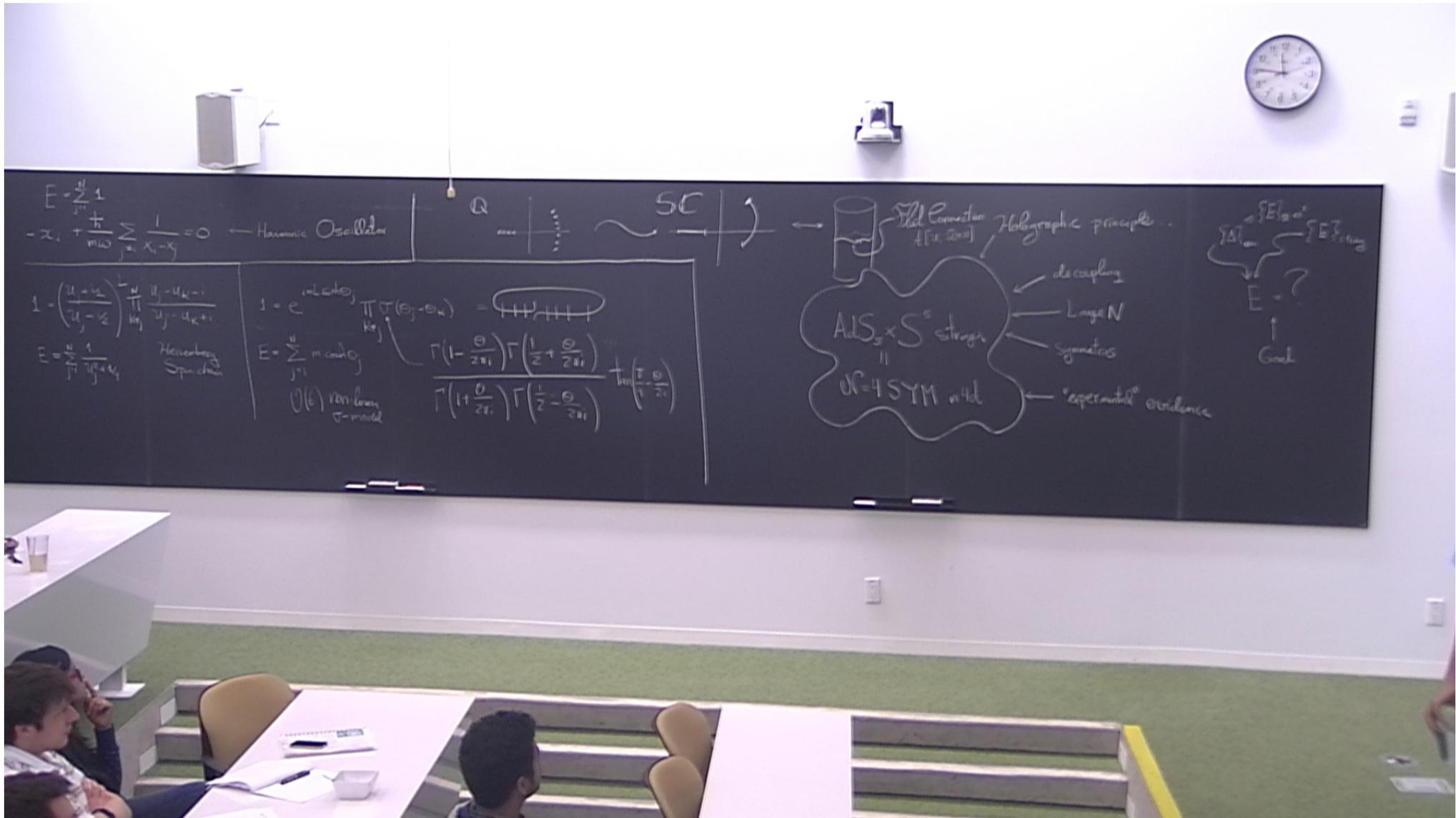












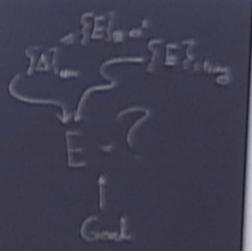
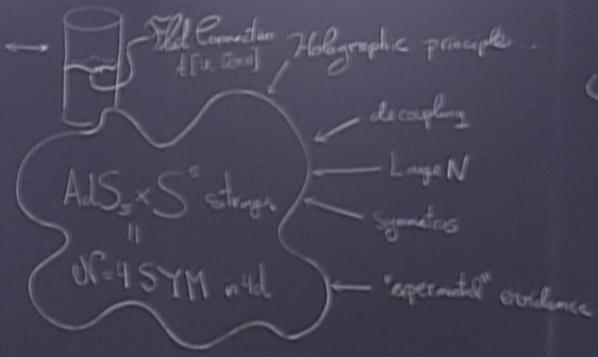
$$E = \sum_{j=1}^M \frac{1}{x_j} - \sum_{j=1}^M \frac{1}{x_j - x_j} = 0 \quad \text{--- Harmonic Oscillator}$$

$$1 = \prod_{j=1}^M \frac{(u_j - u_{k-1})}{(u_j - u_{k+1})}$$

Hiroseki's Spectral

$$1 = e^{-i \sum_{j=1}^M \theta_j} \prod_{j=1}^M \frac{\Gamma(\frac{1}{2} - \frac{\theta_j}{2\pi i})}{\Gamma(\frac{1}{2} + \frac{\theta_j}{2\pi i})}$$

