Title: Higher Spins & Strings

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Abstract:

Higher Spins & Strings

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(Mock) Modularity, Moonshine and String Theory
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based mainly on

MRG, R. Gopakumar, 1406.6103 and 1501.07236

AdS / CFT duality

Much recent progress in string theory has been related to AdS/CFT duality [Maldacena '97, ...]

superstrings on $\mathrm{AdS}_5 \times S^5$	=	SU(N) super Yang-Mills theory in 4 dimensions

4d non-abelian gauge theory similar to that appearing in the standard model of particle physics.

Motivation

One way to get quantitative handle on AdS/CFT correspondence is to consider regime where gauge theory is weakly coupled and at large N:

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

large

small

[Sundborg '01]
[Witten '01]
[Sezgin,Sundell '01]

$l_{\rm s} \to \infty$ \`tensionless strings'

Higher spin theory

Resulting theory has an infinite number of massless higher spin fields, which generate a very large gauge symmetry.

→ effective description in terms of Vasiliev Higher Spin Theory.

maximally unbroken phase of string theory

Leading Regge trajectory

On the dual CFT side, the traces of bilinears of elementary Yang-Mills fields form closed subsector in free theory.

This subsector is believed to correspond to the leading Regge trajectory from the string point of view:

`vector-like' HS -- CFT duality

State of the Art

In the past this idea was taken as a general motivation to consider dualities relating

Vasiliev HS theory on AdS	$\longleftrightarrow$	vector-like weakly coupled CFT

However, recently interesting progress about how these dualities fit actually into stringy AdS/CFT correspondence has been made....

[Chang, Minwalla, Sharma, Yin '12]
[MRG, Gopakumar '14]

3d proposal

Concrete duality of this kind (somewhat similar to Klebanov & Polyakov proposal for AdS4/CFT3)

[MRG,Gopakumar '10]

AdS3:
higher spin theory with a complex scalar of mass M

$\longleftrightarrow$

2d CFT:
$\mathcal{W}_{N,k}$ minimal models in large N 't Hooft limit with coupling λ

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

Outline

This version of the duality is bosonic, but can nevertheless be tested in quite some detail. In particular, we can match

- quantum symmetries
- spectrum

At the end I will also explain how this higher spin duality relates to stringy AdS3 -- CFT2 duality.

Outline

The proof that the quantum symmetries agree requires detailed analysis of the symmetry algebra (VOA) of the dual CFT:

$$\mathcal{W}_\infty[\lambda] \quad \text{algebra}$$

- generated by one spin field for each spin s>1
- apparently characterised by 2 parameters:

$$\lambda \quad \text{and} \quad \text{central charge } c$$

The HS theory on AdS3

The AdS3 HS theory can be described very simply.

Recall that pure gravity in AdS3: Chern-Simons theory based on

$$\mathfrak{sl}(2,\mathbb{R})$$

[Achucarro, Townsend '86]
[Witten '88]

Higher spin description: replace

[Prokushkin, Vasiliev '98]

[one spin field for each spin $s=2,3,\ldots$]

$$\mathfrak{sl}(2,\mathbb{R}) \;\rightarrow\; \mathrm{hs}[\lambda] \cong \mathfrak{sl}(\lambda)$$

where $\mathrm{hs}[\lambda]\oplus\mathbb{C} \cong \dfrac{U(\mathfrak{sl}(2))}{\langle C_2 - \frac{1}{4}(\lambda^2-1)\mathbf{1}\rangle}$

[Bergshoeff et.al. '90]
[Pope, Romans, Shen '90]
[Fradkin, Linetsky '91]

$U(sl(2))$ $\qquad J^+, J^3, J^-$

$\mathbb{1}$

$J^+ \quad J^3 \quad J^-$

$J^+J^+, \; J^+J^3, \; J^+J^-, \; J^3J^3, \; J^3J^-, \; J^-J^-$

Higher spin algebra

Generators of $\mathrm{hs}[\lambda]$:

$$V_n^s \ \text{ with } |n| < s \ , \ s = 2, 3, \ldots$$

`wedge algebra'

