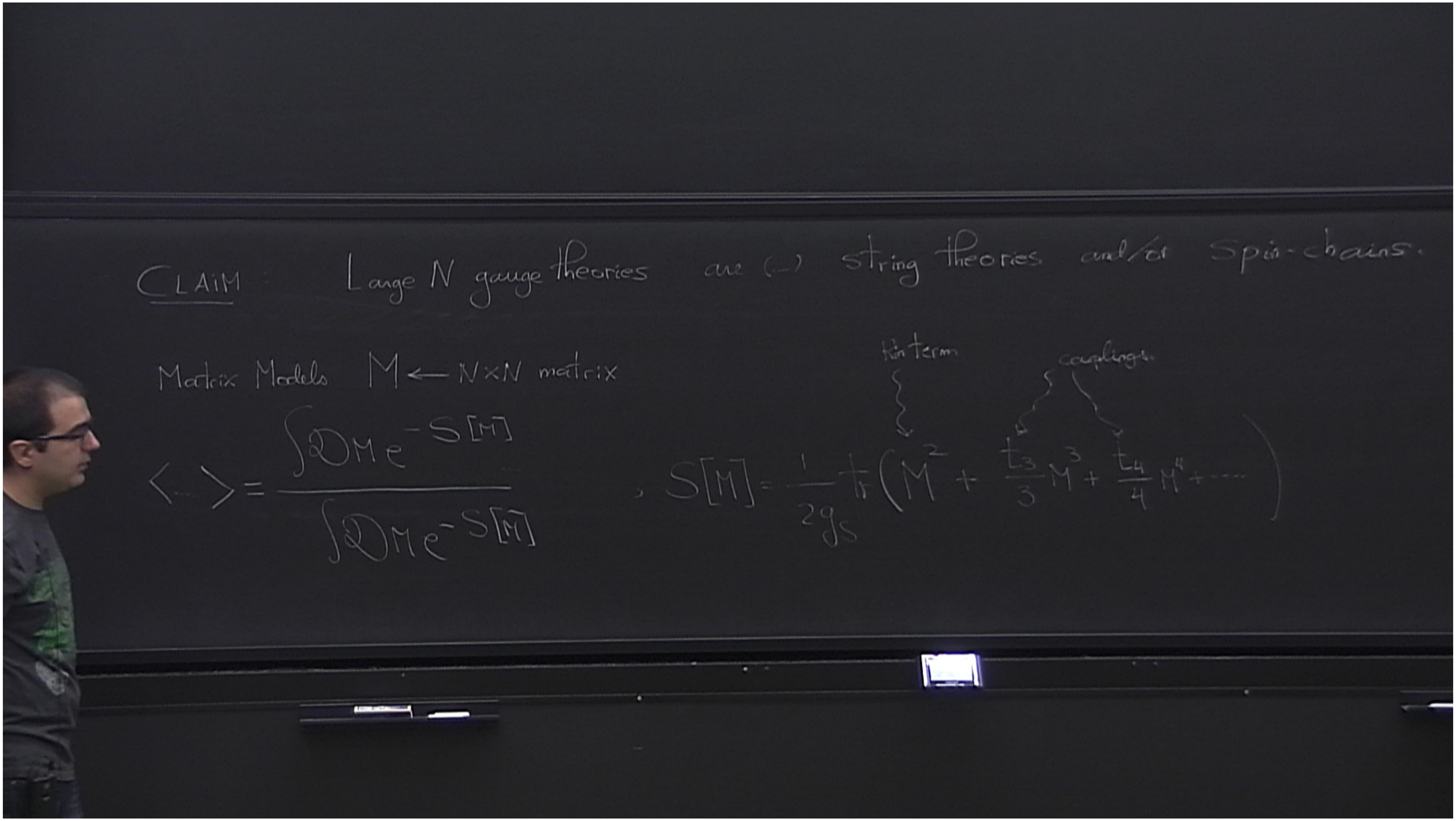
Title: Explorations in String Theory -9

Date: Apr 16, 2015 11:30 AM

URL: http://pirsa.org/15040148

Abstract:


CLAIM: Large N gauge theories are (...) string theories and/or Spin-chains.
Matrix Models $M \leftarrow N\times N$ matrix
$\langle \dots \rangle = \frac{\int \mathcal{D}M\, e^{-S[M]} \dots}{\int \mathcal{D}M\, e^{-S[M]}}$
kin term
couplings
$S[M] = \frac{1}{2g_s} \operatorname{tr}\left(M^2 + \frac{t_3}{3} M^3 + \frac{t_4}{4} M^4 + \dots\right)$

CLAIM: Large N gauge theories are (...) string theories and/or spin-chains.

Matrix Models $M \leftarrow N \times N$ matrix

kin term

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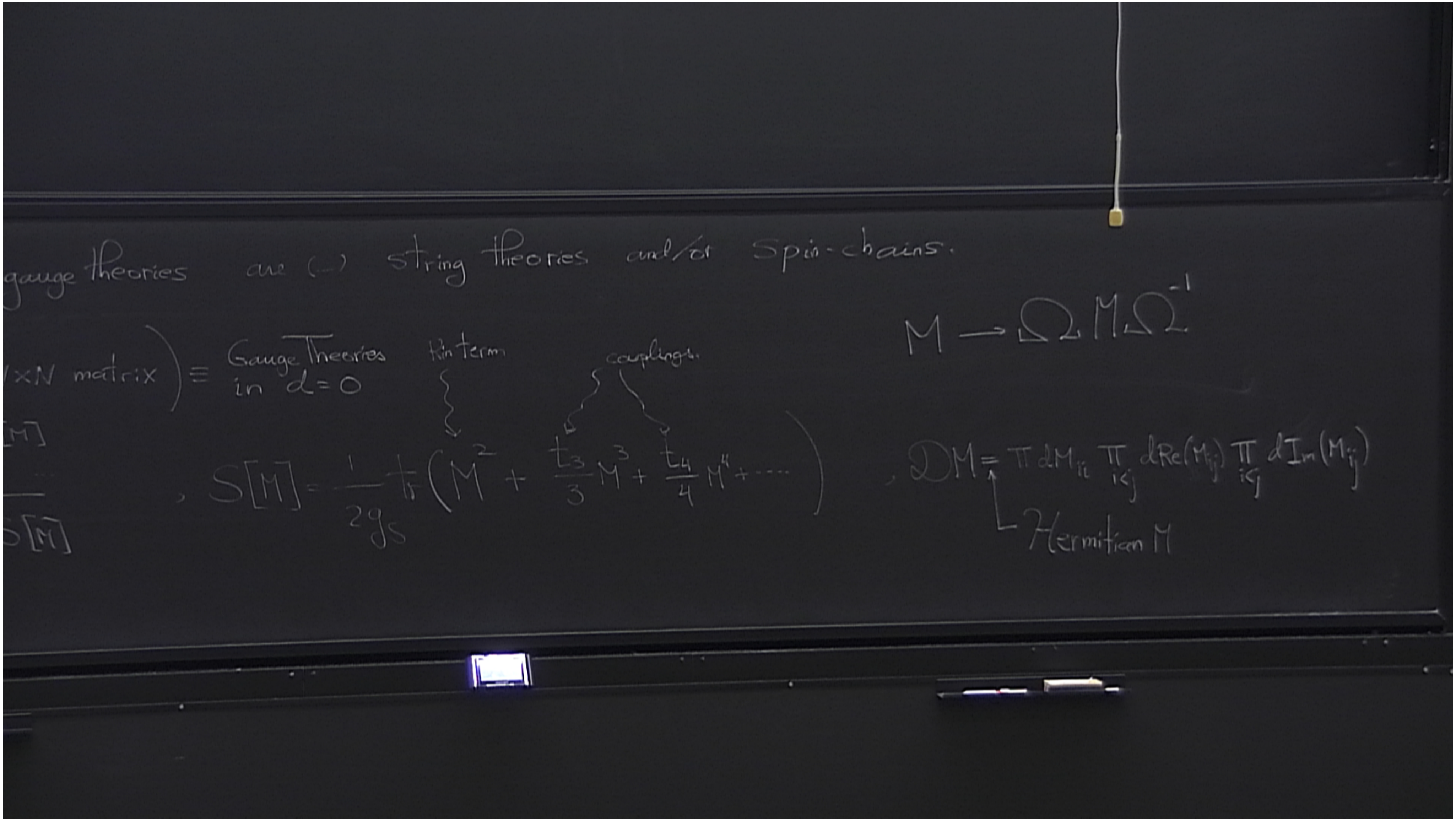
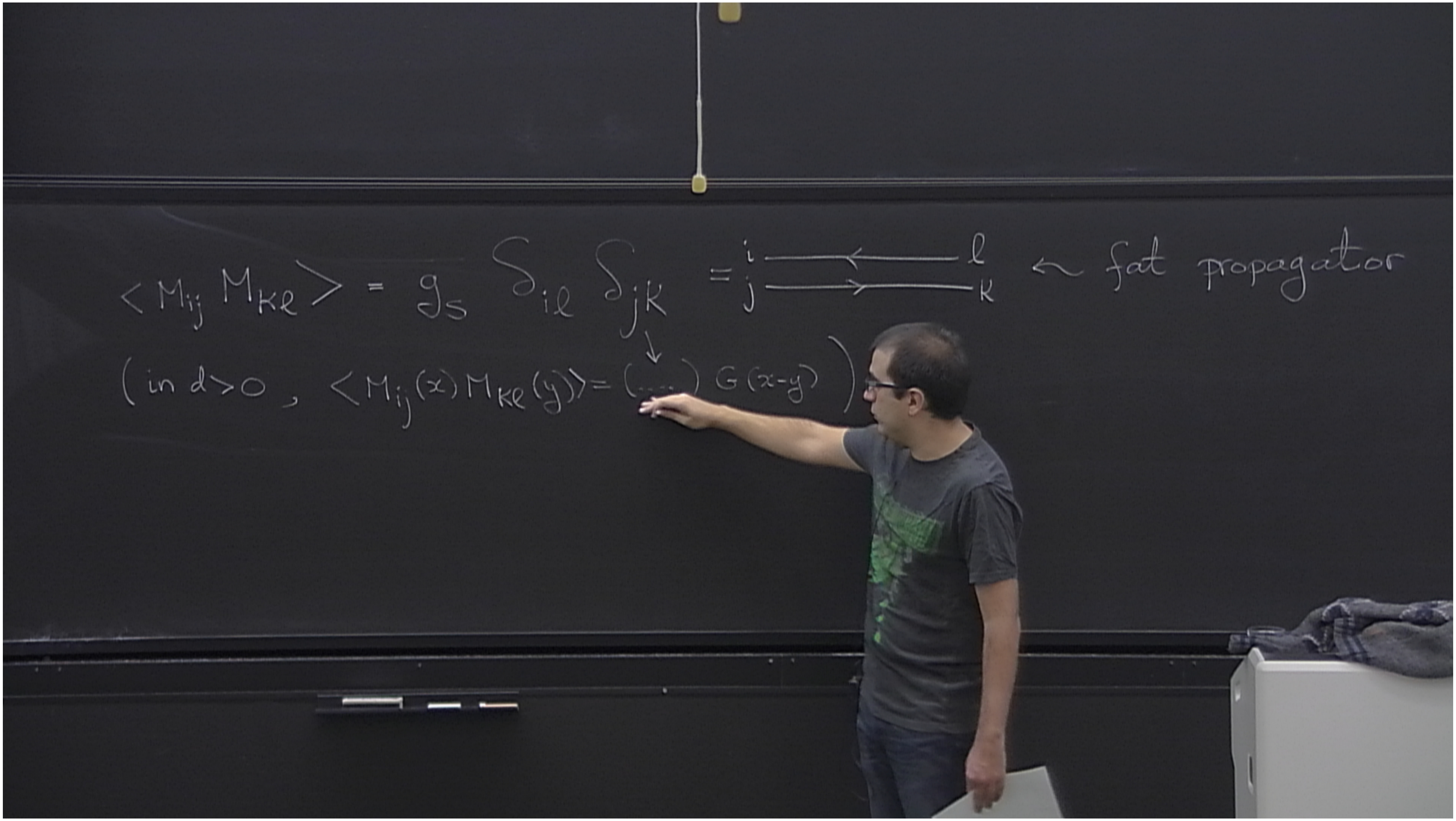
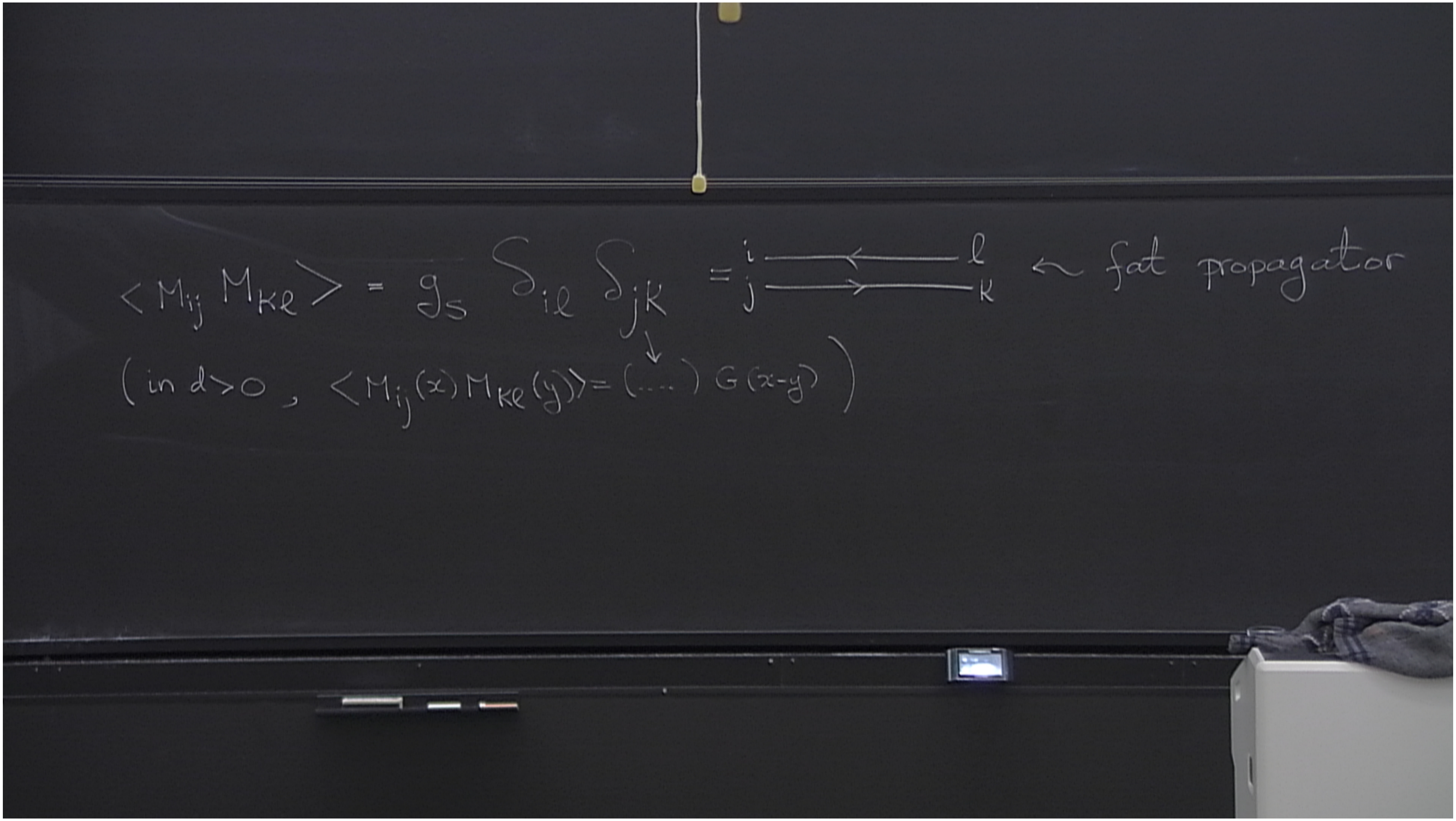
$$\langle \ldots \rangle = \frac{\int \mathcal{D}M\, e^{-S[M]}}{\int \mathcal{D}M\, e^{-S[M]}} \quad , \quad S[M] = \frac{1}{2g_s} \mathrm{tr}\left(M^2 + \frac{t_3}{3} M^3 + \frac{t_4}{4} M^4 + \cdots \right) \quad , \quad \mathcal{D}M =$$


