Title: Explorations in String Theory -3

Date: Apr 08, 2015 11:30 AM

URL: http://pirsa.org/15040142

Abstract:

fixed by $H|\psi\rangle = E\,|\psi\rangle$ by looking

$$\psi(n_1) = e^{ip_1 n_1}$$

$$\psi(n_1, n_2) = e^{ip_1 n_1 + ip_2 n_2} + S(p_1, p_2)\, e^{ip_2 n_1 + ip_1 n_2} = || + X$$

$$\psi(n_1, n_2, n_3) = ||| + X| + |X + XX + XX + XX$$

etc diagonalize H

Periodicity $\Rightarrow$ Bethe equations

here Q_n is enough!

e^{ip_j} $\quad S(p_j, p_k)$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L \prod_{k \neq j}^{N} \frac{u_j - u_k - i}{u_j - u_k + i} = 1, \quad = 1$$

$$E = \sum_{j=1}^{N} \frac{\lambda}{u_j^2 + \frac{1}{4}} \qquad \epsilon(p_j)$$

close-by region

$\sum_{n=1}^{L} (\mathbb{1}-\mathbb{P})_{n\,n+1}$

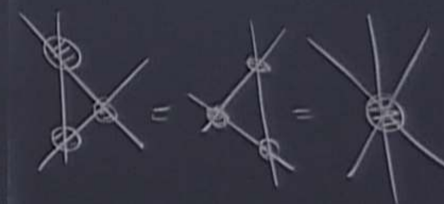
$\boxed{N=1}$ $\longleftarrow$ Always true $\Leftarrow$ translation invariance

$\boxed{N=2}$ $\longleftarrow$ non-trivial for $n_1 = n_2 - 1$ $\quad |\uparrow \dots \uparrow\underbrace{\downarrow\downarrow}_{\text{close-by excitations}}\uparrow \dots \uparrow\rangle$

but otherwise

consequence of Energy + Momentum Conservation

$\boxed{N \geqslant 3}$ $\longleftarrow$ Very non-trivial.

Even if $\exists$ higher charges $\Rightarrow$ 

it is still non-trivial that the ansatz works even for close-by magnons

$$(n_1) = e^{ip_1 n_1}$$

fixed by $H|\psi\rangle = E|\psi\rangle$ by looking at close-by re

$$(n_1, n_2) = e^{ip_1 n_1 + ip_2 n_2} + S(p_1, p_2)\, e^{ip_2 n_1 + ip_1 n_2} = || + X$$

$$(n_1, n_2, n_3) = ||| + X| + |X + XX + XX + |X$$

etc diagonalize $\mathcal{H} = \lambda \sum_{n=1}^{L} (\mathbb{1} - \mathbb{P})_{nn}$

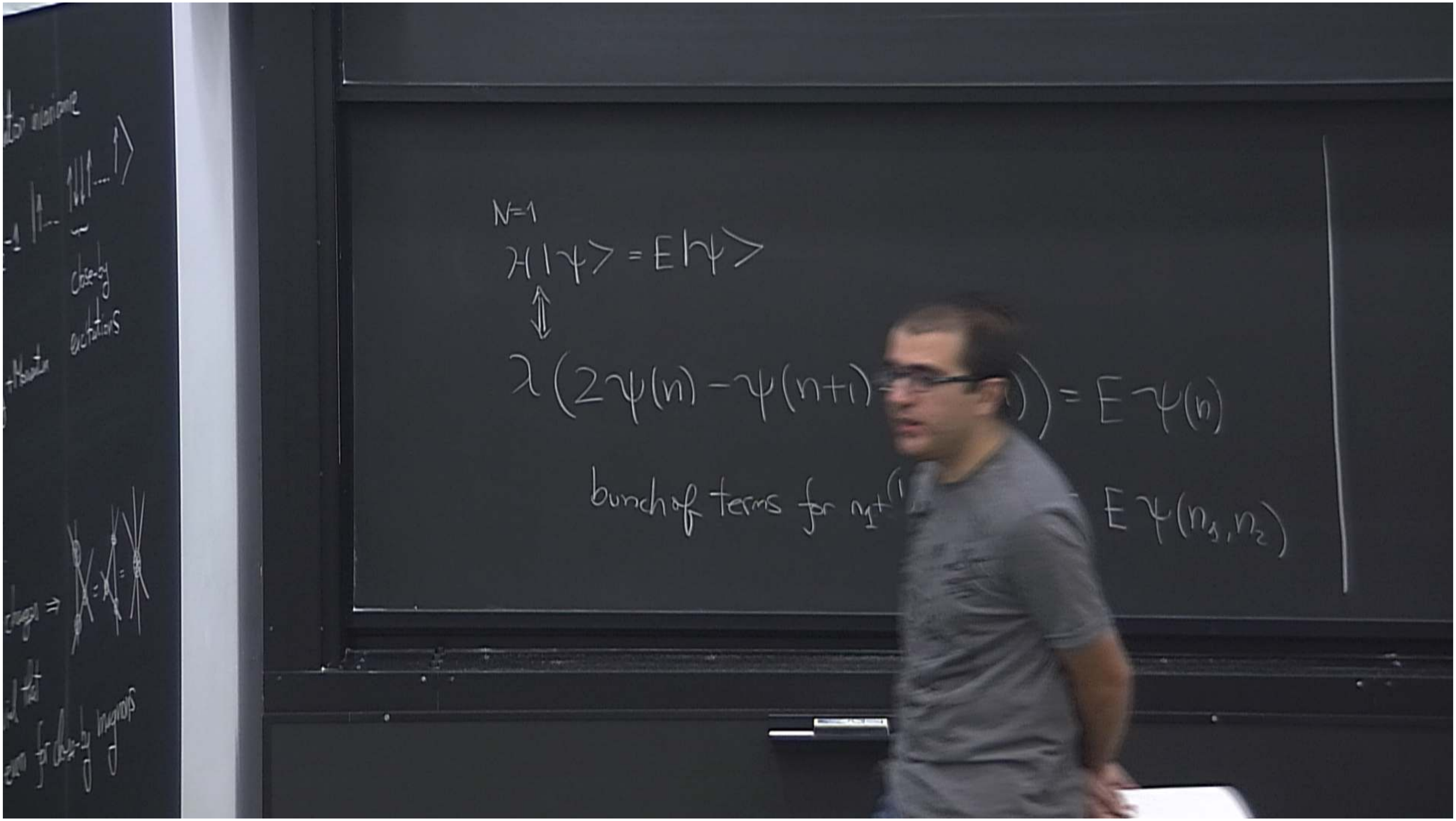
Periodicity $\Rightarrow$ Bethe equations

