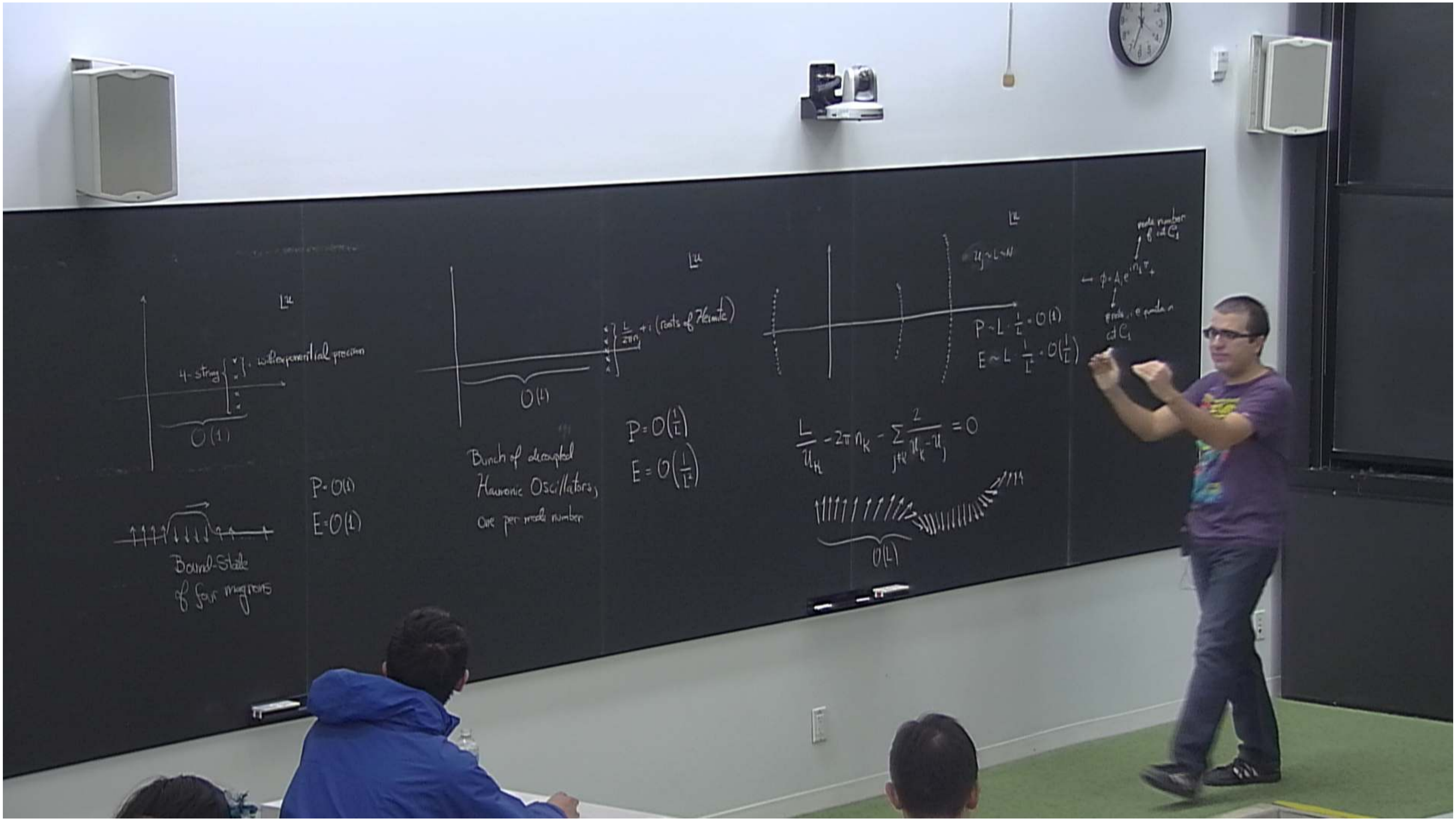
Title: Explorations in String Theory -6

Date: Apr 13, 2015 11:30 AM

URL: http://pirsa.org/15040145

Abstract:


4-string
Bound-State
of four magnons
$P = O(1)$
$E = O(1)$
$\frac{L}{2\pi n_j}$ + (roots of Hermite)
Bunch of decoupled
Harmonic Oscillators,
one per mode number
$P = O\left(\frac{1}{L}\right)$
$E = O\left(\frac{1}{L^2}\right)$
$\frac{L}{u_k} - 2\pi n_k - \sum_{j\neq k} \frac{2}{u_k - u_j} = 0$
$P \sim L \cdot \frac{1}{L} \sim O(1)$
$E \sim L \cdot \frac{1}{L^2} \sim O\left(\frac{1}{L}\right)$
mode number

- Solve the BE in this limit for any number of cuts.
- How to recover that from the effective field theory for low energy excitations around ferro-magnetic vacuum

① Introduce a quasi-momentum

$$p(u) \equiv \sum_{j=1}^{N} \frac{1}{u-u_j} - \frac{L}{2u} \underset{N\to\infty}{\sim} \int_{v\in\mathcal{C}_k} \frac{\rho(v)\,dv}{u-v} - \frac{L}{2u}$$

$p(u+i0)$

$p(u-i0)$

$p(u+i0) - p(u-i0) =$


vacuum
discontinuity of ρ
= 2πi ρ(u)
∓ iπ δ(u−v)
branch points
real cut
silly cut
ρ(u) du real

