Title: Real time simulation of classical Yang-Mills theory and black hole physics

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Abstract: <p>The gauge/gravity enables us to learn about quantum gravity by solving gauge theory. This is not an easy task, of course, and hence numerical techniques should play important roles. So far, properties of super Yang-Mills theories with Euclidean signature, such as the thermodynamic properties, have been studied by using Monte Carlo methods, and good agreement with the dual gravity prediction has been observed, including stringy corrections, both alpha prime and and g_s. Still, the real-time properties are not well understood.

As a modest first step for the real-time study, we consider classical dynamics of the Banks-Fischler-Shenker-Susskind (BFSS) matrix model, which is expected to describe a highly stringy black hole in type IIA superstring theory. It turns out that this classical model has rather rich structure -- qualitative features of the thermalization of a black hole, the fast scrambling proposed by Sekino and Susskind, and a symptom of the evaporation. By taking into account a part of the quantum effect, we give a classical matrix model which can mimic the formation and evaporation of a black hole. We also argue that a similar calculation could be done for classical Yang-Mills theories with nonzero spatial dimension, without suffering from the UV catastrophe.

This talk is based on collaborations with S. Aoki, N. Iizuka (hep-th) and with E. Berkowitz, G. Gur-Ari, J. Maltz, S. Shenker (in progress).</p>

Real time simulation of classical Yang-Mills theory and black hole physics

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Gur Ari-M.H.-Shenker, to appear
+ work in progress with Berkowitz, Gur Ari, Maltz and Shenker

10 April 2015 @ Perimeter Institute

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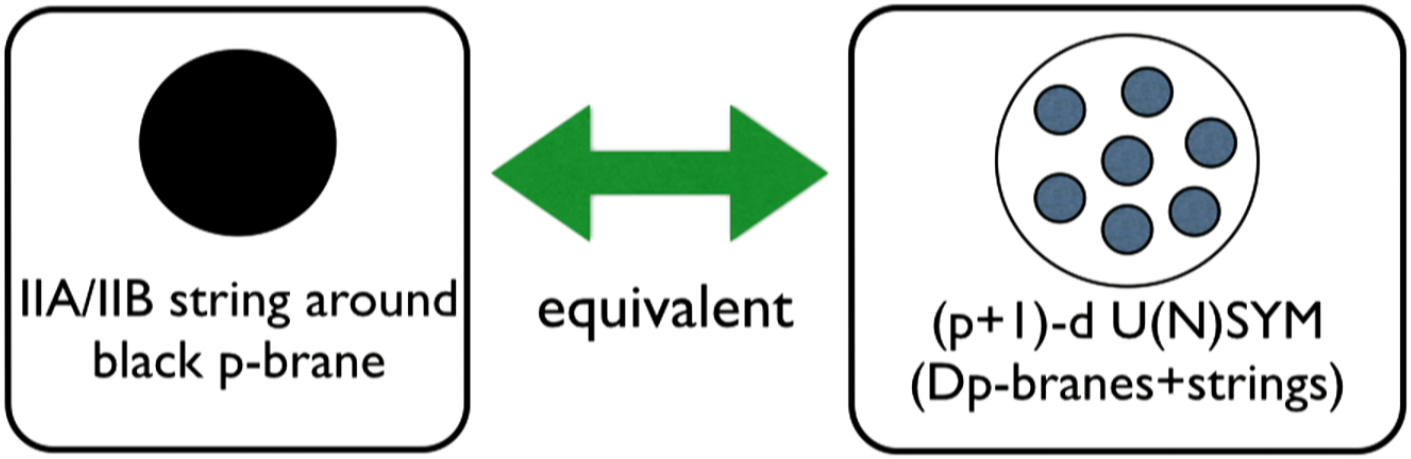
Gauge/Gravity Duality



We can learn about quantum gravity and BH
by solving gauge theory.

