Title: Explorations in String Theory -2

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Abstract:

$$\rho(x) = \frac{\sqrt{2\left(\overbrace{M+\tfrac{1}{2}}^{E} - \frac{x^2}{2}\right)}}{\pi}$$

Spin-Chains

Pauli matrices acting on site n+1

$$H = -\lambda \sum_{n=1}^{L} \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} = \dots = \lambda \sum_{n=1}^{L} \left(\mathbb{1} - \mathbb{P}\right)_{n,n+1} + \text{const.}$$

$$\underbrace{\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\dots\uparrow}_{L}$$

Heisenberg XXX
Spin chain

permutation operator

$$\mathbb{P}|a\,b\rangle = |b\,a\rangle$$

$$\vec{\sigma}_n \cdot \vec{\sigma}_{n+1} = \frac{1}{2}4\left[\underbrace{\left(\frac{\vec{\sigma}_{n+1}+\vec{\sigma}_n}{2}\right)^2}_{j(j+1)} - \left(\frac{\vec{\sigma}}{2}_{n+1}\right)^2 - \underbrace{\left(\frac{\vec{\sigma}_n}{2}\right)^2}_{j(j+1)}\right]$$

$$= 2\left[0 \cdot \mathbb{P}_- + 2\mathbb{P}_+ - 2\frac{3}{4}\right]$$

$$\mathbb{P}_- = \frac{\mathbb{1}-\mathbb{P}}{2} \qquad \mathbb{P}_+ = \frac{\mathbb{1}+\mathbb{P}}{2}$$

$j(j+1)$ with $j = 1/2$

$j(j+1)$ with

$j = 0$ for singlet, i.e. anti-sym

$j = 1$ for triplet, i.e. sym

$\mathbb{P})_{n\,n+1}$ + const.

$\mathbb{P}$: permutation operator

$$\mathbb{P}|a\,b\rangle = |b\,a\rangle$$

onst.

erator

$|b\, a\rangle$

$\dots$

$$\vec{\sigma}_n \cdot \vec{\sigma}_{n+1} = \frac{1}{2}4\left[\underbrace{\left(\frac{\vec{\sigma}_{n+1}+\vec{\sigma}_n}{2}\right)^2}_{j(j+1)} - \left(\frac{\vec{\sigma}_{n+1}}{2}\right)^2 - \underbrace{\left(\frac{\vec{\sigma}_n}{2}\right)^2}_{j(j+1)}\right]$$

$$= 2\left[0\cdot \mathbb{P}_- + 2\mathbb{P}_+ - 2\frac{3}{4}\mathbb{1}\right]$$

$$\mathbb{P}_- = \frac{\mathbb{1}-\mathbb{P}}{2}, \qquad \mathbb{P}_+ = \frac{\mathbb{1}+\tilde{\mathbb{P}}}{2}$$

with $j=0$ for singlet, i.e. anti-sym

$j=1$ for triplet, i.e., sym

with $j=1/2$

Spin-Chains

Pauli matrices acting on site n+1

Hopping

$$H_L = -\lambda/2 \sum_{n=1}^{L} \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} = \dots = \lambda \sum_{n=1}^{L} \left(\mathbb{1} - \mathbb{P} \right)_{n,n+1} + c$$

$$\underbrace{\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\ldots\uparrow}_{L}$$

Heisenberg XXX
Spin chain.

only
nearest neighbour
Hamiltonian.

