

Title: Explorations in String Theory -3

Date: Apr 08, 2015 11:30 AM

URL: <http://pirsa.org/15040142>

Abstract:

$$\psi(n_1) = e^{iP_1 n_1}$$

$$\psi(n_1, n_2) = e^{iP_1 n_1 + iP_2 n_2} + S(P_1, P_2) e^{iP_2 n_1 + iP_1 n_2} = || + \times$$

$$\psi(n_1, n_2, n_3) = ||| + |X| + |X + \cancel{X} + \cancel{X} + \cancel{X} \text{ etc diagonalize } \mathcal{H}$$

fixed by $\mathcal{H}|\psi\rangle = E|\psi\rangle$ by looking

Periodicity \Rightarrow Bethe equations
 here Q_n is enough!

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L \prod_{k \neq j}^N \frac{u_j - u_k - i}{u_j - u_k + i} = 1, \quad \prod_{j=1}^N \frac{u_j + i/2}{u_j - i/2} = 1$$

$$E = \sum_{j=1}^N \frac{\lambda}{u_j^2 + \frac{1}{4}} \in \mathcal{E}(P_j)$$

close-by region

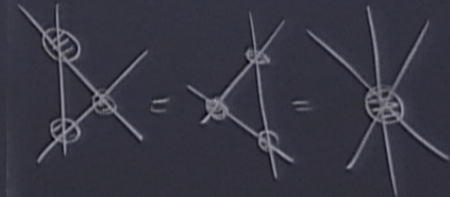
$N=1$ ← Always true ← translation invariance

$N=2$ ← non-trivial for $n_1 = n_2 - 1$ | $\uparrow \dots \uparrow \downarrow \downarrow \uparrow \dots \uparrow$ >
but otherwise
consequence of Energy + Momentum Conservation
close-by excitations

$$\sum_{n=1}^L (1-P)_{n,n+1}$$

$N \geq 3$ ← Very non-trivial.

Even if \exists higher charges \Rightarrow
it is still non-trivial that
the ansatz works even for close-by magnons



fixed by $H|\psi\rangle = E|\psi\rangle$ by looking at close-by re

$(n_1) = e^{iP_1 n_1}$
 $(n_1, n_2) = e^{iP_1 n_1 + iP_2 n_2} + S(P_1, P_2) e^{iP_2 n_1 + iP_1 n_2} = || + \times$
 $(n_1, n_2, n_3) = ||| + \times| + |X + \times\times + \times\times + \times\times$ etc diagonalize $\mathcal{H}(-\lambda \sum_{n=1}^L (1-P)_n)$

Periodicity \Rightarrow Bethe equations
 here Q_n is enough!

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L \frac{\prod_{k \neq j}^N \frac{u_j - u_k - i}{u_j - u_k + i}}{S(P_j, P_k)} = 1, \quad \prod_{j=1}^N \frac{u_j + i/2}{u_j - i/2} = 1$$

$$E = \sum_{j=1}^N \frac{\lambda}{u_j^2 + \frac{1}{4}} \int E(P_j)$$

action invariance
 case-by
 equations
 changes \Rightarrow
 even for case-by invariance

$$N=1$$

$$H|\psi\rangle = E|\psi\rangle$$



$$\lambda (2\psi(n) - \psi(n+1) - \psi(n-1)) = E\psi(n)$$

bunch of terms for $n_1 + 1$

$$= E\psi(n_1, n_2)$$

stationary wave
case-by
equations
crossing
for case-by
inverses

$$N=1$$
$$H|\psi\rangle = E|\psi\rangle$$



$$\lambda(2\psi(n) - \psi(n+1) - \psi(n-1)) = E\psi(n)$$

bunch of terms for $n \pm 1, \dots$ etc = E

So

So, where are Q_n ?

$$-\sum_{n=1}^L P_{n+1} = \frac{1}{i} \frac{d}{du} \log \text{Tr} \left((u+iP)_{01} \dots (u+iP) \right)$$

$$1) = E \psi(n)$$

$$k = E \psi(n_1, n_2)$$

So, where are all Q_n ?

$$-\sum_{n=1}^L P_{nm+1} = \frac{1}{i} \frac{d}{du} \log \text{Tr} \left((u+iP)_{o1} \dots (u+iP)_{oL} \right) \Big|_{u=0}$$

So, where are Q_n ?

$$-\sum_{n=1}^L P_{n+1}$$

$$= \frac{1}{i} \frac{d}{du} \log \text{Tr} \left((u+iP)_{01} \dots (u+iP)_{0L} \right)$$

spectral parameter, $u \in \mathbb{C}$

$u=0$

So, where are Q_n ?