For these higher spin theories asymptotic symmetry algebra can be determined following Brown & Henneaux, leading to classical

$$\mathcal{W}_\infty[\lambda] \ \text{ algebra}$$

[Henneaux & Rey '10]
[Campoleoni et al '10]
[MRG, Hartman '11]

Asymptotic symmetry algebra

Asymptotic symmetry algebra extends hs algebra `beyond the wedge':

pure gravity: $L_0, L_{\pm 1} \;\longrightarrow\; L_n \;,\; n \in \mathbb{Z}$

$\mathfrak{sl}(2,\mathbb{R})$ (Virasoro)

higher spin: $\mathrm{hs}[\lambda] \;\longrightarrow\; \mathcal{W}_\infty[\lambda]$

generated by V_n^s
$s = 2, \ldots, \infty, \; n \in \mathbb{Z}$

[Figueroa-O'Farrill et.al. '92]

Dual CFT

By the usual arguments, dual CFT should therefore have

$$\mathcal{W}_\infty[\lambda] \text{ symmetry.}$$

Basic idea:

$$\mathcal{W}_\infty[\lambda] = \lim_{N\to\infty} \mathcal{W}_{N,k} \qquad \text{with} \qquad \lambda = \frac{N}{N+k} \ .$$

‘t Hooft limit of 2d CFT!

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‘t Hooft limit of 2d CFT!

The minimal models

The minimal model CFTs are the cosets

$$\mathcal{W}_{N,k}: \qquad \frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

[Bais et.al. '88]
[Bouwknegt, Schoutens '92]

e.g. Ising model (N=2, k=1)
tricritical Ising (N=2, k=2)
3-state Potts (N=3,k=1),..

with central charge

$$c_N(k) = (N-1)\left[1 - \frac{N(N+1)}{(N+k)(N+k+1)}\right] .$$

General N: higher spin analogue of Virasoro minimal models. [Spin fields of spin s=2,3,..,N.]

Relation of symmetries

The asymptotic symmetry algebra

$$\mathcal{W}_\infty[\lambda]$$

is a classical (commutative) Poisson algebra.

In order to understand relation to minimal model W-algebras, need to understand how to quantise it.

Quantisation

Quantisation is quite subtle since the Poisson algebra is non-linear --- cannot just replace Poisson brackets by commutators without violating Jacobi identities...

However, there seems to exist a unique way of defining a consistent quantum W-algebra (whose classical limit reduces to Poisson algebra).

[MRG, Gopakumar ‘12]
[Blumenhagen, et.al. ’94]
[Hornfeck ’92-’93]

Quantum symmetry

There are two steps to this argument. To illustrate them consider an example. Naive quantisation of classical algebra leads to

[MRG, Gopakumar '12]

$$[W^3_m, W^3_n] = 2(m-n)W^4_{m+n} + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n}$$
$$+ \frac{8N_3}{c}(m-n)(LL)_{m+n} + \frac{N_3 c}{144} m(m^2-1)(m^2-4)\delta_{m,-n}$$

spin-3 field

non-linear term

Jacobi identity

$$[W_m^3, W_n^3] = 2(m-n)W_{m+n}^4 + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n}$$
$$+ \frac{8N_3}{c}(m-n)(LL)_{m+n} + \frac{N_3 c}{144}m(m^2-1)(m^2-4)\delta_{m,-n}$$

Jacobi identity determines quantum correction

$$[W_m^3, W_n^3] = 2(m-n)W_{m+n}^4 + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n}$$
$$+ \frac{8N_3}{c+\frac{22}{5}}(m-n)\Lambda_{m+n}^{(4)} + \frac{N_3 c}{144}m(m^2-1)(m^2-4)\delta_{m,-n}$$

where

$$\Lambda_n^{(4)} = \sum_p :L_{n-p}L_p: + \tfrac{1}{5}x_n L_n$$

Similar considerations apply for the other commutators.