$\theta_j$  phases  
J-model



$A=1 \dots n$   
 $\in (\theta), p(\theta)$

$\prod_i (z \dots z x z \dots z x z \dots)$

$\overset{x}{\cdot} \in (\varphi)$



$$A = 1 \dots n$$

$$\in (\theta), p(\theta)$$

$$\frac{1}{r} (Z \dots ZXZ \dots ZXZ \dots)$$

$$\frac{X}{r} \in (\mathbb{P})$$

in reality

$$\frac{1}{r} (Z$$



$$A = 1 \dots n$$

$$\in (\theta), p(\theta)$$

$$\prod_i (z \dots z X z \dots z X z \dots)$$

$$X \in (\mathbb{P})$$

in reality

$$\prod_i (z \dots z X z \dots z \nabla z \dots)$$

6 scalars

$$Z = \phi_1 + i$$

$$X = \phi_2 + i$$

$$Y = \phi_5 + i$$

$$A = 1 \dots n$$

$$\in (\theta), p(\theta)$$

$$\frac{1}{r} (z \dots z X z \dots z X z \dots)$$

$$X \in (\mathbb{P})$$

in reality

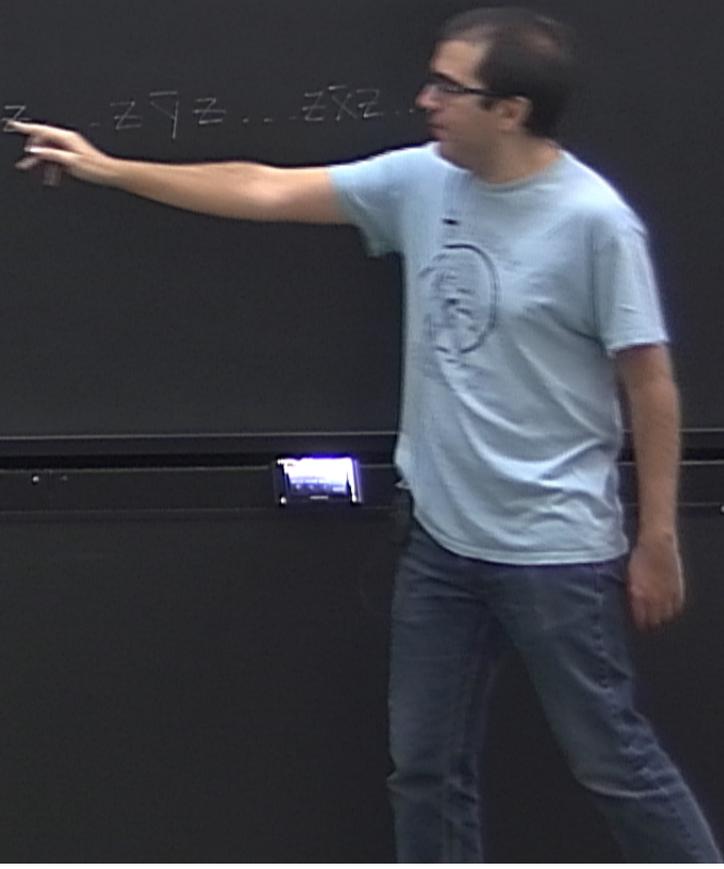
$$\frac{1}{r} (z \dots z X z \dots z Y z \dots z \nabla z \dots z X z \dots)$$

6 Scalars in  $\mathbb{R}^4$

$$Z = \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4 + c\sigma$$

$$Y = \phi_5 + i\phi_6$$



(e)

$$\frac{1}{i} (Z \dots ZXZ \dots ZXZ \dots)$$

$X \rightarrow \in (\mathbb{P})$

in reality

$$\frac{1}{i} (Z \dots ZXZ \dots ZYZ \dots ZYZ \dots ZXZ \dots ZZZ \dots)$$

excitations

unstable

$H$

$\bar{Y}$  modes

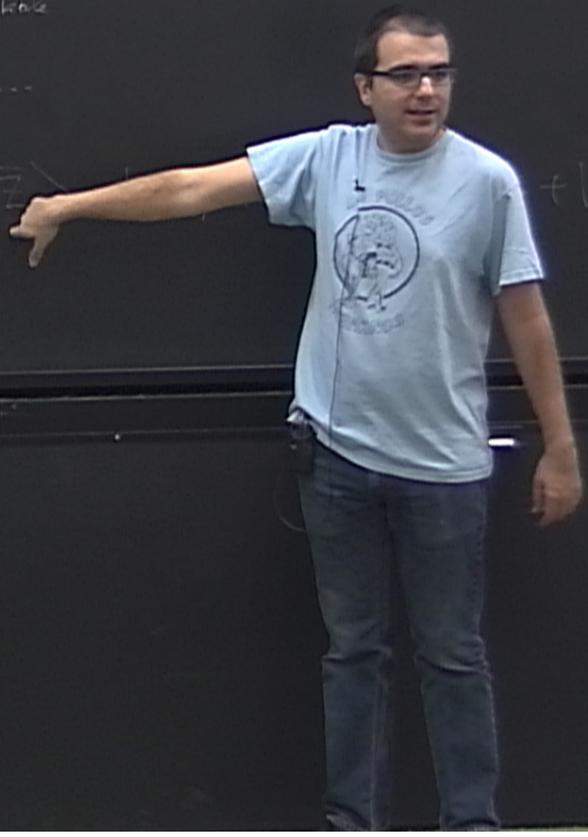
6 scalars in  $\mathcal{O}^2=4$

$$Z = \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4 + c.c.$$

$$Y = \phi_5 + i\phi_6$$

$$H|ZZ\rangle + |YY\rangle + \dots$$



(e)

$$\frac{1}{i} (z \dots z X z \dots z X z \dots)$$

$$X \rightarrow \in (\mathbb{P})$$

in reality

$$\frac{1}{i} (z \dots z)$$

excitations

H

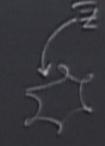
Y rows

unstable

excitation

X

→



← X

Scalars

$$Z = \phi$$

$$X = \phi$$

$$Y = \phi$$

$$H|z\bar{z}\rangle = |xx\rangle + |x\bar{x}\rangle + |y\bar{y}\rangle + |y\bar{y}\rangle + \dots$$

Scalars: SO(4) symmetry

(e)

$X \in \mathbb{P}$

$\frac{1}{i} (Z \dots ZXZ \dots ZXZ \dots)$

in reality

$\frac{1}{i} (Z \dots ZXZ \dots ZYZ \dots ZYZ \dots ZXZ \dots Z\bar{Z}Z \dots ZDZ \dots Z\bar{D}Z \dots)$

excitations

unstable

excitation

$X \rightarrow \bar{X}$

$\bar{Y}$  modes

$H |Z\bar{Z}\rangle = |X\bar{X}\rangle + |X\bar{Y}\rangle + |Y\bar{Y}\rangle + |Z\bar{Z}\rangle$

Scalars:  $SO(4)$  symmetry

derivatives:  $SO(1,3)$  symmetry

$Z = \phi_1 + i\phi_2$

$X = \phi_3 + i\phi_4 + c.c.$

$Y = \phi_5 + i\phi_6$

$\frac{1}{i} (Z \dots ZXZ \dots ZXZ \dots)$   
 $\cdot X \in \mathbb{P}$   
 $\bar{Z}$

in reality  
 $\text{tr} (Z \dots ZXZ \dots ZYZ \dots ZYZ \dots ZXZ \dots ZZZ \dots ZDZ \dots ZDZ \dots Z \psi \dots Z \psi^a \dots Z)$

excitations  
 vacuum  
 $H$   
 $\bar{Y}$  modes

$H|Z\bar{Z}\rangle = |X\bar{X}\rangle + |X\bar{X}\rangle + |Y\bar{Y}\rangle + |Y\bar{Y}\rangle + |Z\bar{Z}\rangle$

Scalars:  $SO(4)$  symmetry =  $SU(2) \times SU(2)$   
 derivatives:  $SO(1,3)$  symmetry =  $SU(2) \times SU(2)$

+ fermions =  $SU(2)^2$  X important details



8 bosons + 8 f

$$\chi = \phi_3 + i\phi_4 + c.c.$$

$$\gamma = \phi_5 + i\phi_6$$

Scalars:  $SO(4)$  symmetry =  $S^3$   
 derivatives:  $SO(4,3)$  symmetry =  $S^4$

8 bosons + 8 fermions  
 $\underbrace{\hspace{1cm}}_{4D + 4 \phi's}$



by  $SO(2|2)^2 \leftarrow$  compact.