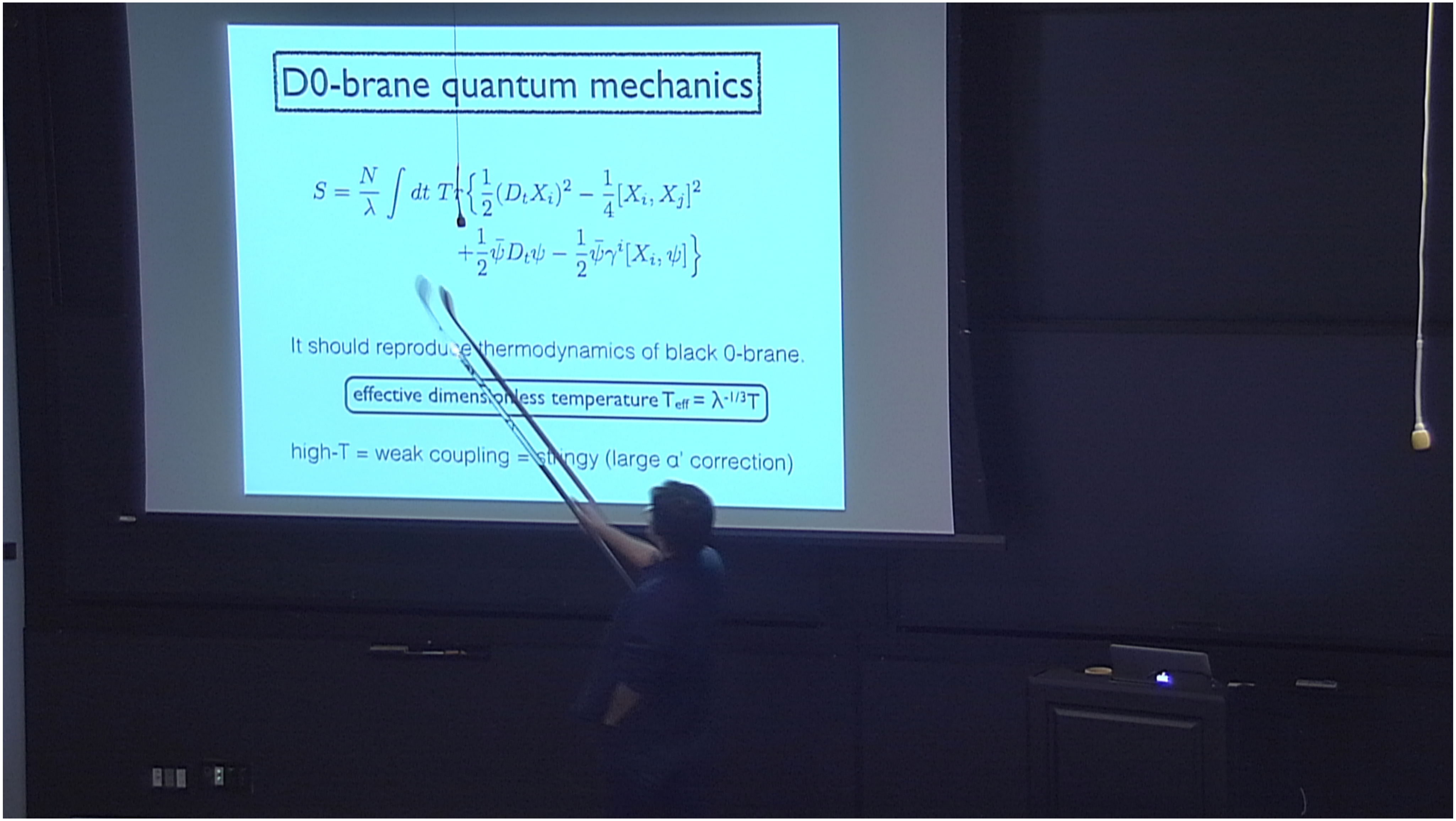
D0-brane quantum mechanics

$$S = \frac{N}{\lambda}\int dt\ Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i[X_i, \psi]\Big\}$$

It should reproduce thermodynamics of black 0-brane.

effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$

high-T = weak coupling = stringy (large α' correction)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Thermodynamics
(imaginary time)

- large-N, strong coupling vs SUGRA

Anagnostopoulos-M.H.-Nishimura-Takeuchi, 2007
Catterall-Wiseman, 2008, Kadoh-Kamata, 2015

- large-N, finite coupling vs SUGRA+α'

M.H.-Hyakutake-Nishimura-Takeuchi, 2008
Kadoh-Kamada, 2015

- finite-N vs SUGRA+α'+g_s

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<u>Seems to be correct.</u>

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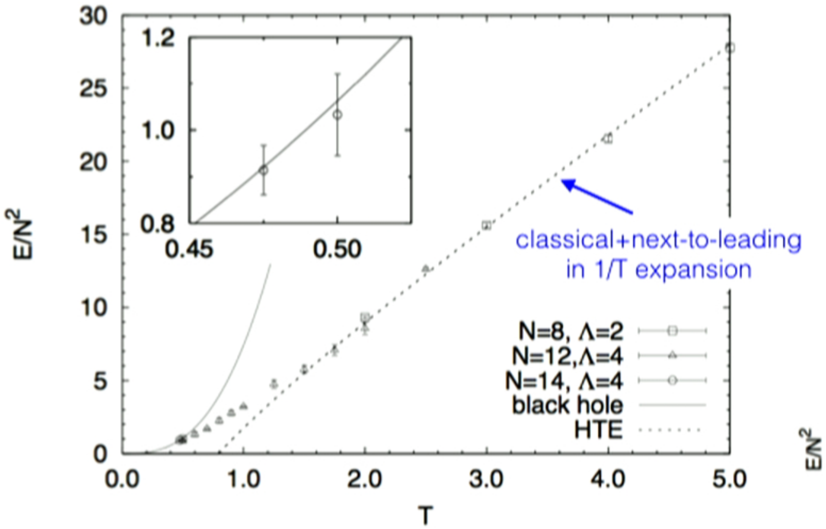
Energy of BH & MQM