Structure constants

The second step concerns structure constants. The fields can be rescaled so that

$$W^{(3)} \cdot W^{(3)} \sim \frac{c}{3} \cdot \mathbf{1} + 2 \cdot L + \frac{32}{(5c+22)} \cdot \Lambda^{(4)} + 4 \cdot W^{(4)}$$

but then coupling constant

$$W^{(3)} \cdot W^{(4)} \sim C_{33}^{4} \cdot W^{(3)} + \cdots$$

characterises algebra. Classical analysis determines

[MRG, Hartman '11]

$$\left(C_{33}^{4}\right)^{2} = \frac{64}{5} \frac{\lambda^2 - 9}{\lambda^2 - 4} + \mathcal{O}\left(\tfrac{1}{c}\right) \,.$$

Structure constants

Classical analysis determines

$$\left(C_{33}^{4}\right)^{2}=\frac{64}{5}\,\frac{\lambda^{2}-9}{\lambda^{2}-4}+\mathcal{O}\left(\tfrac{1}{c}\right)\ .$$

Requirement that representation theory agrees for $\lambda=N$ with $\mathcal{W}_N$:

$$\gamma^{2}\equiv\left(C_{33}^{4}\right)^{2}=\frac{64\,(c+2)\,(\lambda-3)\,\big(c(\lambda+3)+2(4\lambda+3)(\lambda-1)\big)}{(5c+22)\,(\lambda-2)\,\big(c(\lambda+2)+(3\lambda+2)(\lambda-1)\big)}\ .$$

[Note: $\mathrm{hs}[\lambda]\Big|_{\lambda=N}\cong sl(N,\mathbb{R})$ implies $\mathcal{W}_\infty[\lambda]|_{\lambda=N}=\mathcal{W}_N$.]

Explicit check

This formula has also been checked explicitly for the two special cases:

[MRG, Jin, Li '13]

$\lambda = 0:$ N complex fermions with u(1) coset giving c = N-1 [Bergshoeff et.al. '90]

$\lambda = 1:$ N complex bosons giving c=2N [Bakas, Kiritsis '90]

Higher Structure Constants

Similarly, higher structure constants can be determined

[Blumenhagen, et.al.'94]
[Hornfeck '92-93]

$$C_{33}^{4}C_{44}^{4}=\frac{48\big(c^{2}(\lambda^{2}-19)+3c(6\lambda^{3}-25\lambda^{2}+15)+2(\lambda-1)(6\lambda^{2}-41\lambda-41)\big)}{(\lambda-2)(5c+22)\left(c(\lambda+2)+(3\lambda+2)(\lambda-1)\right)}$$

$$(C_{34}^{5})^{2}=\frac{25(5c+22)(\lambda-4)\big(c(\lambda+4)+3(5\lambda+4)(\lambda-1)\big)}{(7c+114)(\lambda-2)\left(c(\lambda+2)+(3\lambda+2)(\lambda-1)\right)}$$

$$\begin{aligned}C_{45}^{5}=&\frac{15}{8(\lambda-3)(c+2)(114+7c)\big(c(\lambda+3)+2(4\lambda+3)(\lambda-1)\big)}C_{33}^{4}\\&\times\Big[c^{3}(3\lambda^{2}-97)+c^{2}(94\lambda^{3}-467\lambda^{2}-483)+c(856\lambda^{3}-5192\lambda^{2}+4120)\\&\quad+216\lambda^{3}-6972\lambda^{2}+6756\Big]\,.\end{aligned}$$

Higher Structure Constants

Actually, can rewrite all of them more simply as

[MRG, Gopakumar '12]

$$C^4_{44} = \frac{9(c+3)}{4(c+2)}\,\gamma - \frac{96(c+10)}{(5c+22)}\,\gamma^{-1}$$

$$(C^5_{34})^2 = \frac{75(c+7)(5c+22)}{16(c+2)(7c+114)}\,\gamma^2 - 25$$

$$C^5_{45} = \frac{15\,(17c+126)(c+7)}{8\,(7c+114)(c+2)}\,\gamma - 240\,\frac{(c+10)}{(5c+22)}\,\gamma^{-1}$$

where $\gamma^2 \equiv \left(C^4_{33}\right)^2$

These structure constants (and probably all) are actually determined in terms of γ^2 by Jacobi identity.

