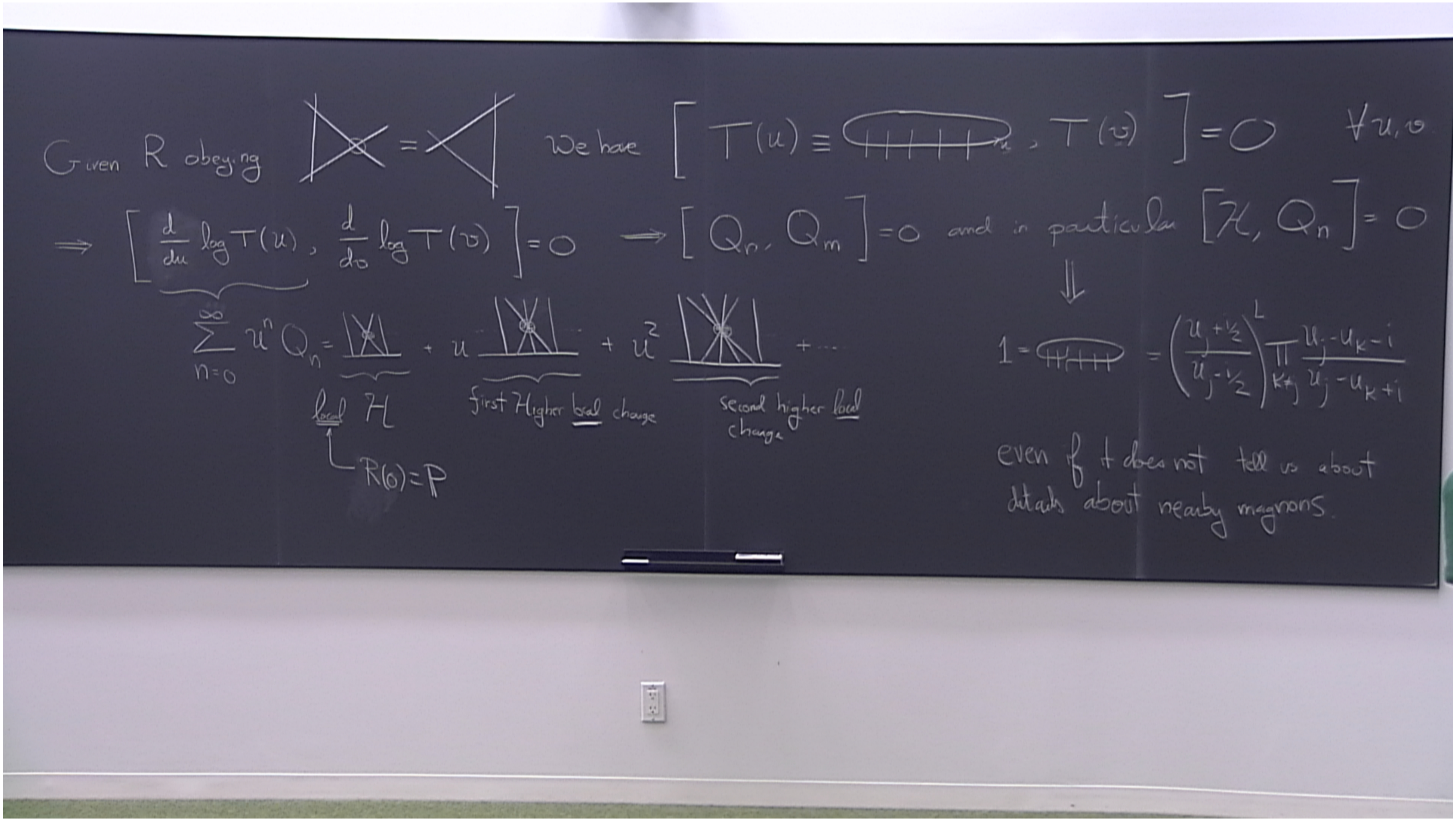
Title: Explorations in String Theory -4

Date: Apr 09, 2015 11:30 AM

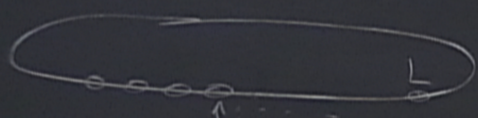
URL: http://pirsa.org/15040143

Abstract:



Given R obeying $=$
We have $\left[T(u) \equiv , T(v) \right] = 0 \quad \forall u, v$
$\Rightarrow \left[\frac{d}{du} \log T(u), \frac{d}{dv} \log T(v) \right] = 0 \Rightarrow \left[Q_n, Q_m \right] = 0$ and in particular $\left[\mathcal{H}, Q_n \right] = 0$
$\sum_{n=0}^{\infty} u^n Q_n = + u + u^2 + \cdots$
local $\mathcal{H}$
first Higher local charge
Second higher local charge
$R(0) = P$
$\Downarrow$
$1 = = \left(\frac{u_j + 1/2}{u_j - 1/2} \right)^L \prod_{k \neq j} \frac{u_j - u_k - i}{u_j - u_k + i}$
even if it does not tell us about details about nearby magnons.