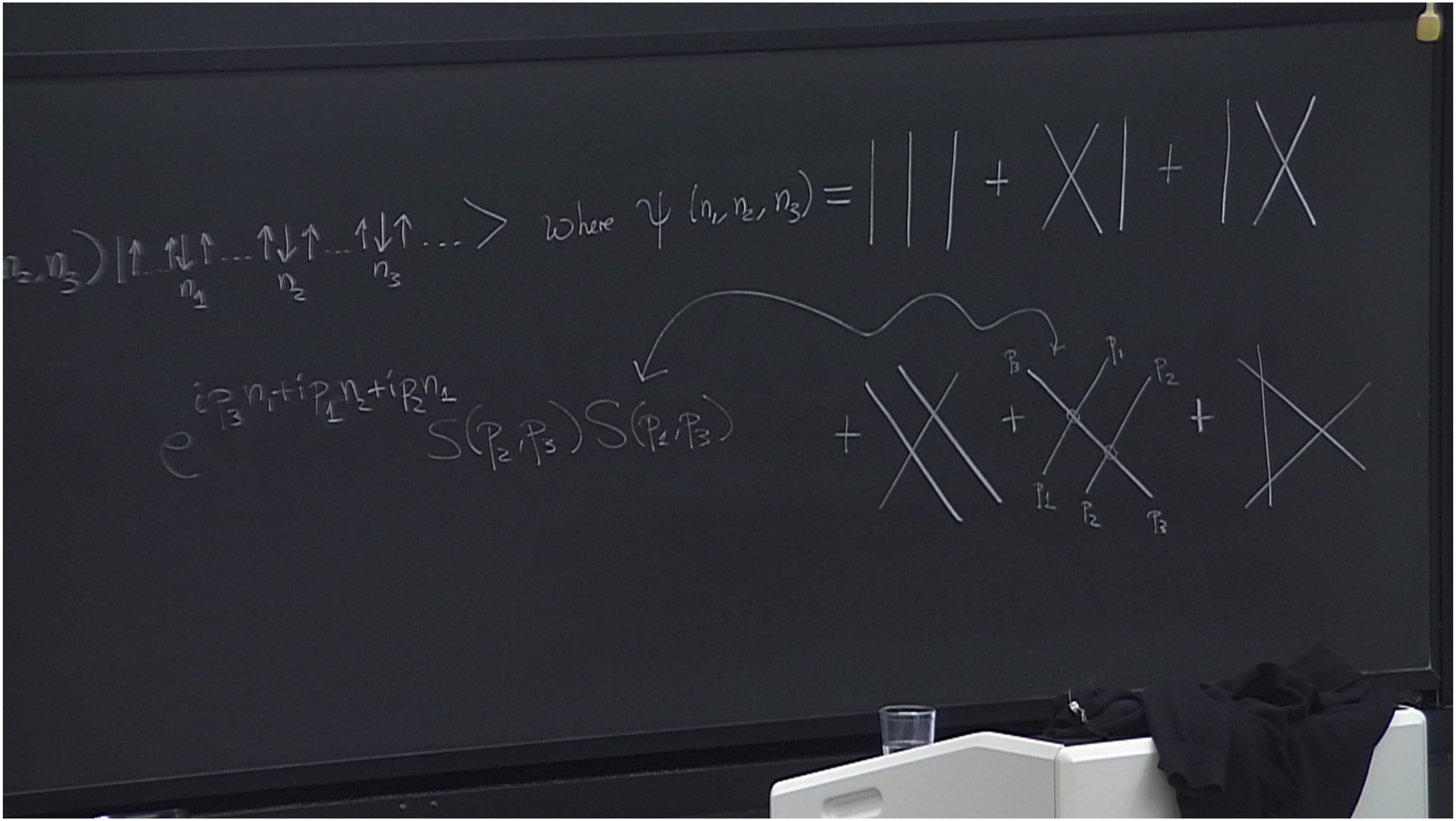
permutation op

$$\mathbb{P}|a\, b\rangle =$$

CLAIM:

$$|\psi\rangle_{3\text{ magnons}} = \sum_{n_1<n_2<n_3}^{L} \psi(n_1,n_2,n_3)\,|\uparrow\ldots\uparrow\underset{n_1}{\downarrow}\uparrow\ldots\uparrow\underset{n_2}{\downarrow}\uparrow\ldots\uparrow\underset{n_3}{\downarrow}\uparrow\ldots\rangle$$

diagonalizes $\mathcal{H}$.