[Candu, MRG, Kelm, Vollenweider, unpublished]

Quantum algebra

Thus full quantum algebra seems to be characterised by two free parameters: central charge c and

$$\gamma^2 \text{ (rather than } \lambda)$$ [MRG, Gopakumar '12]

But

$$(C_{33}^4)^2 \equiv \gamma^2 = \frac{64(c+2)(\lambda-3)\big(c(\lambda+3)+2(4\lambda+3)(\lambda-1)\big)}{(5c+22)(\lambda-2)\big(c(\lambda+2)+(3\lambda+2)(\lambda-1)\big)} .$$

Thus there are three roots that lead to the same algebra:

$$\mathcal{W}_\infty[\lambda_1] \cong \mathcal{W}_\infty[\lambda_2] \cong \mathcal{W}_\infty[\lambda_3] \qquad \text{at fixed } c$$

\`Triality'

Triality

In particular,

$$\mathcal{W}_\infty[N] \cong \mathcal{W}_\infty[\tfrac{N}{N+k}] \cong \mathcal{W}_\infty[-\tfrac{N}{N+k+1}] \quad \text{at } c = c_N(k)$$

minimal model

asymptotic symmetry
algebra of hs theory

This is even true at finite N and k, not just in the 't Hooft limit!

This triality generalises level-rank duality of coset models of [Kuniba, Nakanishi, Suzuki '91] and [Altschuler, Bauer, Saleur '90].

Symmetries

So the symmetries give strong evidence for the duality

HS on AdS3 =CS with $\mathrm{hs}[\lambda]$	$\longleftrightarrow$	2d CFT with $\mathcal{W}_\infty[\lambda]$ symmetry = minimal models
(ℓ, λ)	$\xleftrightarrow[\frac{3\ell}{4G_N} = c_{N,k}]{\lambda = \frac{N}{N+k}}$	(N, k)
size of AdS3		(sugra: c large)

Spectrum

Higher spin fields themselves correspond only to the vacuum representation of the W-algebra!

Contribution from all representations of the form $(\Lambda; 0)$ is accounted for by adding to the hs theory a complex scalar field of mass [MRG, Gopakumar '10]

$$-1 \leq M^2 \leq 0 \quad \text{with} \quad M^2 = -(1-\lambda^2) \ .$$

[Compatible with hs symmetry since hs theory has massive scalar multiplet with this mass.]

[Prokushkin, Vasiliev '98]

Total 1-loop partition function

The perturbative 1-loop partition function of the hs theory is then

$$Z^{(1)}_{\rm pert} = \prod_{s=2}^{\infty}\prod_{n=s}^{\infty}\frac{1}{|1-q^n|^2} \times \prod_{l,l'=0}^{\infty}\frac{1}{(1-q^{h+l}\bar{q}^{h+l'})^2}$$

$\mathcal{W}_N$ modes

$$(\mathrm{f};0)^{\otimes r_1}\otimes(\bar{\mathrm{f}};0)^{\otimes r_2}$$

We have shown analytically that this agrees exactly with CFT partition function of $(\Lambda;0)$ representations in 't Hooft limit!

[MRG,Gopakumar, '10]
[MRG,Gopakumar,Hartman,Raju, '11]

Spectrum

Thus

$$\boxed{Z_{\text{pert}} = \sum_{\Lambda} |\chi_{(\Lambda,0)}|^2 \ .}$$

The remaining states, i.e. those of the form

$$(\Lambda;\nu) \qquad \text{with } \nu \neq 0$$

seem to correspond to conical defect solutions (possibly dressed with perturbative excitations).