① Introduce a quasi-momentum

$$p(u) \equiv \sum_{j=1}^{N} \frac{1}{u-u_j} - \frac{L}{2u} \underset{N\to\infty}{\sim} \int_{v\in\mathcal{C}_k} \frac{\rho(v)\,dv}{u-v} - \frac{L}{2u}$$

$$p(u+i0) - p(u-i0) = 2\pi i\, \rho(u)$$

$$\frac{1}{u-v\pm i0} = P\frac{1}{u-v} \mp i\pi\delta(u-v)$$

$$p(u) \underset{\text{large } u}{\sim} \underbrace{\Big(\underbrace{\int_{v\in\mathcal{C}_k}\rho(v)\,dv}_{N} - \frac{L}{2}\Big)}_{-\left(\frac{L}{2}-N\right) = -S_z}\frac{1}{u}$$

$$p(u) \underset{\text{small } u}{\sim} -\frac{L}{2u} - \left(\int_{v\in\mathcal{C}_k}\frac{\rho(v)}{v}\,dv\right) + u\left(\int \frac{\rho(v)\,dv}{v^2}\right) + O(u^2)$$

① Introduce a quasi-momentum

$$p(u) = \sum_{j=1}^{N} \frac{1}{u-u_j} - \frac{L}{2u} \underset{N\to\infty}{\sim} \int_{\cup \mathcal{C}_k} \frac{\rho(v)\,dv}{u-v} - \frac{L}{2u}$$

$$p(u+i0) - p(u-i0) = 2\pi i\, \rho(u)$$

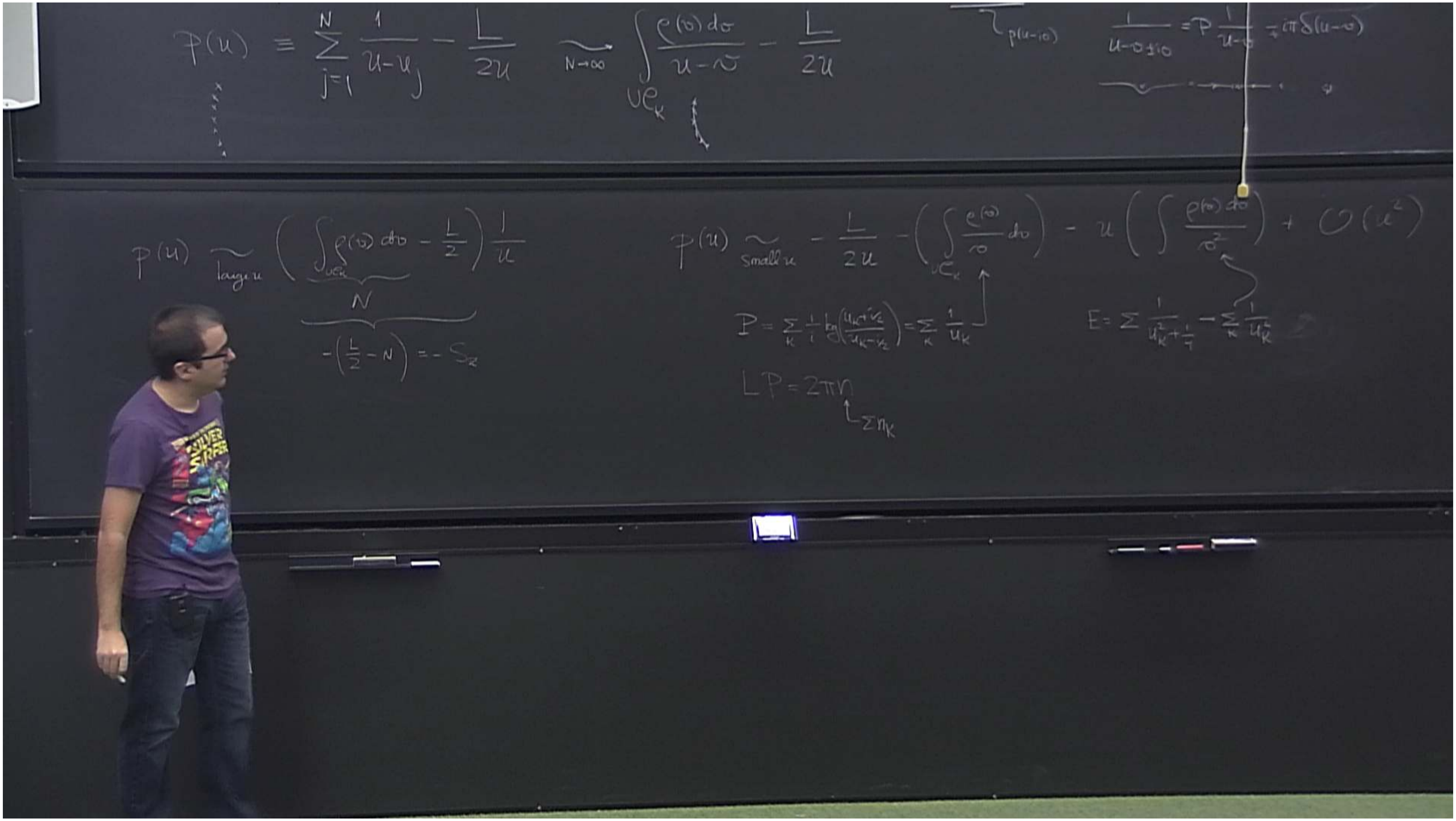
$$\frac{1}{u-v\pm i0} = P\frac{1}{u-v} \mp i\pi\delta(u-v)$$