$\Uparrow$   
 full sym group of  
 Spin-chain



$$\begin{aligned}
 Z &= \phi_1 + i\phi_2 \\
 X &= \phi_3 + i\phi_4 + c.c. \\
 Y &= \phi_5 + i\phi_6
 \end{aligned}$$

$Y$  mod 5

Scalars:  $SO(4)$  symmetry =  $SU(2) \times SU(2)$   
 derivatives:  $SO(4,3)$  symmetry =  $SU(2) \times SU(2)$

8 bosons + 8 fermions  
 $\uparrow$   
 $4D + 4\phi$ 's



by  $SU(2|2)^2 \leftarrow$  compact.



full sym group of  
 Spin-chain  
 $PSU(2,2|4)$

$|site\rangle$

$\uparrow$   
 $Z, X, Y, \bar{Z}, \dots, D, D^2, D^3$   
 $\underbrace{\hspace{10em}}_6$



$$\begin{aligned}
 Z &= \phi_1 + i\phi_2 \\
 X &= \phi_3 + i\phi_4 + c.c. \\
 Y &= \phi_5 + i\phi_6
 \end{aligned}$$

$\bar{Y}$  moves

$$|ZZ\rangle = |XX\rangle + |YY\rangle + |ZZ\rangle$$

Scalars:  $SO(4)$  symmetry =  $SU(2) \times SU(2)$  + fermions  $\equiv SU(2|2)^2$   $\times$  important details  
 derivatives:  $SO(1,3)$  symmetry =  $SU(2) \times SU(2)$

mions  by  $SU(2|2)^2 \leftarrow$  compact.

$\hat{=}$   
 full sym group of  
 Spin-chain  
 $PSU(2,2|4)$

$|site\rangle$   
 $\uparrow$   
 $\underbrace{Z, X, Y, \bar{Z}, \dots}_6 \underbrace{D, D^2, D^3, \dots}_\infty \leftarrow$  super non-compact spin-chain  
 with long range interactions & higher loops



$+i\phi_2$   
 $+i\phi_4 + cc$   
 $+i\phi_6$

$\downarrow$   
 $\gamma$  moves

$|ZZ\rangle = |XX\rangle + |YY\rangle + |YY\rangle + |ZZ\rangle$

Scalars:  $SO(4)$  symmetry =  $SU(2) \times SU(2)$  + fermions  $\equiv SU(2|2)^2$   $\times$  important details

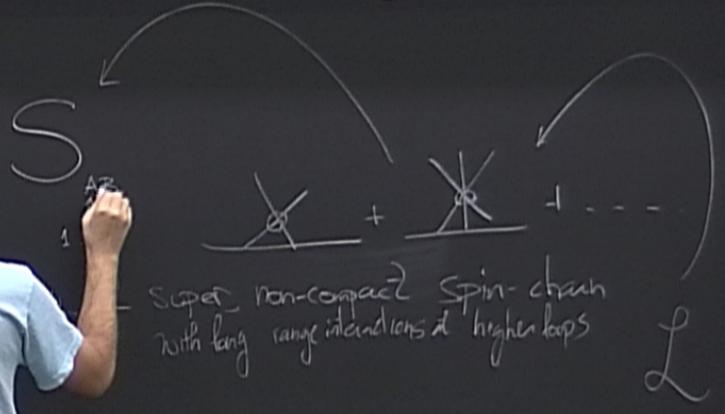
derivatives:  $SO(1,3)$  symmetry =  $SU(2) \times SU(2)$

$\curvearrowright$   
 $\phi$

by  $SO(2|2)^2 \leftarrow$  compact

$\Uparrow$   
 full sym group of  
 Spin-chain  
 $PSU(2,2|4)$

$|site\rangle$   
 $\uparrow$   
 $Z, X, Y, \bar{Z}$



$+i\phi_2$   
 $+i\phi_4 + cc$   
 $+i\phi_6$

$\downarrow$   
 $\gamma$  moves

$|ZZ\rangle = |XX\rangle + |YY\rangle + |YY\rangle + |ZZ\rangle$

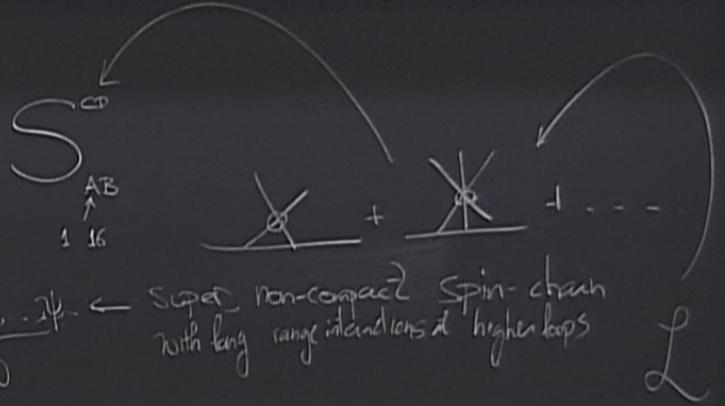
Scalars:  $SO(4)$  symmetry =  $SU(2) \times SU(2)$  + fermions  $\equiv SU(2|2)^2$   $\times$  important details

derivatives:  $SO(1,3)$  symmetry =  $SU(2) \times SU(2)$

$SO(2|2)^2 \leftarrow$  compact.

$\Uparrow$   
 full sym group of  
 Spin-chain  
 $PSU(2,2|4)$

$|site\rangle$   
 $\uparrow$   
 $\underbrace{Z, X, Y, \bar{Z}, \dots}_6$   $\underbrace{D, \bar{D}, \bar{D}, \dots}_\infty$







PSU(2,2|4)

$\underbrace{Z, X, Y, Z, \dots}_6 \quad \underbrace{D, D, D, \dots}_8$

With long range interactions at higher loops

$\mathcal{L}$

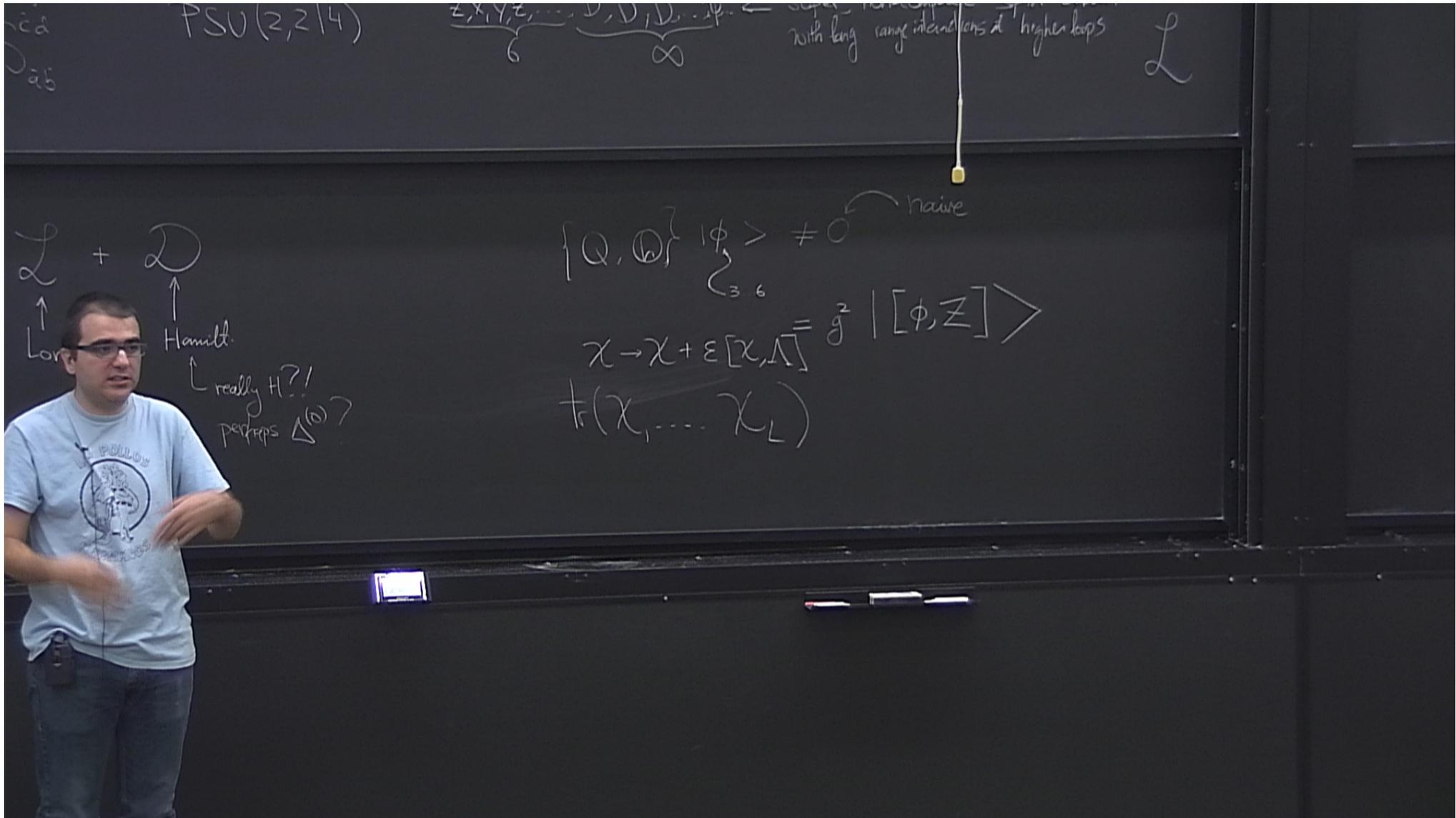
$\mathcal{L} + \mathcal{D}$   
↑ Lorentz

