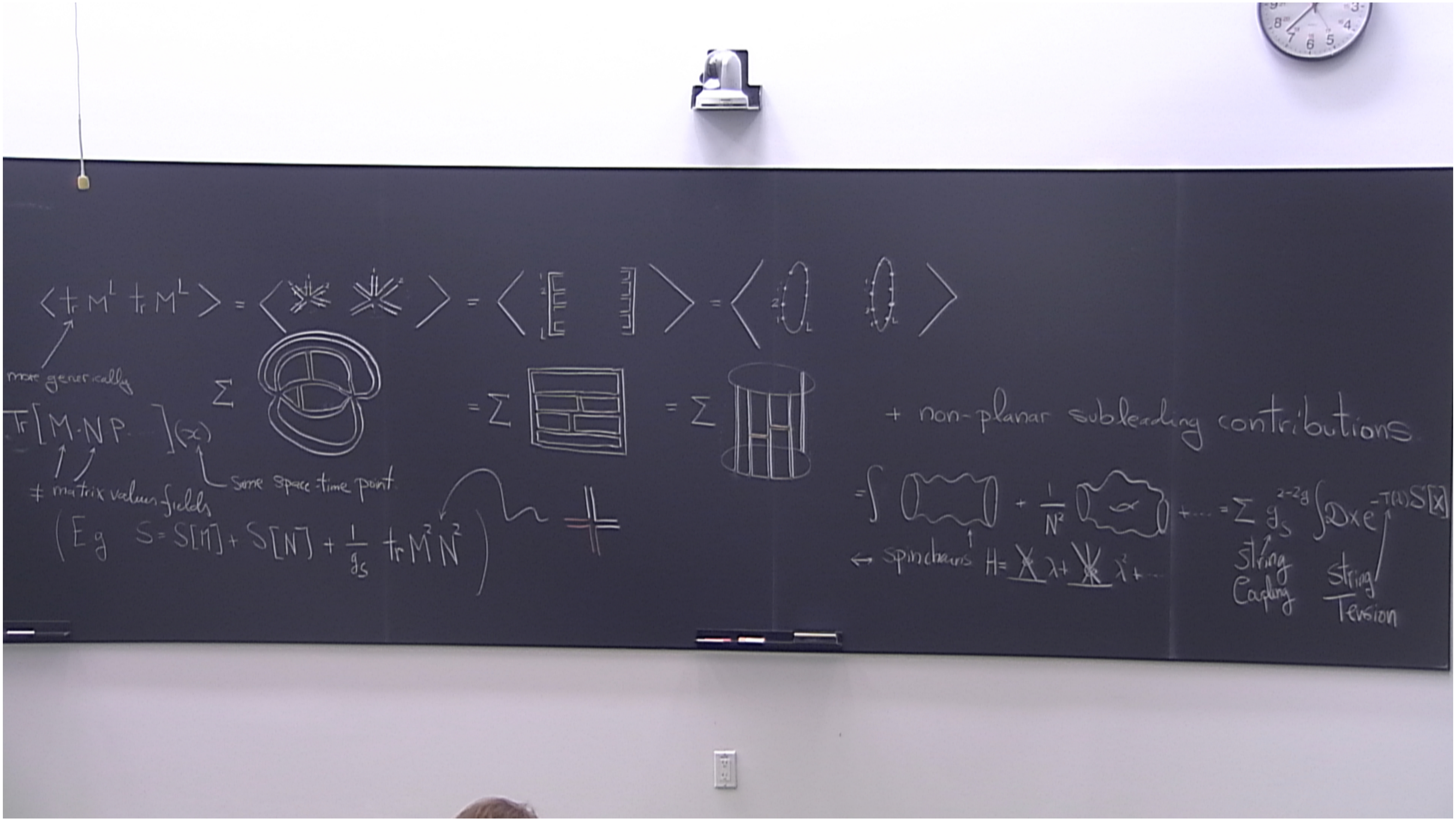
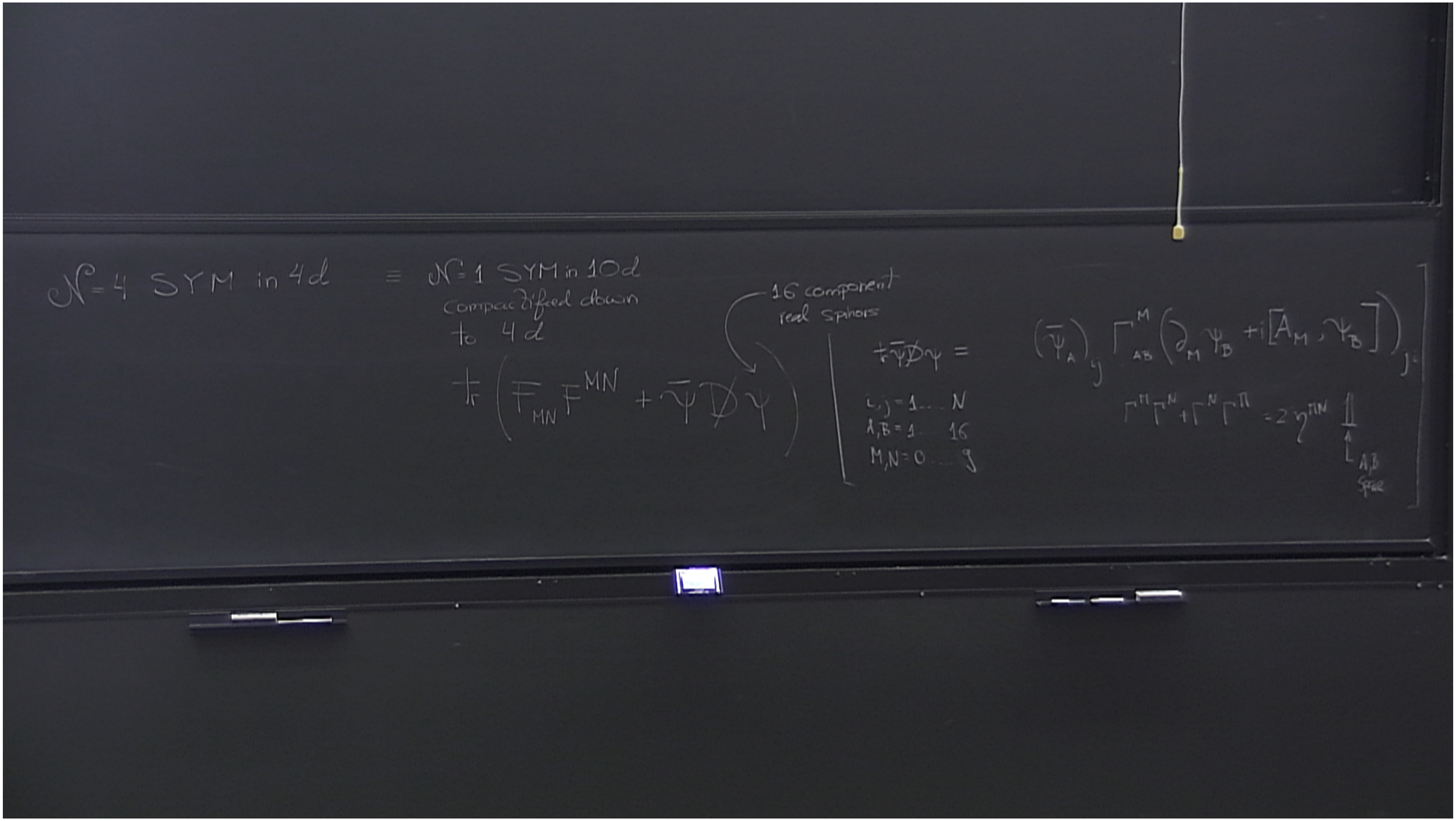
Title: Explorations in String Theory -10

Date: Apr 17, 2015 11:30 AM

URL: http://pirsa.org/15040149

Abstract:


more generically
same space-time point
≠ matrix valued fields
+ non-planar subleading contributions
spinchains
String Coupling
String Tension



$\mathcal{N}=4$ SYM in 4d $\equiv$ $\mathcal{N}=1$ SYM in 10d
compactified down to 4d
16 component real spinors
$\mathrm{tr}\left(F_{MN}F^{MN} + \bar{\psi} \not{D} \psi\right)$
$\mathrm{tr}\,\bar{\psi}\not{D}\psi = (\bar{\psi}_A)_{ij}\,\Gamma^M_{AB}\left(\partial_M \psi_B + i[A_M, \psi_B]\right)_{ji}$
$i,j = 1 \ldots N$
$A,B = 1 \ldots 16$
$M,N = 0 \ldots 9$
$\Gamma^M\Gamma^N + \Gamma^N\Gamma^M = 2\eta^{MN}\,\mathbb{1}_{AB}$
spinor

$10d \to 4d$

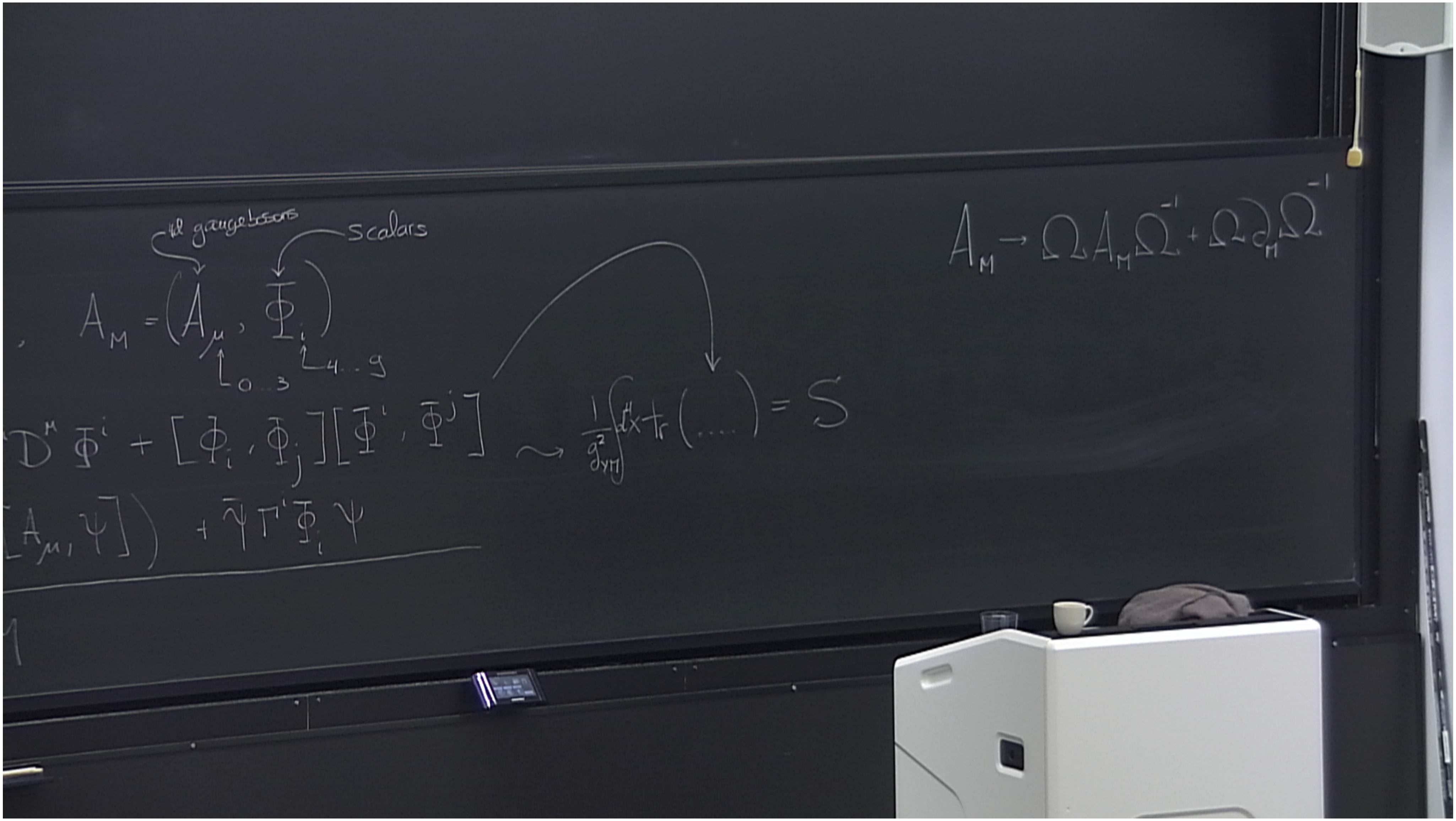
fields do not depend on $x_4 \dots x_9$, $A_M = (A_\mu, \Phi_i)$

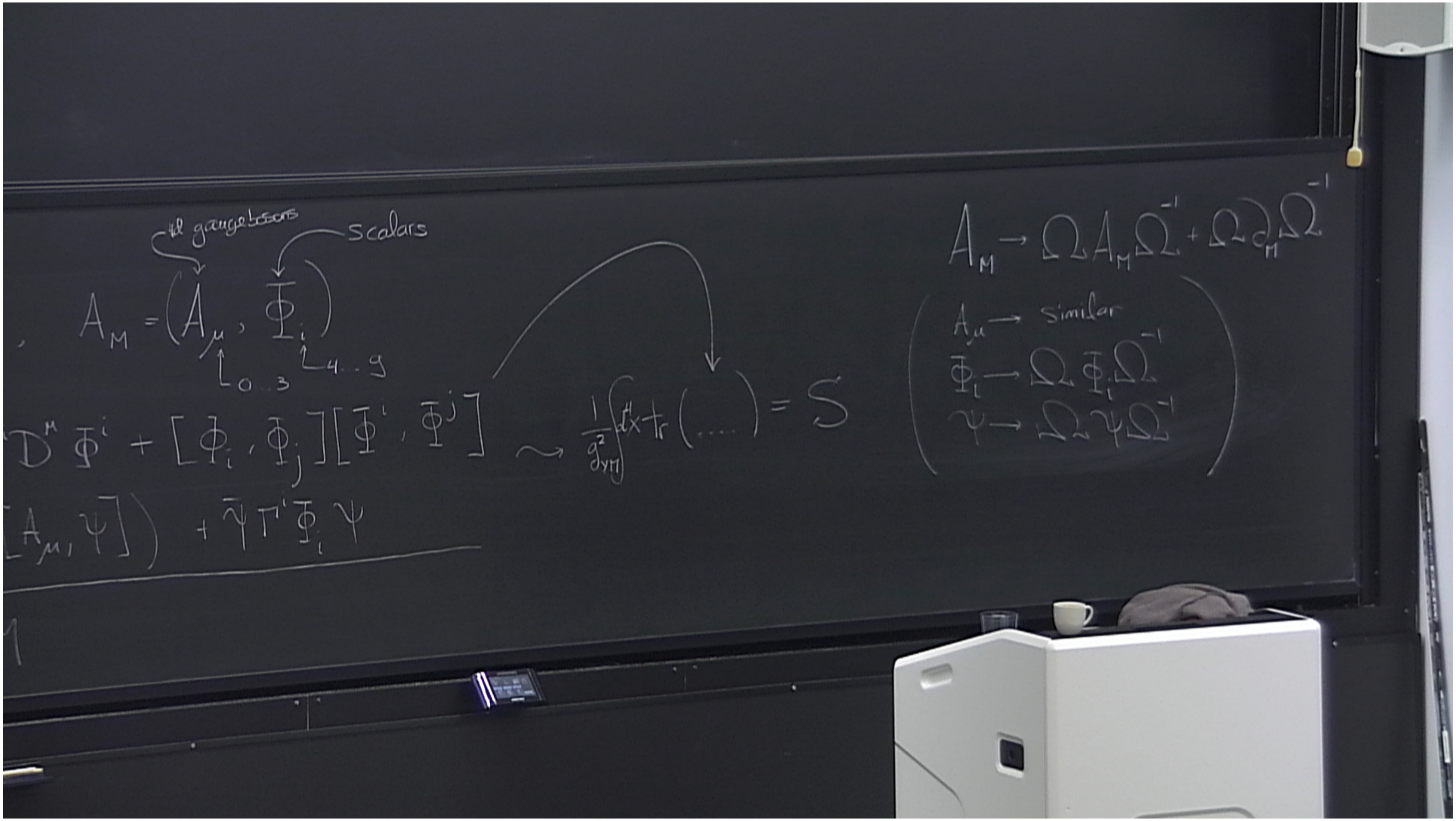
A_μ: 4d gauge bosons, $\mu = 0 \dots 3$; Φ_i: scalars, $i = 4 \dots 9$