$$p(u) \underset{\text{large } u}{\sim} \Big(\underbrace{\underbrace{\int_{\cup \mathcal{C}_k} \rho(v)\,dv}_{N} - \frac{L}{2}}_{-\left(\frac{L}{2}-N\right) = -S_z} \Big) \frac{1}{u}$$

$$p(u) \underset{\text{small } u}{\sim} -\frac{L}{2u} - \left(\int_{\cup \mathcal{C}_k} \frac{\rho(v)}{v}\,dv \right) - u \left(\int \frac{\rho(v)\,dv}{v^2} \right) + \mathcal{O}(u^2)$$

$$P = \sum_k \frac{1}{i} \log\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right) = \sum_k \frac{1}{u_k}$$

$$E = \sum \frac{1}{u_k^2 + \frac{1}{4}} \to \sum_k \frac{1}{u_k^2}$$



$p(u) \equiv \sum_{j=1}^{N} \frac{1}{u-u_j} - \frac{L}{2u} \underset{N\to\infty}{\sim} \int_{v\in\mathcal{C}_k} \frac{\rho(v)\,dv}{u-v} - \frac{L}{2u}$
$p(u-i0)$
$\frac{1}{u-v\pm i0} = P\frac{1}{u-v} \mp i\pi\delta(u-v)$
$p(u) \underset{\text{large } u}{\sim} \Big(\underbrace{\int_{v\in\mathcal{C}_k} \rho(v)\,dv}_{N} - \frac{L}{2} \Big) \frac{1}{u}$
$-\left(\frac{L}{2} - N\right) = -S_z$
$p(u) \underset{\text{small } u}{\sim} -\frac{L}{2u} - \left(\int_{v\in\mathcal{C}_k} \frac{\rho(v)}{v}\,dv \right) - u \left(\int \frac{\rho(v)\,dv}{v^2} \right) + O(u^2)$
$P = \sum_k \frac{1}{i} \log\left(\frac{u_k + i/2}{u_k - i/2}\right) = \sum_k \frac{1}{u_k}$
$E = \sum \frac{1}{u_k^2 + \frac{1}{4}} \to \sum_k \frac{1}{u_k^2}$
$LP = 2\pi n$
$\sum n_k$

$$p(u) \equiv \sum_{j=1}^{N} \frac{1}{u-u_j} - \frac{L}{2u} \underset{N\to\infty}{\sim} \int_{\cup \mathcal{C}_k} \frac{\rho(v)\,dv}{u-v} - \frac{L}{2u}$$

$p(u-i0)$

$$\frac{1}{u-v\pm i0} = \mathcal{P}\frac{1}{u-v} \mp i\pi\,\delta(u-v)$$

$$p(u) \underset{\text{large } u}{\sim} \Bigg(\underbrace{\underbrace{\int_{\cup \mathcal{C}_k} \rho(v)\,dv}_{N} - \frac{L}{2}}_{-\left(\frac{L}{2}-N\right) = -S_z}\Bigg)\frac{1}{u}$$

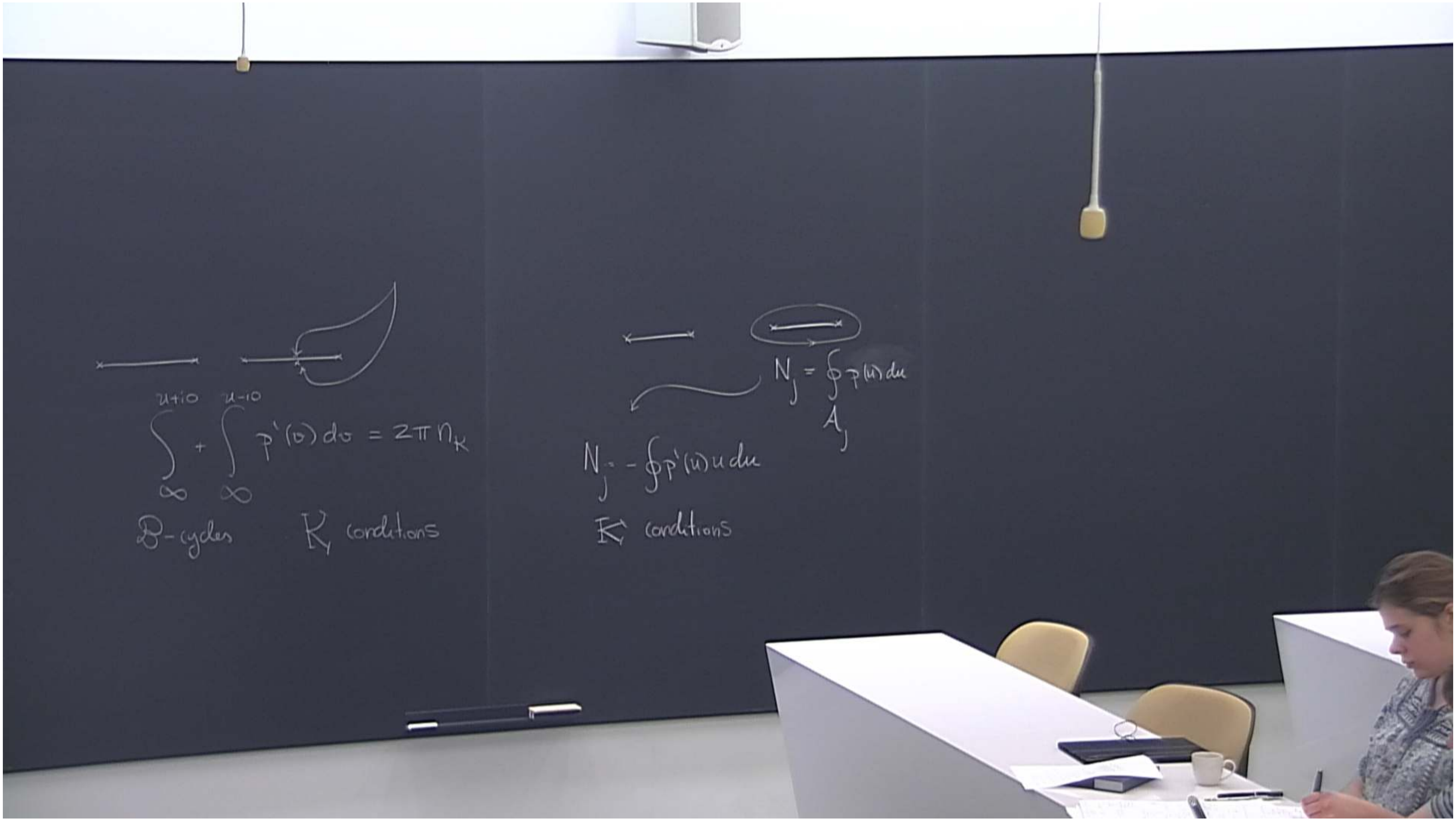
$$p(u) \underset{\text{small } u}{\sim} -\frac{L}{2u} - \left(\int_{\cup \mathcal{C}_k} \frac{\rho(v)}{v}\,dv\right) - u\left(\int \frac{\rho(v)\,dv}{v^2}\right) + \mathcal{O}(u^2)$$

