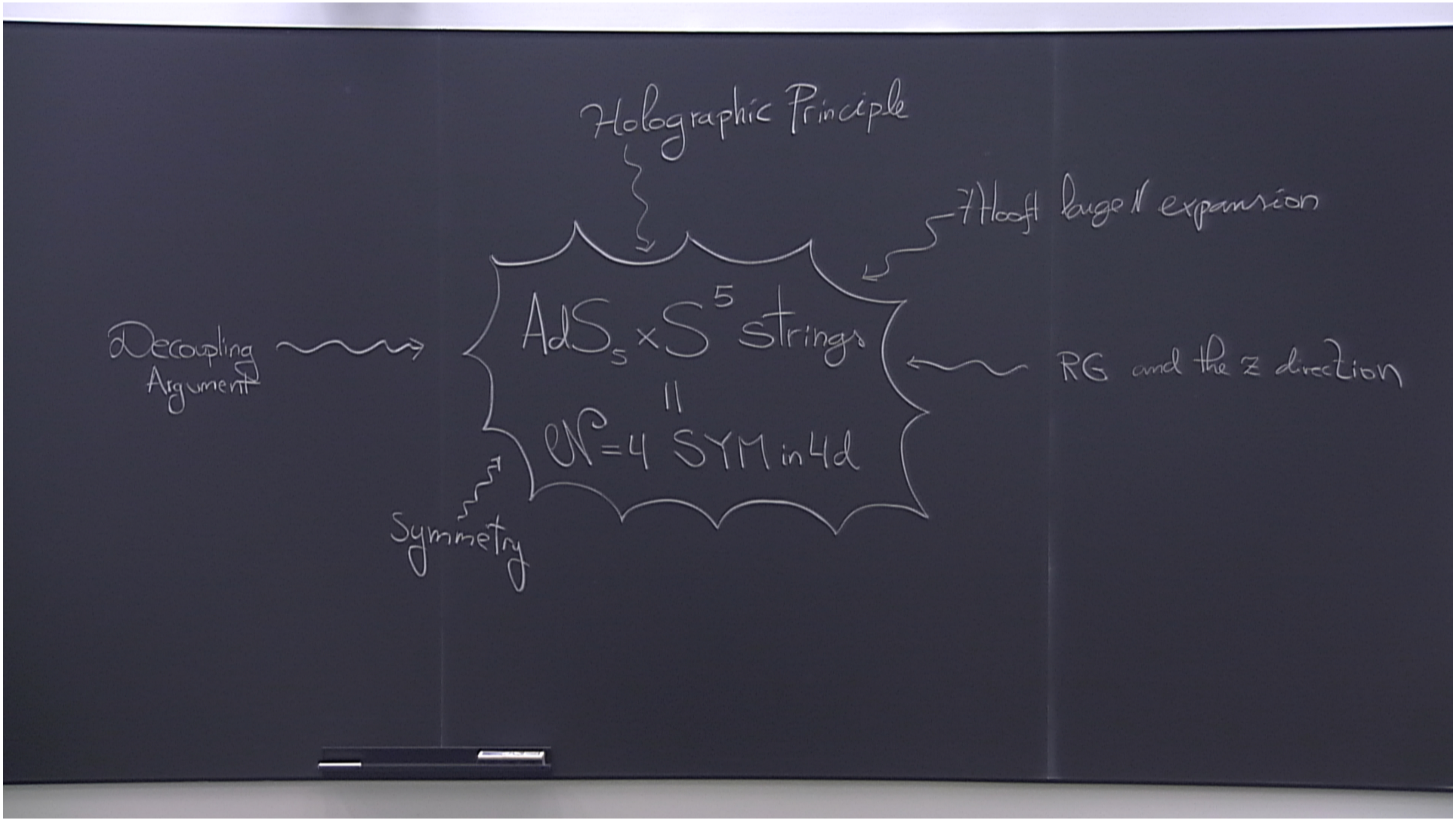
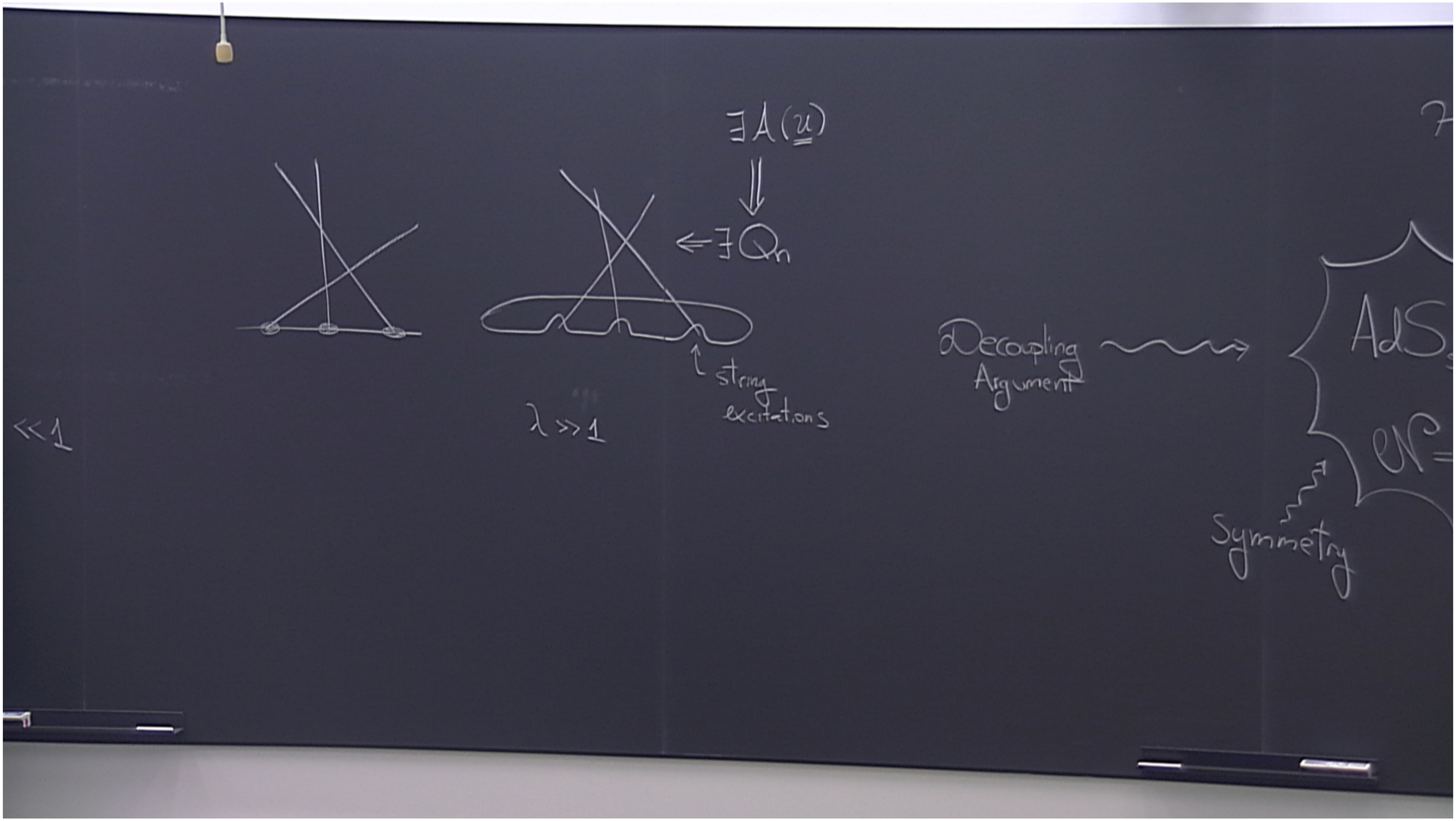
Title: Explorations in String Theory -13