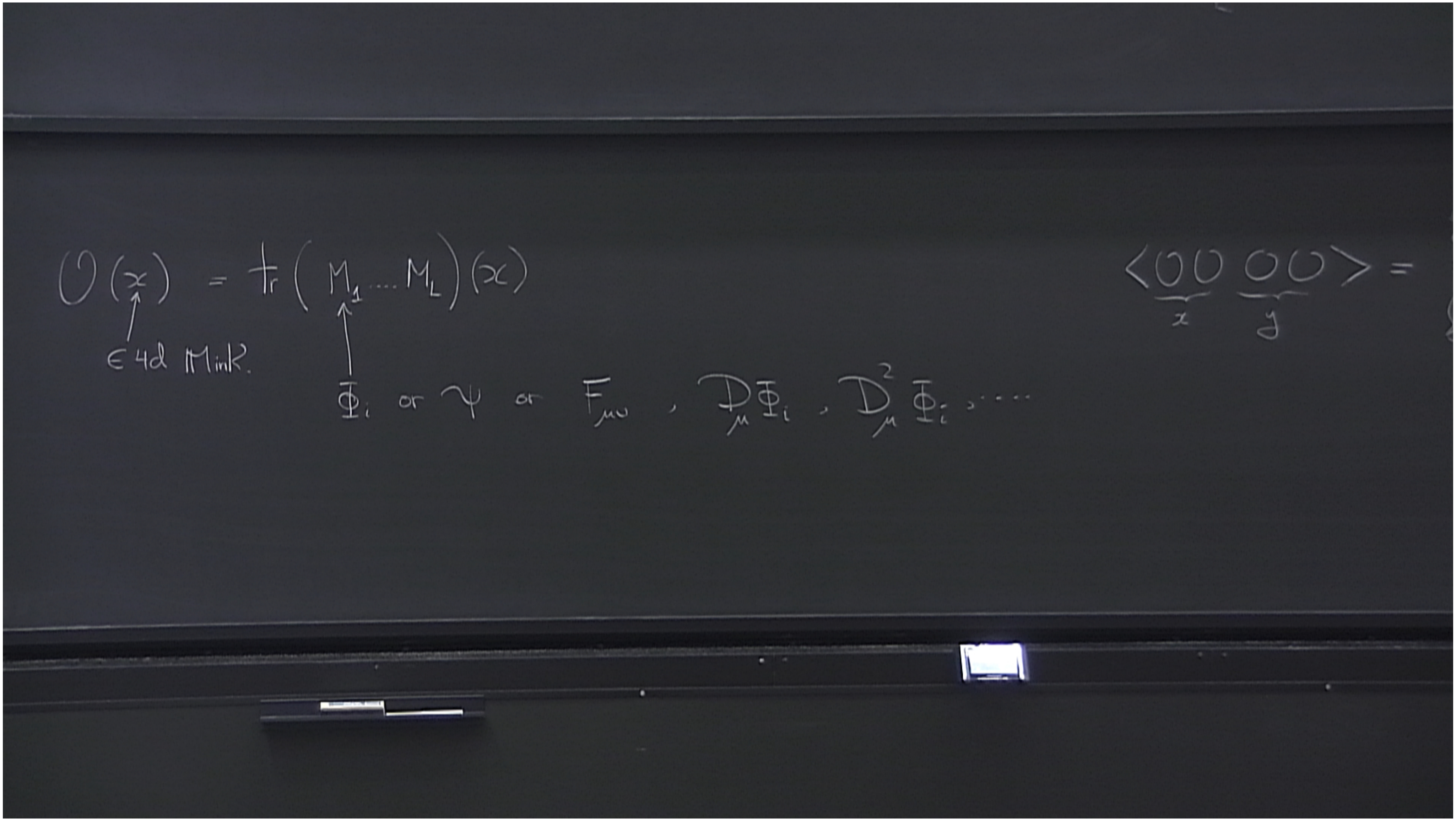
$$F_{MN}F^{MN} = F_{\mu\nu}F^{\mu\nu} + D_\mu \Phi^i D^\mu \Phi^i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j] \sim \frac{1}{g^2_{YM}} \int d^4x \, \mathrm{tr}(\dots) = S$$

$$\bar{\Psi} \not{D} \Psi = \bar{\Psi} \Gamma^\mu (\partial_\mu \Psi + [A_\mu, \Psi]) + \bar{\Psi} \Gamma^i \Phi_i \Psi$$

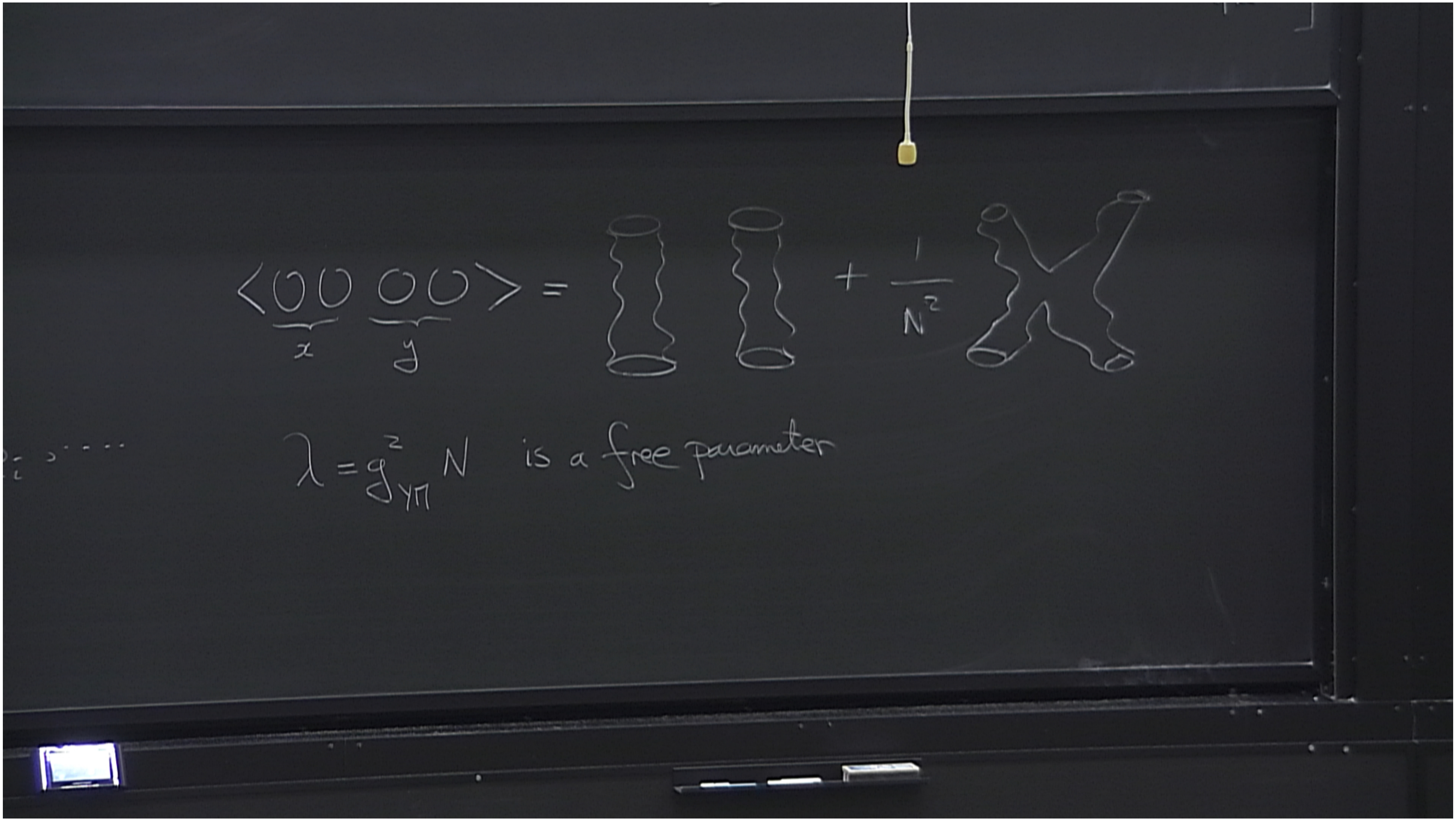
$= \mathcal{N}=4$ SYM


gauge bosons
scalars
A_M = (A_μ , Φ_i)
0...3
4...9
~ 1/g²_YM ∫d⁴x tr (....) = S
A_M → Ω A_M Ω⁻¹ + Ω ∂_M Ω⁻¹


gauge bosons
scalars
$A_M = (A_\mu, \Phi_i)$
$0 \ldots 3$
$4 \ldots 9$
$D^\mu \Phi^i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j]$
$[A_\mu, \Psi]) + \bar{\Psi} \Gamma^i \Phi_i \Psi$
$\sim \frac{1}{g^2_{YM}} \int d^4x \, \mathrm{tr} (\ldots\ldots) = S$
$A_M \to \Omega A_M \Omega^{-1} + \Omega \partial_M \Omega^{-1}$
$A_\mu \to$ similar
$\Phi_i \to \Omega \Phi_i \Omega^{-1}$
$\psi \to \Omega \psi \Omega^{-1}$



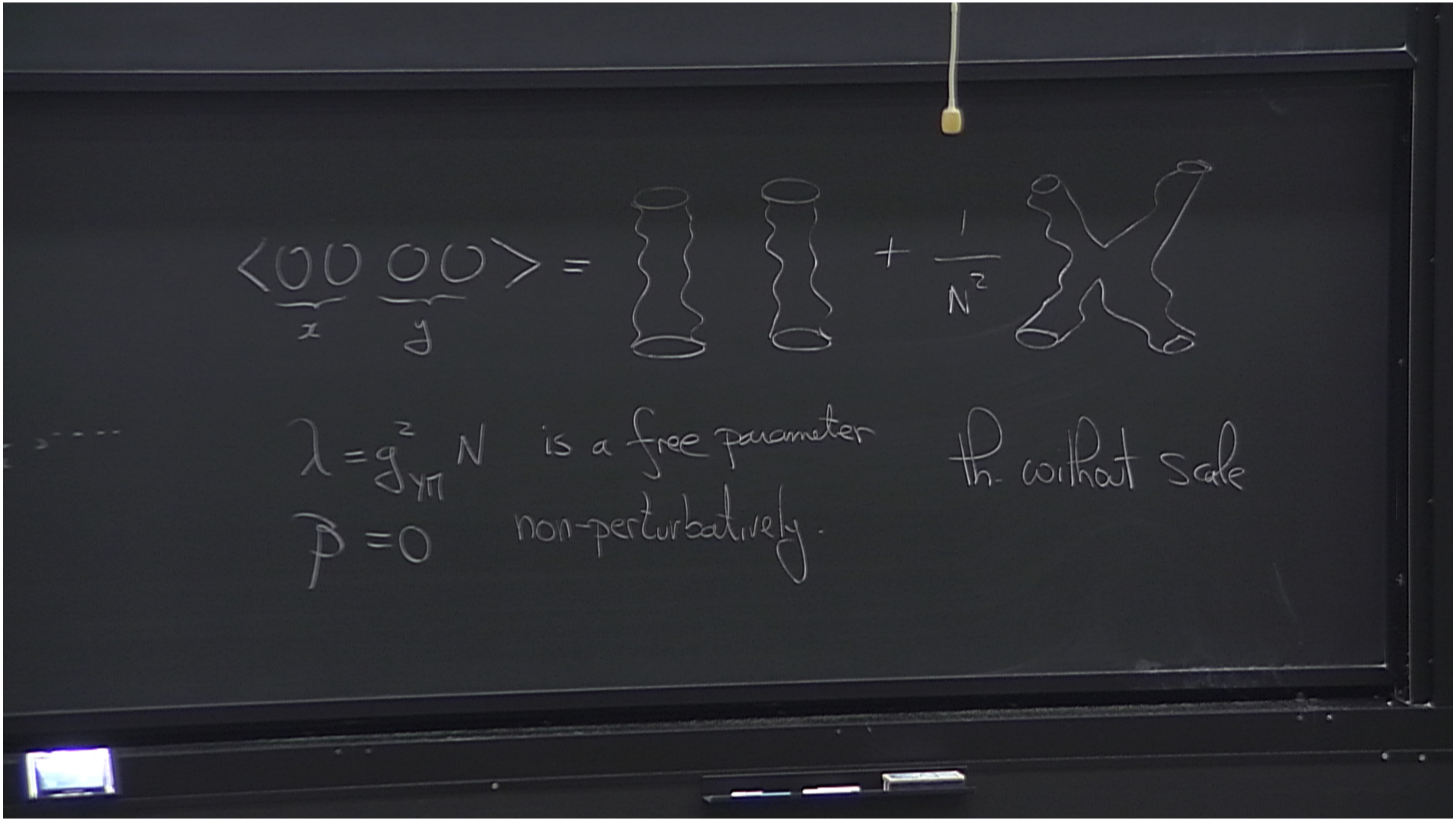
$\mathcal{O}(x) = \mathrm{tr}(M_1 \ldots M_L)(x)$
$\in$ 4d Mink.
Φ_i or ψ or $F_{\mu\nu}$, $D_\mu \Phi_i$, $D_\mu^2 \Phi_i$, ...
$\langle \underbrace{\mathcal{O}\mathcal{O}}_{x} \underbrace{\mathcal{O}\mathcal{O}}_{y} \rangle =$

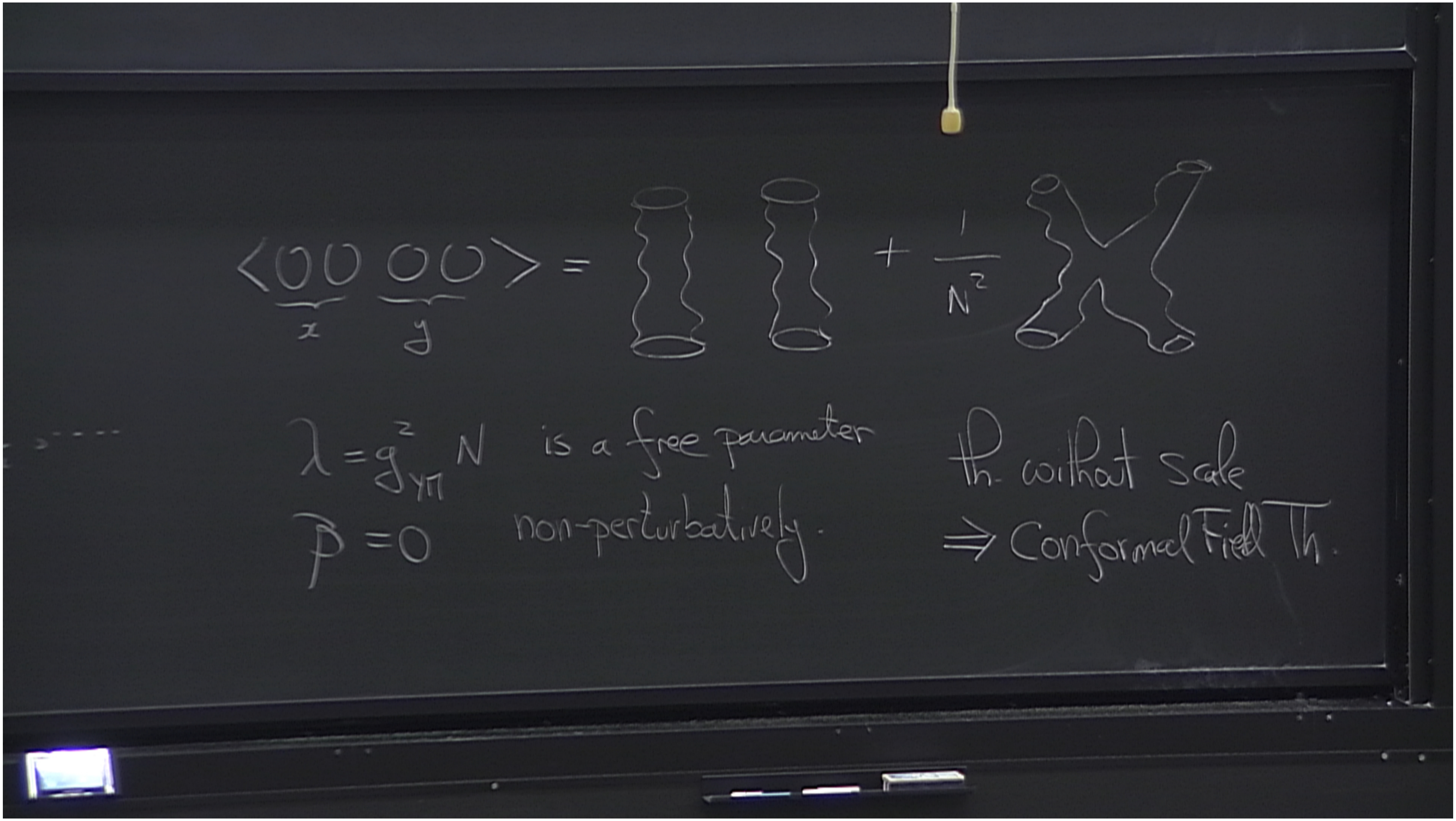

$\langle OO \, OO \rangle = \ldots + \frac{1}{N^2}$
x
y
$\lambda = g^2_{YM} N$ is a free parameter