↑ Hamilt.  
really H?!  
perhaps  $\Delta^{(0)}$ ?

$$\{Q, \mathcal{D}\} |\phi\rangle \neq 0 \quad \text{naive}$$

$\underbrace{\quad}_{3 \ 6}$

$$= g^2 |[\phi, Z]\rangle$$



PSU(2,2|4)

$\underbrace{Z, X, Y, Z, \dots}_6, \underbrace{D, D, D, \dots}_\infty$

with long range interactions at higher loops

$L$

$\mathcal{L} + \mathcal{D}$

↑  
Lor

↑  
Hamilt.

really H?!  
perhaps  $\Delta(6)?$

$\{Q, W\} |\phi\rangle \neq 0$  naive

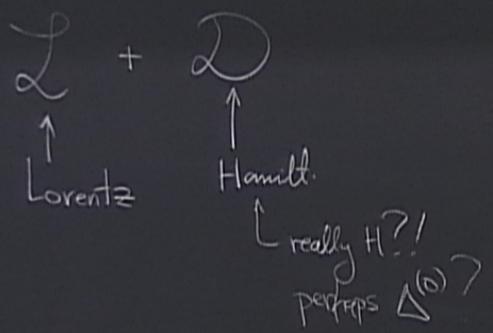
$\chi \rightarrow \chi + \varepsilon [X, \Lambda] = g^2 |[\phi, Z]\rangle$   
 $\text{tr}(\chi, \dots, \chi_L)$

PSU(2,2|4)

$\underbrace{z, x, y, z, \dots}_6 \quad \underbrace{D, D, D, \dots}_\infty \quad \phi$

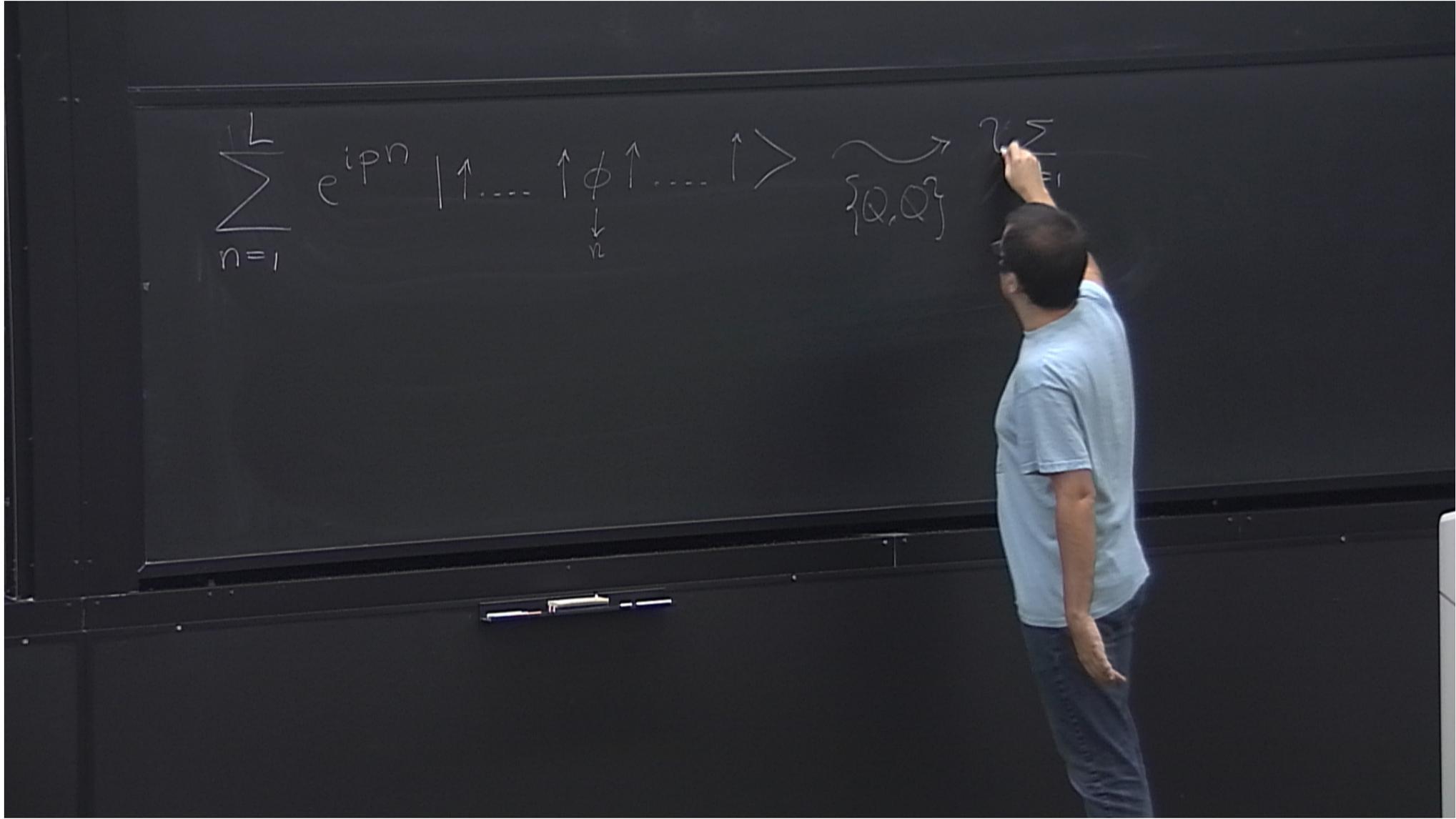
with long range interactions at higher loops