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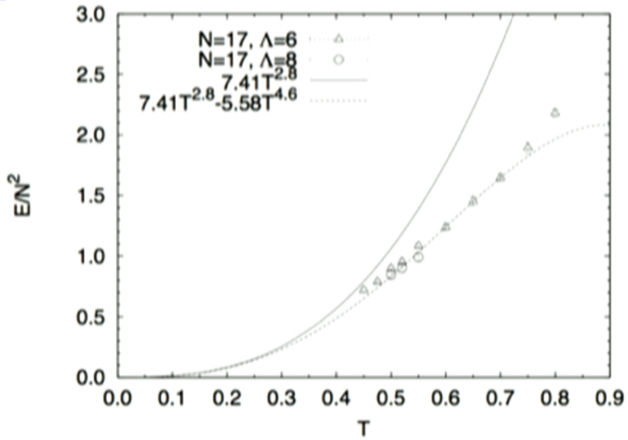

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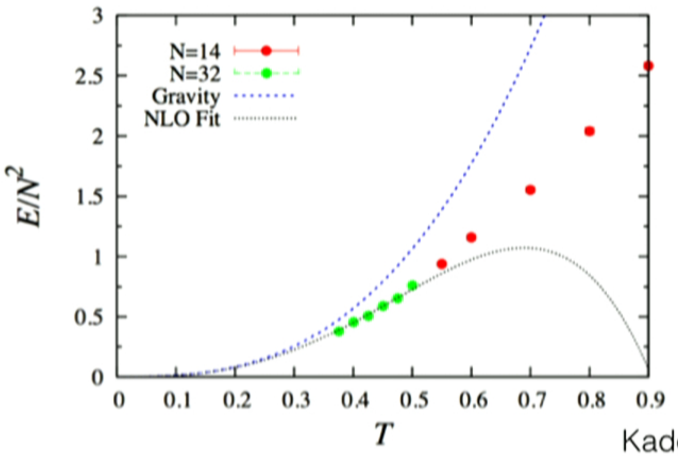

Small deviation?
We will go closer to continuum.
(stay tuned.)

Kadoh-Kamata, 2015

FREE simulation code for BFSS/BMN model

RHMC algorithm + Fourier acceleration (with FFTW3)
Fortran 90/ Fortran 2003; MPI parallelized

Should be useful for learning about BH, M2 and M5.

$0!

Runs on supercomputer, cluster, and macbook

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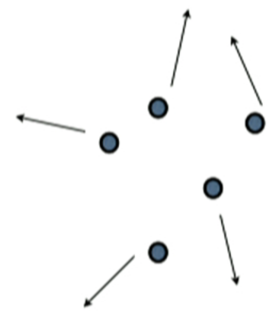
‘eigenvalues’ = D0-branes

bound state of eigenvalues
= black hole



flat direction
= gas of D0-branes

This phase reproduced the dual BH thermodynamics.

Real time study

- Full quantum study is impossible with current technology.

  stochastic quantization (complex Langevin)?
  brute-force diagonalization?
  quantum simulator? → experimental quantum gravity?

- Strong coupling lattice gauge theory (+improvement)

  M.H.-Maltz-Susskind 2014

  stringy d.o.f. is manifest; still numerically demanding, but should be possible in a few years.

- Classical real time evolution

  high temperature = weak coupling
  <u>but</u> highly nonlinear & nonperturbative

  "BH" = soliton (or resonance) of matrix model

We will see the formation & evaporation of "BH" in this limit.

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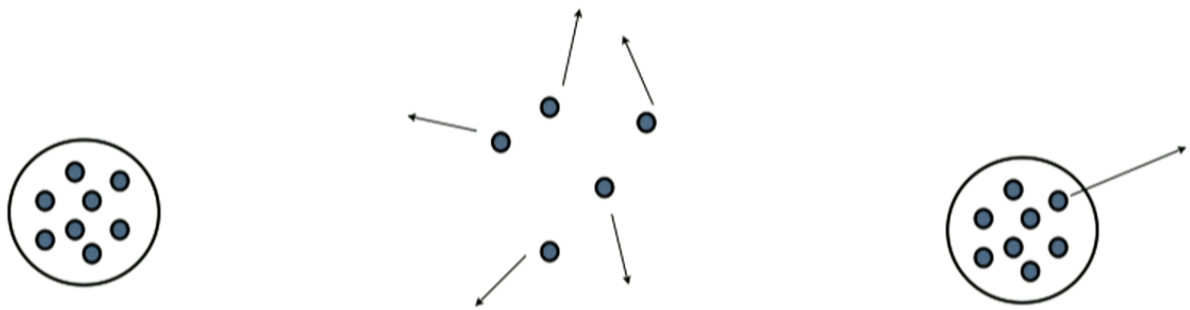

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flat direction
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emission of eigenvalue
= evaporation of BH
(emission of D0)

This model can describe BH evaporation!

This evaporation is suppressed at $N=\infty$.

(The instability has been observed in imaginary time simulation.)