Application : SO(n) spin-chains



$|a\rangle$

$1 \dots n$

$$H = \sum_{n=1}^{L} \left[a\,\mathbb{1} + b\,P + c\,K \right]$$

$\mathbb{1}$: $\delta_{i_1}^{j_1}\delta_{i_2}^{j_2}$

P: $\delta_{i_1}^{j_2}\delta_{i_2}^{j_1}$

K: $\delta_{i_1 i_2}\delta^{j_1 j_2}$

present for SO(n), absent for SU(n).

$$+ c\,\mathbb{K} \Big] = \Big[\alpha\,\mathbb{P}_{anti} + \beta\,\mathbb{P}_{sym\,tr} + \gamma\,\mathbb{P}_{tr} \Big]$$

$$- \delta^{j_2}_{i_1} \delta^{j_1}_{i_2}$$

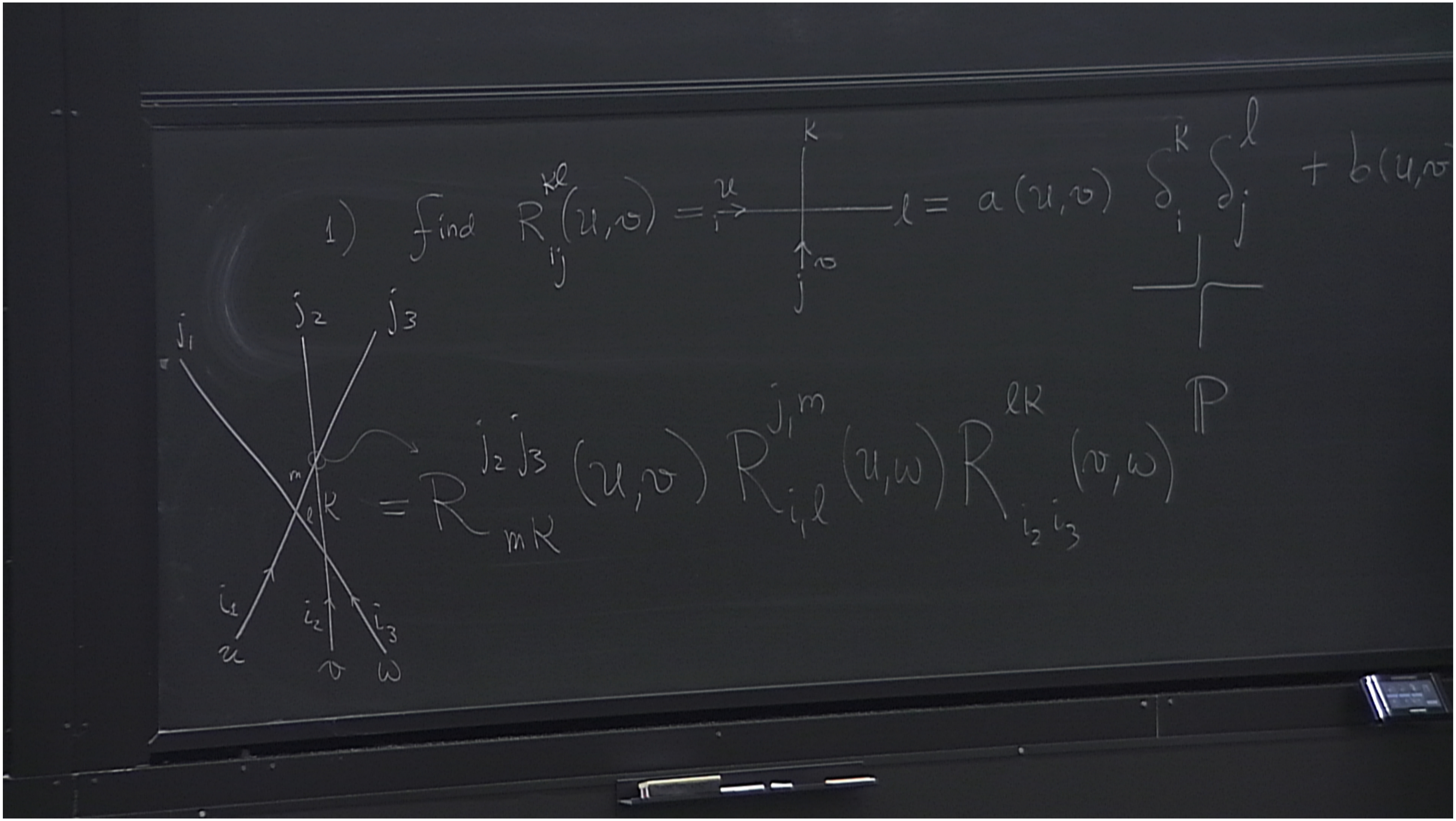
$$\delta_{i_1 i_2} \delta^{j_1 j_2}$$

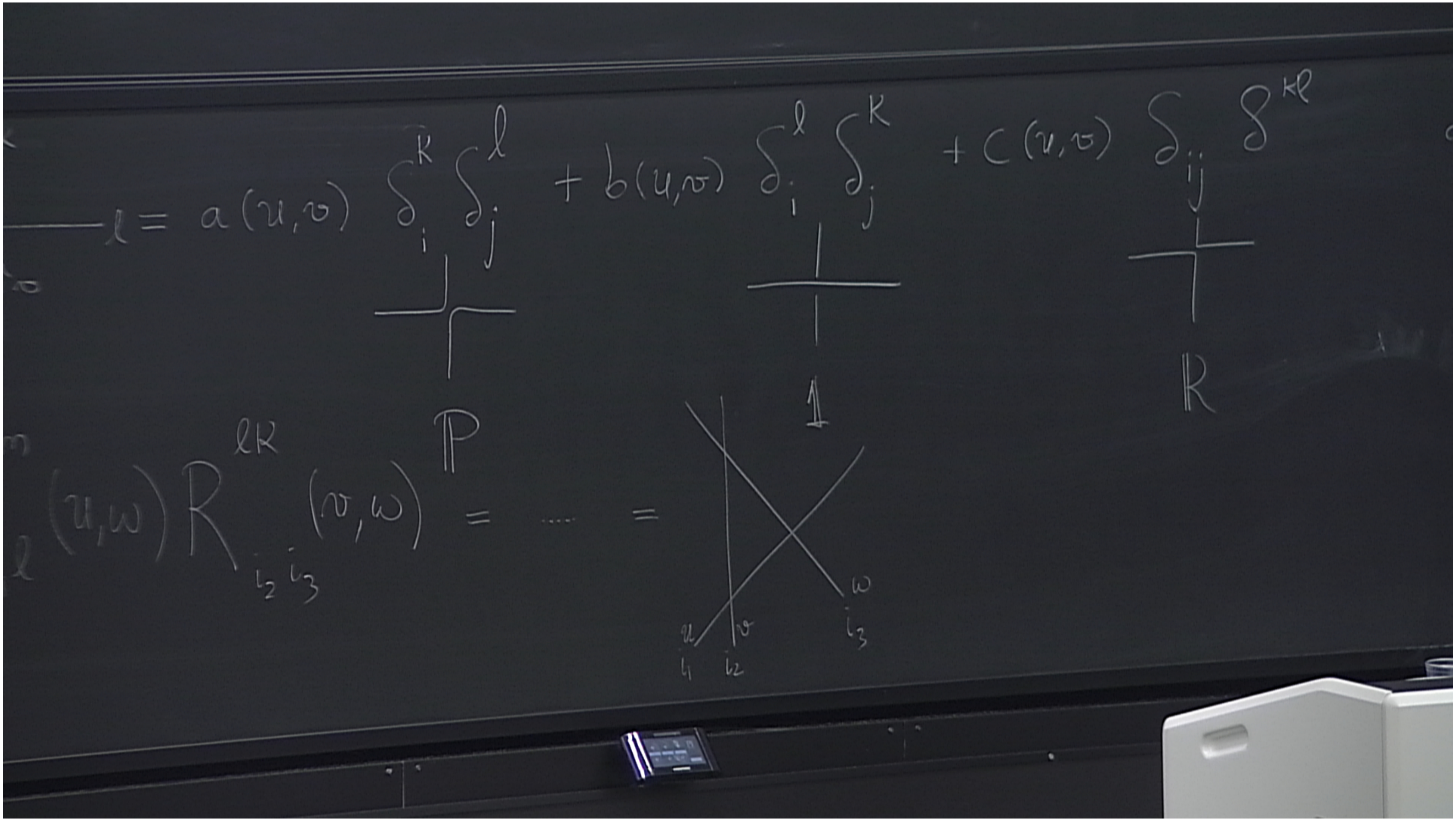
$$\mathbb{1}\,|ab\rangle = |ab\rangle$$

$$\mathbb{P}\,|ab\rangle = |ba\rangle$$

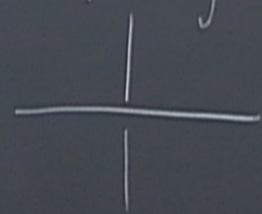
$$\mathbb{K}\,|ab\rangle = \delta_{ab} \sum_{c=1}^{n} |cc\rangle$$