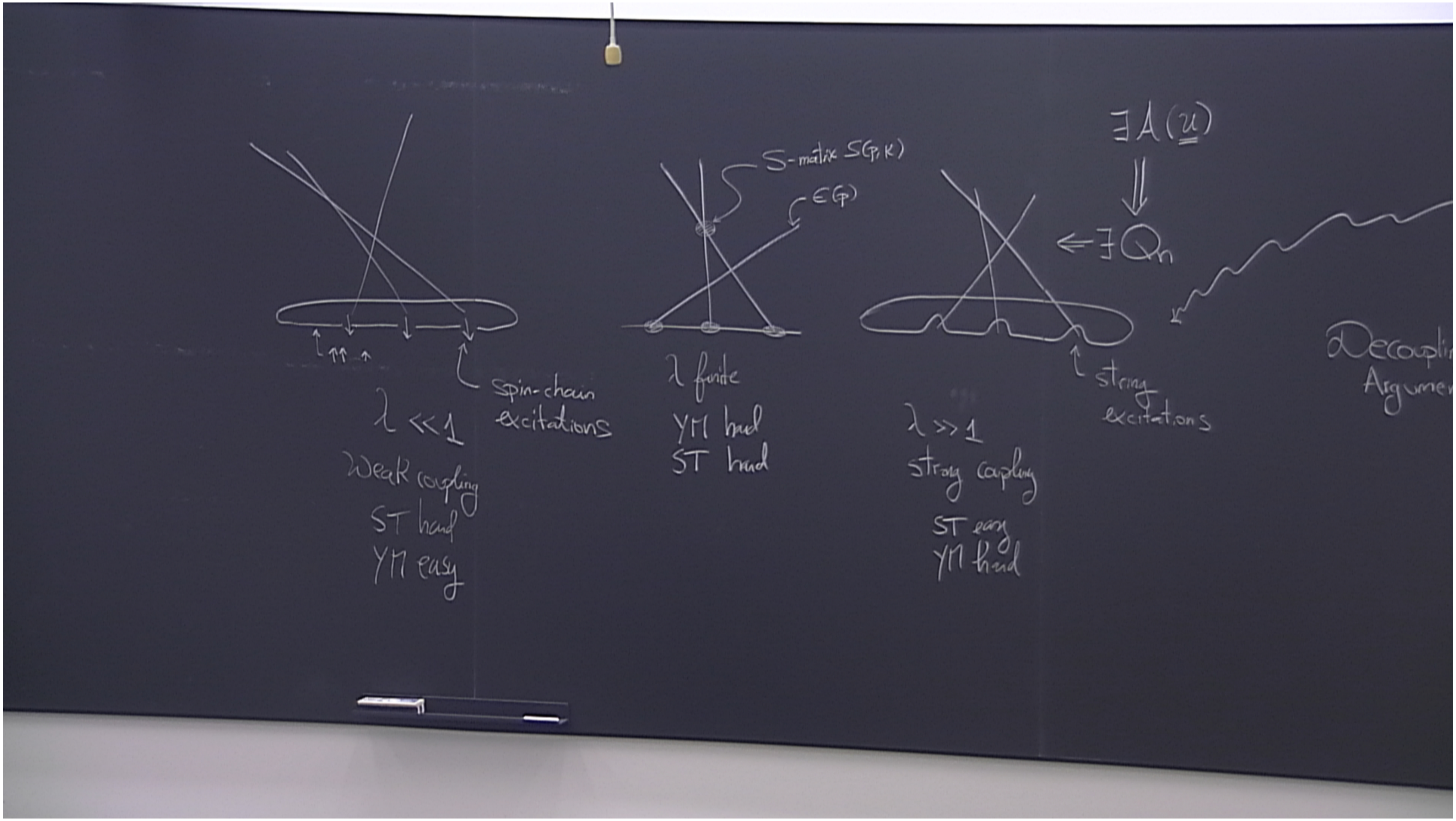
Date: Apr 22, 2015 11:30 AM

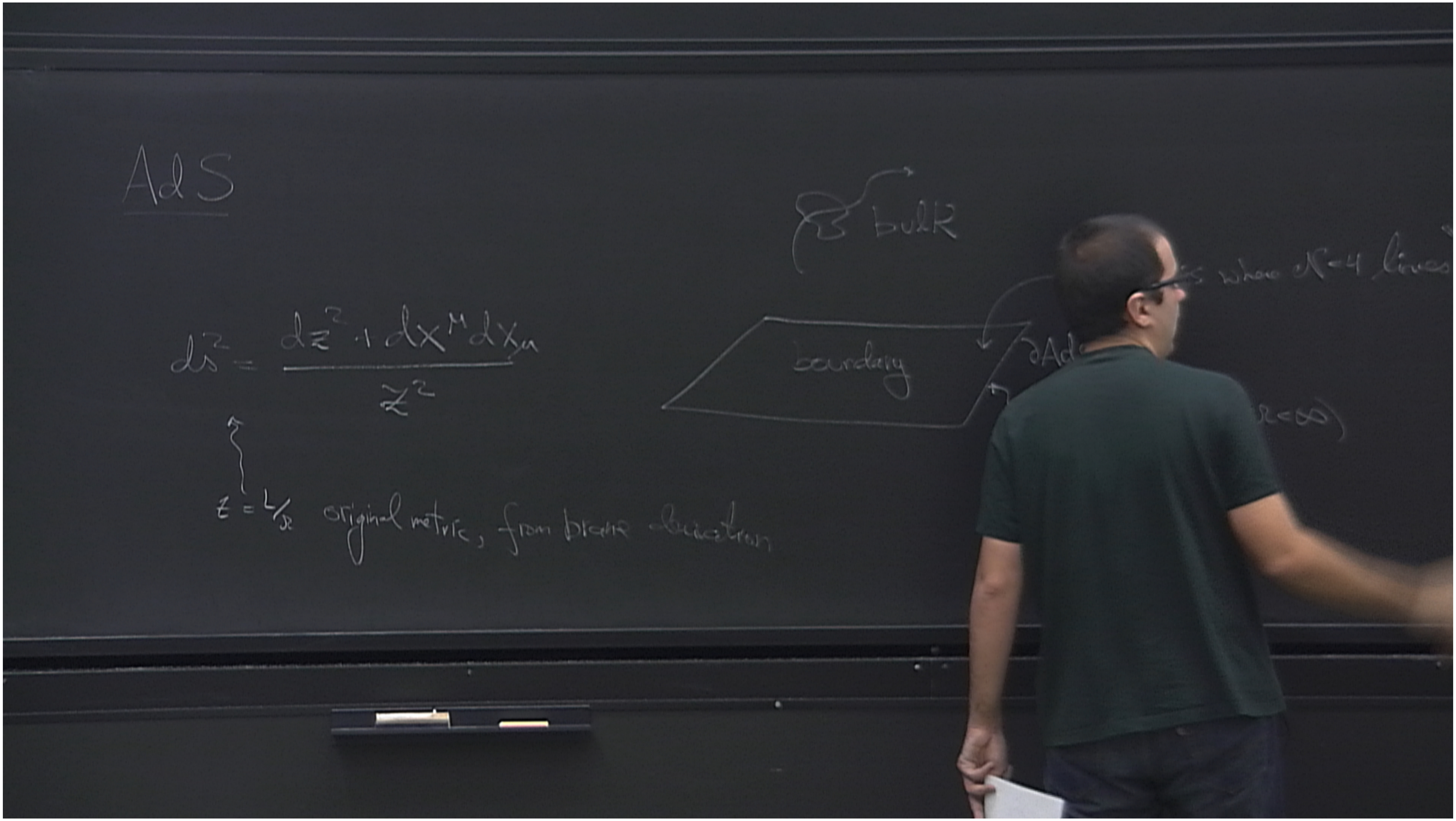
URL: http://pirsa.org/15040152

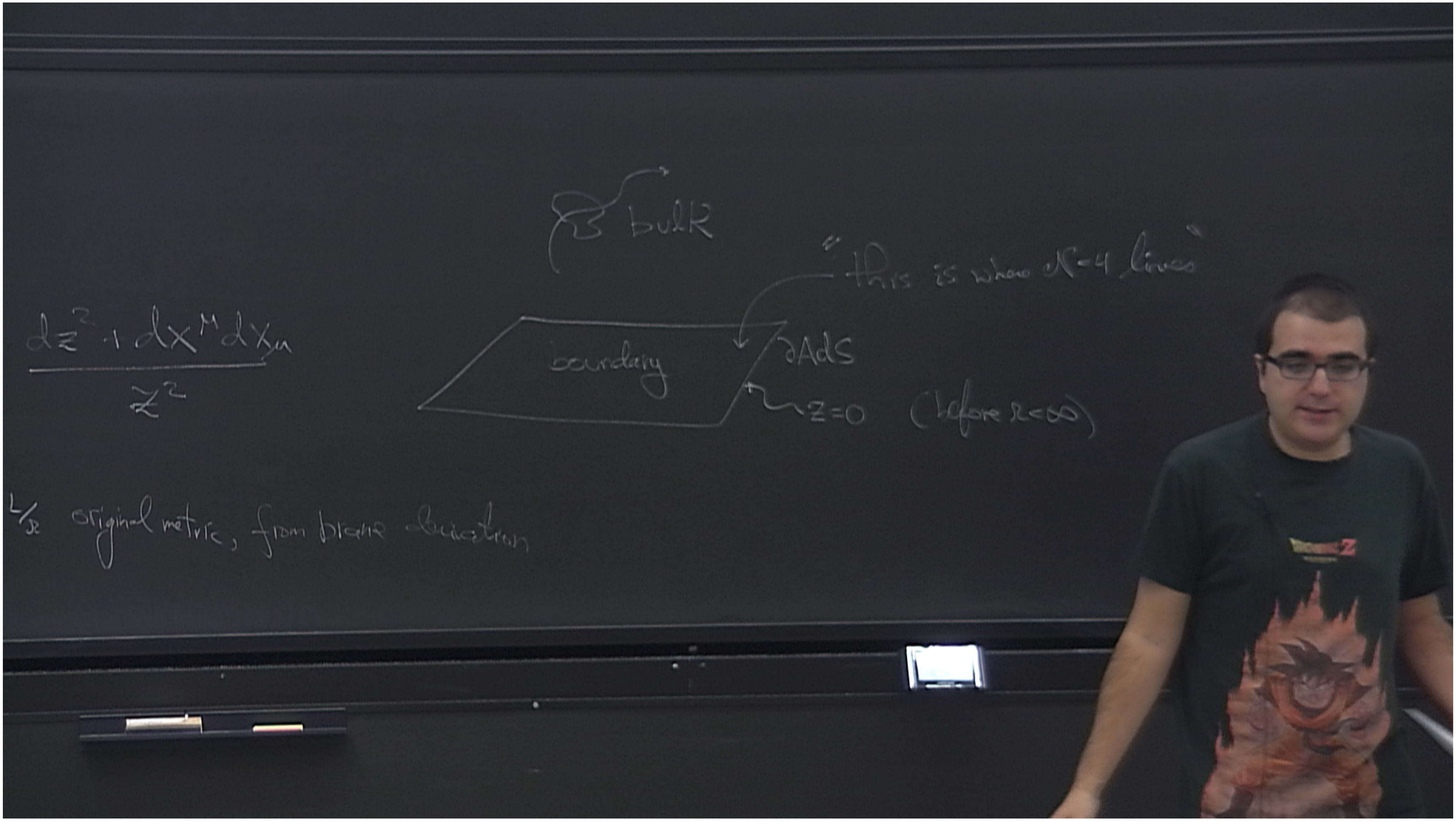
Abstract:

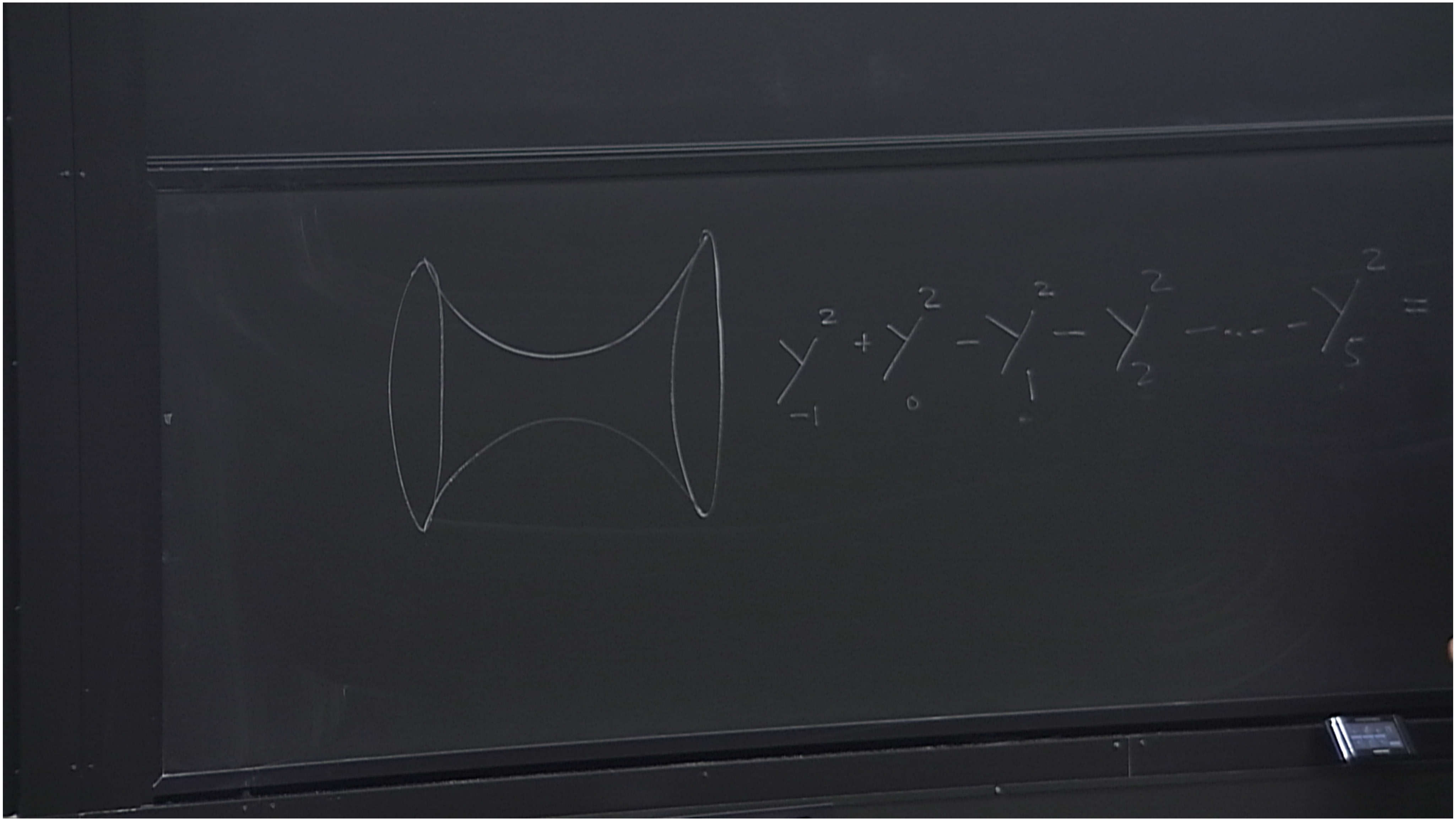

Holographic Principle
'tHooft large N expansion
Decoupling Argument
AdS5 × S5 strings
=
N=4 SYM in 4d
RG and the z direction
Symmetry