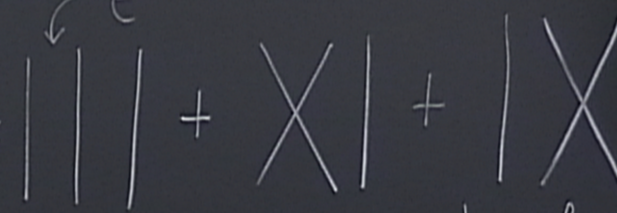
$n_2, n_3) | \uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \rangle$
where $\psi(n_1, n_2, n_3) =$
n_1 n_2 n_3
$e^{ip_3 n_1 + ip_1 n_2 + ip_2 n_1} S(p_2, p_3) S(p_1, p_3)$
p_3 p_1 p_2
p_1 p_2 p_3

$(\ldots n_2, n_3)\,|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots\rangle$

$n_1 \quad n_2 \quad n_3$

where $\psi(n_1, n_2, n_3) =$

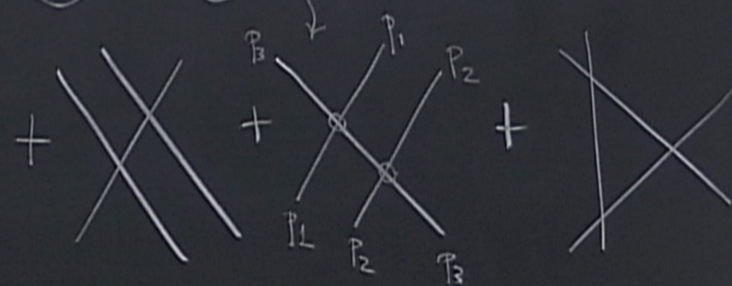
$e^{ip_1 n_1 + ip_2 n_2 + ip_3 n_3}$



incoming

outcome of a scattering process

$e^{ip_3 n_1 + ip_1 n_2 + ip_2 n_1} S(p_2, p_3) S(p_1, p_3)$

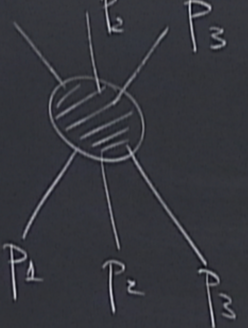


S-matrix $S(p,q) = \dfrac{u(p) - u(q) - i}{u(p) - u(q) + i}$

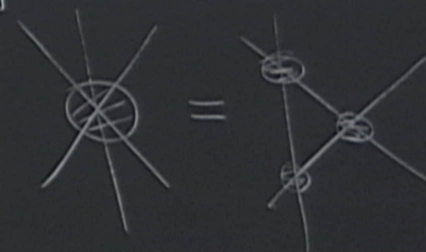
where $u(p) = \frac{1}{2}\cot\frac{p}{2}$

- Gen. for N magnons is obvious. $\psi_2 = || + X$, $\psi_4 =$ sum of $4!$ terms
- Why?! A: Integrability

