

Title: Explorations in String Theory -10

Date: Apr 17, 2015 11:30 AM

URL: <http://pirsa.org/15040149>

Abstract:



$\langle \text{tr} M^L \text{tr} M^L \rangle = \langle \text{tr} M^L \text{tr} M^L \rangle = \langle \text{tr} M^L \text{tr} M^L \rangle = \langle \text{tr} M^L \text{tr} M^L \rangle$

more generically $\sum \text{Tr}[M \cdot NP](x)$

+ matrix values fields (Eg $S = S[M] + S[N] + \frac{1}{g_s} \text{tr} M^2 N^2$)

some space-time point

+ non-planar subleading contributions

$= \int \text{[diagram]} + \frac{1}{N^2} \text{[diagram]} + \dots = \sum g_s^{2-2g} \int \mathcal{D}x e^{-\tau(x) S[x]}$

← spin chains $H = \sum \lambda + \sum \lambda + \dots$

String Caping String Tension

$\mathcal{N}=4$ SYM in 4d \equiv $\mathcal{N}=1$ SYM in 10d
 compactified down
 to 4d

$$\frac{1}{4} \left(F_{MN} F^{MN} + \bar{\psi} \not{D} \psi \right)$$

16 component
 real spinors

$$\frac{1}{4} \bar{\psi} \not{D} \psi =$$

$$\left. \begin{aligned} L, J &= 1 \dots N \\ \lambda, B &= 1 \dots 16 \\ M, N &= 0 \dots 9 \end{aligned} \right\}$$

$$\left(\bar{\psi}_A \right)_y \Gamma_{AB}^M \left(\partial_M \psi_B + i [A_M, \psi_B] \right)_j$$

$$\Gamma^\Pi \Gamma^N + \Gamma^N \Gamma^\Pi = 2 \eta^{\Pi N} \mathbb{1}$$

$\begin{matrix} \uparrow \\ A, B \\ \text{spinors} \end{matrix}$

10d \rightarrow 4d

fields do not depend on $x_4 \dots x_9$

$$A_M = \begin{pmatrix} A_\mu & \Phi_i \end{pmatrix}$$

↑_{0..3} ↑_{4..9}

gauge bosons scalars

$$F_{MN} F^{MN}$$

$$= F_{\mu\nu} F^{\mu\nu}$$

$$+ D_\mu \Phi^i D^\mu \Phi_i$$

$$+ [\Phi_i, \Phi_j][\Phi^i, \Phi^j]$$

$$\sim \frac{1}{g^2} \int d^4x \text{tr}(\dots) = S$$

$$\bar{\Psi} \not{D} \Psi$$

$$= \bar{\Psi} \Gamma^M (\partial_M \Psi + [A_M, \Psi])$$

$$+ \bar{\Psi} \Gamma^i \Phi_i \Psi$$

$$= \mathcal{N}^4 \text{SYM}$$

$A_M = (A_\mu, \Phi_i)$
 gauge bosons scalars
 \uparrow \uparrow
 $0..3$ $4..9$

$$A_M \rightarrow \Omega A_M \Omega^{-1} + \Omega \partial_M \Omega^{-1}$$

$$D^\mu \Phi^i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j] \sim \frac{1}{g^2} \int_{\text{d}^4x} \text{tr}(\dots) = S$$

$$[A_\mu, \Psi] + \bar{\Psi} \Gamma^i \Phi_i \Psi$$

$A_M = \left(\begin{array}{c} \text{gauge bosons} \\ A_\mu \\ \text{scalars} \\ \Phi_i \end{array} \right)$

$\mathcal{D}^\mu \Phi^i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j] \sim \frac{1}{g^2} \int d^4x \text{tr}(\dots) = S$

$(A_\mu, \Psi) + \bar{\Psi} \Gamma^i \Phi_i \Psi$

$A_M \rightarrow \Omega A_M \Omega^{-1} + \Omega \partial_M \Omega^{-1}$

$\left(\begin{array}{l} A_\mu \rightarrow \text{similar} \\ \Phi_i \rightarrow \Omega \Phi_i \Omega^{-1} \\ \Psi \rightarrow \Omega \Psi \Omega^{-1} \end{array} \right)$

$$\mathcal{O}(x) = \text{tr} \left(M_1 \dots M_L \right) (x)$$

↑
 $\in 4d$ Mink.