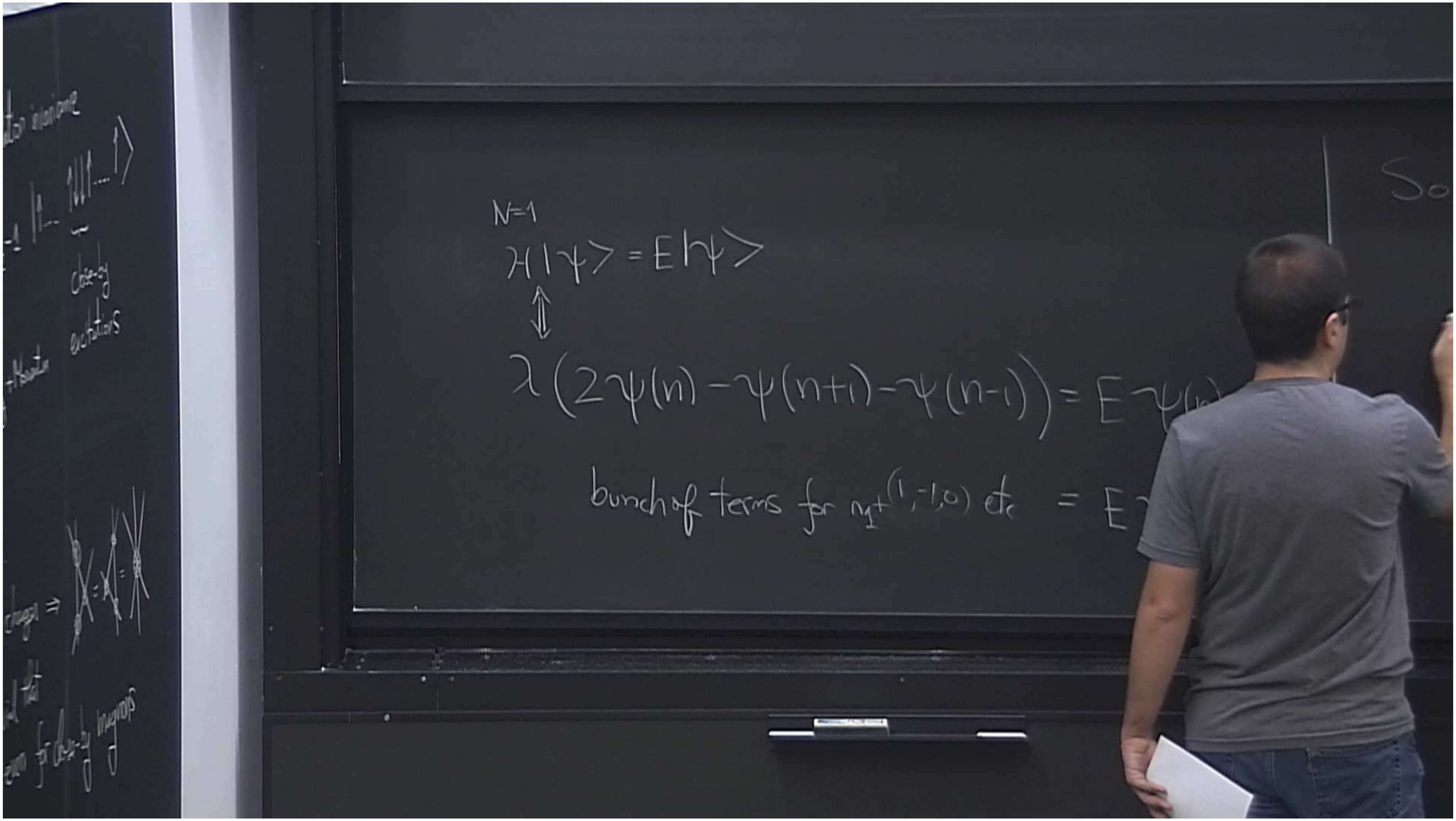
here Q_n is enough!

e^{ip_j} $\quad S(p_j, p_k)$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L \prod_{k\neq j}^{N} \frac{u_j - u_k - i}{u_j - u_k + i} = 1, \quad \ldots = 1 \qquad E = \sum_{j=1}^{N} \frac{\lambda}{u_j^2 + \frac{1}{4}}}$$

$\epsilon(p_j)$


N=1
H|ψ> = E|ψ>
λ(2ψ(n) − ψ(n+1)
) = E ψ(n)
bunch of terms for
E ψ(n1, n2)
close-by
excitations


N=1
$\mathcal{H}|\psi\rangle = E|\psi\rangle$
$\lambda\left(2\psi(n)-\psi(n+1)-\psi(n-1)\right)=E$
bunch of terms for $n_1+(1,-1,0)$ etc $= E$
So

$$\ldots)) = E\,\psi(n)$$

$$\ldots = E\,\psi(n_1, n_2)$$

So, where are Q_n?

$$-\sum_{n=1}^{L} \mathbb{P}_{n\,n+1} = \frac{1}{i}\frac{d}{du}\log \mathrm{Tr}\Big((u+i\mathbb{P})_{01}\cdots\cdots(u+i\mathbb{P}$$

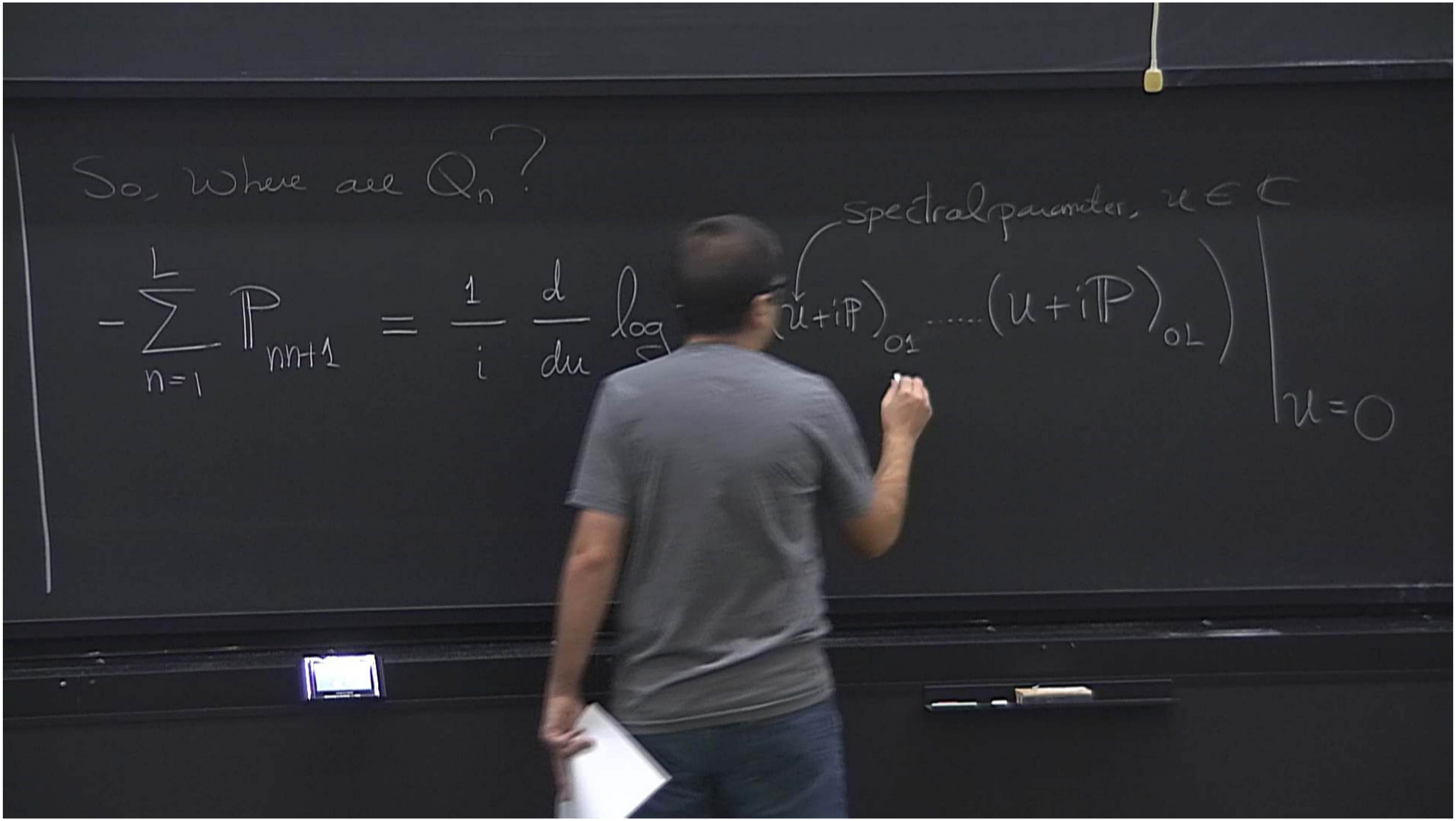
So, where are Q_n?

$$-\sum_{n=1}^{L} \mathbb{P}_{n\,n+1} = \frac{1}{i}\frac{d}{du}\log \mathrm{Tr}\left((u+i\mathbb{P})_{01}\cdots\cdots(u+i\mathbb{P})_{0L}\right)\Bigg|_{u=0}$$

So, where are Q_n?

spectral parameter, $u \in \mathbb{C}$

$$-\sum_{n=1}^{L} \mathbb{P}_{n\,n+1} = \frac{1}{i}\frac{d}{du}\log \mathrm{Tr}\Big((u+i\mathbb{P})_{01}\cdots\cdots(u+i\mathbb{P})_{0L}\Big)\Big|_{u=0}$$


So, where are Q_n?
spectral parameter, $u \in \mathbb{C}$
$-\sum_{n=1}^{L} \mathbb{P}_{n\,n+1} = \frac{1}{i}\frac{d}{du}\log \ldots (u+i\mathbb{P})_{01} \cdots\cdots (u+i\mathbb{P})_{0L} \Big) \Big|_{u=0}$

So, where are Q_n?