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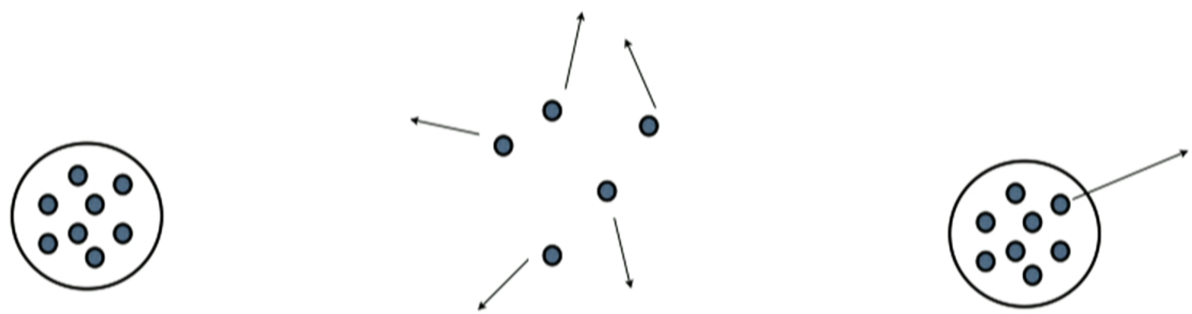

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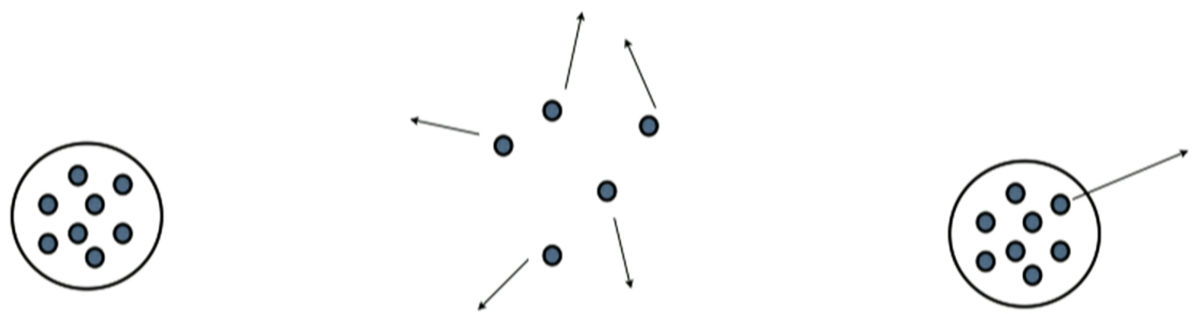

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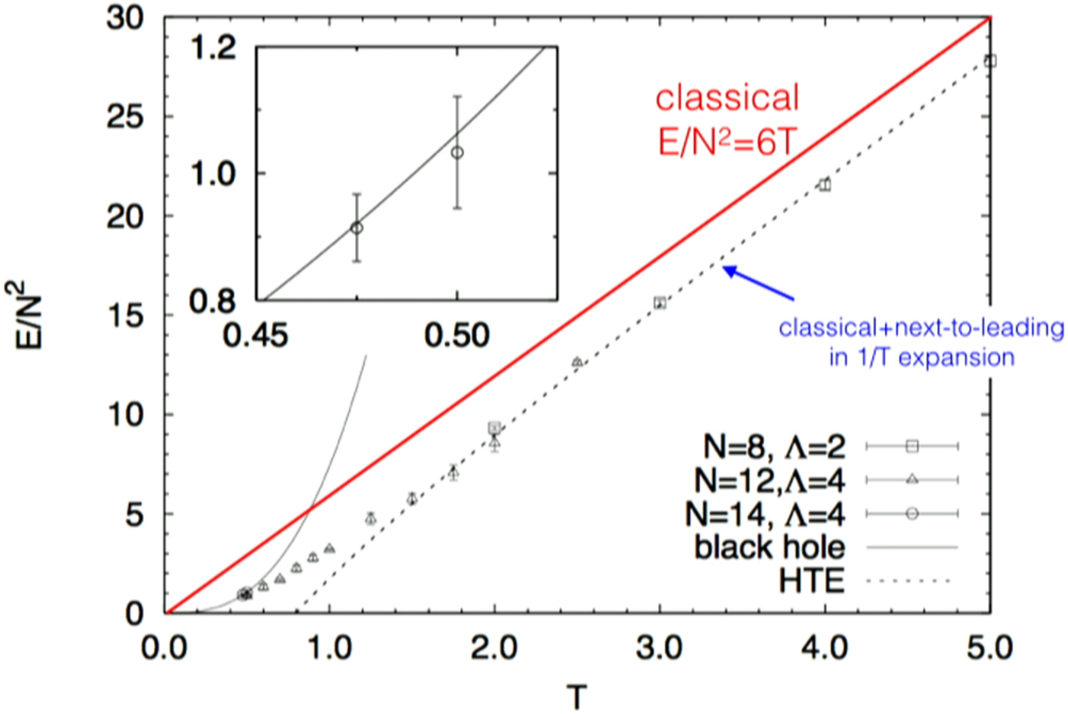

E/N^2 in BFSS (0707.4454[hep-th])