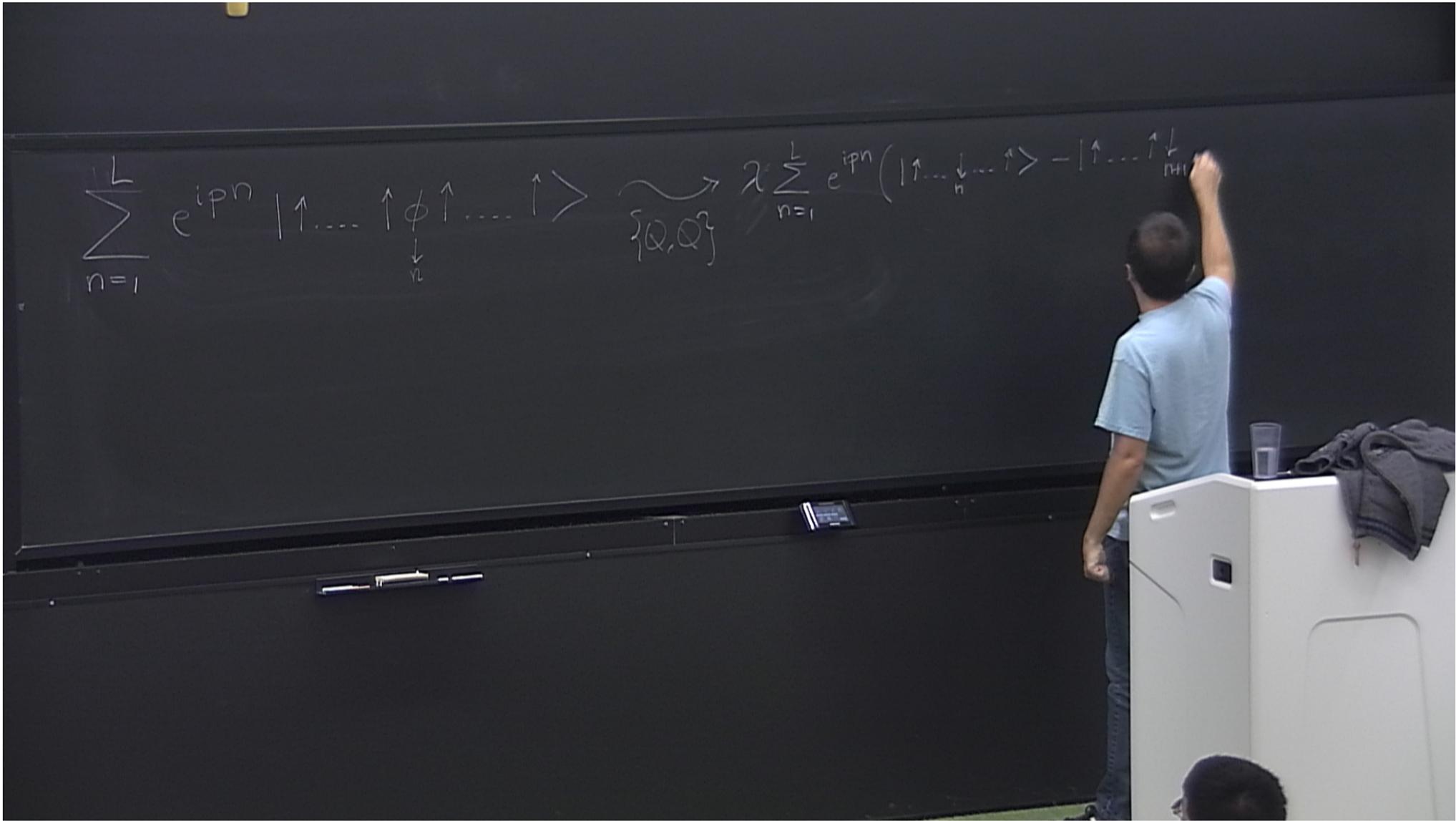
$\mathcal{L}$

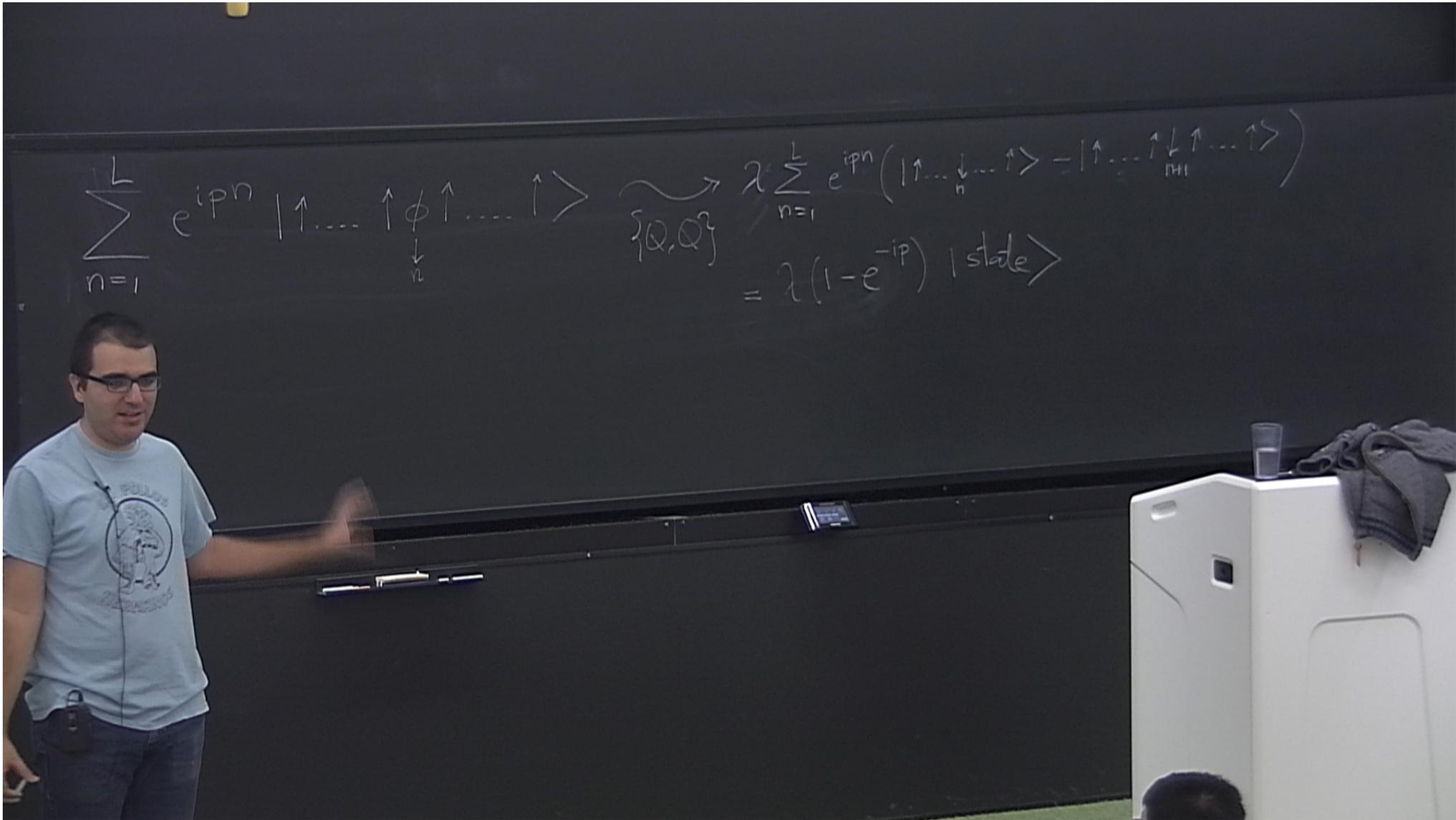


$\{Q, \mathcal{D}\} |\phi\rangle \neq 0$       naive  
 $\underbrace{\quad}_{3,6}$

