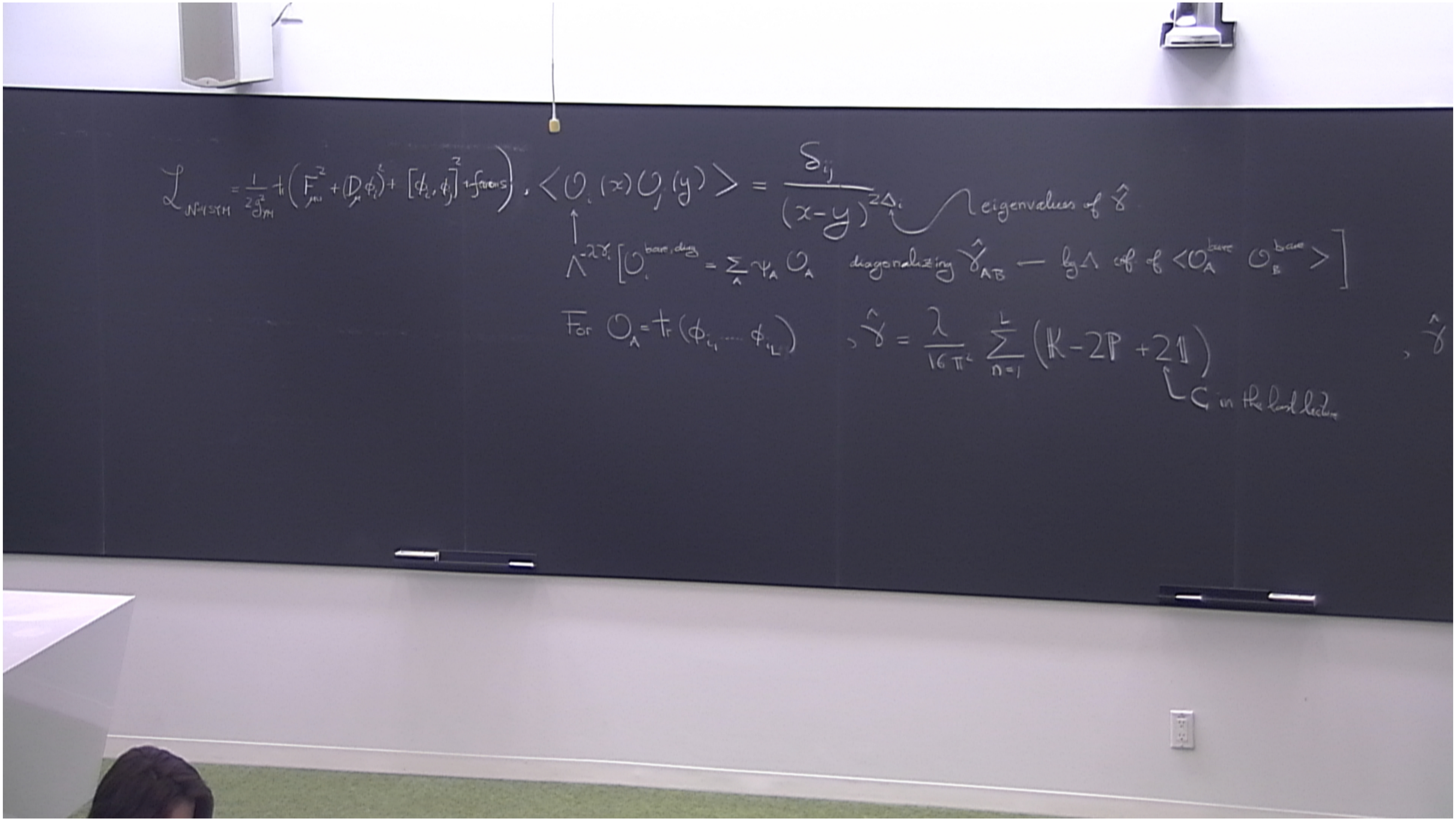
Title: Explorations in String Theory -11

Date: Apr 20, 2015 11:30 AM

URL: http://pirsa.org/15040150

Abstract:



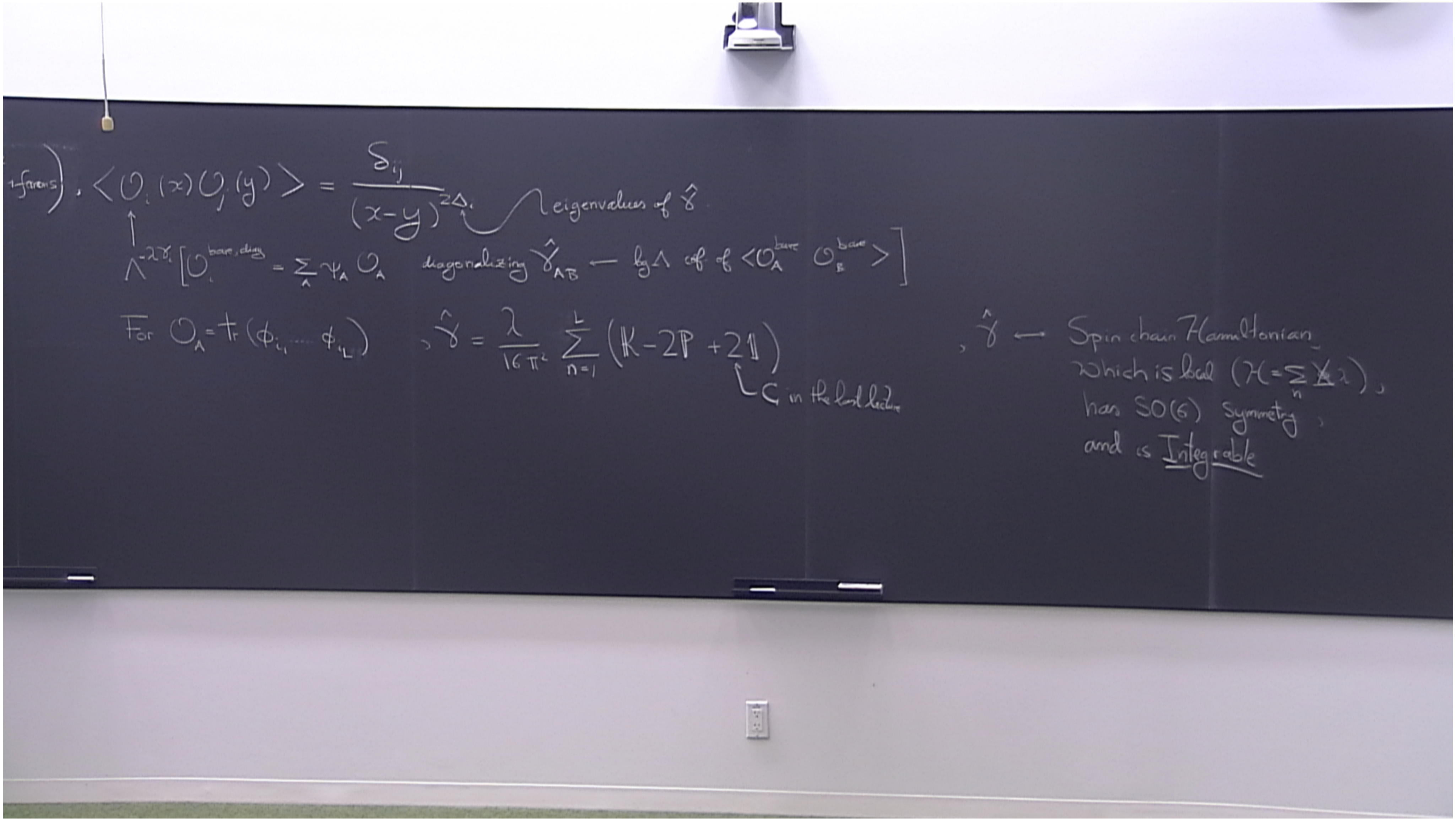
$$\mathcal{L}_{\mathcal{N}=4\,SYM} = \frac{1}{2g_{YM}^2}\mathrm{tr}\left(F_{\mu\nu}^2 + (D_\mu\phi_i)^2 + [\phi_i,\phi_j]^2 + \text{fermions}\right), \quad \langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{(x-y)^{2\Delta_i}}$$

Δ_i ← eigenvalues of $\hat{\gamma}$

$\mathcal{O}_i$ ← $\Lambda^{-2\gamma_i}\left[\mathcal{O}_i^{\text{bare, diag}} = \sum_A \psi_A \mathcal{O}_A\right.$ diagonalizing $\hat{\gamma}_{AB}$ ← $\log\Lambda$ coef of $\left.\langle \mathcal{O}_A^{\text{bare}}\,\mathcal{O}_B^{\text{bare}}\rangle\right]$

For $\mathcal{O}_A = \mathrm{tr}(\phi_{i_1}\cdots\phi_{i_L})$, $\hat{\gamma} = \frac{\lambda}{16\pi^2}\sum_{n=1}^{L}\left(\mathbb{K} - 2\mathbb{P} + 2\mathbb{1}\right)$, $\hat{\gamma}$

$\mathbb{K}$ ← C in the last lecture



fermions) , $\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \dfrac{\delta_{ij}}{(x-y)^{2\Delta_i}}$ eigenvalues of $\hat{\gamma}$

$\Lambda^{-2\gamma_i}[\mathcal{O}_i^{\text{bare, diag}} = \sum_A \psi_A \mathcal{O}_A$ diagonalizing $\hat{\gamma}_{AB} \leftarrow$ logΛ coeff of $\langle \mathcal{O}_A^{\text{bare}}\ \mathcal{O}_B^{\text{bare}}\rangle]$

For $\mathcal{O}_A = \text{tr}(\phi_{i_1} \cdots \phi_{i_L})$, $\hat{\gamma} = \dfrac{\lambda}{16\pi^2}\sum_{n=1}^{L}(\mathbb{K} - 2\mathbb{P} + 2\mathbb{1})$

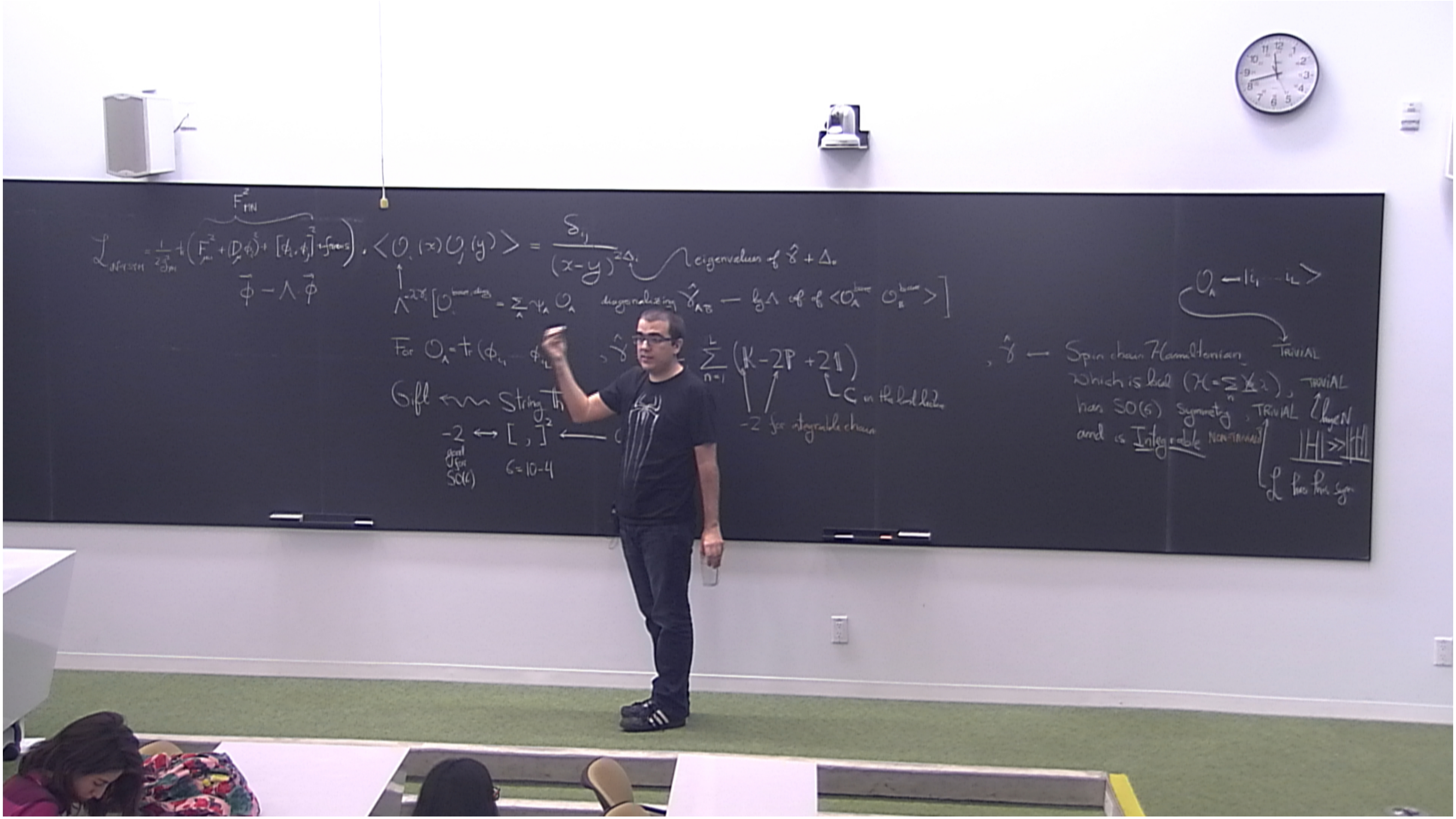
C in the last lecture

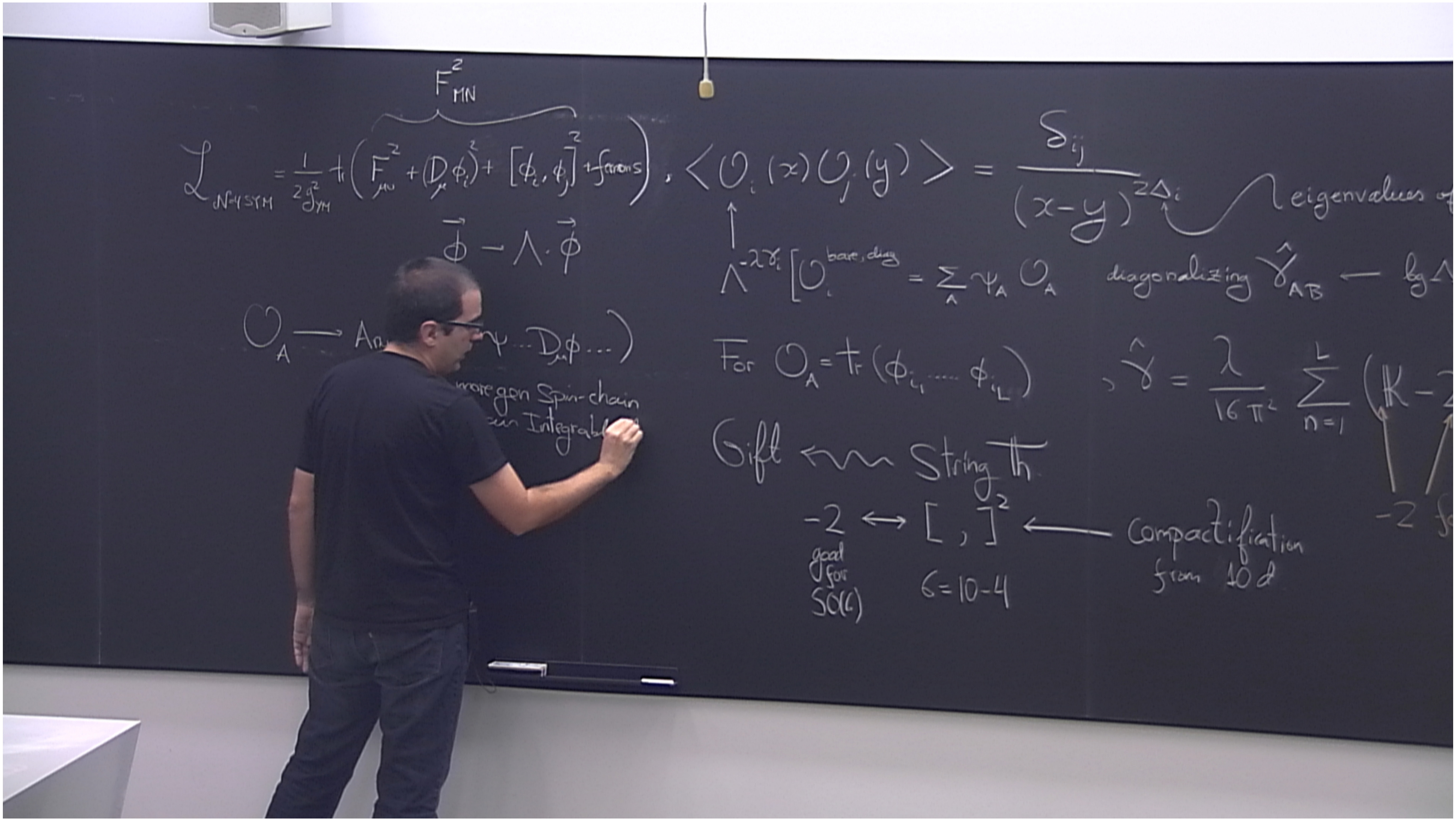
, $\hat{\gamma} \longleftrightarrow$ Spin chain Hamiltonian

which is local ($\mathcal{H} = \sum_n$ [illegible] λ) ,

has SO(6) symmetry ,

and is Integrable


F^2_{MN}
$\mathcal{L}_{N=4\,SYM} = \frac{1}{2g^2_{YM}} \mathrm{tr}\left(F^2_{\mu\nu} + (D_\mu\phi_i)^2 + [\phi_i,\phi_j]^2 + \text{fermions}\right)$, $\langle O_i(x) O_j(y)\rangle = \frac{\delta_{ij}}{(x-y)^{2\Delta_i}}$
eigenvalues of $\hat{\gamma} + \Delta_0$
$\vec{\phi} \to \Lambda \cdot \vec{\phi}$
$\Lambda^{-2\gamma_i}[O_i^{\text{bare, diag}} = \sum_A \psi_A O_A$ diagonalizing $\hat{\gamma}_{AB}$ — by Λ of $\langle O_A^{\text{bare}} O_B^{\text{bare}}\rangle]$
For $O_A = \mathrm{tr}(\phi_{i_1} \cdots \phi_{i_L}$
$\hat{\gamma}$
$\sum_{n=1}^{L} (\mathbb{K} - 2\mathbb{P} + 2\mathbb{1})$
C in the last lecture
−2 for integrable chain
Gift ⟵ String th
$-2 \leftrightarrow [\,,\,]^2 \longleftarrow$
good for SO(6)
6 = 10 − 4
$O_A \leftarrow |i_1 \cdots i_L\rangle$
TRIVIAL
, $\hat{\gamma}$ ⟵ Spin chain Hamiltonian
Which is local ($\mathcal{H} = \sum_n \mathcal{H}_{n,n+1}$), TRIVIAL
has SO(6) symmetry, TRIVIAL
and is Integrable NON-TRIVIAL
large N
$|H| \gg |H|$
$\mathcal{L}$ has this sym



$$\mathcal{L}_{\mathcal{N}=4\,SYM} = \frac{1}{2g^2_{YM}}\,\mathrm{tr}\Big(\overbrace{F_{\mu\nu}^2 + (D_\mu\phi_i)^2 + [\phi_i,\phi_j]^2}^{F^2_{MN}} + \text{fermions}\Big)\,,\quad \langle \mathcal{O}_i(x)\,\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{(x-y)^{2\Delta_i}}$$

eigenvalues of

$$\vec\phi \to \Lambda\cdot\vec\phi$$

$$\Lambda^{-2\gamma_i}\big[\mathcal{O}_i^{\text{bare, diag}} = \sum_A \psi_A\,\mathcal{O}_A$$

diagonalizing $\hat\gamma_{AB} \longleftarrow \log\Lambda$

$\mathcal{O}_A \longrightarrow$ An $(\psi \ldots D_\mu\phi \ldots)$

more gen Spin-chain

Integrabl

For $\mathcal{O}_A = \mathrm{tr}(\phi_{i_1}\cdots\phi_{i_L})$, $\hat\gamma = \frac{\lambda}{16\pi^2}\sum_{n=1}^{L}(\mathbb{K} - 2$

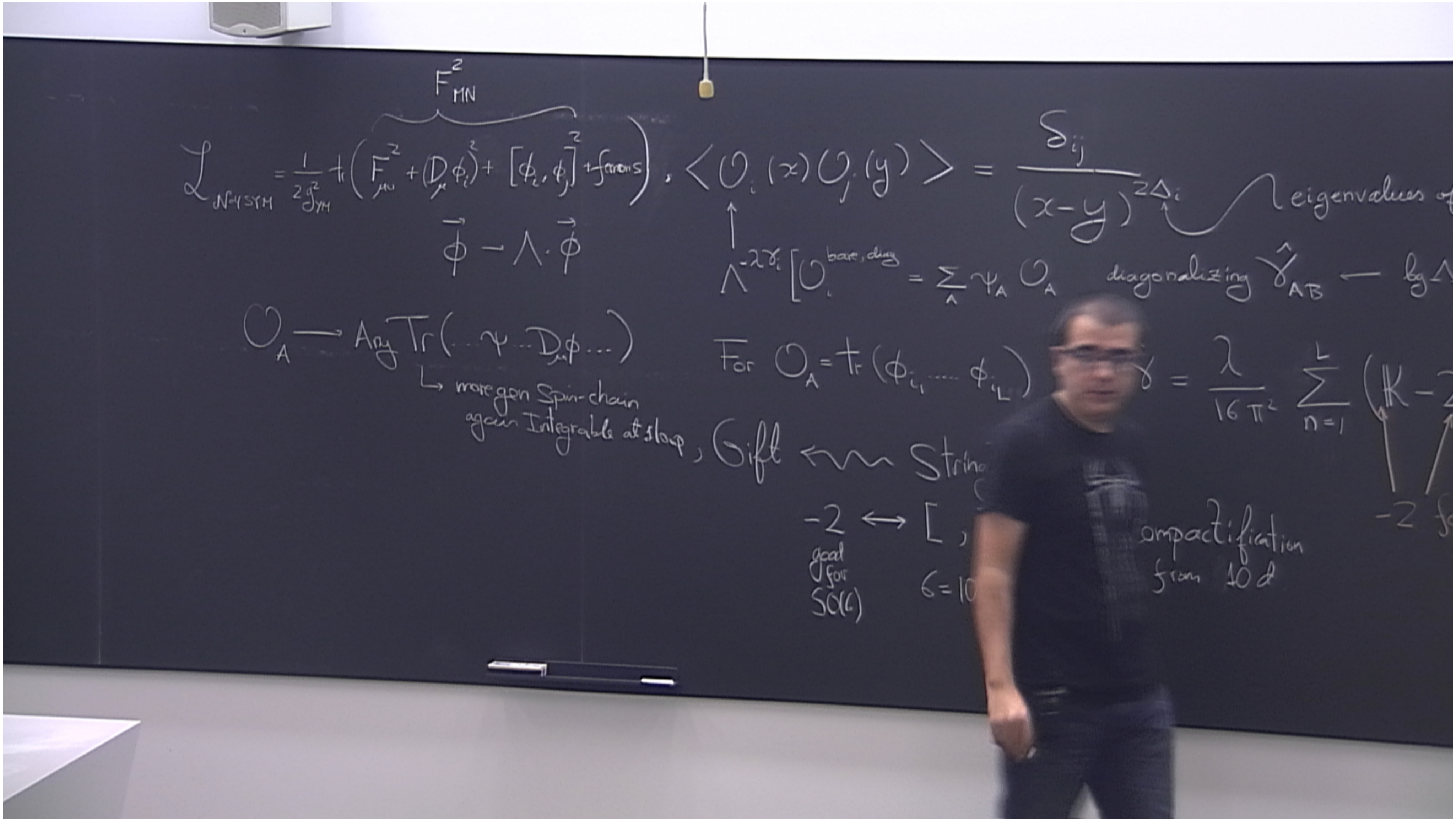
Gift $\leftsquigarrow$ String th

$-2 \leftrightarrow [\ ,\ ]^2 \longleftarrow$ Compactification from 10 d

good for SO(6)

