Title: Explorations in String Theory -14

Date: Apr 23, 2015 11:30 AM

URL: http://pirsa.org/15040153

Abstract:

Toy example:

$SO(n)$ σ-model $\equiv$ unit vector $\vec{u}(\sigma,\tau) \in S^{n-1}$

$$S^{k\ell}_{ij}(\theta \equiv \theta_1 - \theta_2) = \left[\frac{1}{1+\beta\theta}\,\delta^k_i \delta^\ell_j + \frac{\beta\theta}{1+\beta\theta}\,\delta^\ell_i \delta^k_j - \frac{2\beta\theta}{(1+\beta\theta)(2\beta\theta+n-2)}\,\delta_{ij}\,\delta^{k\ell} \right] \times \sigma(\theta)$$

$i = 1, \ldots, n$

$\epsilon = m \cosh\theta$

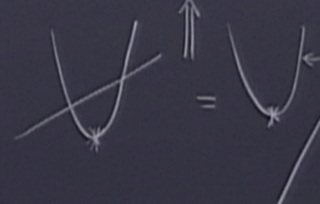
$p = m \sinh\theta$

generated non-perturbatively through dynamical mass transmutation

$\frac{i}{2}\,\frac{n-2}{2}$

$$\sigma(\theta) \underset{n=6}{=} \frac{\Gamma\left(1-\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}+\frac{\theta}{2\pi i}\right)}{\Gamma\left(1+\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}-\frac{\theta}{2\pi i}\right)} \tan\left(\frac{\pi}{4} - \frac{\theta}{2i}\right)$$

with $\sigma\left(\theta + i\frac{\pi}{2}\right)\sigma\left(\theta - i\frac{\pi}{2}\right) = f^n_{\text{applicit}}(\theta) \overset{n=6}{=} \frac{\theta + i\frac{\pi}{2}}{\theta - i\frac{\pi}{2}}$



Spurious particle/ anti-particle singlet pair

Toy example:

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$i,j = 1,\dots,n$

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generated non-perturbatively through dynamical mass transmutation

$\frac{i}{2}\,\frac{n-2}{2}$

$\theta \to \theta + i\pi$

$n=6$

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$=$

Spurious particle/anti-particle singlet pair

$$\sigma\left(\theta + \frac{i\pi}{2}\right)\sigma\left(\theta - \frac{i\pi}{2}\right) = \frac{\theta + \frac{i\pi}{2}}{\theta - \frac{i\pi}{2}}$$

$$g\left(x + \tfrac{i}{2}\right) + g\left(x - \tfrac{i}{2}\right) = \frac{1}{x^2 + \frac{1}{4}}$$

$$f\left(x + \frac{i}{2}\right) f\left(x - \frac{i}{2}\right) = \frac{x + i/2}{x - i/2}$$

$$\log f\left(x + \frac{i}{2}\right) + \log f\left(x - i/2\right) = \log\left(\frac{x + i/2}{x - i/2}\right)$$

$$g \equiv i\, f'/f = i \frac{d}{dx} \log f$$

$$\sigma\left(\theta + \frac{i\pi}{2}\right)\sigma\left(\theta - \frac{i\pi}{2}\right) = \frac{\theta + \frac{i\pi}{2}}{\theta - \frac{i\pi}{2}}$$

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$$g \equiv i f'/f = i \frac{d}{dx} \log f$$

$$\int dx\, e^{ikx}\left[g\left(x + \tfrac{i}{2}\right) + g\left(x - \tfrac{i}{2}\right) = \frac{1}{x^2 + \frac{1}{4}} \right]$$

$$\frac{x+i/2}{x-i/2} \longrightarrow e^{ikx}$$

$$e^{+\frac{k}{2}} \int_{\mathbb{R}} dx\, e^{ik(x+i/2)} g(x + i/2) + e^{-\frac{k}{2}} \int_{\mathbb{R}} dx\, e^{ik(x-i/2)} g(x - i/2) = e^{-|k|/2} = e^{k/2} \int_{\mathbb{R}+i/2} e^{ikx} g(x) + e^{-k/2} \int_{\mathbb{R}-i/2} e^{ikx} g(x)$$

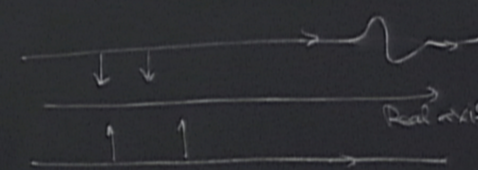
$$\ldots + g\left(x-\tfrac{i}{2}\right) = \frac{1}{x^2+\frac{1}{4}}$$

$$\left(e^{k/2}+e^{-k/2}\right)\hat{g}(k) \Rightarrow \hat{g}(k) = \frac{e^{-|k|/2}}{2\cosh\frac{k}{2}}$$

$x+\frac{i}{2}$

$x-\frac{i}{2}$ $\quad e^{ikx}$

!!



$$= e^{-|k|/2} = e^{k/2}\int_{\mathbb{R}+\frac{i}{2}} e^{ikx} g(x) + e^{-k/2}\int_{\mathbb{R}-\frac{i}{2}} e^{ikx} g(x)$$

$\left(x-\frac{i}{2}\right)$

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$i=1,\dots,n$

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generated non-perturbatively through dynamical mass transmutation