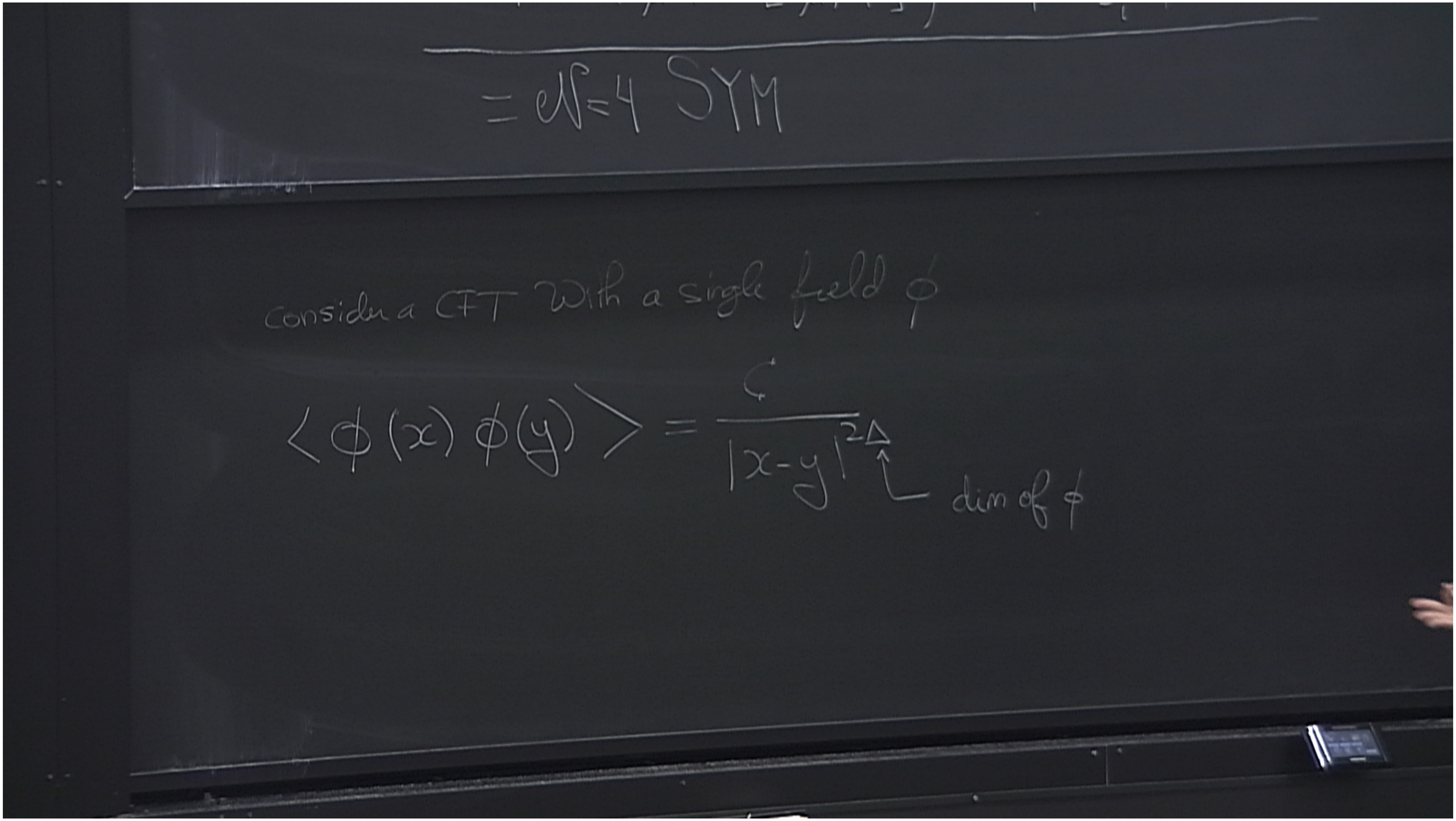
$$\mathcal{O}(x) = \mathrm{tr}\left(M_1 \cdots M_L\right)(x) \quad \leftarrow \text{single trace gauge invariant operators}$$

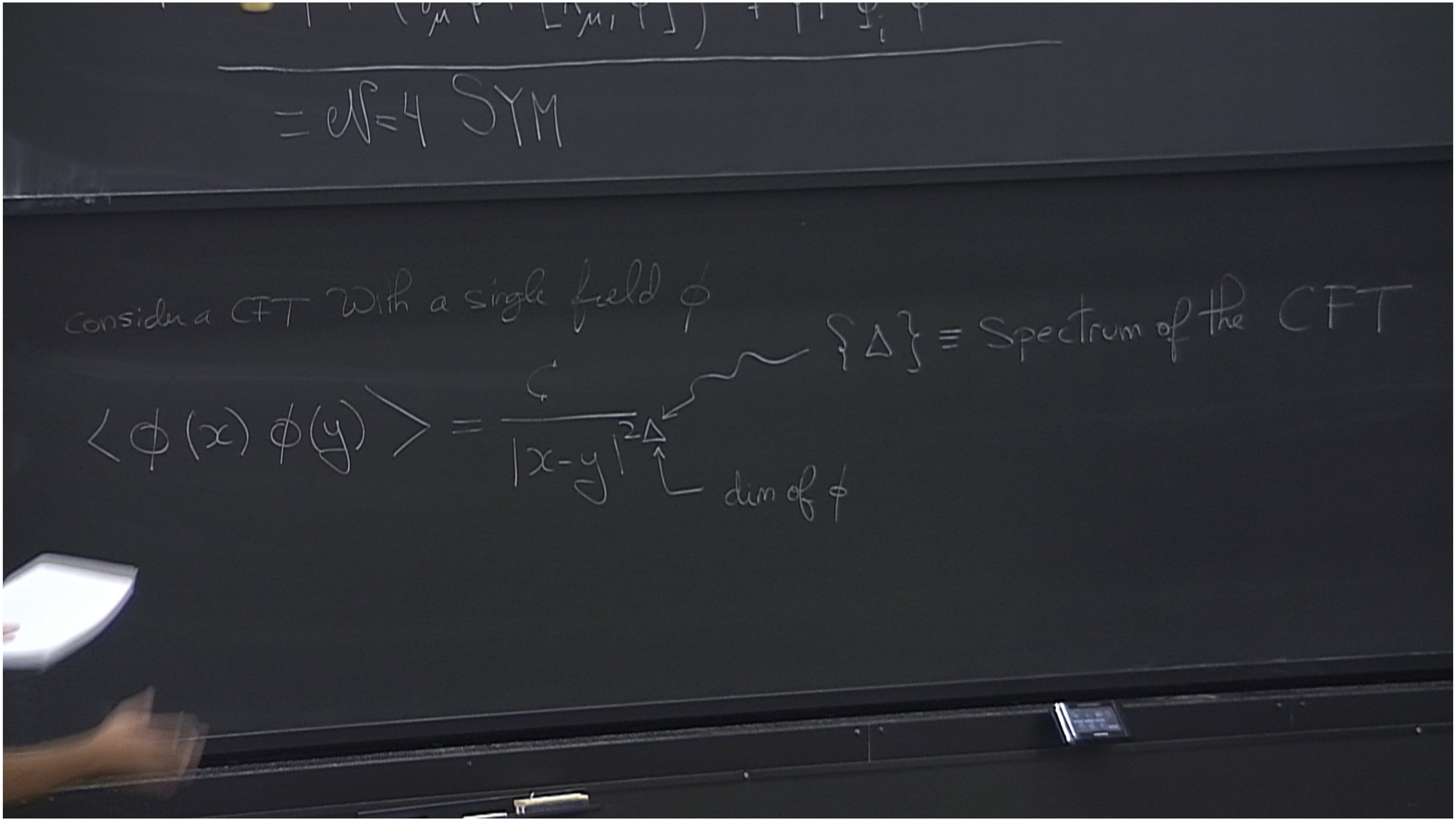
$x \in$ 4d Mink.

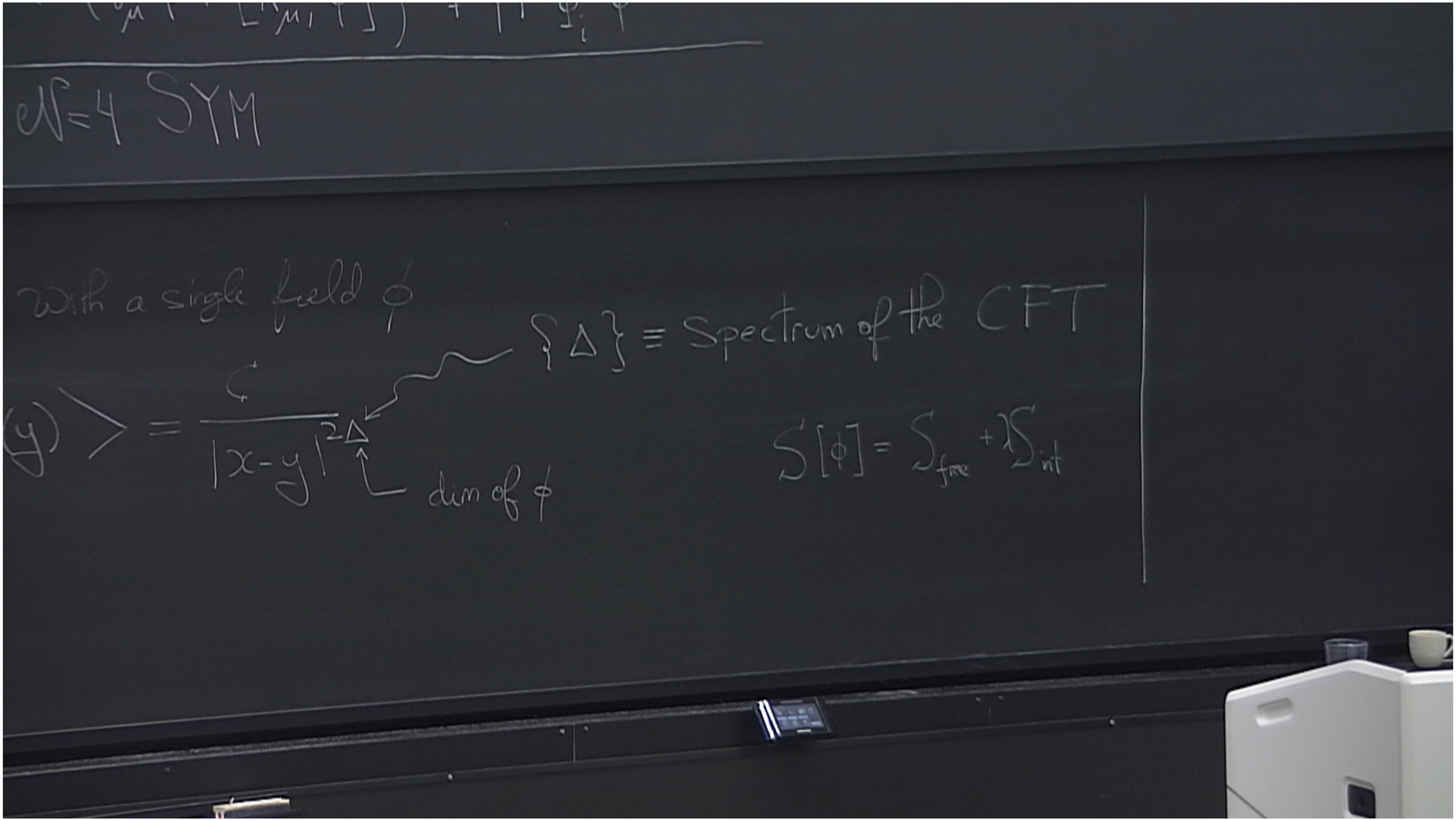
M_1: Φ_i or ψ or $F_{\mu\nu}$, $D_\mu \Phi_i$, $D_\mu^2 \Phi_i$, ...

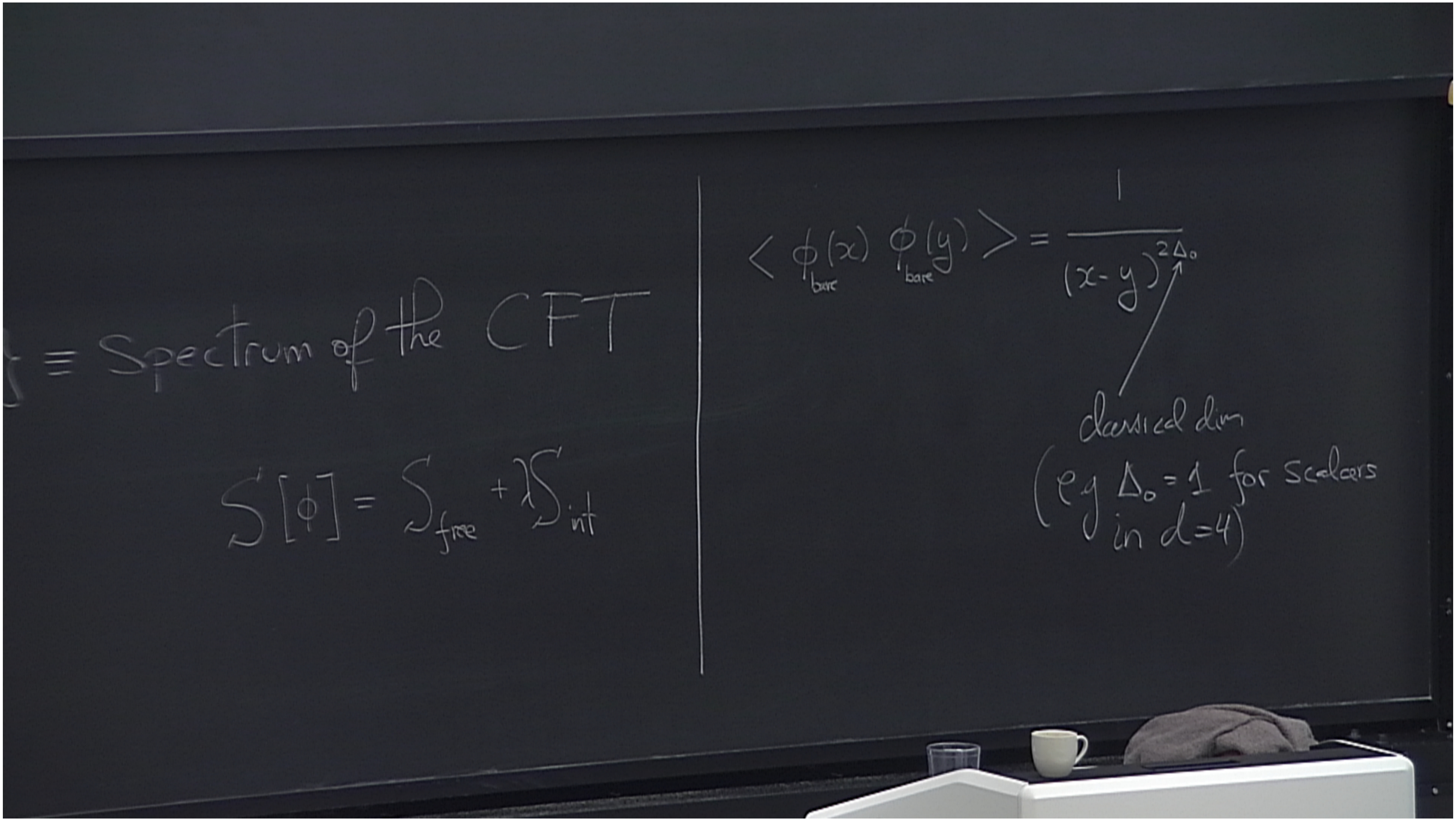

$\langle \underbrace{OO}_{x} \underbrace{OO}_{y} \rangle = \; + \frac{1}{N^2}$
$\lambda = g_{YM}^2 N$ is a free parameter
th. without scale
$\beta = 0$ non-perturbatively.


⟨OO OO⟩ = + 1/N²
x y
λ = g²YM N is a free parameter
β = 0 non-perturbatively.
th. without scale
⇒ Conformal Field Th.