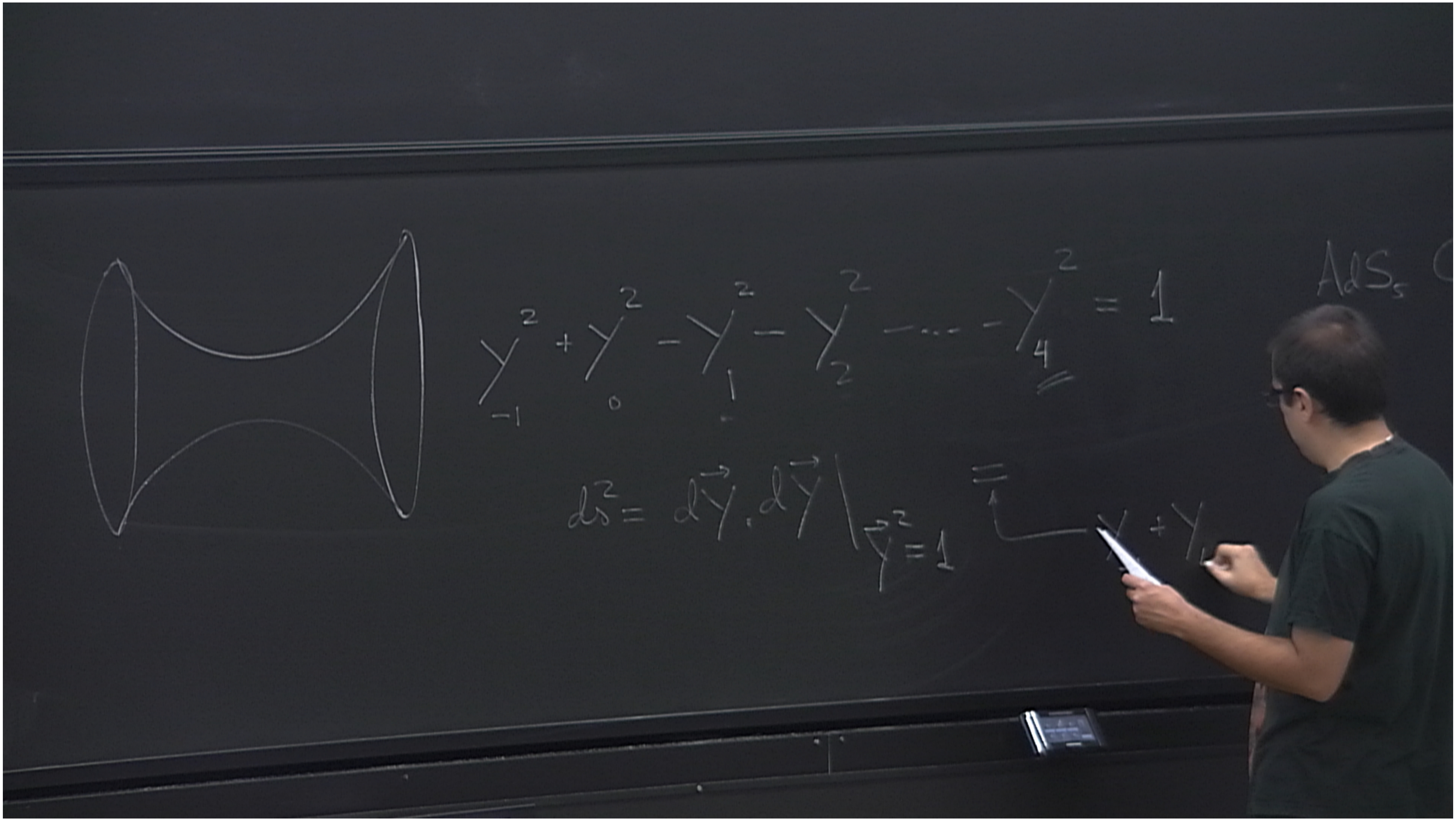

∃ A(u)
∃ Qn
string
excitations
<<1
λ >> 1
Decoupling
Argument
AdS
Symmetry

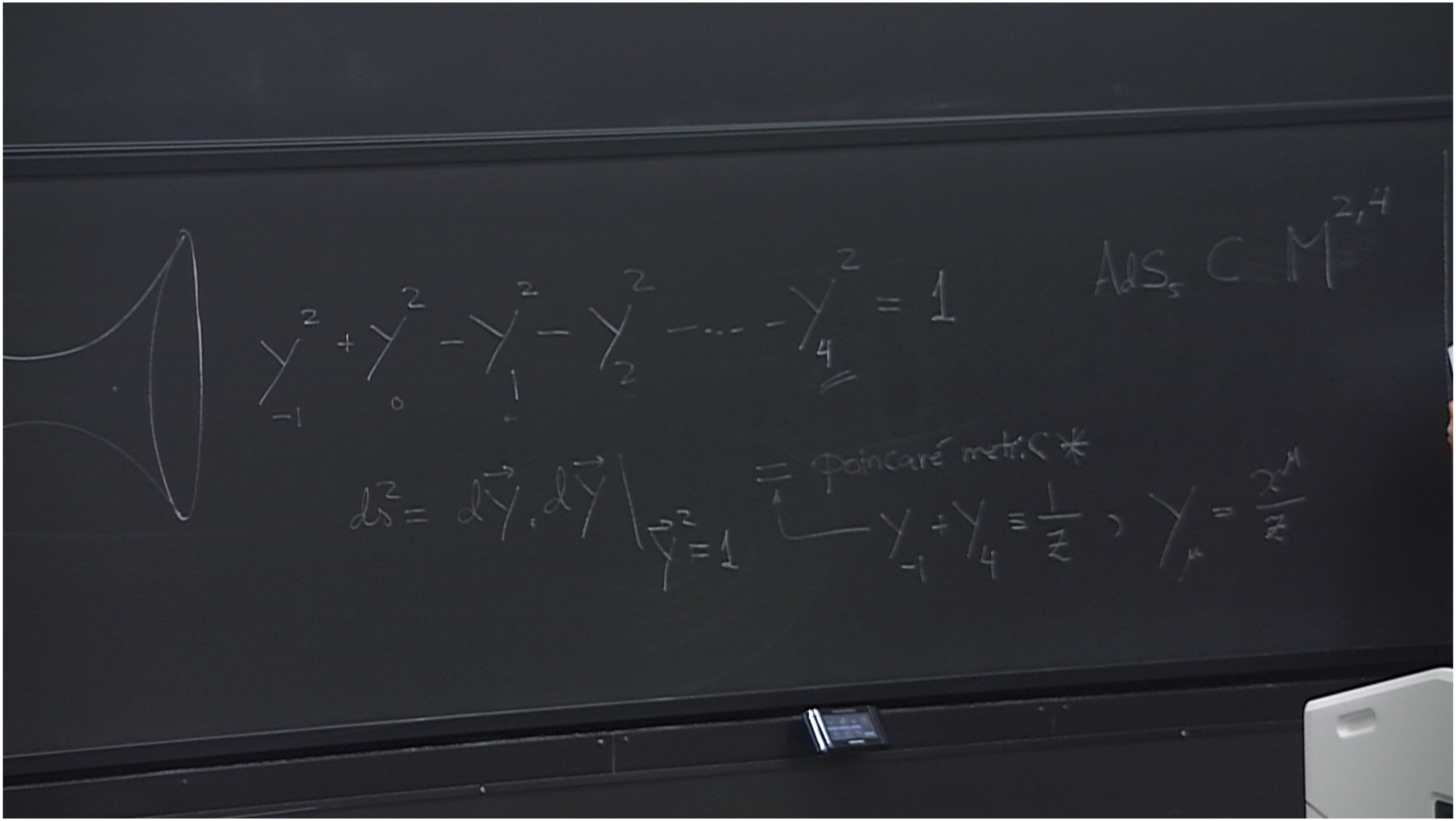

S-matrix S(p,k)
ε(p)
∃ A(u)
⇐ ∃ Q_n
spin-chain excitations
λ << 1
Weak coupling
ST hard
YM easy
λ finite
YM hard
ST hard
λ >> 1
strong coupling
ST easy
YM hard
string excitations

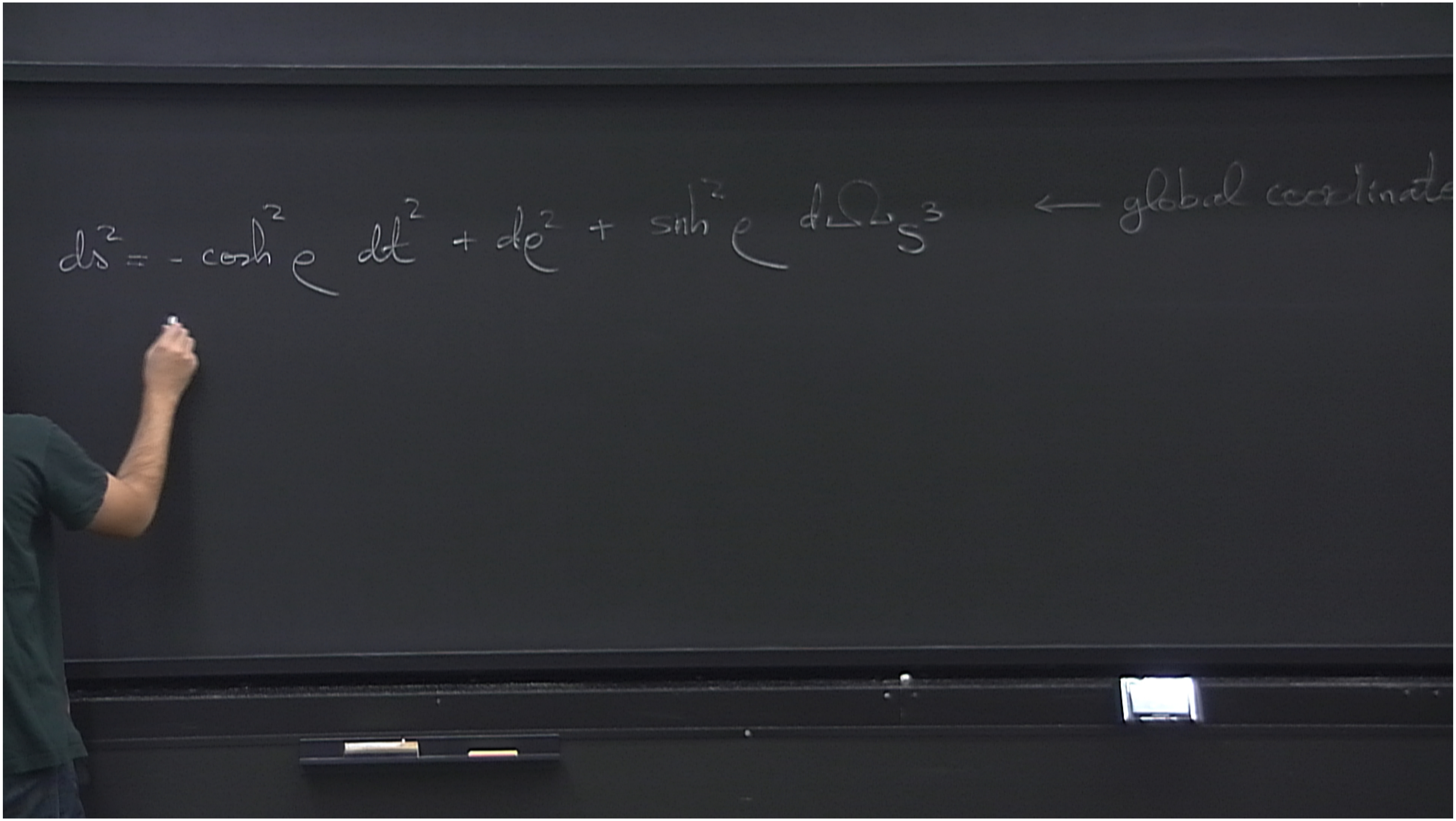

AdS
bulk
boundary
$ds^2 = \frac{dz^2 + dx^\mu dx_\mu}{z^2}$
original metric, from brane


bulk
"this is where N=4 lives"
boundary
∂AdS
z=0
$\frac{dz^2 + dx^\mu dx_\mu}{z^2}$
original metric, from brane

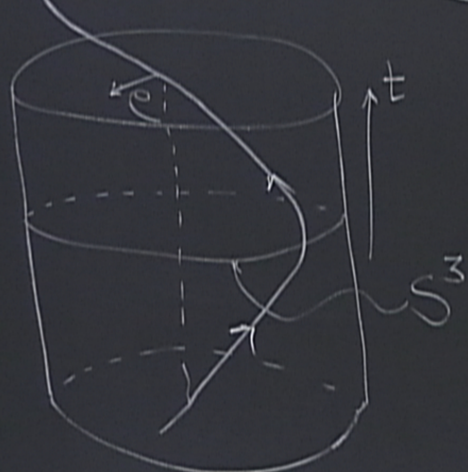

$Y_{-1}^2 + Y_0^2 - Y_1^2 - Y_2^2 - \dots - Y_5^2 =$


$y_{-1}^2 + y_0^2 - y_1^2 - y_2^2 - \dots - y_4^2 = 1$
AdS_5
$ds^2 = d\vec{y} \cdot d\vec{y} \big|_{\vec{y}^2 = 1} =$


$Y_{-1}^2 + Y_0^2 - Y_1^2 - Y_2^2 - \dots - Y_4^2 = 1$
$AdS_5 \subset M^{2,4}$
$ds^2 = d\vec{Y} \cdot d\vec{Y} \Big|_{\vec{Y}^2 = 1}$ = Poincaré metric *
$Y_{-1} + Y_4 = \frac{1}{z}, \quad Y_\mu = \frac{x^\mu}{z}$


$ds^2 = -\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \, d\Omega_{S^3}$
← global coordinate

$$ds^2 = -\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \, d\Omega_{S^3}$$ ← global coordin



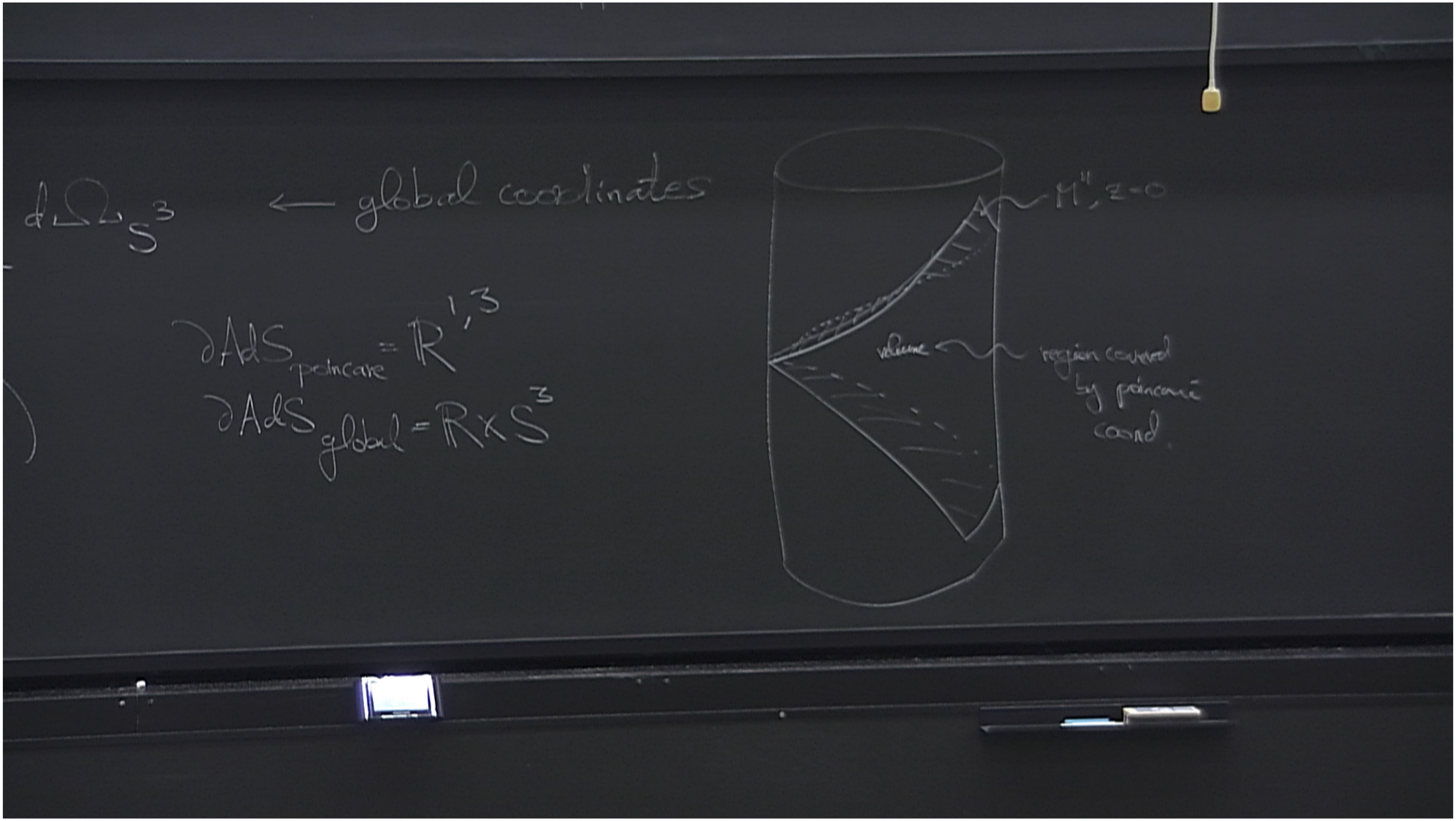
$$\downarrow \rho \to \infty$$

$$+e^{2\rho}\big(\underbrace{-dt^2 + d\Omega^2_{S^3}}\big)$$

$$\partial AdS_{global} = \mathbb{R} \times S^3$$

$$\partial AdS_{poincare} = \mathbb{R}^{1,3}$$

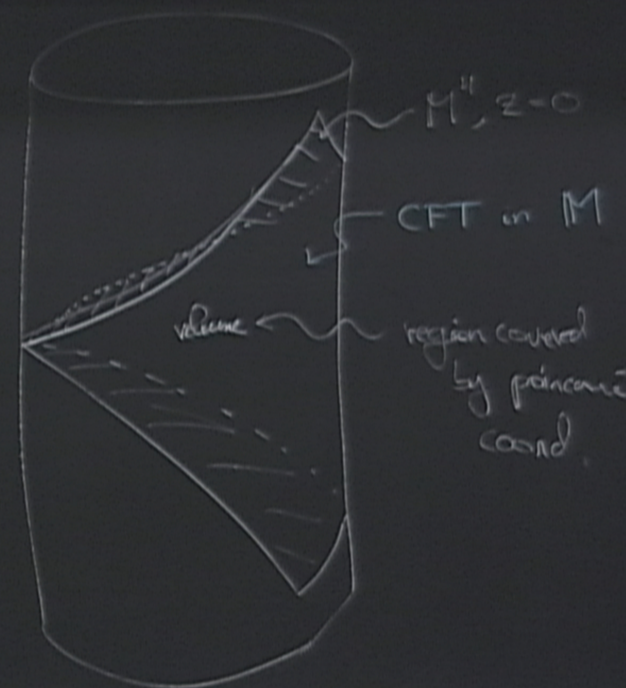
$$\partial AdS_{global} = \mathbb{R} \times S^3$$