gauge theories are (...) string theories and/or Spin-chains.
M → Ω M Ω⁻¹
×N matrix) ≡ Gauge Theories in d=0
kin term
couplings.
S[M] = 1/(2g_s) tr(M² + t₃/3 M³ + t₄/4 M⁴ + ...)
DM = Π dM_ii Π_{i<j} dRe(M_ij) Π_{i<j} dIm(M_ij)
Hermitian M


<M_ij M_kl> = g_s δ_il δ_jk = i ← l, j → k
← fat propagator
(in d>0 , <M_ij(x) M_kl(y)> = (....) G(x-y))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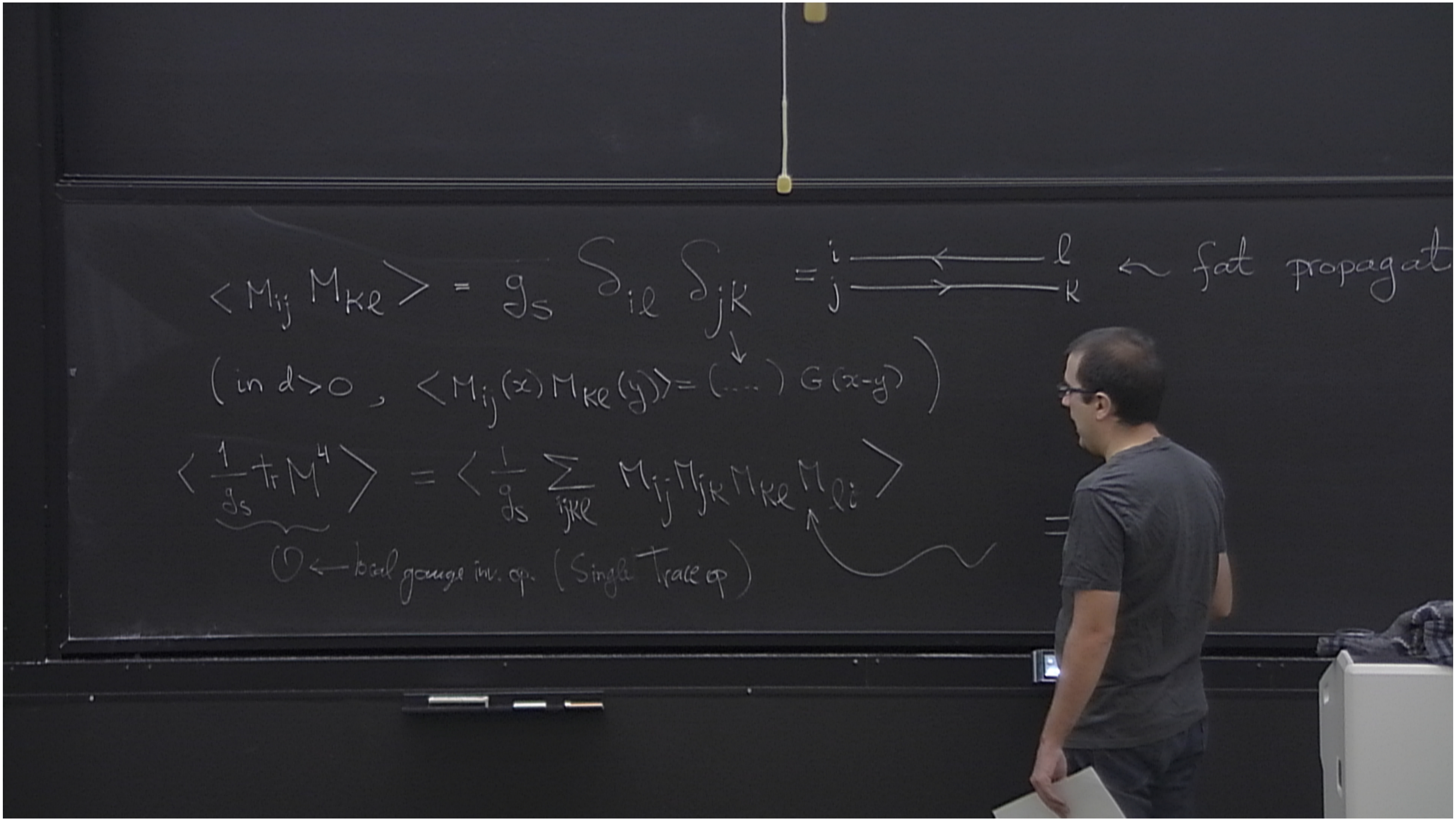
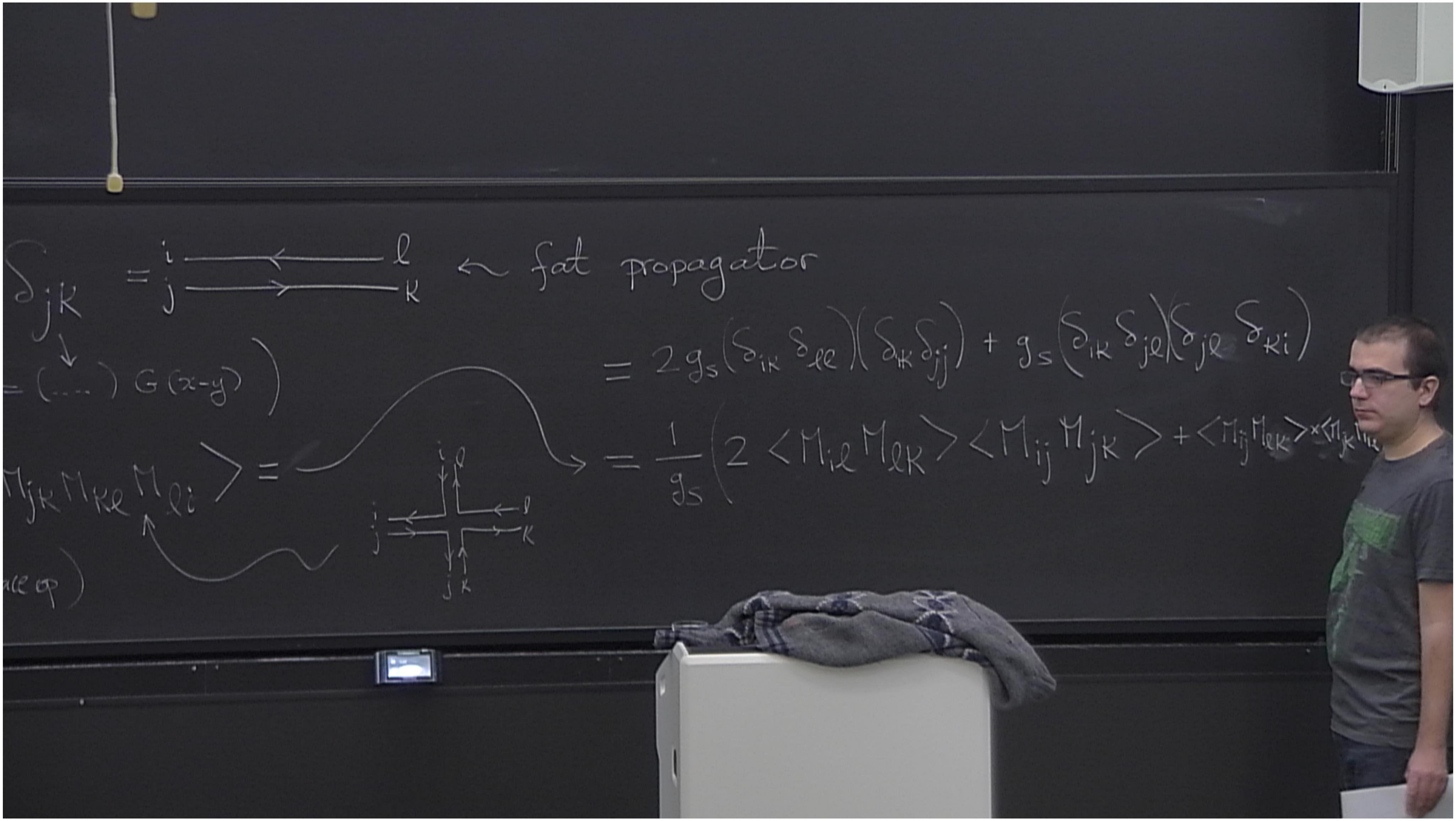
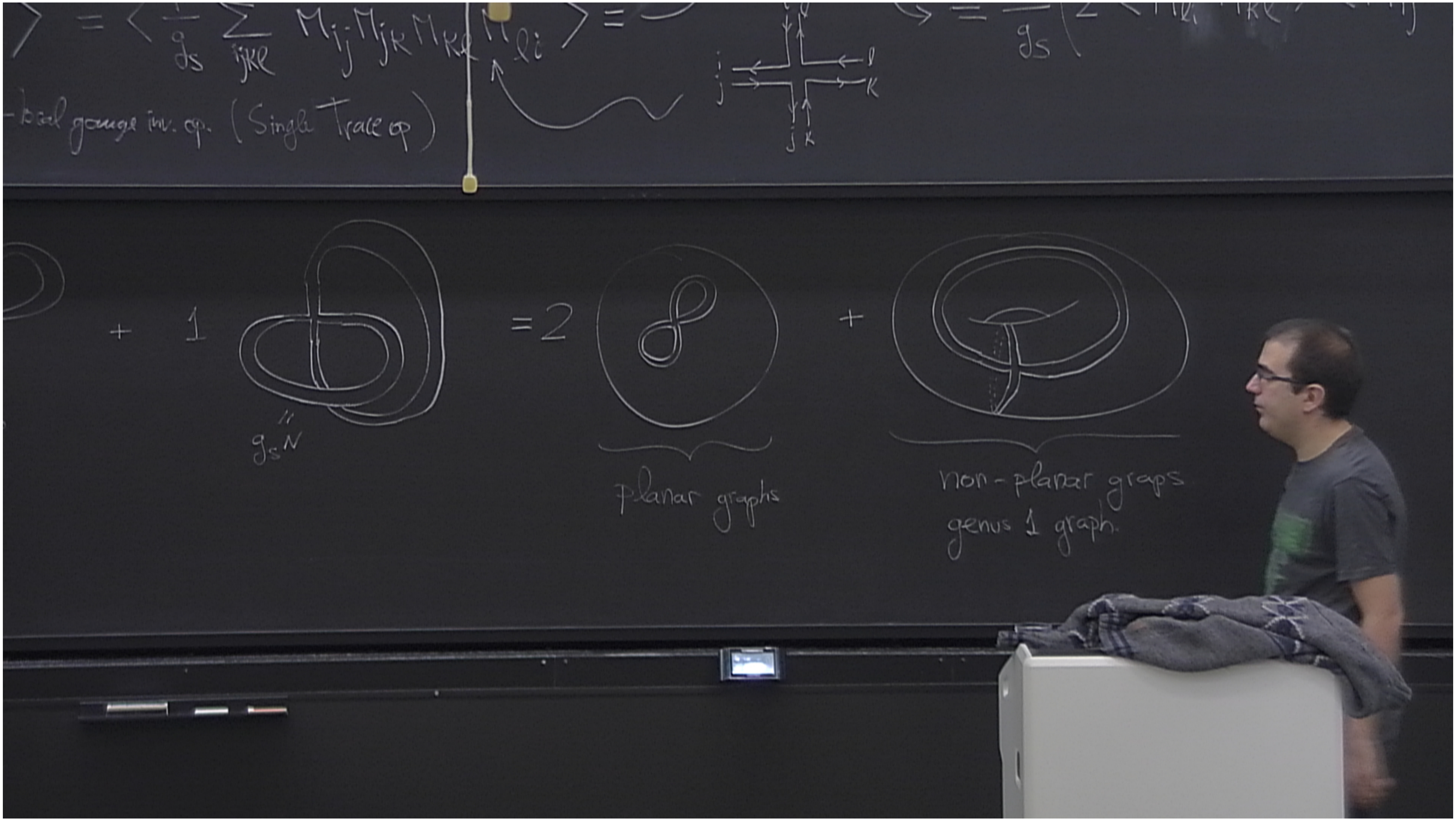