$$-\sum_{n=1}^L P_{n+1}$$

$$= \frac{1}{i} \frac{d}{du} \log$$

spectral parameter, $u \in \mathbb{C}$
 $(u+iP)_{01} \dots (u+iP)_{0L}$

$u=0$

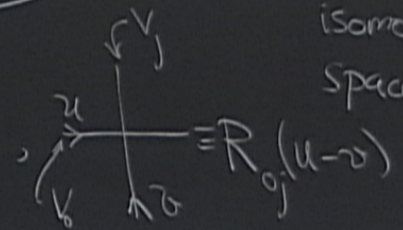
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spectral parameter, $u \in \mathbb{C}$

$u=0$

← R-matrix
 $R_{oj} = u + iP_{oj}$



auxiliary space
 isomorphic to phys
 space $\cong \mathbb{C}^2$

$$\begin{aligned} \mathcal{L}(u) &\equiv T(u) \\ (\mathbb{C}^2)^{\otimes L} : \mathbb{H} &\rightarrow \mathbb{H} \end{aligned}$$

$$\begin{array}{c}
 j_1 \quad j_2 \\
 a \text{---} | \text{---} c \text{---} | \text{---} b \\
 | \quad | \\
 o \quad o \\
 i_1 \quad i_2 \\
 \uparrow \text{ or } \downarrow
 \end{array}
 = R_{a i_1}^{j_1 c} (u-o) R_{c i_2}^{j_2 b} (u-o)$$

$$= R(u) \cdot R(u)$$

$$\text{Tr}(\dots) = \begin{array}{c} \text{---} \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \end{array} \equiv \text{Tr}(\mathcal{L}(u)) \equiv T(u)$$

\leftarrow Hilbert Space $(\mathbb{C}^2)^{\otimes L} : \mathbb{H} \rightarrow \mathbb{H}$

$$\left. \frac{d}{du} \log \hat{T}(u) \right|_{u=0} \equiv T^{-1}(0) T'(0)$$

$$\equiv \text{Tr}(\mathcal{L}(u)) \equiv T(u)$$

Hilbert Space $(\mathbb{C}^2)^{\otimes L} : \mathbb{H} \rightarrow \mathbb{H}$

A diagram showing a 2x2 grid with horizontal axis labeled j_1 and vertical axis labeled j_2 . The origin is labeled u . The right edge is labeled c and the right vertex is labeled b . The bottom edge is labeled i_1 and the bottom vertex is labeled i_2 . A vertical arrow labeled L points up or down between i_1 and i_2 .

$$= R_{\substack{j_1 \\ a, i_1}}^{j_1, c} (u-o) R_{i_2}^{j_2} = R(u) \cdot R$$

(0)

A diagram showing a 2x2 grid with horizontal axis labeled j_1 and vertical axis labeled j_2 . The origin is labeled u . The bottom edge is labeled i_1 and the bottom vertex is labeled i_2 .

$$R : \mathbb{C}^2 \otimes \mathbb{C}^2 \rightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$R_{\substack{j_1, j_2 \\ i_1, i_2}}^{j_1, j_2} (u-o) = u \int_{i_1}^{j_1} \int_{i_2}^{j_2} + i \int_{i_1}^{j_1} \int_{i_2}^{j_2} = u$$

Proof:

$$\frac{1}{iL} R(0) = P = \begin{array}{c} \text{---} \text{---} \\ | \\ \text{---} \end{array}$$

$$\frac{1}{iL} \hat{T}(0) =$$

$$R'(0) = \mathbb{1} = \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array}$$

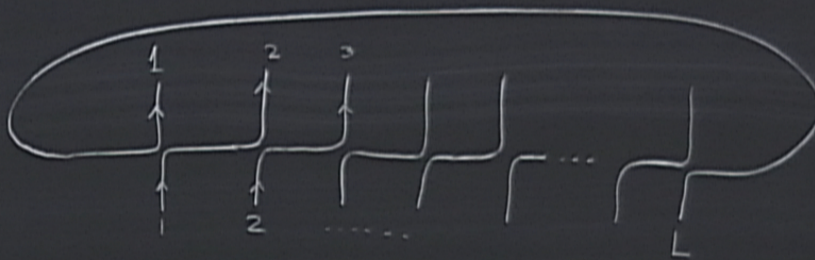
Proof:

$$\frac{1}{i} R(0) = P = \begin{array}{c} \text{---} \downarrow \\ \text{---} \end{array}$$

$$\frac{1}{iL} \hat{T}(0) = \begin{array}{c} \text{---} \downarrow \\ \text{---} \end{array}$$