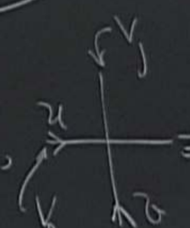
spectral parameter, $u \in \mathbb{C}$

$$-\sum_{n=1}^{L} \mathbb{P}_{n\,n+1} = \frac{1}{i}\,\frac{d}{du} \log \mathrm{Tr}\Big((u+i\mathbb{P})_{01} \cdots\cdots (u+i\mathbb{P})_{0L} \Big)\Big|_{u=0}$$

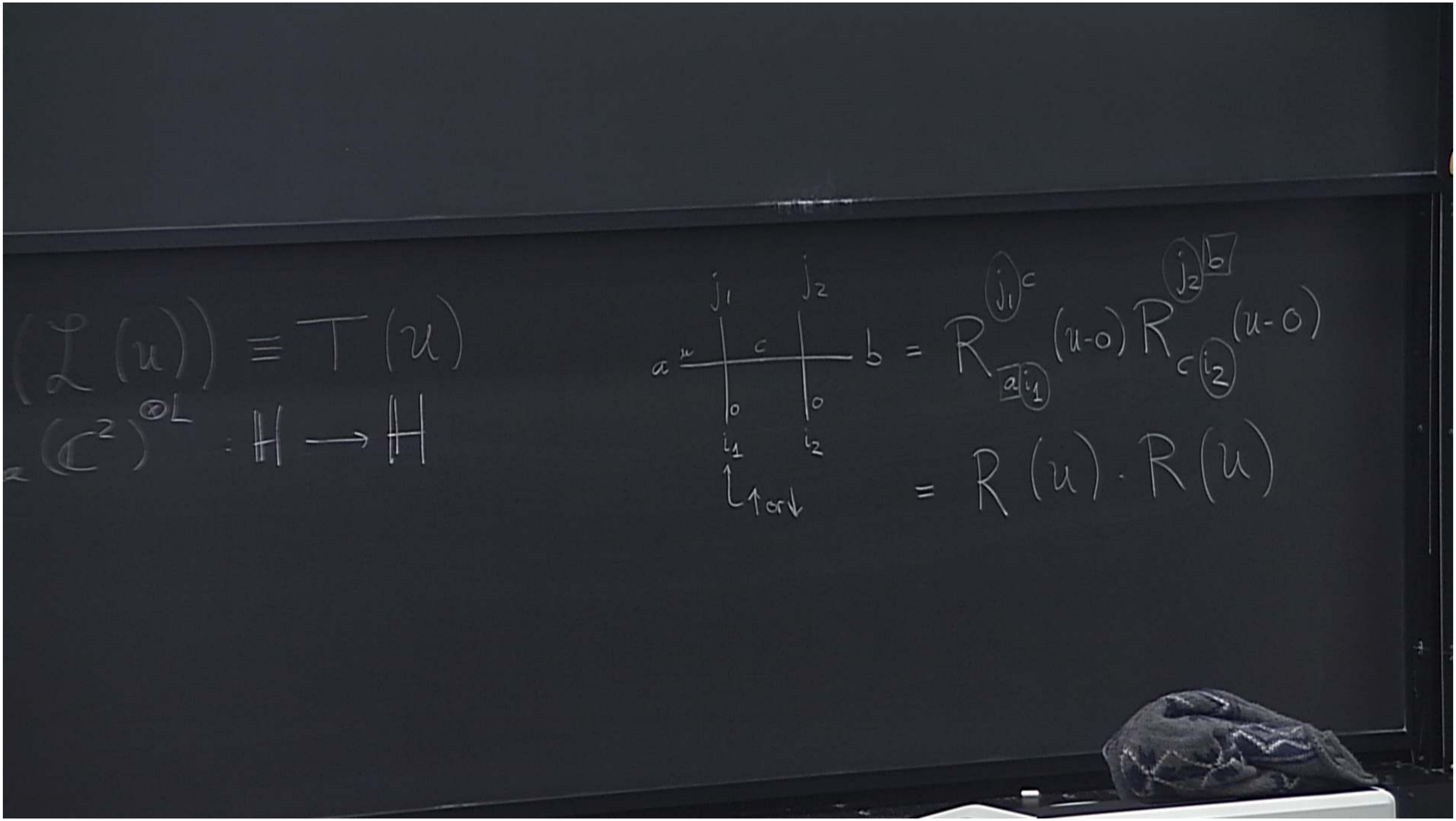
auxiliary space isomorphic to phys space $\cong \mathbb{C}^2$

R-matrix

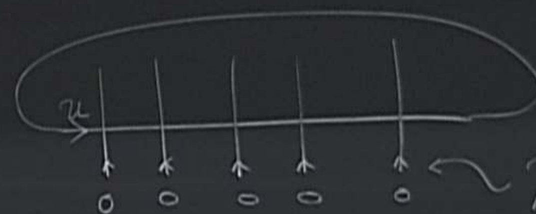
$$R_{0j} = u + i\mathbb{P}_{0j} \,,$$



$\equiv R_{0j}(u-v)$


$(\mathcal{L}(u)) \equiv T(u)$
$(\mathbb{C}^2)^{\otimes L} : \mathbb{H} \longrightarrow \mathbb{H}$
j_1 j_2
a u c b
0 0
i_1 i_2
$\uparrow$ or $\downarrow$
$= R^{j_1 c}_{a i_1}(u-0)\, R^{j_2 b}_{c i_2}(u-0)$
$= R(u) \cdot R(u)$