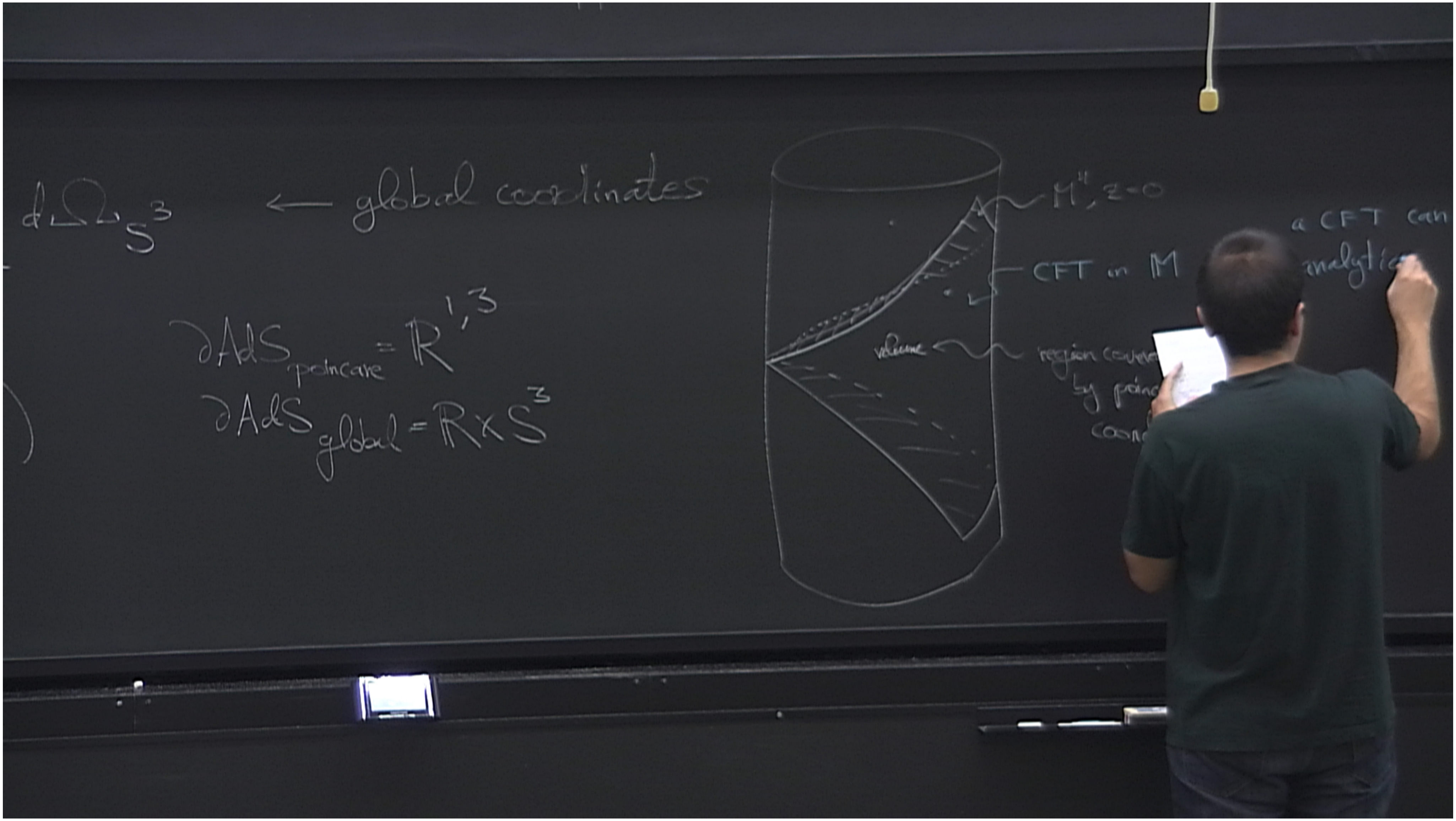

$d\Omega_{S^3}$
← global coordinates
$\partial AdS_{poincare} = \mathbb{R}^{1,3}$
$\partial AdS_{global} = \mathbb{R} \times S^3$
$M^4, z=0$
volume
region covered by poincaré coord.

$d\Omega_{S^3}$ ← global coordinates

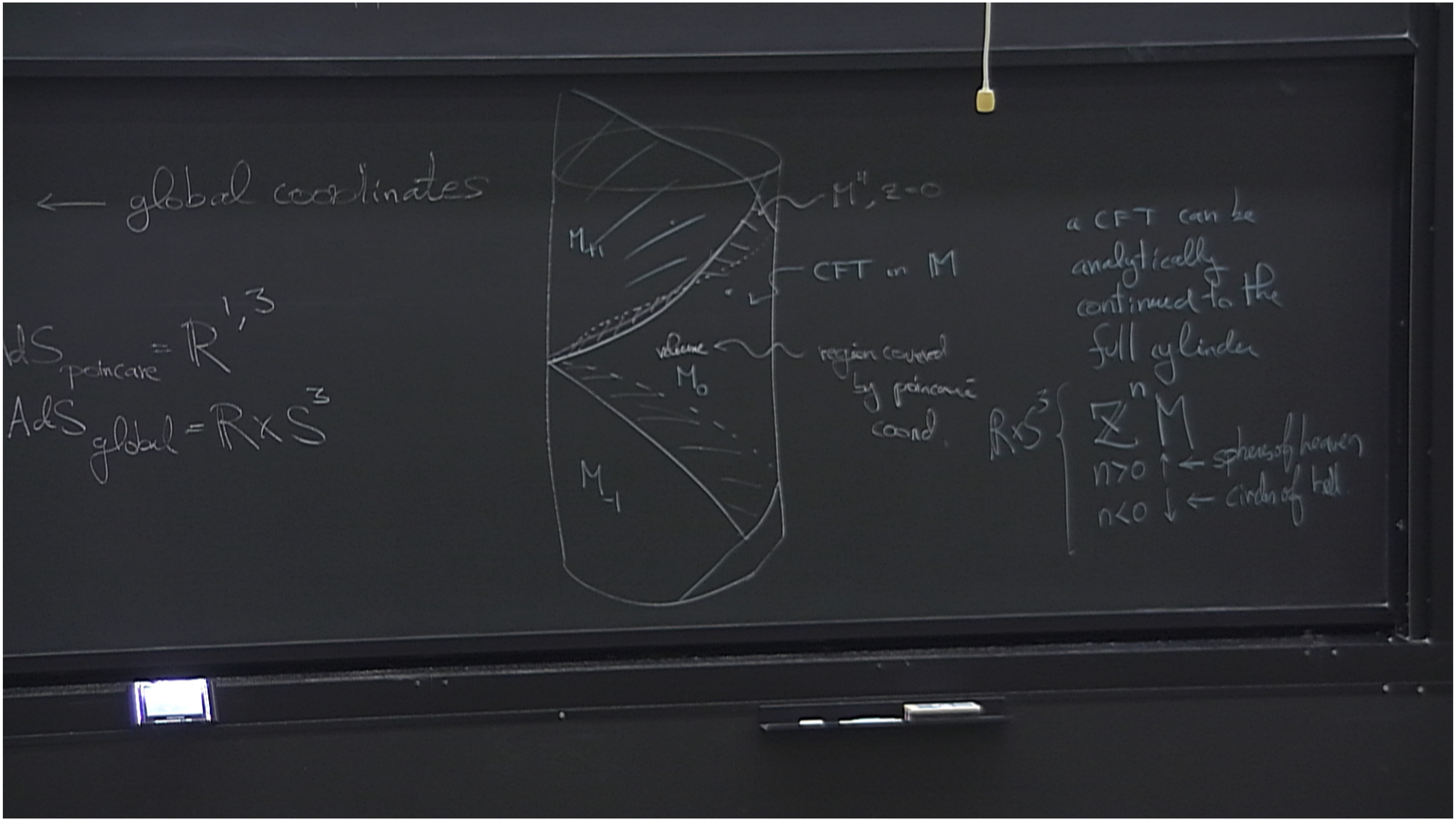
$$\partial AdS_{poincare} = \mathbb{R}^{1,3}$$

$$\partial AdS_{global} = \mathbb{R} \times S^3$$





← global coordinates
∂AdS poincare = ℝ^1,3
∂AdS global = ℝ×S^3
M^4, z=0
CFT in M
volume
a CFT can


← global coordinates
dS poincare = R 1,3
AdS global = R×S 3
M+1
M0
M-1
M4, z=0
CFT on M
volume
region covered by poincaré coord.
a CFT can be analytically continued to the full cylinder
R×S3
Zn M
n>0 ← spheres of heaven
n<0 ← circles of hell

$\mathcal{N}=4$ SYM in $M^4 \longleftrightarrow AdS_5 \times S^5$ strings in poincaré space.

gl
nsf.

$\mathcal{N}=4$ SYM in $\mathbb{R}\times S^3 \longleftrightarrow AdS_5 \times S^5$ strings in global AdS.

$$\{\Delta\}_{M^4} = \{E\}_{\substack{\mathcal{N}=4\,\text{SYM} \\ \text{in } \mathbb{R}\times S^3}}$$

1

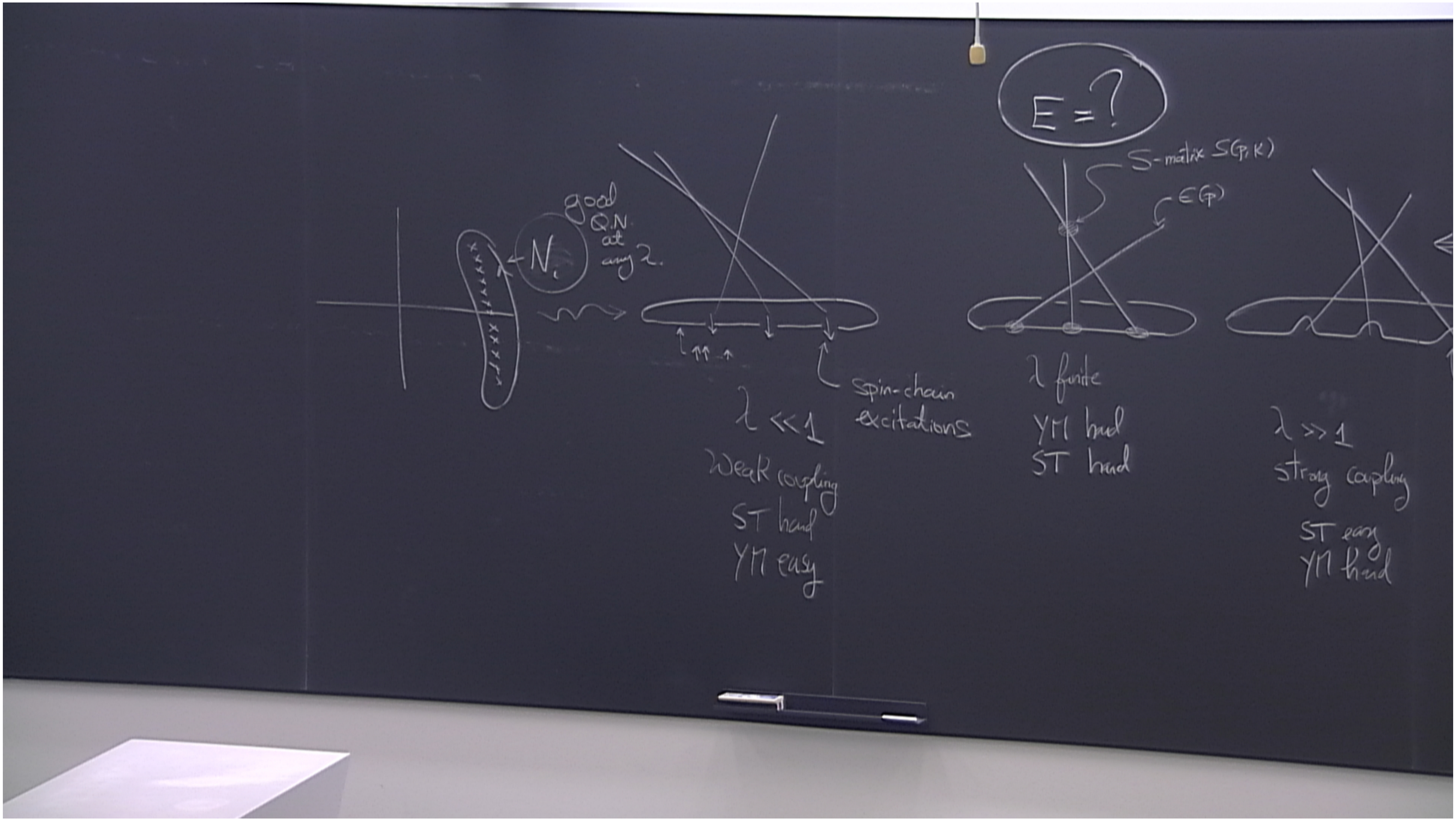
gl
nsf.