$\frac{i}{2}\,\frac{n-2}{2}$

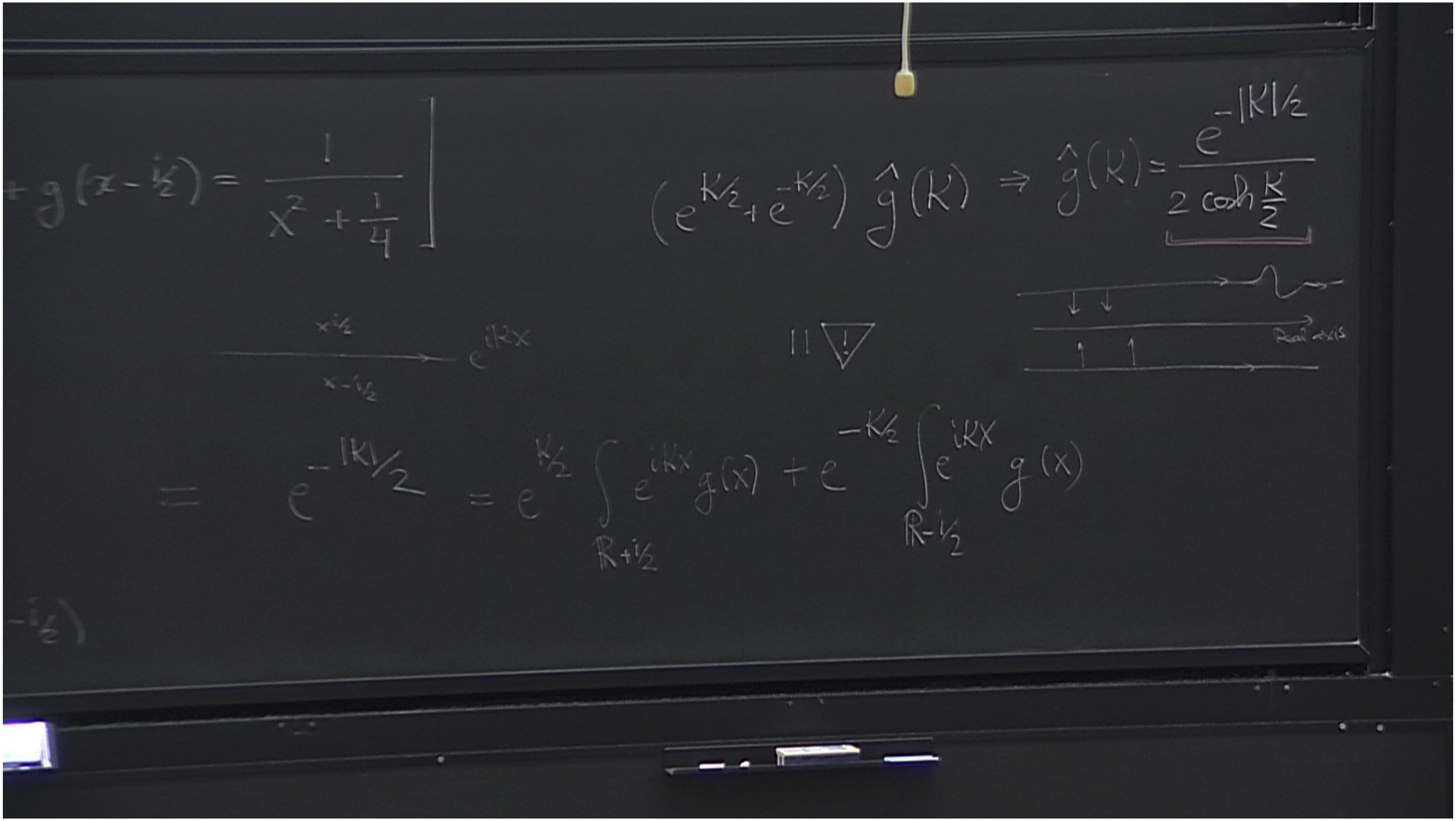
$\theta \to \theta + i\pi$

$$\sigma(\theta) = \frac{\Gamma\left(1-\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}+\frac{\theta}{2\pi i}\right)}{\Gamma\left(1+\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}-\frac{\theta}{2\pi i}\right)} \tan\left(\frac{\pi}{4} - \frac{\theta}{2i}\right)$$

$n=6$

with $\sigma(\theta + i\frac{\pi}{2})\,\sigma(\theta - i\frac{\pi}{2}) = f^n_{\text{explicit}}(\theta) \overset{n=6}{=} \dfrac{\theta + i\frac{\pi}{2}}{\theta - i\frac{\pi}{2}}$

Spurious particle/anti-particle singlet pair

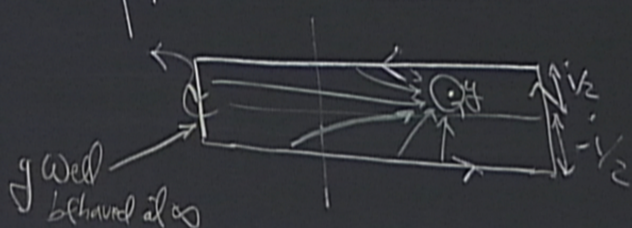


$+ g(x - \frac{i}{2}) = \frac{1}{x^2 + \frac{1}{4}}$
$(e^{k/2} + e^{-k/2})\, \hat{g}(k) \Rightarrow \hat{g}(k) = \frac{e^{-|k|/2}}{2\cosh\frac{k}{2}}$
$x + i/2$
e^{ikx}
$x - i/2$
!!
Real axis
$= e^{-|k|/2} = e^{k/2} \int_{\mathbb{R}+i/2} e^{ikx} g(x) + e^{-k/2} \int_{\mathbb{R}-i/2} e^{ikx} g(x)$
$-\frac{i}{2})$

$$\int \left[g\left(x+\tfrac{i}{2}\right) + g\left(x-\tfrac{i}{2}\right) = F(x) \right] \frac{1}{\cosh[\pi(y-x)]}\, dx$$

no sing inside strip

$$\text{LHS} = \int_{\Gamma} \frac{g(x)\,dx}{\sinh \pi(x-y)} \overset{!}{=} \boxed{g(y) = \tilde{F}(y)} \text{ explicit.}$$



$g(x - i/2)$

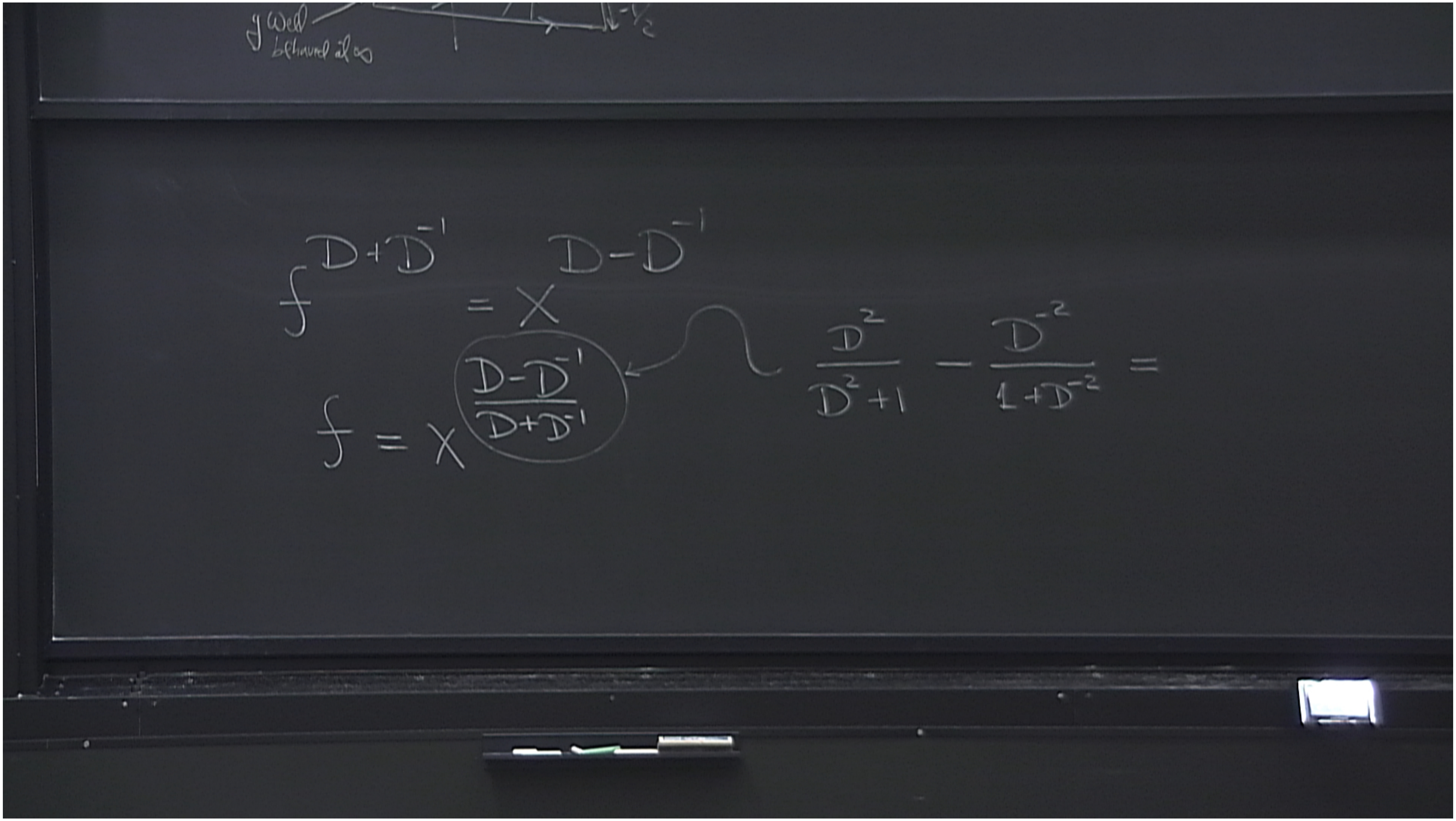
$$\frac{1}{\cosh \frac{k}{2}} \xleftrightarrow{\text{FT}} \frac{1}{\cosh \pi\theta}$$