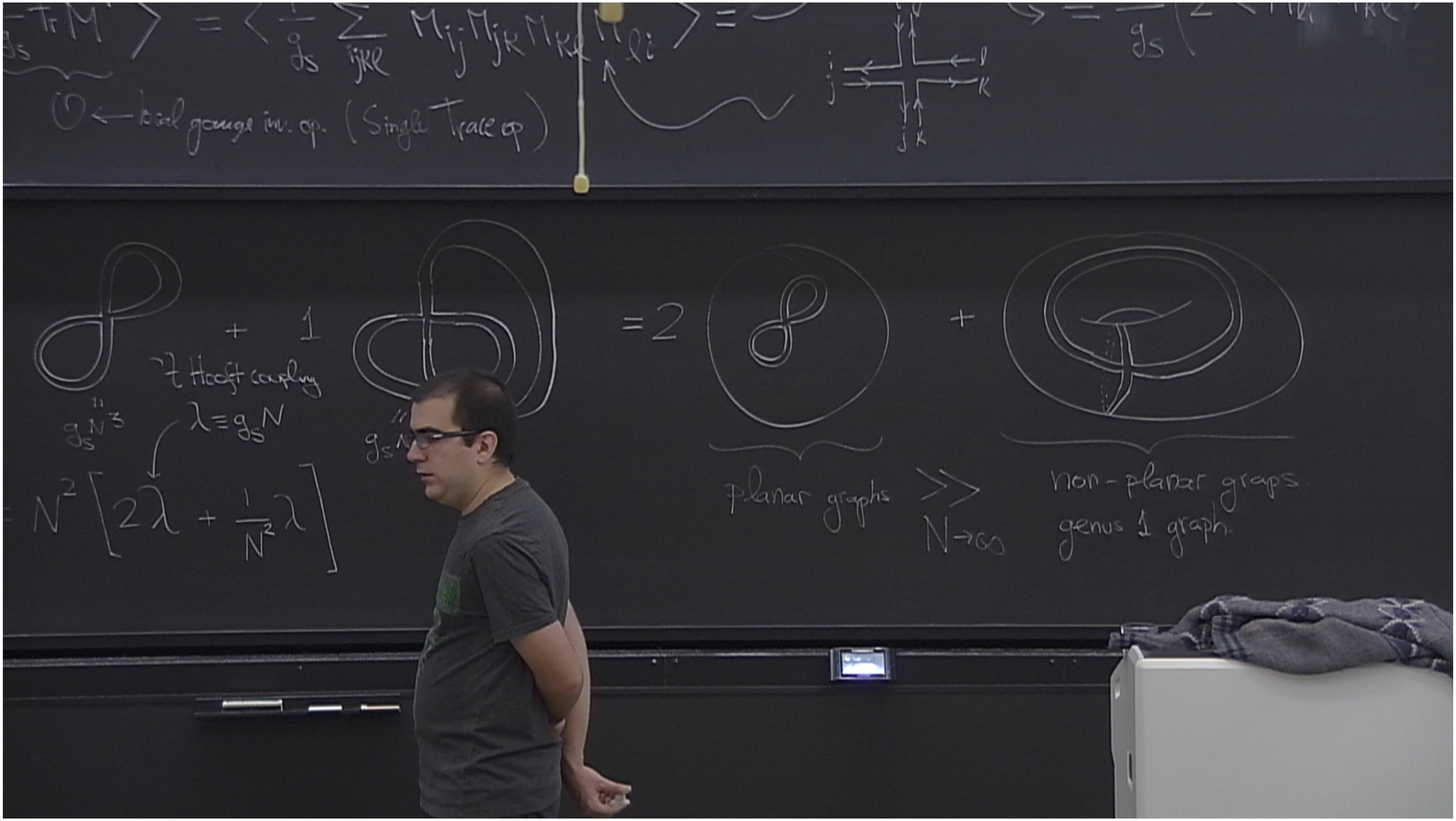

⟨M_ij M_kl⟩ = g_s δ_il δ_jk = i ← l, j → k ← fat propagat
(in d>0 , ⟨M_ij(x) M_kl(y)⟩ = (....) G(x-y))
⟨1/g_s tr M^4⟩ = ⟨1/g_s Σ_ijkl M_ij M_jk M_kl M_li⟩
O ← local gauge inv. op. (Single Trace op)


← fat propagator
= 2 g_s (δ_ik δ_ll)(δ_lk δ_jj) + g_s (δ_ik δ_jl)(δ_jl δ_ki)
= 1/g_s (2 <M_il M_lk><M_ij M_jk> + <M_ij M_lk>×<M_jk M
= (....) G(x-y)


local gauge inv. op. (Single Trace op)
planar graphs
non-planar graps
genus 1 graph.