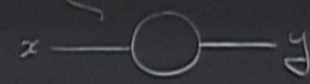

= N=4 SYM
Consider a CFT with a single field φ
⟨φ(x) φ(y)⟩ = C / |x-y|^2Δ
dim of φ


$= \mathcal{N}=4$ SYM
Consider a CFT with a single field ϕ
$\{\Delta\} \equiv$ Spectrum of the CFT
$\langle \phi(x)\,\phi(y) \rangle = \frac{C}{|x-y|^{2\Delta}}$
dim of ϕ


N=4 SYM
With a single field φ
{Δ} ≡ Spectrum of the CFT
(y) > = C / |x-y|^2Δ
dim of φ
S[φ] = S_free + λS_int


≡ Spectrum of the CFT
S[φ] = S_free + λ S_int
< φ_bare(x) φ_bare(y) > = 1 / (x-y)^2Δ₀
classical dim
(eg Δ₀ = 1 for scalars in d=4)

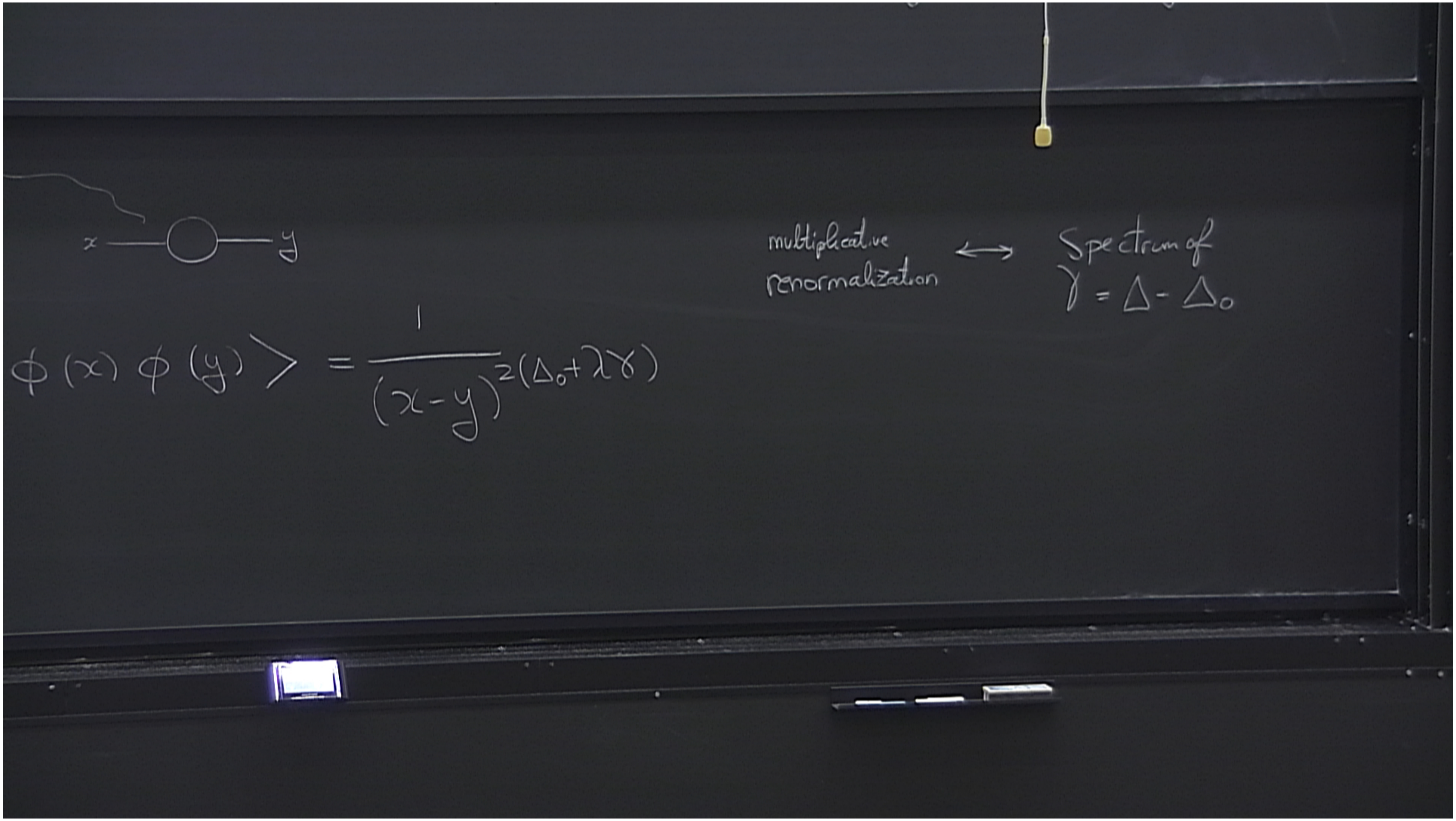
$$\lambda(\ldots) = -\lambda \underbrace{\gamma}_{\text{const}} \log\left(\Lambda^2 (x-y)^2\right)$$

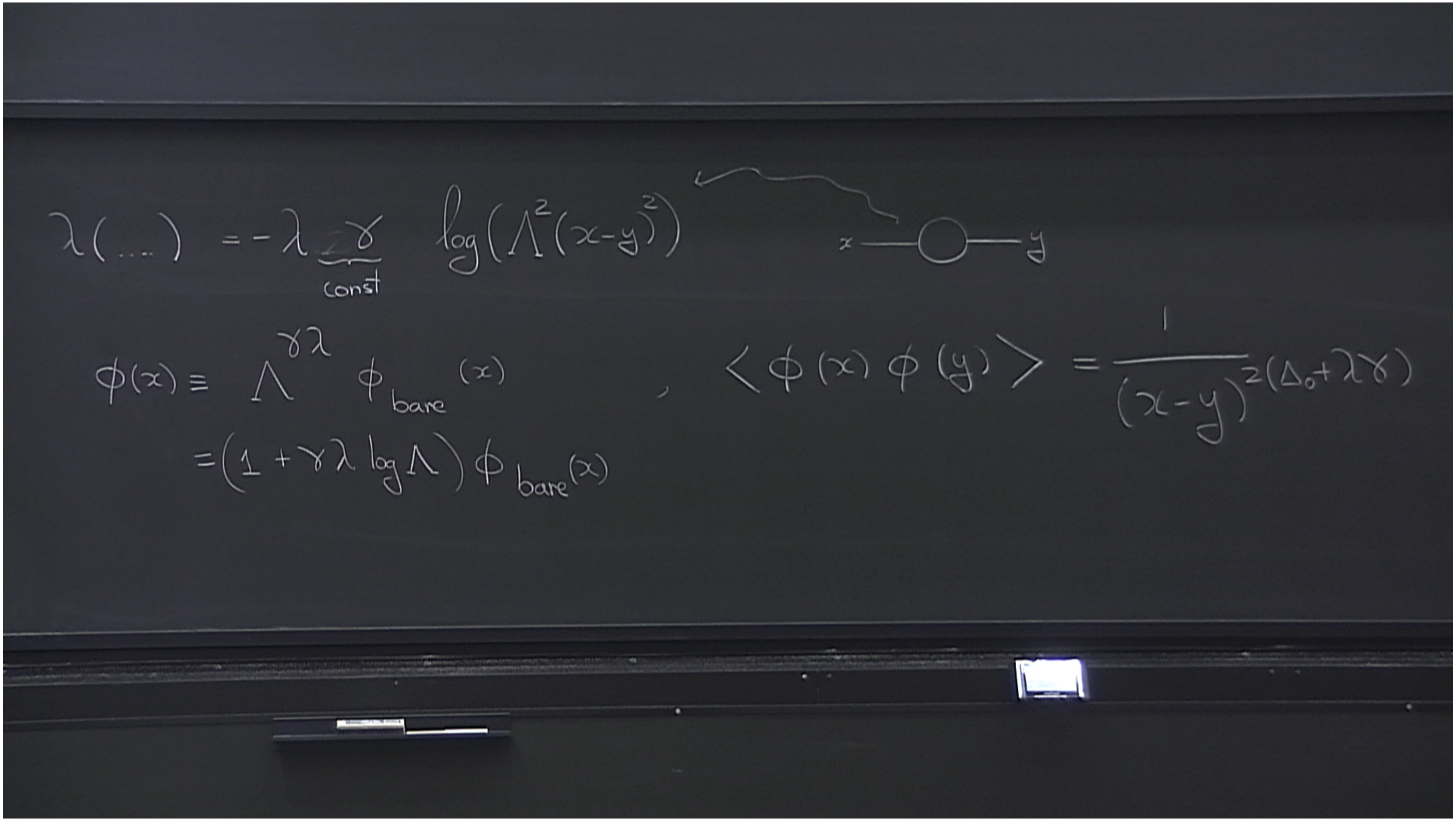


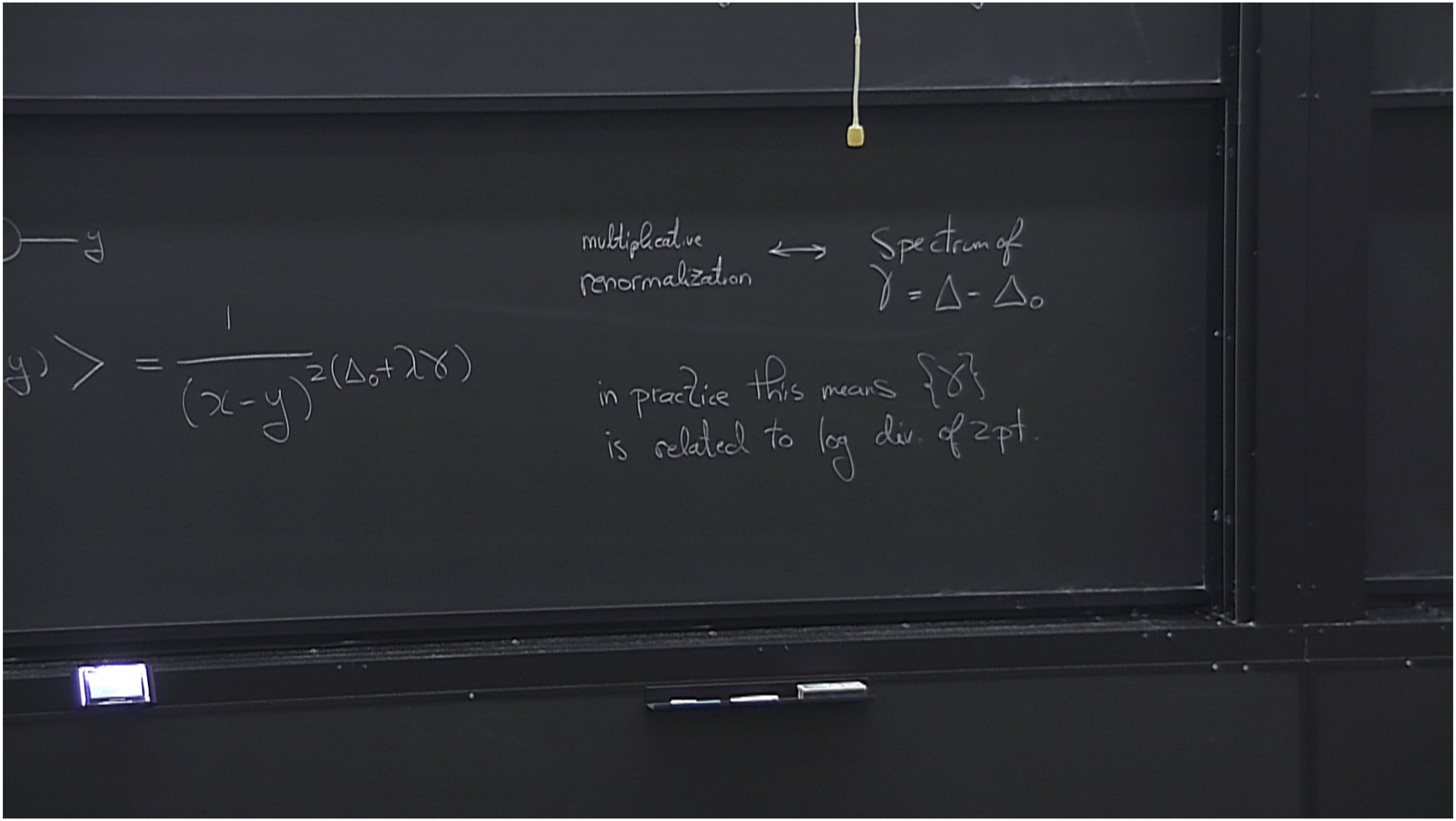
$$\phi(x) = \Lambda^{\gamma\lambda} \phi_{\text{bare}}(x)$$

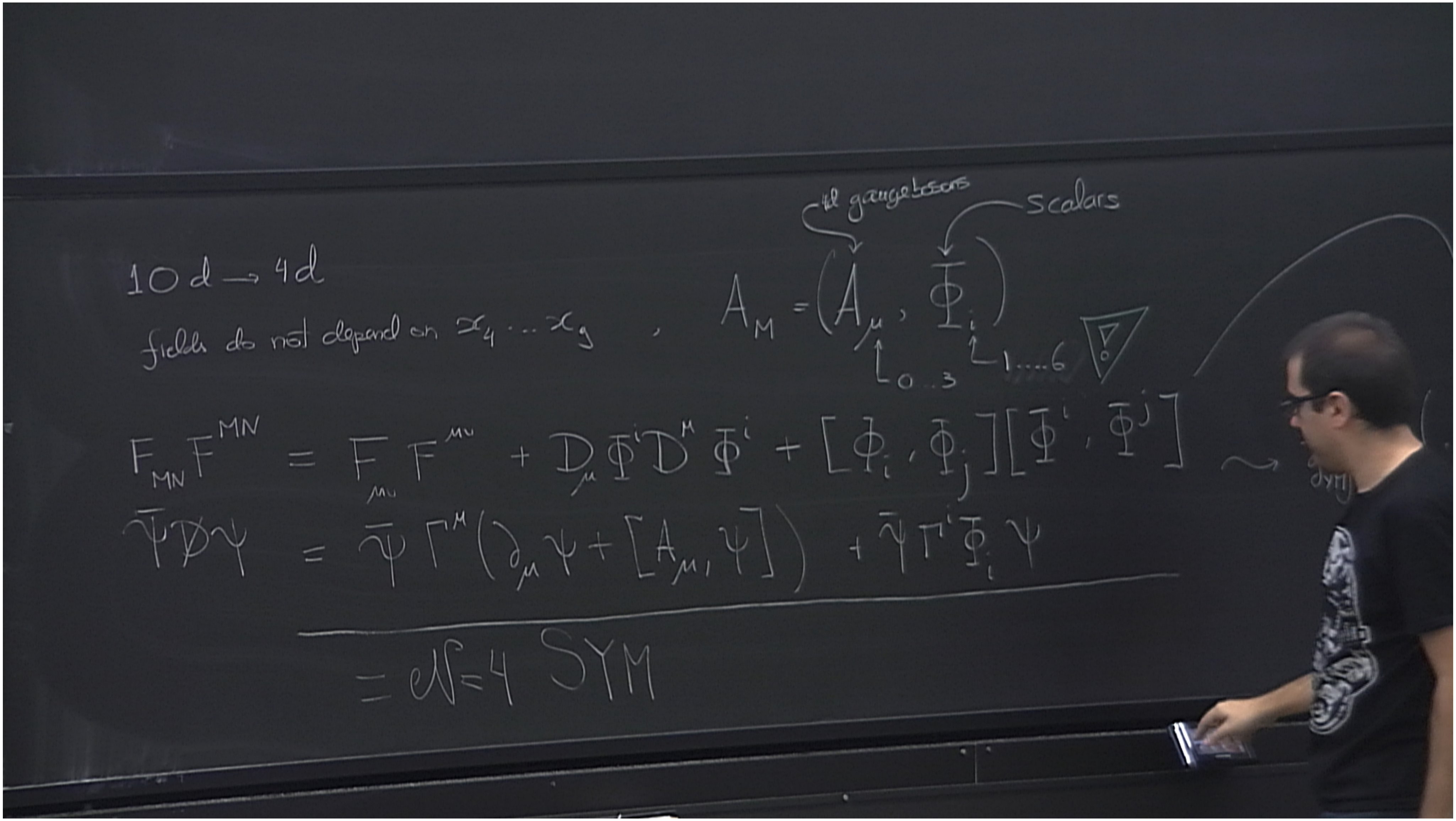
$$= \left(1 + \gamma\lambda \log\Lambda\right) \phi_{\text{bare}}(x)$$

$$\langle \phi(x)\, \phi(y) \rangle = \frac{1}{(x-y)^{2(\Delta_0 + \lambda\gamma)}}$$


x
y
multiplicative renormalization ⟷ Spectrum of
γ = Δ - Δ₀
φ(x) φ(y) ⟩ = 1 / (x-y)^{2(Δ₀+λγ)}


$\lambda(\ldots) = -\lambda \underbrace{\gamma}_{\text{const}} \log(\Lambda^2 (x-y)^2)$
x
y
$\phi(x) \equiv \Lambda^{\gamma\lambda} \phi_{bare}(x)$
$= (1 + \gamma\lambda \log\Lambda)\, \phi_{bare}(x)$
$\langle \phi(x)\, \phi(y) \rangle = \frac{1}{(x-y)^{2(\Delta_0 + \lambda\gamma)}}$


y
y) > = 1 / (x-y)^2(Δ₀+λγ)
multiplicative renormalization ⟷ Spectrum of γ = Δ - Δ₀
in practice this means {γ}
is related to log div. of 2pt.