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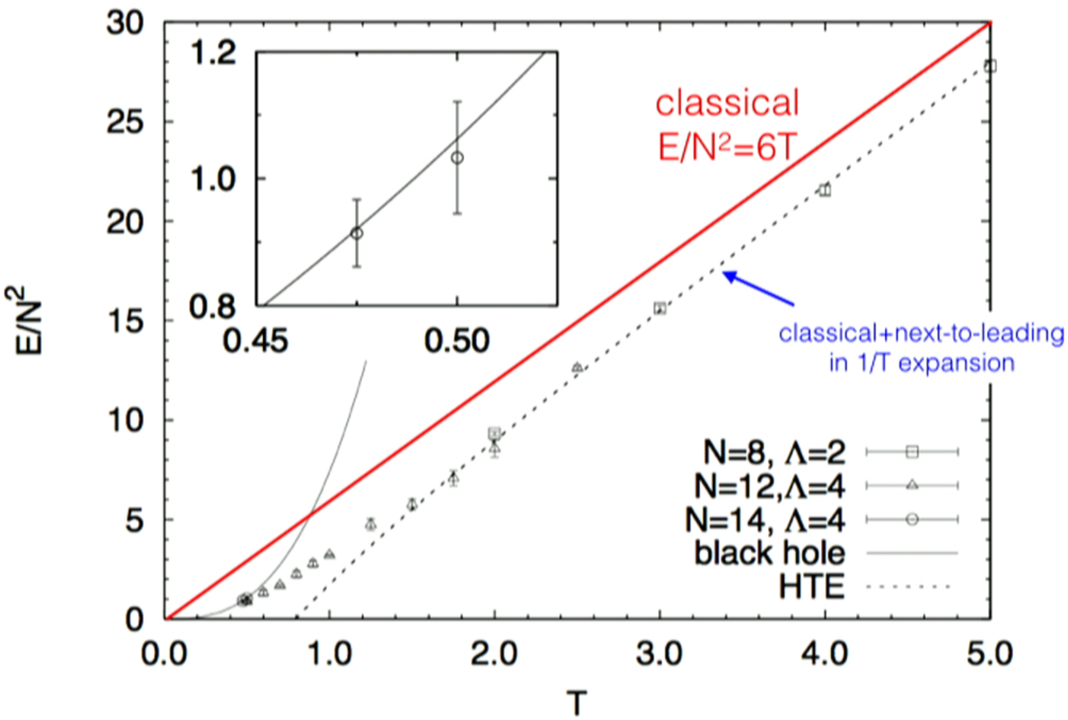

E/N^2 in BFSS (0707.4454[hep-th])

$$L = \frac{1}{2g_{YM}^2} \mathrm{Tr} \left(\sum_i (D_t X^i)^2 + \frac{1}{2} \sum_{i \neq j} [X_i, X_j]^2 \right)$$

$$\longrightarrow \begin{cases} \dfrac{d^2 X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0 \\ \sum_i \left[X^i, \dfrac{dX^i}{dt} \right] = 0 \end{cases}$$

discretize & solve it numerically.
(straightforward.)

$$\frac{d^2 X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0$$

Invariant under the scaling $t \to t/\alpha,\ X_M \to \alpha X_M$

All values of the energy (or ‘temperature’) are equivalent.

$E, T \to \alpha^4 E,\ \alpha^4 T$

When is classical approximation valid?

off diagonal element
= open string

diagonal element
= D0-brane

open string mass
= distance between D0's
$\sim$ $T^{1/4}$ at high-T, for 'BH'

↑

It must be smaller than T.

➡ classical approximation
is valid at high-T BH.

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Formation & thermalization of BH

- This system is chaotic. (Savvidy, 1984; Berenstein et al, 2012)
- Almost all initial conditions end up with

'typical' matrix configurations — BH.

BH

open strings
(off-diagonal elements)
are excited

entropy ∼ N^2

gas

open strings are suppressed

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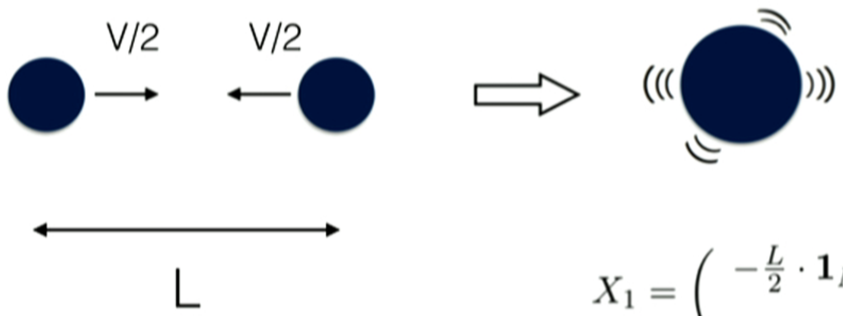
Collision of 2 BHs



L, V: fix

$\sigma \sim 1/(\sqrt{N}L)$

$\Longrightarrow \quad E \sim N^2$

't Hooft limit

$$X_1 = \begin{pmatrix} -\frac{L}{2} \cdot \mathbf{1}_{N/2} & \\ & \frac{L}{2} \cdot \mathbf{1}_{N/2} \end{pmatrix},$$