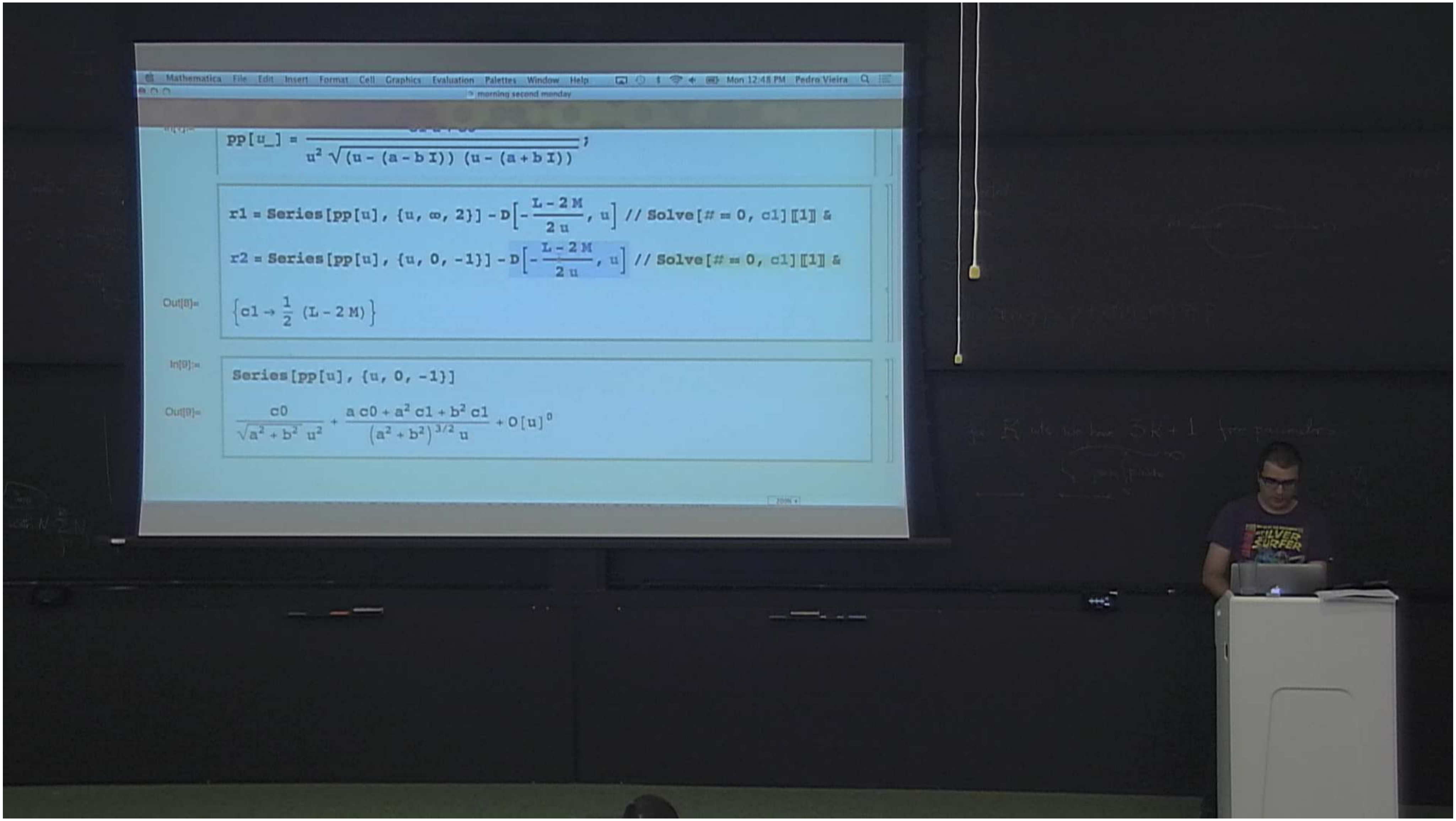
$$P = \sum_k \frac{1}{i}\log\left(\frac{u_k+\frac{i}{2}}{u_k-\frac{i}{2}}\right) = \sum_k \frac{1}{u_k}$$