Gaussian $\longleftrightarrow$ Gaussian

$$D = e^{\frac{i}{2}\partial_x}$$

$$D\,F(x) = F(x + i/2)$$

$$F^D \equiv e^{D \log F} = e^{\log F(x + i/2)}$$


g well
behaved at ∞
$f^{D+D^{-1}} = x^{D-D^{-1}}$
$f = x^{\frac{D-D^{-1}}{D+D^{-1}}}$
$\frac{D^2}{D^2+1} - \frac{D^{-2}}{1+D^{-2}} =$

$$f^{D+D^{-1}} = x^{D-D^{-1}}$$

$$f = x^{\frac{D-D^{-1}}{D+D^{-1}}}$$

$$\sim \frac{D^2}{D^2+1} - \frac{D^{-2}}{1+D^{-2}} = \sum_{n=1}^{\infty} (-1)^{n-1} D^{2n} - \sum_{n=1}^{\infty} (-1)^{n-1} D^{-2n}$$

$$D^{-1}$$

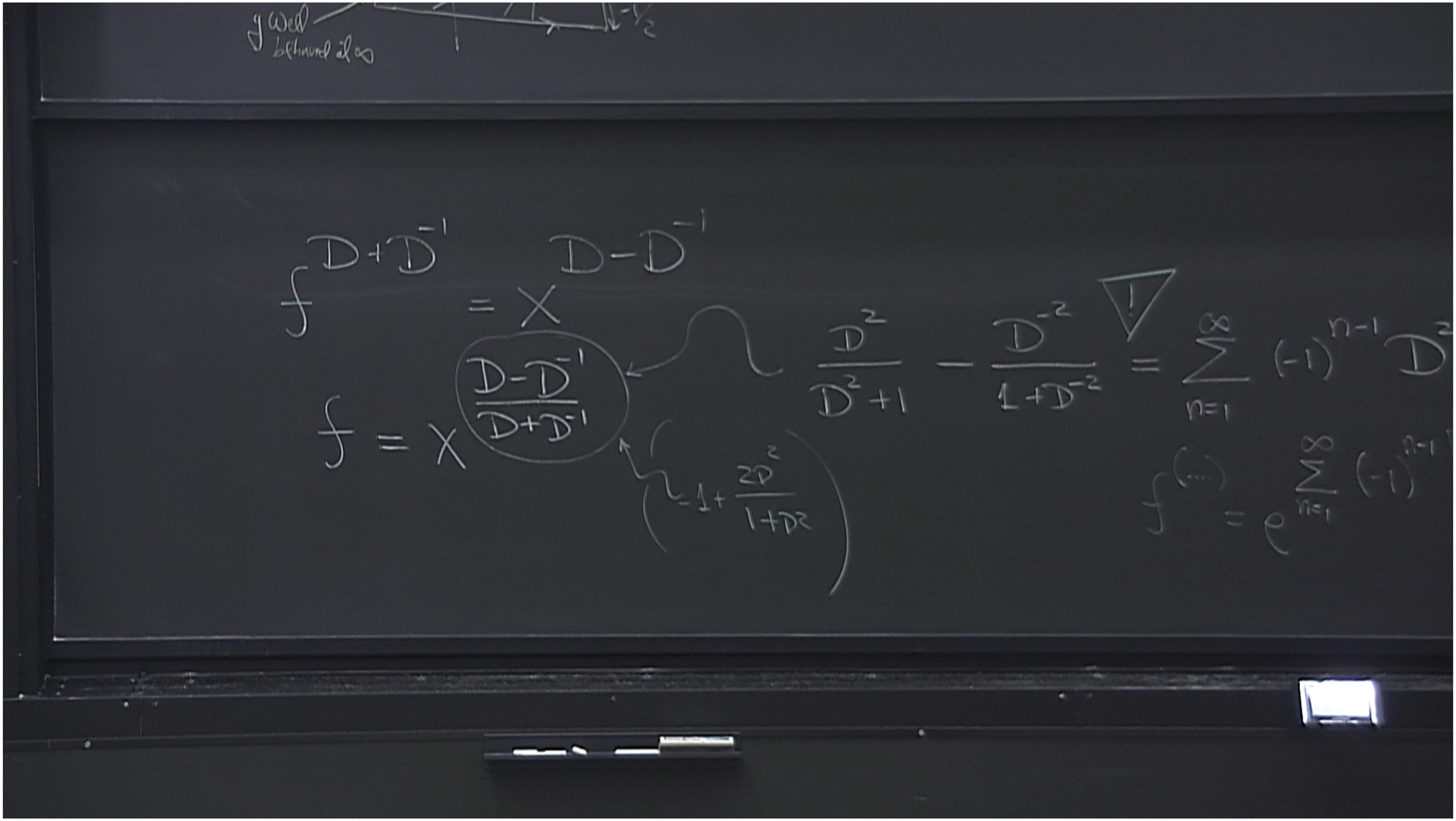
$$\sim \frac{D^2}{D^2+1} - \frac{D^{-2}}{1+D^{-2}} = \sum_{n=1}^{\infty} (-1)^{n-1} D^{2n} - \sum_{n=1}^{\infty} (-1)^{n-1} D^{-2n}$$