$6 = 10-4$

-2 fo



$$\mathcal{L}_{\mathcal{N}=4\,SYM} = \frac{1}{2g^2_{YM}} \mathrm{tr}\Big(\underbrace{F^2_{\mu\nu} + (D_\mu \phi_i)^2}_{F^2_{MN}} + [\phi_i, \phi_j]^2 + \text{fermions}\Big) \,, \quad \langle \mathcal{O}_i(x)\, \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{(x-y)^{2\Delta_i}}$$

eigenvalues of

$$\vec{\phi} \to \Lambda \cdot \vec{\phi}$$

$$\Lambda^{-2\gamma_i} [\mathcal{O}_i^{\text{bare, diag}} = \sum_A \psi_A \mathcal{O}_A$$

diagonalizing $\hat{\gamma}_{AB} \leftarrow \log \Lambda$

$$\mathcal{O}_A \longrightarrow \text{Any Tr}(\ldots \psi \ldots D_\mu \phi \ldots)$$

↳ more gen Spin-chain
again Integrable at 1 loop , Gift ⟵ String

For $\mathcal{O}_A = \mathrm{tr}(\phi_{i_1} \ldots \phi_{i_L})$ $\gamma = \frac{\lambda}{16\pi^2} \sum_{n=1}^{L} (\mathbb{K} - 2$

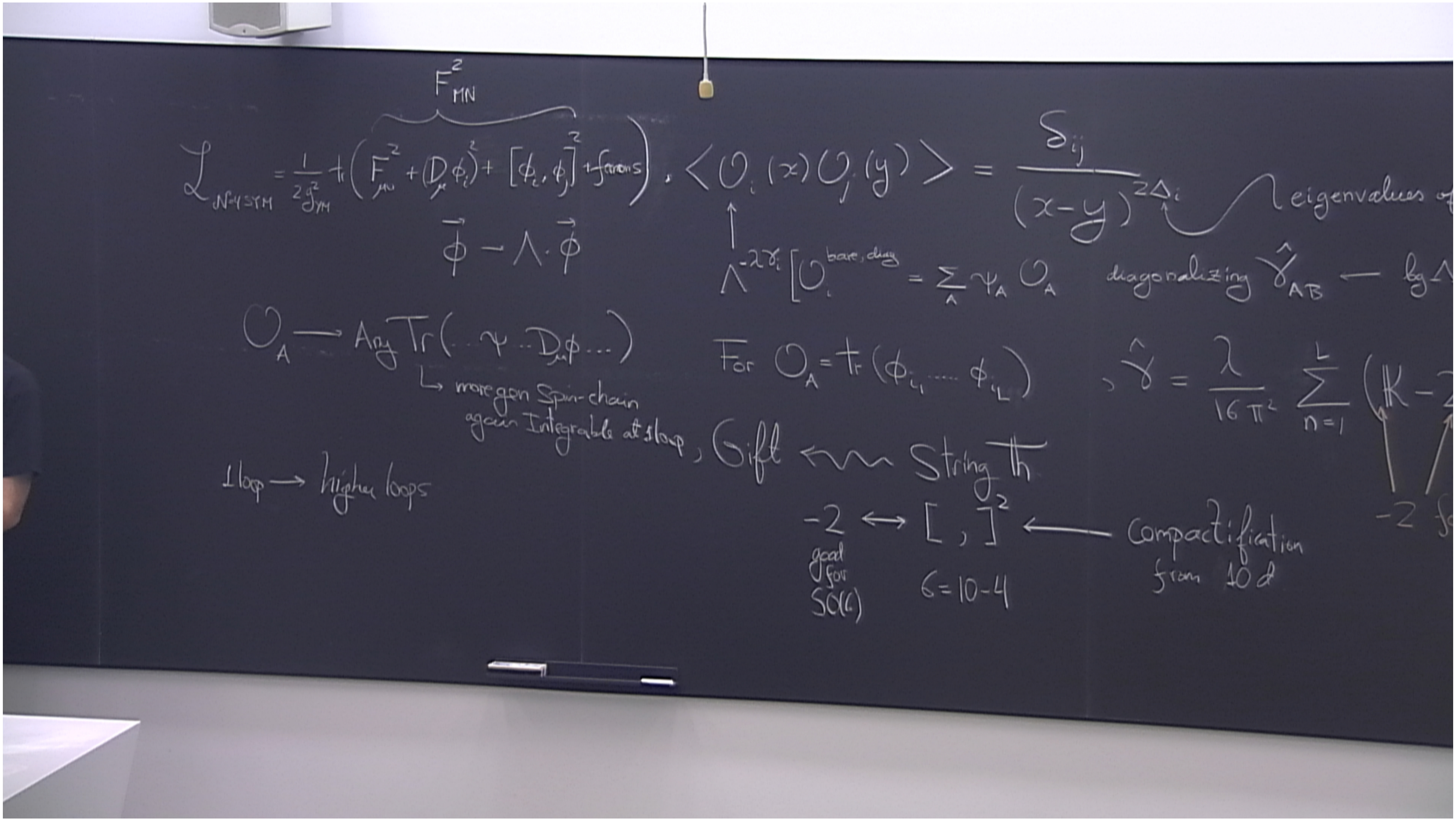
$-2 \leftrightarrow [\ ,$

good for SO(6)

$6 = 10$

ompactification from 10 d

-2 fo



$$\mathcal{L}_{\mathcal{N}=4\,SYM} = \frac{1}{2g^2_{YM}} \mathrm{tr}\Big(\underbrace{F^2_{\mu\nu} + (D_\mu \phi_i)^2 + [\phi_i, \phi_j]^2}_{F^2_{MN}} + \text{fermions} \Big) , \quad \langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{(x-y)^{2\Delta_i}}$$

eigenvalues o

$$\vec{\phi} \to \Lambda \cdot \vec{\phi}$$

$$\Lambda^{-2\gamma_i} [\mathcal{O}_i^{\text{bare, diag}} = \sum_A \psi_A \mathcal{O}_A \quad \text{diagonalizing } \hat{\gamma}_{AB} \longleftarrow \text{lg} \Lambda$$

$$\mathcal{O}_A \longrightarrow \text{Any } \mathrm{Tr}(\ldots \psi \ldots D_\mu \phi \ldots)$$

$$\text{For } \mathcal{O}_A = \mathrm{tr}(\phi_{i_1} \cdots \phi_{i_L}) , \quad \hat{\gamma} = \frac{\lambda}{16\pi^2} \sum_{n=1}^{L} (\mathbb{K} -$$

↳ more gen Spin-chain
again Integrable at 1loop, Gift ⟵ String th

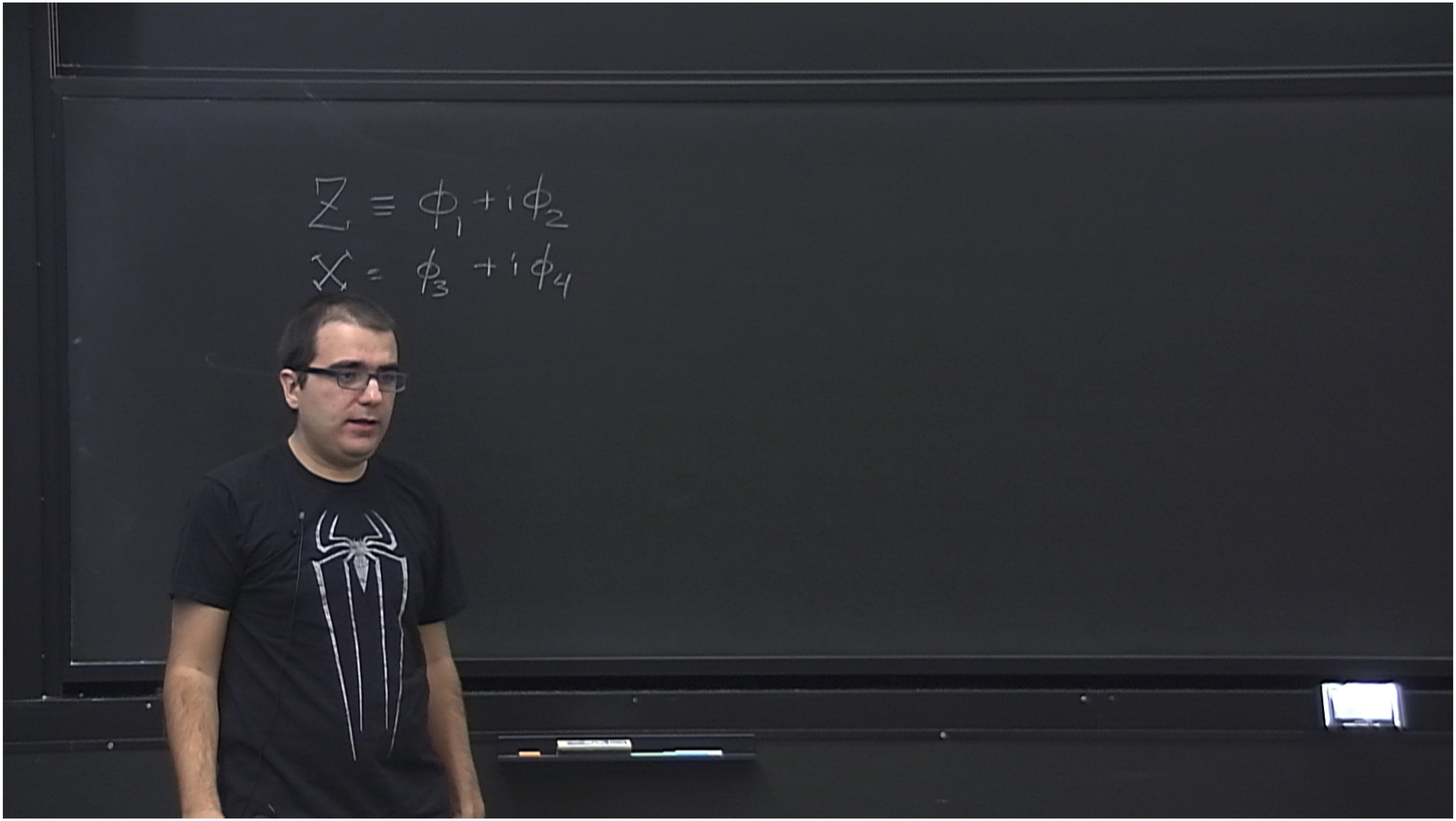
1loop ⟶ higher loops

$$-2 \longleftrightarrow [\ ,\ ]^2 \longleftarrow \text{compactification from } 10\,d$$

good for SO(6)

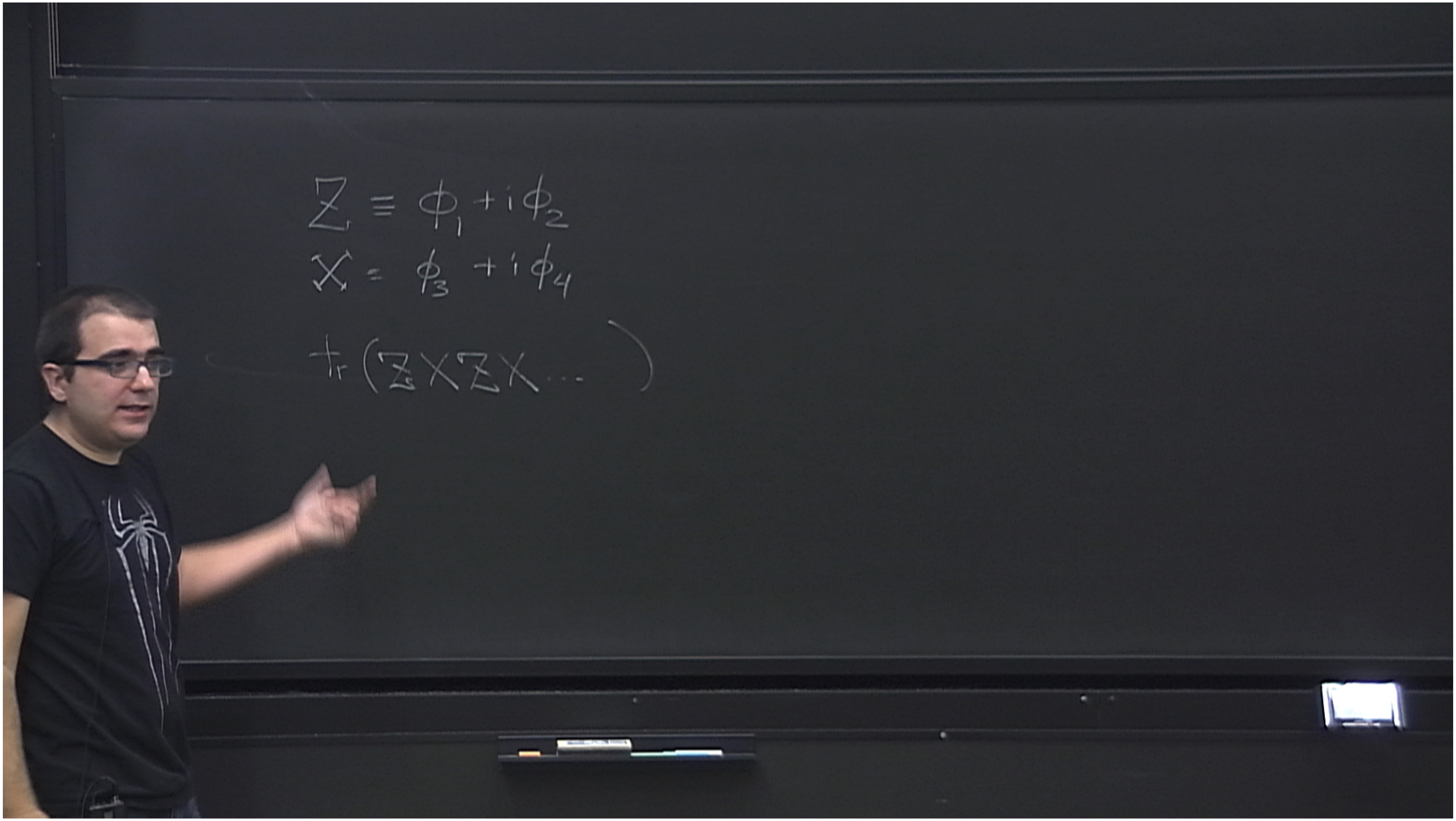
$6 = 10 - 4$

-2 fo



$$Z \equiv \phi_1 + i\phi_2$$

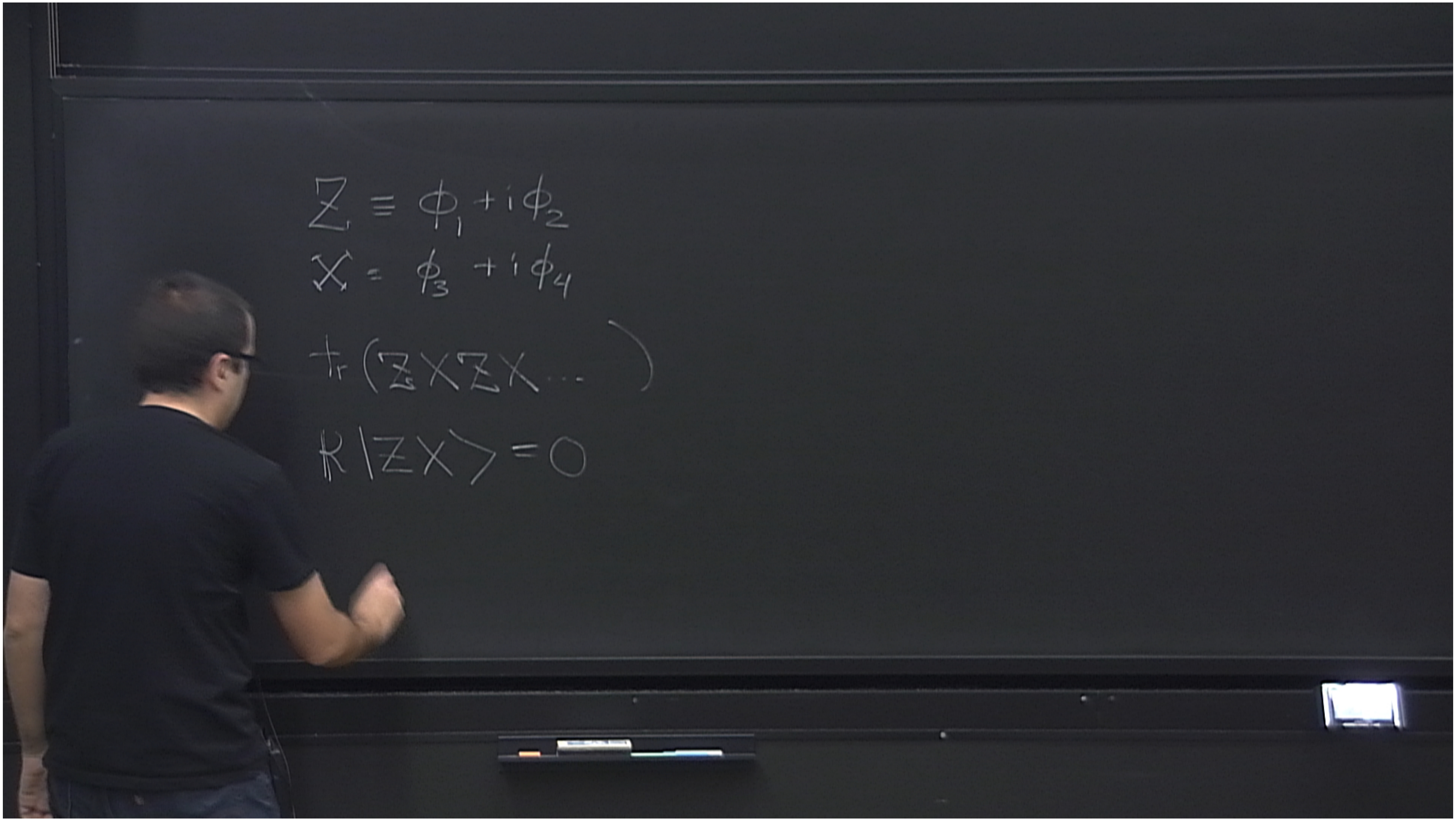
$$X = \phi_3 + i\phi_4$$



$$Z \equiv \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$\mathrm{tr}\left(ZXZX\ldots\right)$$

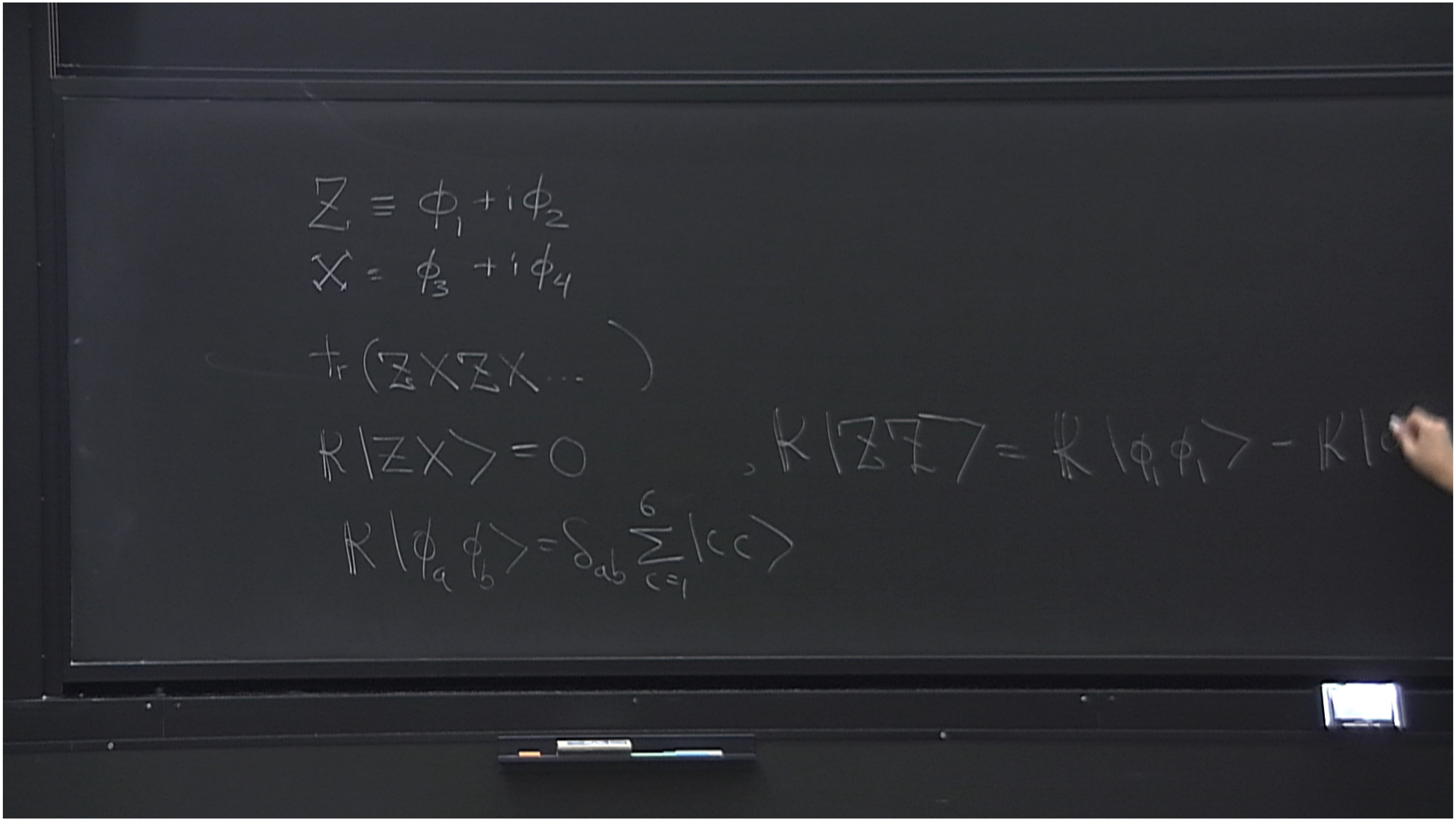


$$Z \equiv \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$\mathrm{tr}(ZXZX\ldots)$$