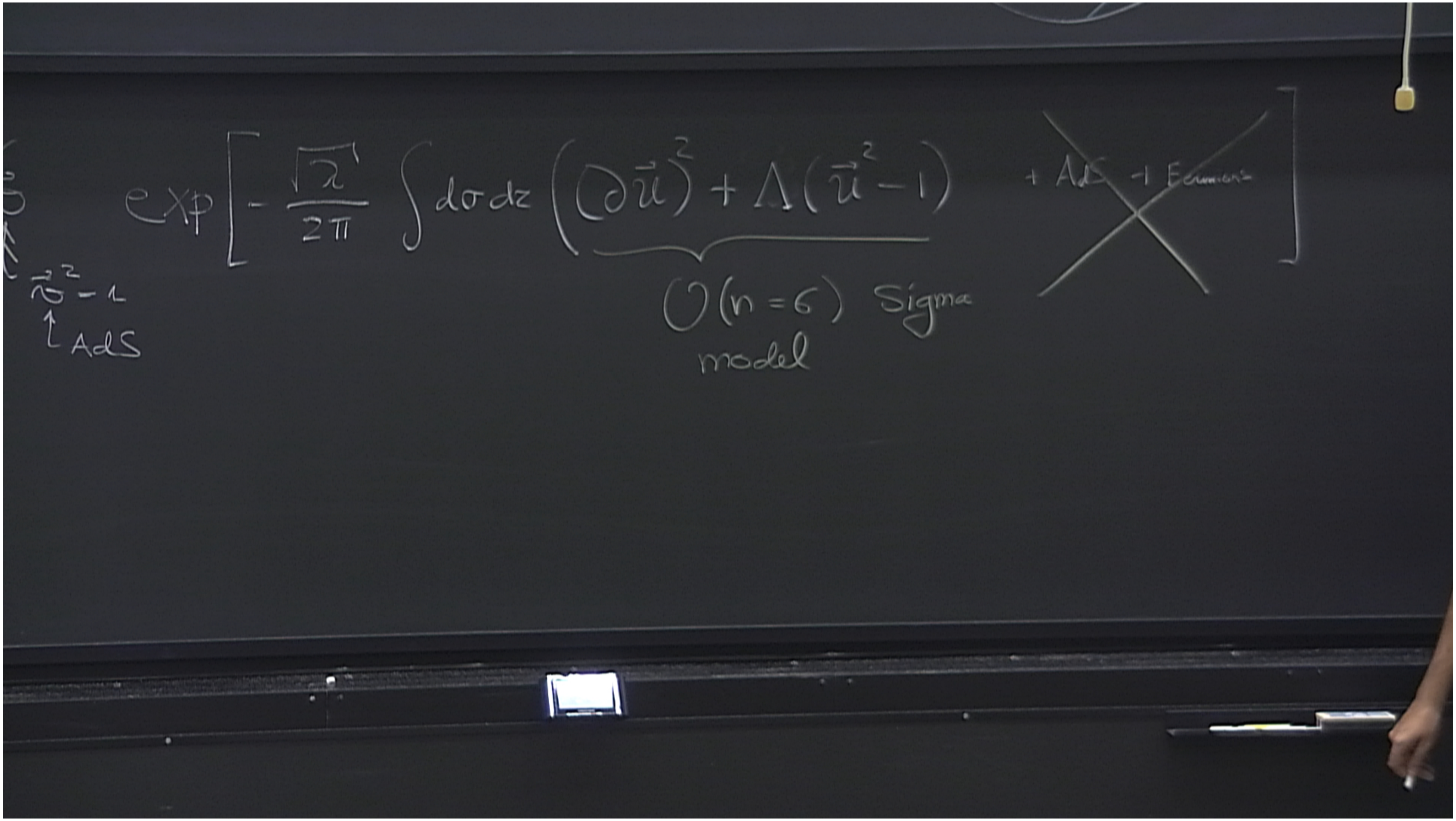
$\mathcal{N}=4$ SYM in $M^4 \longleftrightarrow AdS_5 \times S^5$ strings in poincaré space.

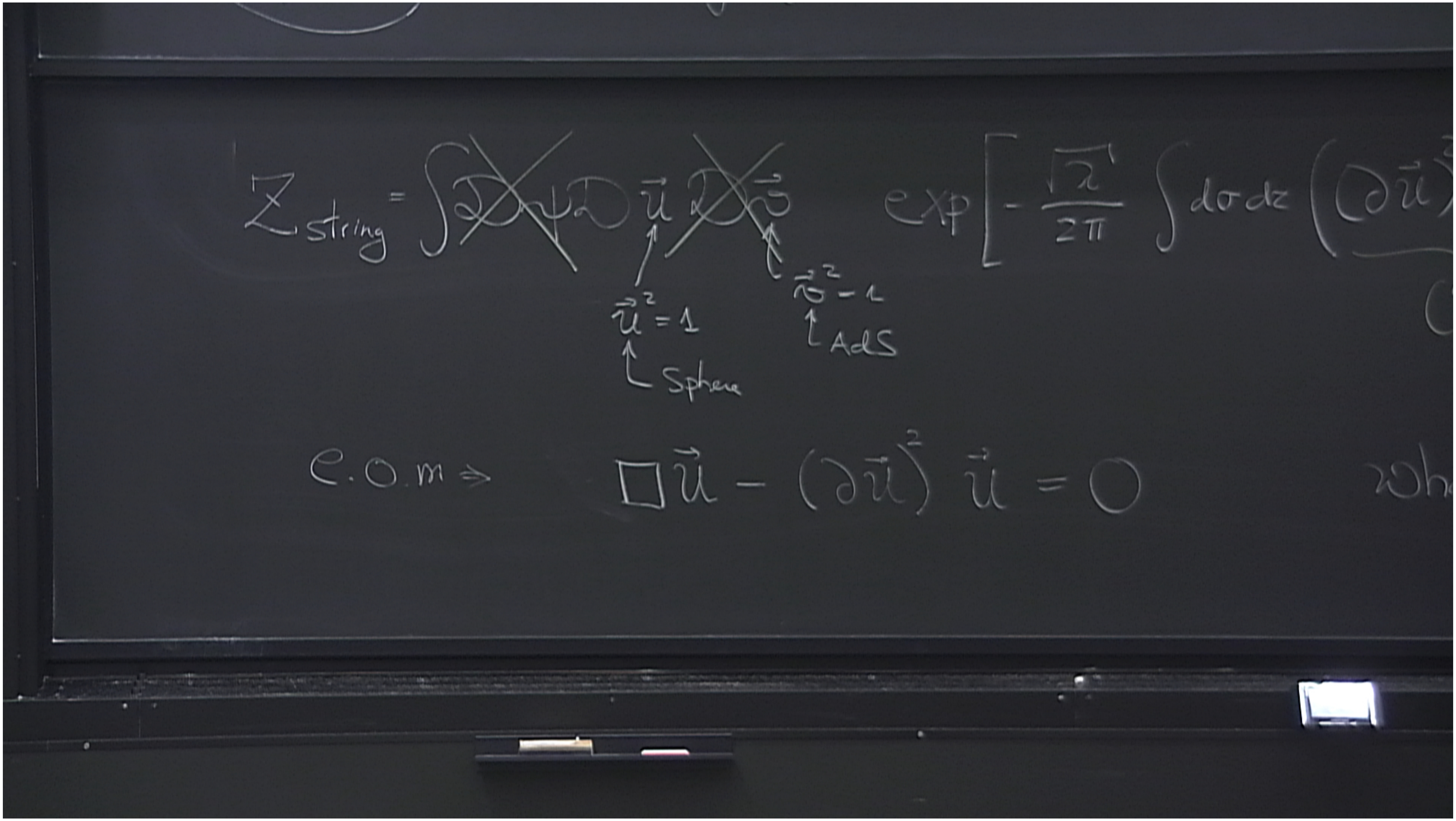
$\mathcal{N}=4$ SYM in $\mathbb{R}\times S^3 \longleftrightarrow AdS_5 \times S^5$ strings in global AdS.

$$\underbrace{\{\Delta\}}_{M^4} = \{E\}_{\substack{\mathcal{N}=4\,\mathrm{SYM} \\ \mathrm{in}\ \mathbb{R}\times S^3}} = \underbrace{\{E\}}_{\mathrm{strings}}$$

1


E = ?
S-matrix S(p,k)
ε(p)
good Q.N. at any λ.
N_c
spin-chain excitations
λ << 1
weak coupling
ST hard
YM easy
λ finite
YM hard
ST hard
λ >> 1
strong coupling
ST easy
YM hard


$\exp\left[-\frac{\sqrt{\lambda}}{2\pi}\int d\sigma\, dz \left(\underbrace{(\partial\vec{u})^2+\Lambda(\vec{u}^2-1)}_{O(n=6)\ \text{Sigma model}}\right) \cancel{+\ \text{AdS} + \text{Fermions}}\right]$
$\vec{v}^2=1$
AdS


Z string = ∫ Dψ D u D v exp[- √λ / 2π ∫ dσ dτ ((∂u)
u² = 1
Sphere
v² = 1
AdS
e.o.m ⇒ □u − (∂u)² u = 0

in $\mathbb{R}\times S^3$

1

6 is very big. $n \gg 1$.

$$= \int \mathcal{D}\Lambda(\sigma,\tau)\ \exp\left[\frac{\sqrt{\lambda}}{2\pi}\int \Lambda\right.$$

in $\mathbb{R}\times S^3$

1

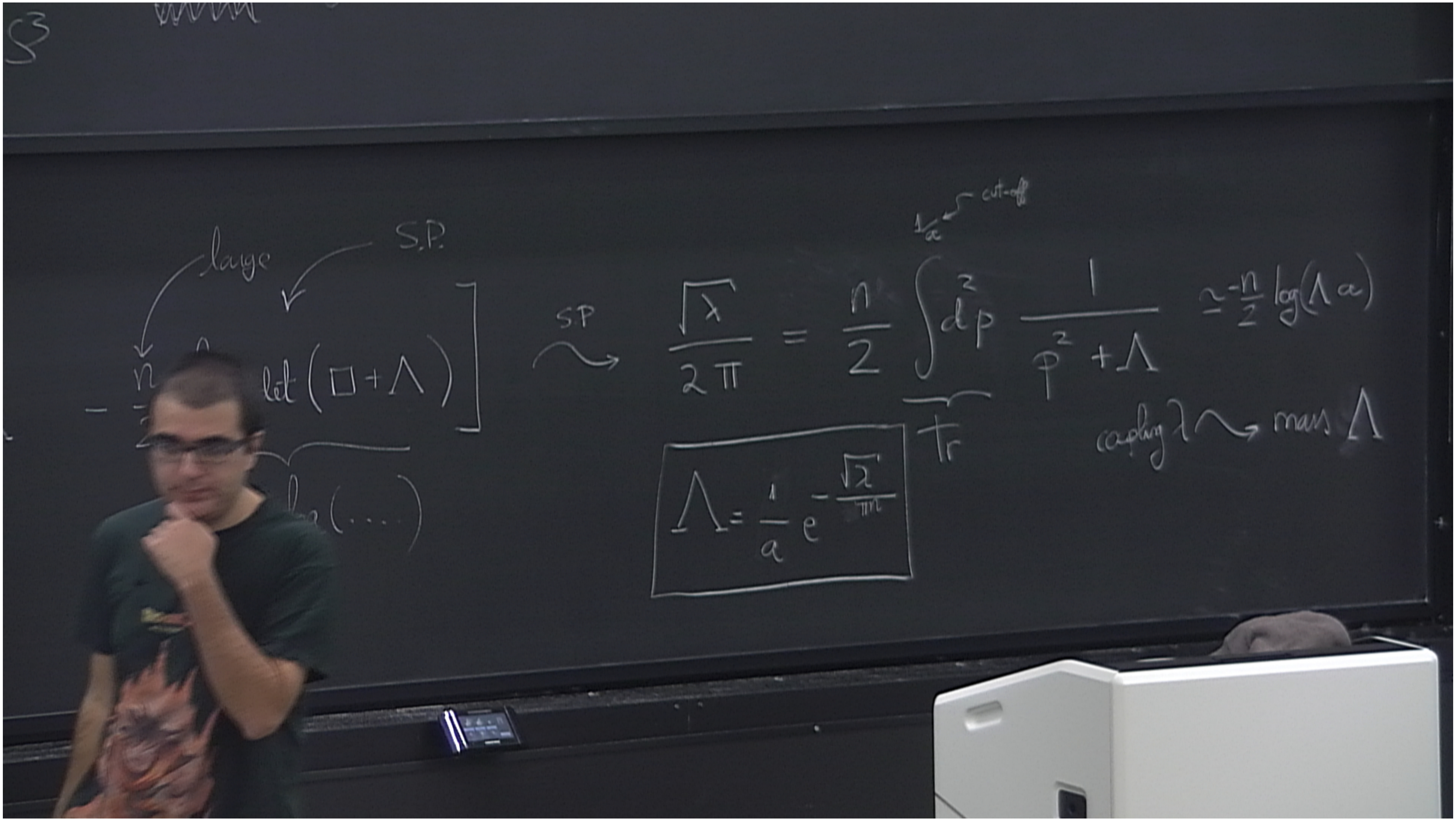
6 is very big. $n \gg 1$.