1) find $R(u,v) = {}_i\xrightarrow{u}\!\!\!\overset{k}{\underset{j}{+}}\!\!\!{}^{\uparrow v}\, l = a(u,v)\, \underbrace{\delta_i^k \delta_j^l}_{\mathbb{P}} + b(u,v)\, \underbrace{\delta_i^l \delta_j^k}_{\mathbb{1}} + \ldots$

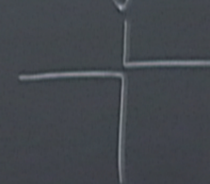

1) find $R_{ij}^{kl}(u,v) = a(u,v)\,\delta_i^k \delta_j^l + b(u,v$
P
$= R_{mk}^{j_2 j_3}(u,v)\, R_{i_1 l}^{j_1 m}(u,w)\, R_{i_2 i_3}^{lk}(v,w)$


$\ell = a(u,v)\, \delta_i^k \delta_j^\ell + b(u,v)\, \delta_i^\ell \delta_j^k + c(u,v)\, \delta_{ij}\, \delta^{k\ell}$
P
1
R
$\ell\, (u,w) R^{\ell k}_{i_2 i_3} (v,w) = \dots = $
u i_1
v i_2
w i_3

ℓ j $+ \, b(u,v)\, \delta_i^{\ell}\, \delta_j^{k} \; + \, c(u,v)\, \delta_{ij}\, \delta^{k\ell}$ obeying YB