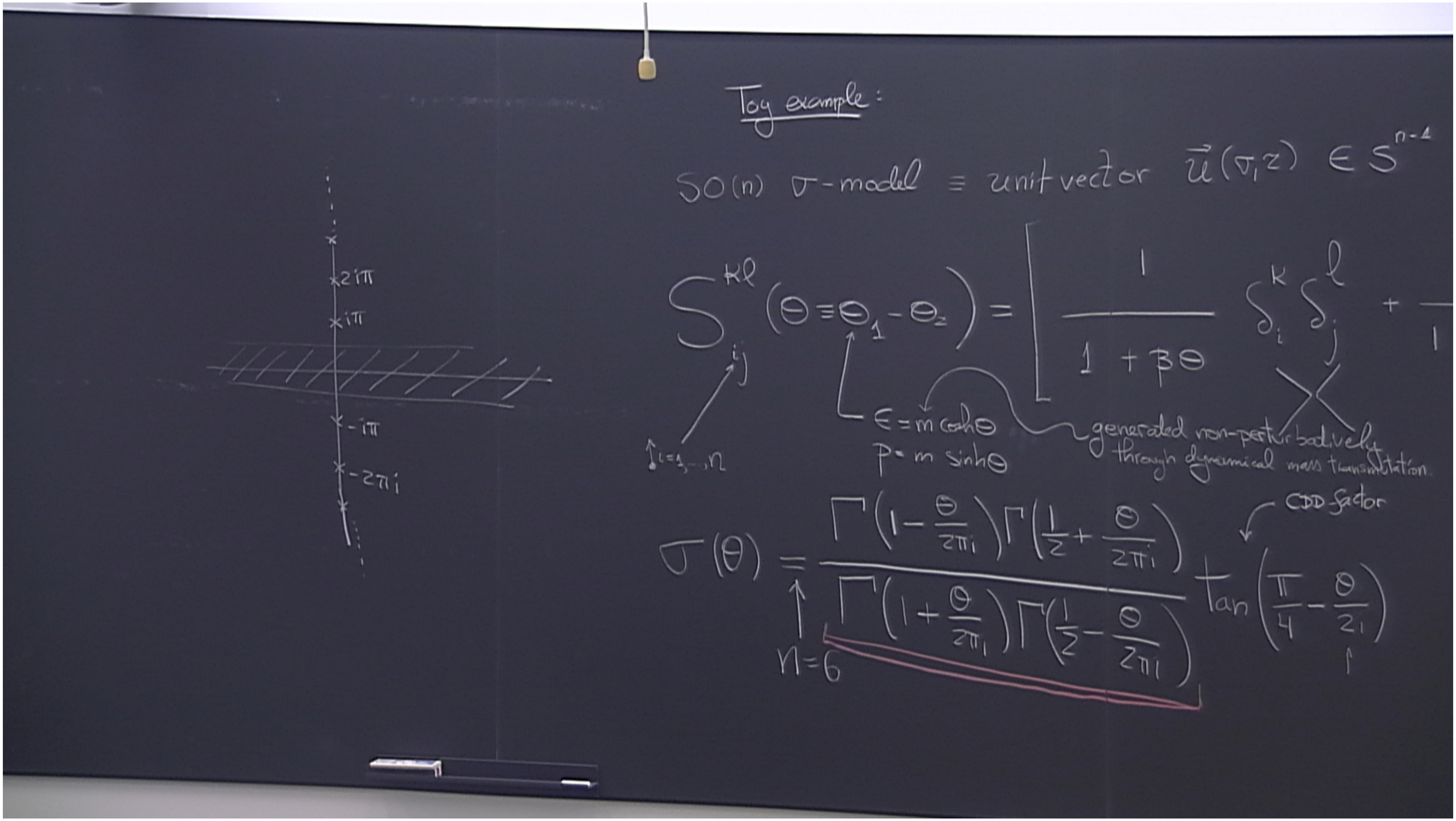
$$f^{(\dots)} = e^{\sum_{n=1}^{\infty} (-1)^{n-1} \left[\log\left(x+2n\frac{i}{2}\right) - \log\left(x-2n\frac{i}{2}\right) \right]}$$

$$\frac{D^{-2}}{\ldots + D^{-2}} = \sum_{n=1}^{\infty} (-1)^{n-1} D^{2n} - \sum_{n=1}^{\infty} (-1)^{n-1} D^{-2n}$$

$\frac{d}{dx}$, compute, $\int$ back

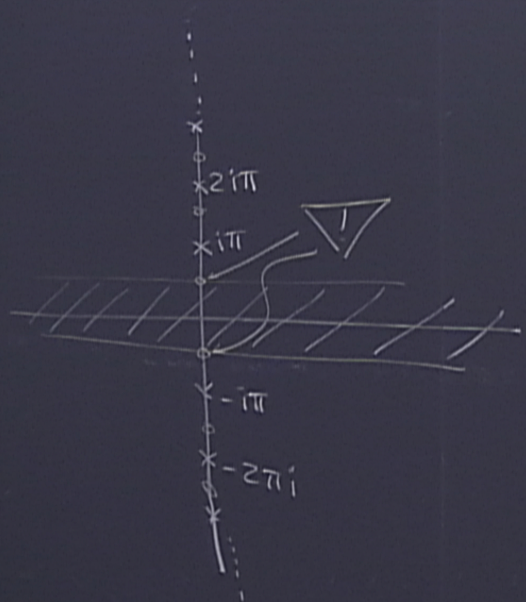
$$f^{(\ldots)} = e^{\sum_{n=1}^{\infty} (-1)^{n-1} \left[\log\left(x + 2n\frac{i}{2}\right) - \log\left(x - 2n\frac{i}{2}\right) \right]}$$


if well behaved at ∞
$-1/2$
$f^{D+D^{-1}} = x^{D-D^{-1}}$
$f = x^{\frac{D-D^{-1}}{D+D^{-1}}}$
$-1 + \frac{2D^2}{1+D^2}$
$\frac{D^2}{D^2+1} - \frac{D^{-2}}{1+D^{-2}} = \sum_{n=1}^{\infty} (-1)^{n-1} D^2$
$f^{(\dots)} = e^{\sum_{n=1}^{\infty} (-1)^{n-1}}$


Toy example:
SO(n) σ-model ≡ unit vector $\vec{u}(\sigma,\tau) \in S^{n-1}$
$S^{kl}_{ij}(\theta \equiv \theta_1 - \theta_2) = \Big[\frac{1}{1+\beta\theta} \delta^k_i \delta^l_j + \frac{}{1}$
$i = 1,\dots,n$
$\epsilon = m \cosh\theta$
$p = m \sinh\theta$
generated non-perturbatively through dynamical mass transmutation
CDD factor
$\sigma(\theta) = \frac{\Gamma(1-\frac{\theta}{2\pi i})\Gamma(\frac{1}{2}+\frac{\theta}{2\pi i})}{\Gamma(1+\frac{\theta}{2\pi i})\Gamma(\frac{1}{2}-\frac{\theta}{2\pi i})} \tan\left(\frac{\pi}{4}-\frac{\theta}{2i}\right)$
$n=6$
$2i\pi$
$i\pi$
$-i\pi$
$-2\pi i$

Toy example:

SO(n) σ-model ≡ unit vector $\vec{u}(\sigma,\tau) \in S^{n-1}$



$$S_{ij}^{kl}(\theta \equiv \theta_1 - \theta_2) = \Big[\frac{1}{1+\beta\theta}\,\delta_i^k\,\delta_j^l + \frac{\beta\theta}{1+\beta\theta}$$

$i = 1, \ldots, n$

$\epsilon = m\cosh\theta$
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generated non-perturbatively through dynamical mass transmutation.

CDD-factor

$$\sigma(\theta) = \frac{\Gamma\left(1-\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}+\frac{\theta}{2\pi i}\right)}{\Gamma\left(1+\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}-\frac{\theta}{2\pi i}\right)}\tan\left(\frac{\pi}{4}-\frac{\theta}{2i}\right)$$

$n = 6$

$\frac{i}{2}\,\frac{n}{}$

$\theta \rightarrow$

Toy example:

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$i = 1, \dots, n$

$\epsilon = m \cosh\theta$

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generated non-perturbatively through dynamical mass transmutation.