$$R|ZX\rangle = 0$$



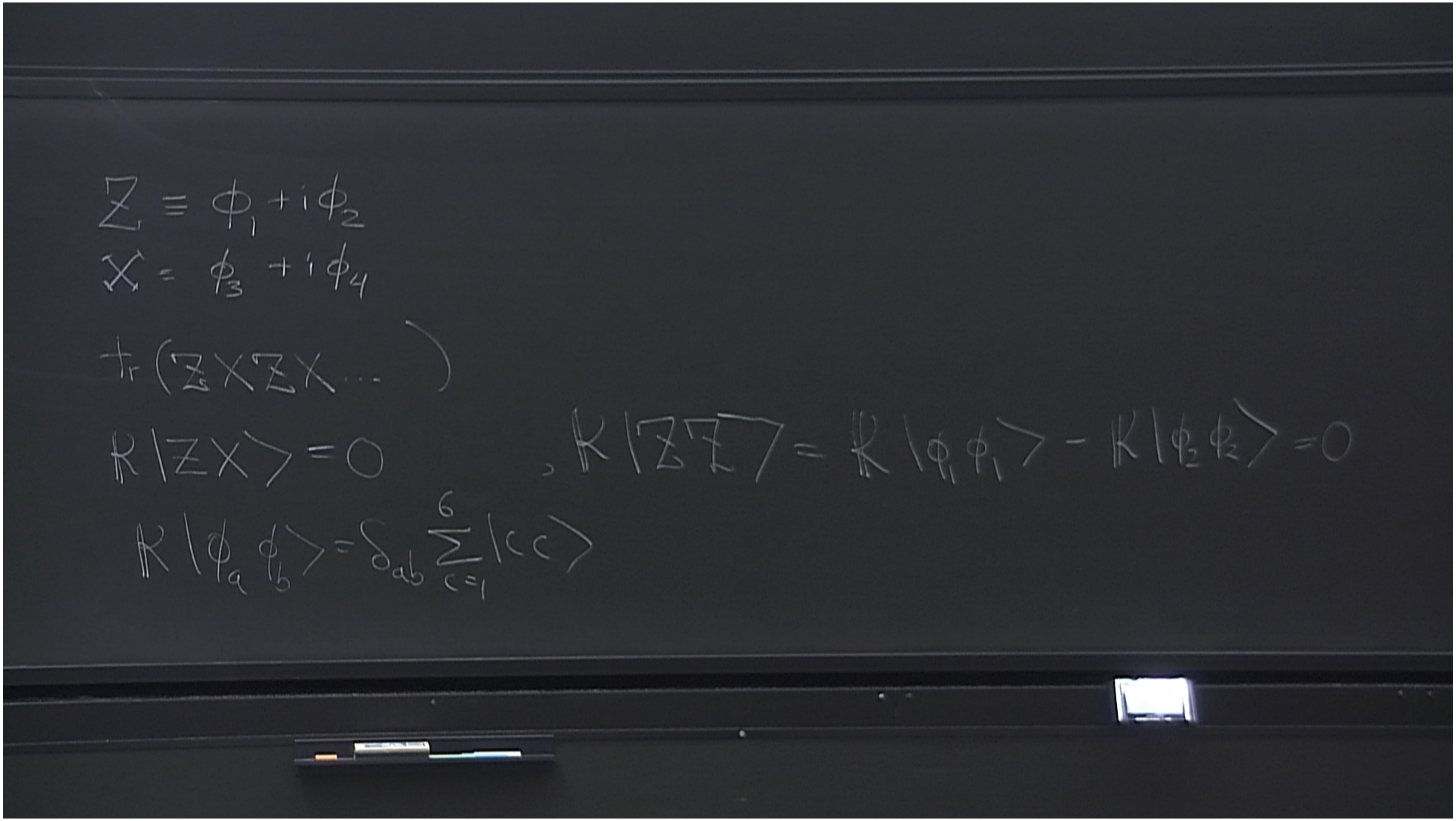
$Z \equiv \phi_1 + i\phi_2$

$X = \phi_3 + i\phi_4$

$\mathrm{tr}(ZXZX\ldots)$

$K|ZX\rangle = 0$, $K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|$

$K|\phi_a\phi_b\rangle = \delta_{ab}\sum_{c=1}^{6}|cc\rangle$



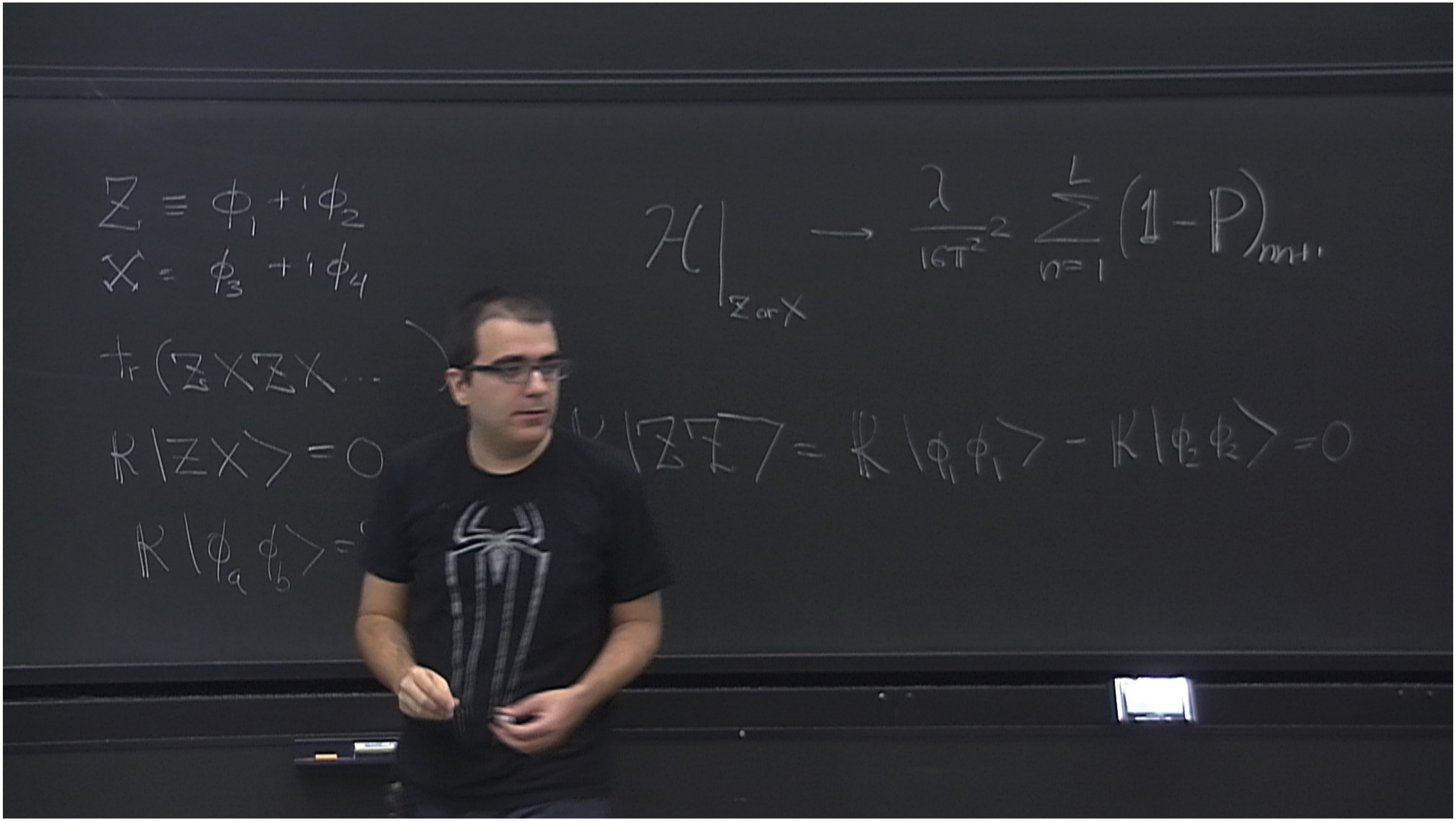
$$Z \equiv \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$\mathrm{tr}(ZXZX\ldots)$$

$$K|ZX\rangle = 0 \quad , K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$

$$K|\phi_a\phi_b\rangle = \delta_{ab}\sum_{c=1}^{6}|cc\rangle$$



$$Z \equiv \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

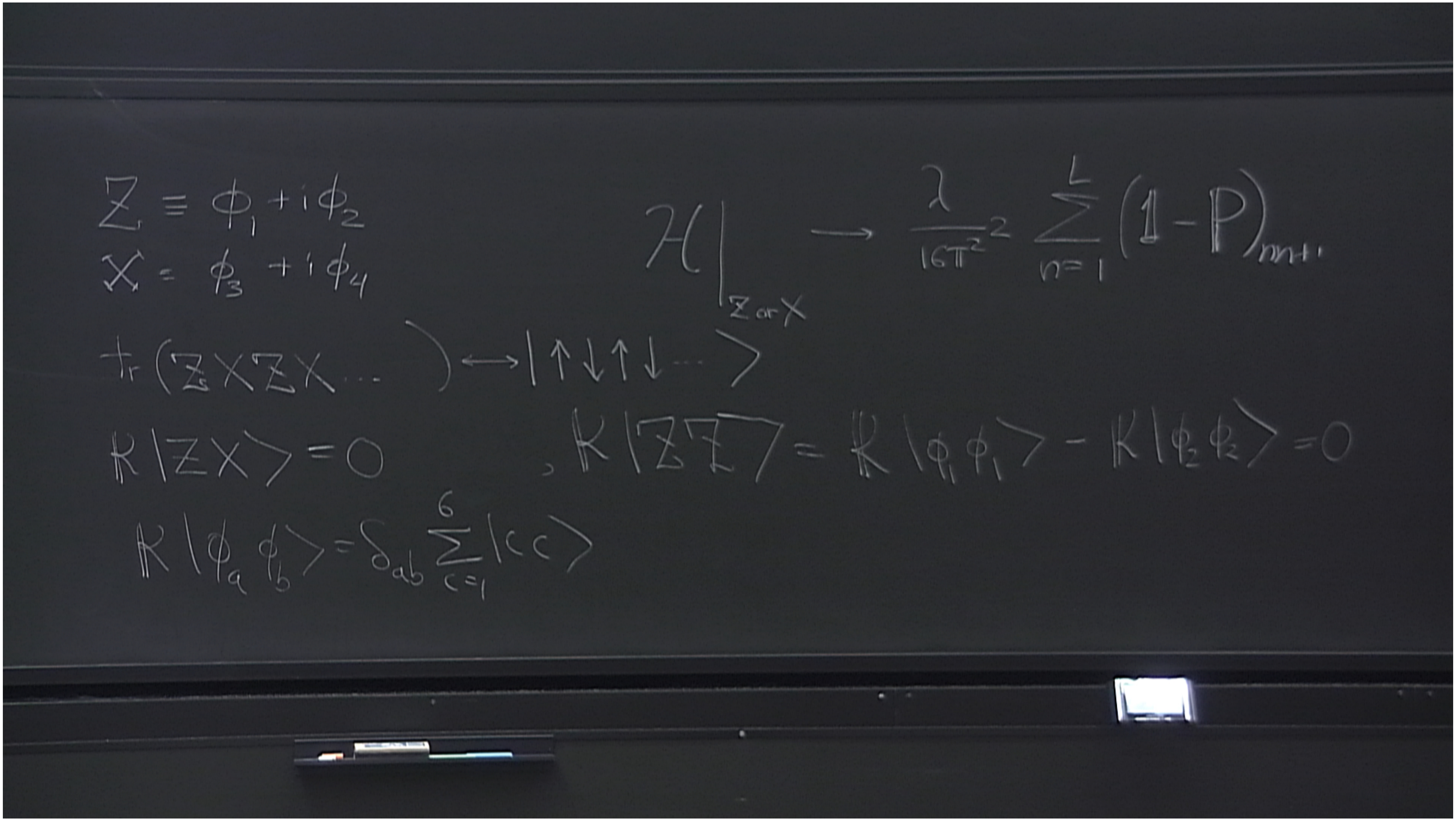
$$\mathrm{tr}(ZXZX\ldots)$$

$$K|ZX\rangle = 0$$

$$K|\phi_a \phi_b\rangle =$$

$$\mathcal{H}\Big|_{Z\text{ or }X} \longrightarrow \frac{\lambda}{16\pi^2} 2 \sum_{n=1}^{L} (1 - P)_{n\,n+1}$$

$$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$



$$Z \equiv \phi_1 + i\phi_2$$

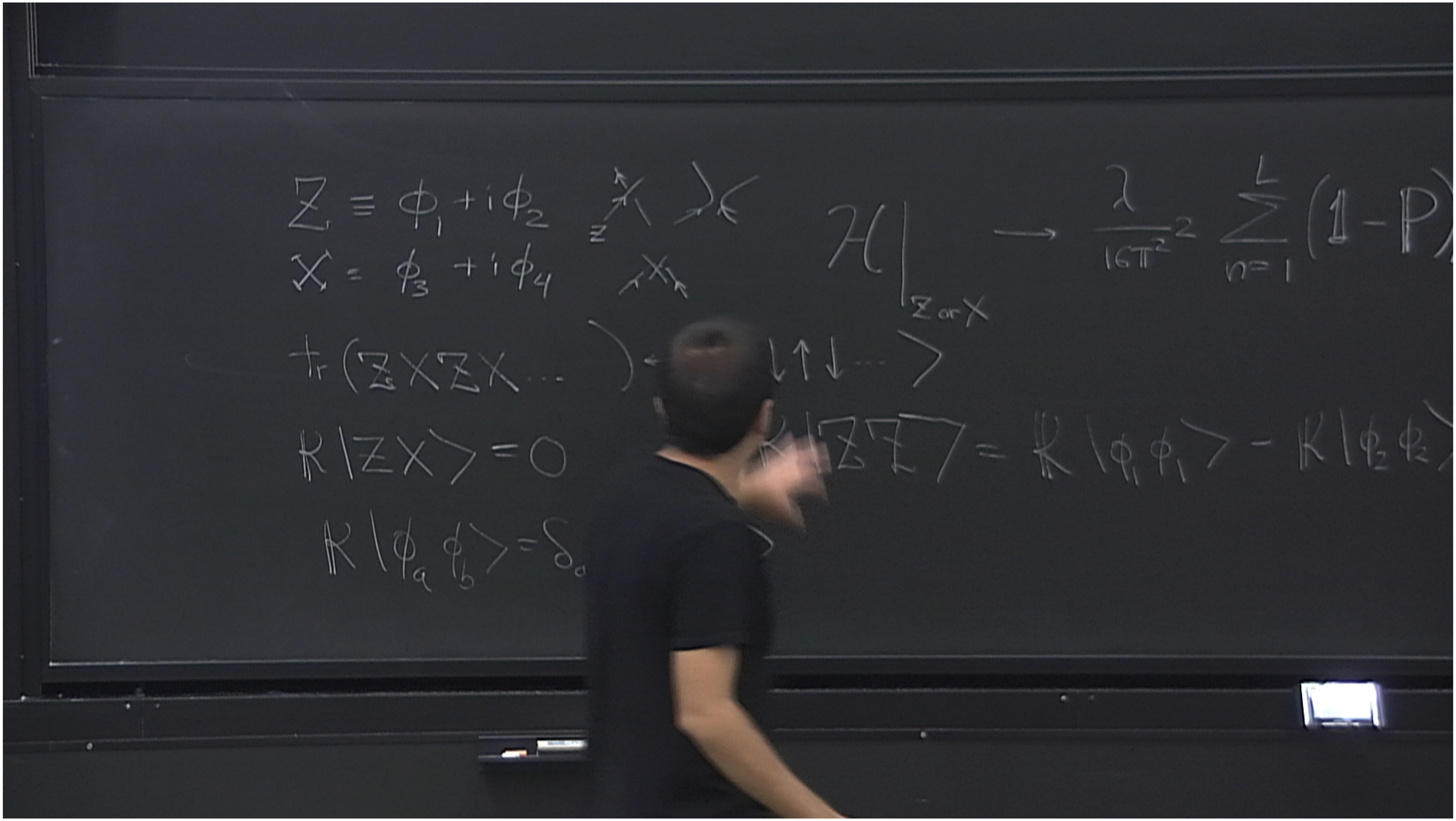
$$X = \phi_3 + i\phi_4$$

$$\mathcal{H}\Big|_{Z \text{ or } X} \longrightarrow \frac{\lambda}{16\pi^2}2 \sum_{n=1}^{L} (1 - P)_{n\,n+1}$$

$$\mathrm{tr}(ZXZX\ldots) \longleftrightarrow |\uparrow\downarrow\uparrow\downarrow\cdots\rangle$$

$$K|ZX\rangle = 0 \quad , K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$

$$K|\phi_a\phi_b\rangle = \delta_{ab}\sum_{c=1}^{6}|cc\rangle$$



$Z \equiv \phi_1 + i\phi_2$

$X = \phi_3 + i\phi_4$

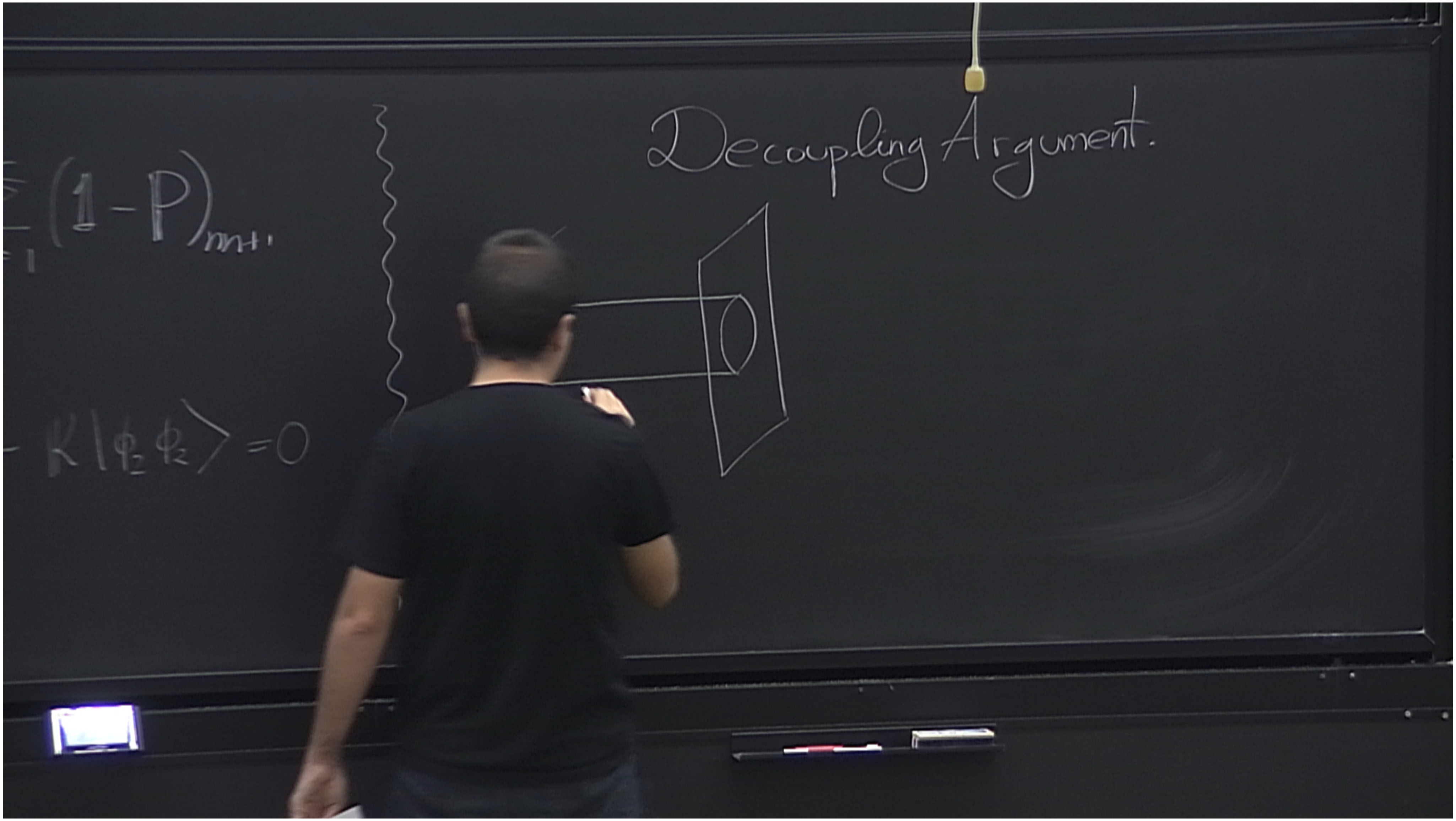
$\mathcal{H}\Big|_{Z \text{ or } X} \longrightarrow \frac{\lambda}{16\pi^2}2 \sum_{n=1}^{L} (\mathbb{1} - \mathbb{P})$

$\mathrm{tr}(ZXZX\ldots) \leftarrow$ $\downarrow\uparrow\downarrow\ldots\rangle$

$\mathbb{K}|ZX\rangle = 0$

$\mathbb{K}|ZZ\rangle = \mathbb{K}|\phi_1\phi_1\rangle - \mathbb{K}|\phi_2\phi_2\rangle$

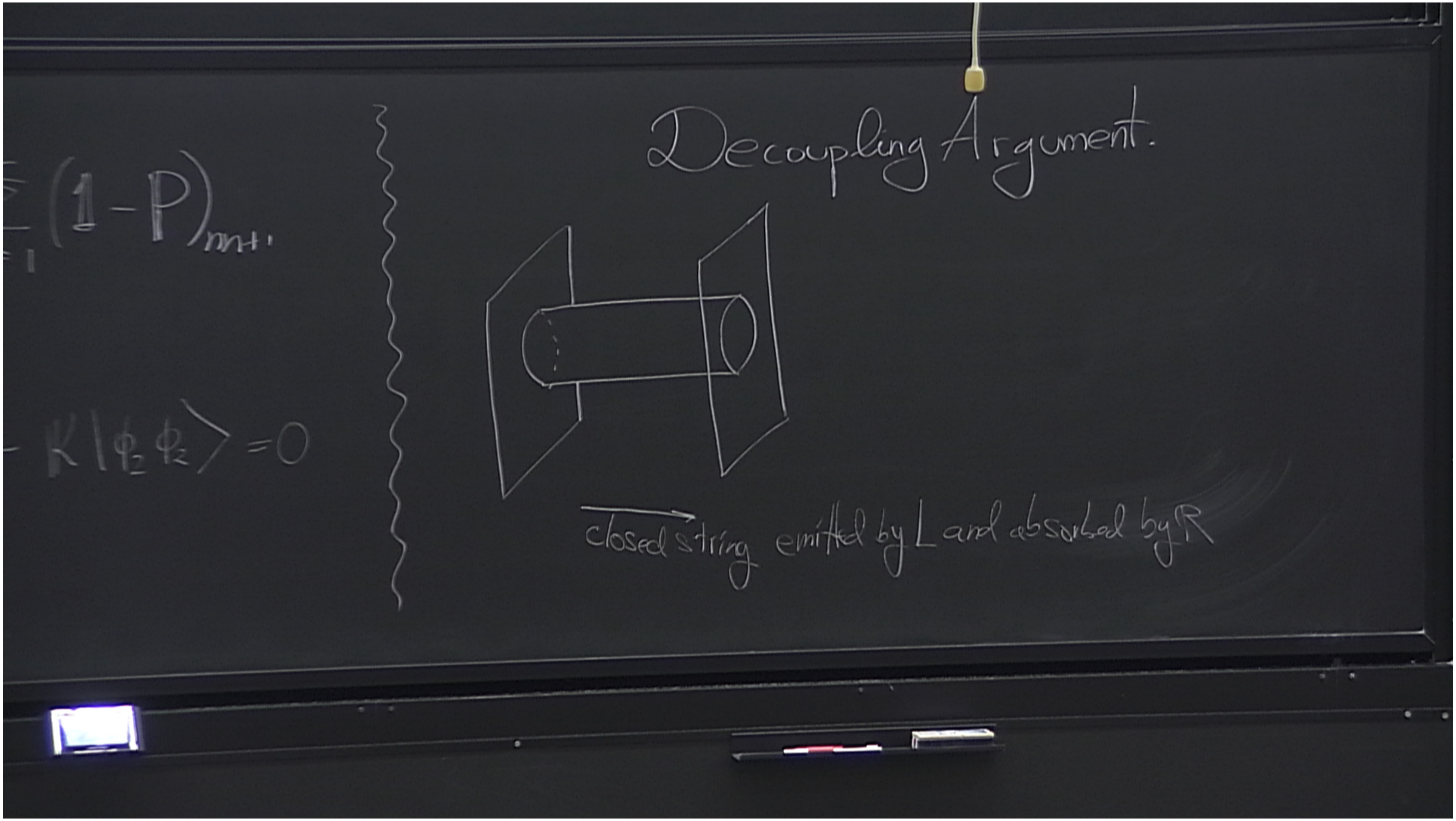
$\mathbb{K}|\phi_a\phi_b\rangle = \delta_a$



Decoupling Argument.