local gauge inv. op. (Single Trace op)
't Hooft coupling
λ ≡ g_s N
g_s N^3
g_s N
= N^2 [2λ + 1/N λ]
= 2
planar graphs ≫
N→∞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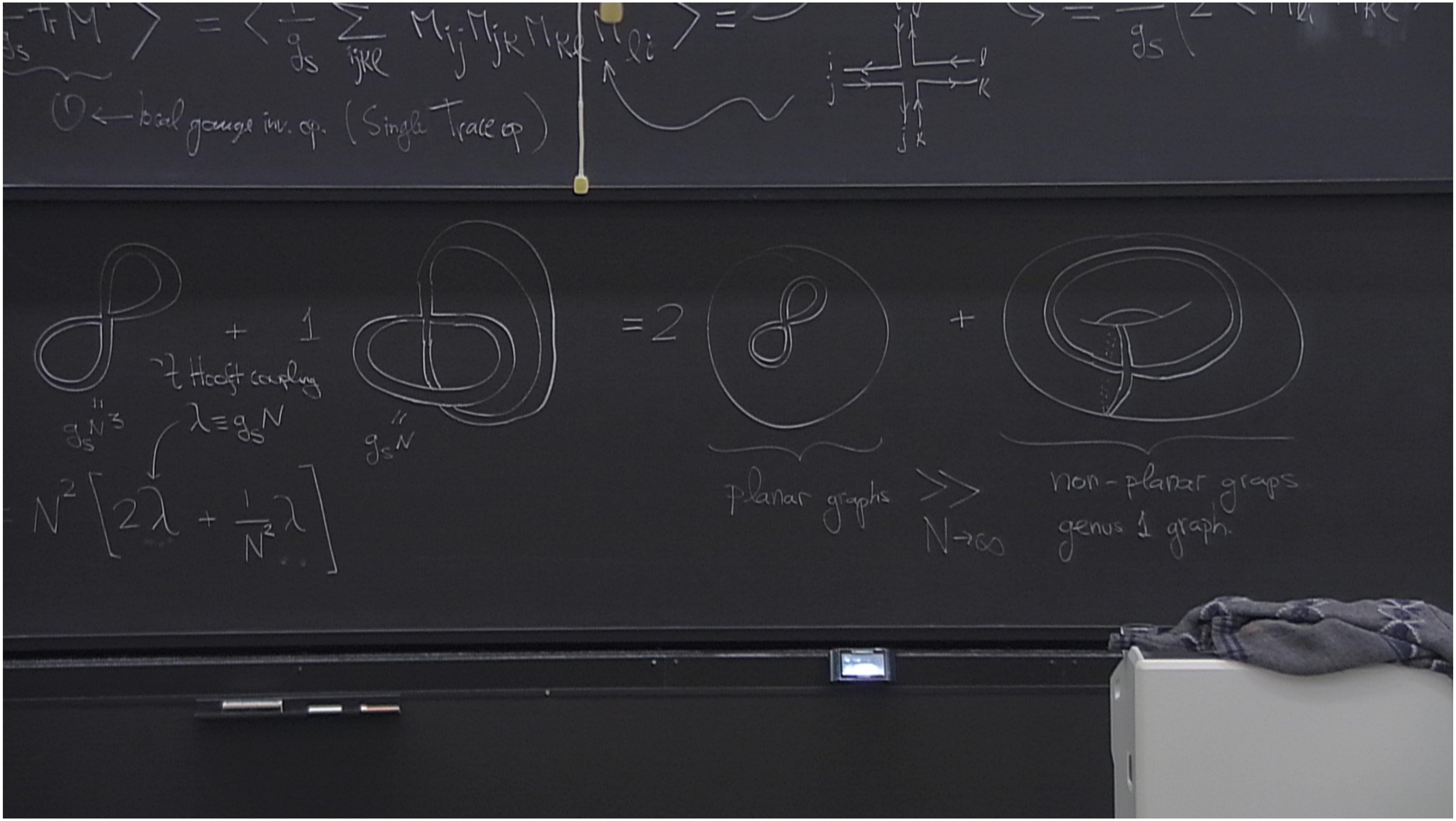