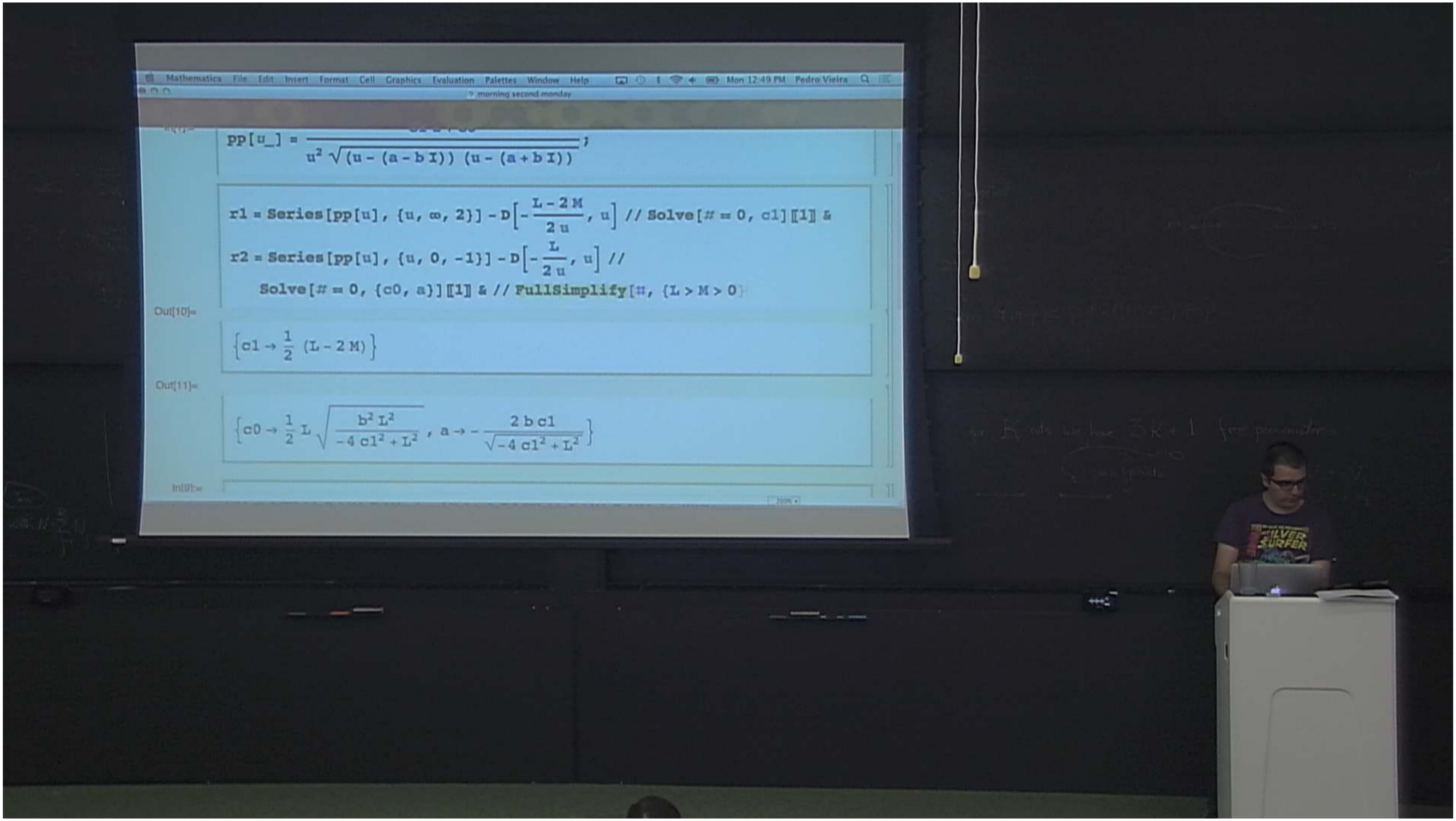
$$E = \sum \frac{1}{u_k^2+\frac{1}{4}} \to \sum_k \frac{1}{u_k^2}$$

$$LP = 2\pi n$$

$\sum n_k$


u+i0   u-i0
∫ + ∫ p'(v) dv = 2π n_K
∞   ∞
B-cycles   K_y conditions
N_j = ∮ p(u) du
A_j
N_j = -∮ p'(u) u du
K conditions


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:48 PM Pedro Vieira
morning second monday
pp[u_] =
u^2 √((u - (a - b I)) (u - (a + b I)))
r1 = Series[pp[u], {u, ∞, 2}] - D[-(L - 2 M)/(2 u), u] // Solve[# == 0, c1][[1]] &
r2 = Series[pp[u], {u, 0, -1}] - D[-(L - 2 M)/(2 u), u] // Solve[# == 0, c1][[1]] &
Out[8]=
{c1 → 1/2 (L - 2 M)}
In[9]:=
Series[pp[u], {u, 0, -1}]
Out[9]=
c0/(√(a^2 + b^2) u^2) + (a c0 + a^2 c1 + b^2 c1)/((a^2 + b^2)^(3/2) u) + O[u]^0