$\sum_{=1} (\mathbb{1}-\mathbb{P})_{m+1}$

$K|\phi_2\phi_2\rangle = 0$



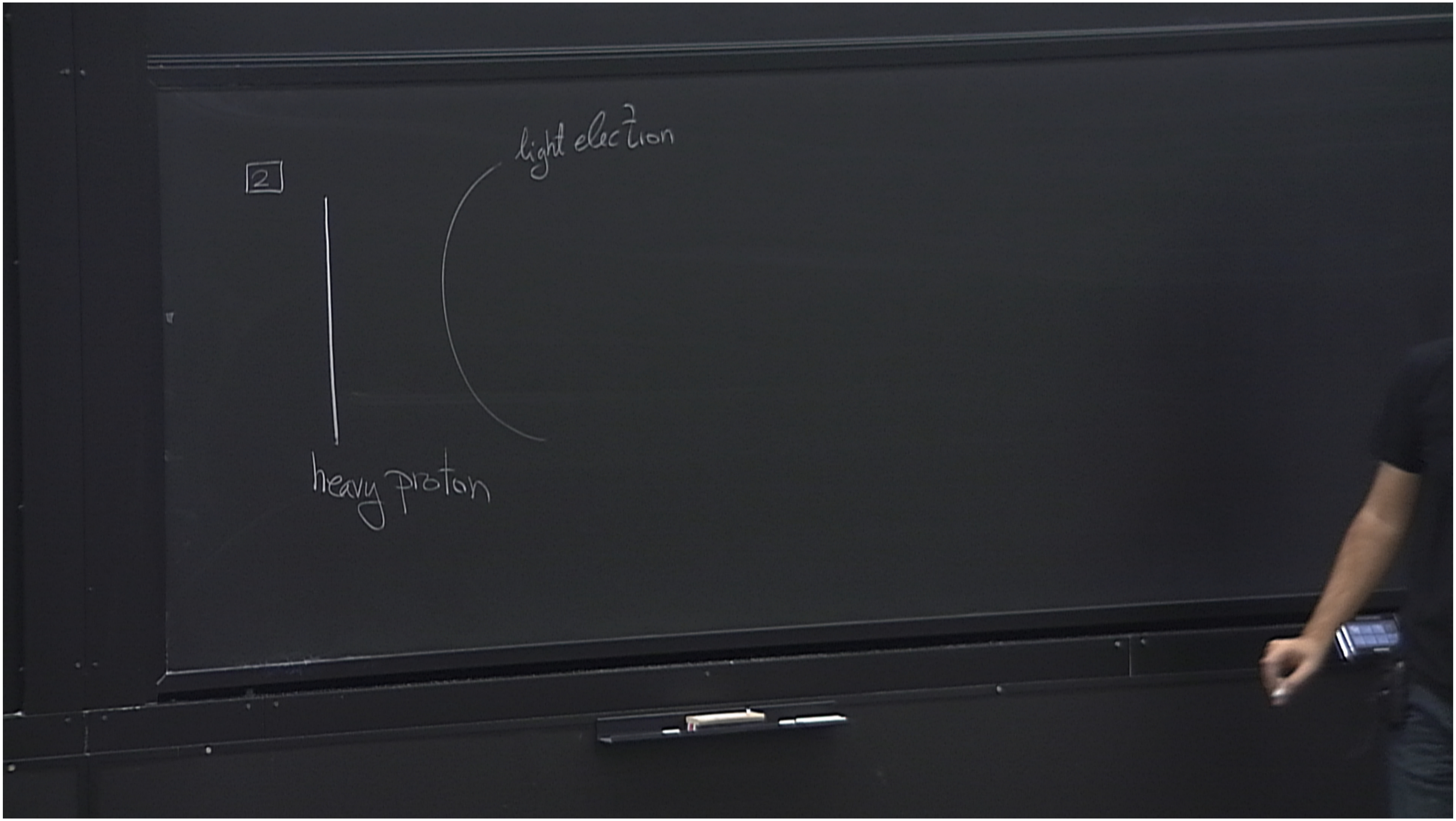
$\sum_{=1}(\mathbb{1}-P)_{nn+1}$

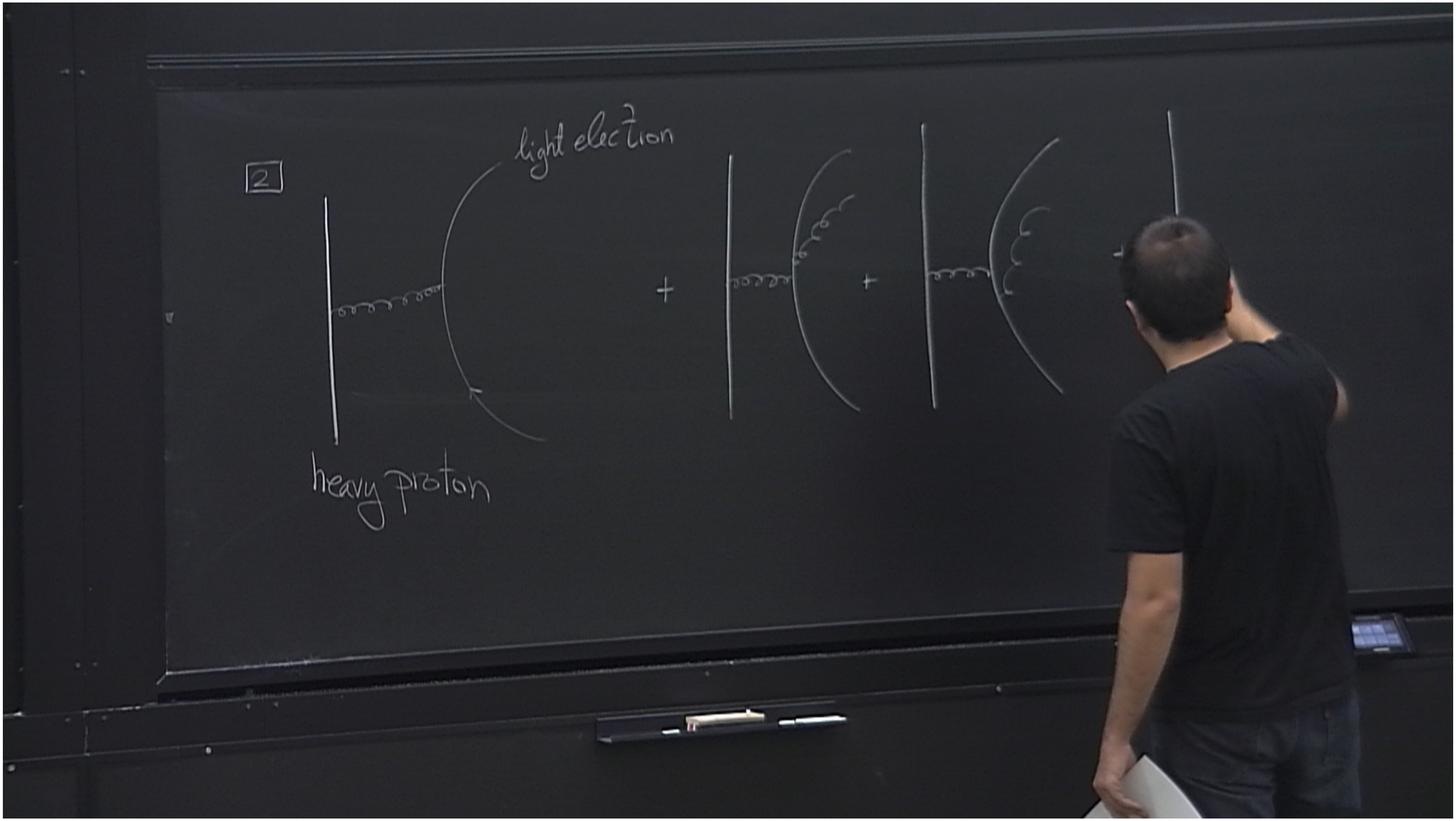
$-K|\phi_2\phi_2\rangle=0$

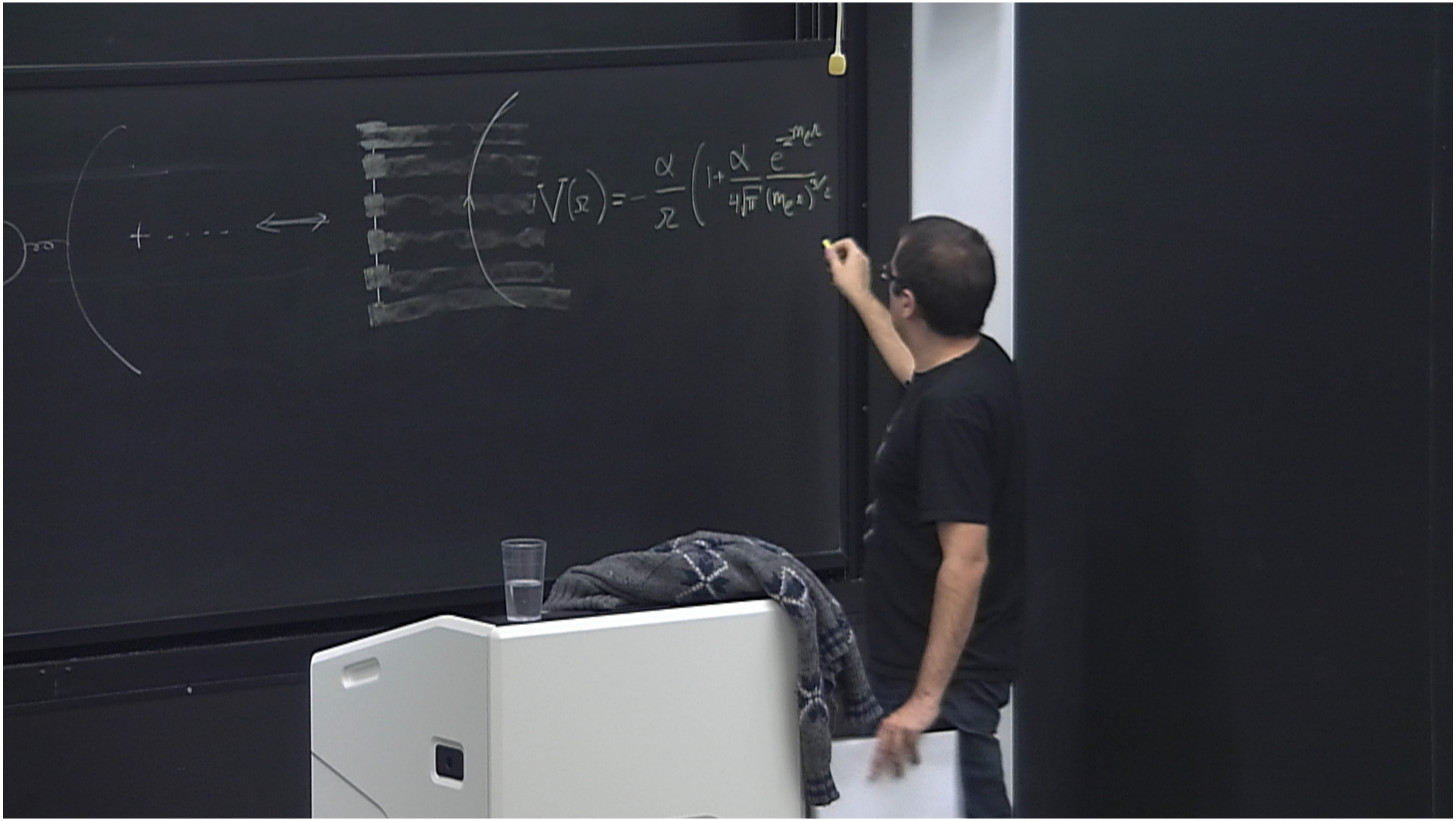
Decoupling Argument.

$\xrightarrow{\hspace{2cm}}$

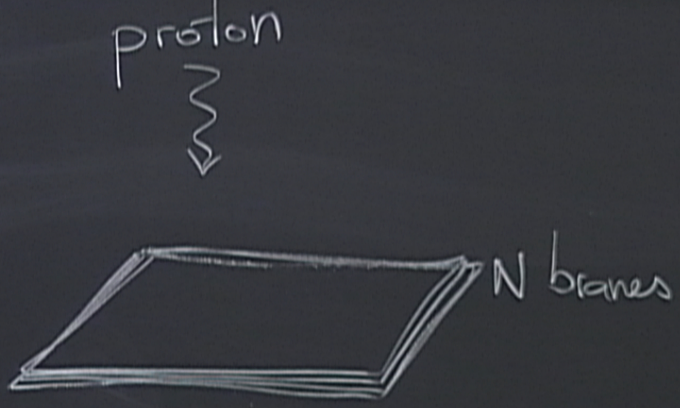
closed string emitted by L and absorbed by R

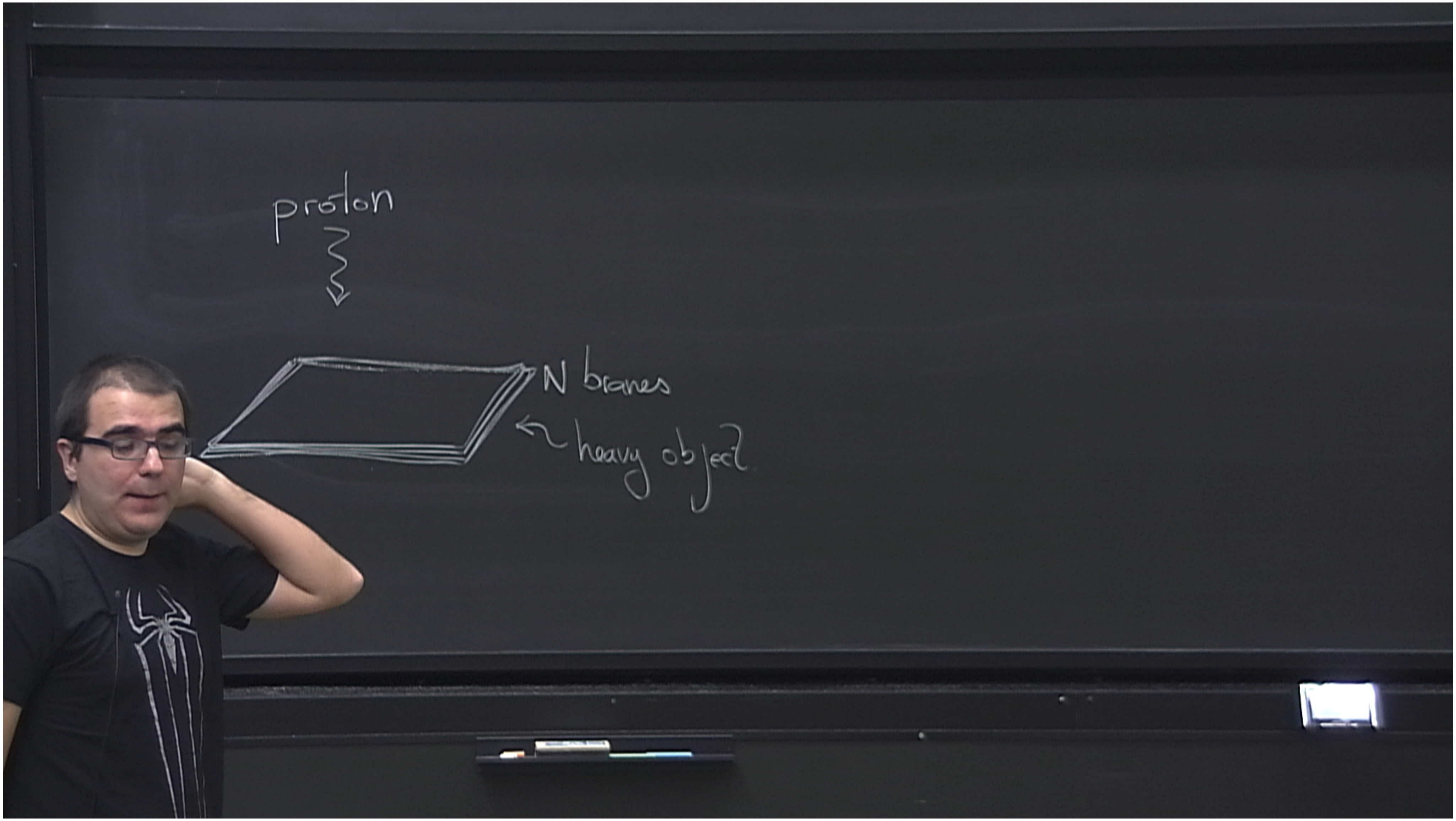

2
light electron
heavy proton

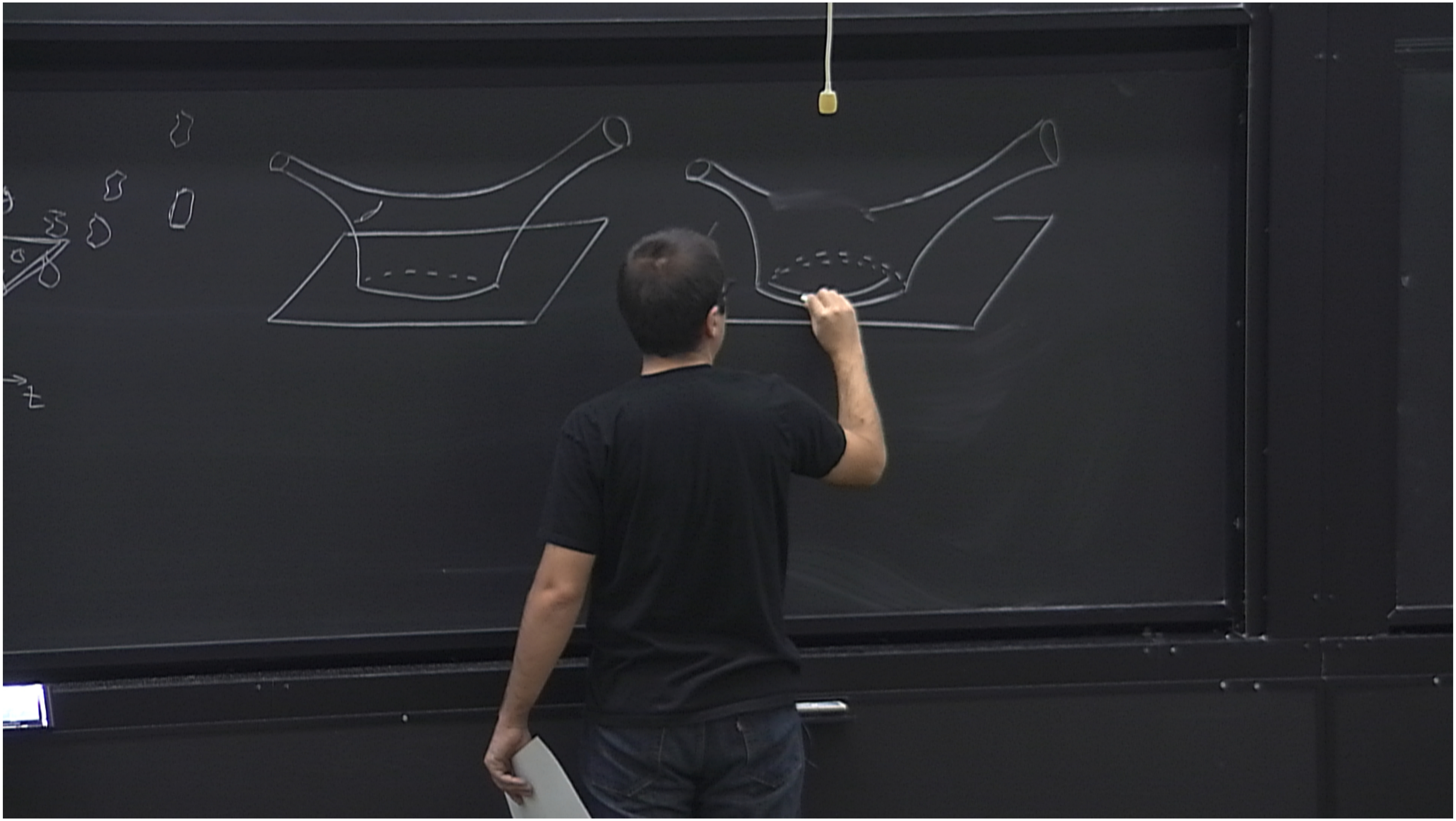

2
light electron
heavy proton
+
+
+

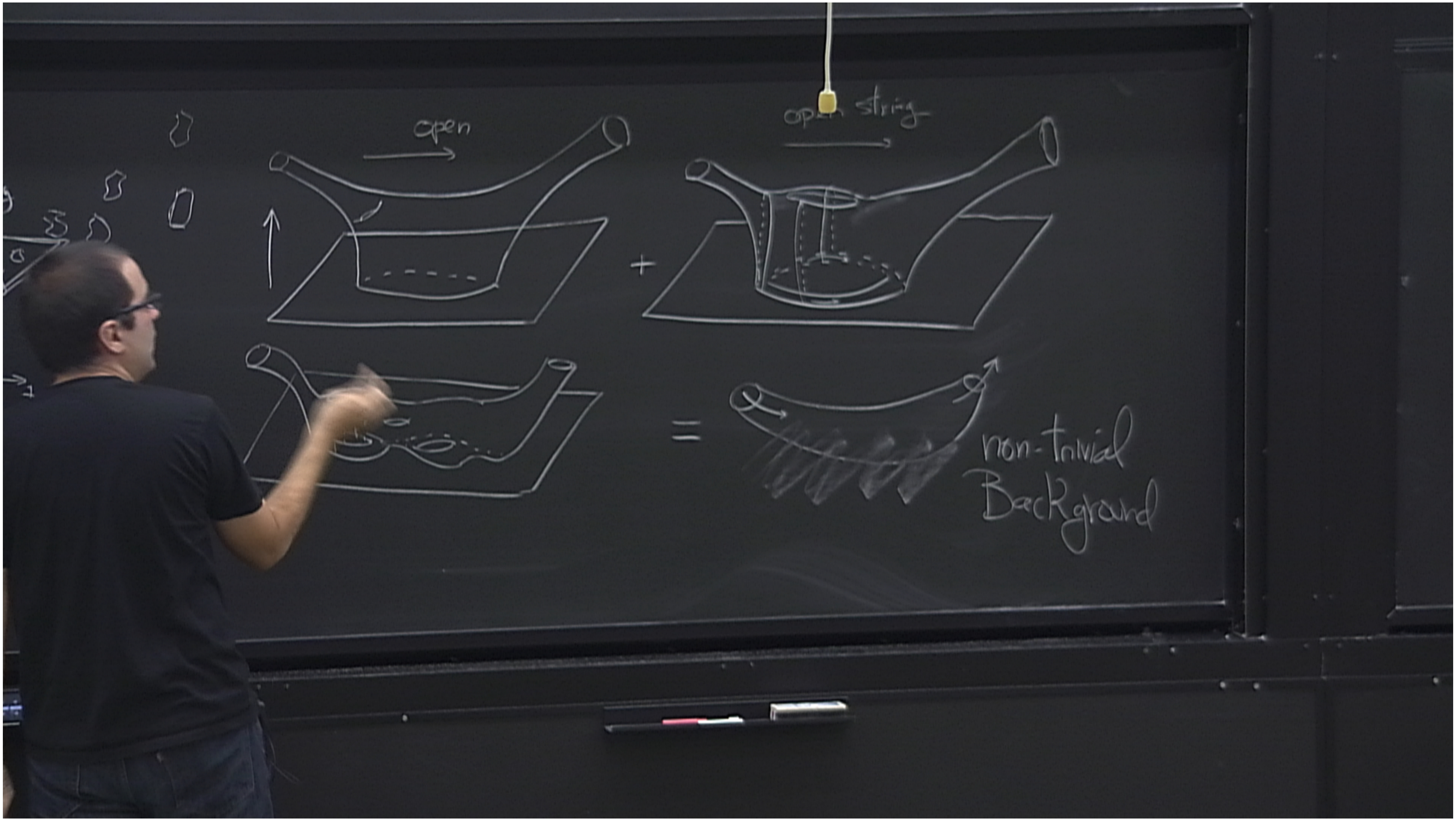


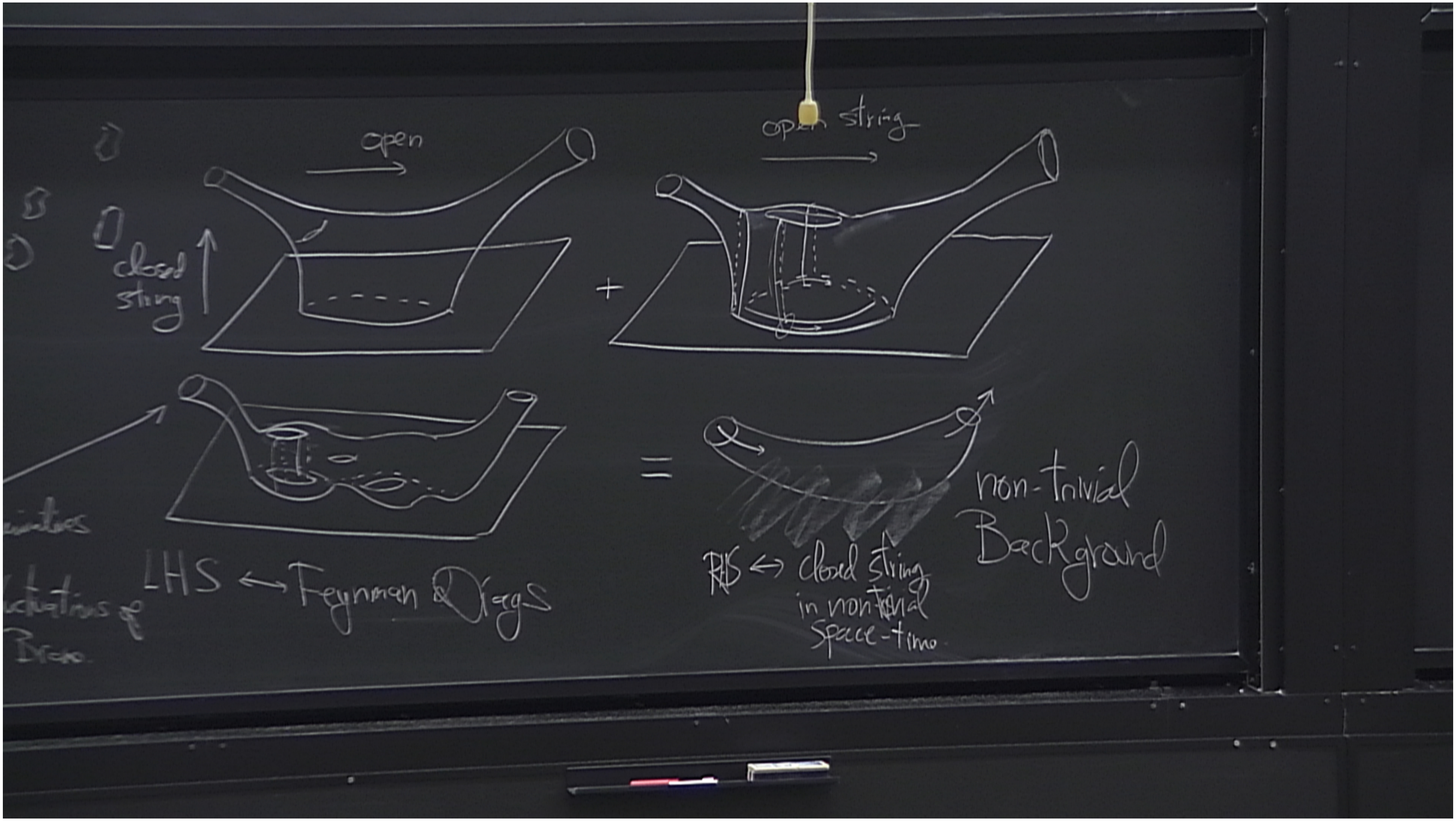
$$V(r) = -\frac{\alpha}{r}\left(1+\frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2m_e r}}{(m_e r)^{3/2}}\right.$$