[Castro, Gopakumar, Gutperle, Raeymaekers '11]
[MRG, Gopakumar '12]
[Perlmutter, Prochazka, Raeymaekers '12]

Recent developments

[MRG, Gopakumar '13]

In order to understand relation to string theory have studied supersymmetric version of duality, in particular, case with large $\mathcal{N} = 4$ superconformal symmetry.

hs theory based on

$$\mathrm{shs}_2[\lambda] \quad \longleftrightarrow \quad \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa \ .$$

Wolf space cosets

in 't Hooft limit with $\lambda = \dfrac{N}{N+k+2}$.

Symmetries

The Wolf space coset CFTs have same symmetry as dual CFT of string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

Large $\mathcal{N} = 4$

$$\boxed{\mathrm{Vir} \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)}$$

with 4 supercharges

[Boonstra, Peeters, Skenderis '98; Elitzur, Feinerman, Giveon, Tsabar '99; de Boer, Pasquinucci, Skenderis '99; Gukov, Martinec, Moore, Strominger '04; ...]

Relation to string theory

large $\mathcal{N}=4$	small $\mathcal{N}=4$
$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$	$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$
hs theory based on $\mathrm{shs}_2[\lambda]$	string theory
$\updownarrow$	$\updownarrow$
Wolf space cosets	symmetric orbifold $\left(\mathbb{T}^4\right)^{\otimes(N+1)}/S_{N+1}$

Contraction

While we cannot compare these two dualities directly, the large superconformal symmetry contracts to the small superconformal symmetry in the limit in which the level k goes to infinity

$$\lambda = \frac{N}{N+k+2} \to 0$$

Indeed, this just describes the case where the radius of one of the two 3-spheres goes to infinity, and hence the sphere approximates flat space.

hs theory in string theory

$\boxed{\text{large } \mathcal{N} = 4}$ $\boxed{\text{small } \mathcal{N} = 4}$

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1 \xrightarrow{\lambda \to 0} \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$

hs theory based on $\mathrm{shs}_2[\lambda]$

string theory

$\updownarrow$ $\updownarrow$

Wolf space cosets $\subset$ symmetric orbifold

$\mathrm{Sym}_{N+1}(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes(N+1)}/S_{N+1}$

Wolf space cosets

What happens to the Wolf space cosets in this limit?

$$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa \ .$$

[MRG, Suchanek '11]

As in bosonic case, the `perturbative' part of the spectrum can be identified with the U(N)-singlet sector

$$\mathcal{H}_{\rm pert} = \bigoplus_{\Lambda} (0;\Lambda) \otimes (0;\Lambda^*) = \begin{pmatrix} 4(N+1) \text{ free bosons} \\ 4(N+1) \text{ free fermions} \end{pmatrix} / \mathrm{U}(N)$$

[MRG, Gopakumar '14]

Untwisted sector

Here the free bosons and fermions transform as

$$\text{bosons:} \qquad 2 \cdot (\mathbf{N}, \mathbf{1}) \oplus 2 \cdot (\bar{\mathbf{N}}, \mathbf{1}) \oplus 4 \cdot (\mathbf{1}, \mathbf{1})$$

$$\text{fermions:} \qquad (\mathbf{N}, \mathbf{2}) \oplus (\bar{\mathbf{N}}, \mathbf{2}) \oplus 2 \cdot (\mathbf{1}, \mathbf{2})$$

$$\mathrm{U(N)} \quad \mathfrak{su}(2)$$

The other coset representations can be interpreted as twisted sectors (and descendants) of this continuous orbifold --- actually, can give very concrete identification....