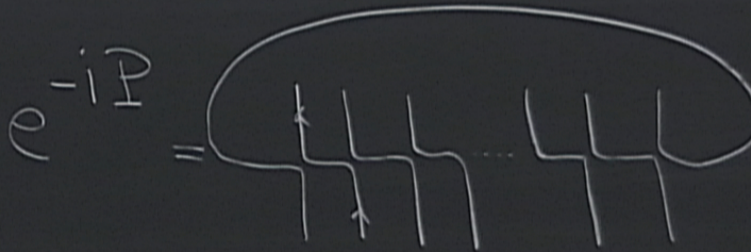
$$R'(0) = 1 = \begin{array}{c} \uparrow \\ \text{---} \\ \downarrow \end{array}$$

$$\frac{1}{i^L} \hat{T}(0) =$$



$= e^{iP}$
 translation by 1 unit
 mom

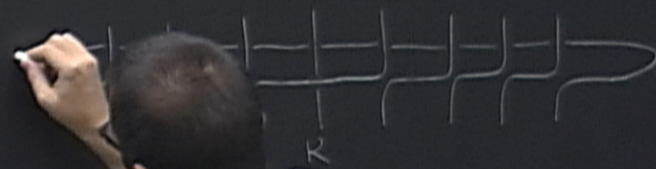
$$i^L \hat{T}(0)^{-1} =$$



$$= e^{-iP}$$

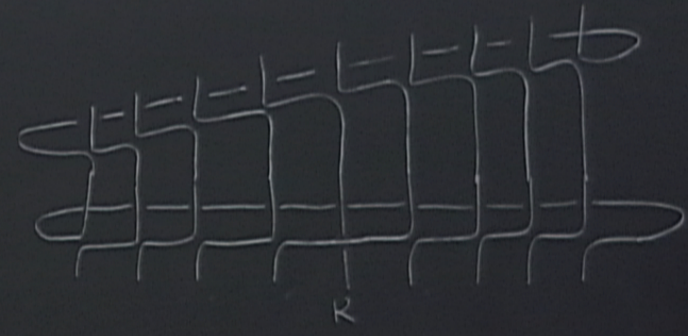
$$\hat{T}(0)^{-1} \cdot \hat{T}'(0) = \sum_{K=1}^L$$

\uparrow
 Which R-matrix
 $\frac{d}{du}$ acts



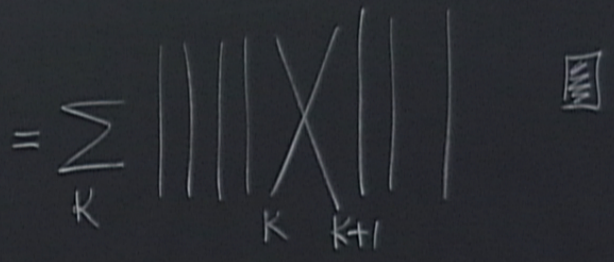
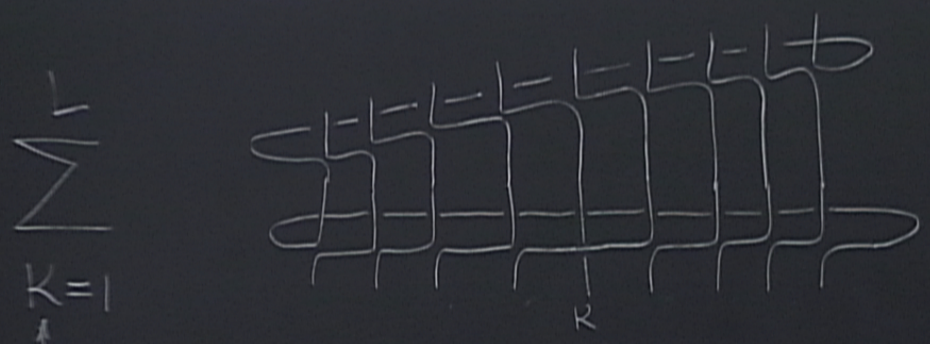
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\uparrow
 Which R-matrix
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$$= \sum_{K=1}^L$$