$$\frac{i}{2}\;\frac{n-2}{2}$$

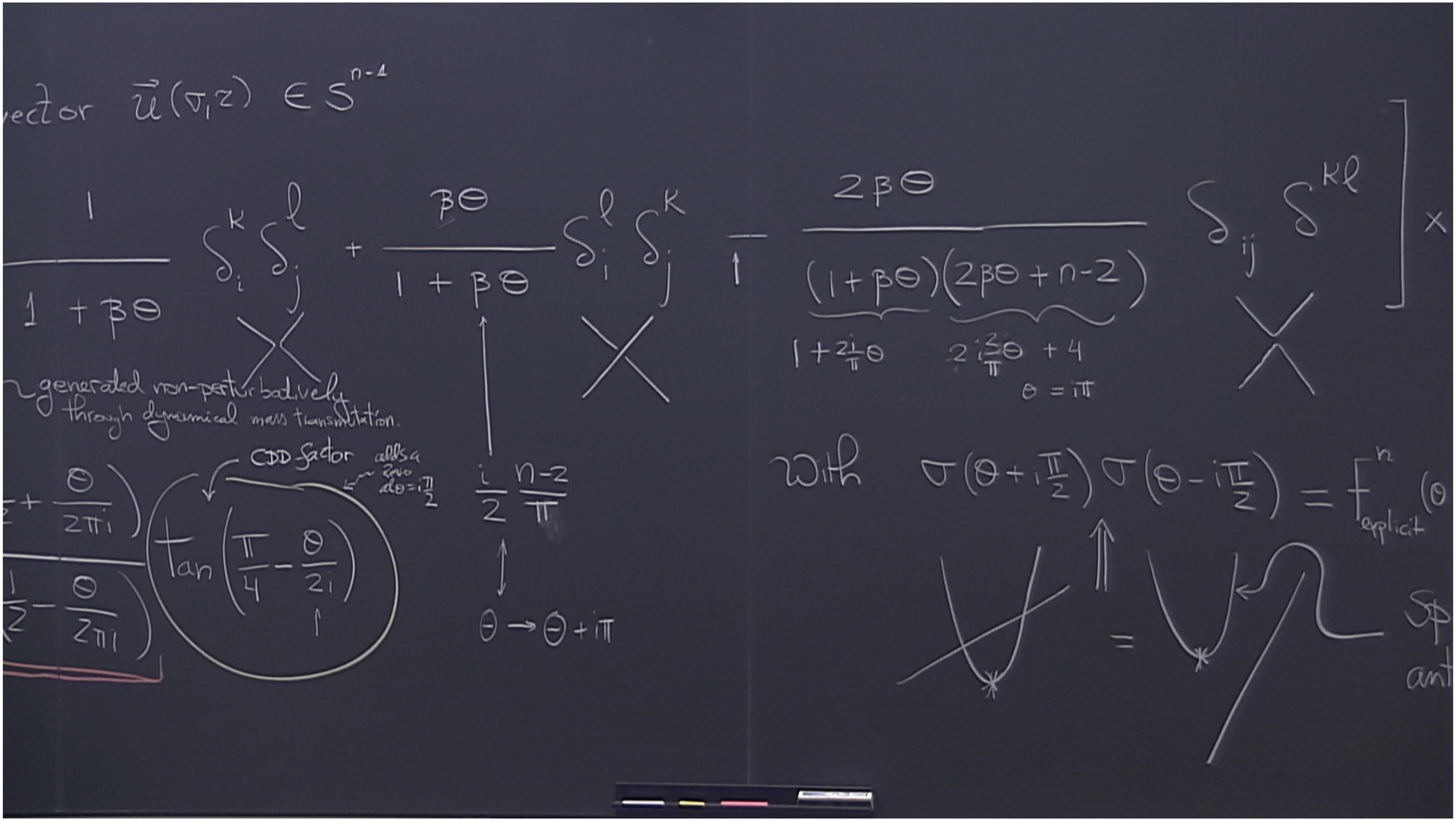
$$\theta \to \theta + i\pi$$

CDD-factor

allows a zero at $\theta = i\frac{\pi}{2}$

$$\sigma(\theta) = \frac{\Gamma\left(1 - \frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2} + \frac{\theta}{2\pi i}\right)}{\Gamma\left(1 + \frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2} - \frac{\theta}{2\pi i}\right)} \tan\left(\frac{\pi}{4} - \frac{\theta}{2i}\right)$$

$n = 6$

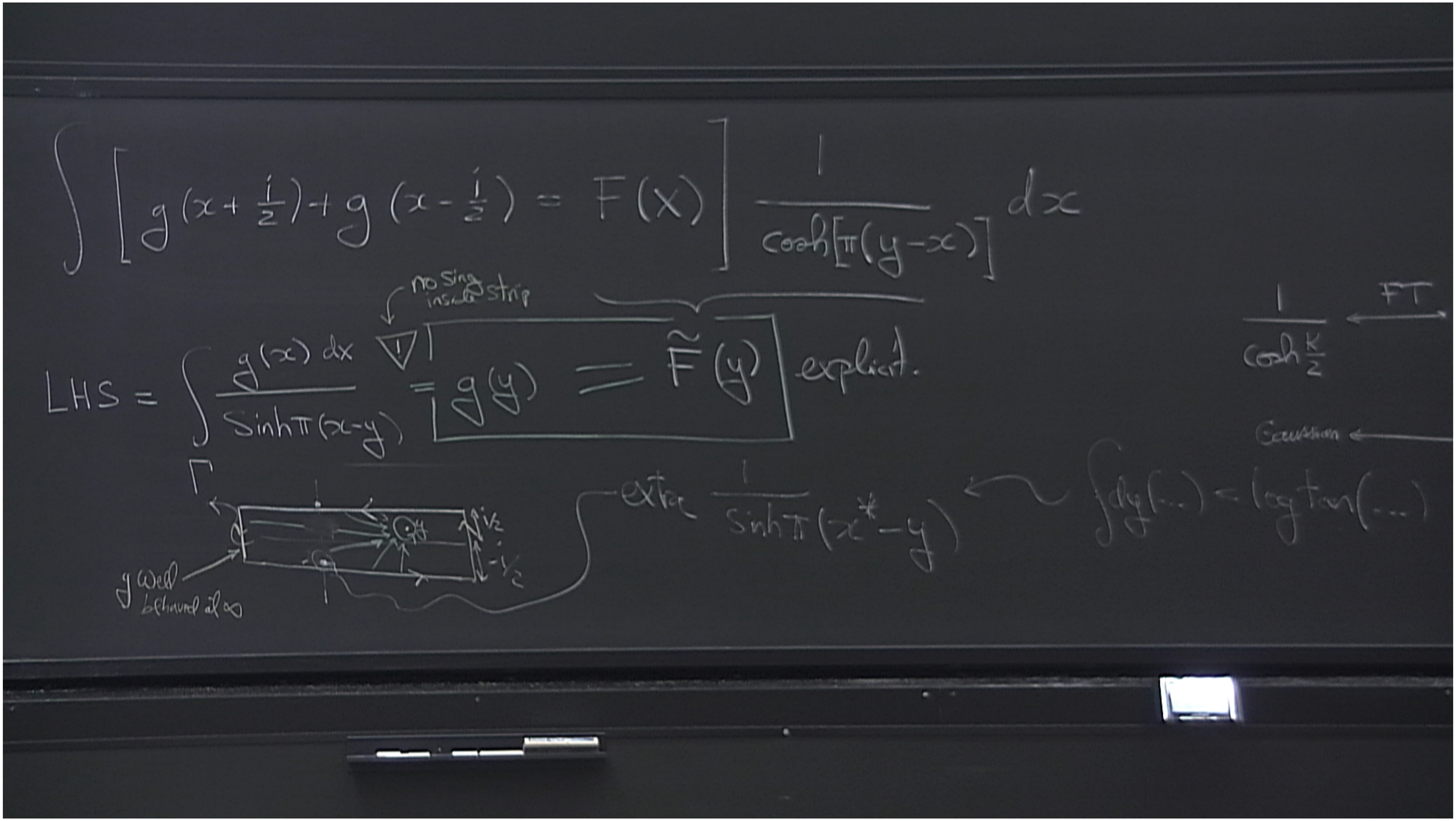

vector $\vec{u}(\sigma, \tau) \in S^{n-1}$
$\frac{1}{1+\beta\theta} \delta_i^k \delta_j^l + \frac{\beta\theta}{1+\beta\theta} \delta_i^l \delta_j^k - \frac{2\beta\theta}{(1+\beta\theta)(2\beta\theta+n-2)} \delta_{ij} \delta^{kl}$
$1+\frac{2i}{\pi}\theta$
$2\frac{i}{\pi}\theta+4$
$\theta = i\pi$
generated non-perturbatively through dynamical mass transmutation.
CDD-factor
adds a zero at $\theta = i\frac{\pi}{2}$
$\tan\left(\frac{\pi}{4} - \frac{\theta}{2i}\right)$
$\frac{i}{2}\frac{n-2}{\pi}$
$\theta \to \theta + i\pi$
With $\sigma\left(\theta + i\frac{\pi}{2}\right)\sigma\left(\theta - i\frac{\pi}{2}\right) = F^n_{\text{explicit}}(\theta$
$=$