↑
 Φ_i or ψ or $F_{\mu\nu}$, $\mathcal{D}_\mu \Phi_i$, $\mathcal{D}_\mu^2 \Phi_i$, ...

$$\langle \underbrace{00}_x \underbrace{00}_y \rangle =$$

$$\langle \underbrace{00}_x \underbrace{00}_y \rangle = \text{[Diagram 1]} + \frac{1}{N^2} \text{[Diagram 2]}$$

The diagram shows the decomposition of a two-loop correlator. The left side is the expectation value of two loops, labeled x and y . The right side is the sum of two diagrams: two separate loops and a connected diagram with a cross-like structure, weighted by $1/N^2$.

$\lambda = g_{YM}^2 N$ is a free parameter

$$\mathcal{O}(x) = \text{tr} (M_1 \dots M_L)(x)$$

\uparrow
 $\in 4d \text{ Mink.}$

← Single trace gauge invariant operators

$$\Phi_i \text{ or } \psi \text{ or } F_{\mu\nu}, \quad \mathcal{D}_\mu \Phi_i, \quad \mathcal{D}_\mu^2 \Phi_i, \dots$$

$$\langle \underbrace{00}_x \underbrace{00}_y \rangle = \text{[Diagram 1]} + \frac{1}{N^2} \text{[Diagram 2]}$$

The diagram shows the expectation value of the product of two operators, each consisting of two circles. The first term is the sum of two diagrams: one with a wavy vertical line connecting the two circles, and another with a straight vertical line. The second term is a diagram with two crossing lines connecting the four circles, multiplied by a factor of 1/N^2.

$\lambda = g_{\text{eff}}^2 N$ is a free parameter
 $\beta = 0$ non-perturbatively.

th. without scale

$$\langle \underbrace{00}_x \underbrace{00}_y \rangle = \text{[Diagram 1]} + \frac{1}{N^2} \text{[Diagram 2]}$$

The equation shows a correlation function of two pairs of operators, each pair labeled with a subscript x and y . The result is a sum of two diagrams. The first diagram consists of two vertical tubes, one for x and one for y , each with wavy lines representing internal interactions. The second diagram is a four-point interaction where the two x legs and two y legs meet at a central point, forming a cross-like shape with wavy lines.

$\lambda = g_{\text{eff}}^2 N$ is a free parameter
 $\beta = 0$ non-perturbatively.

th. without scale
 \Rightarrow Conformal Field Th.

$$= d\mathcal{F} = 4 \text{ SYM}$$

consider a CFT with a single field ϕ

$$\langle \phi(x) \phi(y) \rangle = \frac{c}{|x-y|^{2\Delta}} \quad \begin{array}{l} \uparrow \\ \text{dim of } \phi \end{array}$$

$= \mathcal{N}^4 \text{ SYM}$

consider a CFT with a single field ϕ

$$\langle \phi(x) \phi(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

$\Delta \leftarrow \text{dim of } \phi$

$\{\Delta\} \equiv \text{Spectrum of the CFT}$

$d=4$ SYM

with a single field ϕ

$\{\Delta\} \equiv$ Spectrum of the CFT

$$\langle \phi(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

dim of ϕ

$$S[\phi] = S_{\text{free}} + \lambda S_{\text{int}}$$

$S \equiv$ Spectrum of the CFT

$$S[\phi] = S_{\text{free}} + \lambda S_{\text{int}}$$

$$\langle \phi_{\text{bare}}(x) \phi_{\text{bare}}(y) \rangle = \frac{1}{(x-y)^{2\Delta_0}}$$

classical dim
(e.g. $\Delta_0 = 1$ for scalars
in $d=4$)

$$\lambda(\dots) = -\lambda \underbrace{\gamma}_{\text{const}} \log(\Lambda^2(x-y)^2)$$