$\chi \rightarrow \chi + \varepsilon [\chi, \Lambda] = g^2 |[\phi, Z]\rangle$   
 $\text{tr}(\chi_1, \dots, \chi_L)$  } trivial on full state but perhaps not on indiv. cc.







$$\sum_{n=1}^L e^{ipn} |\uparrow \dots \uparrow \downarrow_n \uparrow \dots \uparrow\rangle \xrightarrow{\{Q, Q^\dagger\}} \lambda \sum_{n=1}^L e^{ipn} (|\uparrow \dots \downarrow_n \uparrow \dots \uparrow\rangle - |\uparrow \dots \uparrow \downarrow_{n+1} \uparrow \dots \uparrow\rangle)$$

$$= \lambda (1 - e^{-ip}) |\text{state}\rangle$$

$$\sum_{n=1}^L e^{ipn}$$

$$|\uparrow \dots \uparrow \phi \uparrow \dots \uparrow\rangle$$

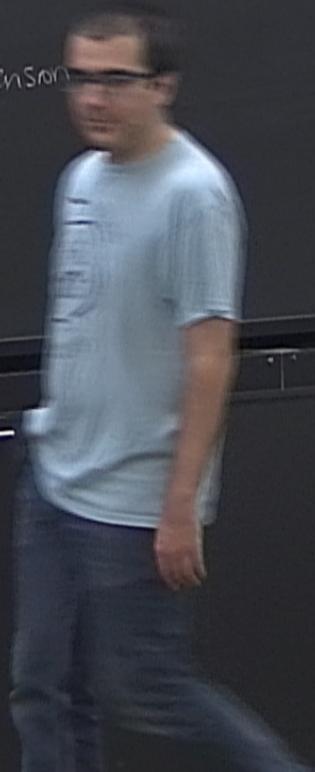
$\{Q, Q^\dagger\}$

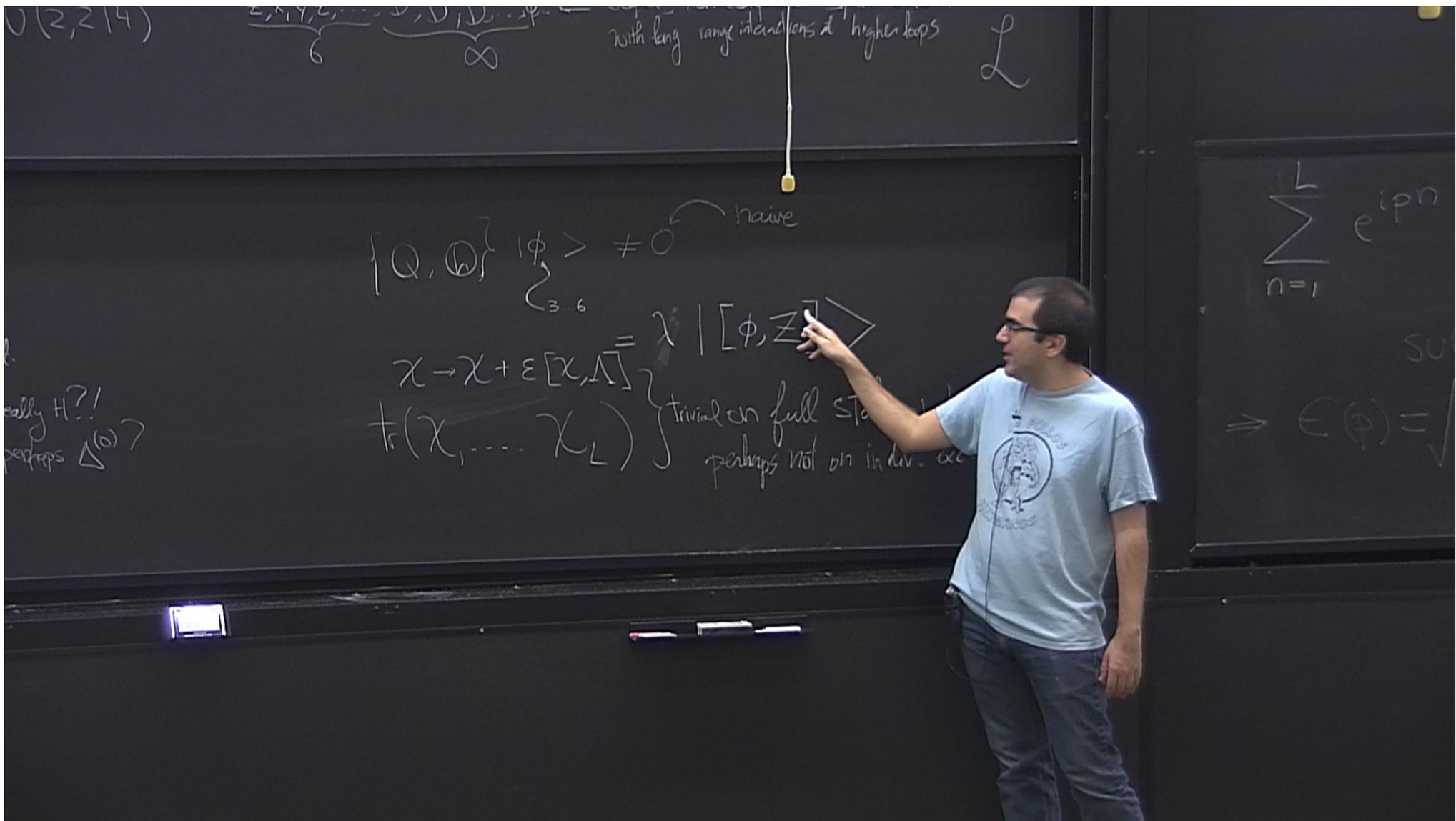
$$\lambda \sum_{n=1}^L e^{ipn} (|\uparrow \dots \downarrow_n \dots \uparrow\rangle - |\uparrow \dots \uparrow_{n+1} \dots \uparrow\rangle)$$