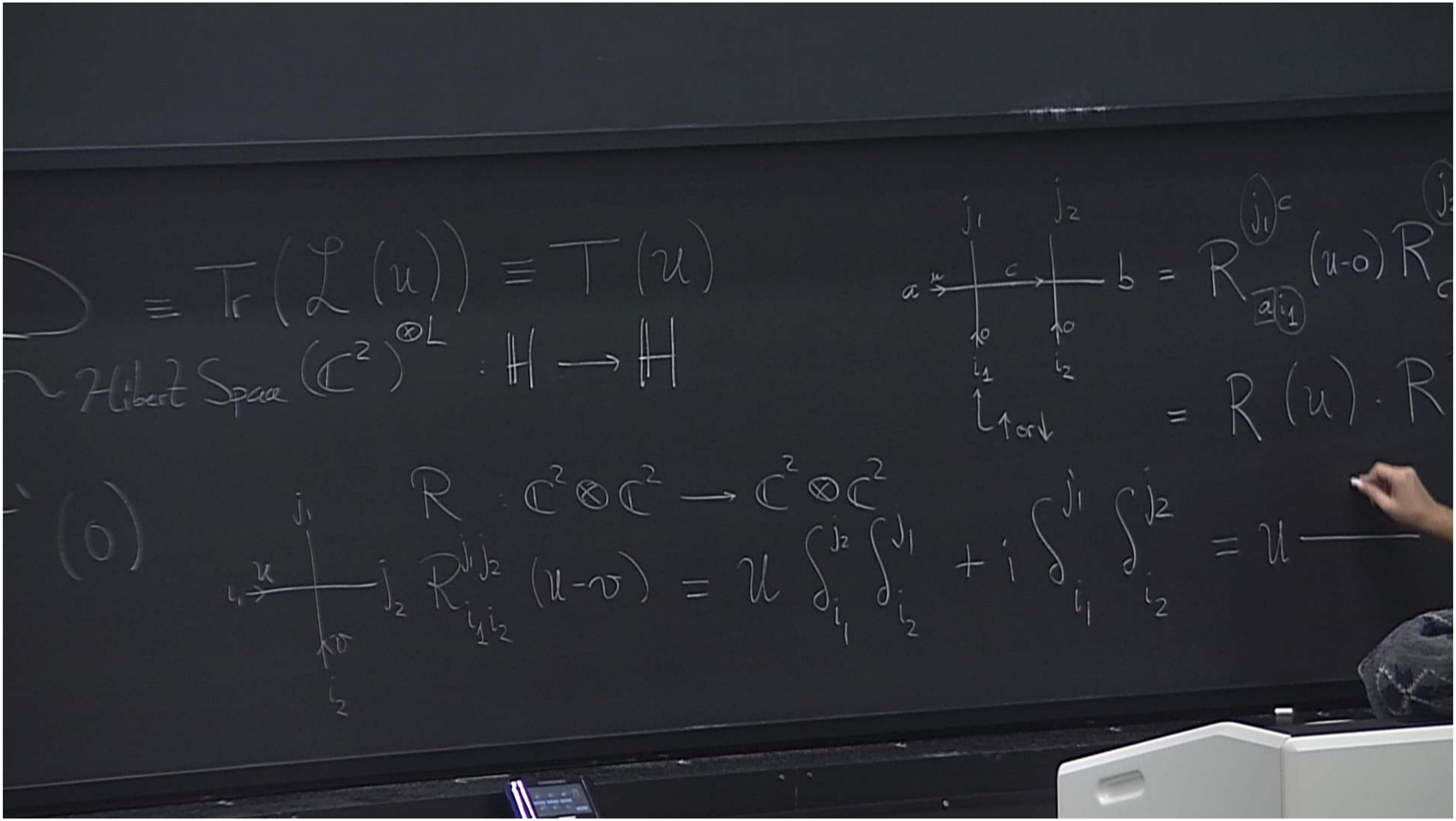
$\mathrm{Tr}(\ldots\ldots) =$



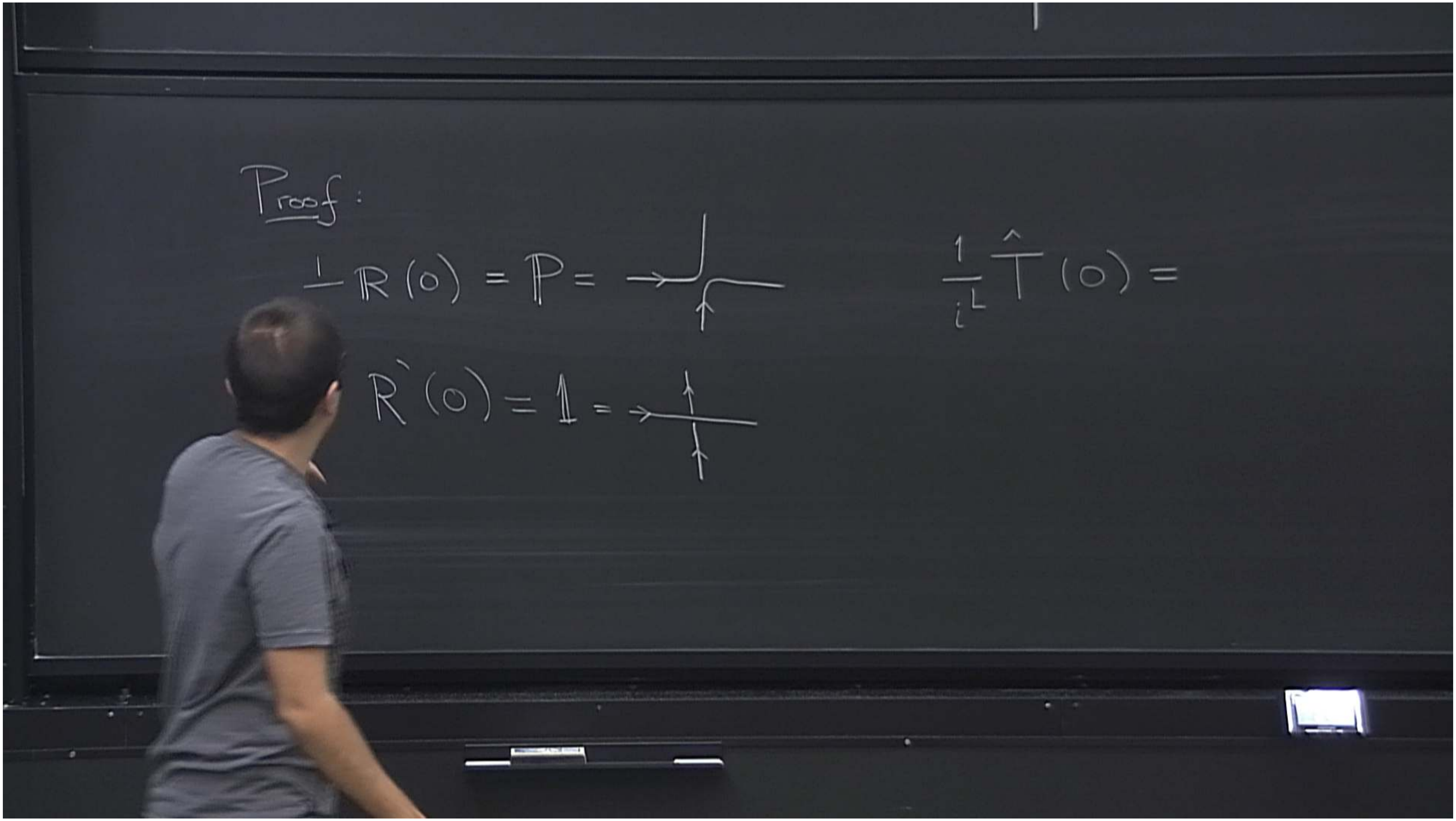
$\equiv \mathrm{Tr}(\mathcal{L}(u)) \equiv T(u$

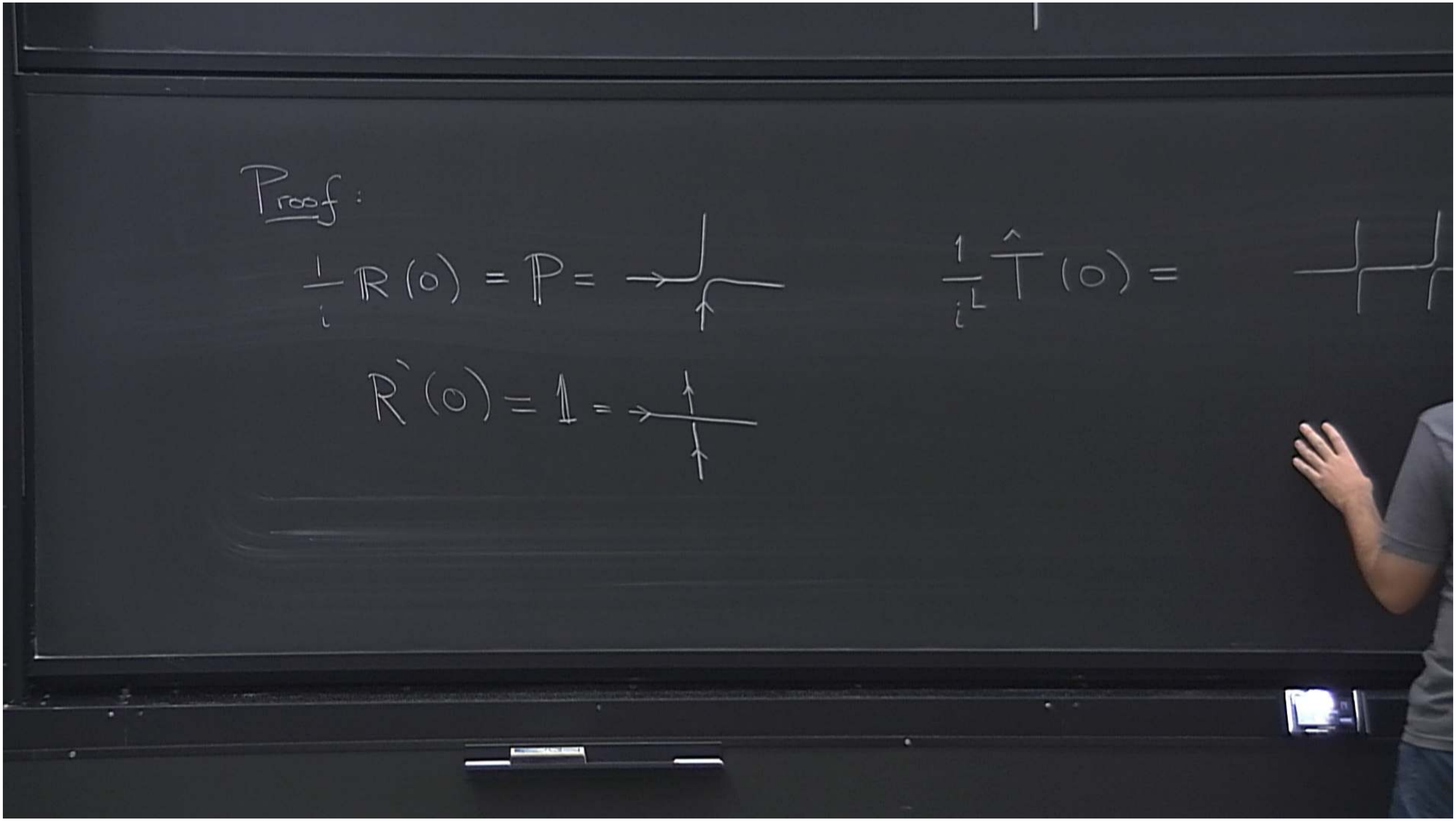
Hilbert Space $(\mathbb{C}^2)^{\otimes L}$: $\mathbb{H} \longrightarrow \mathbb{H}$