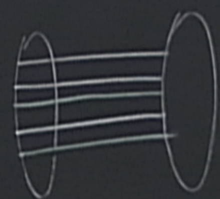

10d → 4d
fields do not depend on x_4 ... x_9
4d gauge bosons
scalars
A_M = (A_μ , Φ_i)
0...3
1....6
F_MN F^MN = F_μν F^μν + D_μ Φ^i D^μ Φ^i + [Φ_i , Φ_j][Φ^i , Φ^j]
Ψ̄ D̸ Ψ = Ψ̄ Γ^μ (∂_μ Ψ + [A_μ , Ψ]) + Ψ̄ Γ^i Φ_i Ψ
= N=4 SYM

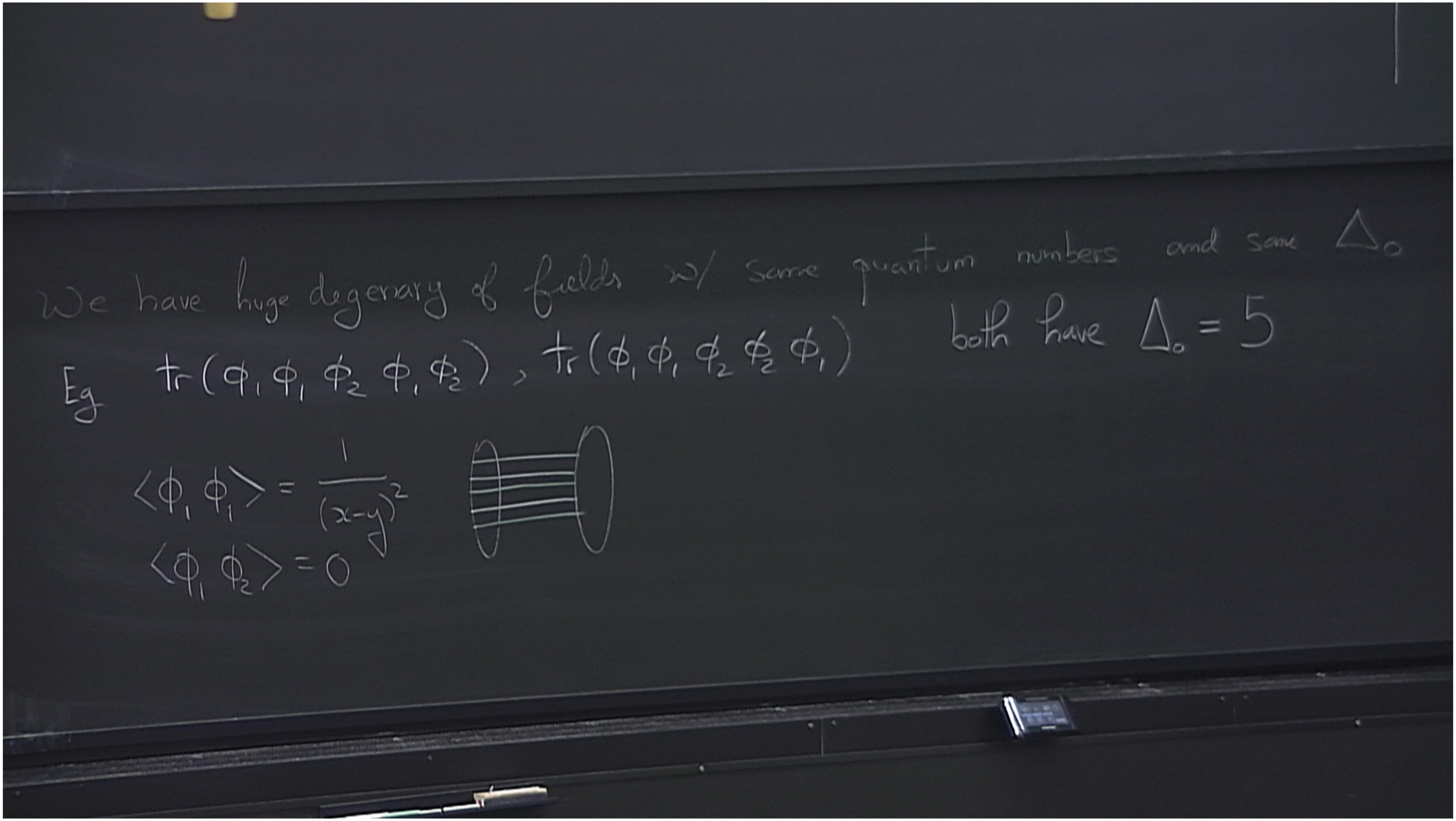
We have huge degeneracy of fields w/ same quantum numbers and same Δ_0

Eg $\mathrm{tr}(\phi_1 \phi_1 \phi_2 \phi_1 \phi_2)$, $\mathrm{tr}(\phi_1 \phi_1 \phi_2 \phi_2 \phi_1)$ both have $\Delta_0 = 5$

$$\langle \phi_1 \phi_1 \rangle = \frac{1}{(x-y)^2}$$

$$\langle \phi_1 \phi_2 \rangle = 0$$

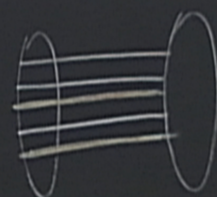



We have huge degeneracy of fields w/ same quantum numbers and same Δ0
Eg tr(φ1 φ1 φ2 φ1 φ2), tr(φ1 φ1 φ2 φ2 φ1) both have Δ0 = 5
⟨φ1 φ1⟩ = 1/(x-y)^2
⟨φ1 φ2⟩ = 0

We have huge degeneracy of fields w/ same quantum numbers and same Δ_0

Eg $\mathrm{tr}(\phi_1 \phi_1 \phi_2 \phi_1 \phi_2)$, $\mathrm{tr}(\phi_1 \phi_1 \phi_2 \phi_2 \phi_1)$ both have $\Delta_0 = 5$

$$\langle \phi_1 \phi_1 \rangle = \frac{1}{(x-y)^2}$$



$$\sim \left(\frac{1}{(x-y)^2} \right)^5, \quad \Delta_0 = 5$$

$$\langle \phi_1 \phi_2 \rangle = 0$$

$$\langle O_A^{bare} O_B^{bare} \rangle = \frac{1}{(x-y)^{2\Delta_0}} \Big(\delta_{AB} - \lambda \gamma_{AB} \log \Lambda^2 (x-y)^2$$

$$\sum \psi^A O_A^{bare} \equiv O_A^{diag,\,bare}$$

↑ wavefunction

diagonalize $\hat{\gamma}_{AB}$

with eigenvalue γ_A

$$\hat{\gamma}\, O^{diag.\,bare} = \gamma_A\, O^{diag.\,bare}$$

$$\langle \mathcal{O}_A^{bare} \mathcal{O}_B^{bare} \rangle = \frac{1}{(x-y)^{2\Delta_0}} \left(\delta_{AB} - \lambda \gamma_{AB} \log \Lambda^2 (x-y)^2 \right.$$

$$\sum \psi^A \mathcal{O}_A^{bare} \equiv \mathcal{O}_A^{diag,\,bare}$$

↑ wavefunction

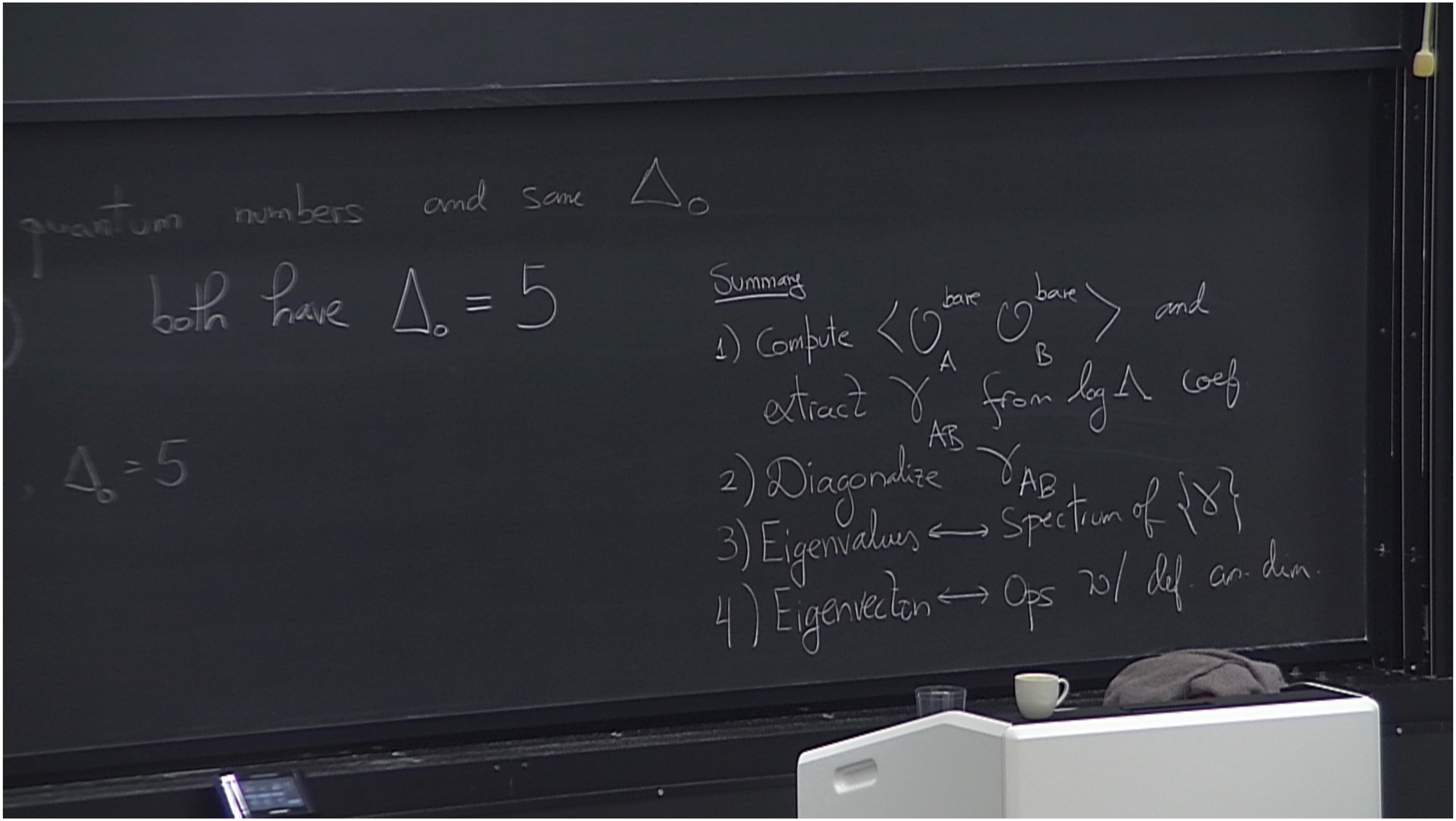
diagonalize $\hat{\gamma}_{AB}$

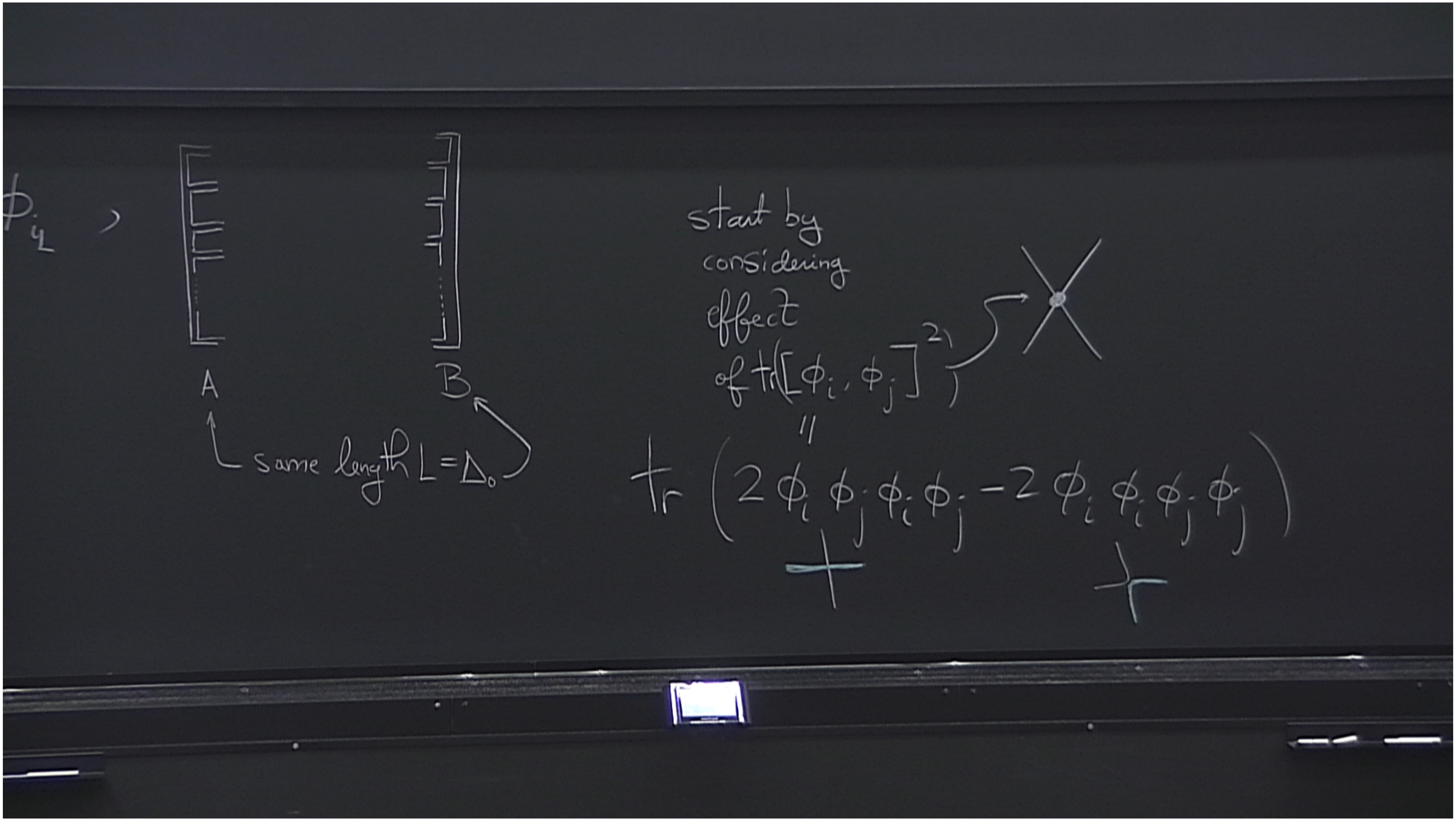
with eigenvalue γ_A

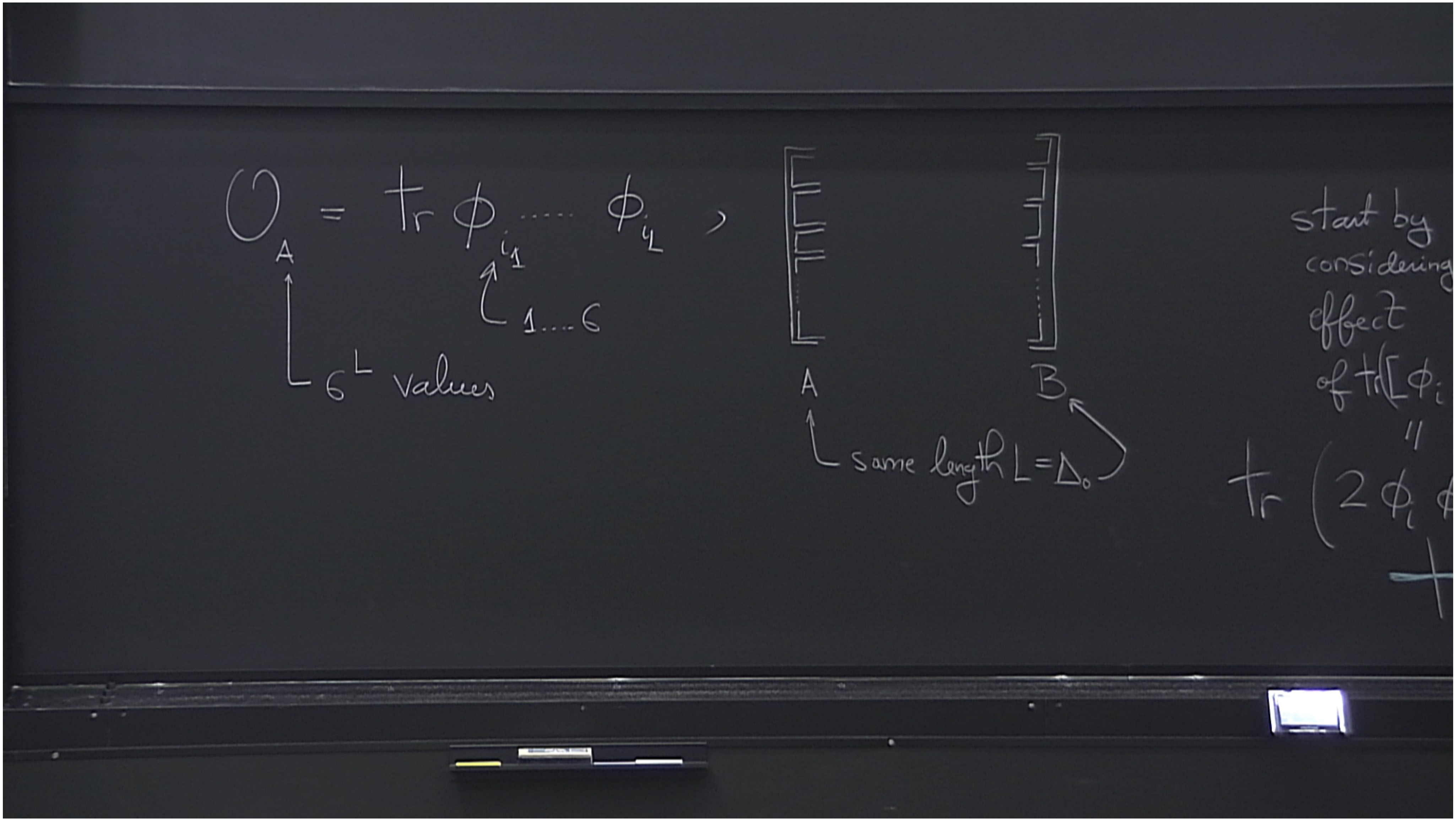
$$\hat{\gamma}\, \mathcal{O}^{diag.\,bare} = \gamma_A\, \mathcal{O}^{diag.\,bare}$$