$$\begin{aligned} \phi(x) &= \Lambda^{\gamma\lambda} \phi_{\text{bare}}(x) \\ &= (1 + \gamma\lambda \log \Lambda) \phi_{\text{bare}}(x) \end{aligned}$$

$$\langle \phi(x) \phi(y) \rangle = \frac{1}{(x-y)^{2(\Delta_0 + \gamma\lambda)}}$$



multiplicative
renormalization



Spectrum of
 $\gamma = \Delta - \Delta_0$

$$\langle \phi(x) \phi(y) \rangle = \frac{1}{(\lambda - y)^{2(\Delta_0 + \lambda \gamma)}}$$

$$\lambda(\dots) = -\lambda \underbrace{\gamma}_{\text{const}} \log(\Lambda^2(x-y)^2)$$



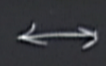
$$\begin{aligned} \phi(x) &\equiv \Lambda^{\gamma\lambda} \phi_{\text{bare}}(x) \\ &= (1 + \gamma\lambda \log \Lambda) \phi_{\text{bare}}(x) \end{aligned}$$

$$\langle \phi(x) \phi(y) \rangle = \frac{1}{(x-y)^{2(\Delta_0 + \gamma\lambda)}}$$

ϕ

$$\langle \phi \rangle = \frac{1}{(\lambda - y)^{2(\Delta_0 + \lambda \gamma)}}$$

multiplicative renormalization



Spectrum of $\gamma = \Delta - \Delta_0$

in practice this means $\{\gamma\}$ is related to log div. of 2pt.

10d \rightarrow 4d

fields do not depend on $x_4 \dots x_9$

gauge bosons scalars

$$A_M = \left(\underset{\substack{\uparrow \\ 0 \dots 3}}{A_\mu}, \underset{\substack{\uparrow \\ 1 \dots 6}}{\Phi^i} \right)$$

∇

$$F_{MN} F^{MN} = F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi^i D^\mu \Phi^i + [\Phi^i, \Phi^j][\Phi^i, \Phi^j]$$

$$\bar{\Psi} \not{D} \Psi = \bar{\Psi} \Gamma^M (\partial_\mu \Psi + [A_\mu, \Psi]) + \bar{\Psi} \Gamma^i \Phi^i \Psi$$

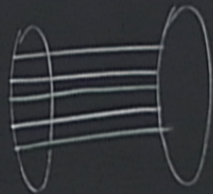
= d=4 SYM

We have huge degeneracy of fields w/ same quantum numbers and same Δ_0

Eg $\text{tr}(\phi_1 \phi_1 \phi_2 \phi_1 \phi_2)$, $\text{tr}(\phi_1 \phi_1 \phi_2 \phi_2 \phi_1)$ both have $\Delta_0 = 5$

$$\langle \phi_1 \phi_1 \rangle = \frac{1}{(x-y)^2}$$

$$\langle \phi_1 \phi_2 \rangle = 0$$

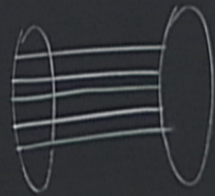


We have huge degeneracy of fields w/ same quantum numbers and same Δ_0

Eg $\text{tr}(\phi_1 \phi_1 \phi_2 \phi_1 \phi_2)$, $\text{tr}(\phi_1 \phi_1 \phi_2 \phi_2 \phi_1)$ both have $\Delta_0 = 5$

$$\langle \phi_1 \phi_1 \rangle = \frac{1}{(x-y)^2}$$

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We have huge degeneracy of fields w/ same quantum numbers and same Δ_0