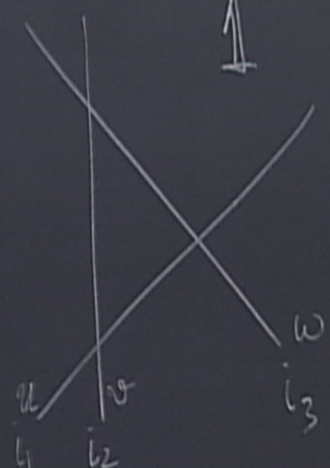


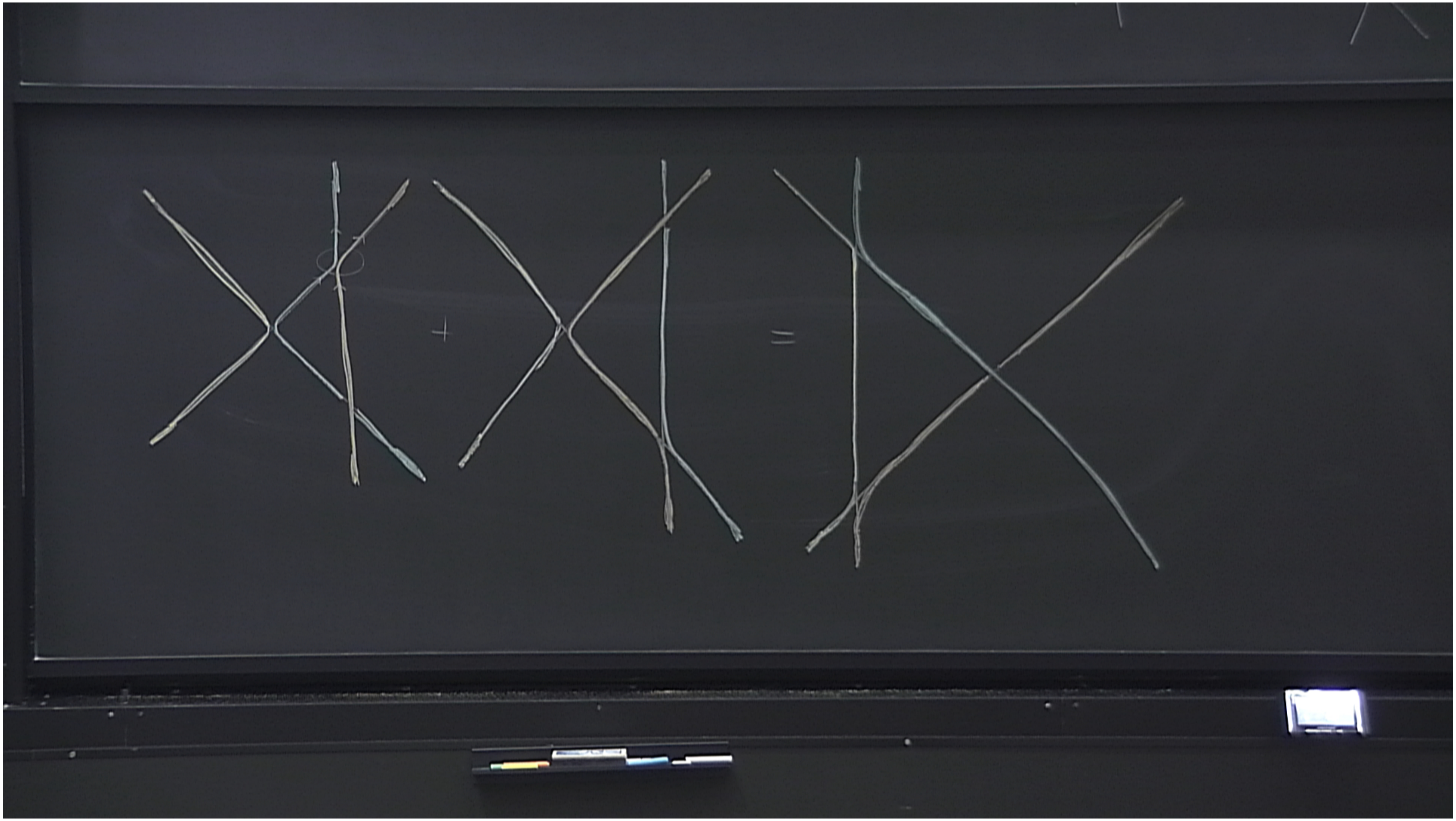
$\mathbb{1}$



$\mathbb{R}$

$= \; \cdots \; =$





write eq, divide by $a(u,v)\, a(u,w)\, a(v,w)$

$$\hat{b}(v,w) + \hat{b}(u,v) = \hat{b}(u,w)$$

$$\hat{b}(v,w) = \frac{b(v,w)}{a(v,w)}$$

write eq, divide by $a(u,v)\,a(u,w)\,a(v,w)$

$$\hat{b}(v,w) + \hat{b}(u,v) = \hat{b}(u,w) \Rightarrow \hat{b}(u,v) = f(u) - f(v)$$

$$\frac{b(v,w)}{a(v,w)}$$

indeed we can set $a(u,v) = 1$

* can set

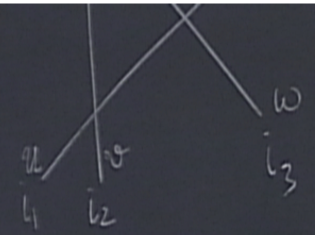
$a(u,v) = 1$

$b(u,v) = u - v$

idea: plug this into YB and see if we can fix $c(u,v)$.

outcome

$$c(u,v) = -\frac{2(u-v)}{2(u-v)+n-2}$$



$$\ldots v) = -\frac{2(u-v)}{2(u-v)+n-2}$$

or $\quad C(u,v) = 0$

$$R(u,v) = (u-v)\mathbb{1} + \mathbb{P}$$

outcome

$$C(u,v) = -\frac{2(u-v)}{2(u-v)+n-2} \quad \underline{\underline{\text{or}}} \quad C(u,v)=0$$

$$R(u,v) = (u-v)\mathbb{1} + \mathbb{P}$$

$R(u) =$  $= u\,\mathbb{1} + \mathbb{P} - \frac{2u}{2u+n-2}\,\mathbb{K}$

$$\mathcal{H} = \# \frac{d}{du} \log T(u) \Big|_{u=0} = \sum \left(\# \, \mathbb{1} + \# \left(\mathbb{P} - \frac{2}{n-2} \mathbb{K} \right)_{n,n+1} \right)$$