[MRG, Gopakumar '14]
[MRG, Kelm '14]

Comparison

This now looks very similar to the untwisted sector of the symmetric orbifold

$$\mathrm{Sym}_{N+1}(\mathbb{T}^4) \equiv \left(\mathbb{T}^{4(N+1)}\right)/S_{N+1}$$

Indeed, this sector is generated by free bosons and fermions in

$$\begin{aligned}
&\text{bosons:} \qquad 4\cdot(\mathbf{N+1},\mathbf{1}) = 4\cdot(\mathbf{N},\mathbf{1}) \oplus 4\cdot(\mathbf{1},\mathbf{1}) \\
&\text{fermions:} \qquad 2\cdot(\mathbf{N+1},\mathbf{2}) = 2\cdot(\mathbf{N},\mathbf{2}) \oplus 2\cdot(\mathbf{1},\mathbf{2})
\end{aligned}$$

S_{N+1} $\quad \mathfrak{su}(2)$

Branching rule

In fact

$$S_{N+1} \subset \mathrm{U}(N)$$

and under this embedding, we have the branching rules

$$\mathbf{N}_{\mathrm{U}(N)} \to \mathbf{N}_{S_{N+1}} \qquad \bar{\mathbf{N}}_{\mathrm{U}(N)} \to \mathbf{N}_{S_{N+1}}$$

Comparison

Wolf coset:

$$\text{bosons:} \qquad 2\cdot(\mathbf{N},\mathbf{1})\oplus 2\cdot(\bar{\mathbf{N}},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1})$$

$$\text{fermions:} \qquad (\mathbf{N},\mathbf{2})\oplus(\bar{\mathbf{N}},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})$$

Symmetric orbifold:

$$\text{bosons:} \qquad 4\cdot(\mathbf{N},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1})$$

$$\text{fermions:} \qquad 2\cdot(\mathbf{N},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})$$

Thus the action of the permutation group on the free bosons and fermions is induced from the U(N) action!

Subtheory

It therefore follows that [MRG, Gopakumar '14]

U(N) invariant states of free theory $\subset$ untwisted sector of sym. orbifold i.e. S_{N+1} invariant states of free theory

↑ perturbative part of CFT dual of hs theory for $\lambda \to 0$

↑ (part of) CFT dual of string theory in this limit

hs theory is closed subsector of string theory!

Stringy symmetry

From the hs point of view, the symmetric orbifold (i.e. the stringy CFT dual) is characterised by an extended chiral algebra.

The character of this stringy algebra equals

$$Z_{\text{stringy}}(q,y) = \sum_{\Lambda} \underbrace{D(\Lambda)}_{\text{multiplicity of singlet representation of } S_{N+1}} \chi_{(0;\Lambda)}(q,y)$$

Stringy chiral algebra

Explicitly, we find

$$
\begin{aligned}
Z_{\text{stringy}}(q,y) \quad = \quad & \chi_{(0;0)}(q,y) + \chi_{(0;[2,0,\ldots,0])}(q,y) + \chi_{(0;[0,0,\ldots,0,2])}(q,y) \\
& + \chi_{(0;[3,0,\ldots,0,0])}(q,y) + \chi_{(0;[0,0,0,\ldots,0,3])}(q,y) \\
& + \chi_{(0;[2,0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,0,\ldots,0,2])}(q,y) \\
& + 2\cdot\chi_{(0;[4,0,\ldots,0,0])}(q,y) + 2\cdot\chi_{(0;[0,0,0,\ldots,0,4])}(q,y) \\
& + \chi_{(0;[0,2,0,\ldots 0,0])}(q,y) + \chi_{(0;[0,0,\ldots 0,2,0])}(q,y) \\
& + \chi_{(0;[3,0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,0,\ldots,0,3])}(q,y) \\
& + 2\cdot\chi_{(0;[2,0,0,\ldots,0,2])}(q,y) + \cdots
\end{aligned}
$$

This reproduces precisely vacuum character of symmetric orbifold (from DMVV).

Light States

The stringy extension does not contain any of the `light' states any longer since they do not give rise to allowed (untwisted) representations of the extended chiral algebra.