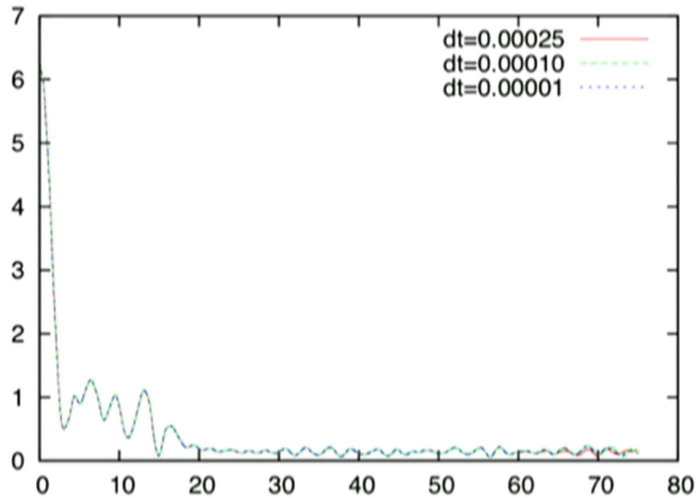
$$V_1 = \begin{pmatrix} \frac{V}{2} \cdot \mathbf{1}_{N/2} & \\ & -\frac{V}{2} \cdot \mathbf{1}_{N/2} \end{pmatrix},$$

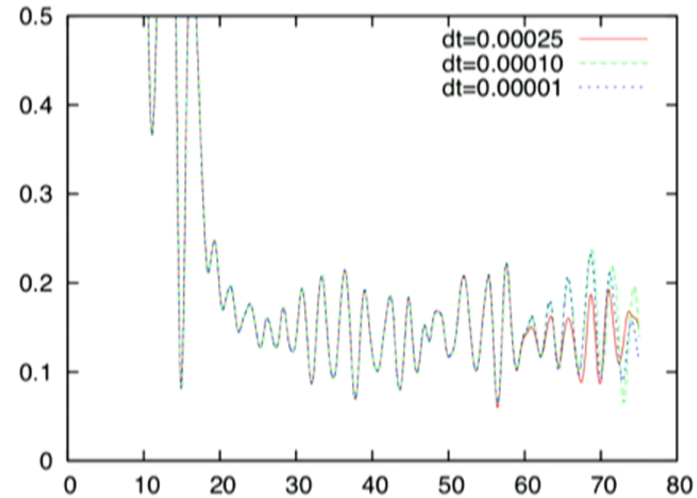
$$V_2 = \cdots = V_d = 0,$$

Gaussian random number

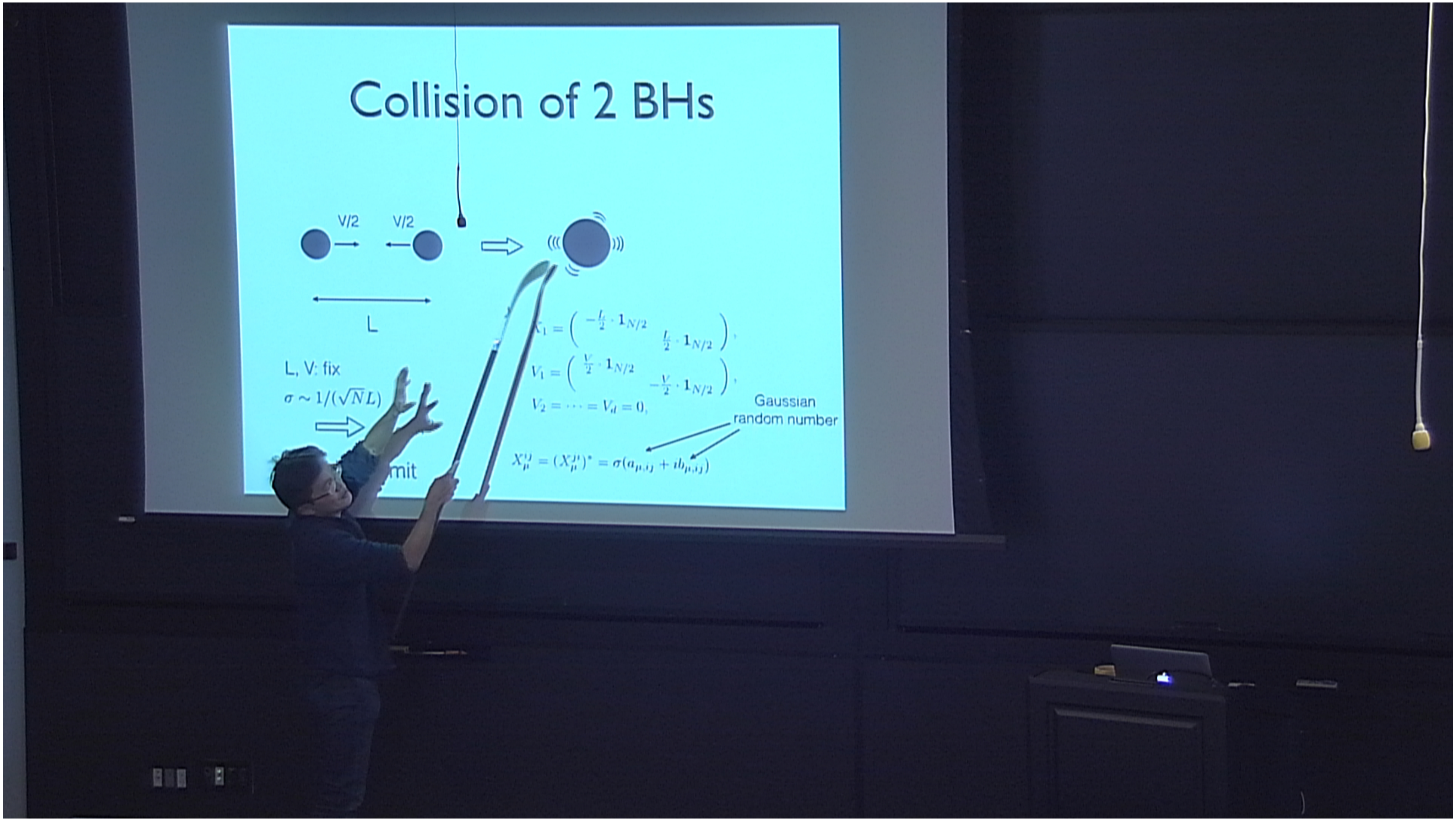
$$X_\mu^{ij} = (X_\mu^{ji})^* = \sigma(a_{\mu,ij} + ib_{\mu,ij})$$