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:49 PM Pedro Vieira
morning second monday
pp[u_] = ... / (u^2 √((u - (a - b I)) (u - (a + b I))));
r1 = Series[pp[u], {u, ∞, 2}] - D[-(L - 2 M)/(2 u), u] // Solve[# == 0, c1][[1]] &
r2 = Series[pp[u], {u, 0, -1}] - D[-L/(2 u), u] //
Solve[# == 0, {c0, a}][[1]] & // FullSimplify[#, {L > M > 0}
Out[10]=
{c1 → 1/2 (L - 2 M)}
Out[11]=
{c0 → 1/2 L √(b^2 L^2/(-4 c1^2 + L^2)), a → -2 b c1/√(-4 c1^2 + L^2)}
SILVER SURFER

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morning second monday

$2\,u^2\sqrt{L^2+4\,L\,n\,\pi\,u+4\,n\,\pi\,u\,(-2\,M+n\,\pi\,u)}$

In[50]:=

```
Series[p[u], {u, 0, 0}] // FullSimplify[#, {L > 0}] &
```

Out[50]=

$$\frac{L}{2\,u^2}+\frac{4\,M\,(-L+M)\,n^2\,\pi^2}{L^3}+O[u]^1$$

$$-\frac{4\,M\,(-L+M)\,n^2\,\pi^2}{L^3} == \text{Energy of the state with one cut !}$$