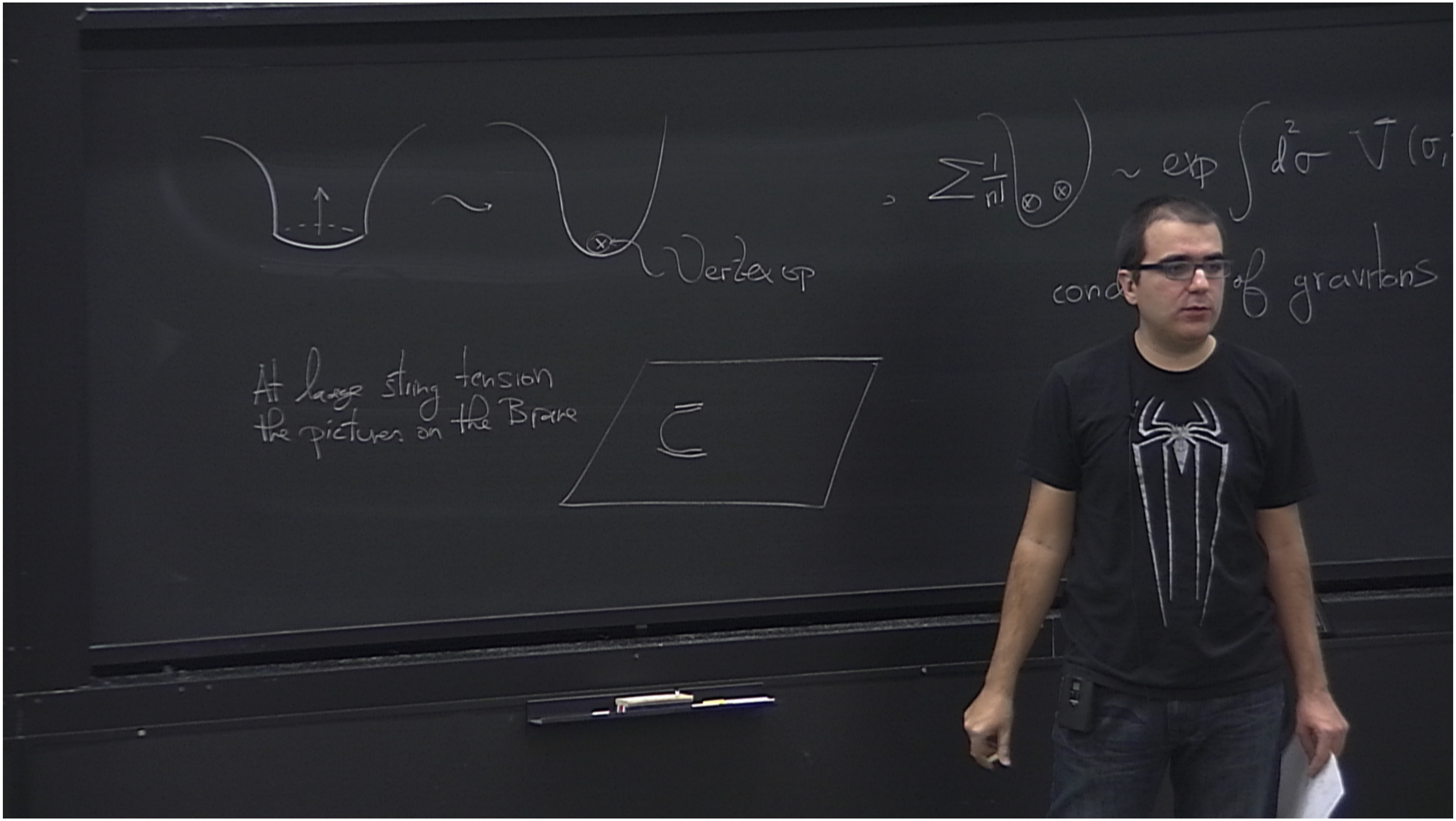

proton
N branes


proton
N branes
heavy object?




open
open string
+
=
non-trivial
Background

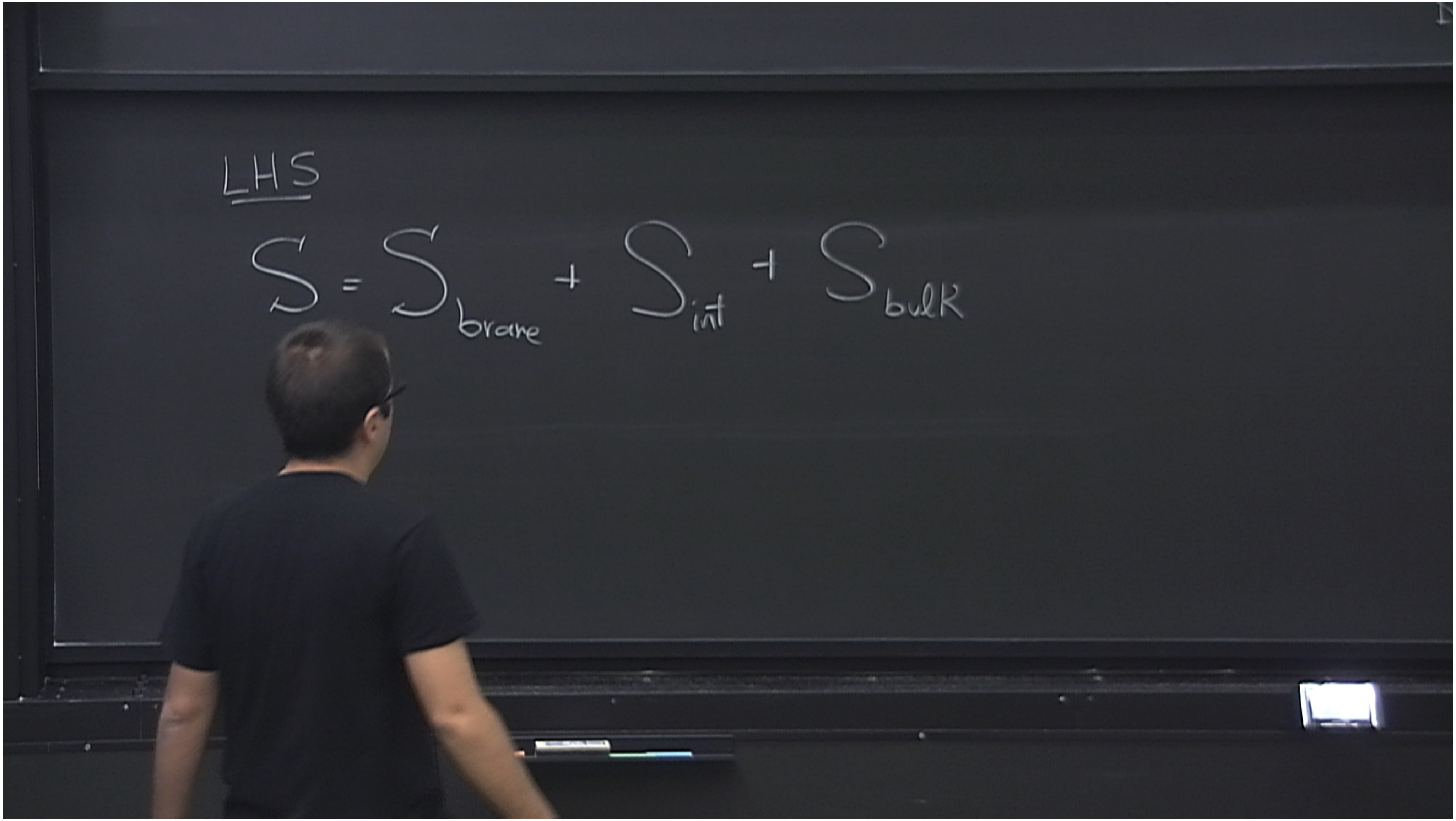

open
closed string
+
open string
=
non-trivial Background
LHS ↔ Feynman Diags
RHS ↔ closed string in nontrivial Space-time



$\sum \frac{1}{n!}$ $\sim \exp \int d^2\sigma \; V(\sigma,$

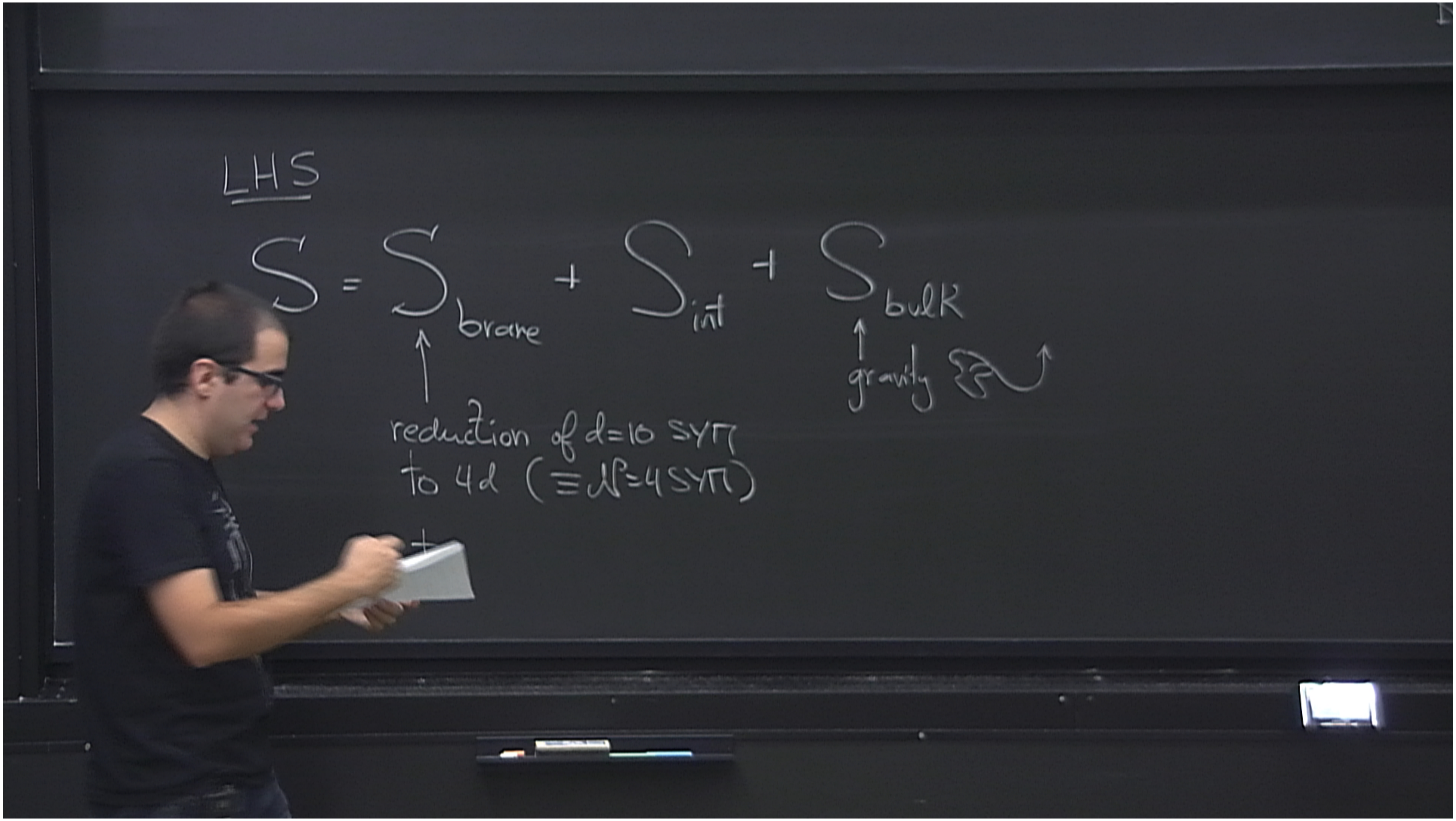
cond of gravitons

At large string tension
the picture on the Brane



LHS

$$S = S_{brane} + S_{int} + S_{bulk}$$



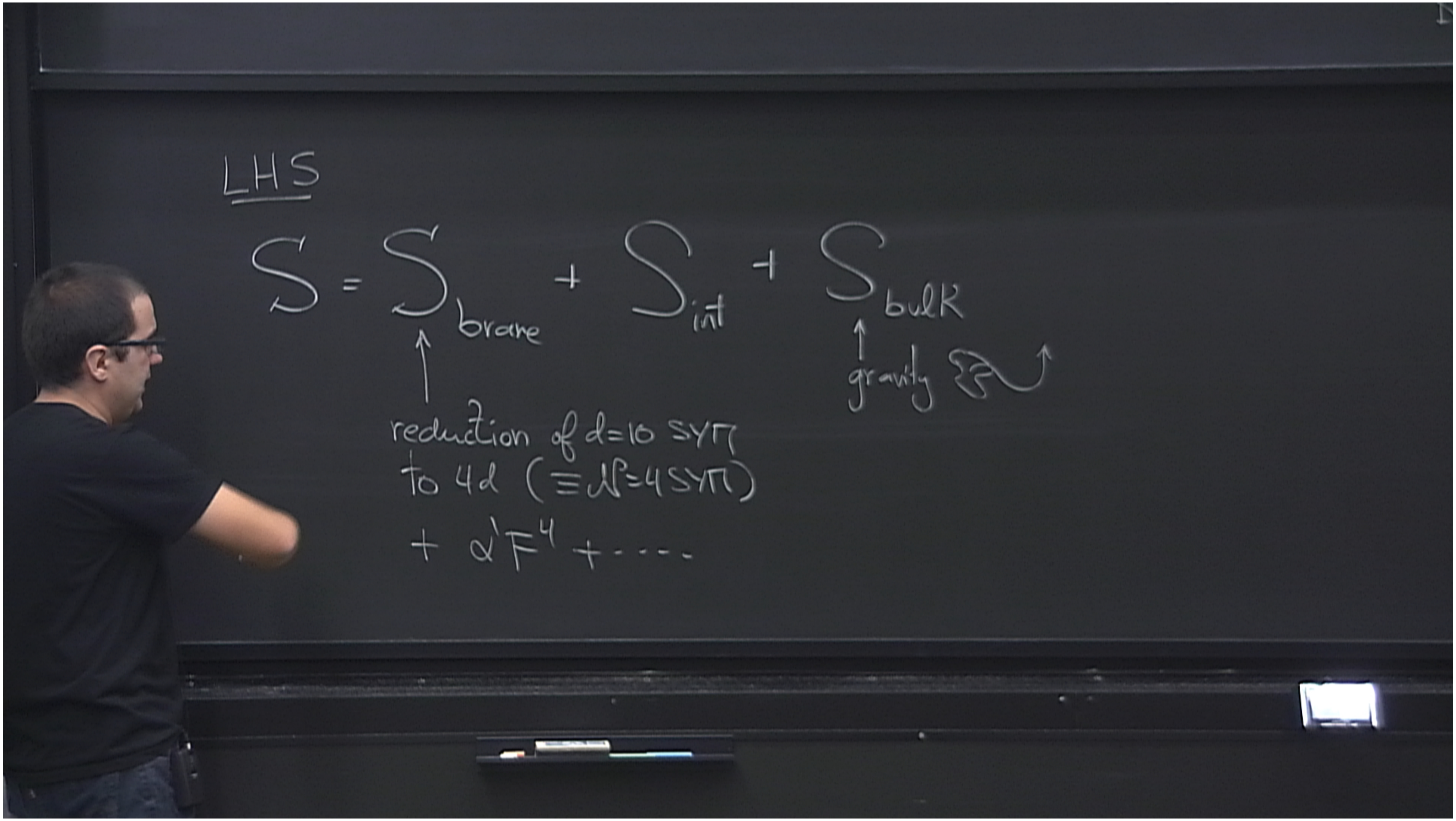
LHS

$$S = S_{brane} + S_{int} + S_{bulk}$$

reduction of $d=10$ SYM to 4d ($\equiv \mathcal{N}=4$ SYM)

gravity

+

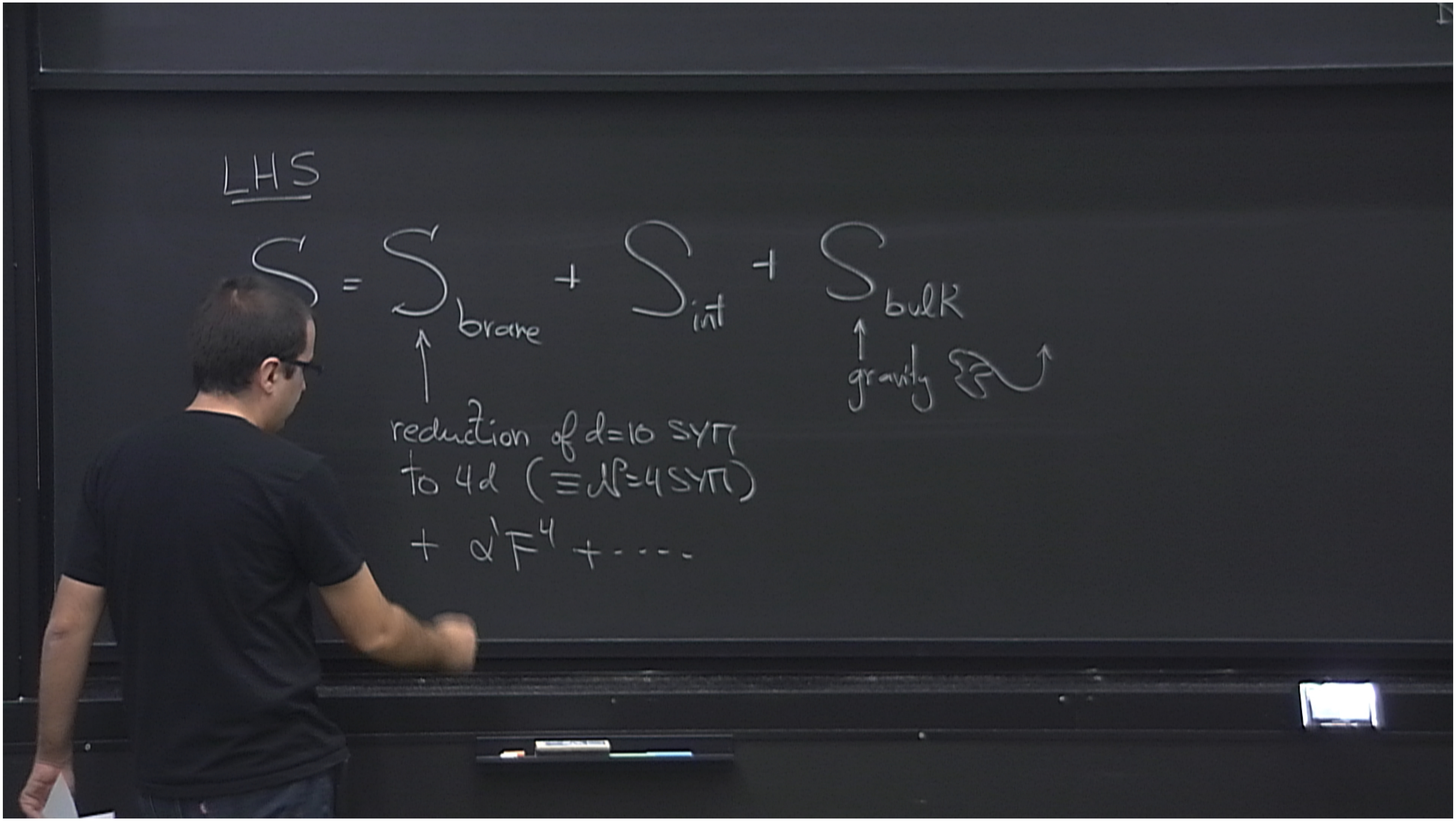


<u>LHS</u>

$$S = S_{brane} + S_{int} + S_{bulk}$$

$\uparrow$ reduction of $d=10$ SYM to $4d$ ($\equiv \mathcal{N}=4$ SYM) $+ \alpha' F^4 + \cdots$