Eg $\text{tr}(\phi_1 \phi_1 \phi_2 \phi_1 \phi_2)$, $\text{tr}(\phi_1 \phi_1 \phi_2 \phi_2 \phi_1)$ both have $\Delta_0 = 5$

$$\langle \phi_1 \phi_1 \rangle = \frac{1}{(x-y)^2} \quad \text{Diagram} \sim \left(\frac{1}{(x-y)^2} \right)^5, \quad \Delta_0 = 5$$

$$\langle \phi_1 \phi_2 \rangle = 0$$

$$\langle \textcircled{A}^{\text{bare}} \textcircled{B}^{\text{bare}} \rangle = \frac{1}{(x-y)^{2\Delta_0}} \left(S_{AB} - 2\gamma_{AB} \log \Lambda^2 (x-y)^2 \right)$$

$$\sum \psi^A \textcircled{A}^{\text{bare}} \equiv \textcircled{A}^{\text{diag, bare}}$$

↑ wavefunction

diagonalize $\hat{\gamma}_{AB}$
 with eigenvalue γ_A
 $\hat{\gamma} \textcircled{A}^{\text{diag, bare}} = \gamma_A \textcircled{A}^{\text{diag, bare}}$

$$\langle \textcircled{A}^{\text{bare}} \textcircled{B}^{\text{bare}} \rangle = \frac{1}{(x-y)^{2\Delta_0}} \left(S_{AB} - 2\gamma_{AB} \log \Lambda^2 (x-y)^2 \right)$$

$$\sum \psi^A \textcircled{A}^{\text{bare}} \equiv \textcircled{A}^{\text{diag, bare}}$$

↑ wavefunction

diagonalize $\hat{\gamma}_{AB}$
 with eigenvalue γ_A
 $\hat{\gamma} \textcircled{A}^{\text{diag, bare}} = \gamma_A \textcircled{A}^{\text{diag, bare}}$

$$\left. \begin{array}{l} \text{)}^2 \\ \text{)} \end{array} \right)$$

$$\langle \textcircled{A}^{\text{diag. base}}(x) \textcircled{B}(y) \rangle = \frac{S_{AB}}{(x-y)^{2\Delta_0}} \left(1 - 2\gamma_A \log \sqrt{x-y} \right)$$

quantum numbers and same Δ_0

both have $\Delta_0 = 5$

$\Delta_0 = 5$

Summary

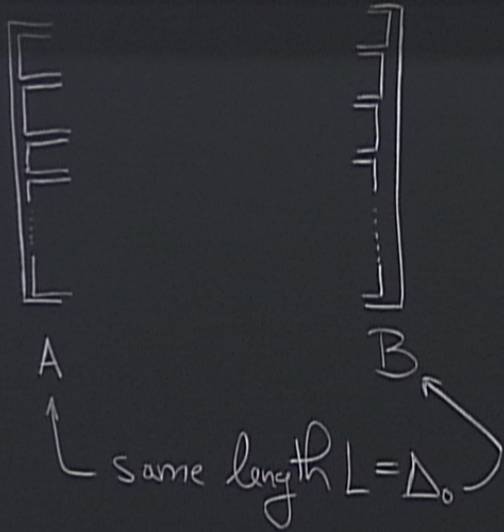
1) Compute $\langle \mathcal{O}_A^{\text{bare}} \mathcal{O}_B^{\text{bare}} \rangle$ and
extract γ_{AB} from $\log \Delta$ coef

2) Diagonalize γ_{AB}

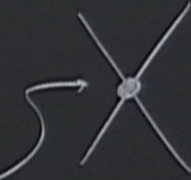
3) Eigenvalues \leftrightarrow Spectrum of $\{\gamma\}$

4) Eigenvector \leftrightarrow Ops w/ def. an. dim.

ϕ_i



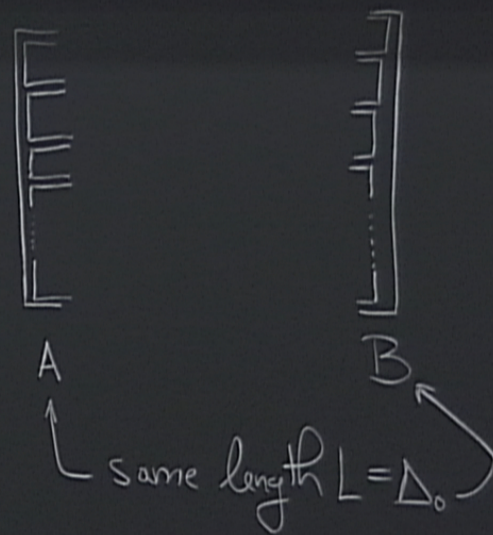
start by
considering
effect
of $\text{tr}[\phi_i, \phi_j]^2$