$\langle \mathrm{Tr} X_1^2 / N \rangle$





N=8, $L = 5.0$ $V =$ 0 $\sqrt{N}\sigma =$ 0.12



Collision of 2 BHs
V/2
V/2
L
L, V: fix
$\sigma \sim 1/(\sqrt{N}L)$
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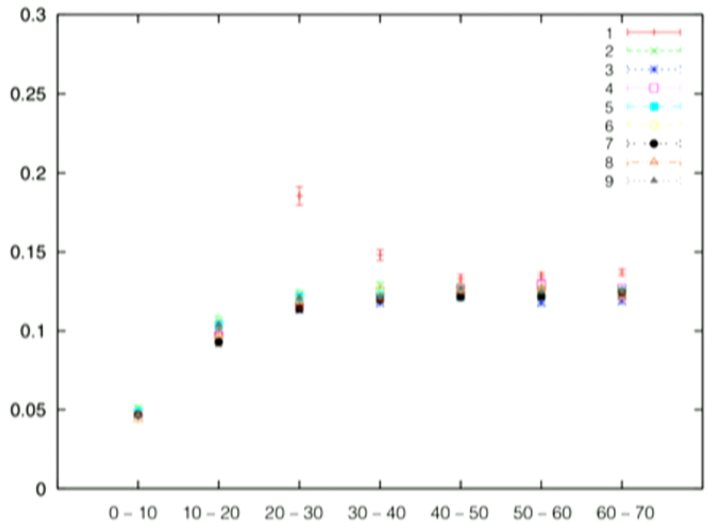
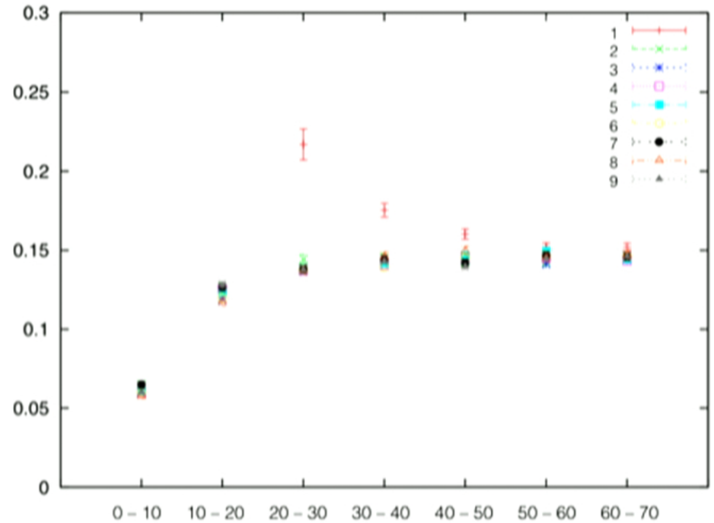