$R(0) = P$

Example SO(6) Spin-chain

$$\sum \left(\mathbb{1} - \mathbb{P} \pm \frac{\mathbb{K}}{2} \right)$$

:-D

:-(

$$- \frac{2}{n-2}\mathbb{K}\Big)_{n,n+1} \Big)$$

Spin-chain:

$$+\frac{\mathbb{K}}{2}\Big)$$

:-D

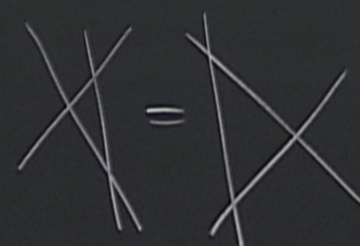
:-(

R-matrix $\neq$ S-matrix

XXX spin chain

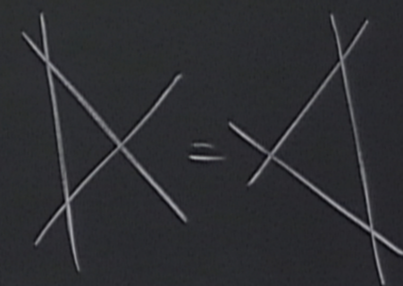
R-matrix is a 4x4 matrix

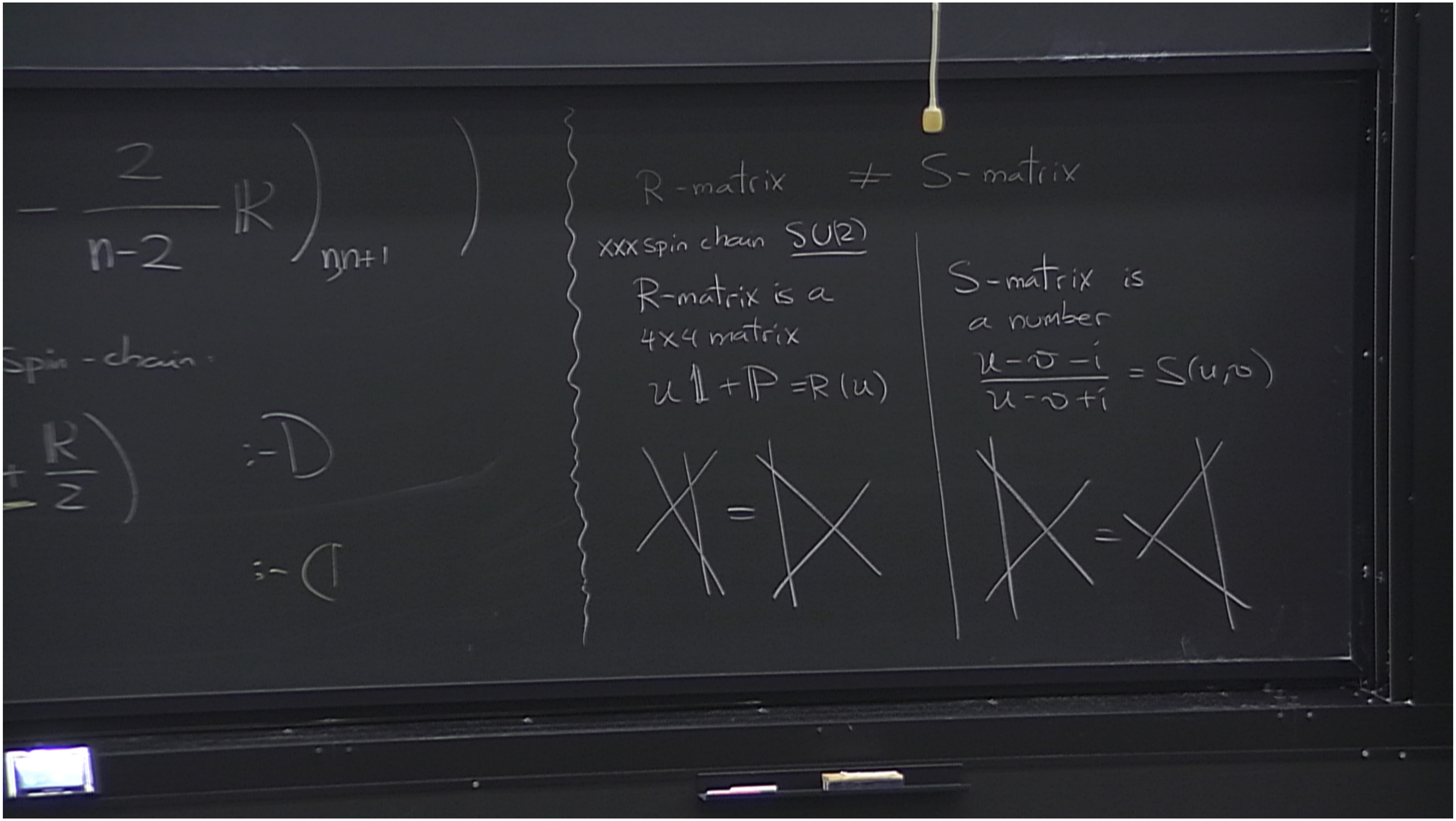
$$u\mathbb{1} + \mathbb{P} = R(u)$$