$$\text{tr} \left(\begin{array}{c} 2\phi_i \phi_j \phi_i \phi_j \\ + \end{array} - \begin{array}{c} 2\phi_i \phi_i \phi_j \phi_j \\ + \end{array} \right)$$

$$C = \text{tr} \phi_{i_1} \dots \phi_{i_L}$$

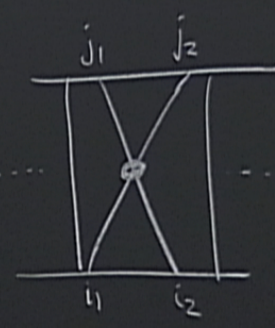
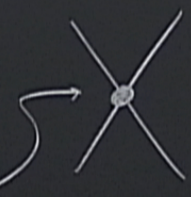
\uparrow \uparrow
 A $1 \dots L$
 \uparrow
 6^L values



start by
 considering
 effect
 of $\text{tr}[\phi_i$

$$\text{tr} (2 \phi_i \phi_i)$$

start by
considering
effect
of $\text{tr}[\phi_i, \phi_j]^2$



$$= 4 \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} - 2 \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} - 2 \begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array}$$

$$= 4 \int_{i_1}^{j_2} \int_{i_2}^{j_1} - 2 \int_{i_1}^{j_2} \int_{i_2}^{j_1} - 2 \int_{i_1}^{j_2} \int_{i_2}^{j_1}$$

$$\text{tr} \left(\begin{array}{c} 2 \phi_i \phi_j \phi_i \phi_j \\ + \\ - 2 \phi_i \phi_i \phi_j \phi_j \\ + \end{array} \right)$$

$$\begin{aligned}
 & \text{Diagram} = \# \left(\underbrace{g_{\text{YM}}^2 N}_{\lambda} \right) \sum_{\text{I}} \left(\text{Diagram 1} + \text{Diagram 2} - 2 \text{Diagram 3} \right)_{n, n+1} \\
 & \text{4 vertex}
 \end{aligned}$$

4) Eigenvector ops

$$\left(\begin{array}{c} \times \\ \dots \\ \times \end{array} \right)_{n+1} \times \int \frac{d^4 z}{\left((x-z)^2 \right)^2 \left((y-z)^2 \right)^2} \times \text{all other prop} \left(\left| \dots \right| \left| \times \right| \left| \dots \right| \right)$$

this
ones

$2 \left(\begin{matrix} X \\ 1 \end{matrix} \right)_{n+1}$

$\times \int \frac{d^4 z}{\left((x-z)^2 \right)^2 \left((y-z)^2 \right)^2}$ \times all other prop $\left| \dots \right| \left| X \right| \left| \dots \right|$
 this ones

div for $z \rightarrow x$ or y

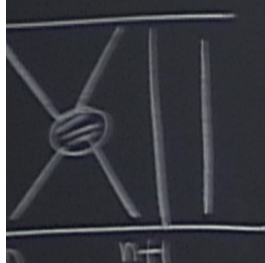
$\frac{1}{\left((x-y)^2 \right)^2} \int \frac{d^4 z}{\left(x-z \right)^4} \sim \int_{\frac{1}{\Lambda}}^{|x-y|} \frac{r^3 dr}{r^4} \sim \log \Lambda (x-y)$

$$N) \sum \left(\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} + \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} - 2 \begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array} \right)_{n, n+1} \times \int \frac{d^4 z}{((x-z)^2)^2 ((y-z)^2)^2} \times \text{all other}$$

tree level props
x log Λ

div for $z \rightarrow x$ or y

$$\frac{1}{((x-y)^2)^2} \int \frac{d^4 z}{(x-z)^4} \sim \int_{\frac{1}{\Lambda}}^{|x-y|} \frac{r^3 dr}{r^4}$$