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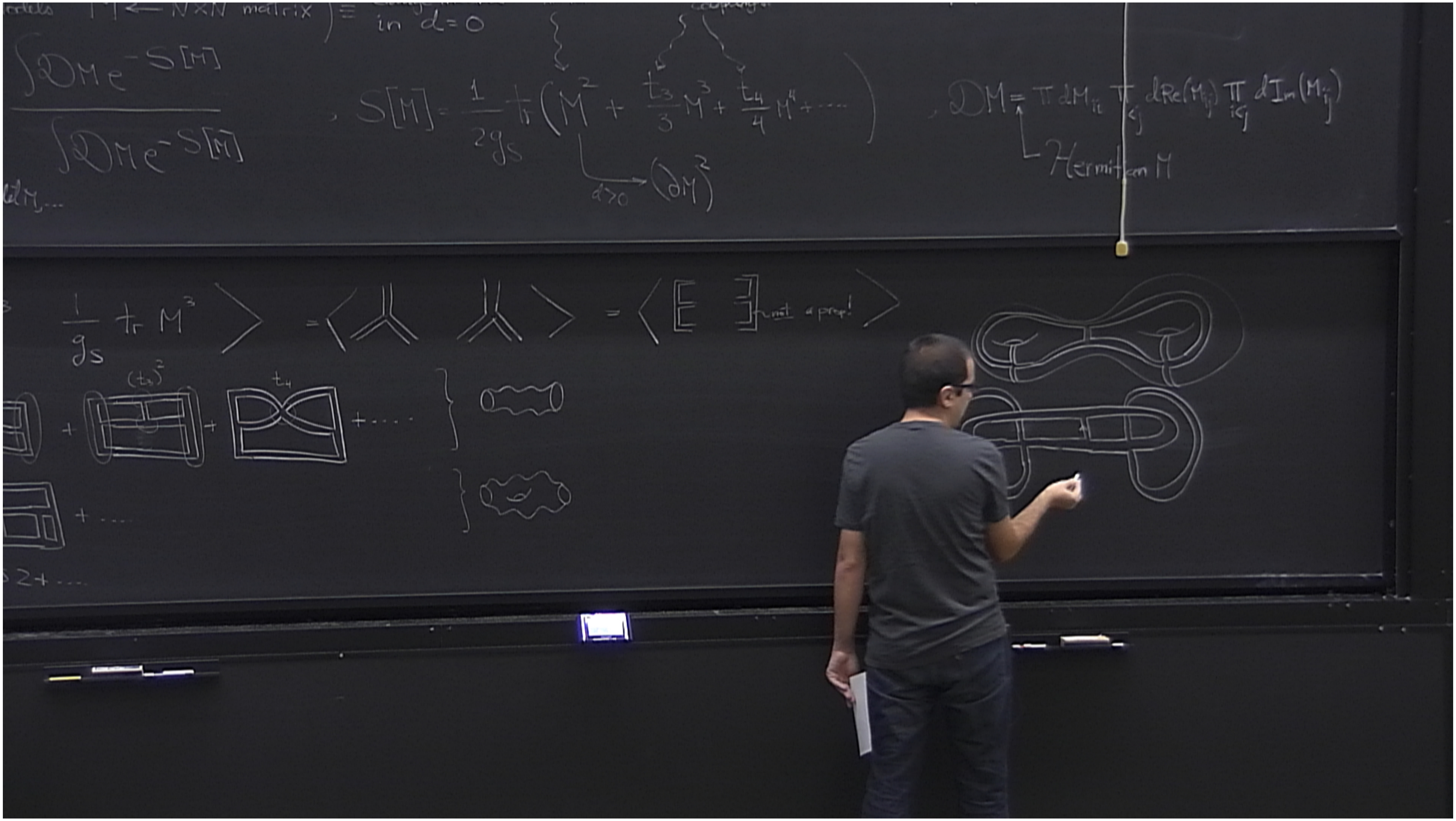
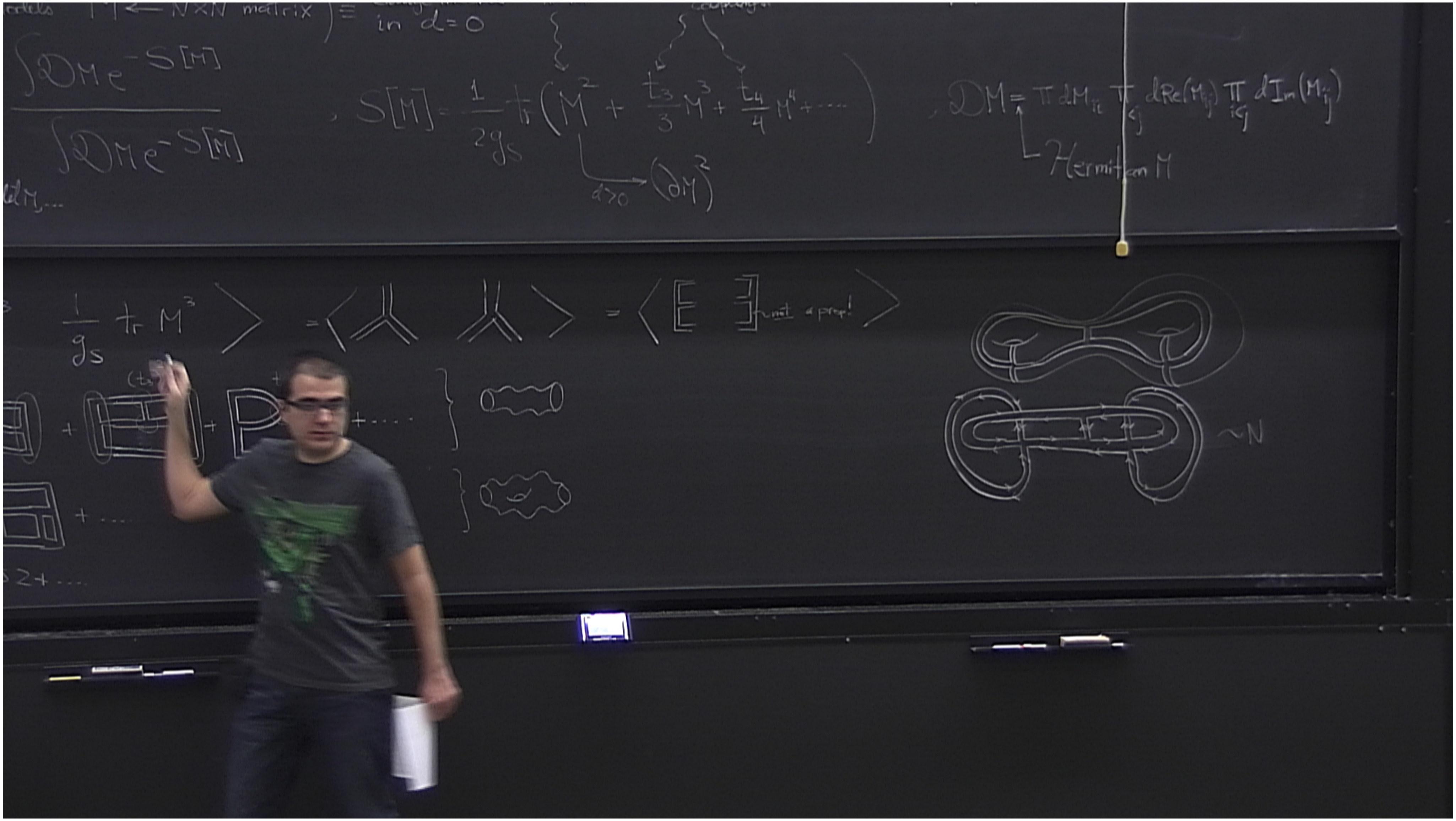
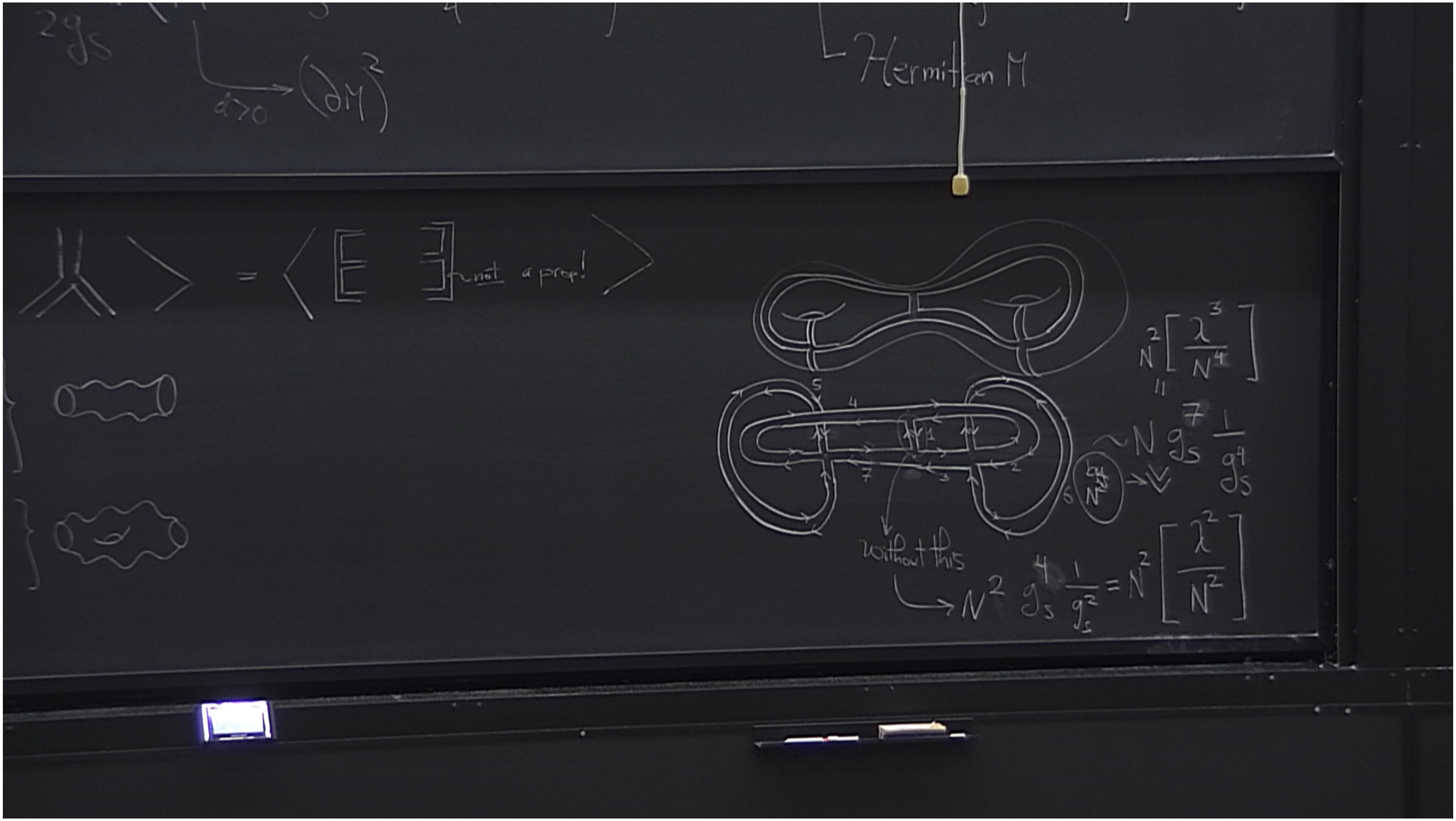
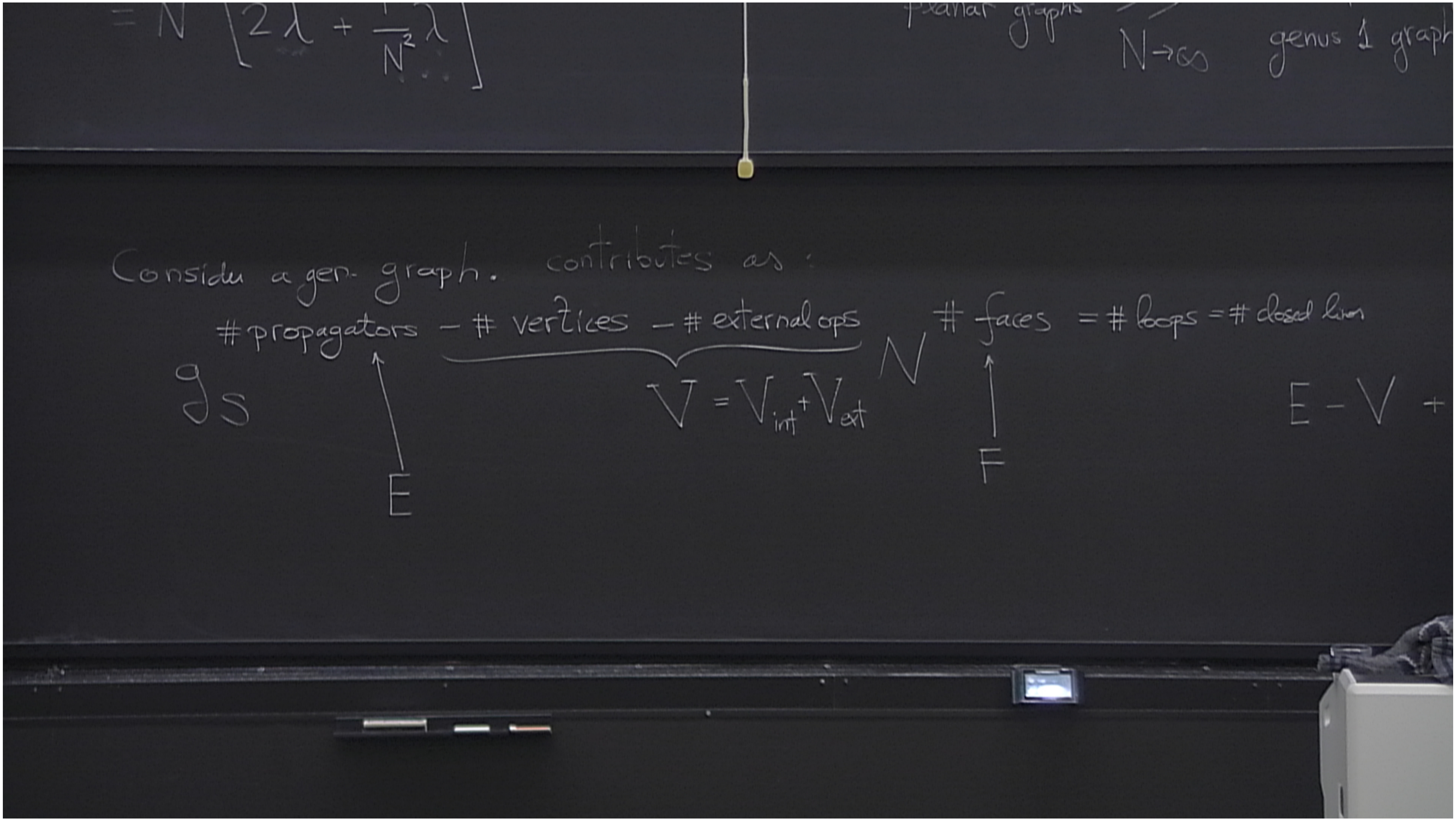
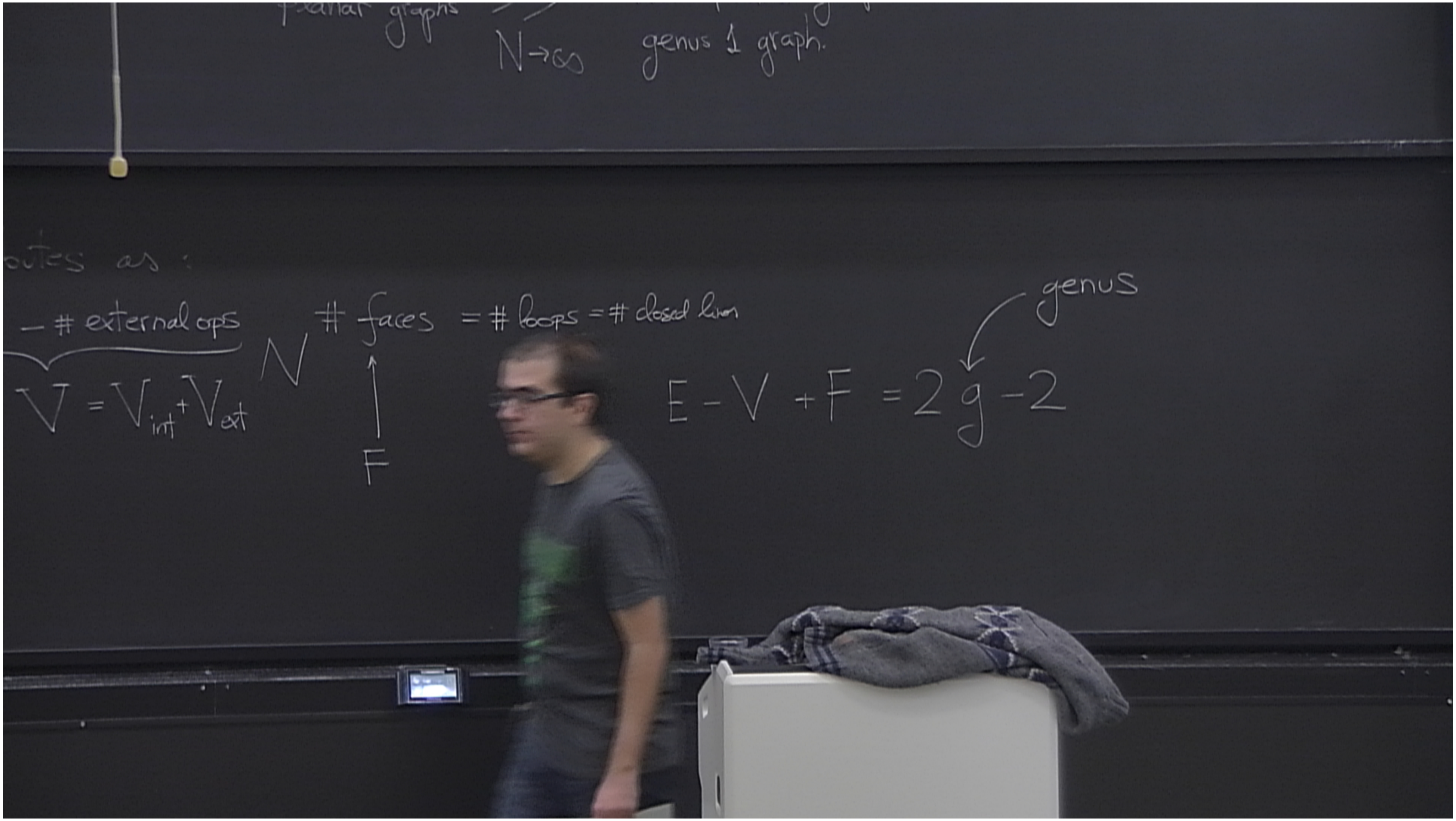