$y)^2 \Big)$

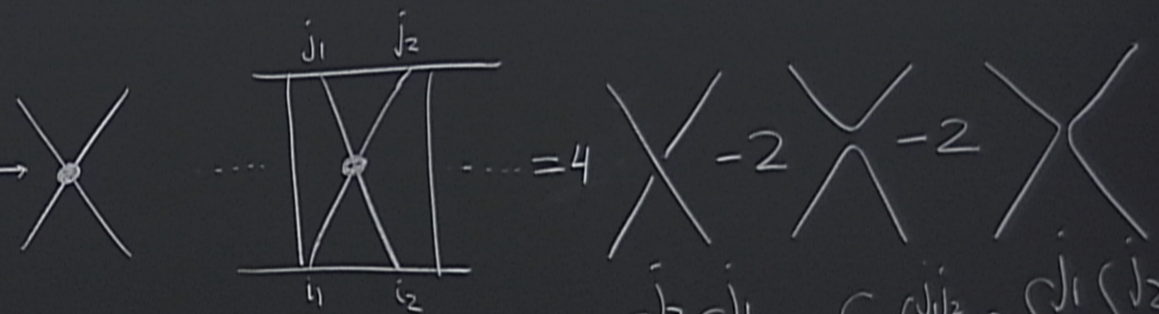
$$\langle \mathcal{O}_A^{\text{diag. bare}}(x)\, \mathcal{O}_B^{\cdots}(y) \rangle = \frac{\delta_{AB}}{(x-y)^{2\Delta_0}} \left(1 - \lambda \gamma_A \log \Lambda^2 (x-y)^2 \right)$$


quantum numbers and same Δ_0
both have $\Delta_0 = 5$
$\Delta_0 = 5$
Summary
1) Compute $\langle \mathcal{O}_A^{bare} \mathcal{O}_B^{bare} \rangle$ and
extract γ_{AB} from $\log \Lambda$ coef
2) Diagonalize γ_{AB}
3) Eigenvalues $\longleftrightarrow$ Spectrum of $\{\gamma\}$
4) Eigenvectors $\longleftrightarrow$ Ops w/ def. an. dim.


A
B
same length $L = \Delta_0$
start by considering effect of $\mathrm{tr}([\phi_i, \phi_j]^2)$
$=$
$\mathrm{tr}\left(2\phi_i\phi_j\phi_i\phi_j - 2\phi_i\phi_i\phi_j\phi_j\right)$

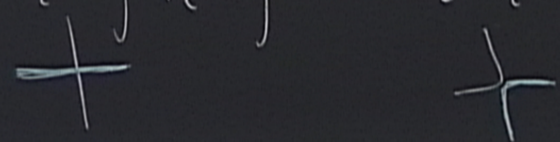

$\mathcal{O}_A = \mathrm{tr}\,\phi_{i_1} \cdots\cdots \phi_{i_L}$,
1....6
6^L values
A
B
same length $L = \Delta_0$
start by
considering
effect
of tr[φ_i
"
tr (2 φ_i

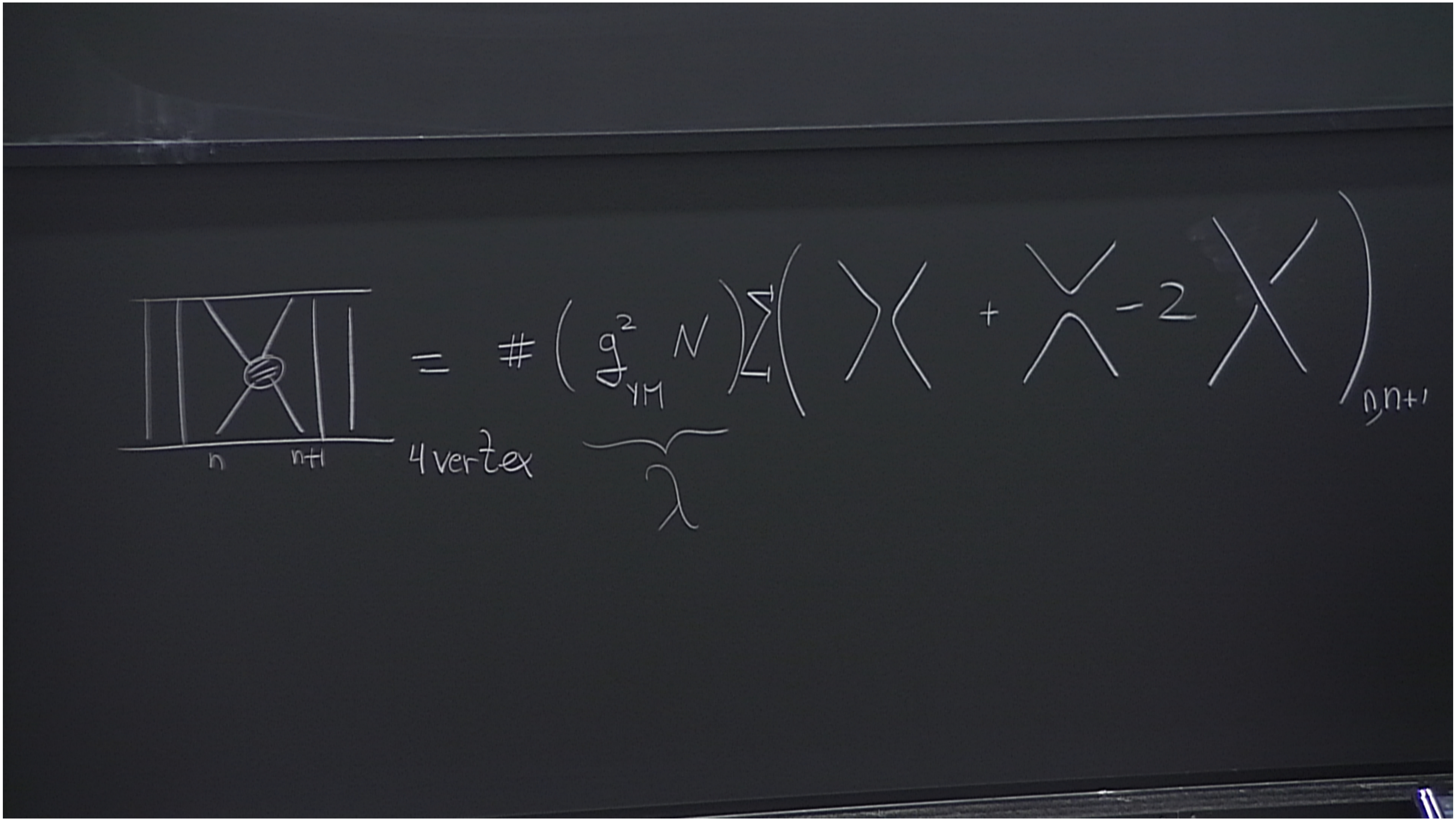
start by considering effect of $\mathrm{tr}([\phi_i, \phi_j]^2)$

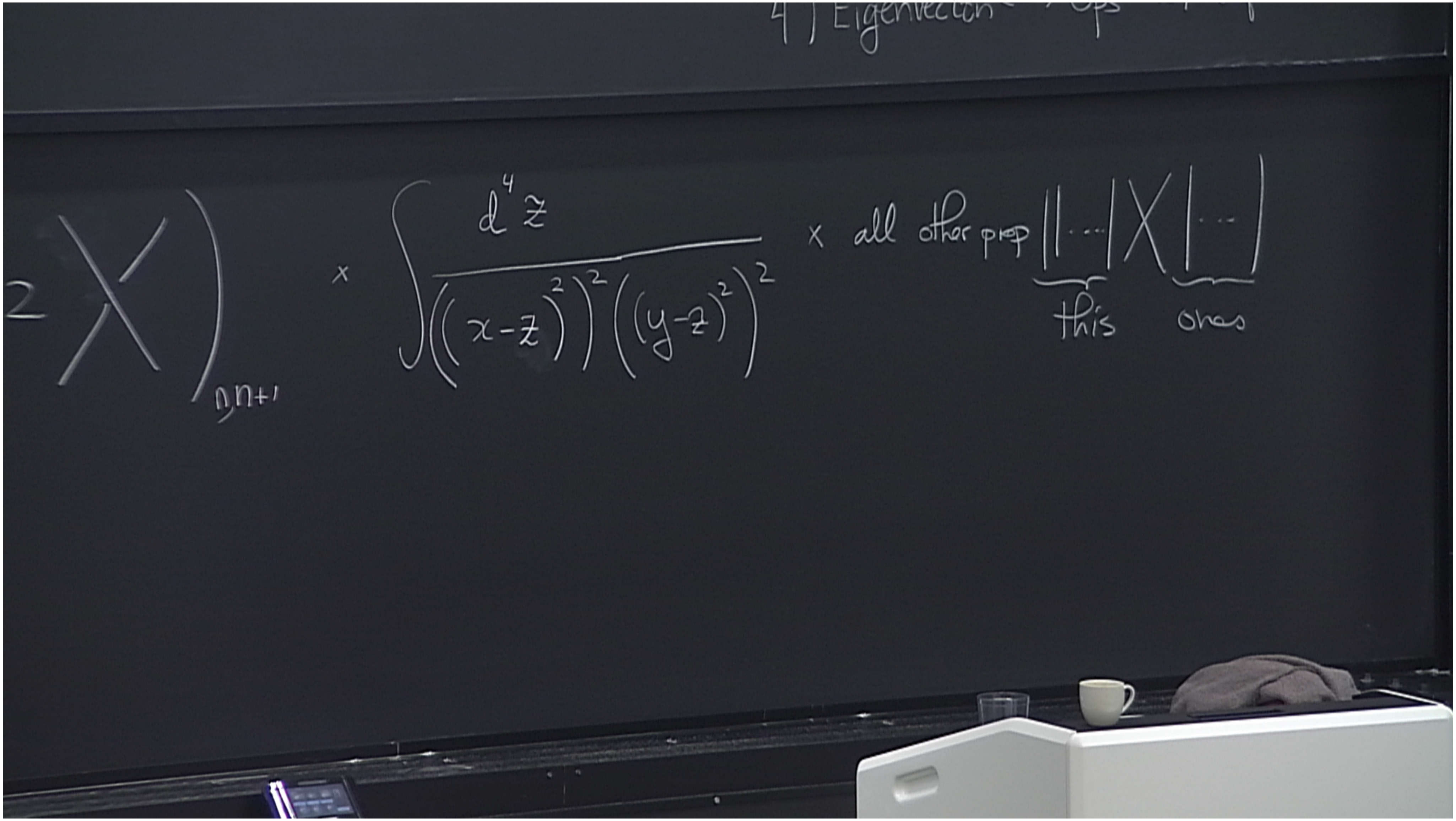


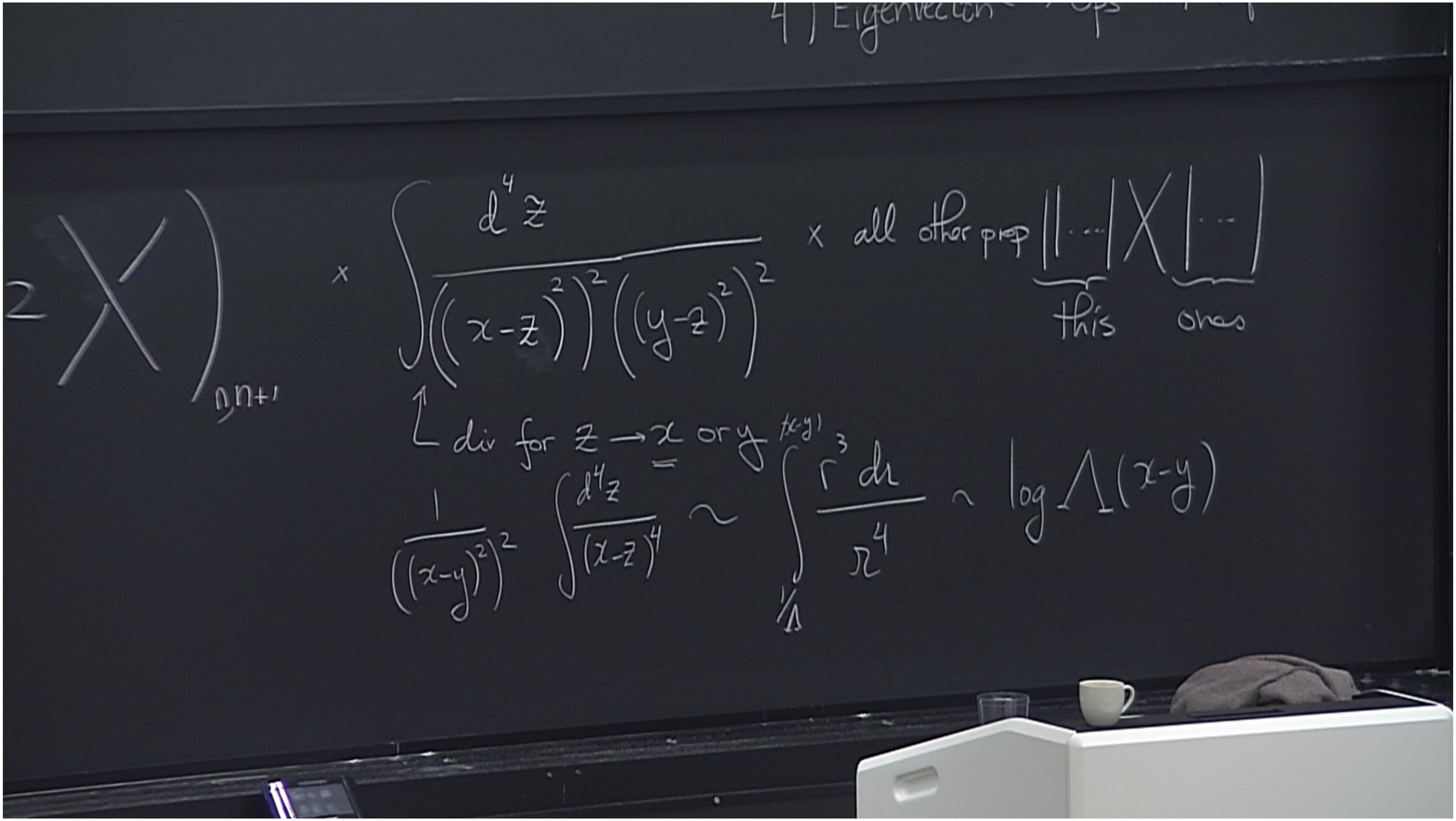
$$= 4\,\delta_{i_1}^{j_2}\delta_{i_2}^{j_1} - 2\,\delta_{i_1 i_2}\delta^{j_1 j_2} - 2\,\delta_{i_1}^{j_1}\delta_{i_2}^{j_2}$$