4 vertex

$$= \# \left(g_{\text{YM}}^2 N \right) \sum_{\Gamma} \left(\text{diagram 1} + \text{diagram 2} - 2 \text{diagram 3} \right)_{n, n+1} \times \int \frac{d^d x}{((x-y)^2)^2}$$

λ

part of δ
coming from
 $[\phi, \phi]^2$

tree level props
 $\times \log \Lambda$

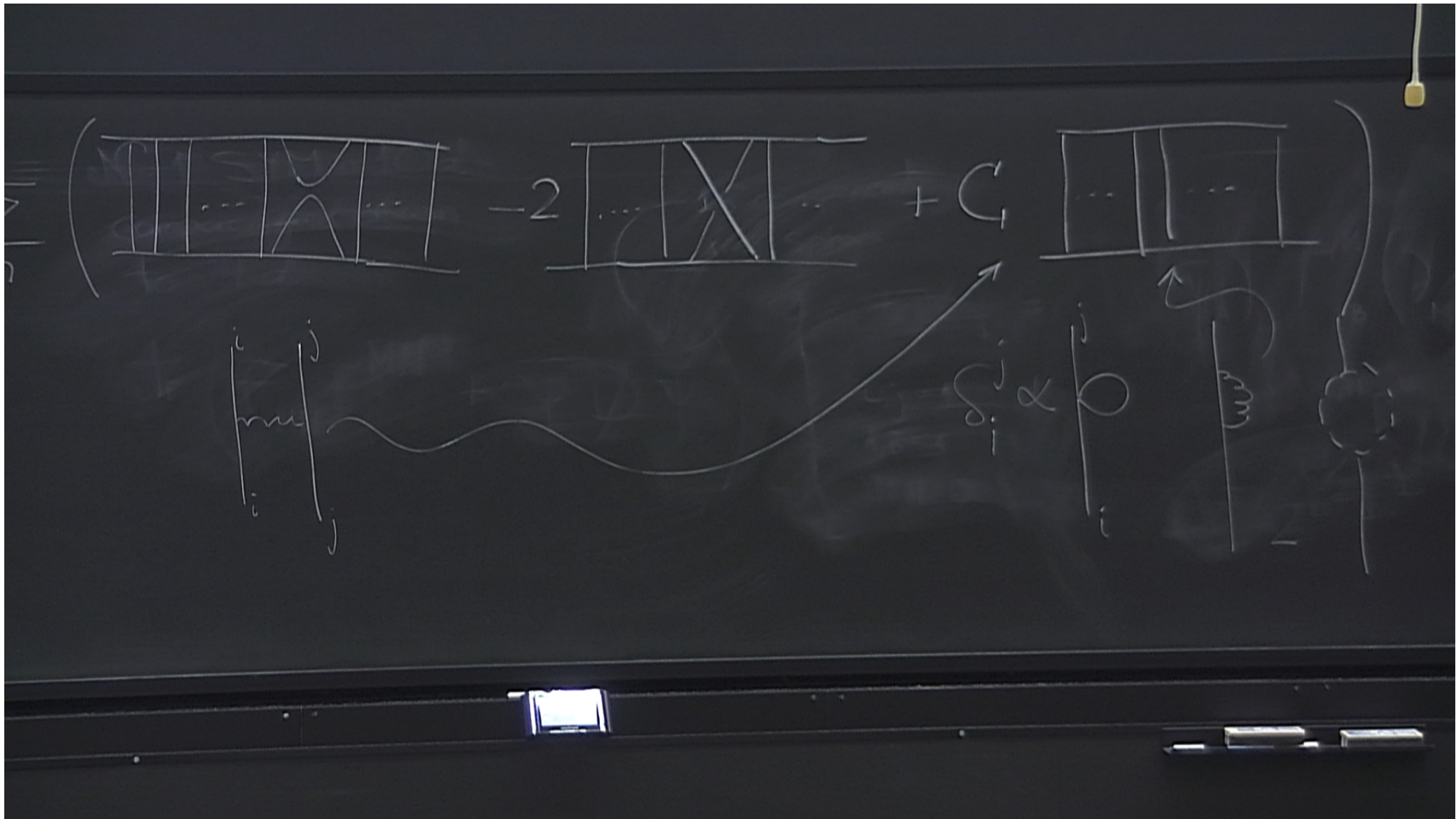
div

$\frac{1}{((x-y)^2)^2}$

$$\chi_{\substack{j_1 \dots j_L \\ i_1 \dots i_L}} = \underbrace{(\dots)}_{\text{num const}} \lambda \sum_n \left(\text{Diagram 1} - 2 \text{Diagram 2} \right)$$

The first diagram is a rectangular box with two horizontal lines and two vertical lines. Inside, there are two vertical lines, and between them are two vertices connected by a horizontal line.

The second diagram is a rectangular box with two horizontal lines and two vertical lines. Inside, there are two vertical lines, and between them are two vertices connected by a horizontal line.



$\frac{1}{k} \phi_{i_1} \dots \phi_{i_L} \longleftrightarrow \text{SO}(6) \text{ spin-chain } \mathbb{G}^L$

$|i_1 \dots i_L\rangle \in \mathcal{H}^{\mathbb{G}^L}$

$\gamma_{i_1 \dots i_L}^{j_1 \dots j_L} \longleftrightarrow \hat{\gamma} = H = \frac{\lambda}{16\pi^2} \sum_{n=1}^L (K - 2P + c\mathbb{1})$

↓ loop
Spectrum
{ γ }

$H|\gamma\rangle = E|\gamma\rangle$

$$\frac{1}{r} \phi_{i_1} \dots \phi_{i_L} \longleftrightarrow \text{SO}(6) \text{ spin-chain } \mathcal{H}^G$$