$p_1'\ p_2'\ p_3'$ = original set (eventually permuted) $\Longleftarrow$

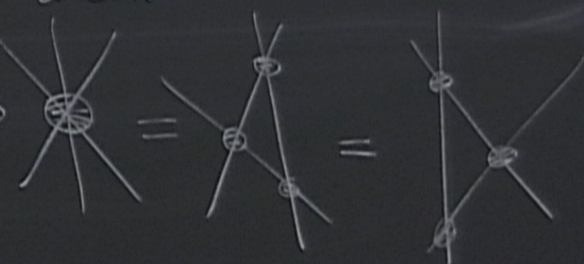


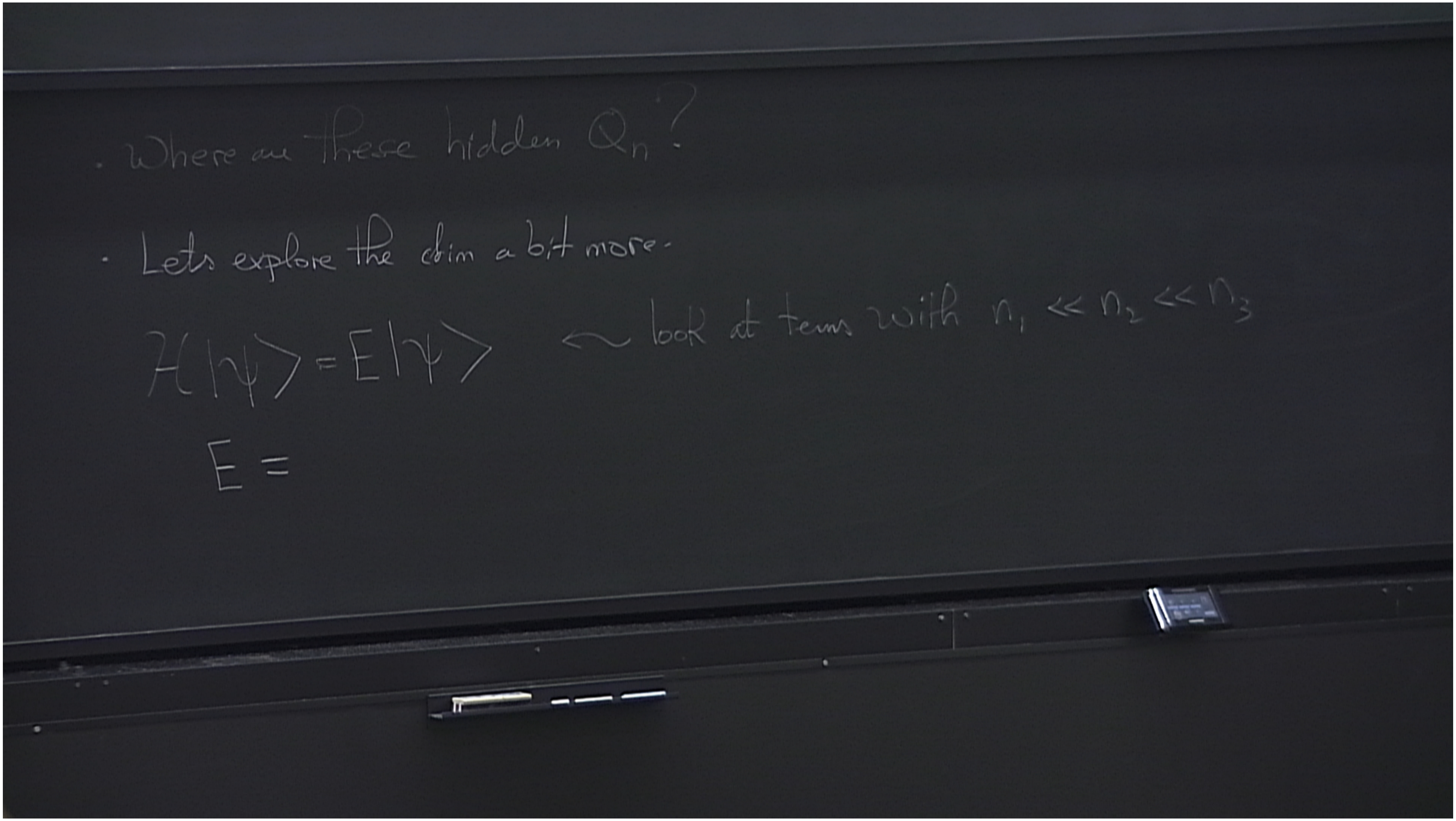
Only reasonable explanation



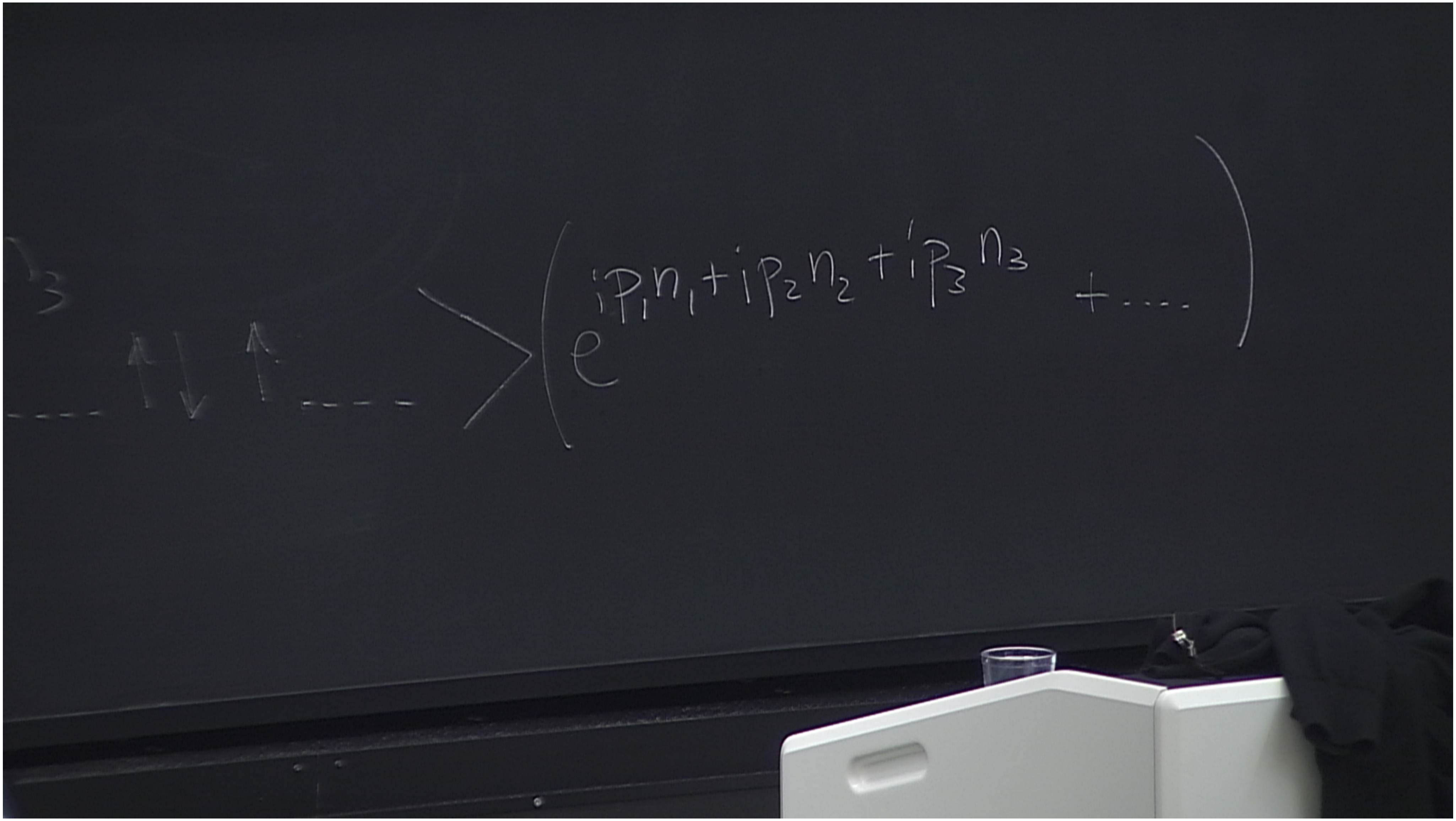
sum of 4! terms  + ...

$\exists$ hidden charges like $Q_3 = \sum p_j^3$ $\rightsquigarrow$ shifts wave packet by p-dep amount

$e^{i\alpha Q_3}$ 



· Where are these hidden Q_n?
· Lets explore the claim a bit more.
$\mathcal{H}|\psi\rangle = E|\psi\rangle$ ← look at terms with $n_1 \ll n_2 \ll n_3$
$E =$


$\left(e^{ip_1 n_1 + ip_2 n_2 + ip_3 n_3} + \dots \right)$

den Q_n ?

a bit more.