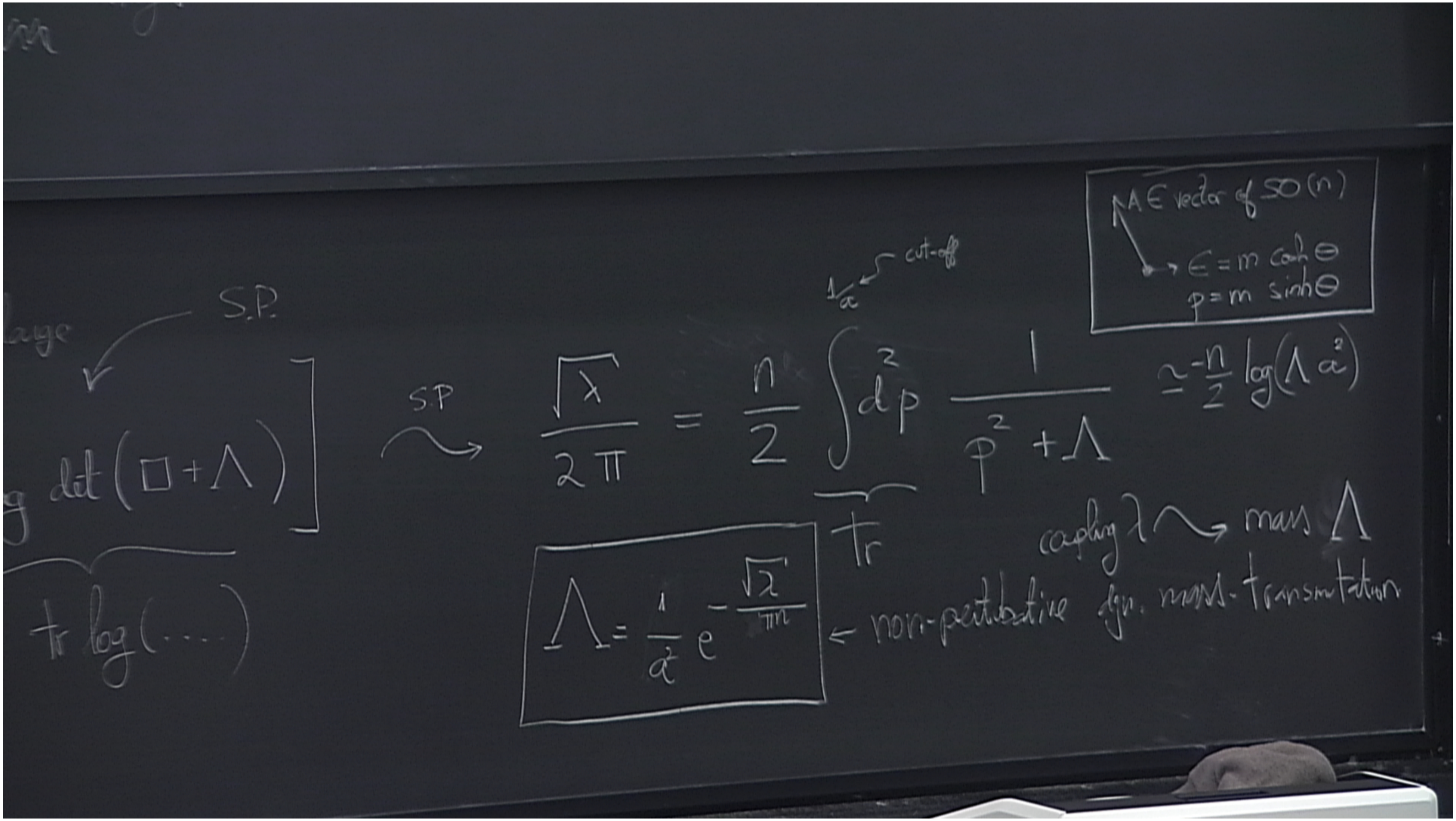
large

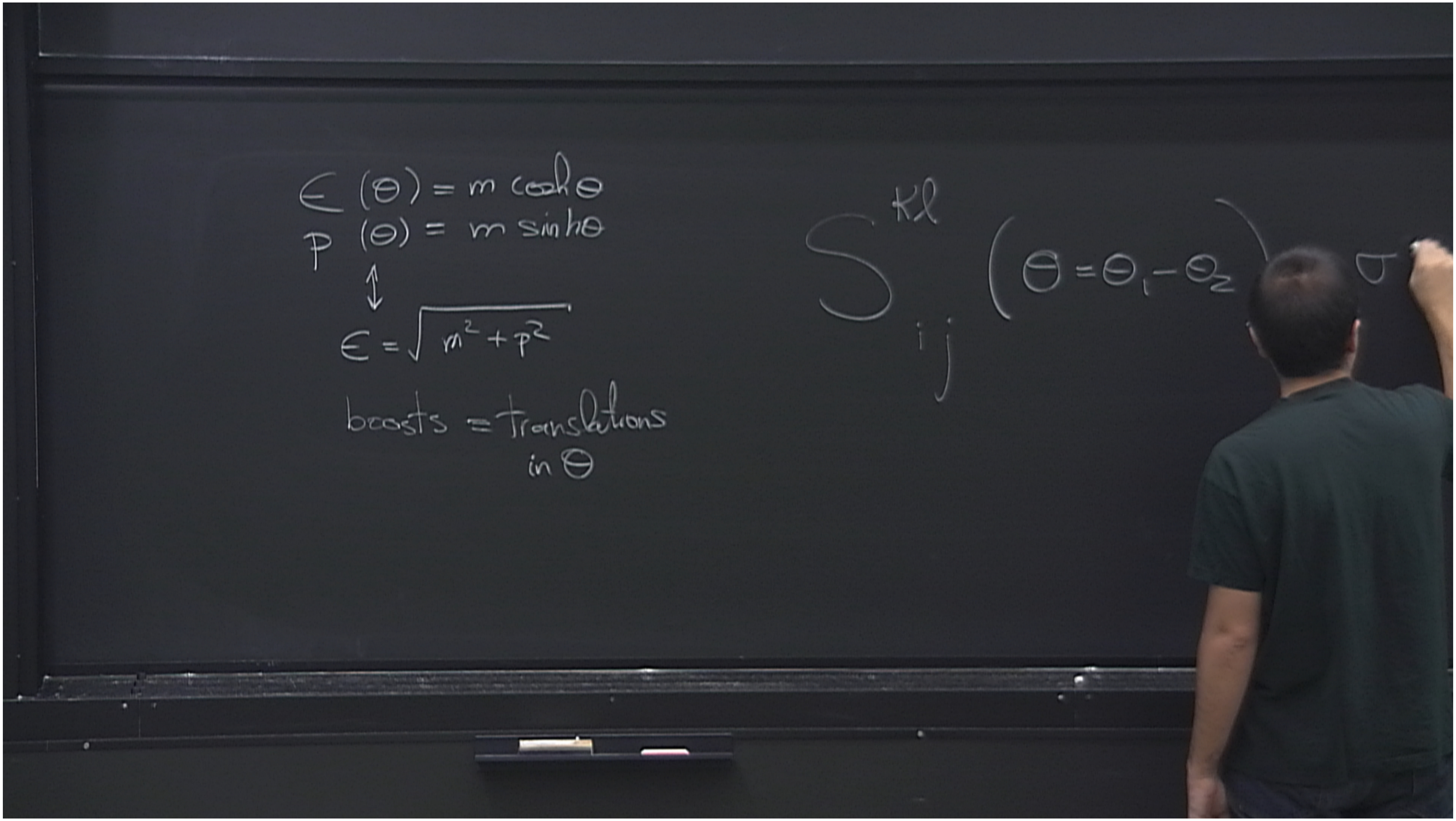
S.P.

SP

$$= \int \mathcal{D}\Lambda(\sigma,\tau)\, \exp\left[\frac{\sqrt{\lambda}}{2\pi}\int \Lambda - \frac{n}{2}\log\det(\Box+\Lambda)\right] \rightsquigarrow$$


large
S.P.
cut-off
1/a
SP
$\frac{\sqrt{\lambda}}{2\pi} = \frac{n}{2} \int d^2p \, \frac{1}{p^2 + \Lambda} \simeq -\frac{n}{2} \log(\Lambda a)$
Tr
coupling λ ⟶ mass Λ
$\Lambda = \frac{1}{a} e^{-\frac{\sqrt{\lambda}}{\pi n}}$
det (□ + Λ)

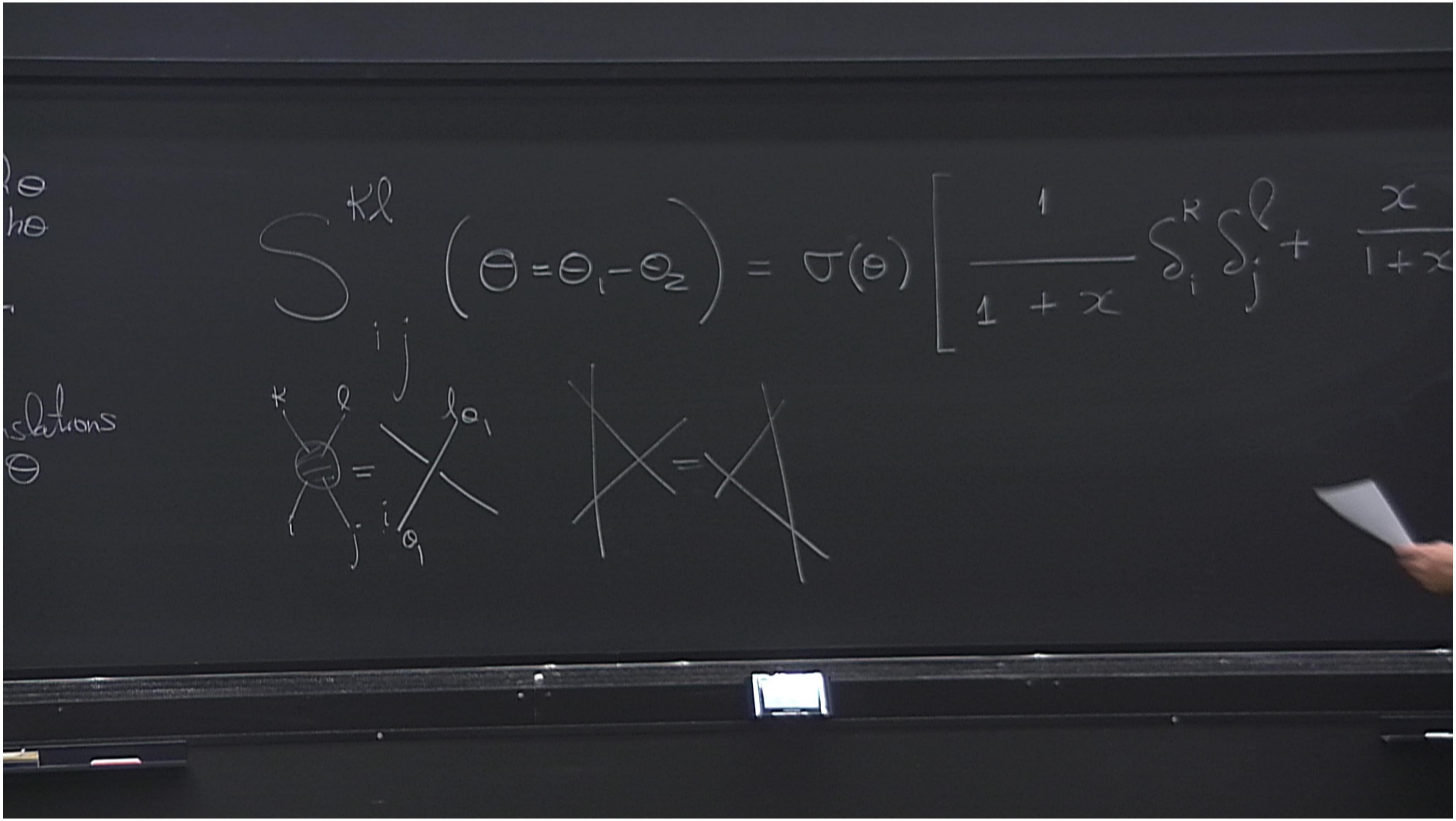

A ∈ vector of SO(n)
ε = m cosh θ
p = m sinh θ
cut-off
1/a
S.P.
large
S.P
√λ / 2π = n/2 ∫ d²p 1/(p² + Λ) ≃ -n/2 log(Λ a²)
log det(□ + Λ)
tr log(. . . .)
tr
coupling λ ⇝ mass Λ
Λ = 1/a² e^(-√λ/πn)
← non-perturbative dyn. mass-transmutation

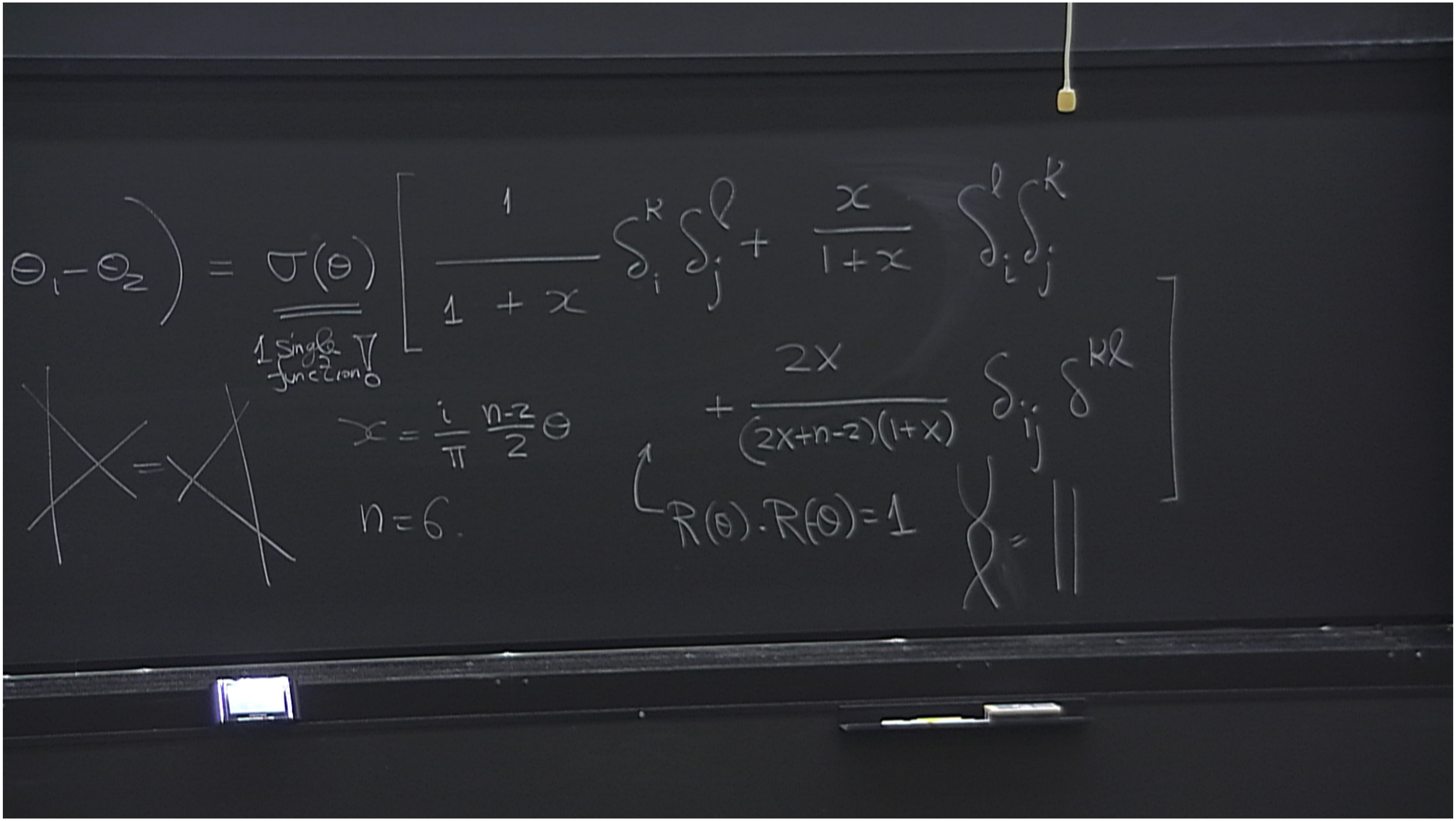

$\epsilon(\theta) = m \cosh\theta$
$p(\theta) = m \sinh\theta$
$\epsilon = \sqrt{m^2 + p^2}$
boosts = translations in θ
$S^{kl}_{ij}(\theta = \theta_1 - \theta_2)$

$$\epsilon(\theta) = m\cosh\theta$$
$$p(\theta) = m\sinh\theta$$
$$\updownarrow$$
$$\epsilon = \sqrt{m^2 + p^2}$$

boosts = translations in θ

$$S_{ij}^{kl}(\theta = \theta_1 - \theta_2) = \sigma(\theta)\Big[\qquad \delta_{i}^{k}$$