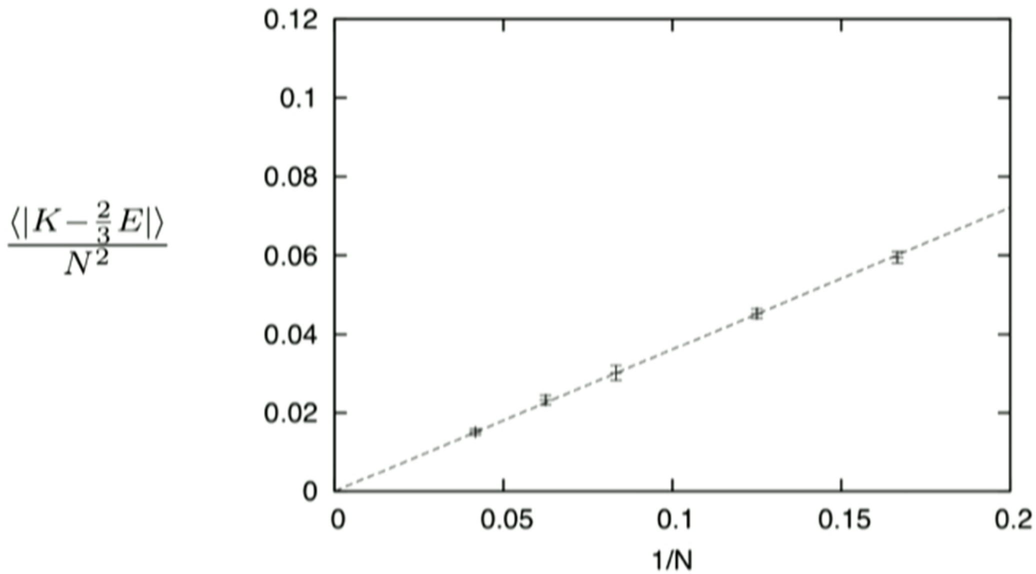
average of $\mathrm{Tr}\, X_1^2/N, \ldots, \mathrm{Tr}X_9^2/N$
50 samples

N=8, $L = 5.0$ $V = 0$ $\sqrt{N}\sigma = 0.12, 0.14$

Rotational symmetric at late time, already at finite-N

Virial theorem: $\langle K \rangle = 2\langle V \rangle = \frac{2}{3}E$

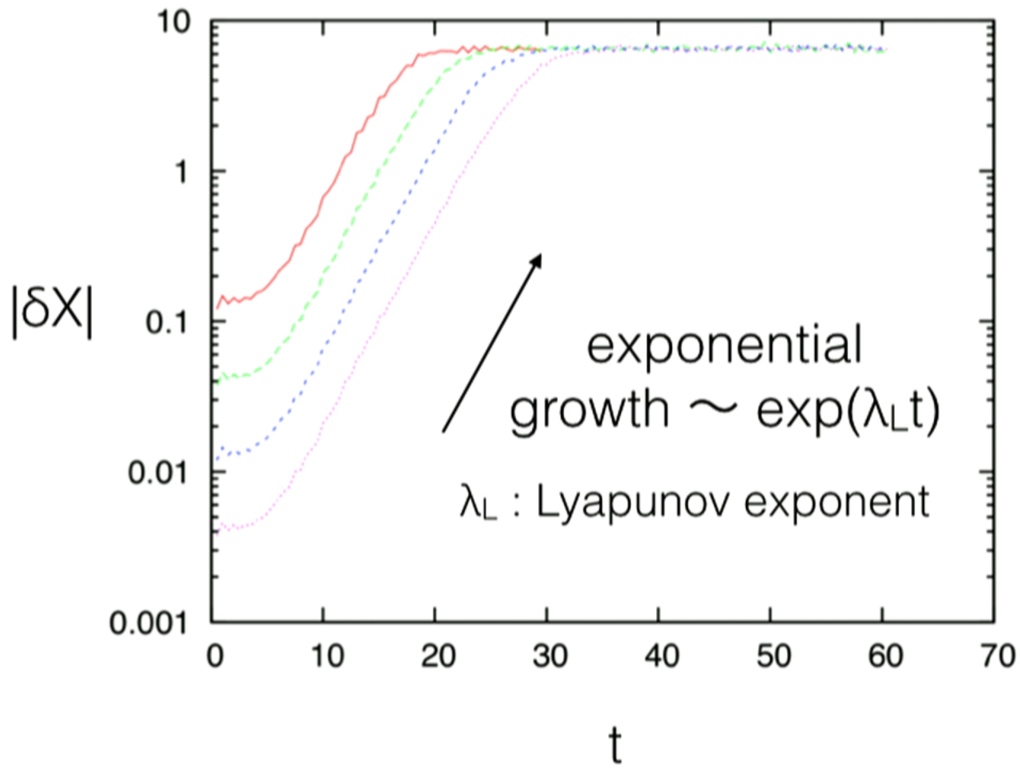
(For bound state. Time average, any N.)



Fluctuation disappears at large-N. → 'Thermalization' at large-N.

Fast scrambling

- Add a small perturbation:
  $X \to X+\delta X$, $V \to V+\delta V$.
- The information should be scrambled in 'scrambling time' $t_s \sim \log N$. (Sekino-Susskind, 2008.)
- Let's test this conjecture.


10
1
0.1
0.01
0.001
|δX|
0
10
20
30
40
50
60
70
t
exponential
growth ∼ exp(λLt)
λL : Lyapunov exponent

N=8
$E/N^2=10$