$$X^{\frac{D-D^{-1}}{D-D^{-1}}-1} = 1$$

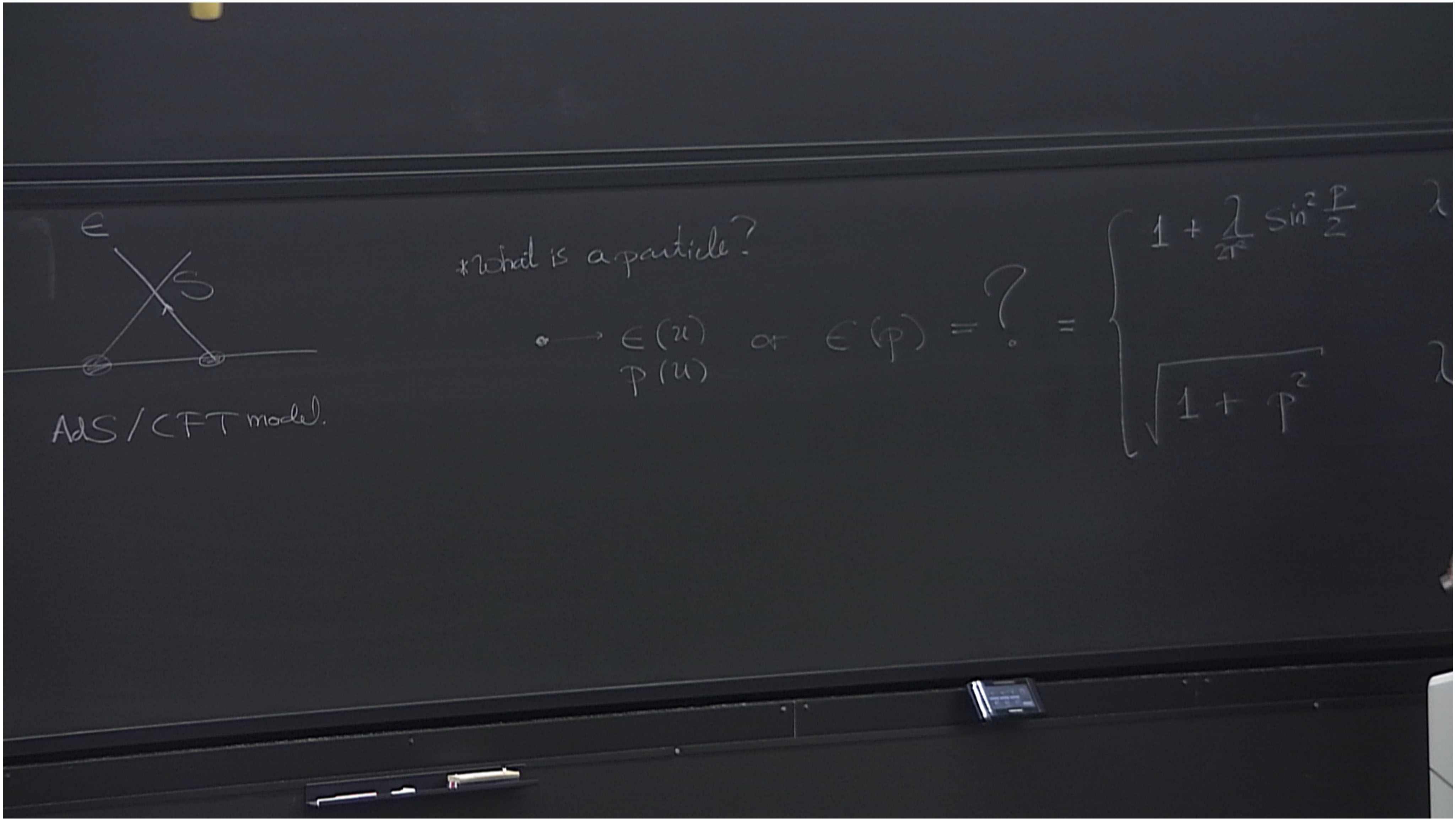
$$\ldots(-1)^{n-1} D^{2n} - \sum_{n=1}^{\infty} (-1)^{n-1} D^{-2n}$$