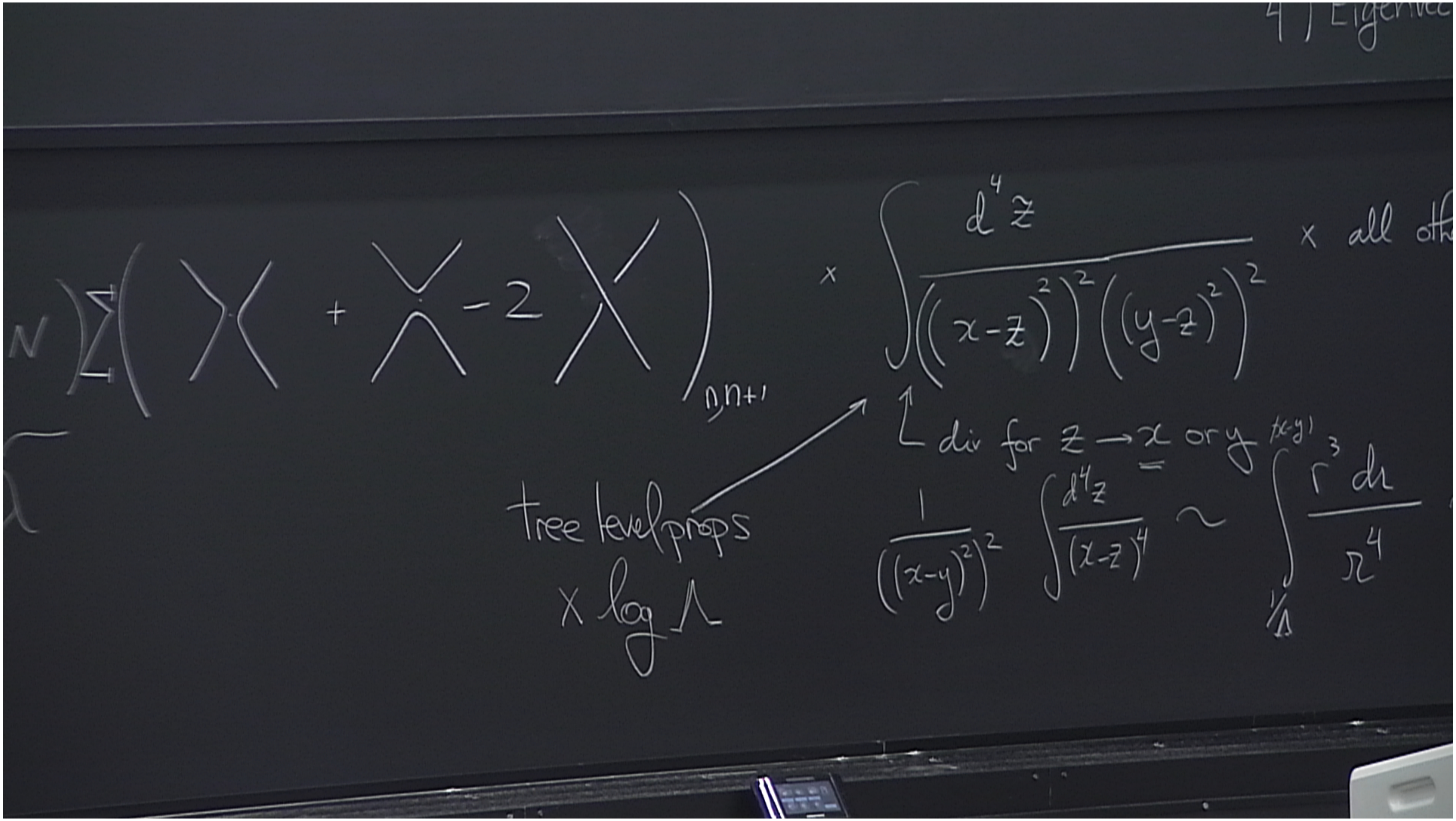
$$\mathrm{tr}([\phi_i, \phi_j]^2) = \mathrm{tr}\left(2\phi_i\phi_j\phi_i\phi_j - 2\phi_i\phi_i\phi_j\phi_j\right)$$

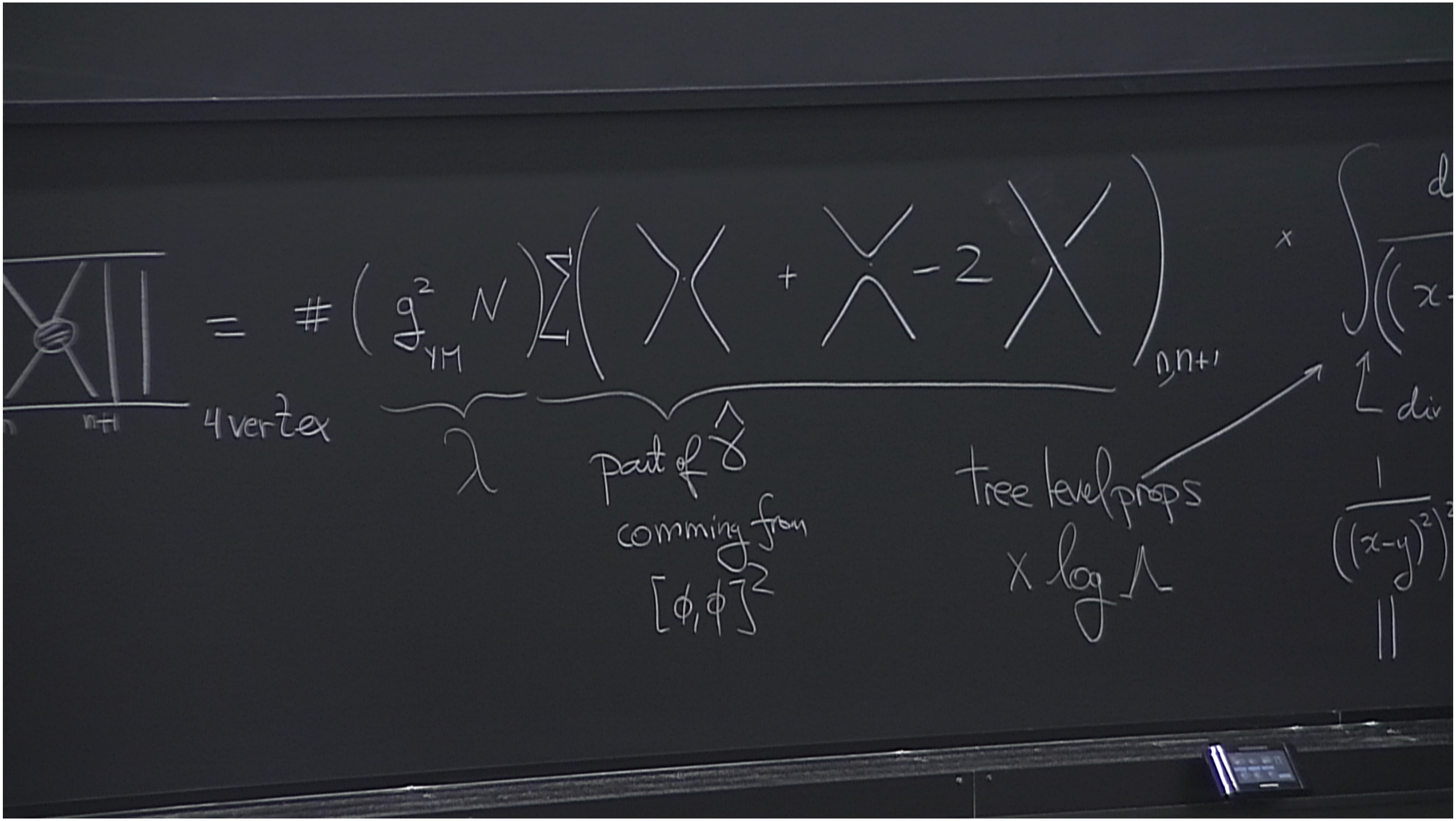


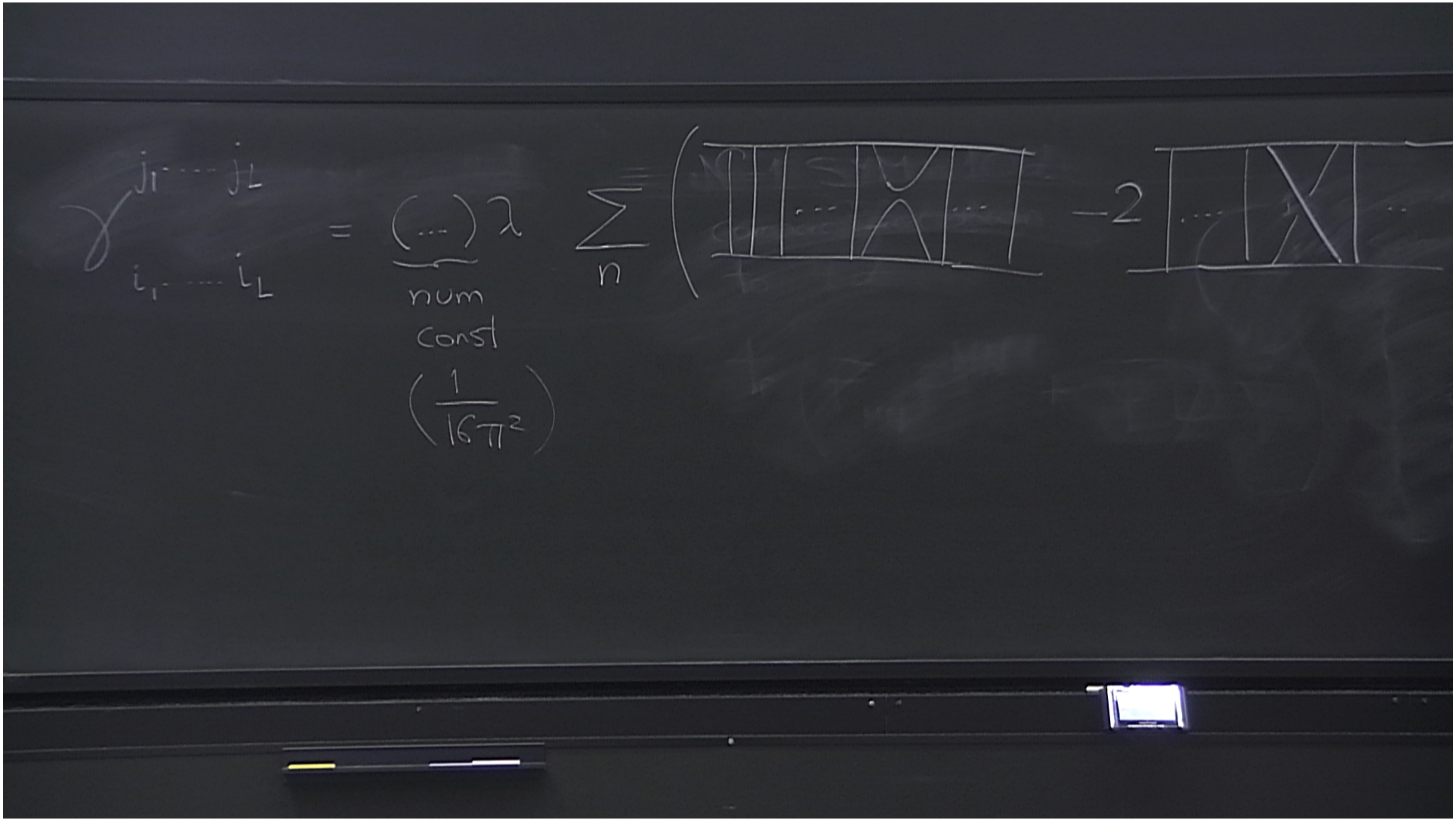

n
n+1
4 vertex
= # (g²YM N) Σ (+ − 2)n,n+1
λ

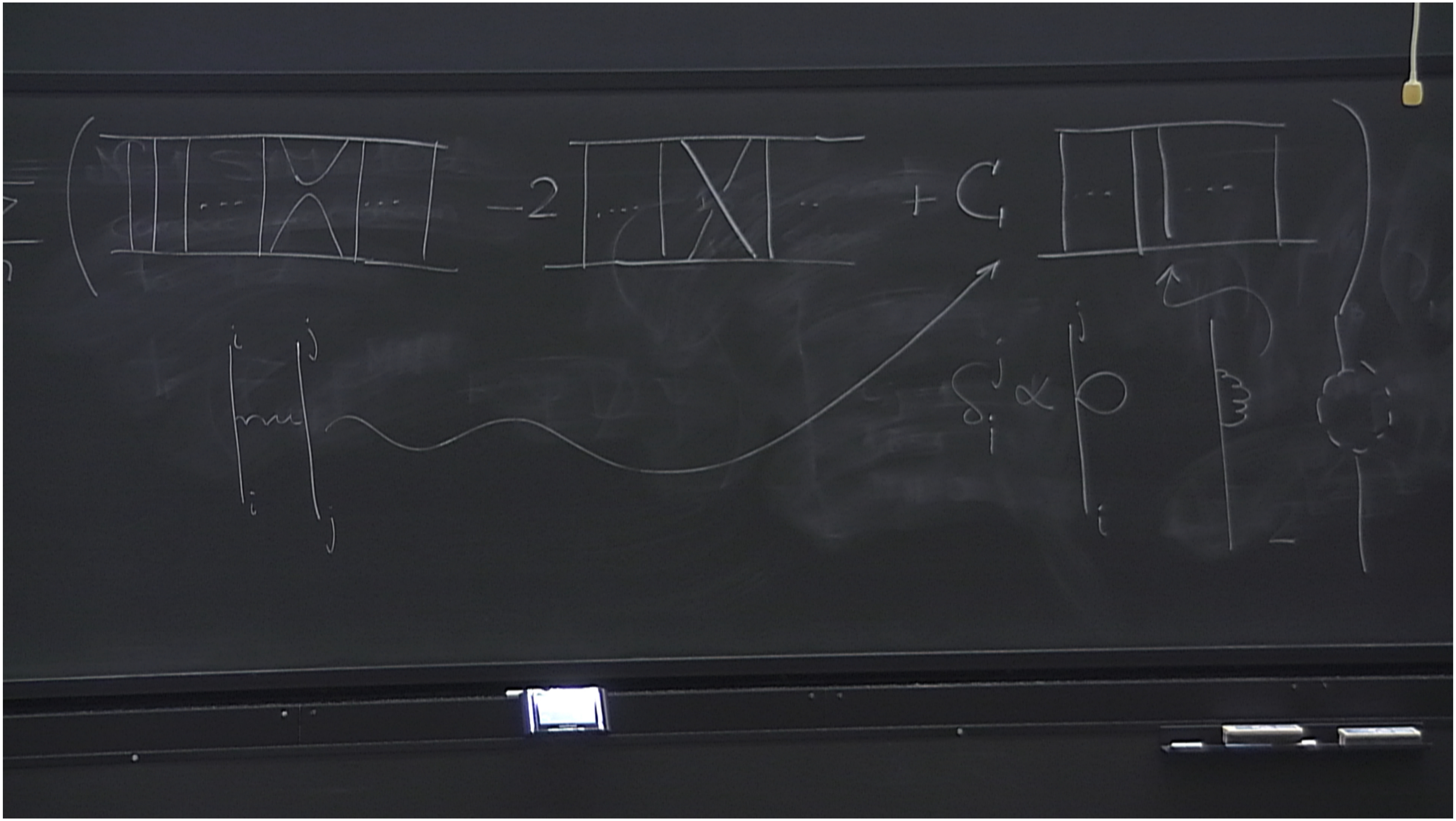

4) Eigenvector
× ∫ d⁴z / ((x-z)²)² ((y-z)²)²
× all other prop
this
ones
n,n+1

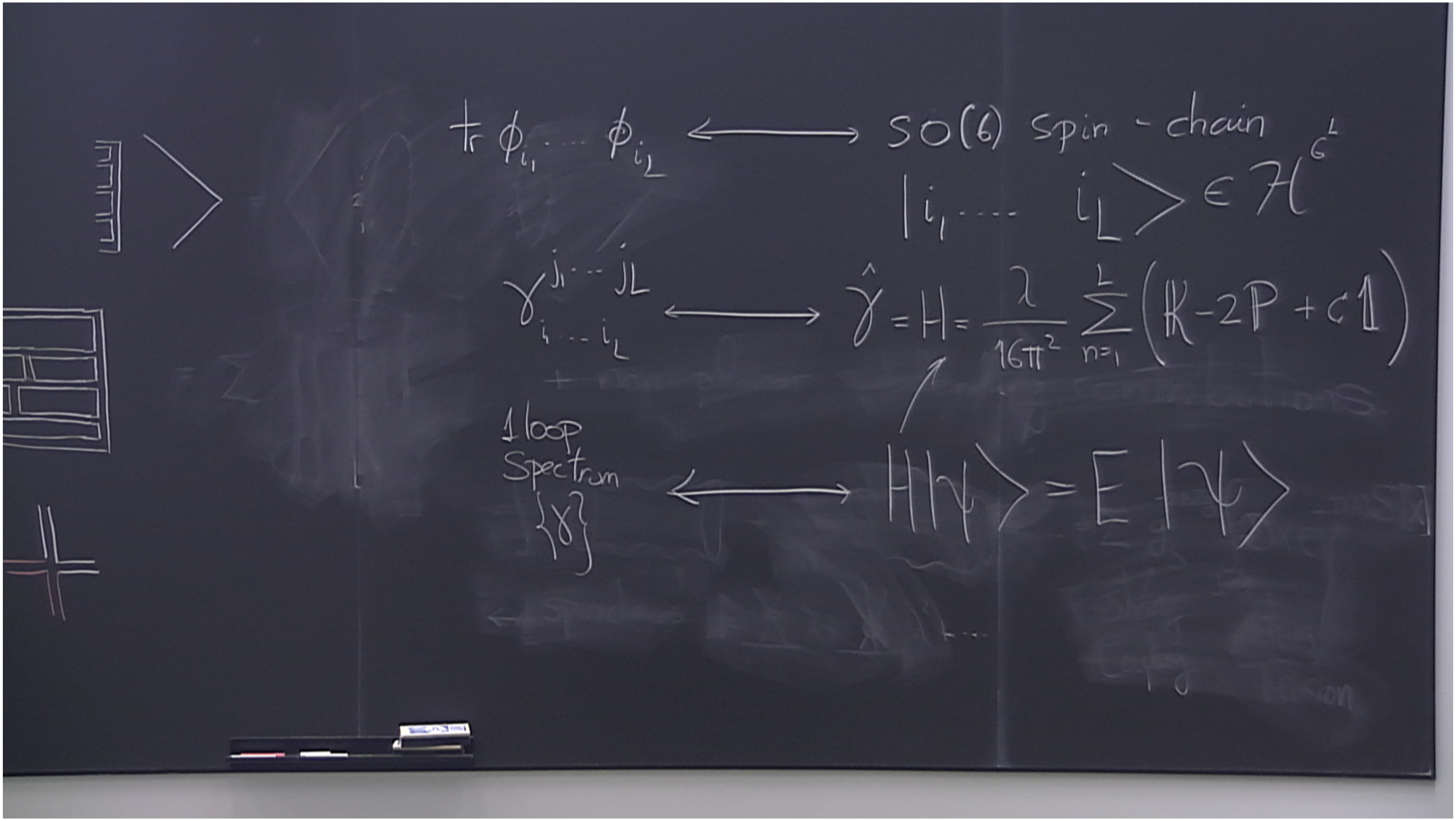

4) Eigenvector
$\times \int \frac{d^4 z}{((x-z)^2)^2 ((y-z)^2)^2}$ × all other prop
this
ones
$n, n+1$
div for $z \to x$ or y
$\frac{1}{((x-y)^2)^2} \int \frac{d^4 z}{(x-z)^4} \sim \int_{1/\Lambda}^{(x-y)} \frac{r^3 dr}{r^4} \sim \log \Lambda (x-y)$