M ← N×N matrix
in d=0
∫DM e^{-S[M]}
∫DM e^{-S[M]}
S[M] = 1/(2g_s) tr(M² + t_3/3 M³ + t_4/4 M⁴ + ...)
d>0
(∂M)²
DM = Π dM_ii Π_{i<j} dRe(M_ij) Π_{i<j} dIm(M_ij)
Hermitian M
1/g_s tr M³
not a prop!
(t_3)²
t_4
+ ....
2+ ....

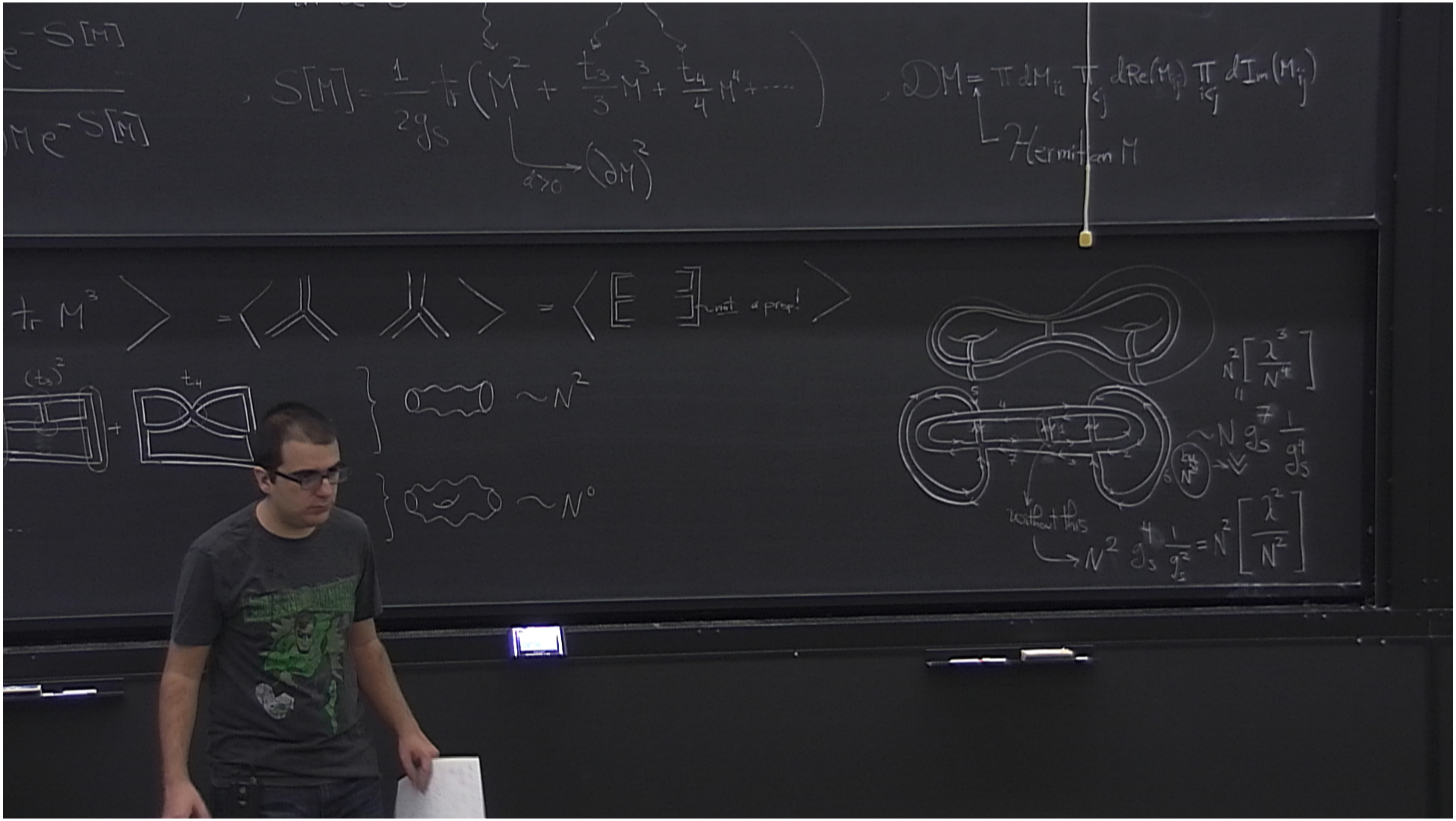

N×N matrix
in d=0
Hermitian M
not a prop!
~N

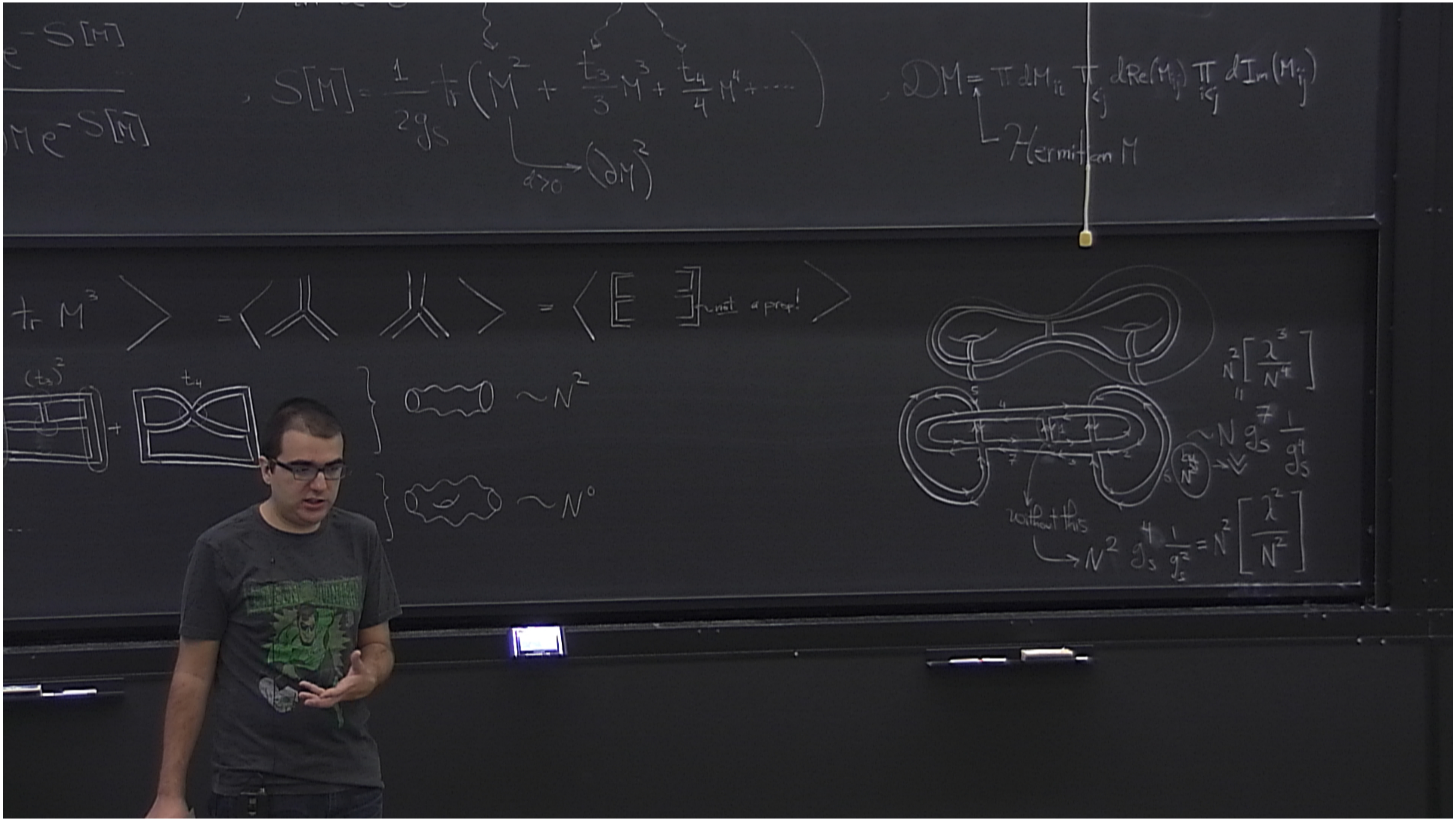


2gs
d>0
(∂M)²
Hermitian M
not a prop!
N²[λ³/N⁴]
~N gs⁷ 1/gs⁴
without this
N² gs⁴ 1/gs² = N²[λ²/N²]

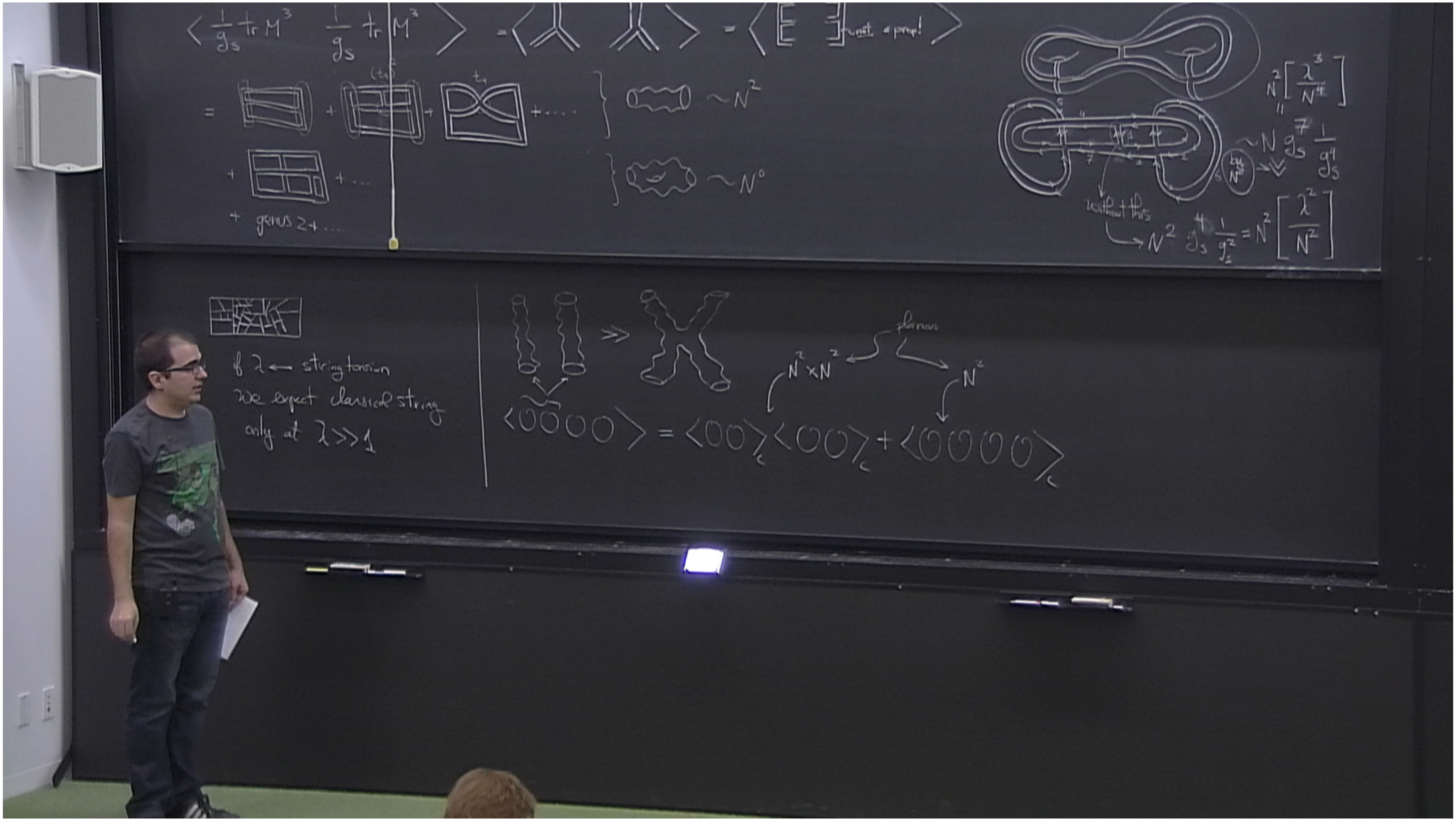

genus 1 graph
N→∞
Consider a gen. graph. contributes as:
propagators − # vertices − # external ops
faces = # loops = # closed lines
g_s
N
V = V_int + V_ext
E
F
E − V +