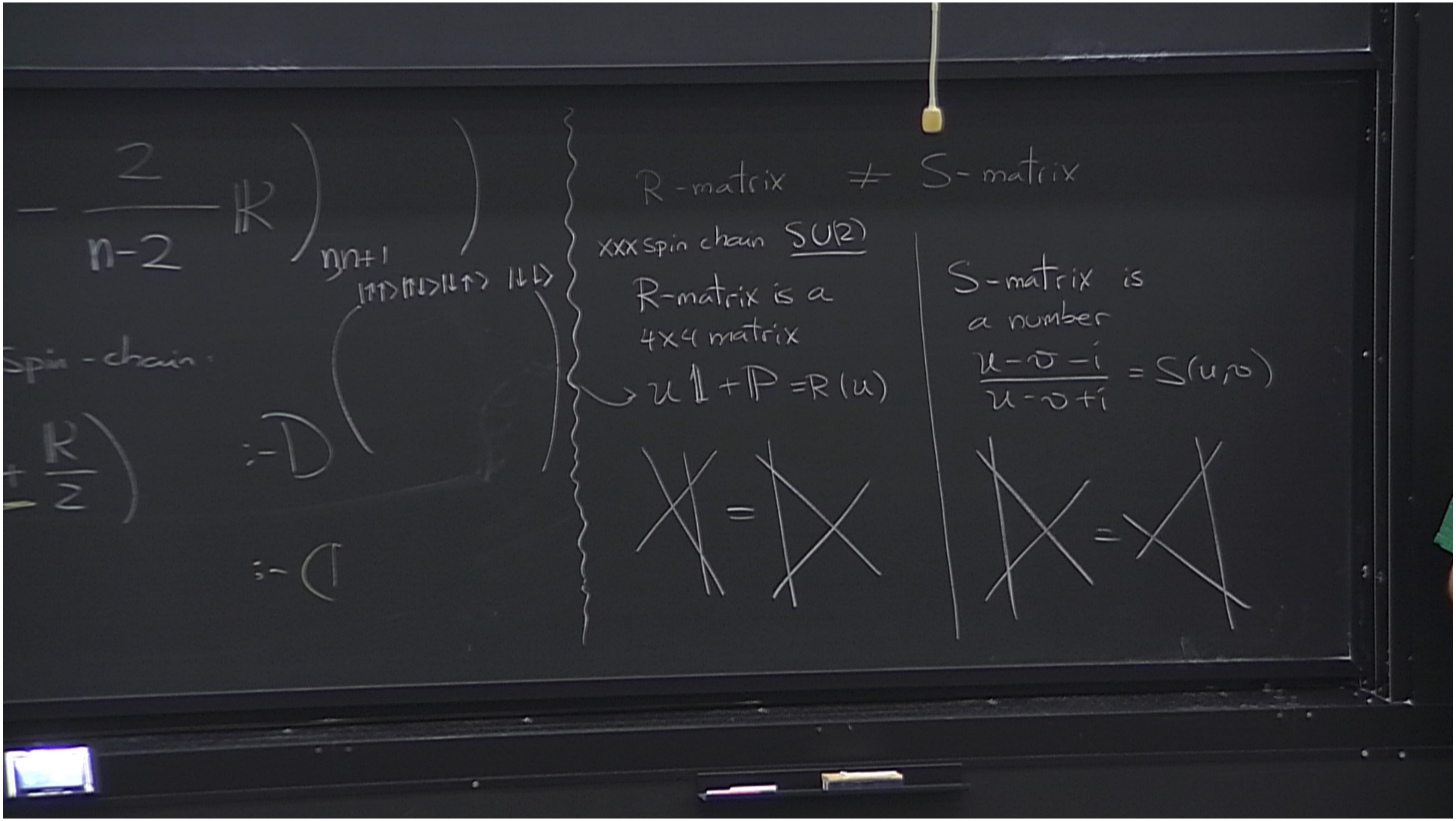


S-matrix is a number

$$\frac{u-v-i}{u-v+i} = S(u,v)$$




R-matrix ≠ S-matrix
XXX spin chain SU(2)
R-matrix is a
4×4 matrix
u 𝟙 + ℙ = R(u)
S-matrix is
a number
(u − v − i)/(u − v + i) = S(u,v)
Spin-chain:



R-matrix ≠ S-matrix
XXX spin chain SU(2)
R-matrix is a
4x4 matrix
u 𝟙 + P = R(u)
S-matrix is
a number
(u - v - i)/(u - v + i) = S(u,v)
Spin-chain:
|↑↑⟩ |↑↓⟩ |↓↑⟩ |↓↓⟩