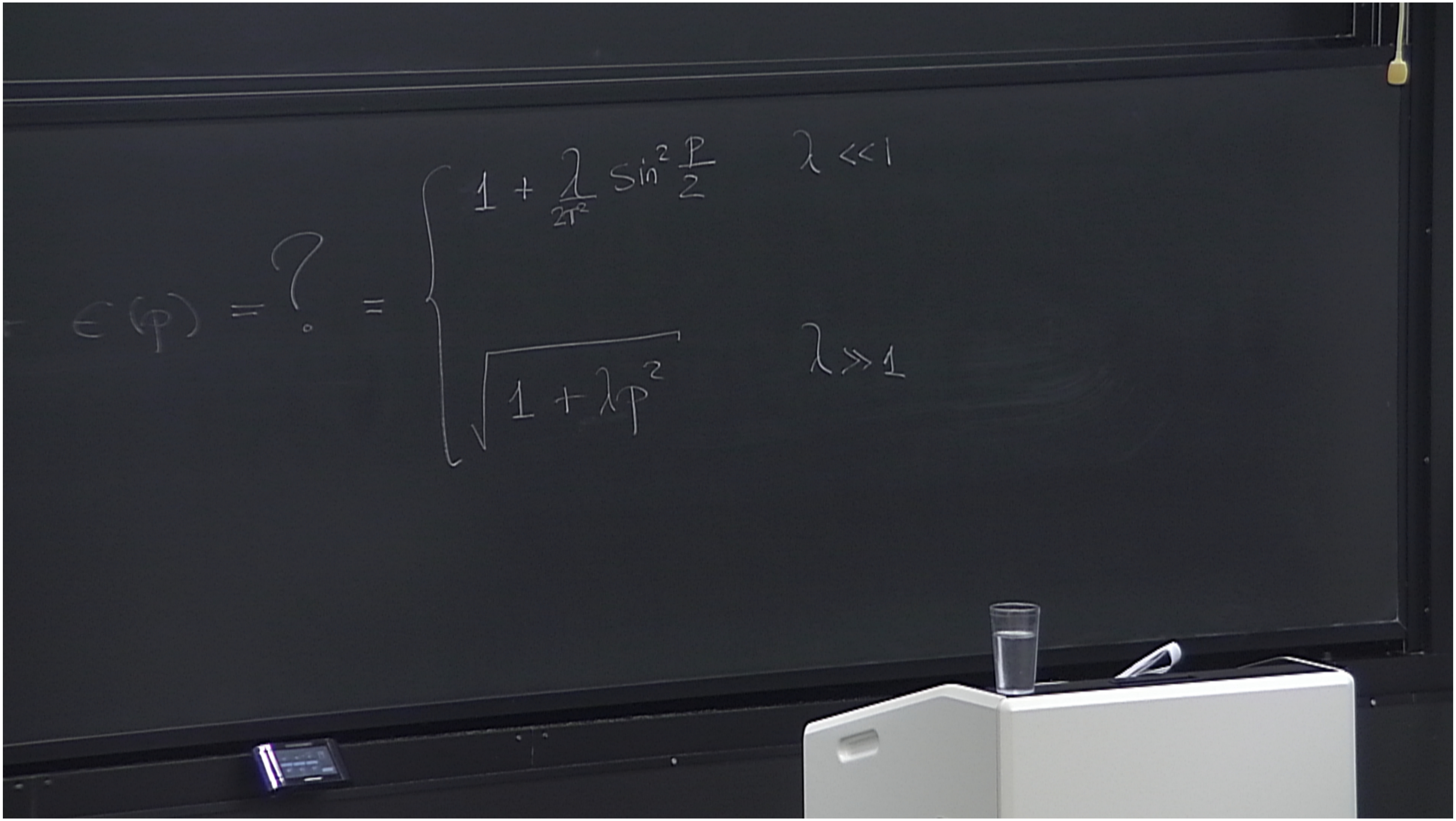
$\frac{d}{dx}$, compute, $\int$ back

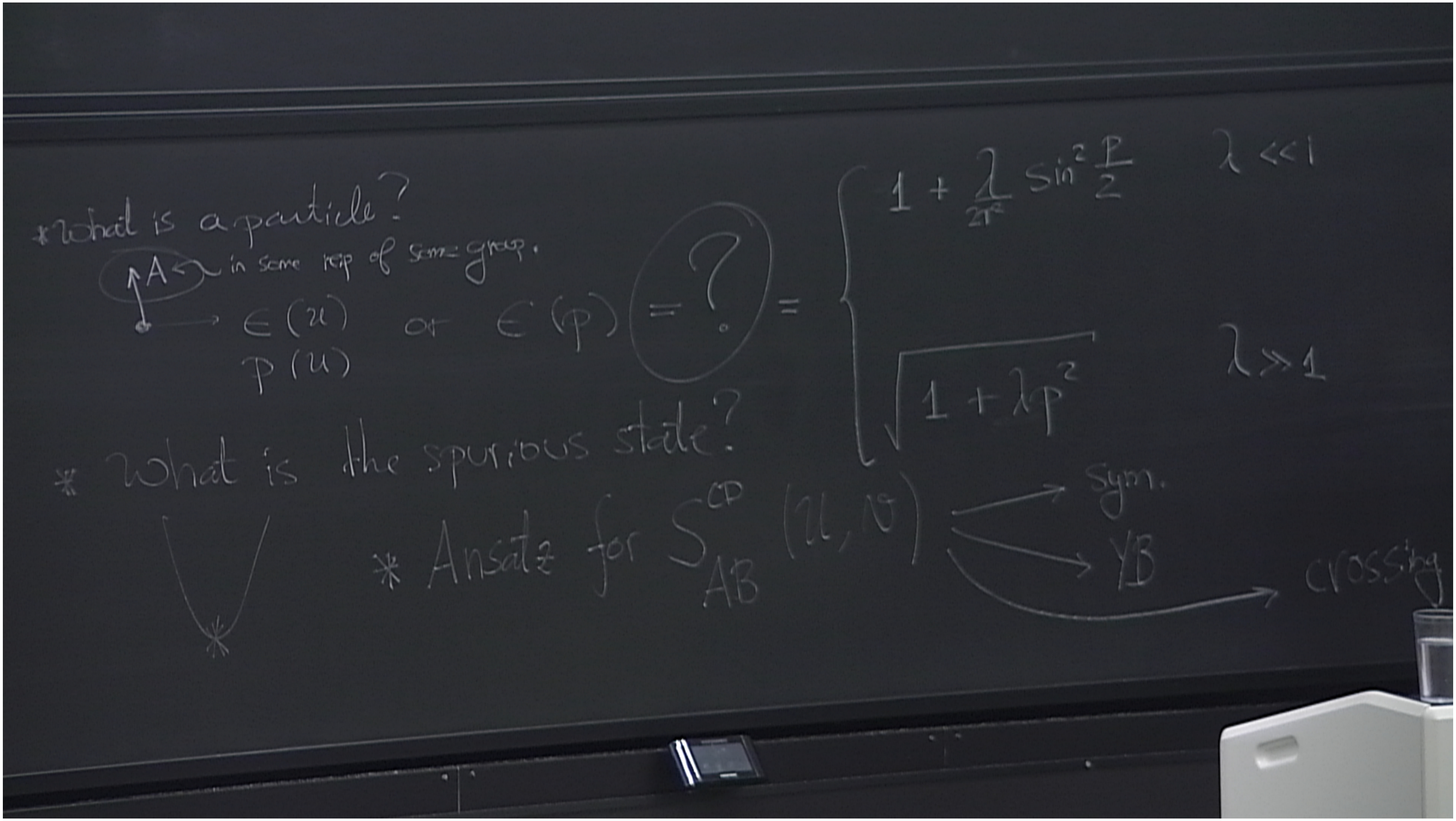
$$\ldots\sum_{n=1}^{\infty} (-1)^{n-1} \left[\log\left(x+2n\frac{i}{2}\right) - \log\left(x-2n\frac{i}{2}\right) \right] \Rightarrow \text{real part.}$$