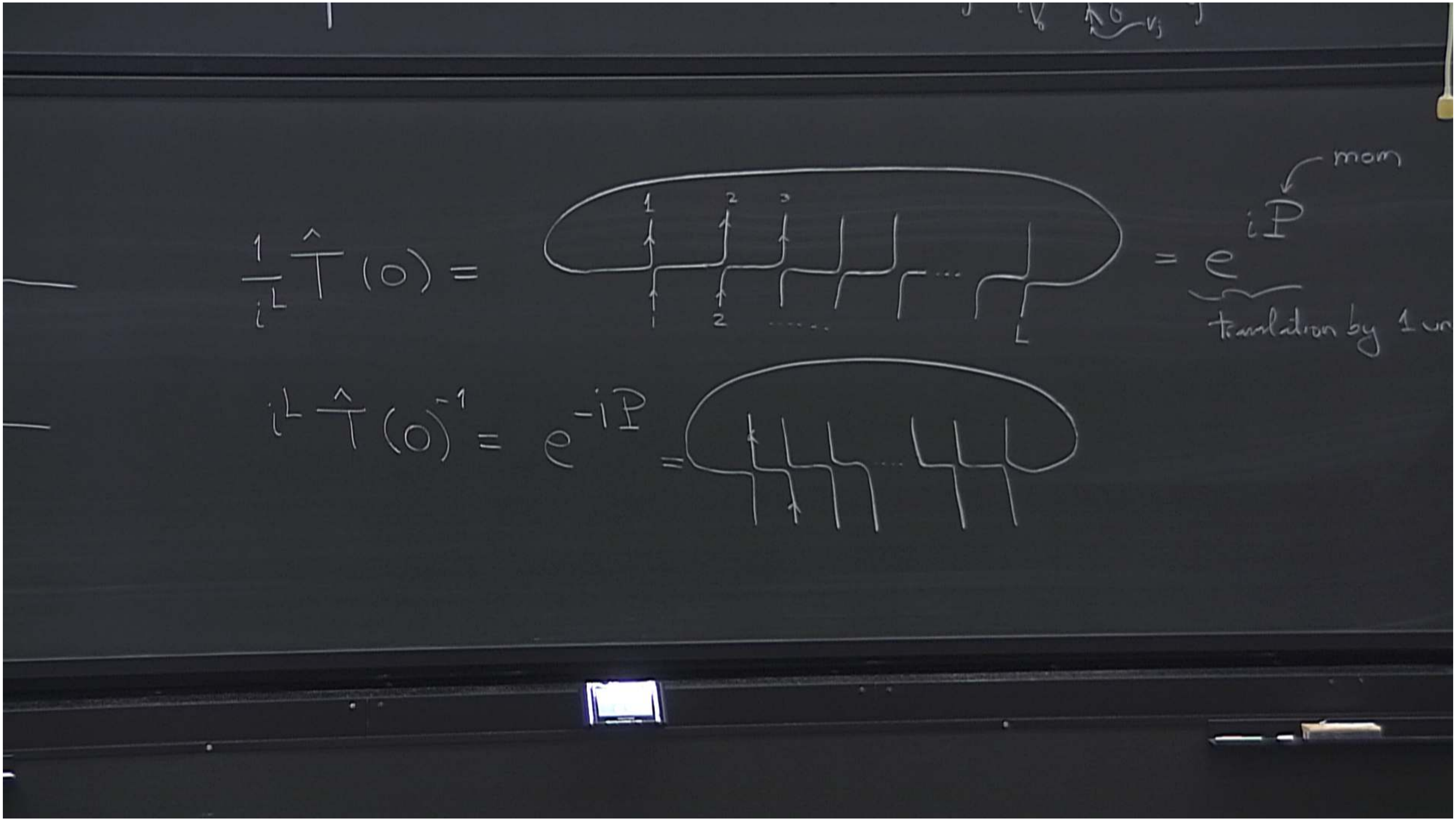
$$\frac{d}{du} \log \hat{T}(u)\Big|_{u=0} \equiv T^{-1}(0)\, T'(0)$$

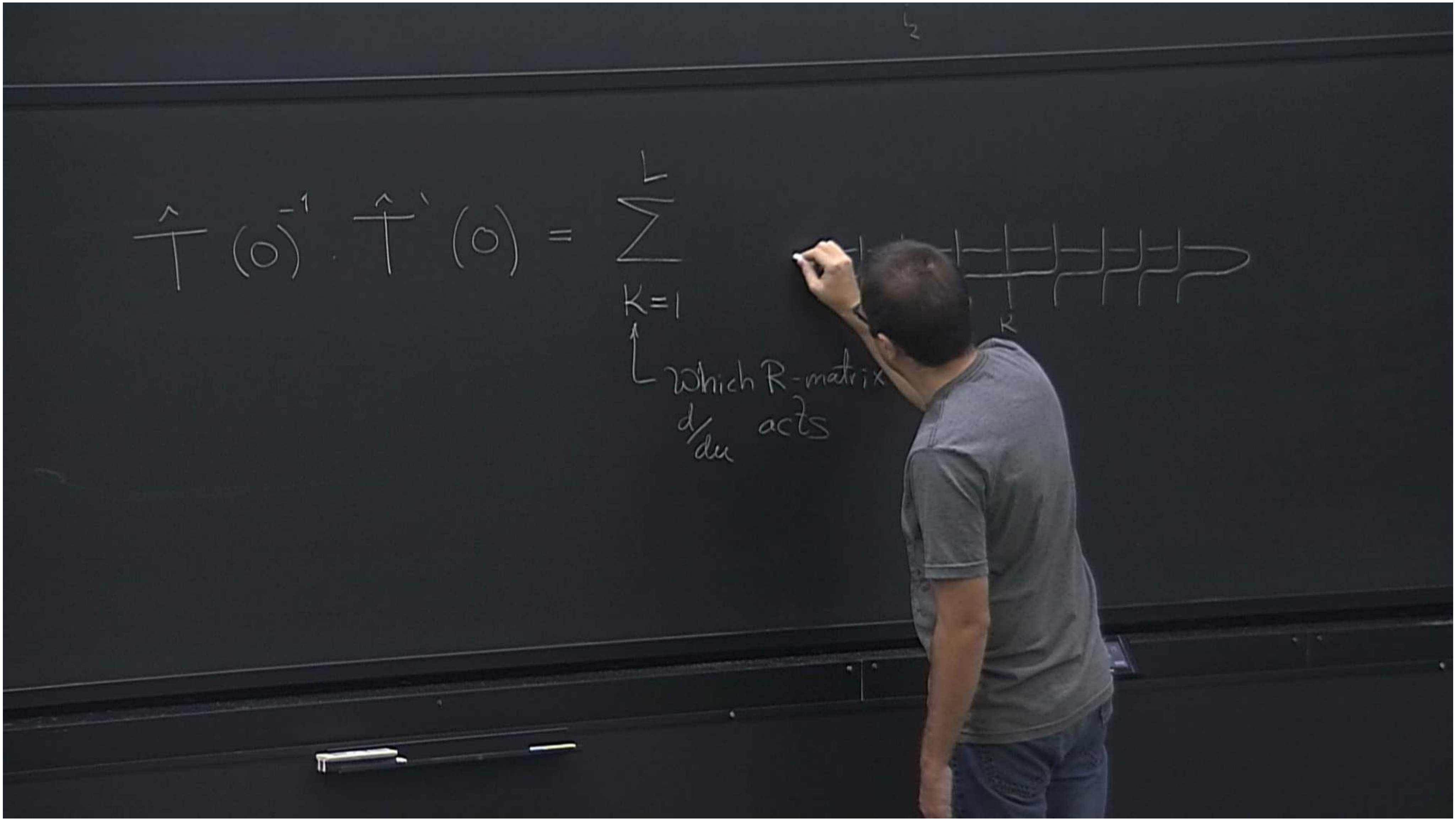


≡ Tr(ℒ(u)) ≡ T(u)
Hilbert Space (ℂ²)^⊗L : ℍ ⟶ ℍ
(0)
R : ℂ²⊗ℂ² ⟶ ℂ²⊗ℂ²
R^{j1 j2}_{i1 i2} (u − v) = u δ^{j2}_{i1} δ^{j1}_{i2} + i δ^{j1}_{i1} δ^{j2}_{i2} = u
j1 j2
a u c b = R^{j1 c}_{a i1} (u−0) R
i1 i2
↑ or ↓
= R(u) · R