$R(u) = \ldots - u\,\mathbb{1} + \mathbb{P} - \frac{\ldots}{2u+n-2}\mathbb{K}$

SU(3) 0 0 0 0 0 1 1 0 0 2 0 0 ... 0

R-matrix

$u\,\mathbb{1} + \mathbb{P} = R_{SU(3)}$

9×9 matrix

S-matrix

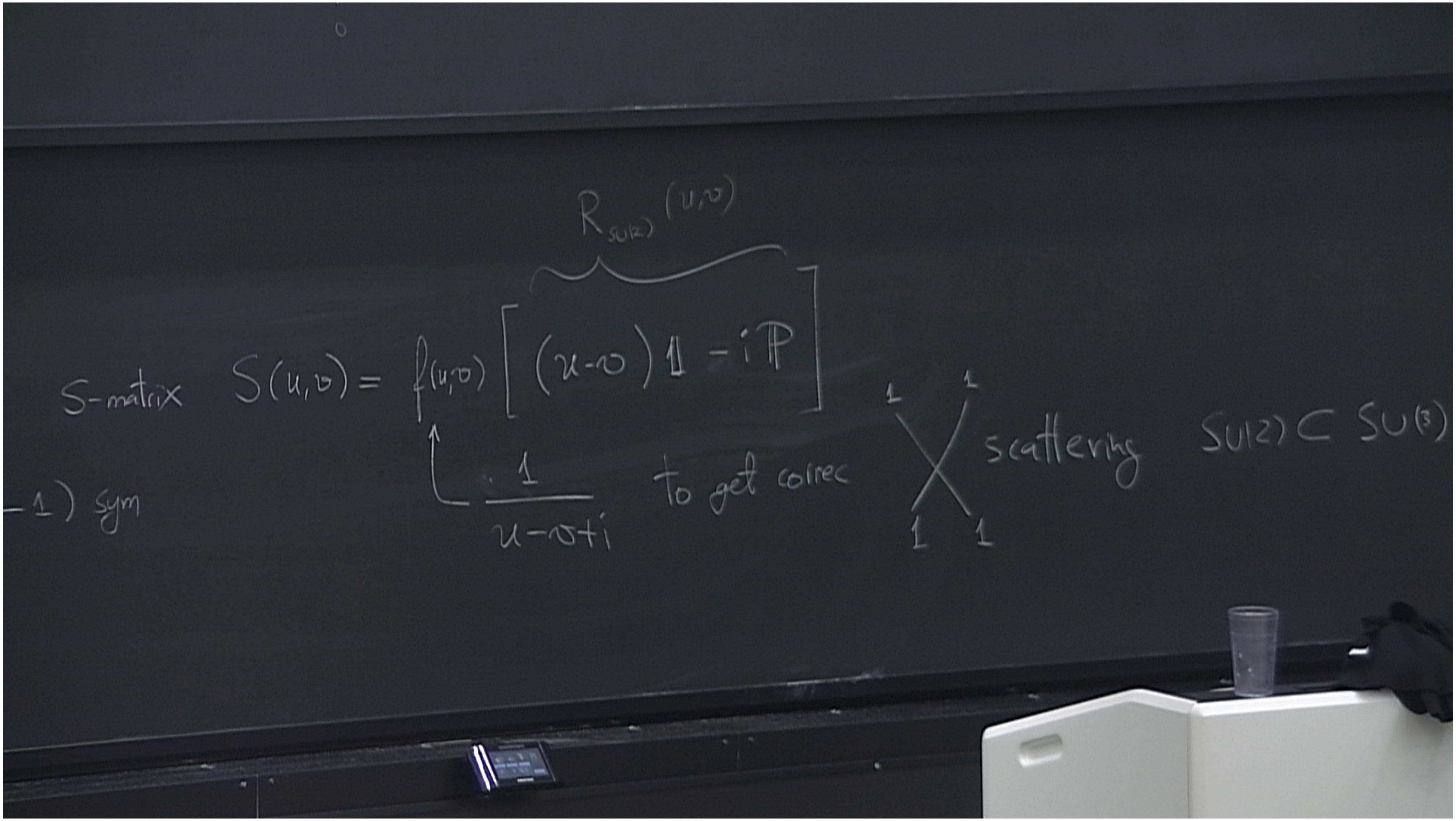
a', b', a, b

$\begin{pmatrix} a = 1,2 \\ b = 1,2 \end{pmatrix}$

4×4 matrix with SU(3−1) sym

S-matrix $S(u,v) = f(u,v)\left[\overbrace{(u-v)\,\mathbb{1} - i\,\mathbb{P}}^{R_{SU(2)}(u,v)}\right]$

$f(u,v) = \frac{1}{u-v+i}$ to get correc


S-matrix $S(u,v) = f(u,v)\left[\overbrace{(u-v)\mathbb{1} - i\mathbb{P}}^{R_{SU(2)}(u,v)}\right]$
$\frac{1}{u-v+i}$ to get correc
scattering $SU(2) \subset SU(3)$
-1) sym

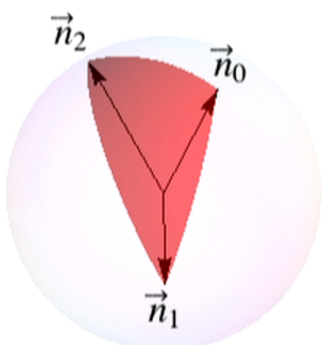
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 12:29 PM Pedro Vieira

spheretriangle

```
In[369]:= triangle = SphericalPlot3D[Boole[0 < ϕ < π / 4 ∧ θ > π / 2 + ϕ / 2], {θ, 0, Pi}, {ϕ, 0, 2 Pi}, PlotStyle → {Opacity[0.5], Red}, Mesh → False,
   Boxed → False, Axes → False, PlotPoints → 40];

In[414]:= picture = Show[triangle, Graphics3D[{Opacity[0.1], Sphere[{0, 0, 0}, 99 / 100]}], Graphics3D[Arrow[{{0, 0, 0}, {1, 0, 0}}]],
  Graphics3D[Arrow[{{0, 0, 0}, {1, 0, 0}}]],
  Graphics3D[Arrow[{{0, 0, 0}, {Sin[π / 2 + π / 8] Cos[π / 4], Sin[π / 2 + π / 8] Sin[π / 4], Cos[π / 2 + π / 8]}}]],
  Graphics3D[Arrow[{{0, 0, 0}, {Sin[π] Cos[π / 4], Sin[π] Sin[π / 4], Cos[π]}}]],
  Graphics3D[Text[Style["n⃗1", Large], 12 / 10 {Sin[π] Cos[π / 4], Sin[π] Sin[π / 4], Cos[π]}]],
  Graphics3D[Text[Style["n⃗0", Large], 12 / 10 {Sin[π / 2 + π / 8] Cos[π / 4], Sin[π / 2 + π / 8] Sin[π / 4], Cos[π / 2 + π / 8]}]],
  Graphics3D[Text[Style["n⃗2", Large], 12 / 10 {1, 0, 0}]], PlotRange → All, ViewPoint -> {1, 1 / 2, -2}]
```