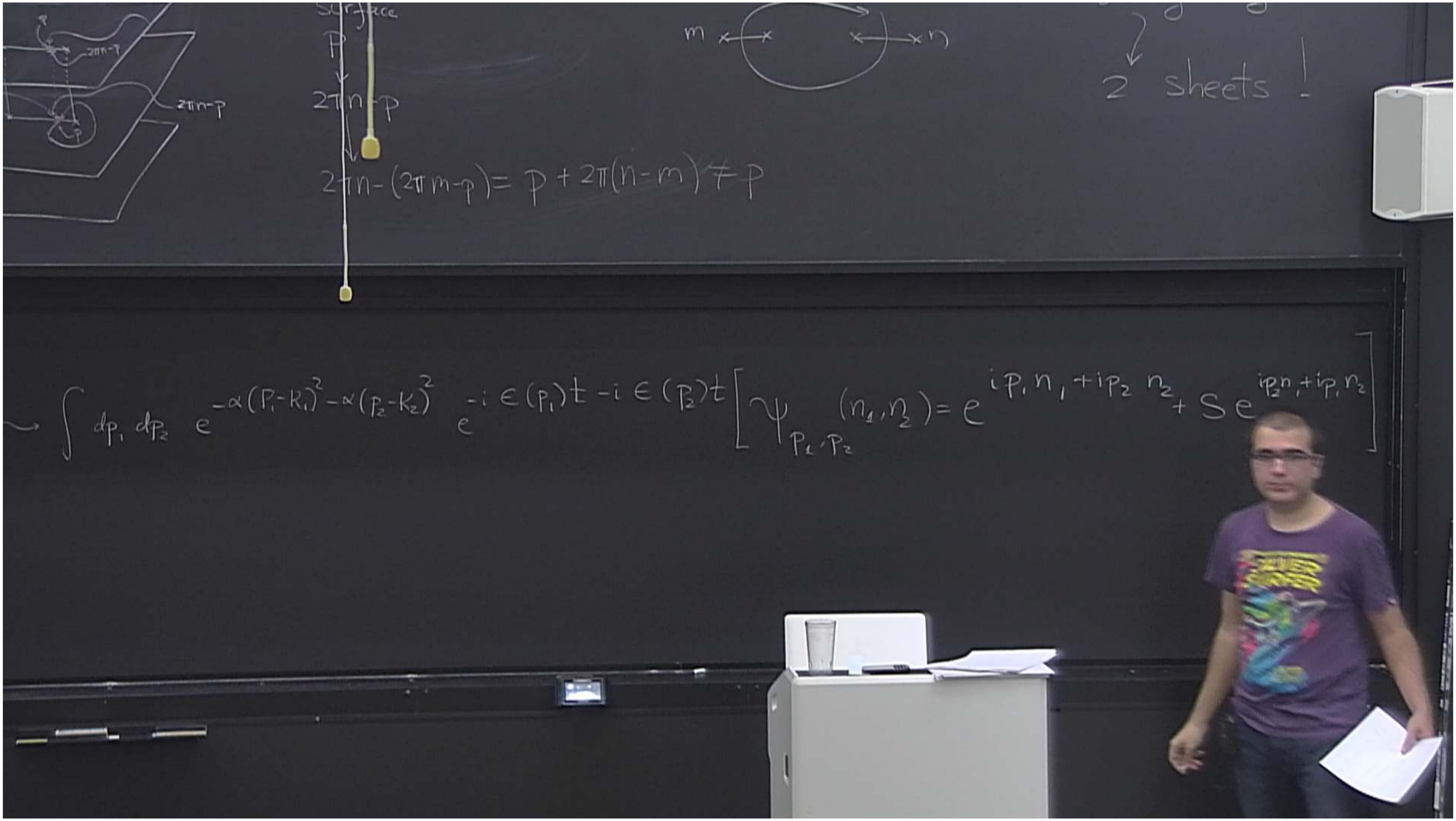
In[51]:=

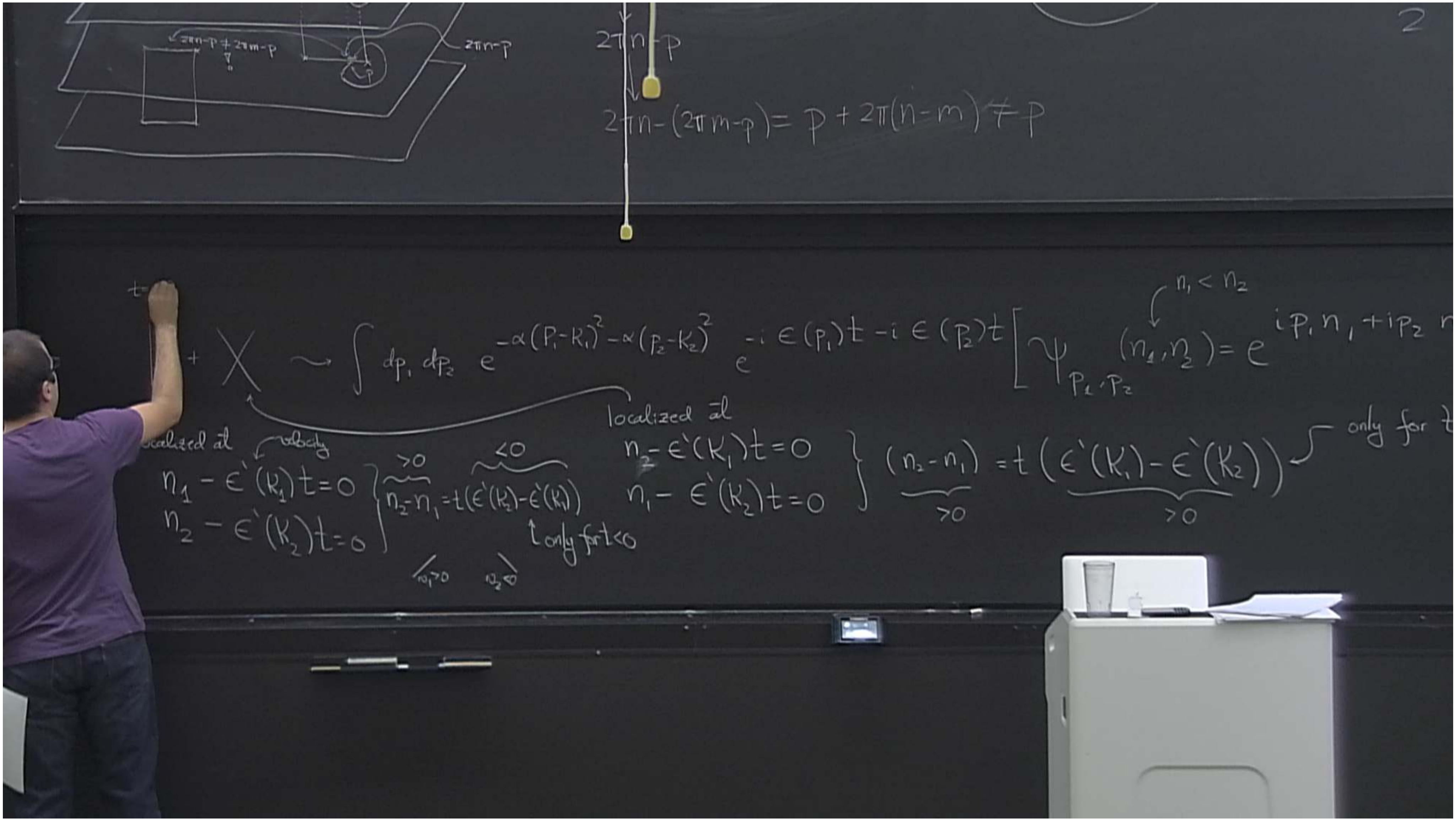
$$-\frac{4\,M\,(-L+M)\,n^2\,\pi^2}{L^3}\;/.\;M\to\alpha\,L\;//\;\text{Simplify}$$

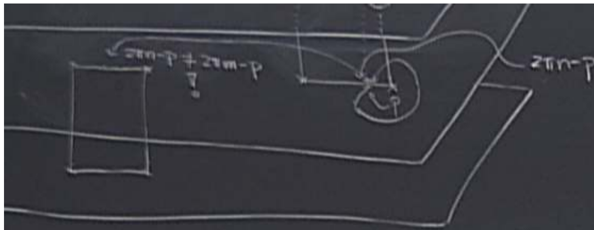
Out[51]=

$$-\frac{4\,n^2\,\pi^2\,(-1+\alpha)\,\alpha}{L}$$

200%


surface
2 sheets !
$2\pi n-(2\pi m-p)=p+2\pi(n-m)\neq p$
$\int dp_1\, dp_2\, e^{-\alpha(p_1-k_1)^2-\alpha(p_2-k_2)^2}\, e^{-i\epsilon(p_1)t-i\epsilon(p_2)t}\left[\psi_{p_1,p_2}(n_1,n_2)=e^{ip_1n_1+ip_2n_2}+S\,e^{ip_2n_1+ip_1n_2}\right]$


2
2πn-p
2πn − (2πm − p) = p + 2π(n − m) ≠ p
2πn-p ≠ 2πm-p
2πn-p
n₁ < n₂
X ~ ∫ dp₁ dp₂ e^{−α(p₁−k₁)² − α(p₂−k₂)²} e^{−i ε(p₁)t − i ε(p₂)t} [ψ_{p₁,p₂}(n₁,n₂) = e^{ip₁n₁ + ip₂n₂}
localized at
velocity
n₁ − ε′(k₁)t = 0
n₂ − ε′(k₂)t = 0
>0
<0
n₂ − n₁ = t(ε′(k₂) − ε′(k₁))
only for t<0
n₁>0
n₂<0
localized at
n₂ − ε′(k₁)t = 0
n₁ − ε′(k₂)t = 0
(n₂ − n₁) = t(ε′(k₁) − ε′(k₂))
>0
>0
only for t