$\leftsquigarrow$ look at terms with $n_1 \ll n_2 \ll n_3$

r.h.s. $E \, |\uparrow \dots \uparrow\uparrow\downarrow\uparrow \dots \uparrow\downarrow\uparrow \dots \uparrow\downarrow\uparrow \dots \rangle$

l.h.s

$$E = \sum_{j=1}^{3} \epsilon(p_j)$$

$$\lambda(2-2\cos p) = \lambda\, 4 \sin^2 \frac{p}{2} = \frac{\lambda}{u^2+\frac{1}{4}}$$

$$n_3 \quad \uparrow \downarrow \uparrow \ldots\ldots \rangle \left(e^{ip_1 n_1 + ip_2 n_2 + ip_3 n_3} + \ldots\ldots \right)$$

$$p_3)] \,|\ldots\rangle \left(\ldots \right)$$

1 magnon

$$|\psi\rangle = \sum e^{ipn} \,|\uparrow \cdots\cdots \uparrow\downarrow\uparrow \cdots\cdots\rangle$$

$$\mathcal{H}|\psi\rangle = \sum_n \lambda 2 e^{ipn} \,|\uparrow \cdots \uparrow \underset{n}{\downarrow} \uparrow \cdots\rangle \quad = \left(2\lambda - \lambda e^{ip} - \lambda e^{-ip}\right)$$

$$- \sum_n \lambda e^{ipn} \,|\uparrow \cdots\cdots \overset{n+1}{\uparrow\downarrow\uparrow} \cdots\rangle$$

$$- \sum \lambda e^{ipn} \,|\uparrow \cdots\cdots \overset{n-1}{\uparrow\downarrow}\, \uparrow \cdots\rangle$$

$$\cdots\uparrow\cdots\cdots\rangle$$

$$\underset{n}{\downarrow}\uparrow\cdots\cdots\rangle \quad = \underbrace{\left(2\lambda - \lambda e^{-ip} - \lambda e^{+ip}\right)}_{\epsilon(p)} \overbrace{\sum_n e^{ipn}\,|\uparrow\cdots\cdots\underset{n}{\downarrow}\cdots\cdots\uparrow\rangle}^{|\psi\rangle}$$