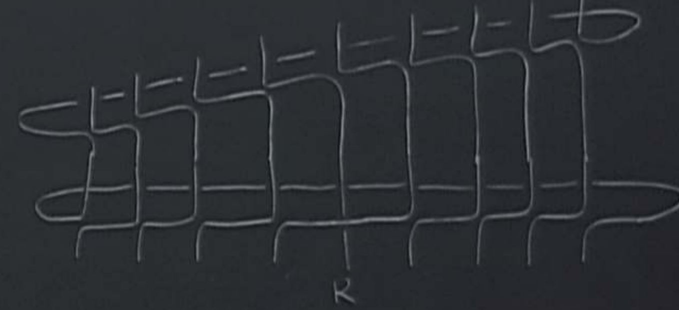

Proof:
$\frac{1}{i^L} \mathbb{R}(0) = \mathbb{P} =$
$\check{R}(0) = \mathbb{1} =$
$\frac{1}{i^L} \hat{T}(0) =$


Proof:
$\frac{1}{i} \mathbb{R}(0) = \mathbb{P} =$
$R^{`}(0) = \mathbb{1} =$
$\frac{1}{i^L} \hat{T}(0) =$


$\frac{1}{i^L}\hat{T}(0) =$
$= e^{iP}$
mom
translation by 1 un
$i^L\hat{T}(0)^{-1} = e^{-iP} =$

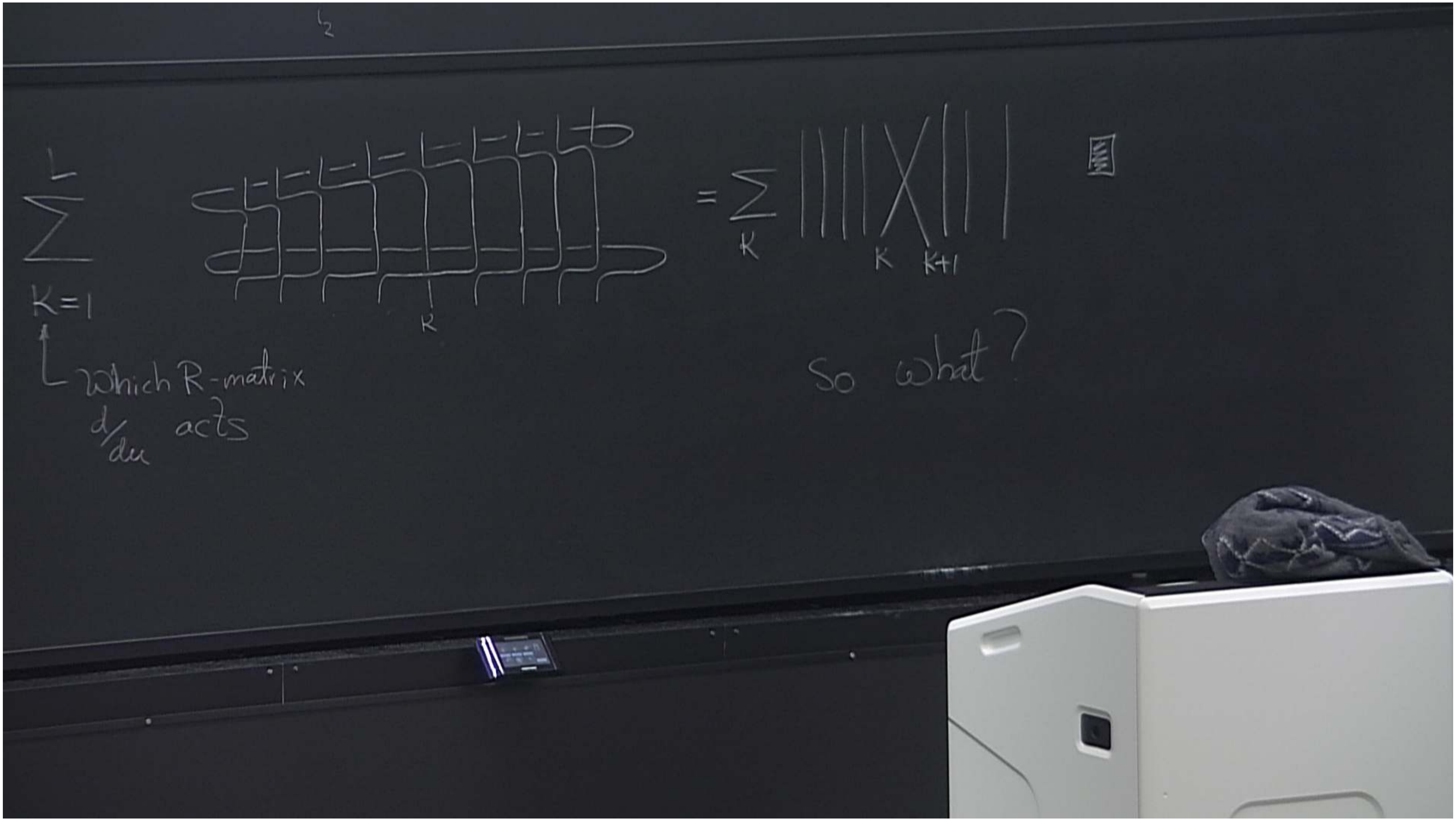

$\hat{T}(0)^{-1} \cdot \hat{T}'(0) = \sum_{K=1}^{L}$
Which R-matrix
d/du acts
R

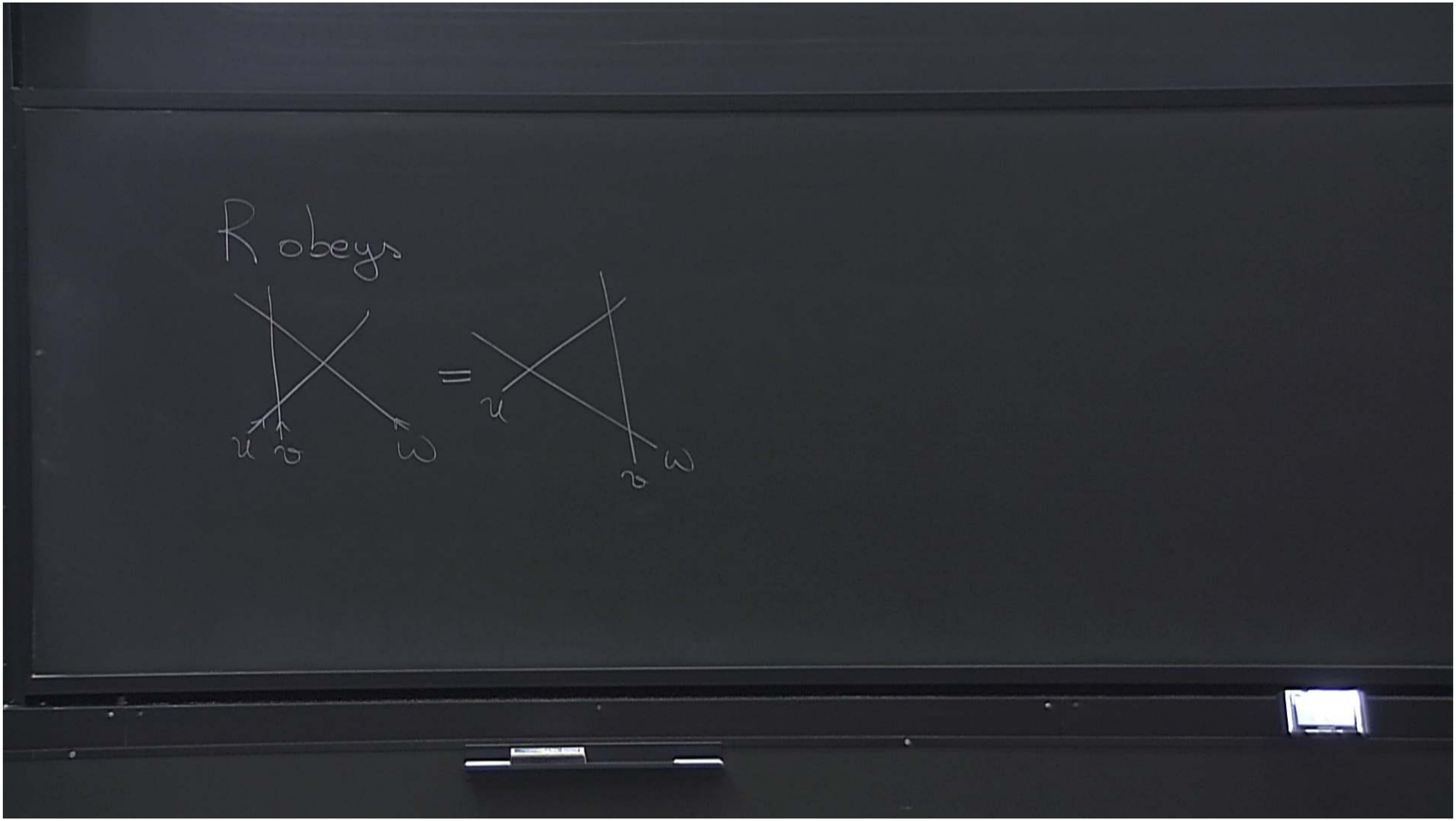
$$\hat{T}(0)^{-1} \cdot \hat{T}'(0) = \sum_{k=1}^{L}$$