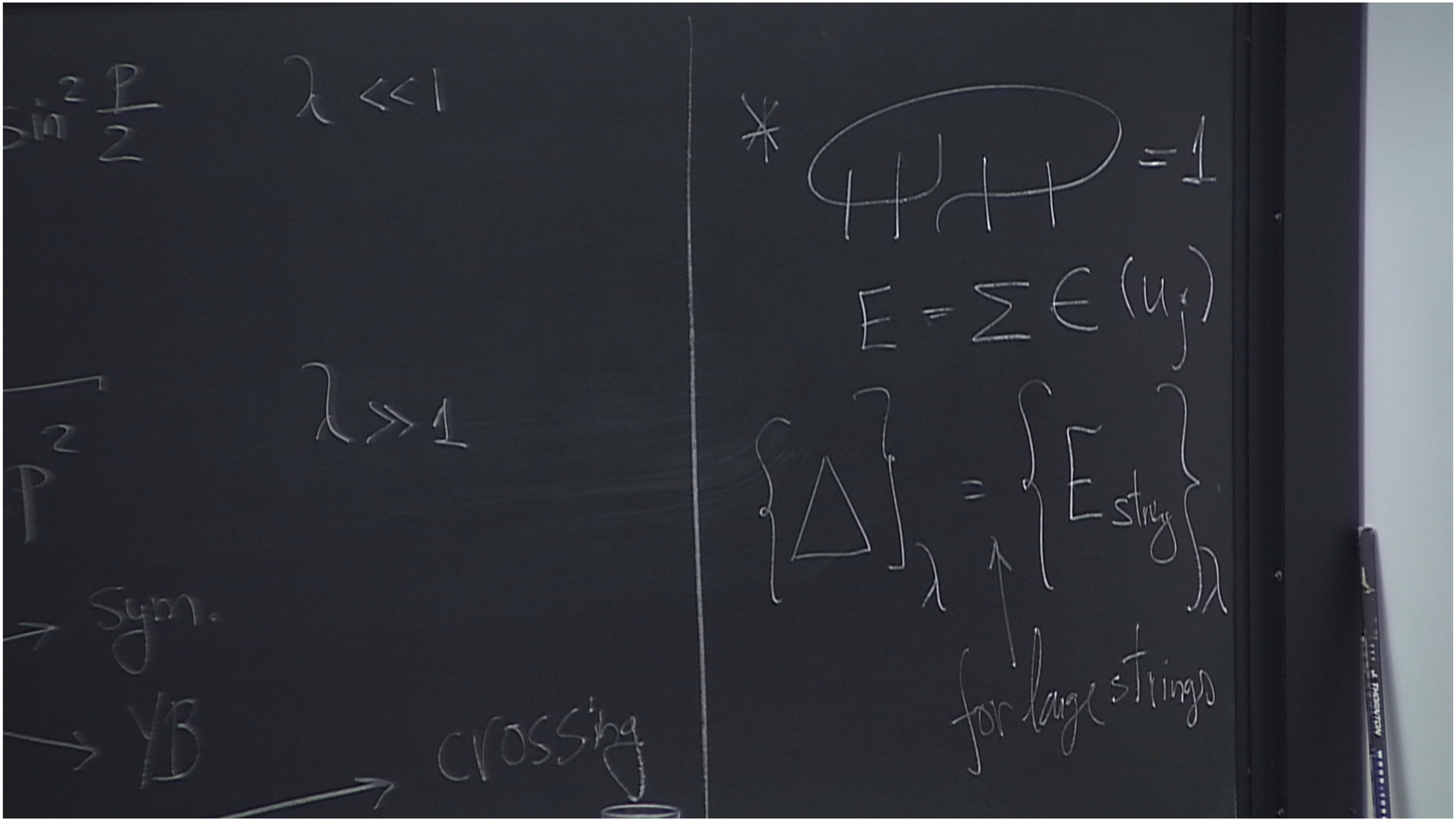


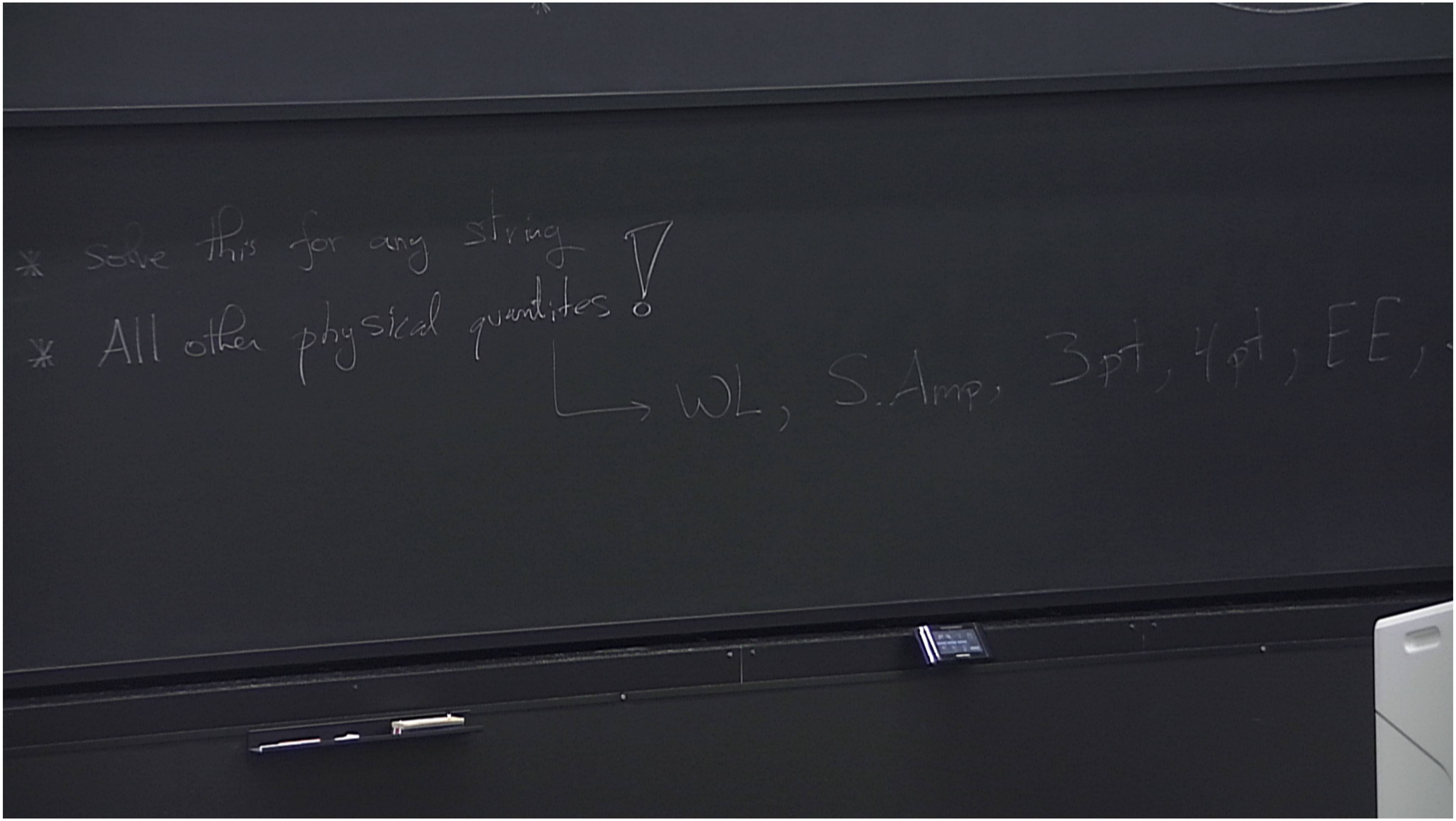
$\int \left[g(x+\tfrac{i}{2}) + g(x-\tfrac{i}{2}) = F(x) \right] \frac{1}{\cosh[\pi(y-x)]} dx$
no sing inside strip
LHS $= \int \frac{g(x)\,dx}{\sinh\pi(x-y)} = \boxed{g(y) = \tilde{F}(y)}$ explicit.
$\frac{1}{\cosh\frac{k}{2}} \xleftrightarrow{FT}$
Gaussian $\leftarrow$
Γ
g well behaved at ∞
$i/2$
$-i/2$
extra $\frac{1}{\sinh\pi(x^*-y)}$
$\int dy(\ldots) \sim \log\tan(\ldots)$


E
S
AdS / CFT model.
* What is a particle?
$\bullet \longrightarrow \epsilon(x)$
$p(x)$
or $\epsilon(p) = ? =$
$1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2}$
$\sqrt{1 + p^2}$
λ
λ


$\epsilon(\varphi) = ? = \begin{cases} 1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2} & \lambda \ll 1 \\ \sqrt{1 + \lambda p^2} & \lambda \gg 1 \end{cases}$


* What is a particle?
A in some rep of some group.
$\epsilon(u)$ or $\epsilon(p)$ = ? =
$p(u)$
$1 + \frac{\lambda}{2\pi^2} \sin^2 \frac{p}{2}$
$\lambda \ll 1$
$\sqrt{1 + \lambda p^2}$
$\lambda \gg 1$
* What is the spurious state?
* Ansatz for $S_{AB}^{CD}(u, v)$
sym.
YB
crossing


sin² p/2
λ << 1
λ >> 1
p²
→ Sym.
→ YB
crossing
* = 1
E = Σ ∈ (u_j)
{Δ}_λ = {E_string}_λ
for large strings


* solve this for any string !
* All other physical quantites !
→ WL, S.Amp, 3pt, 4pt, EE,