$\uparrow$ gravity

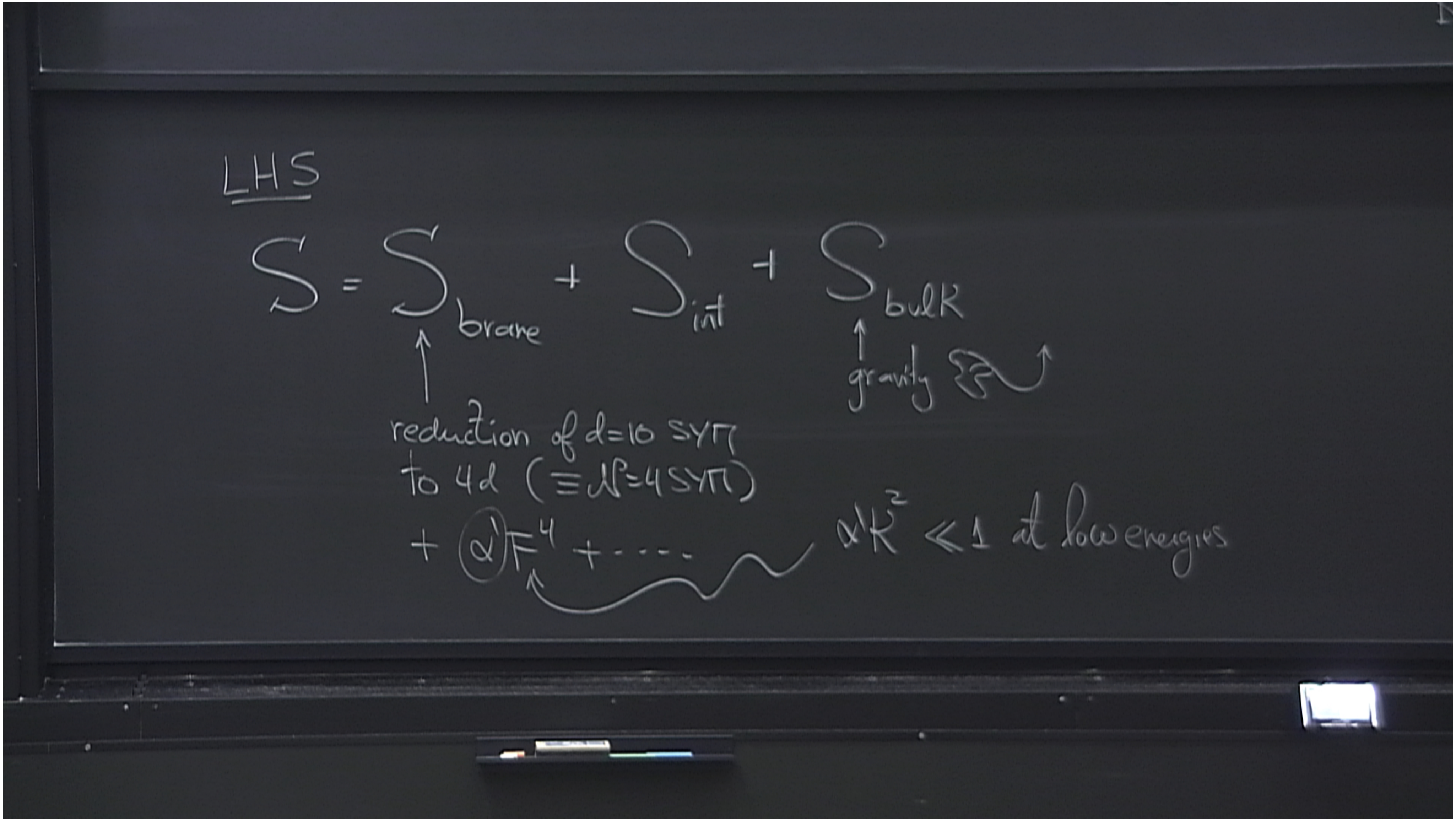


LHS

$$S = S_{brane} + S_{int} + S_{bulk}$$

S_{brane}: reduction of $d=10$ SYM to 4d ($\equiv \mathcal{N}=4$ SYM) $+ \alpha' F^4 + \cdots$

S_{bulk}: gravity



LHS

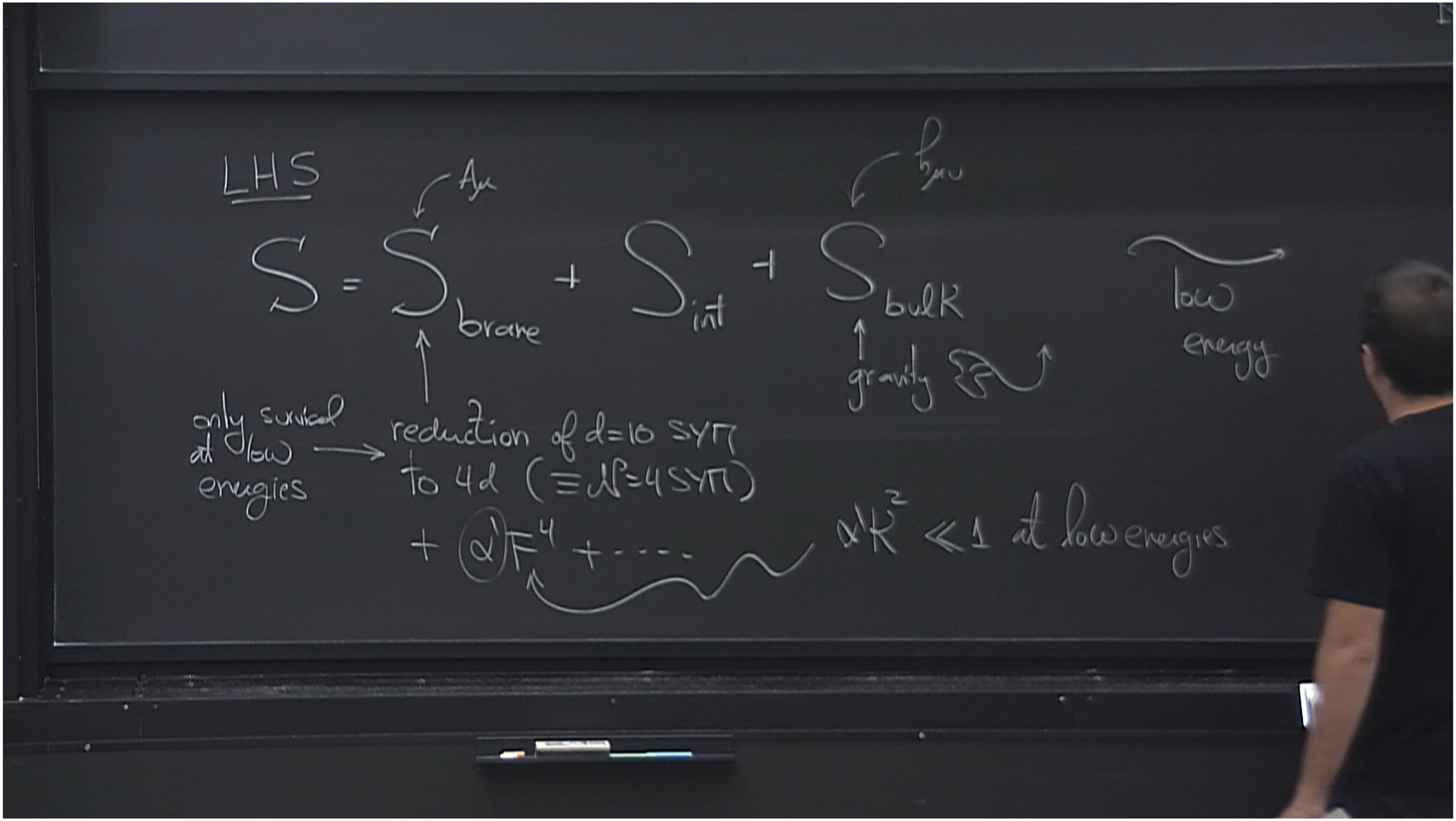
$$S = \underset{\uparrow}{S_{brane}} + S_{int} + \underset{\uparrow}{S_{bulk}}$$

gravity

reduction of d=10 SYM
to 4d ($\equiv \mathcal{N}=4$ SYM)

$+ \ \alpha' F^4 + \cdots$

$\alpha' k^2 \ll 1$ at low energies



LHS

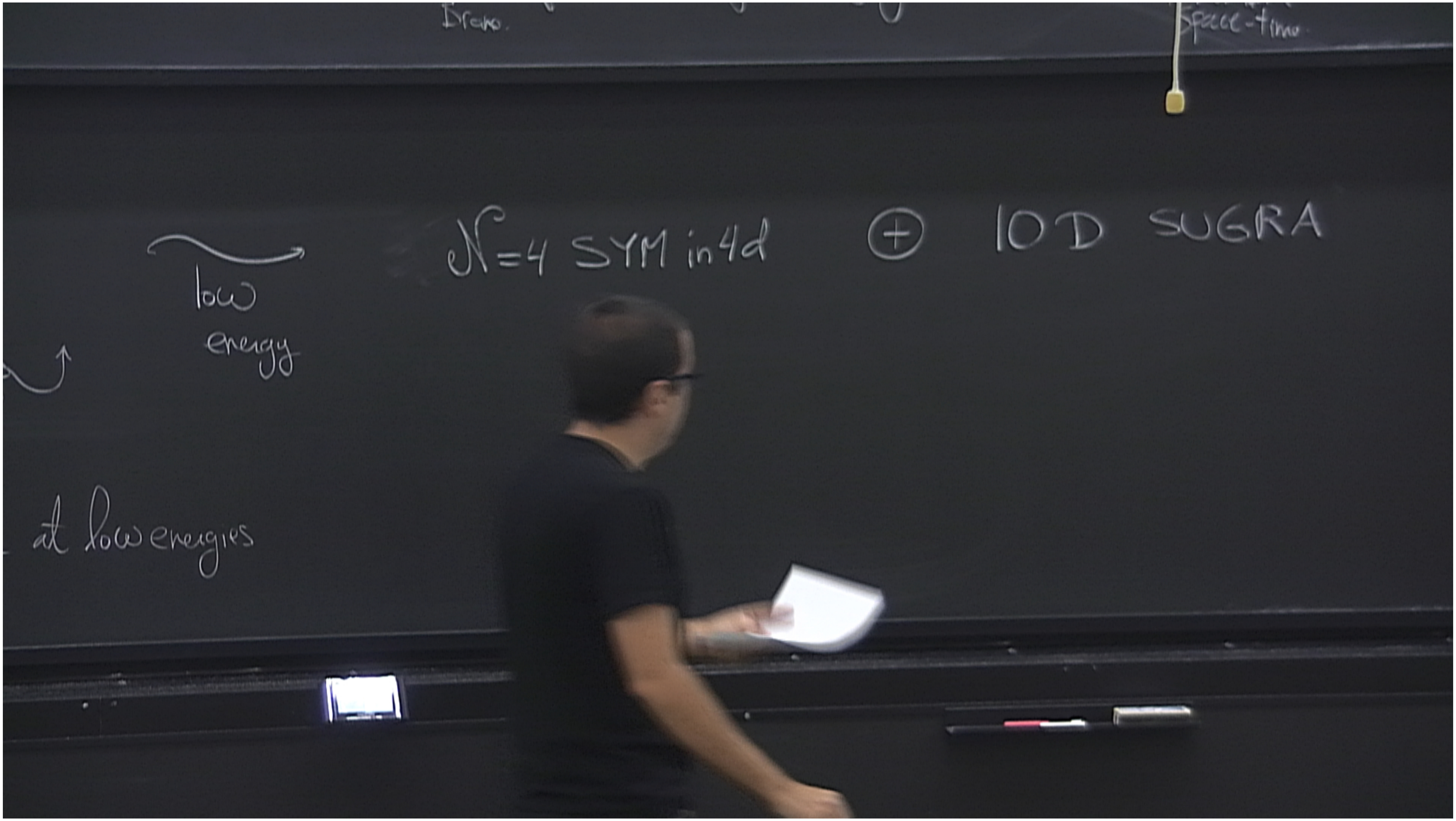
$$S = \overset{A_\mu}{S}_{brane} + S_{int} + \overset{h_{\mu\nu}}{S}_{bulk} \xrightarrow[\text{low energy}]{}$$

gravity

only survived at low energies → reduction of $d=10$ SYM to 4d ($\equiv \mathcal{N}=4$ SYM)

$+ \boxed{\alpha'} F^4 + \cdots$

$\alpha' k^2 \ll 1$ at low energies

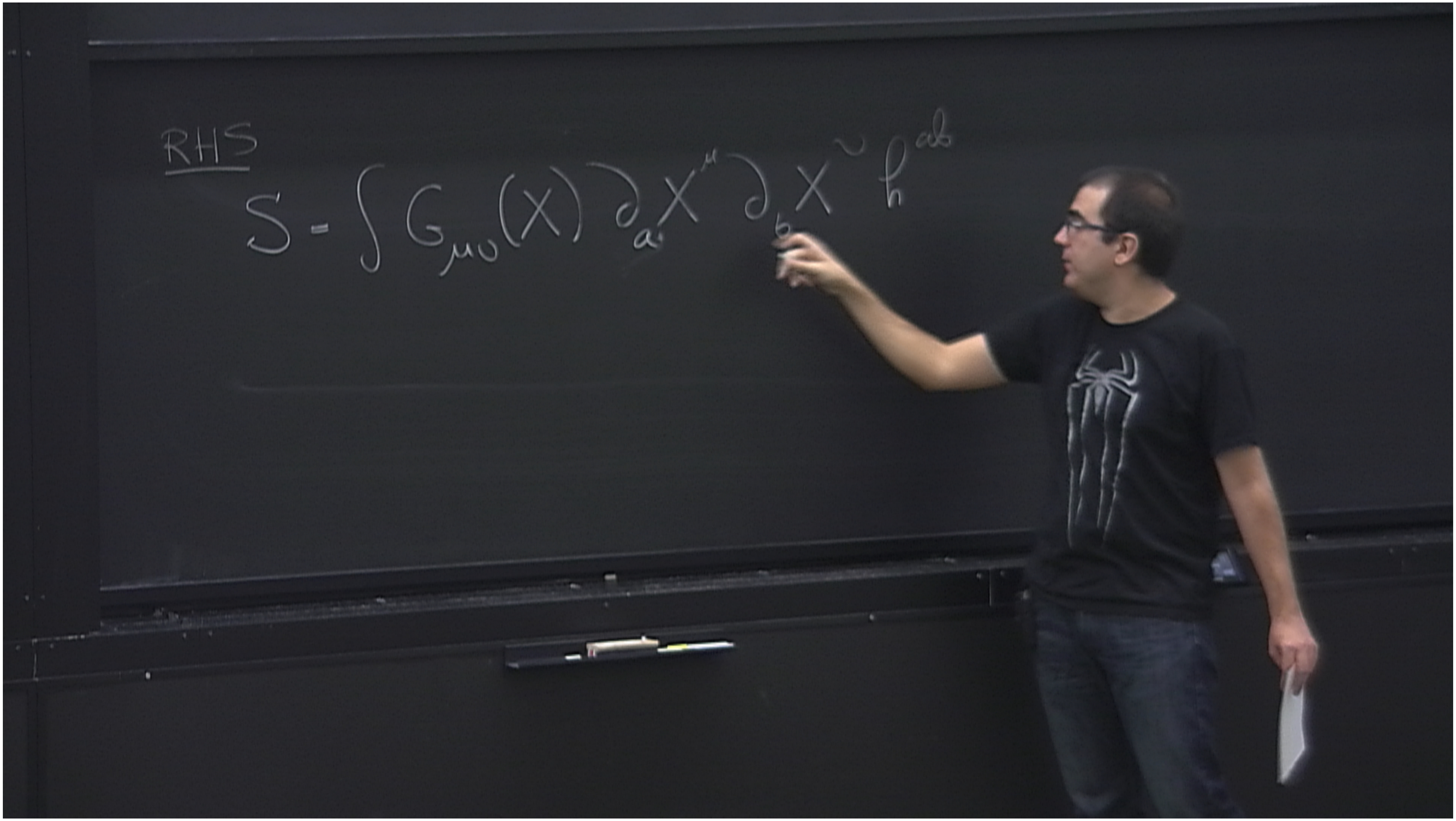


Brane.

Space-time.

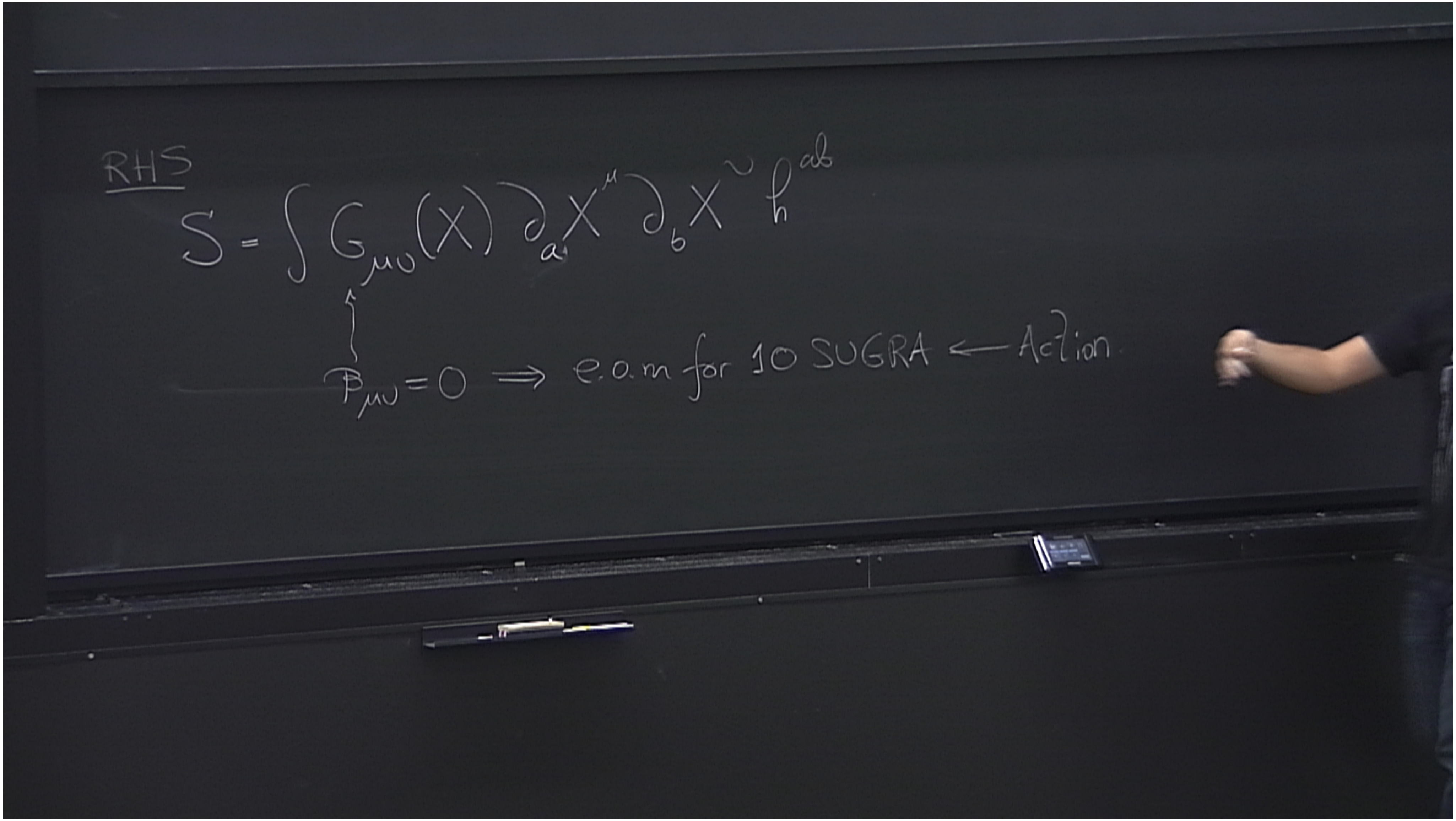
$\xrightarrow[\text{low energy}]{}$ $\mathcal{N}=4$ SYM in 4d $\oplus$ 10D SUGRA

at low energies



RHS

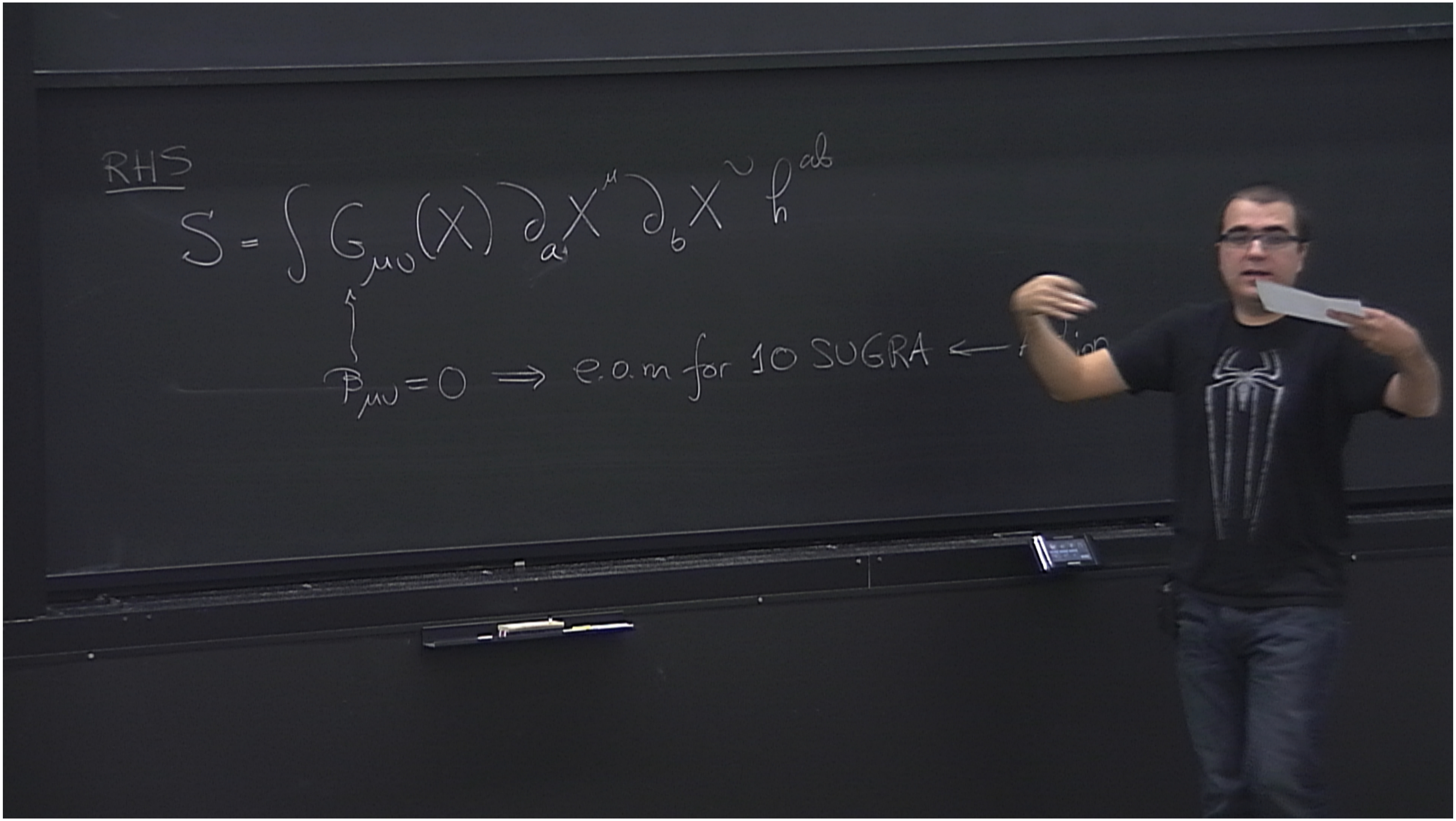
$$S = \int G_{\mu\nu}(X)\,\partial_a X^\mu\,\partial_b X^\nu\,h^{ab}$$