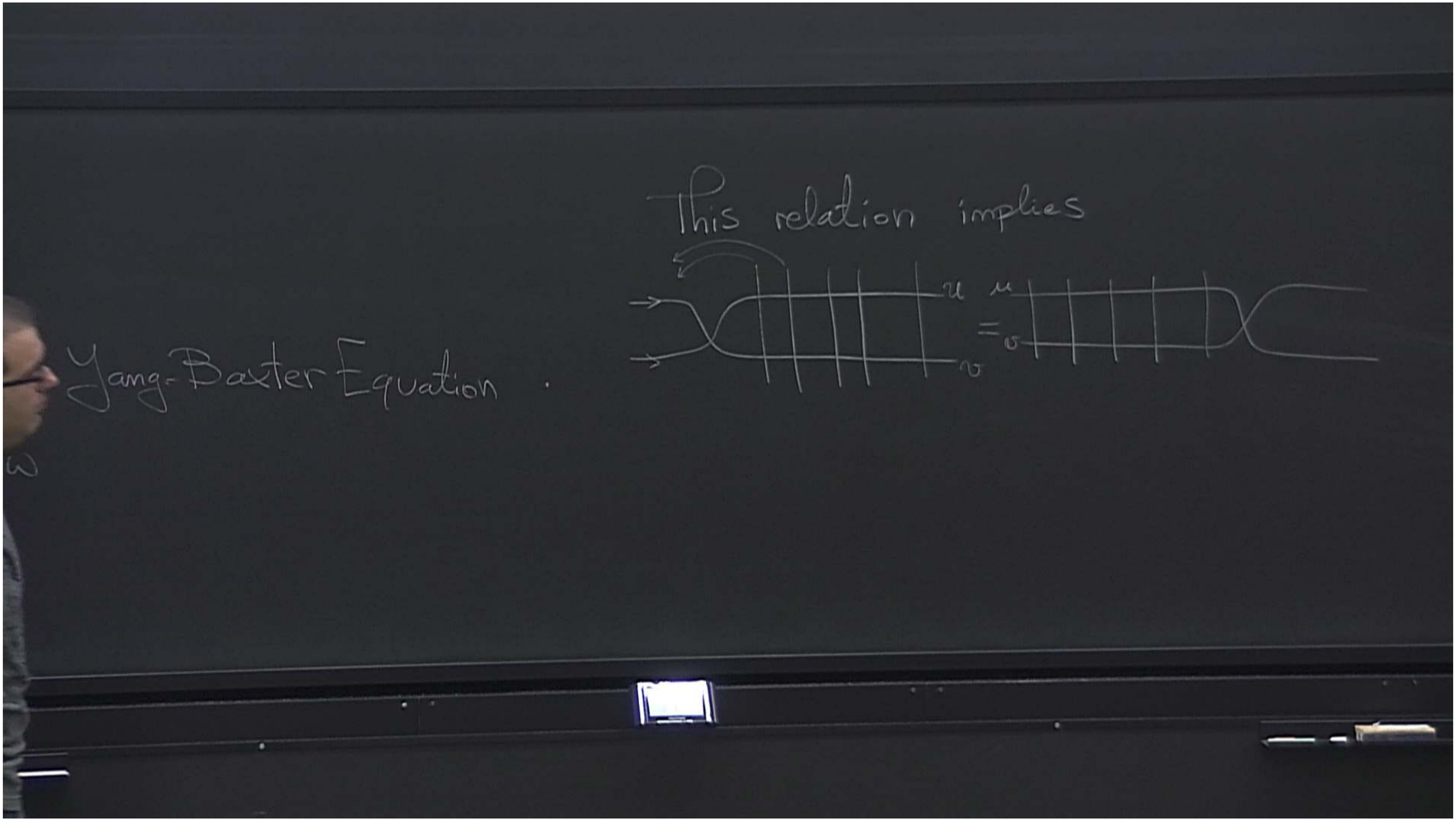


$$= \sum_{k}$$

which R-matrix d/du acts

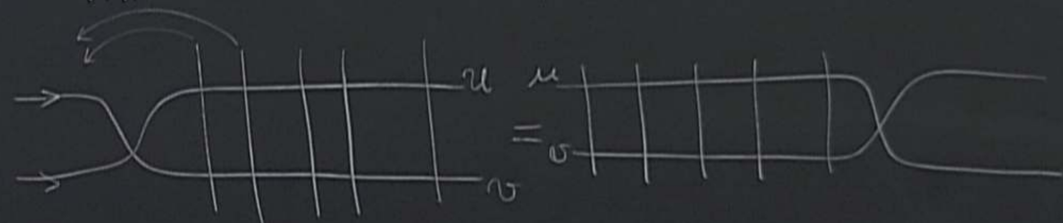

L
K=1
K
= Σ
K
K K+1
So what?
Which R-matrix
d/du acts


R obeys
u v w
=
u
v w


Yang-Baxter Equation .
This relation implies
u
v
=
u
v

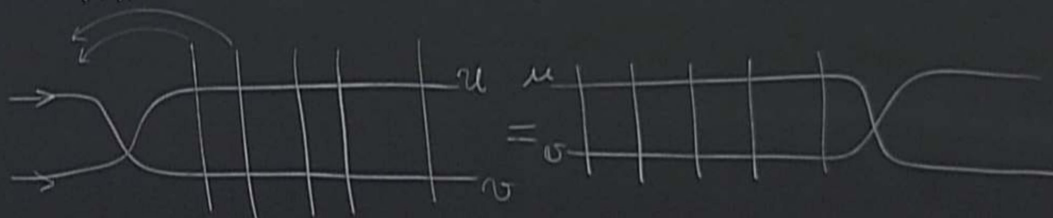
ation .