$2\pi n - p$

$2\pi n - (2\pi m - p) = p + 2\pi(n-m) \neq p$

2 sheet

$t=-\infty \sim S\times(t\to+\infty)$

$n_1 < n_2$

$$|| + X \sim \int dp_1\, dp_2\; e^{-\alpha(p_1-k_1)^2-\alpha(p_2-k_2)^2}\, e^{-i\epsilon(p_1)t - i\epsilon(p_2)t}\Big[\psi_{p_1,p_2}(n_1,n_2) = e^{ip_1n_1+ip_2n_2} + Se$$

localized at   velocity

$$\left.\begin{aligned} n_1 - \epsilon'(k_1)t &= 0 \\ n_2 - \epsilon'(k_2)t &= 0 \end{aligned}\right\} \underbrace{n_2-n_1}_{>0} = t\,\underbrace{(\epsilon'(k_2)-\epsilon'(k_1))}_{<0}$$

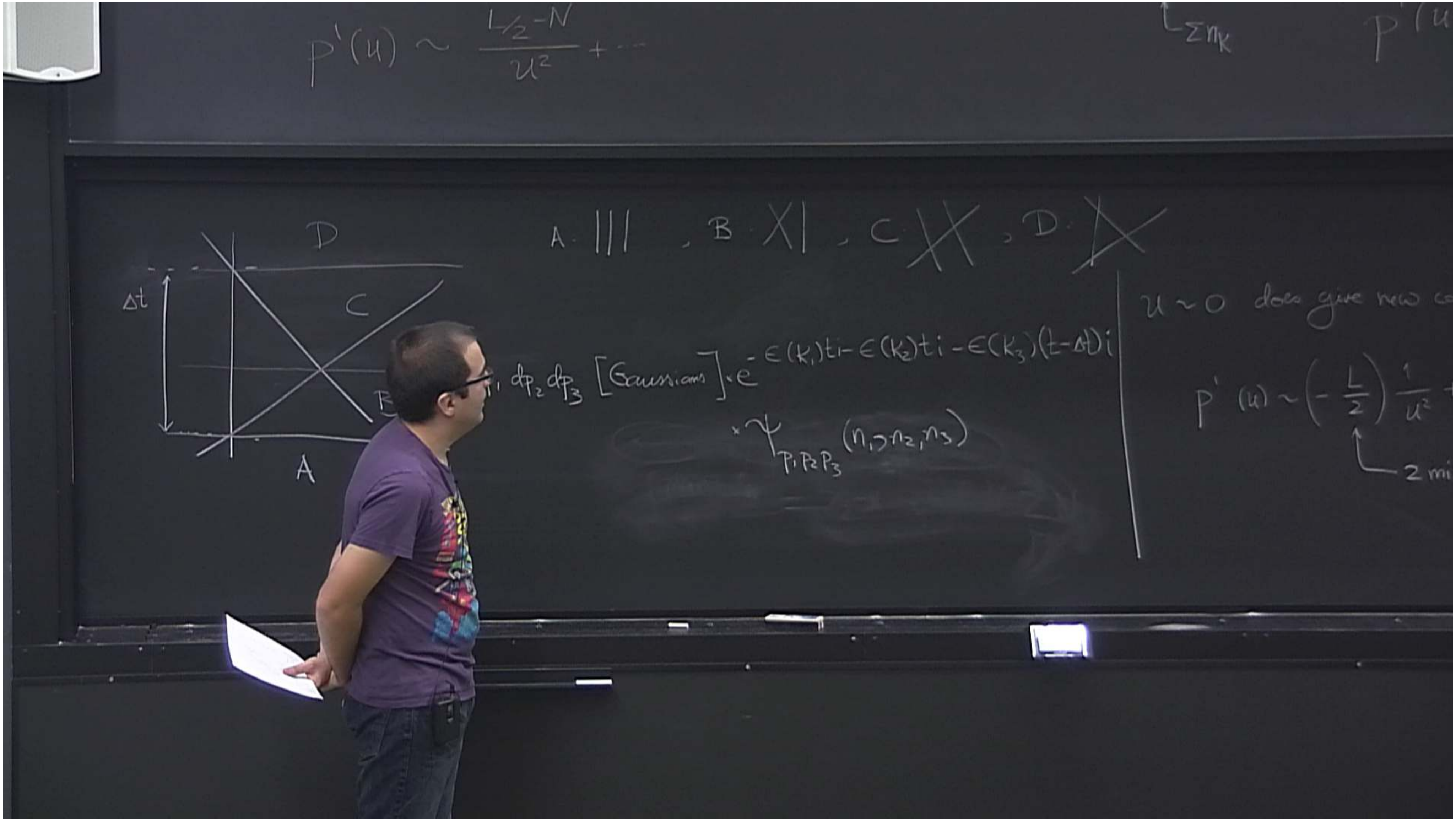
t only for $t<0$

$n_1>0$ $n_2<0$

localized at

$$\left.\begin{aligned} n_2 - \epsilon'(k_1)t &= 0 \\ n_1 - \epsilon'(k_2)t &= 0 \end{aligned}\right\} \underbrace{(n_2-n_1)}_{>0} = t\,\underbrace{\big(\epsilon'(k_1)-\epsilon'(k_2)\big)}_{>0}$$

only for $t>0$


A
B
C
D
Δt
[Gaussians]
u ~ 0 does give new co
2 mi