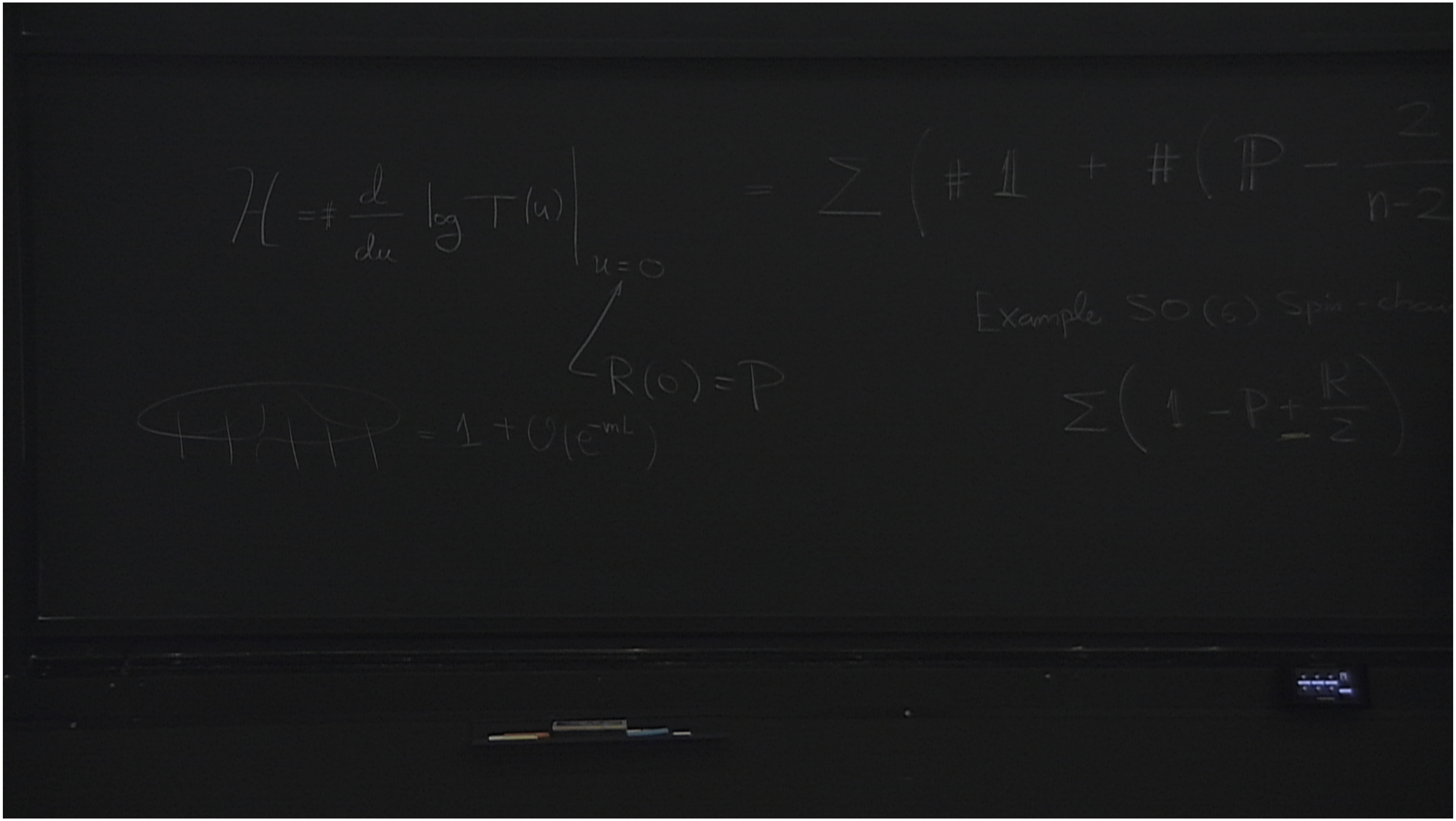
Out[414]=




```
In[415]:= Export[(NotebookDirectory[] <> "triangle.pdf"), picture, "AllowRasterization" → True, ImageSize → 360, ImageResolution → 600]

Out[415]= /Users/pvieira/Dropbox/Physics/Courses/PSI course/triangle.pdf
```

100%



$\mathcal{H} = \# \frac{d}{du} \log T(u) \Big|_{u=0}$
$= \sum \left(\# \mathbb{1} + \# \left(\mathbb{P} - \frac{2}{n-2}\right.\right.$
$R(0) = P$
$= 1 + \mathcal{O}(e^{-mL})$
Example SO(6) Spin-chain
$\sum \left(\mathbb{1} - \mathbb{P} \pm \frac{\mathbb{K}}{2} \right)$