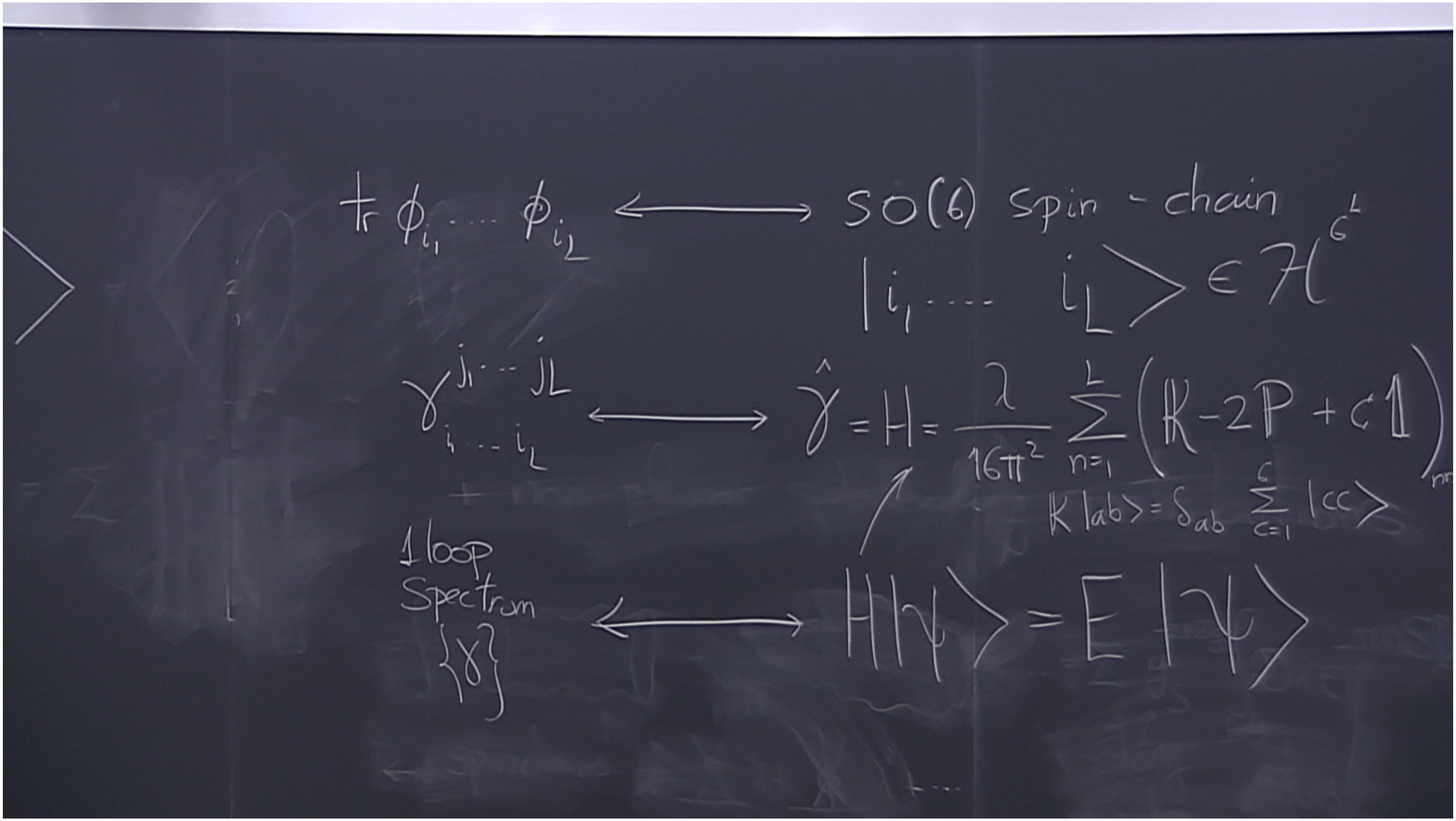

4) Eigenva
× all oth
n,n+1
div for z → x or y
tree level props
× log Λ

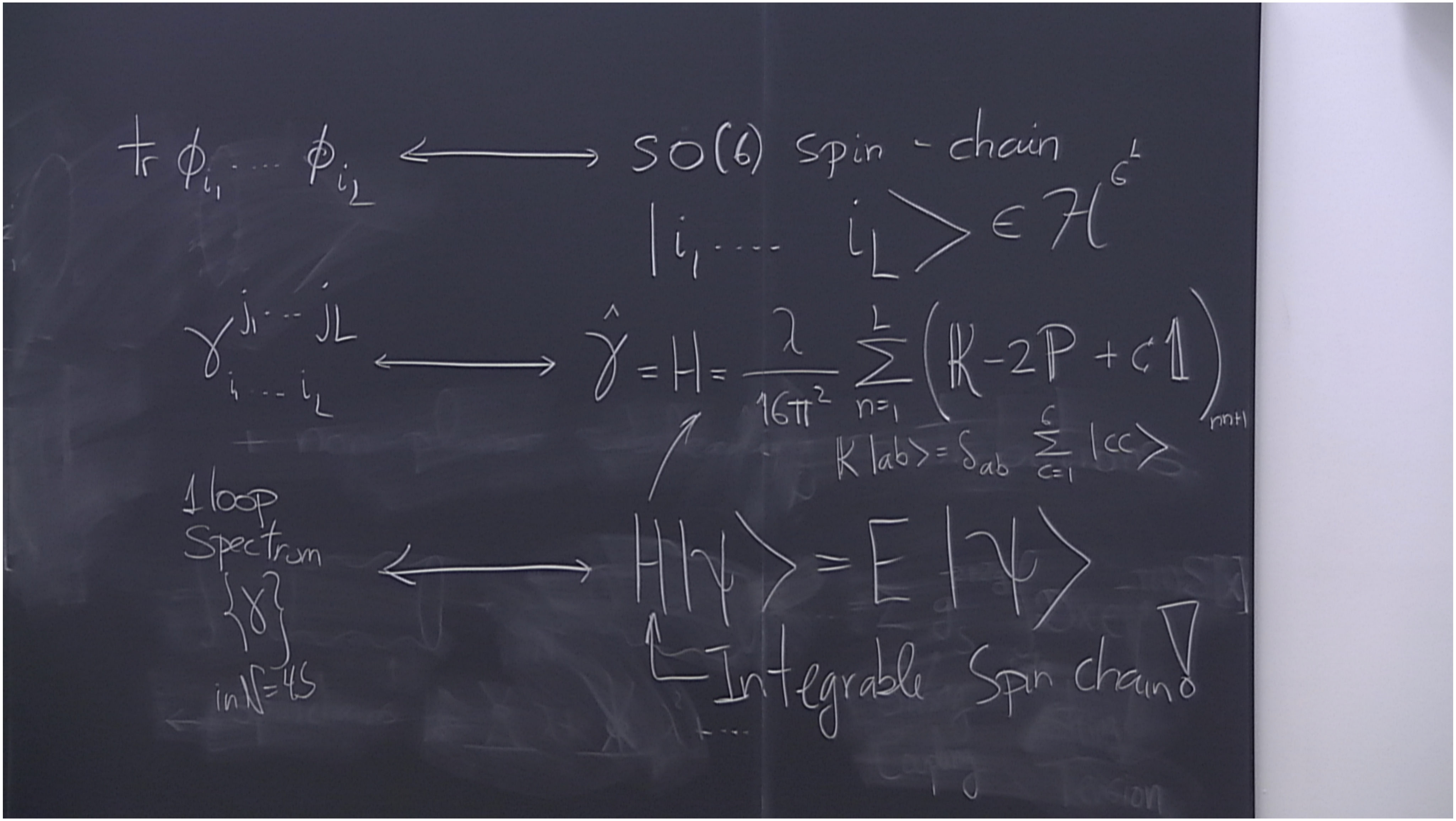

= # (g^2_{YM} N) Σ (+ − 2)
n,n+1
4 vertex
λ
part of $\hat{\gamma}$
comming from
$[\phi,\phi]^2$
tree level props
× log Λ
div

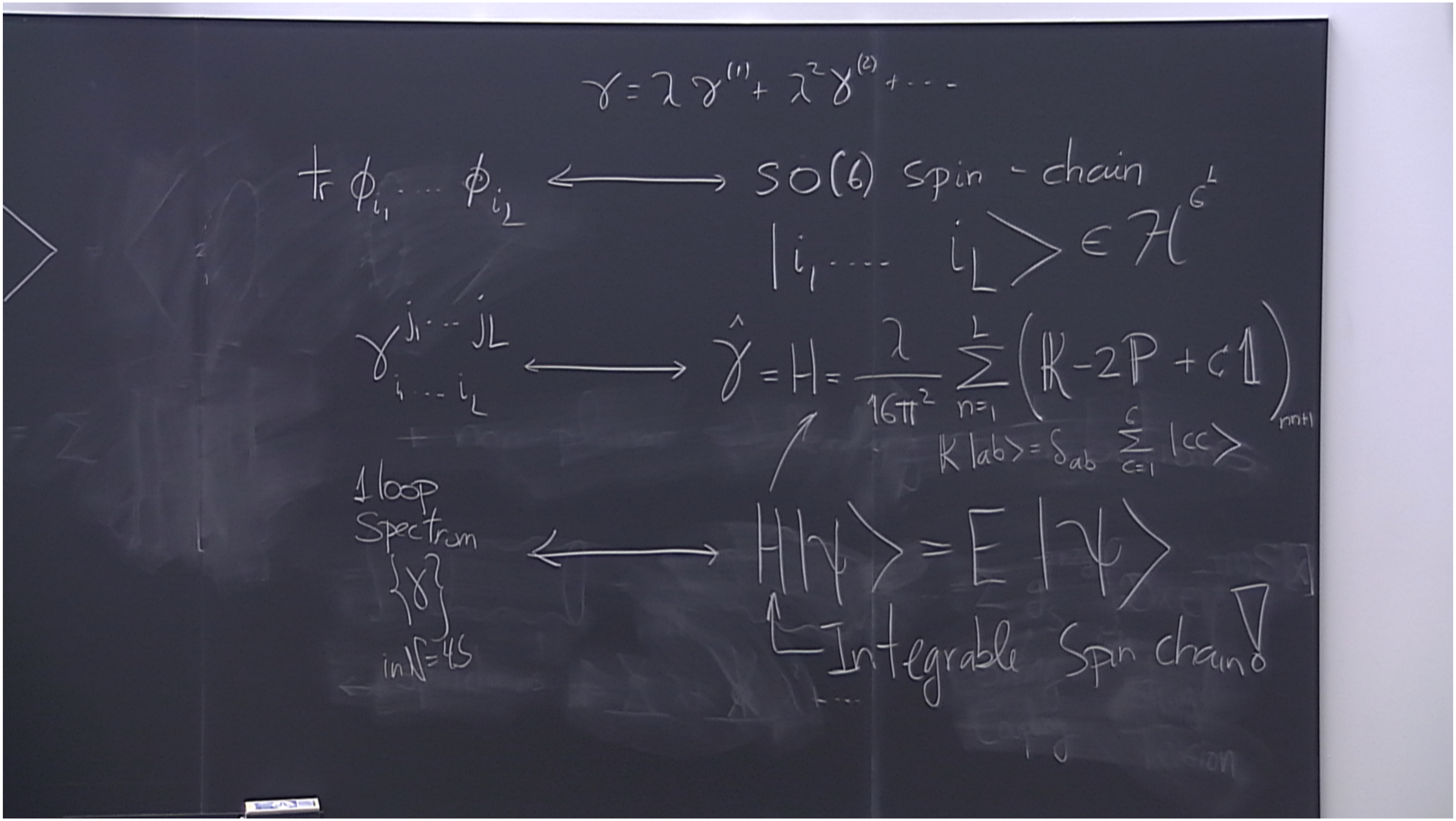

$\gamma^{j_1 \cdots j_L}_{i_1 \cdots i_L} = (\ldots)\lambda \sum_n ($
num
const
$\left(\frac{1}{16\pi^2}\right)$
-2




$\mathrm{tr}\,\phi_{i_1}\cdots\phi_{i_L} \longleftrightarrow$ SO(6) spin - chain
$|i_1 \cdots i_L\rangle \in \mathcal{H}^{6^L}$
$\gamma^{j_1\cdots j_L}_{i_1\cdots i_L} \longleftrightarrow \hat{\gamma} = H = \frac{\lambda}{16\pi^2}\sum_{n=1}^{L}\left(\mathbb{K} - 2\mathbb{P} + c\mathbb{1}\right)$
1 loop Spectrum $\{\gamma\}$ $\longleftrightarrow H|\psi\rangle = E|\psi\rangle$


tr φ_{i_1} .... φ_{i_L} ⟷ SO(6) spin-chain
|i_1 .... i_L⟩ ∈ H^{6^L}
γ^{j_1 ... j_L}_{i_1 ... i_L} ⟷ γ̂ = H = λ/(16π²) Σ_{n=1}^{L} (K − 2P + c 1)_{nn}
K|ab⟩ = δ_ab Σ_{c=1}^{6} |cc⟩
1 loop Spectrum {γ} ⟷ H|ψ⟩ = E|ψ⟩


$\mathrm{tr}\, \phi_{i_1} \cdots \phi_{i_L} \longleftrightarrow$ SO(6) spin - chain
$|i_1 \cdots i_L\rangle \in \mathcal{H}_6^L$
$\gamma^{j_1 \cdots j_L}_{i_1 \cdots i_L} \longleftrightarrow \hat{\gamma} = H = \frac{\lambda}{16\pi^2} \sum_{n=1}^{L} \left(\mathbb{K} - 2\mathbb{P} + c\,\mathbb{1} \right)_{n\,n+1}$
$\mathbb{K}|ab\rangle = \delta_{ab} \sum_{c=1}^{6} |cc\rangle$
1 loop
Spectrum
$\{\gamma\}$
in $\mathcal{N}=4$ S
$\longleftrightarrow H|\psi\rangle = E|\psi\rangle$
Integrable Spin chain!


$\gamma = \lambda \gamma^{(1)} + \lambda^2 \gamma^{(2)} + \cdots$
$\mathrm{tr}\, \phi_{i_1} \cdots \phi_{i_L} \longleftrightarrow$ SO(6) spin-chain
$|i_1 \cdots i_L\rangle \in \mathcal{H}_6^L$
$\gamma^{j_1 \cdots j_L}_{i_1 \cdots i_L} \longleftrightarrow \hat{\gamma} = H = \frac{\lambda}{16\pi^2} \sum_{n=1}^{L} \left(\mathbb{K} - 2\mathbb{P} + c\mathbb{1} \right)_{n n+1}$
$\mathbb{K}|ab\rangle = \delta_{ab} \sum_{c=1}^{6} |cc\rangle$
1 loop
Spectrum
$\{\gamma\}$
in $\mathcal{N}=4$ S
$\longleftrightarrow H|\psi\rangle = E|\psi\rangle$
Integrable Spin chain!