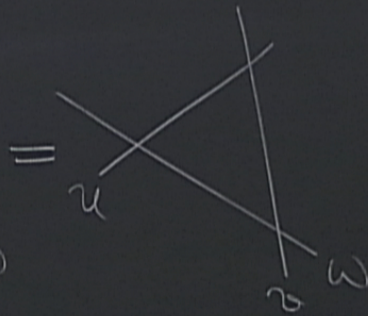
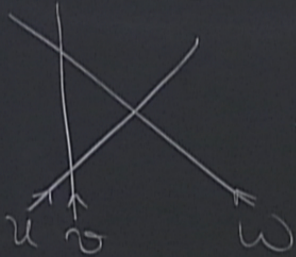
2



Which R-matrix
 $\frac{d}{du}$ acts

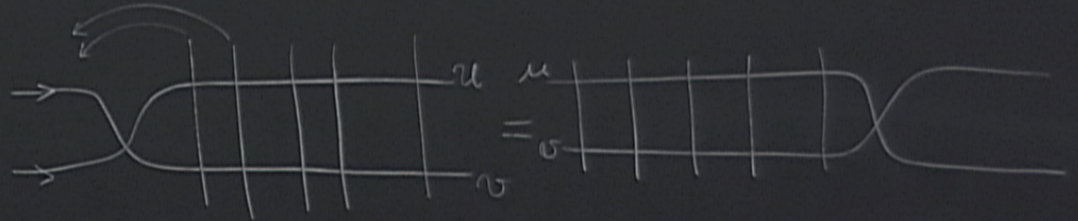
So what?

R obeys

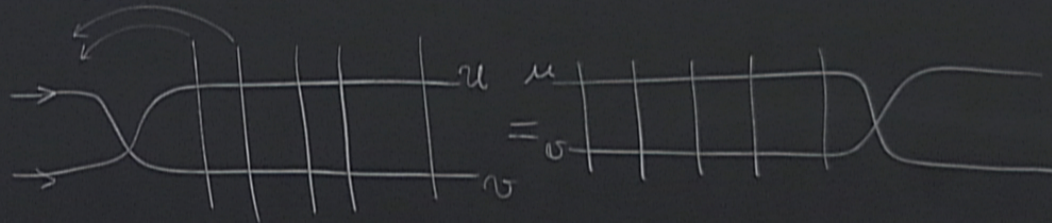


Yang-Baxter Equation

This relation implies

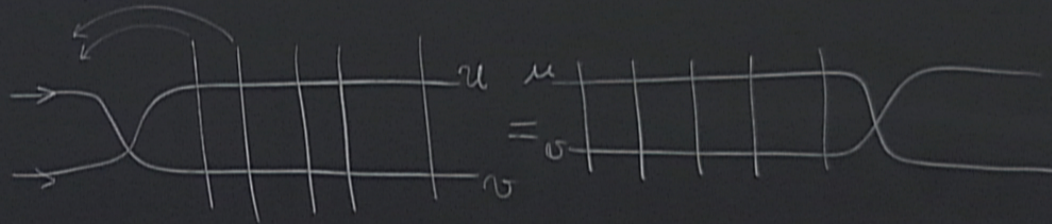


This relation implies



$$R_{o_1 o_2}(u-v) L_{o_1}(u) L_{o_2}(v) = L_{o_2}(v) L_{o_1}(u) R_{o_1 o_2}(u-v)$$

This relation implies



$$R_{\sigma_1 \sigma_2}(u-v) L_{\sigma_1}(u) L_{\sigma_2}(v) = L_{\sigma_2}(v) L_{\sigma_1}(u) R_{\sigma_1 \sigma_2}(u-v) \Rightarrow$$

multiply

$$[\hat{T}(u), \hat{T}(v)] = 0$$



$$0 = [\hat{H}, \log \hat{T}(v) = \sum v^n Q_n], \quad [H, Q_n]$$

$Q_1 = H, Q_2 = \text{higher charge}$

high

$$] = 0$$

higher non-local charges we were after!

$$) = \sum \alpha^n Q_n], \quad [H, Q_n] = 0$$

$Q_1 = H, Q_2 = \text{higher charge etc.}$

higher non-local charges we were after!

$$[H, Q_n] = 0$$

Main Ingredients:

① R-matrix with right sym.

= higher charge etc.

higher non-local charges we were after!

Main Ingredients:

- ① R-matrix With right sym.
- ② R obeys YB (fixes rel coeff of inv tensors)

(our case)
 $R = aI + bP$

$$[H, Q_n] = 0$$

gen charge etc.

$$O = [Q_n, Q_m]$$

U_1, U_2 right angle