This relation implies



$$R_{o_1 o_2}(u-v)\, L_{o_1}(u)\, L_{o_2}(v) = L_{o_2}(v)\, L_{o_1}(u)\, R_{o_1 o_2}(u-v)$$

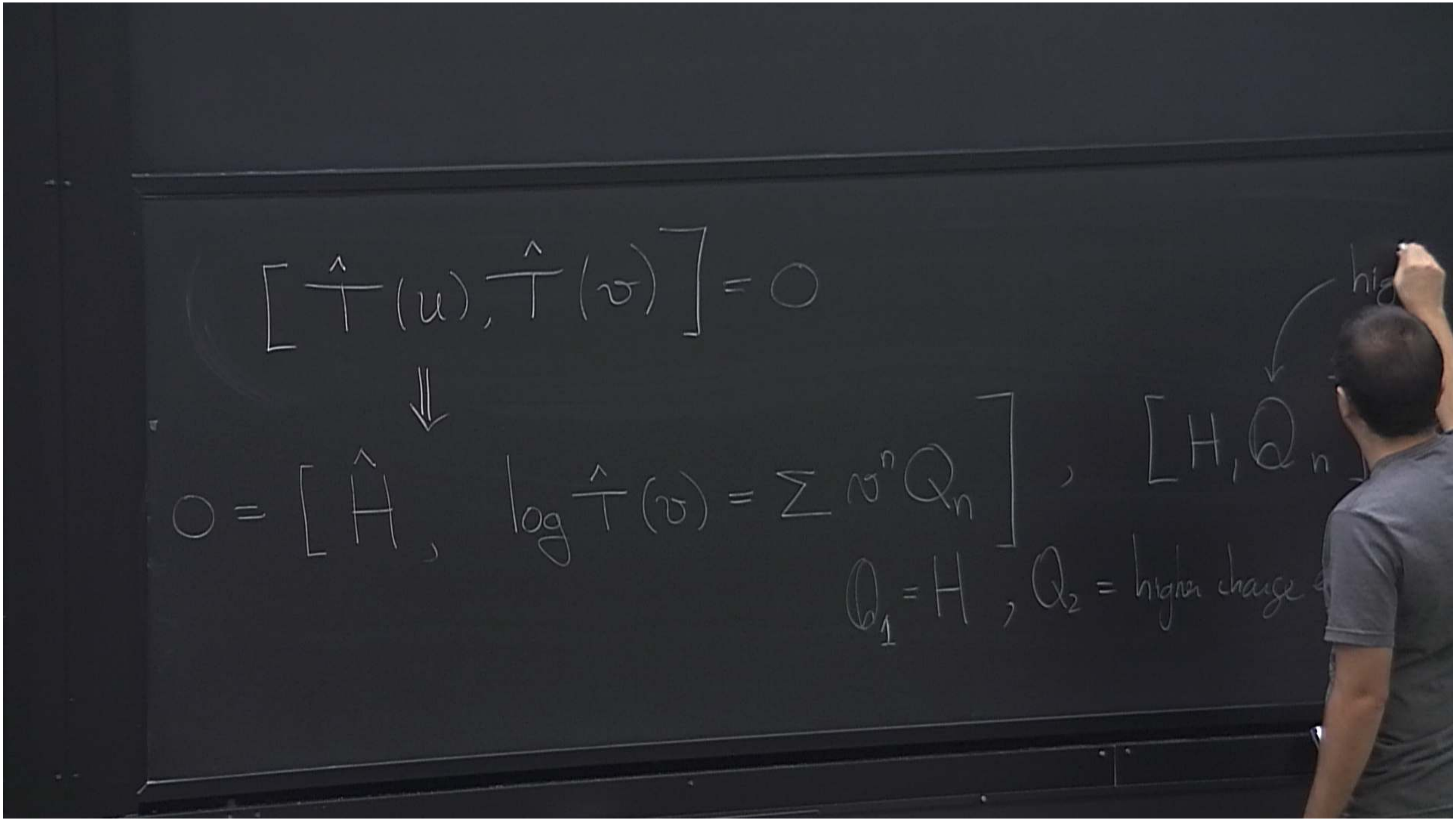
This relation implies



vation .

$$R_{o_1 o_2}(u-v)\, L_{o_1}(u)\, L_{o_2}(v) = L_{o_2}(v)\, L_{o_1}(u)\, R_{o_1 o_2}(u-v) \Rightarrow$$

multiply


$[\hat{T}(u), \hat{T}(v)] = 0$
$\Downarrow$
$0 = [\hat{H}, \log \hat{T}(v) = \sum v^n Q_n]$, $[H, Q_n$
$Q_1 = H$, $Q_2 =$ higher charge

$$] = 0$$

higher non-local charges we were after!

$$) = \sum \nu^n Q_n \Big] , \quad [H, Q_n] = 0$$

$Q_1 = H$, Q_2 = higher charge etc. . . .

higher non-local charges we were after!

$$[H, Q_n] = 0$$

Main Ingredients:

① R-matrix with right sym.

= higher charge etc....

higher non-local charges we were after!

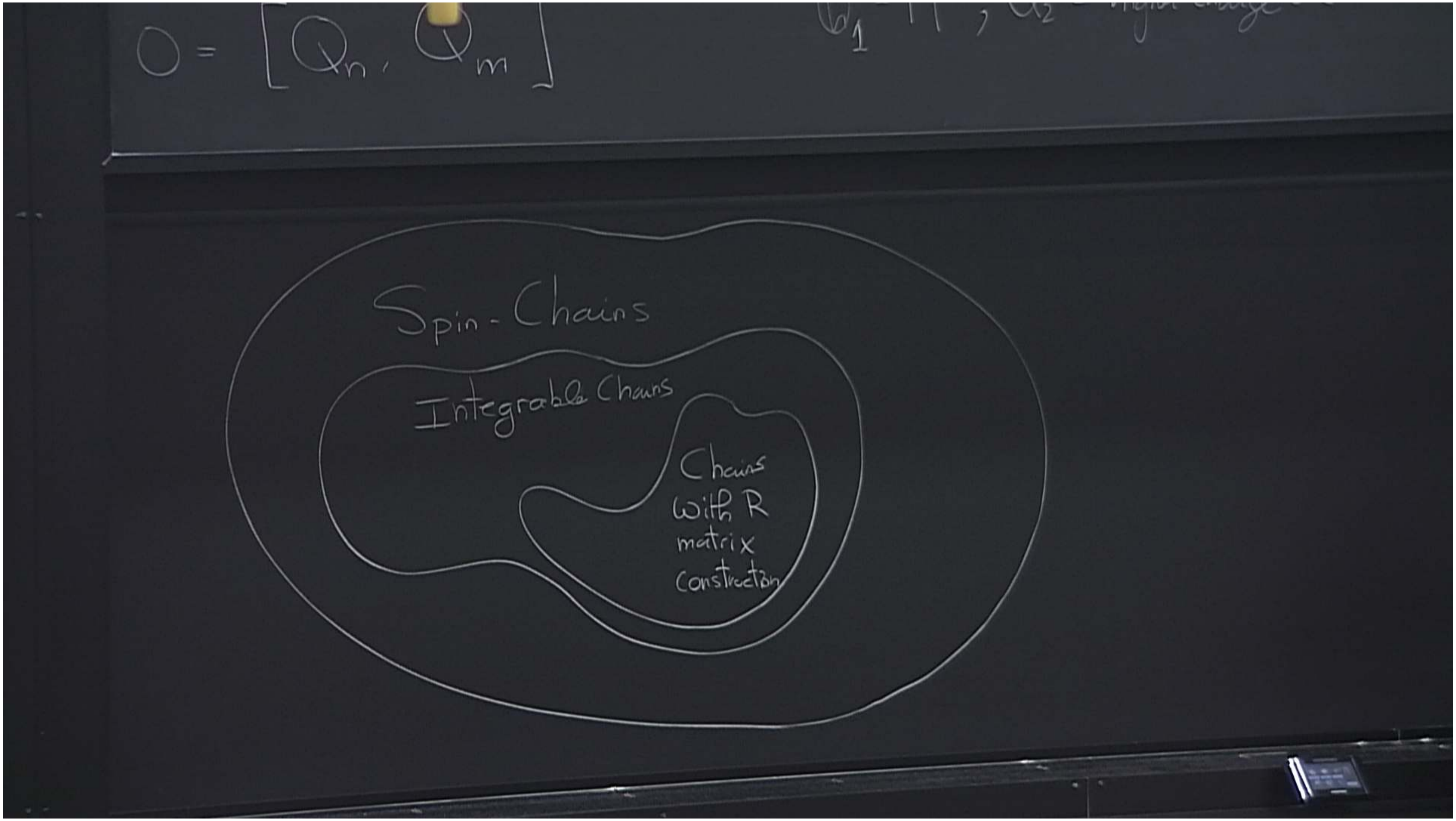
$$[H, Q_n] = 0$$

higher charge etc ....

(our case
$R = a\mathbb{1} + bP$)

Main Ingredients:

1. R-matrix with right sym.
2. R obeys YB (fixes rel coeff of inv tensors)


$O = [Q_n , Q_m]$
Spin-Chains
Integrable Chains
Chains with R matrix construction