$S^{kl}_{ij}(\theta = \theta_1 - \theta_2) = \sigma(\theta)\left[\frac{1}{1+x}\delta^k_i \delta^l_j + \frac{x}{1+x}\right.$
nslations


$\theta_1-\theta_2) = \sigma(\theta)\left[\frac{1}{1+x}\delta_i^k\delta_j^l + \frac{x}{1+x}\delta_i^l\delta_j^k + \frac{2x}{(2x+n-2)(1+x)}\delta_{ij}\delta^{kl}\right]$
1 single function!
$x = \frac{i}{\pi}\frac{n-2}{2}\theta$
$n=6.$
$R(\theta)\cdot R(-\theta)=1$
=
=

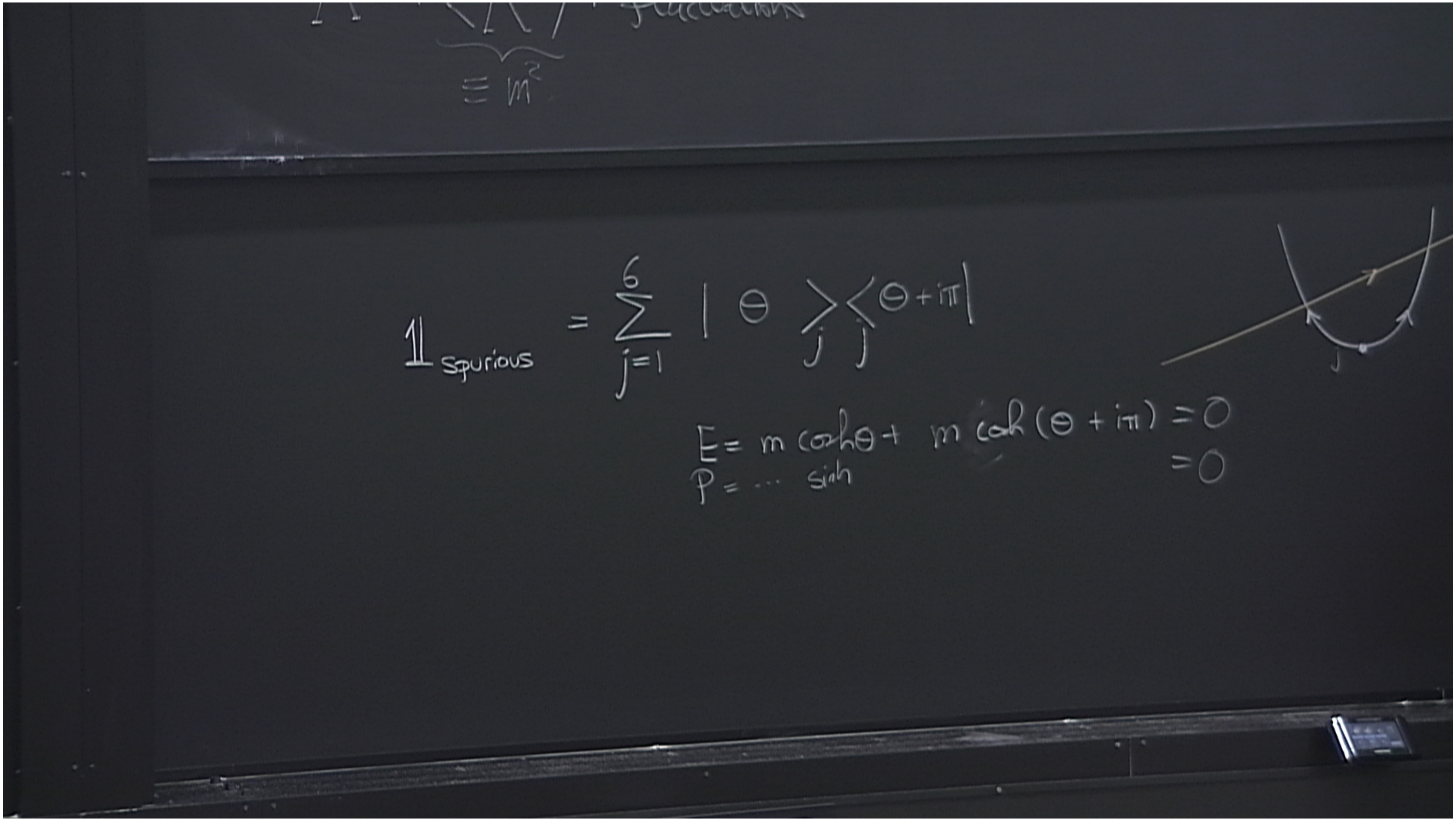
fluctuations

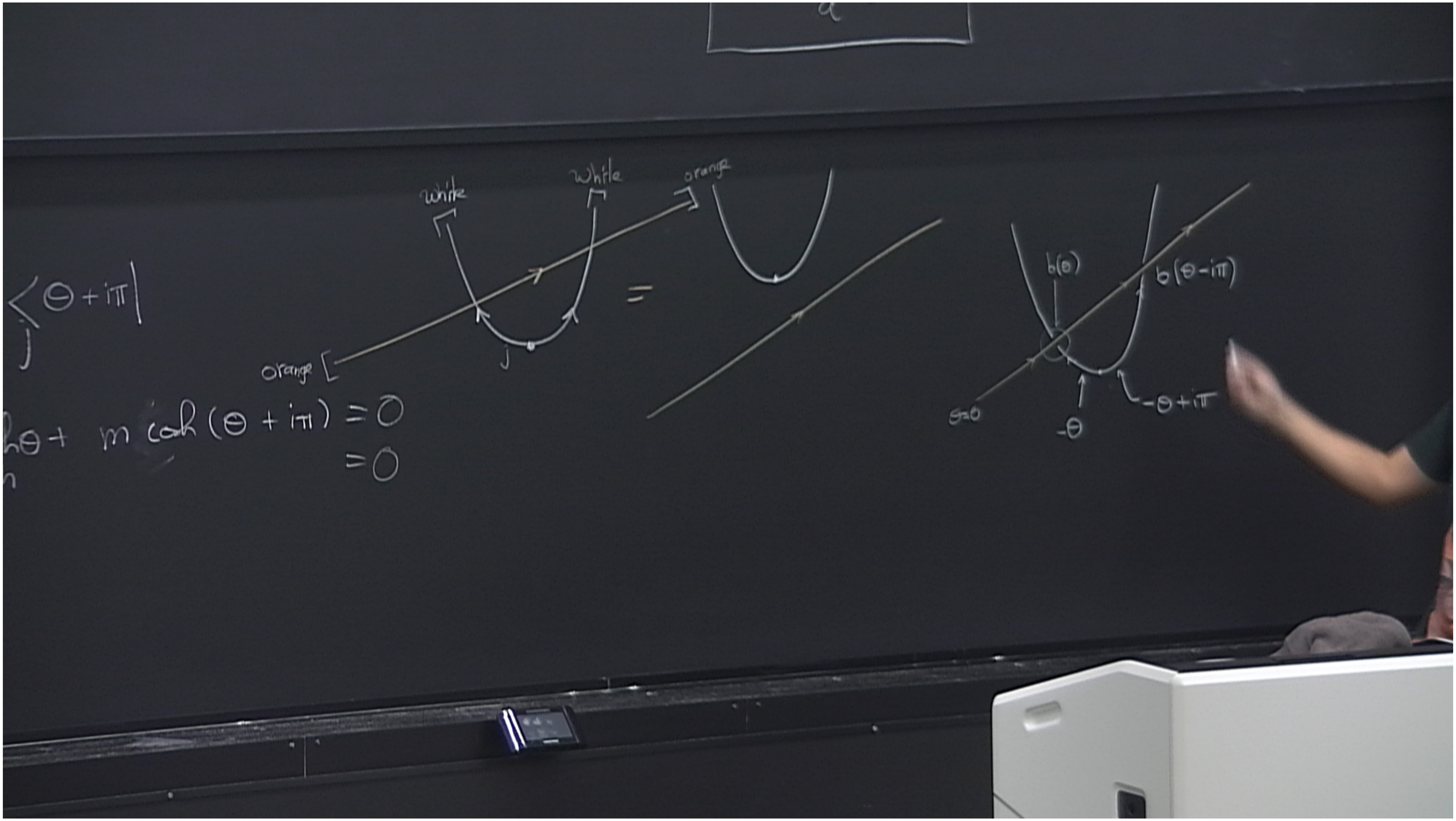
$\equiv m^2$

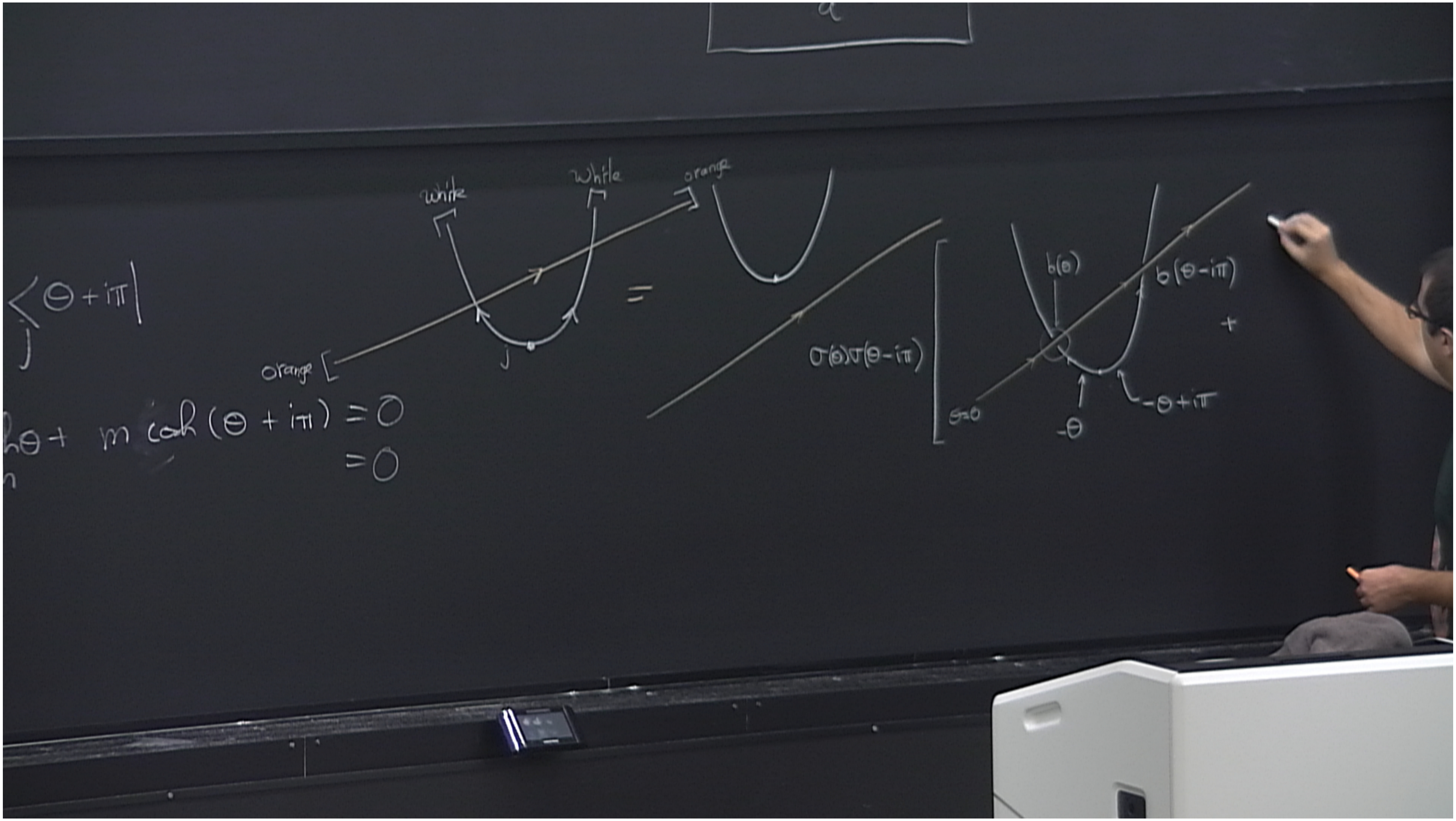
$$\mathbb{1}_{\text{spurious}} = \sum_{j=1}^{6} |\,\theta\,\rangle_j \; {}_j\langle\,\theta + i\pi\,|$$