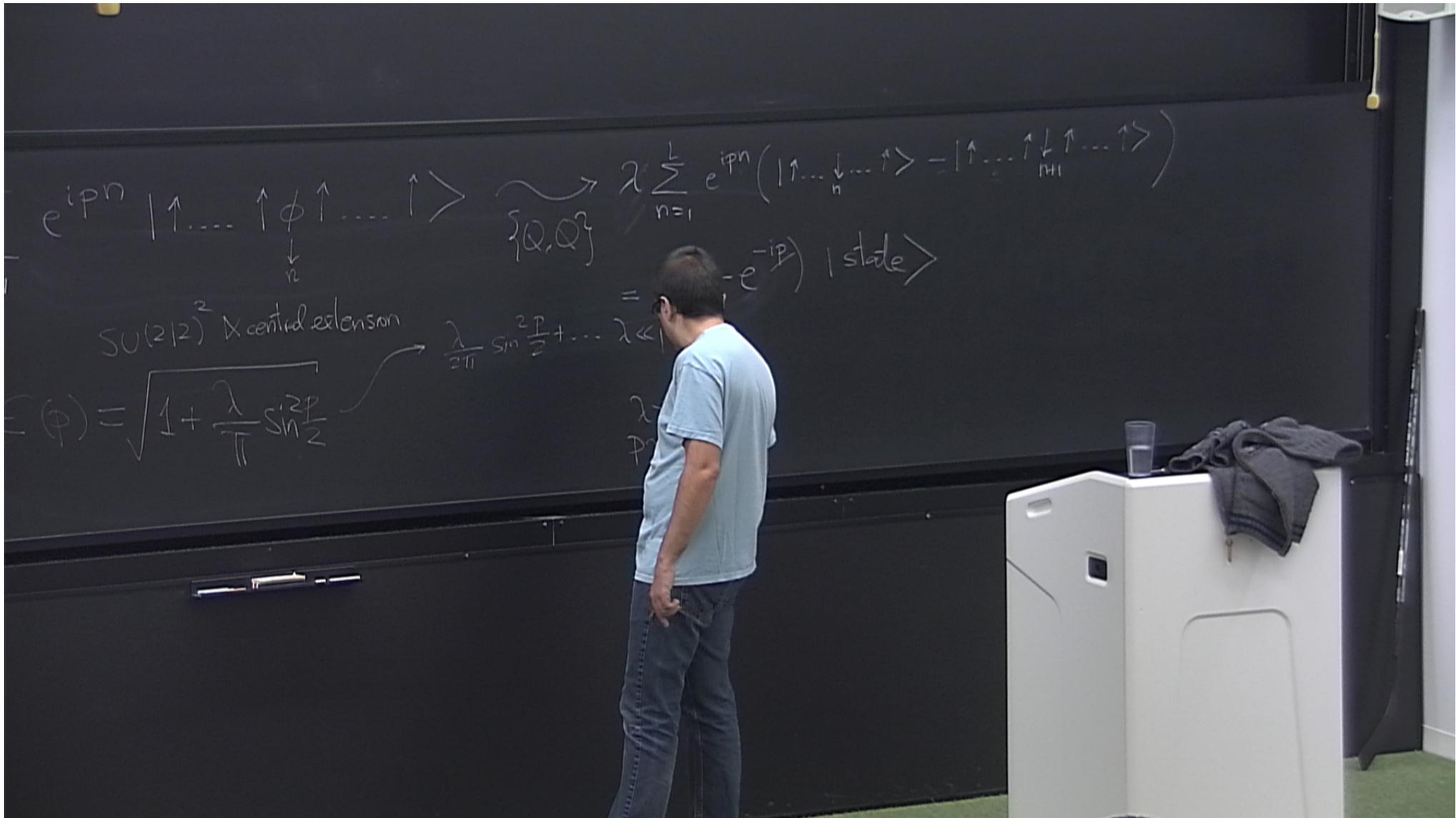
$$= \lambda (1 - e^{-ip}) |\text{state}\rangle$$

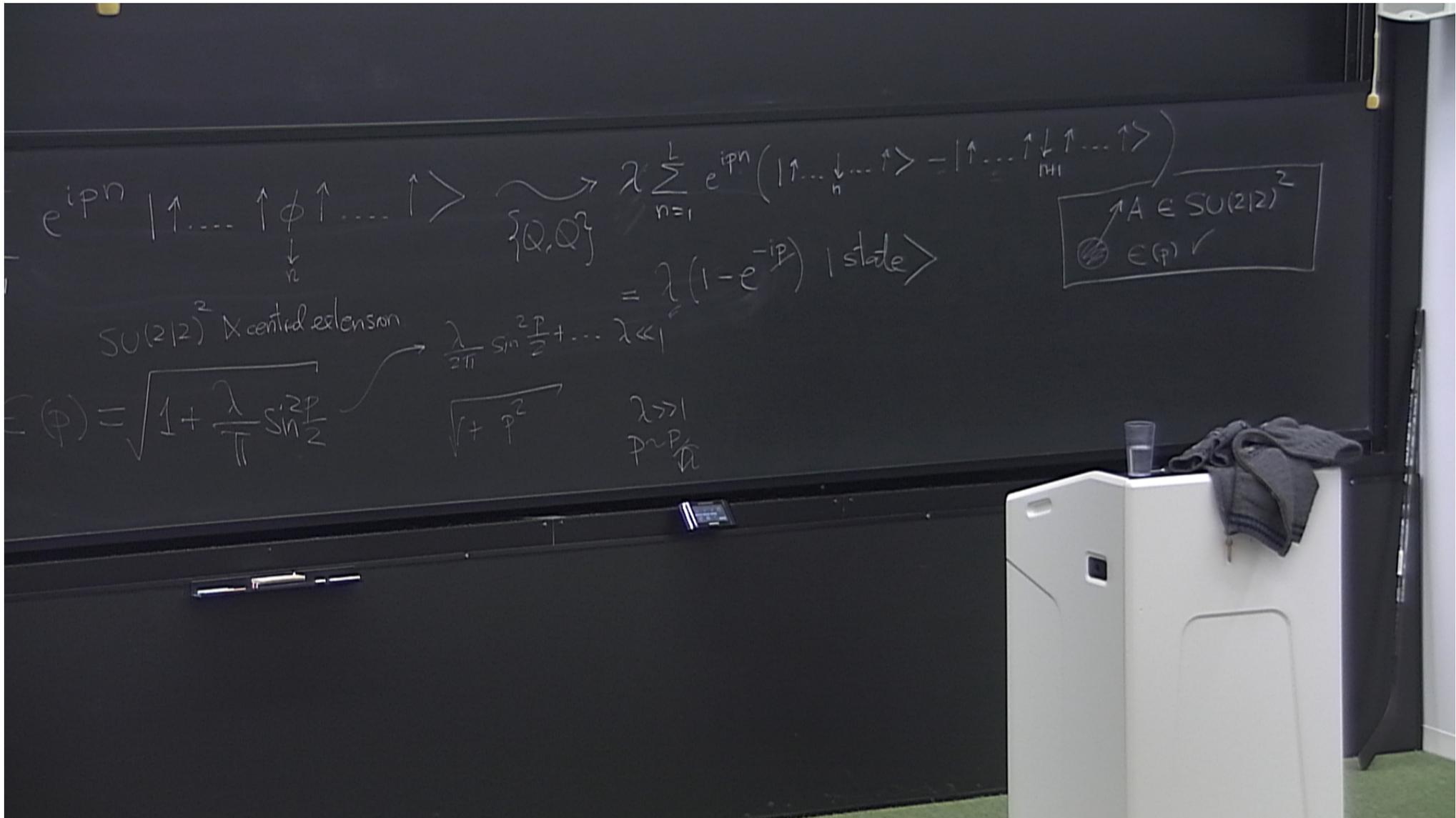
$SU(2|2)^2$  x central extension

$$E(p) = \sqrt{1 + \frac{\lambda}{\pi} \sin^2 \frac{p}{2}}$$









$$e^{ipn} |\uparrow \dots \uparrow \phi \uparrow \dots \uparrow\rangle$$

$\downarrow$   
 $n$

$\{Q, Q^2\}$

$$\lambda \sum_{n=1}^L e^{ipn} (|\uparrow \dots \downarrow_n \dots \uparrow\rangle - |\uparrow \dots \uparrow \downarrow_{n+1} \dots \uparrow\rangle)$$