$$\cdots\uparrow\overset{n+1}{\downarrow}\uparrow\cdots\rangle$$

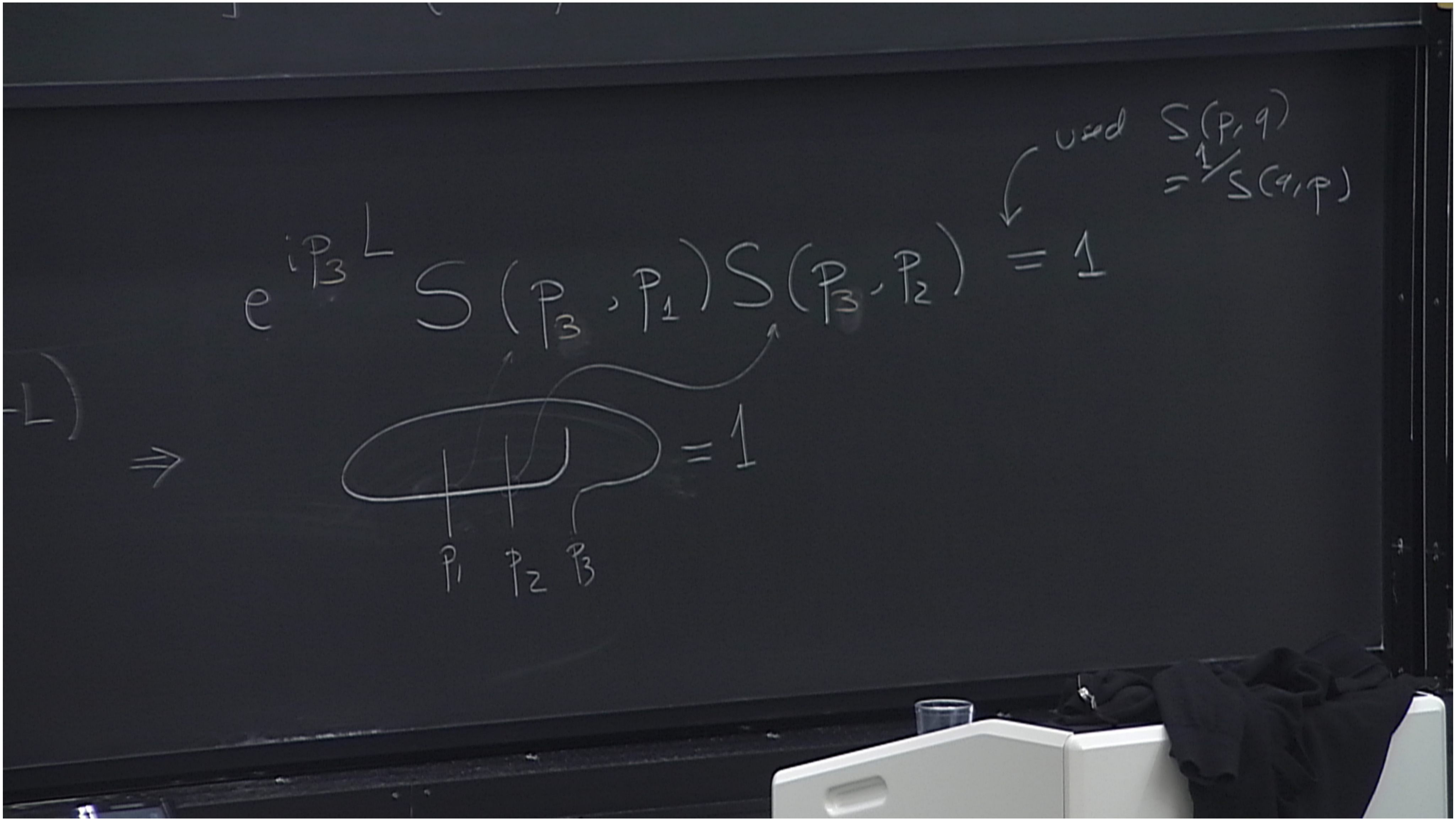
$$\cdots\uparrow\overset{n-1}{\downarrow}\uparrow\cdots\cdots\rangle$$

Need to impose periodic b.c.

$$\psi(n_1, n_2, n_3) = \psi(n_2, n_3, n_1 + L) \quad \forall\, n_1, n_2, n_3$$

$$\equiv e^{ip_1 n_2 + ip_2 n_3 + ip_3 (n_1 + L)}$$

$$e^{ip_1 n_2 + ip_2 n_3 + ip_3 n_1} \; SS$$

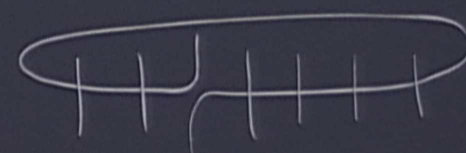

used $S(p,q) = 1/S(q,p)$
$e^{ip_3 L} S(p_3, p_1) S(p_3, p_2) = 1$
$\Rightarrow$
$= 1$
p_1 p_2 p_3

$$\rho(x) = \frac{\sqrt{2\big(\overbrace{M+\tfrac{1}{2}}^{E} - \tfrac{x^2}{2}\big)}}{\pi}$$

more generally

Bethe Eqs.

$$K = 1 \dots N : \quad \boxed{\begin{gathered} e^{i p_k L} \prod_{j \neq k}^{N} S(p_k, p_j) = 1 \\ E = \sum_{j=1}^{N} 4\lambda \sin^2 \frac{p_j}{2} \end{gathered}}$$

 $= 1$

In term u_j :

$$\boxed{\begin{gathered} E = \sum_{j=1}^{N} \frac{\lambda}{u_j^2 + \frac{1}{4}} \\ \left(\frac{u_j + i/2}{u_j - i/2}\right)^L \prod_{k \neq j}^{N} \frac{u_j - u_k - i}{u_j - u_k + i} = 1 \end{gathered}}$$

$K = 1$