Spacetime intepretation:

classical hs solutions do not lift to string theory.

Stringy completion

At $\lambda \to 0$ stringy description characterised by extended chiral algebra (which could be directly obtained from symmetric orbifold).

It is natural to believe that the same idea should also work away from this special point --- this suggests a new avenue for how to find the CFT dual of string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

Stringy symmetries

Our description gives a very explicit description of `maximal' stringy symmetry.

We are currently trying to understand its algebraic structure, i.e., the wedge subalgebra that forms a conventional Lie algebra (extending the higher spin algebra). It has the structure of a `Higher Spin Square', i.e., it contains two higher spin algebras that generate the full algebra…..

[MRG, Gopakumar, '15]

Regge trajectories

There is also a natural idea for how to identify the different `Regge trajectories' in this description:

quadratic terms = original higher spin fields

= leading Regge trajectory

symmetric orbifold

cubic terms = first subleading Regge trajectory

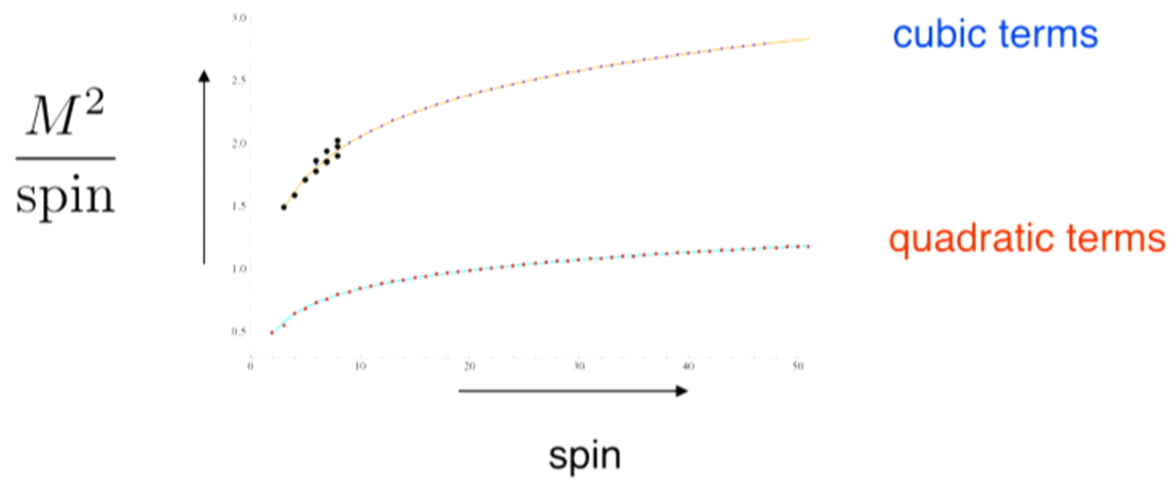
quartic terms = 2nd subleading Regge trajectory

etc…

Tension perturbation

Evidence from analysis of perturbation by exactly marginal operator that corresponds to switching on string tension

[MRG, Peng, Zadeh, to appear]



Conclusions

- Explained evidence for bosonic minimal model holography and sketched large $\mathcal{N} = 4$ generalisation.
- In the supersymmetric case found natural embedding of CFT dual of hs theory into CFT dual of string theory.
- Gives first concrete realisation of idea that hs theory emerges as a subsector of string theory in tensionless limit.

Open problems & future directions

Open problems:

- Interpretation from D1-D5 viewpoint
- Relation to spin chain picture

[Babichenko, Stefanski, Zarembo '09]
[Borsato, Ohlsson Sax, Sfondrini, Stefanski '14]

HS viewpoint: new perspective on stringy CFT

- Find Lie algebra structure of stringy symmetry
- Find other stringy modular invariants
- higher dimensional analogue?

cf. [Chang, Minwalla, Sharma, Yin '12]
cf. [Beisert,Bianchi,Morales,Samtleben '04]