$A \in SU(2|2)^2$   
 $\in \mathfrak{g}$  ✓

$$= \lambda (1 - e^{-ip}) |\text{state}\rangle$$

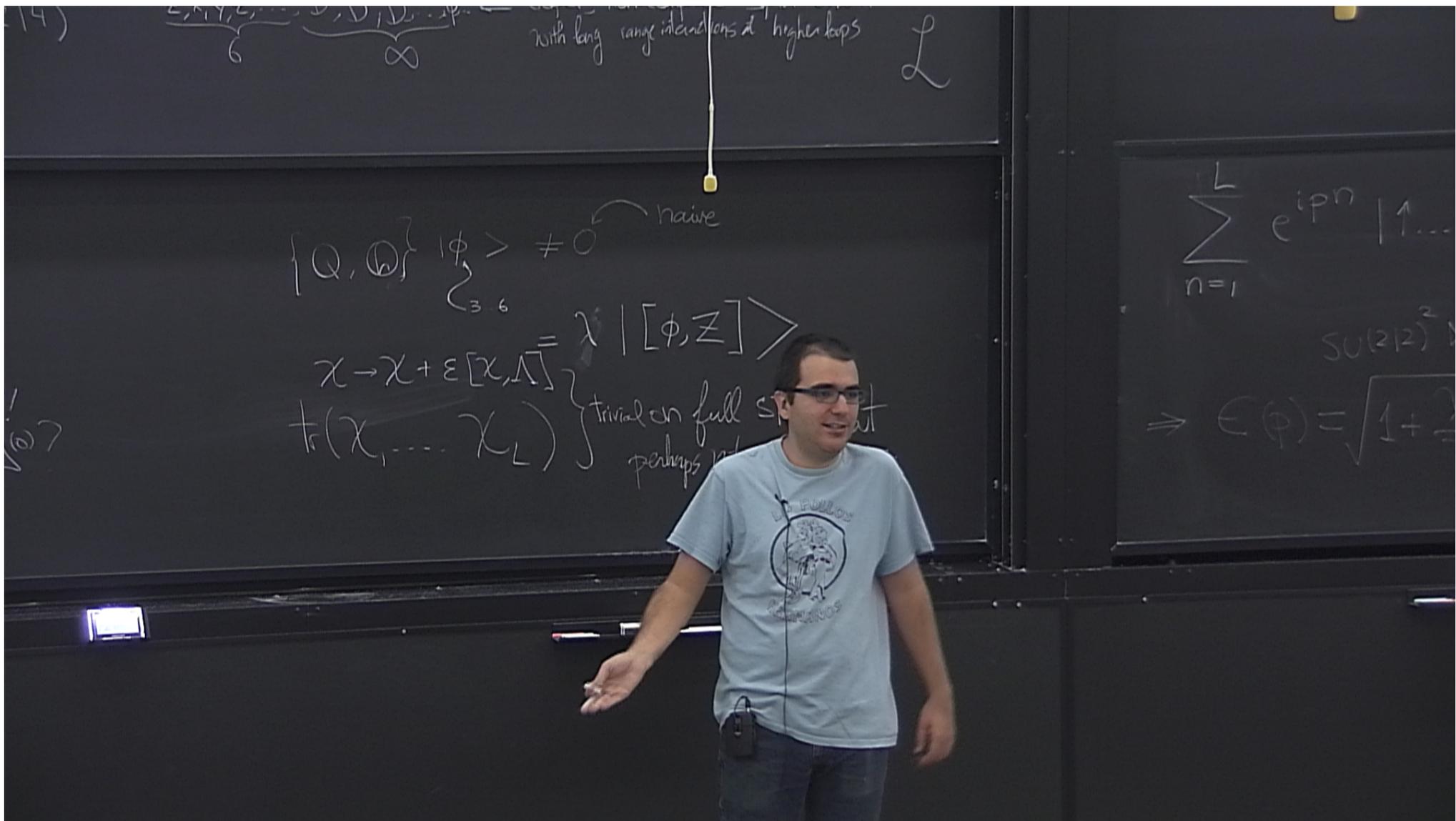
$SU(2|2)^2$  X central extension

$$\frac{\lambda}{2\pi} \sin^2 \frac{p}{2} + \dots \quad \lambda \ll 1$$

$$E(p) = \sqrt{1 + \frac{\lambda}{\pi} \sin^2 \frac{p}{2}}$$

$$\sqrt{1 + p^2} \quad \lambda \gg 1$$

$p \sim \frac{p}{\lambda}$



(14)

$\underbrace{2, 4, 6, \dots}_{6}$   $\underbrace{3, 5, 7, \dots}_{\infty}$  with long range interactions at higher loops  $L$

naive

$$\{Q, \Phi\} |\phi\rangle \neq 0$$

3 6

$$X \rightarrow X + \epsilon [X, \Lambda]$$

$$\chi \rightarrow \chi + \epsilon [\chi, \Lambda]$$

$$\chi \rightarrow \chi + \epsilon [\chi, \Lambda]$$

trivial on full space  
perhaps not

$$\sum_{n=1}^L e^{i p n} |1 \dots$$

$SU(2|2)^2$

$$\Rightarrow E(\Phi) = \sqrt{1 + \dots}$$

PSU(2,2|4)

$\underbrace{z, x, y, z, \dots}_6 \quad \underbrace{D, D, D, \dots}_\infty$

with long range interactions at higher loops

$\mathcal{L}$

$\mathcal{L} + \mathcal{D}$

↑  
Lorentz

↑  
Hamilt.

really H?!  
perhaps  $\Delta^{(6)}$ ?

$\Rightarrow S = \hat{S} \times \mathcal{J}(u_1, u_2)$   
YB

$\{Q, \mathcal{D}\} |\phi\rangle \neq 0$  naive

$\chi \rightarrow \chi + \epsilon [\chi, \Lambda] = \lambda |[\phi, Z]\rangle$

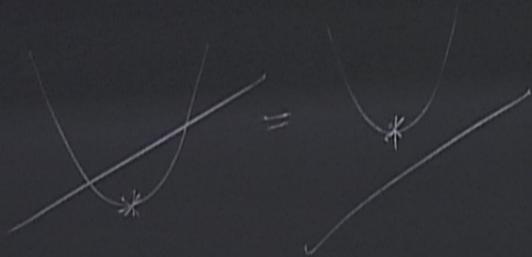
$\text{tr}(\chi_1, \dots, \chi_L)$  } trivial on full state but perhaps not on indiv. exc.

$$\Rightarrow E(\varphi) = \sqrt{1 + \frac{\lambda^2 p^2}{\hbar^2} \sin^2 \frac{\varphi}{2}}$$

$$\sqrt{1 + p^2}$$

$$\lambda \gg 1$$

$$p \sim \frac{p_0}{\hbar}$$



$$\sigma(\bar{u}, \tau) \sigma(u, \tau) = \text{something explicit}$$

Soln.



$$e^{i p L} \quad T S = 1, E = 2E$$

+ corr.  
small

$$\langle \text{Tr}(Z \dots Z X Z \dots Z X) \text{Tr}(\dots) \rangle$$

23560

$$\langle \text{Tr}(ZX)^2 \text{Tr}(ZX)^2 \rangle$$

$$\boxed{\text{good}} + O(\lambda^{23561})$$