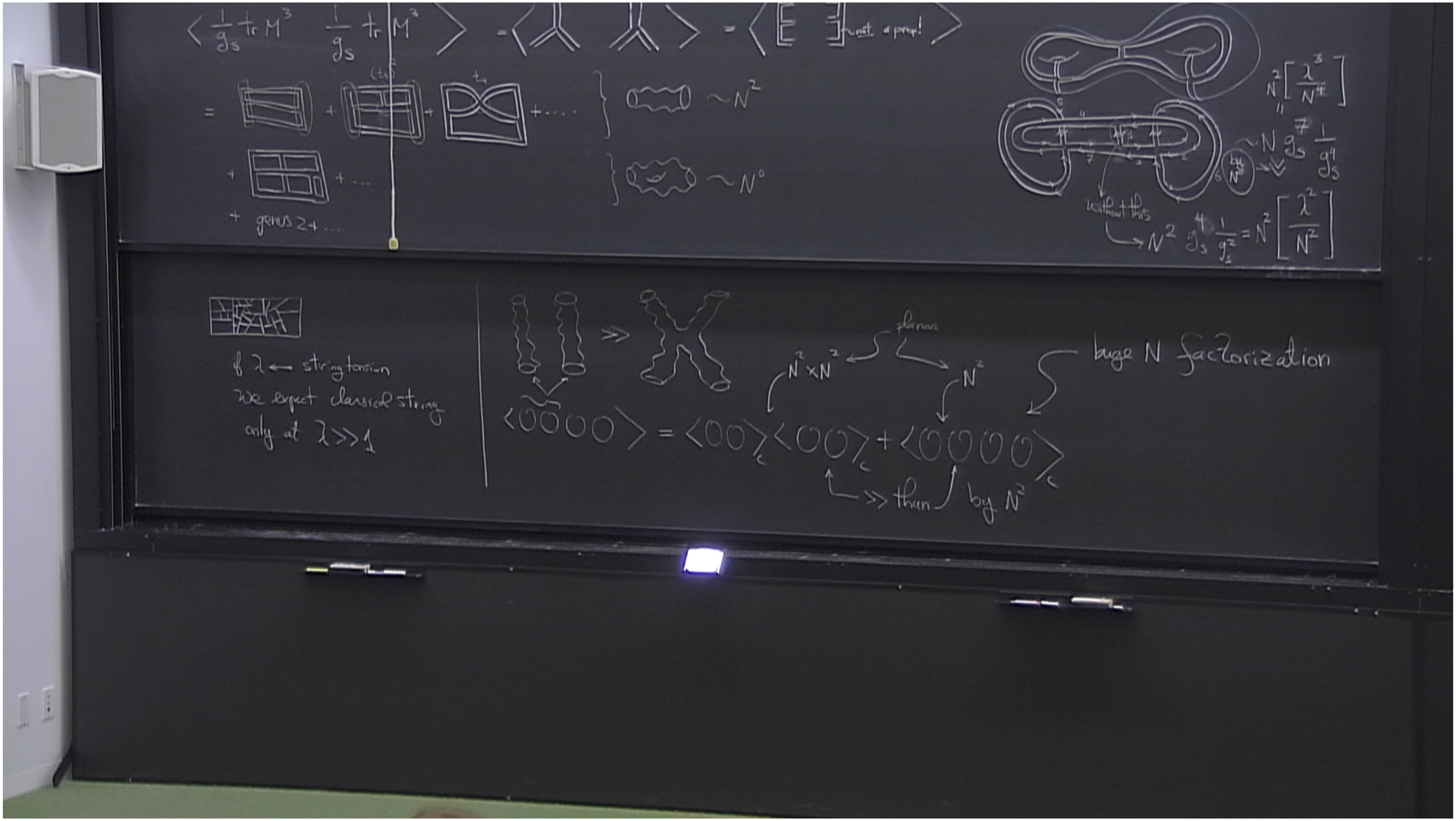

genus 1 graph.
faces = # loops = # closed lines
genus
E − V + F = 2g − 2
V = V_int + V_ext
# external ops

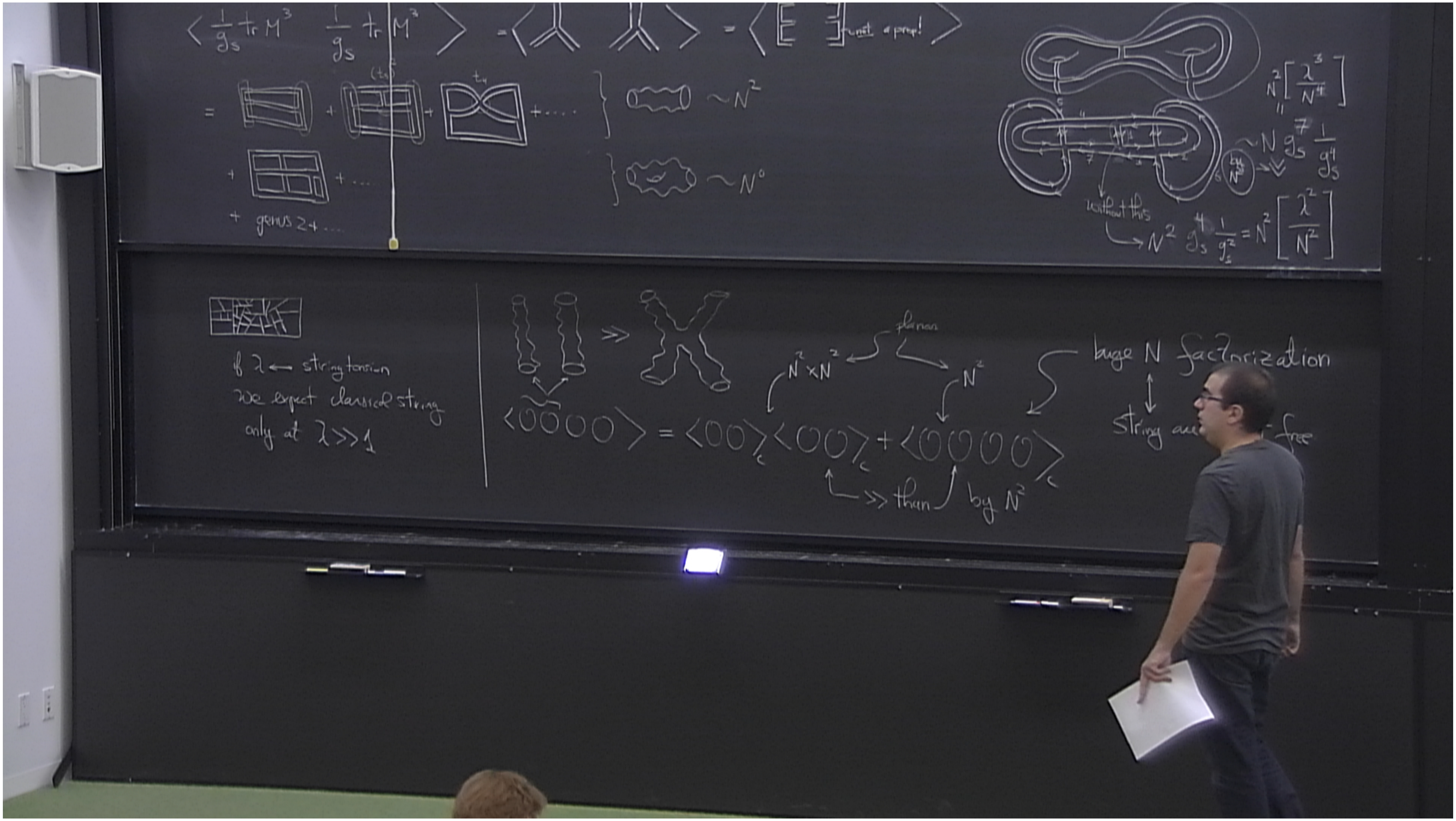

S[M] = 1/2g_s tr(M^2 + t_3/3 M^3 + t_4/4 M^4 + ...)
Hermitian M
not a prop!
without this



S[M] = 1/(2g_s) tr(M^2 + t_3/3 M^3 + t_4/4 M^4 + ...)
d>0
(∂M)^2
DM = Π dM_ii Π_{i<j} dRe(M_ij) Π_{i<j} dIm(M_ij)
Hermitian M
tr M^3
not a prop!
~N^2
~N^0
without this


without this
planar
if λ ← string tension
we expect classical string
only at λ >> 1
+ genus 2 + ...