$$E = m\cosh\theta + m\cosh(\theta + i\pi) = 0$$

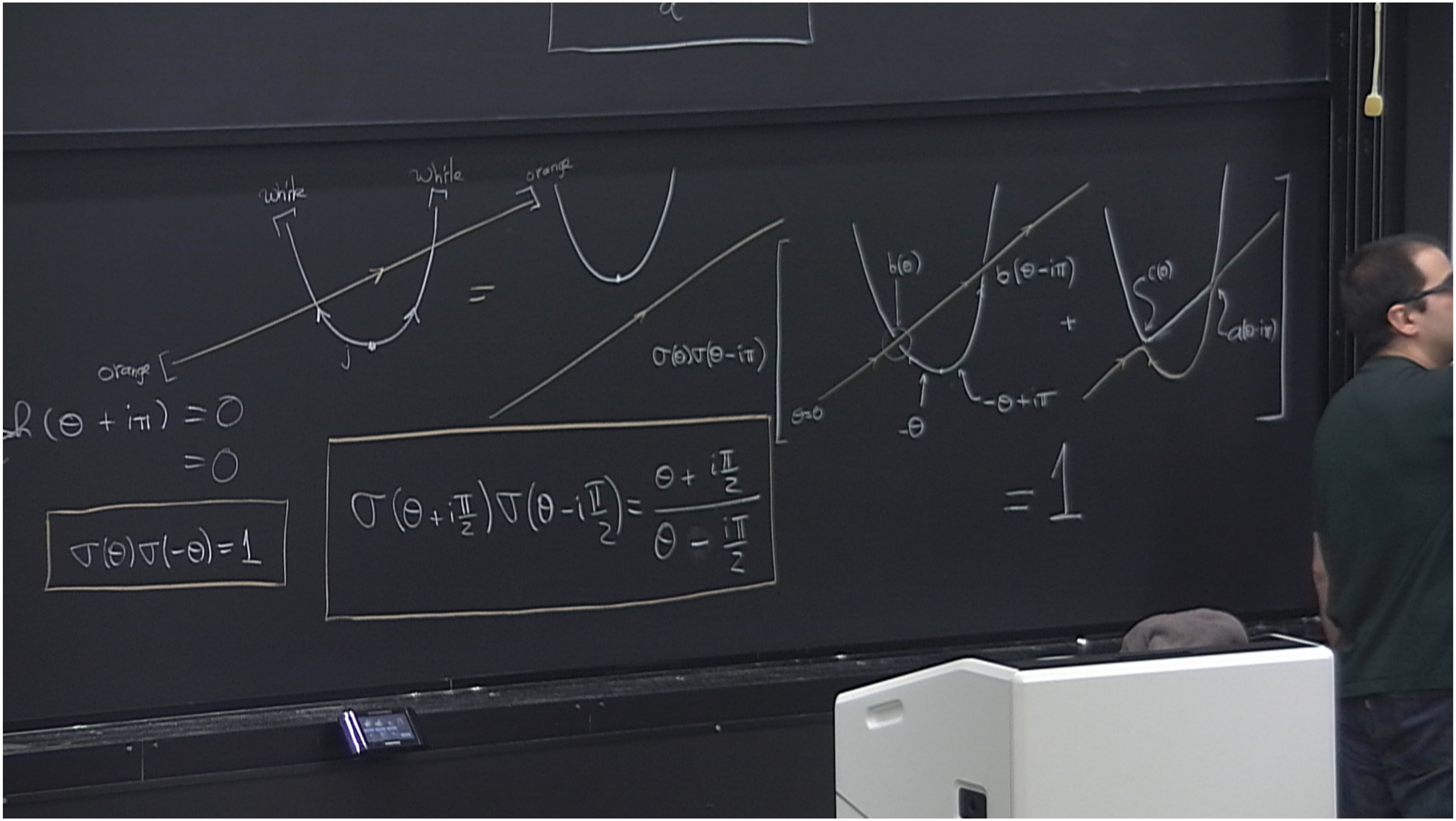
$$P = \cdots \sinh \qquad\qquad = 0$$

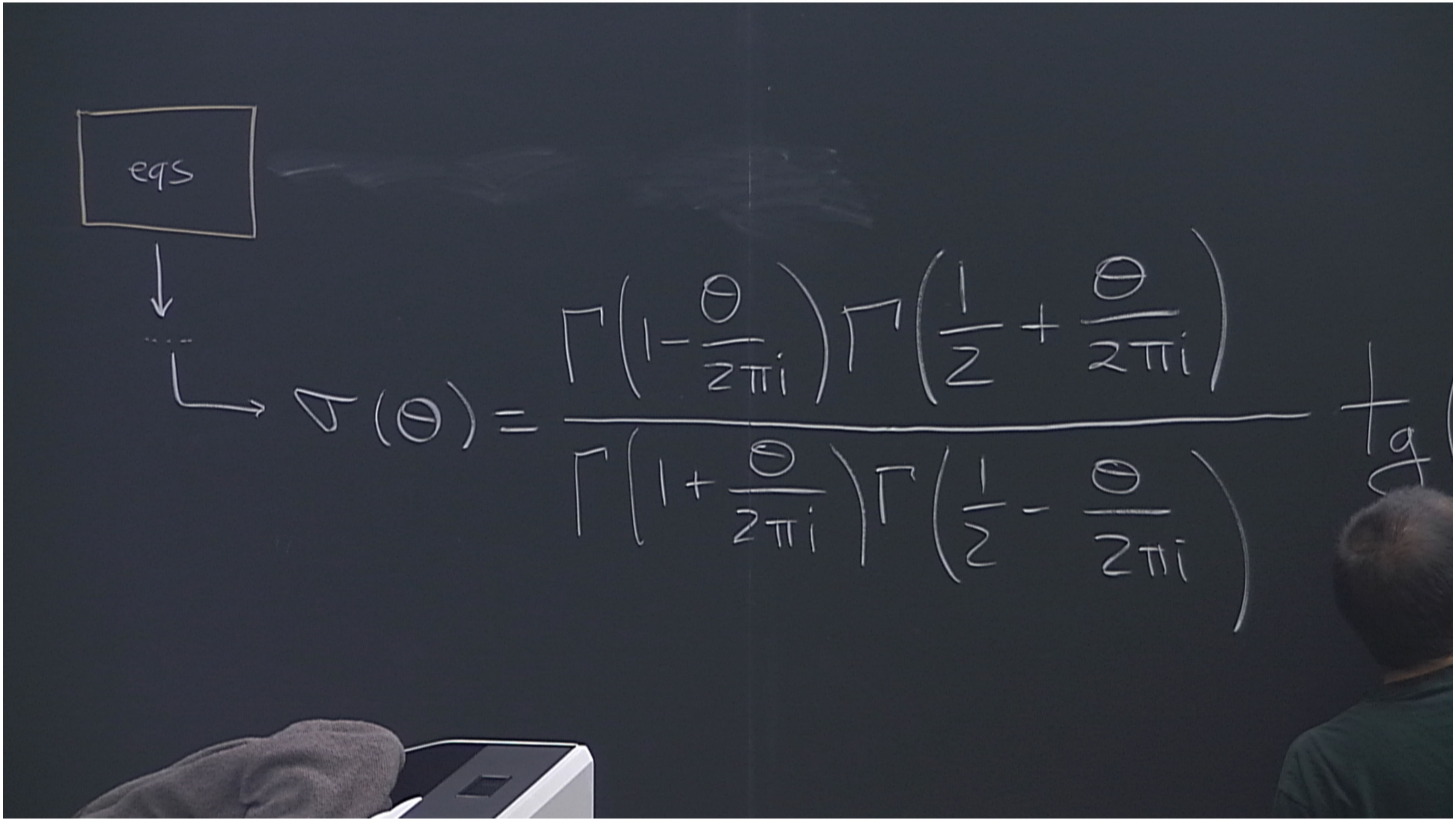

$\equiv m^2$
$\mathbb{1}_{\text{spurious}} = \sum_{j=1}^{6} |\theta\rangle_j \langle\theta + i\pi|_j$
$E = m\cosh\theta + m\cosh(\theta + i\pi) = 0$
$P = \cdots \sinh \quad = 0$
j

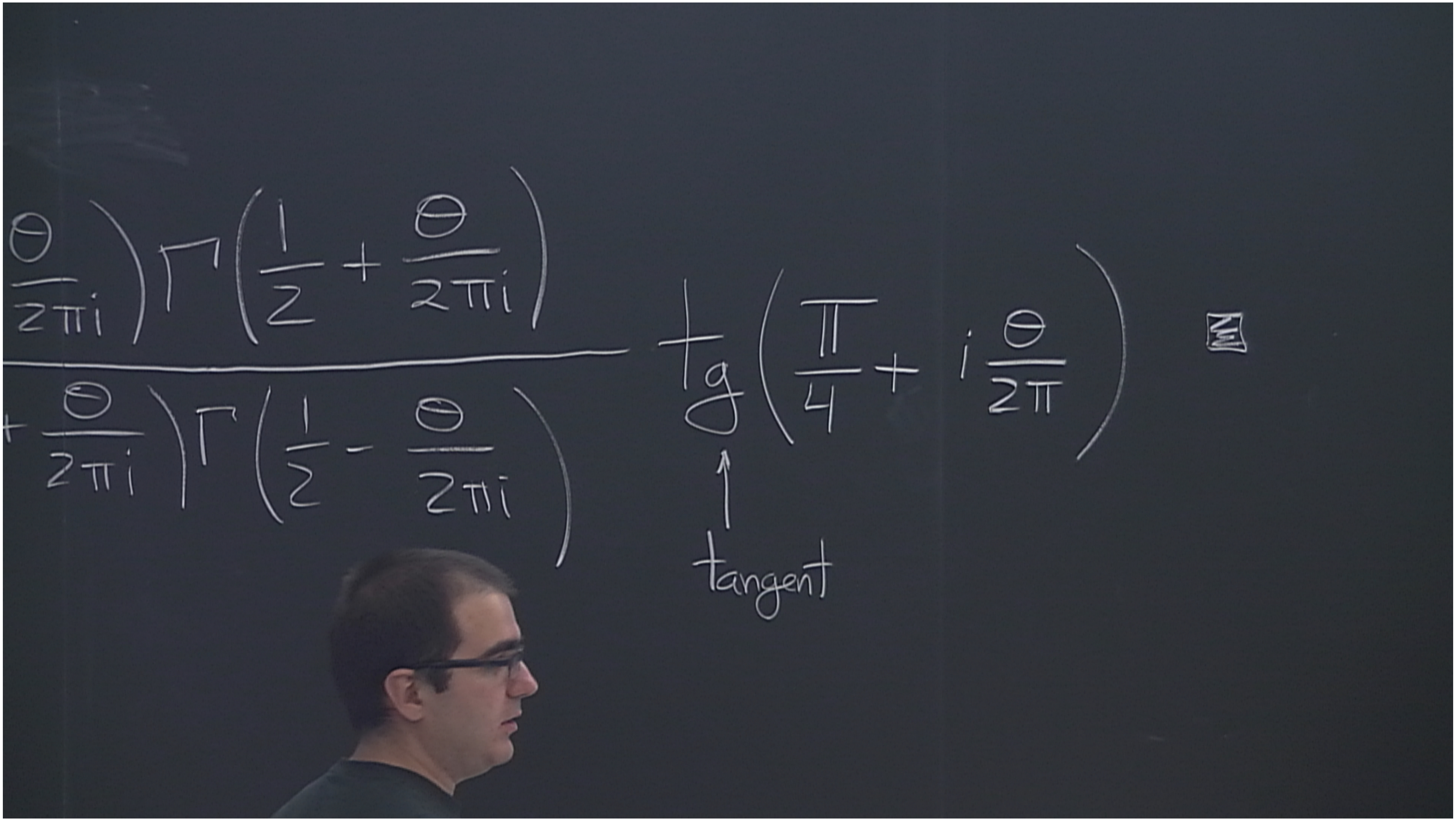

white
White
orange
orange
j
b(θ)
b(θ − iπ)
θ=0
−θ
−θ + iπ
⟨θ + iπ|
m cosh (θ + iπ) = 0
= 0



⟨θ + iπ|
m cosh(θ + iπ) = 0
= 0
white
white
orange
orange
σ(θ)σ(θ − iπ)
b(θ)
b(θ − iπ)
+
σ=0
−θ
−θ + iπ


white
white
orange
orange
j
=
σ(θ)σ(θ−iπ)
b(θ)
b(θ−iπ)
+
c(θ)
a(θ−iπ)
θ=0
−θ
−θ+iπ
= 1
(θ + iπ) = 0
= 0
σ(θ)σ(−θ) = 1
σ(θ + iπ/2)σ(θ − iπ/2) = (θ + iπ/2)/(θ − iπ/2)

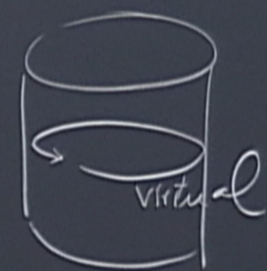

eqs
$\sigma(\theta) = \dfrac{\Gamma\left(1-\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}+\frac{\theta}{2\pi i}\right)}{\Gamma\left(1+\frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2}-\frac{\theta}{2\pi i}\right)}$ tg


$$\frac{\ldots\frac{\theta}{2\pi i}\Big)\Gamma\Big(\frac{1}{2}+\frac{\theta}{2\pi i}\Big)}{\ldots+\frac{\theta}{2\pi i}\Big)\Gamma\Big(\frac{1}{2}-\frac{\theta}{2\pi i}\Big)}\quad \mathrm{tg}\Big(\frac{\pi}{4}+i\frac{\theta}{2\pi}\Big)$$
tangent

$$\boxed{E = \sum_{j=1}^{N} m \cosh\theta_j}$$

Where

$$e^{i m \mathcal{L} \sinh\theta_j} \prod_{k \neq j}^{N} \sigma(\theta_j - \theta_k) = 1$$



$$+ \mathcal{O}\left(e^{-m\mathcal{L}}\right)$$