RHS

$$S = \int G_{\mu\nu}(X)\, \partial_a X^\mu\, \partial_b X^\nu\, h^{ab}$$

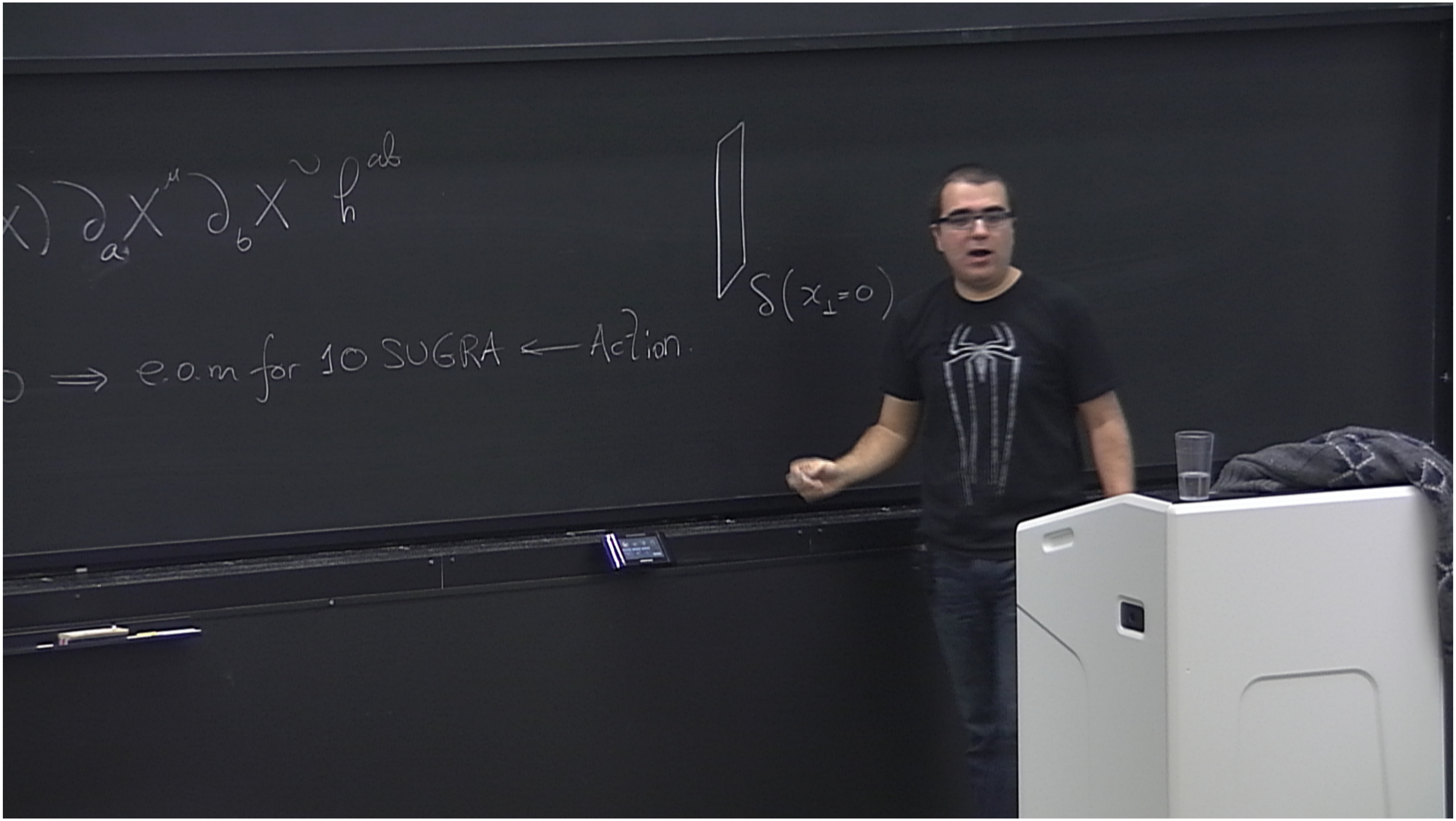
$$\beta_{\mu\nu} = 0 \Rightarrow \text{e.o.m for } 10 \text{ SUGRA} \leftarrow \text{Action.}$$

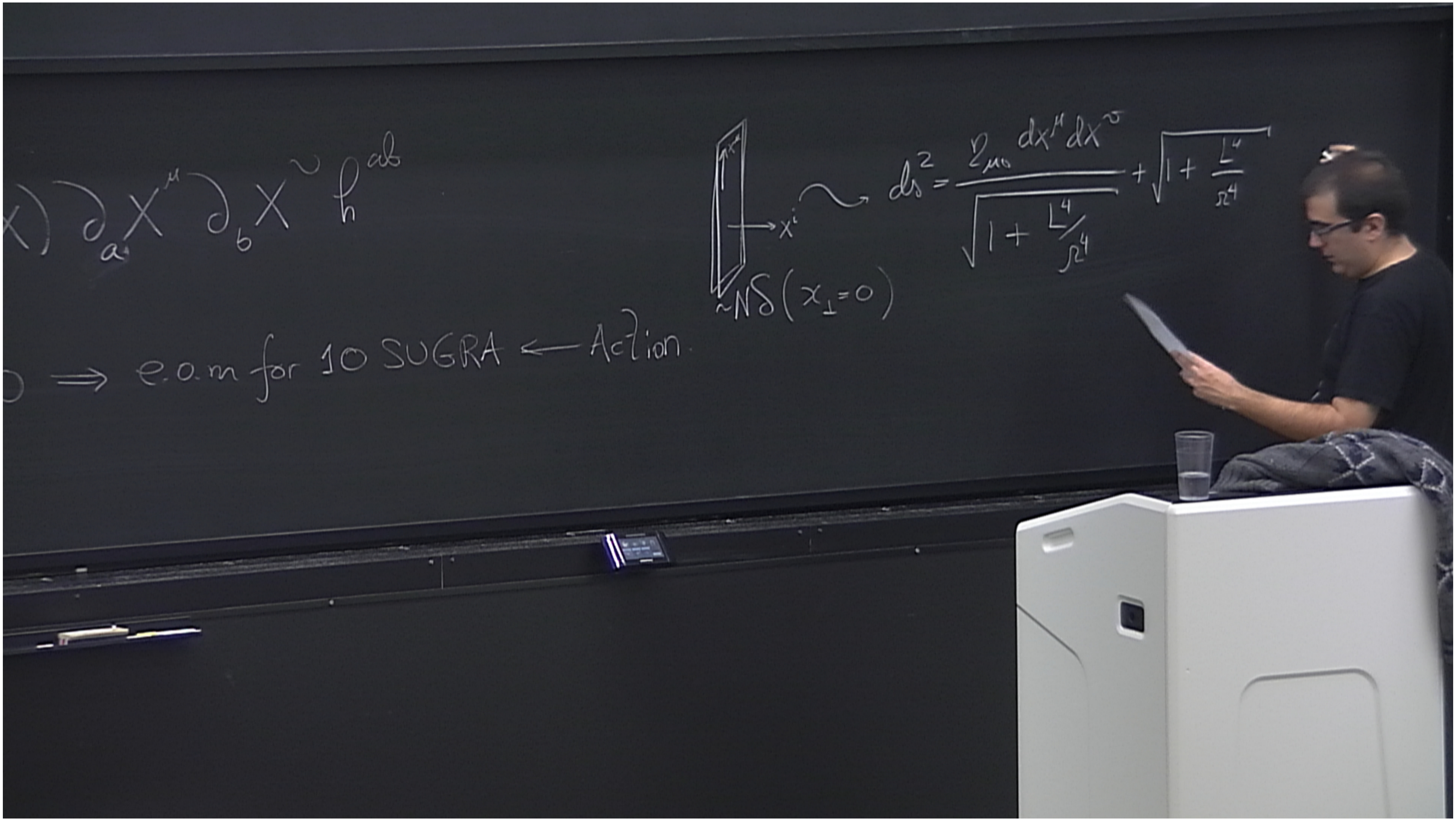


RHS

$$S = \int G_{\mu\nu}(X)\,\partial_a X^\mu \partial_b X^\nu h^{ab}$$

$\beta_{\mu\nu} = 0 \Rightarrow$ e.o.m for 10 SUGRA ←


$x) \partial_a X^\mu \partial_b X^\nu h^{ab}$
$\delta(x_\perp = 0)$
$0 \Rightarrow$ e.o.m for 10 SUGRA ← Action.

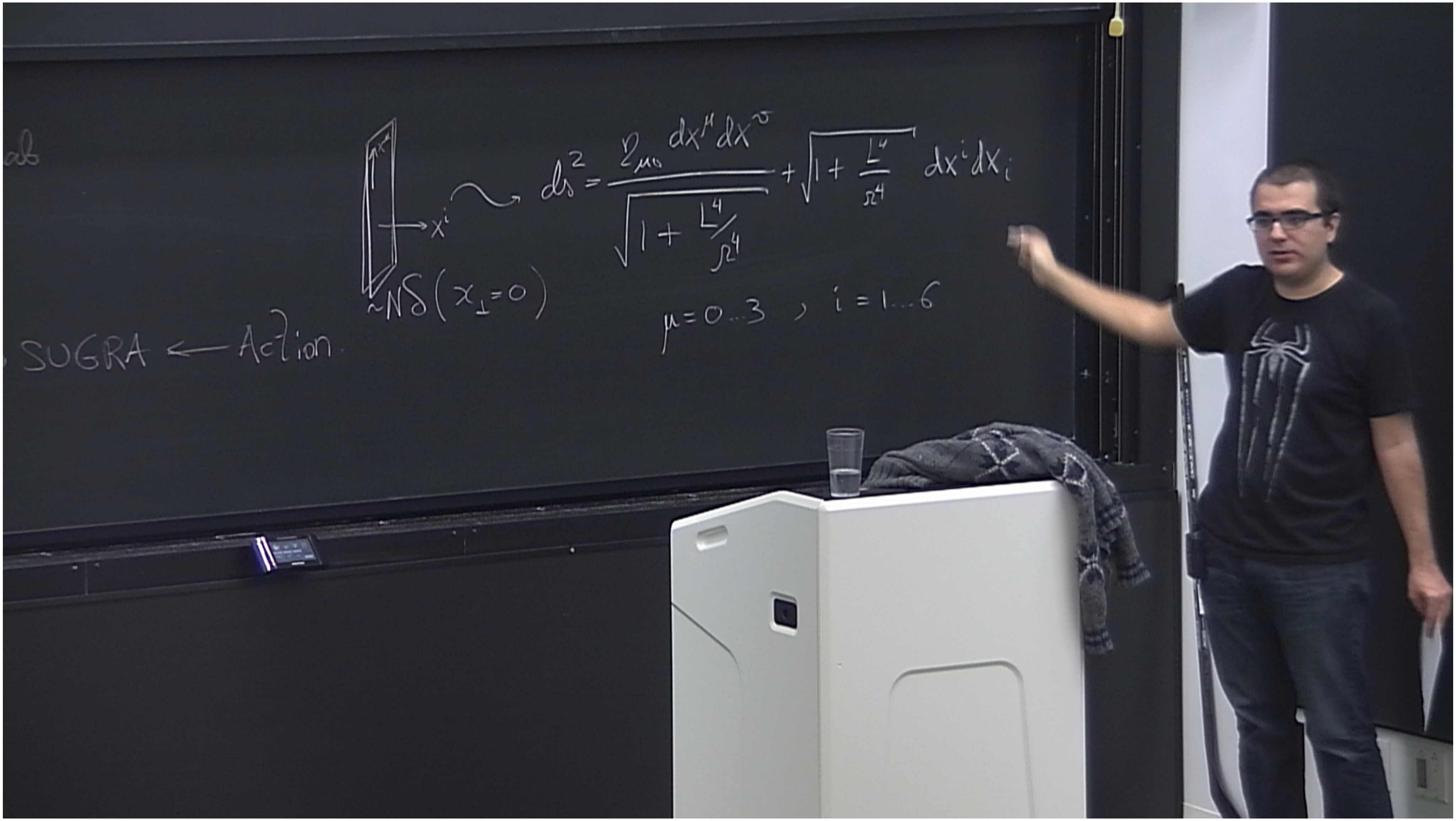


$x)\,\partial_a X^\mu \partial_b X^\nu h^{ab}$

$0 \Rightarrow$ e.o.m for 10 SUGRA $\leftarrow$ Action.

$\tilde{\ } N\delta(x_\perp = 0)$

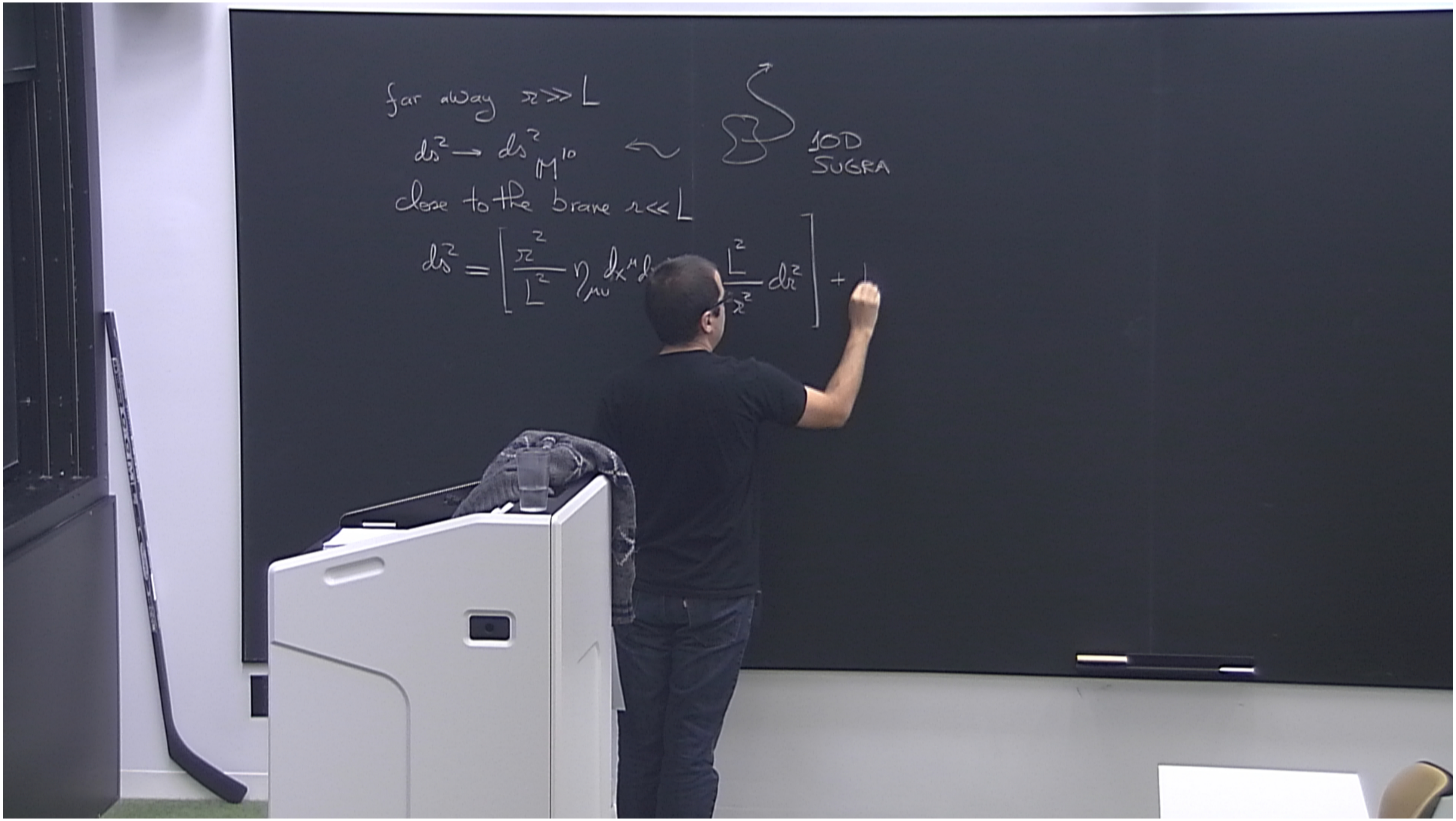
$\tilde{\ } \; ds^2 = \dfrac{\eta_{\mu\nu} dx^\mu dx^\nu}{\sqrt{1 + L^4/r^4}} + \sqrt{1 + \dfrac{L^4}{r^4}}$



$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu}{\sqrt{1 + L^4/r^4}} + \sqrt{1 + \frac{L^4}{r^4}}\, dx^i dx_i$$

$\mu = 0 \ldots 3 \quad , \quad i = 1 \ldots 6$

SUGRA ← Action.

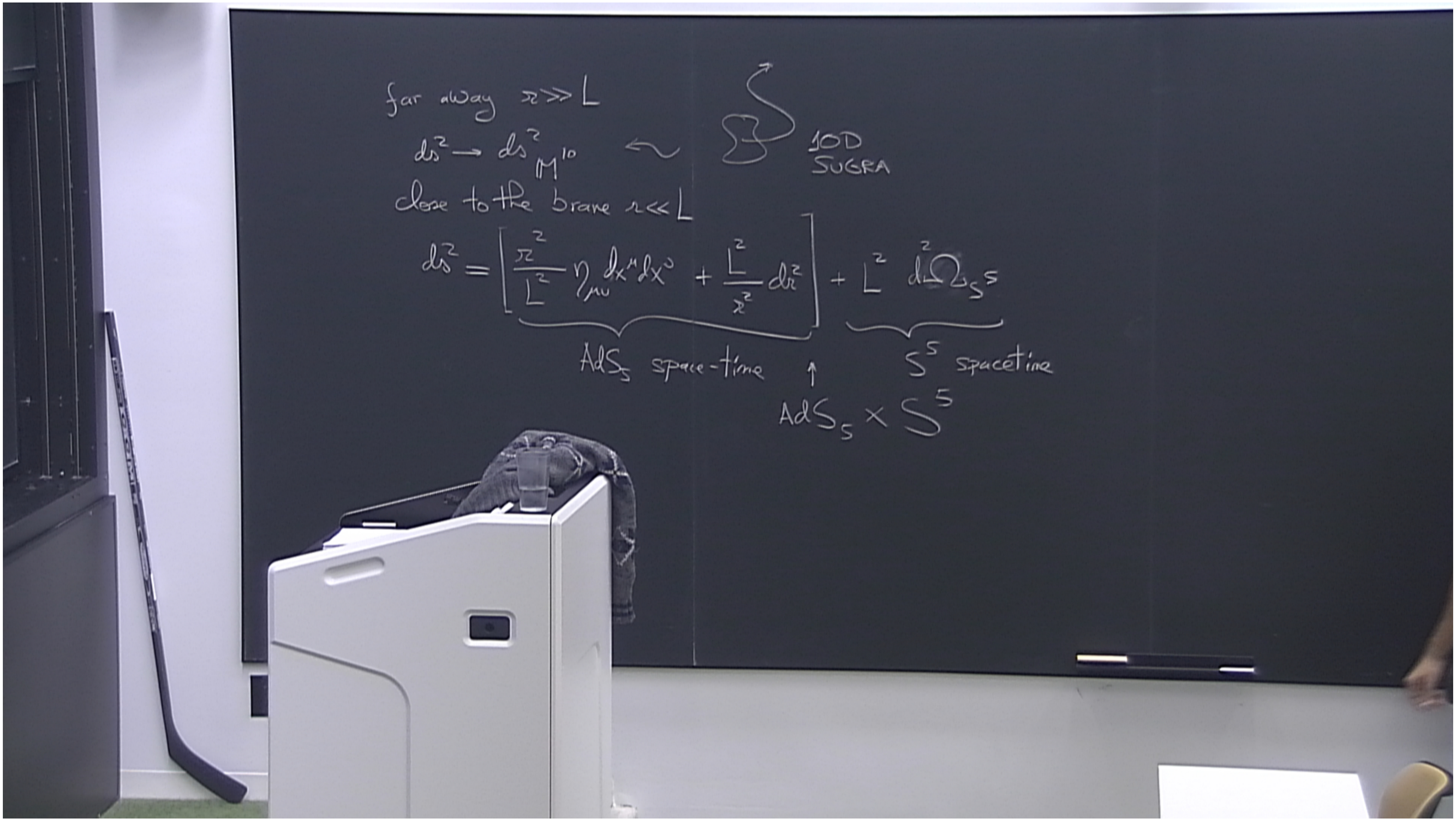


far away $r \gg L$

$ds^2 \to ds^2_{M^{10}}$ $\leftarrow \sim$

close to the brane $r \ll L$

$$ds^2 = \Big[\frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu d \quad \frac{L^2}{r^2}dr^2 \Big] +$$



far away $r \gg L$

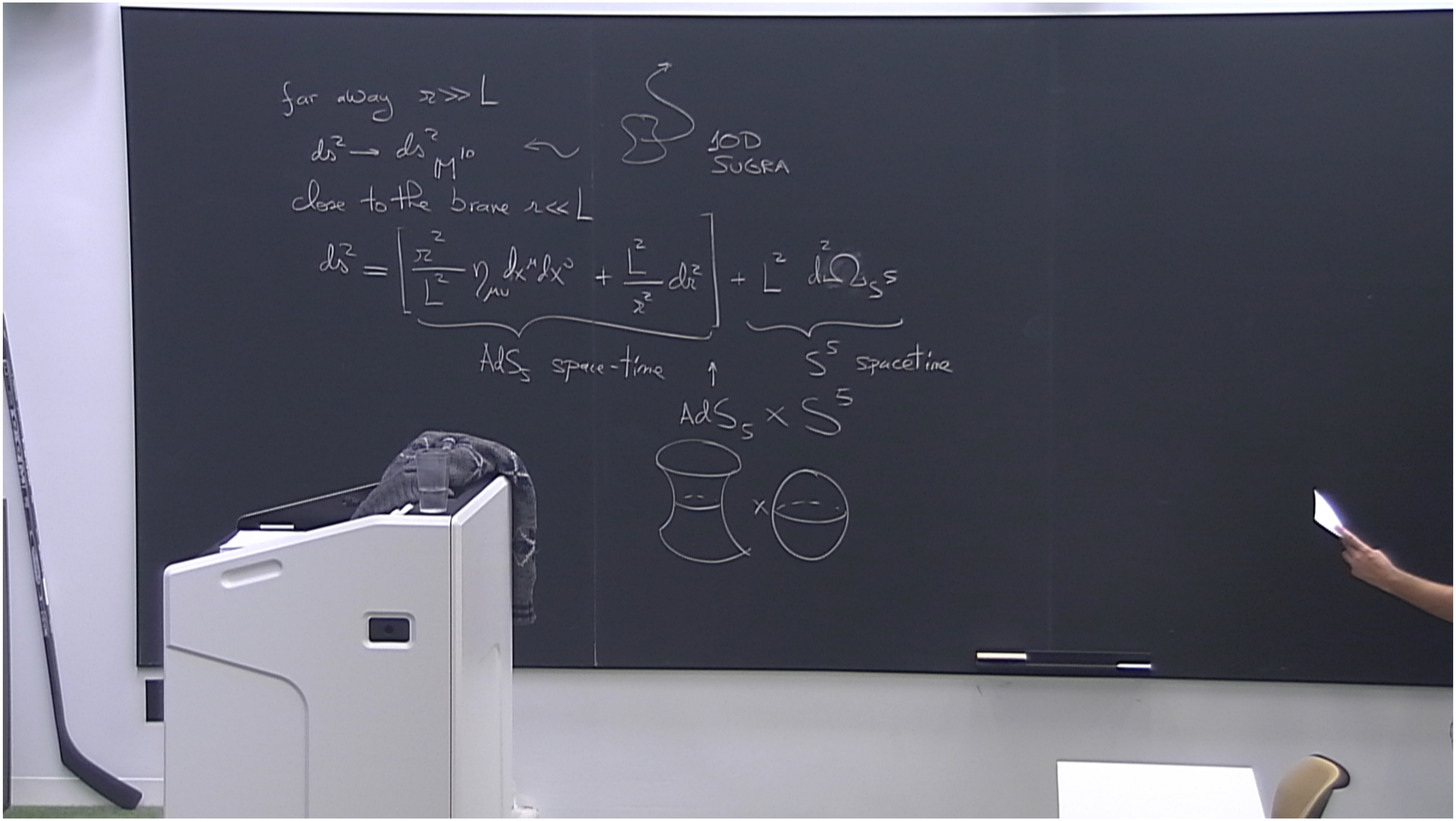
$ds^2 \to ds^2_{M^{10}}$ $\leftsquigarrow$ 10D SUGRA

close to the brane $r \ll L$

$$ds^2 = \underbrace{\left[\frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2\right]}_{AdS_5 \text{ space-time}} + \underbrace{L^2 \, d\Omega^2_{S^5}}_{S^5 \text{ spacetime}}$$