genus 2+ ...
If λ ← string tension
we expect classical string
only at λ >> 1
planar
large N factorization
then
by N²
without this



genus 2+ ...
if λ ← string tension
we expect classical string
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large N factorization
without this
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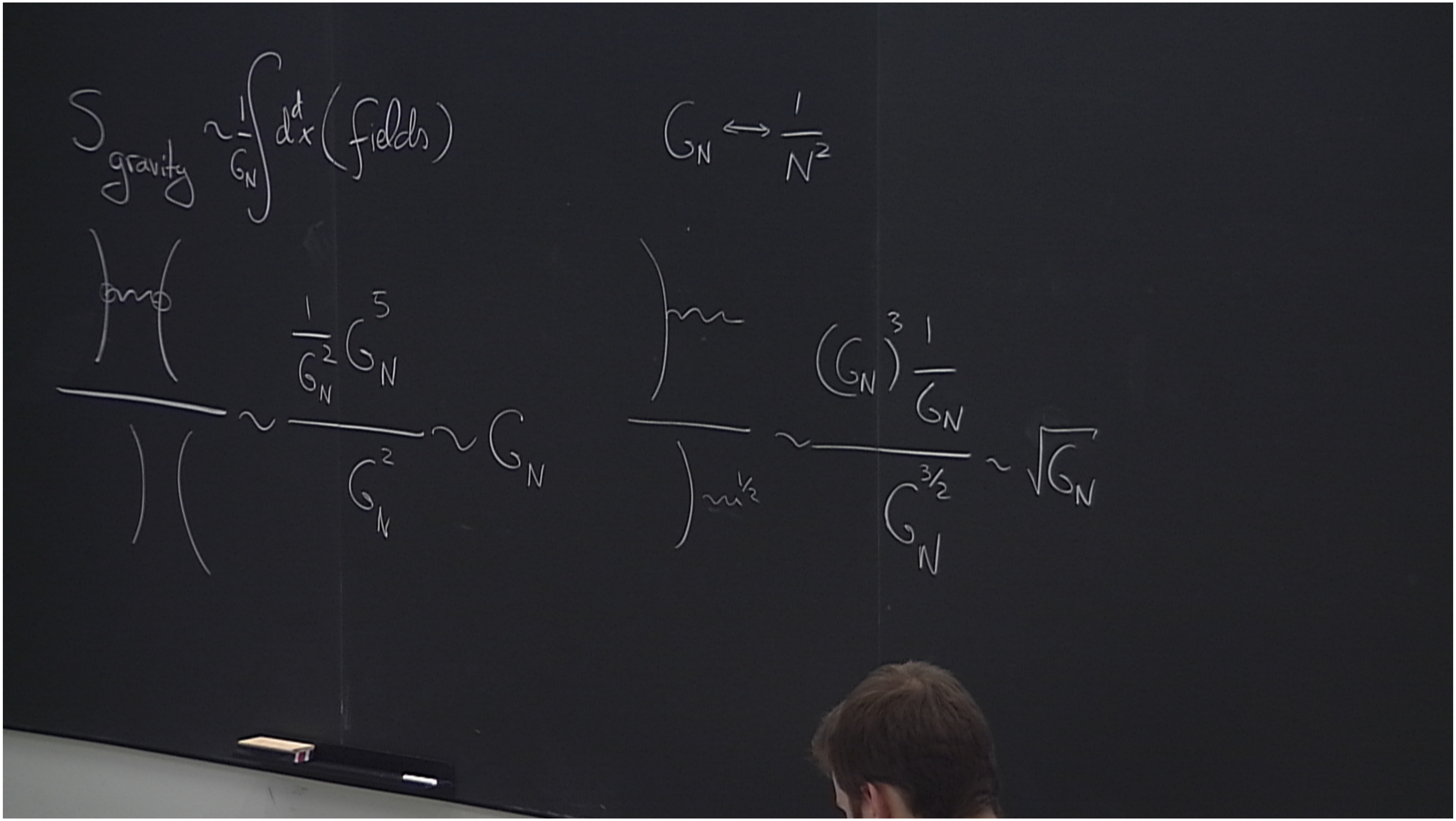
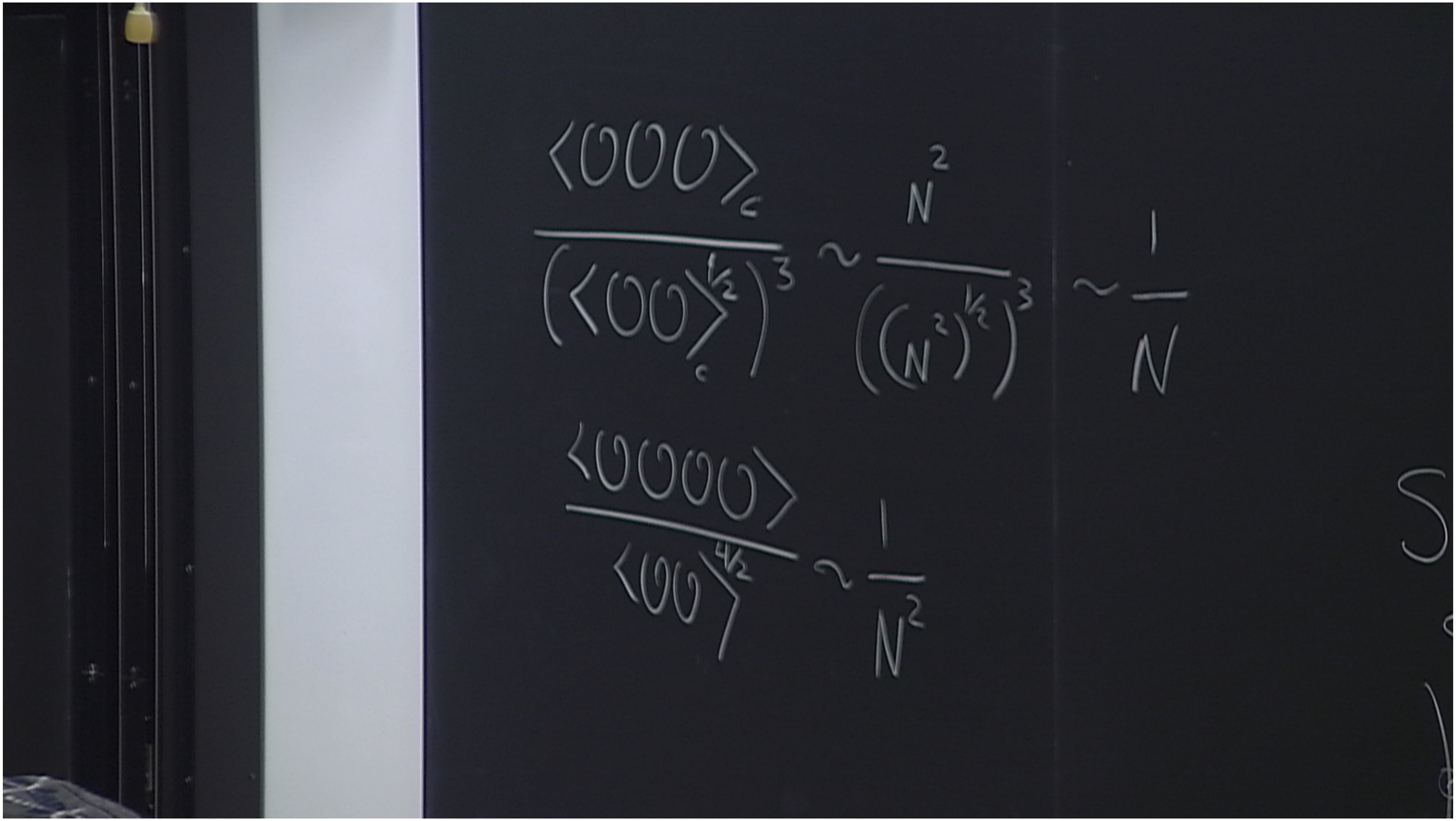
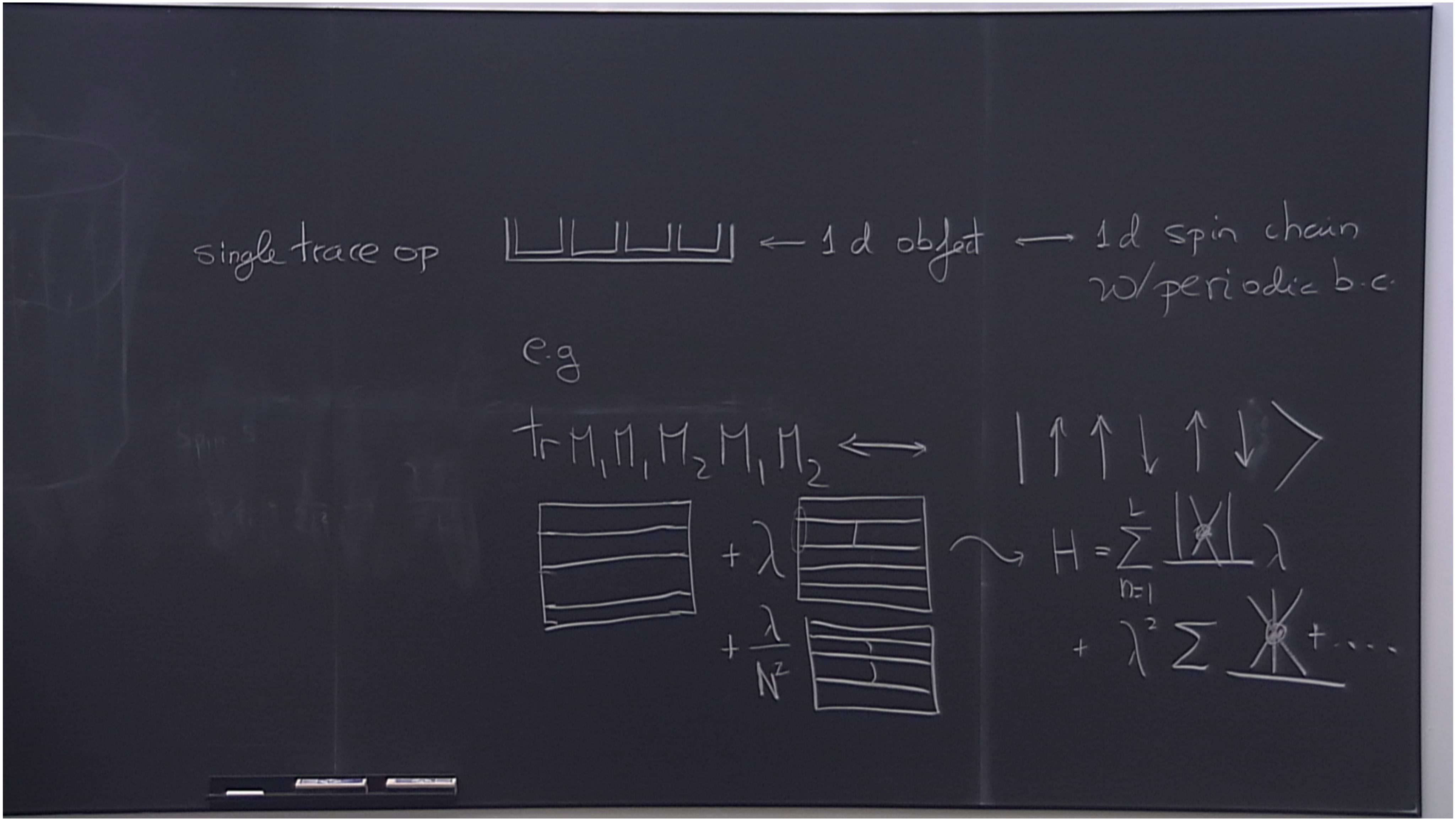

S_gravity ~ 1/G_N ∫ d^d x (fields)
G_N ↔ 1/N^2
~ (1/G_N^2) G_N^5 / G_N^2 ~ G_N
~ (G_N)^3 (1/G_N) / G_N^{3/2} ~ √G_N
1/2


$\frac{\langle OOO \rangle_c}{(\langle OO \rangle_c^{1/2})^3} \sim \frac{N^2}{((N^2)^{1/2})^3} \sim \frac{1}{N}$
$\frac{\langle OOOO \rangle}{\langle OO \rangle^{4/2}} \sim \frac{1}{N^2}$


single trace op
← 1d object ⟷ 1d spin chain
w/ periodic b.c.
e.g
tr M1 M1 M2 M1 M2 ⟷ |↑ ↑ ↓ ↑ ↓⟩
+ λ
+ λ/N²
H = Σ_{n=1}^{L} λ
+ λ² Σ + ...