$$|i_1 \dots i_L\rangle \in \mathcal{H}^G$$

$$\gamma_{i_1 \dots i_L}^{j_1 \dots j_L} \longleftrightarrow \hat{\gamma} = H = \frac{\lambda}{16\pi^2} \sum_{n=1}^L \left(K - 2P + c\mathbb{1} \right)$$

$$K|ab\rangle = S_{ab} \sum_{c=1}^6 |cc\rangle$$

1 loop
Spectrum
{ γ }

$$H|\Psi\rangle = E|\Psi\rangle$$

$\frac{1}{r} \phi_{i_1} \dots \phi_{i_L} \longleftrightarrow \text{SO}(6) \text{ spin-chain}$

$$|i_1 \dots i_L\rangle \in \mathcal{H}^G$$

$\gamma_{i_1 \dots i_L}^{j_1 \dots j_L}$

$$\hat{\gamma} = H = \frac{\lambda}{16\pi^2} \sum_{n=1}^L (K - 2P + c\mathbb{1})_{n+1}$$

$$K|ab\rangle = \delta_{ab} \sum_{c=1}^6 |cc\rangle$$

1 loop
Spectrum

$\{\gamma\}$
in $N=4S$

$$H|\psi\rangle = E|\psi\rangle$$

Integrable Spin chain!

$$\gamma = \lambda \gamma^{(1)} + \lambda^2 \gamma^{(2)} + \dots$$

$$\frac{1}{\hbar} \phi_{i_1} \dots \phi_{i_L} \longleftrightarrow \text{SO}(6) \text{ spin-chain } \mathfrak{g}^L$$

$$|i_1 \dots i_L\rangle \in \mathcal{H}^{\mathfrak{g}^L}$$

$$\gamma_{i_1 \dots i_L}^{j_1 \dots j_L} \longleftrightarrow \hat{\gamma} = H = \frac{\lambda}{16\pi^2} \sum_{n=1}^L \left(K - 2P + c \mathbb{1} \right)_{n+1}$$

$$K |ab\rangle = \delta_{ab} \sum_{c=1}^6 |cc\rangle$$

1 loop
Spectrum
{ γ }
in $\mathfrak{F} = 4S$

$$H|\psi\rangle = E|\psi\rangle$$

Integrable spin chain!