↑

$AdS_5 \times S^5$



far away $r \gg L$

$ds^2 \to ds^2_{M^{10}} \leftsquigarrow$

10D SUGRA

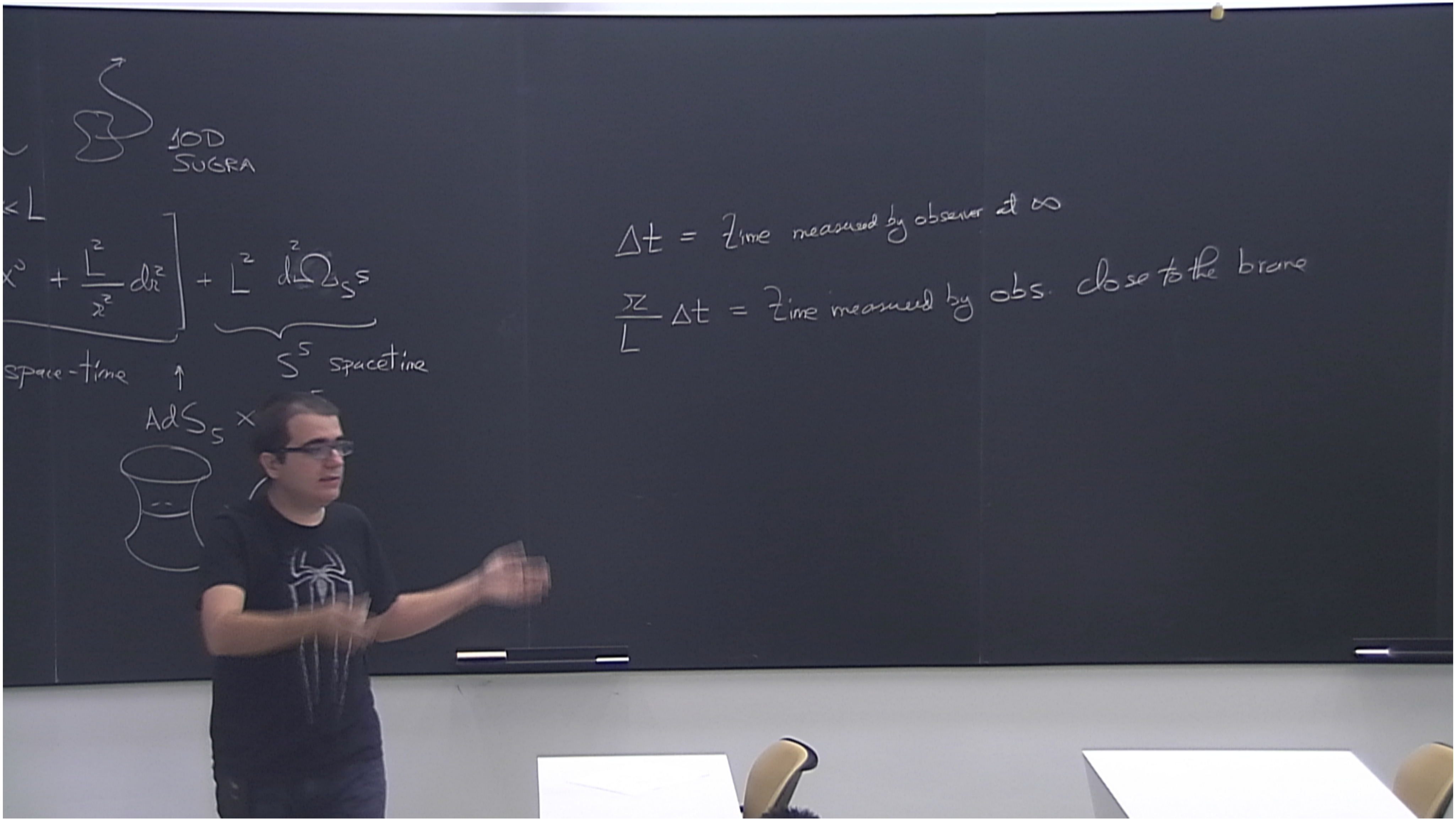
close to the brane $r \ll L$

$$ds^2 = \underbrace{\left[\frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2\right]}_{AdS_5 \text{ space-time}} + \underbrace{L^2\, d\Omega^2_{S^5}}_{S^5 \text{ spacetime}}$$

↑

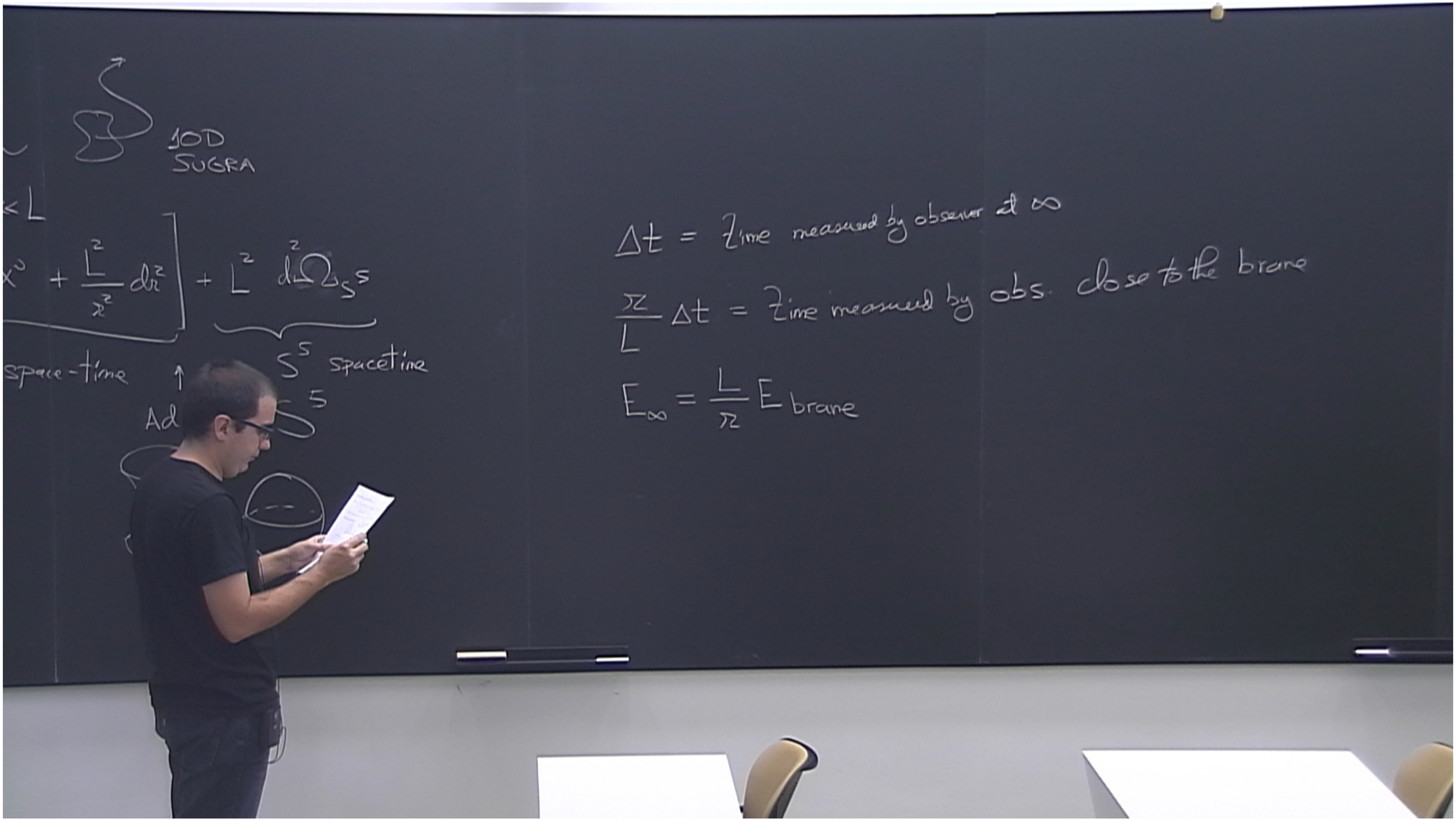
$AdS_5 \times S^5$

×



Δt = time measured by observer at ∞

$\frac{r}{L} \Delta t$ = time measured by obs. close to the brane

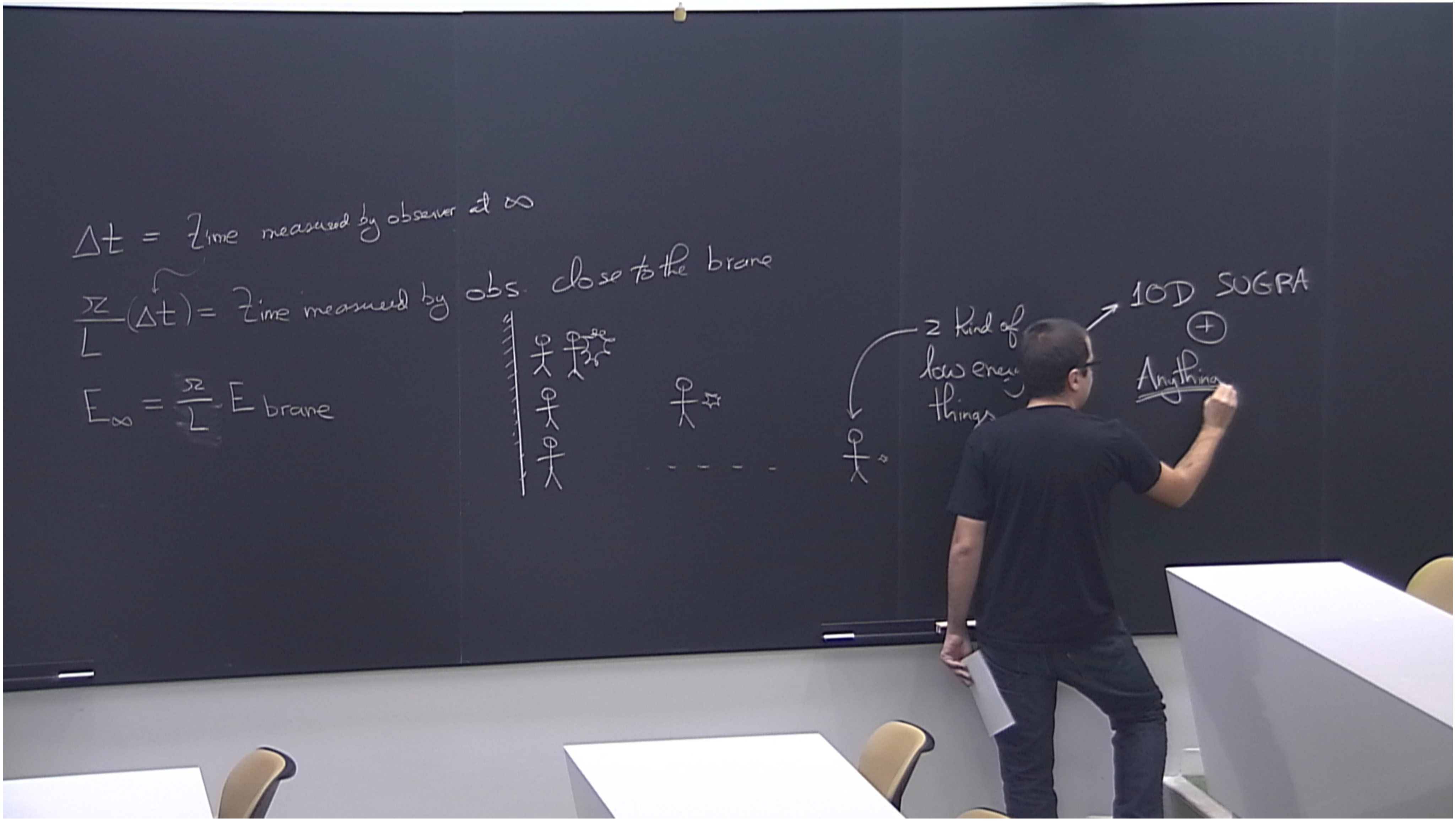


$$x^\nu + \frac{L^2}{r^2} dr^2 \Big] + L^2 d\Omega^2_{S^5}$$

Δt = time measured by observer at ∞

$\frac{r}{L} \Delta t$ = time measured by obs. close to the brane

$$E_\infty = \frac{L}{r} E_{\text{brane}}$$



Δt = time measured by observer at ∞

$\frac{r}{L}(\Delta t)$ = time measured by obs. close to the brane

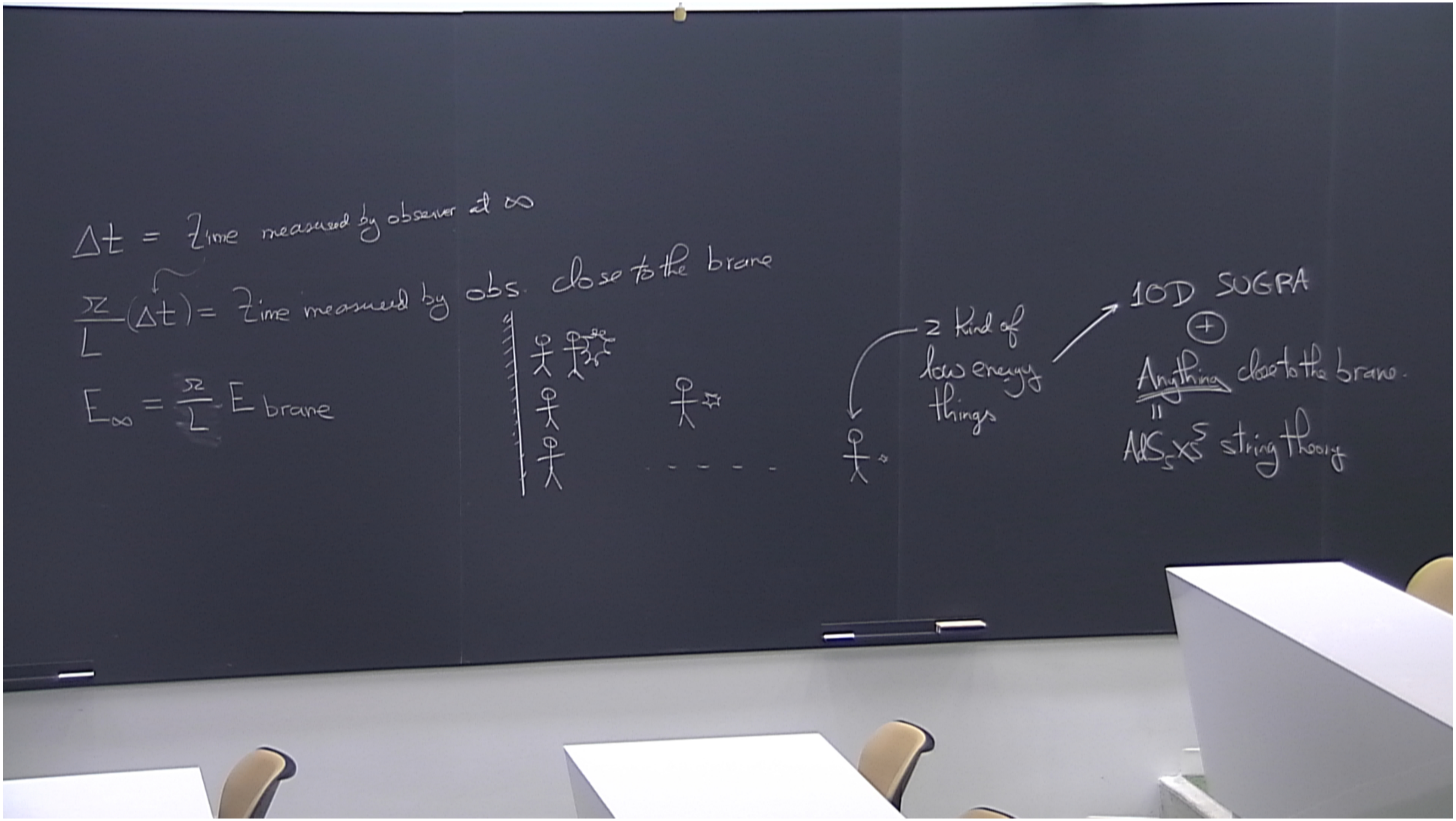
$E_\infty = \frac{r}{L} E_{\text{brane}}$

2 kind of low ener things

10D SUGRA

⊕

Anything



Δt = time measured by observer at ∞

$\frac{r}{L}(\Delta t)$ = time measured by obs. close to the brane

$E_\infty = \frac{r}{L} E_{brane}$

2 kind of low energy things

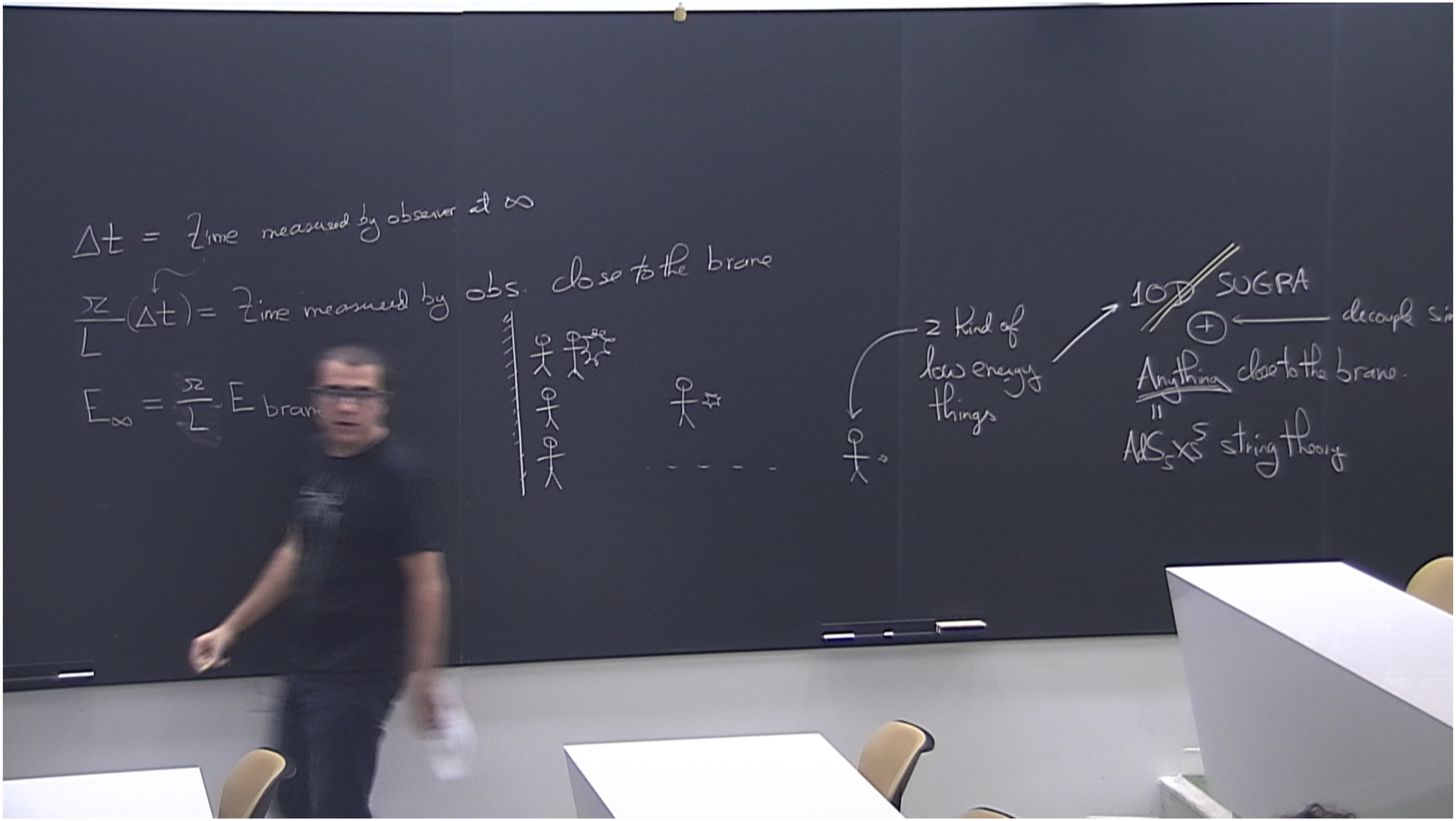
10D SUGRA

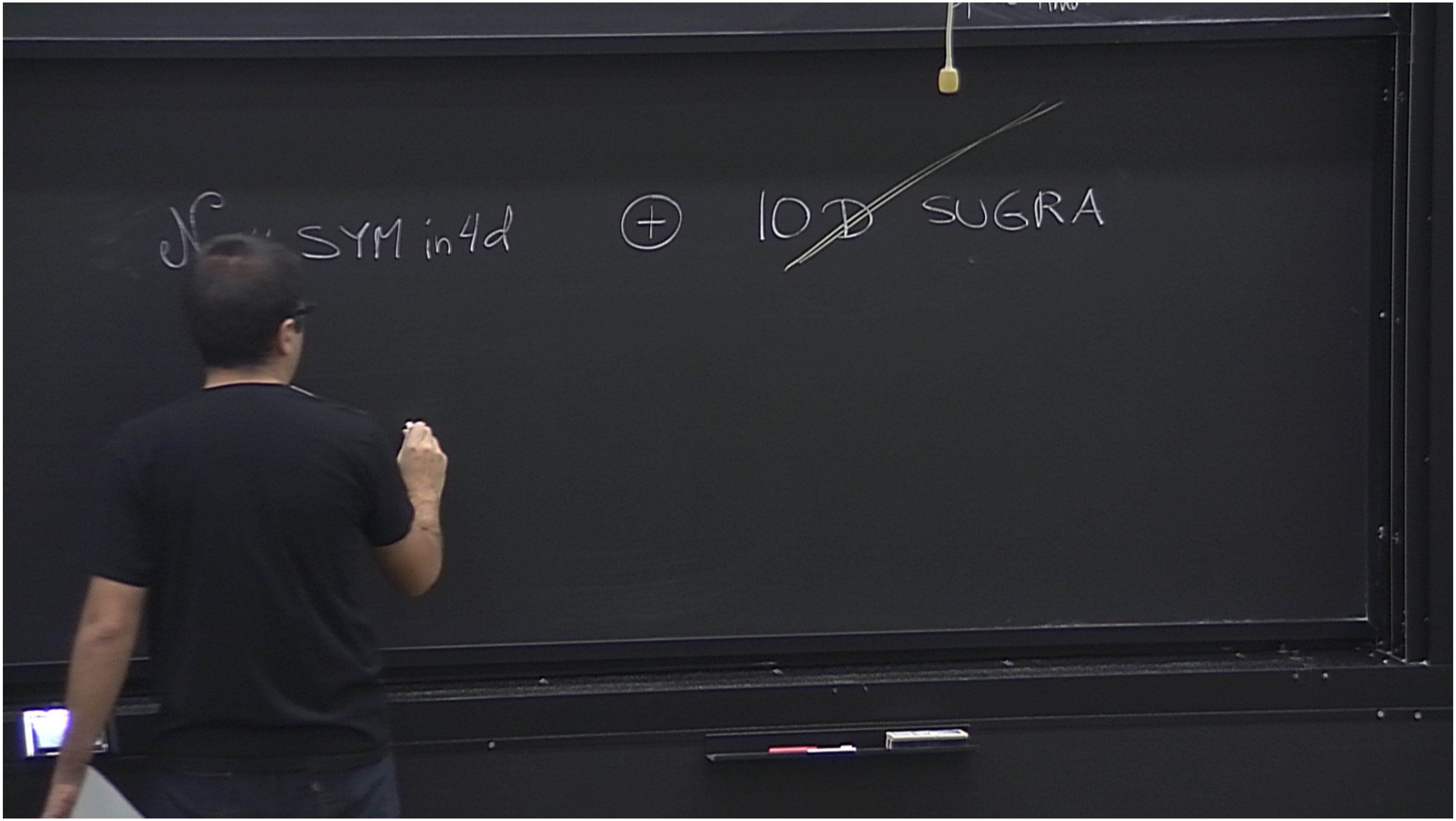
$\oplus$

Anything close to the brane.

$\parallel$

$AdS_5 \times S^5$ string theory





$\mathcal{N}$ SYM in 4d $\oplus$ 10~~D~~ SUGRA

$\mathcal{N}=4$ SYM in 4d $\oplus$ 10 ~~D~~ SUGRA

$$\Rightarrow \boxed{\mathcal{N}=4 \text{ SYM in 4d} \equiv AdS_5 \times S^5 \text{ string th}}$$