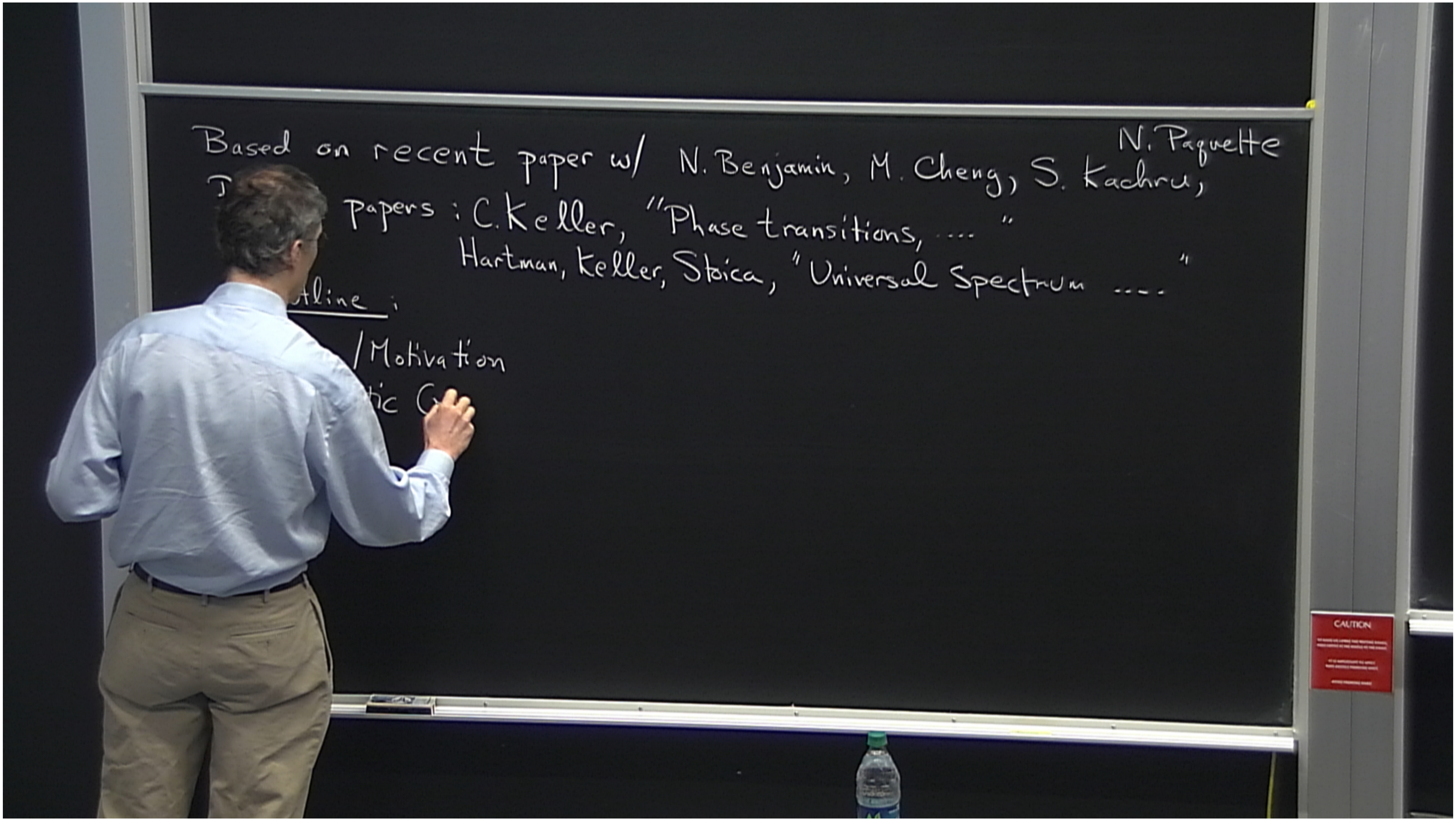
Title: Measuring the Elliptic Genus

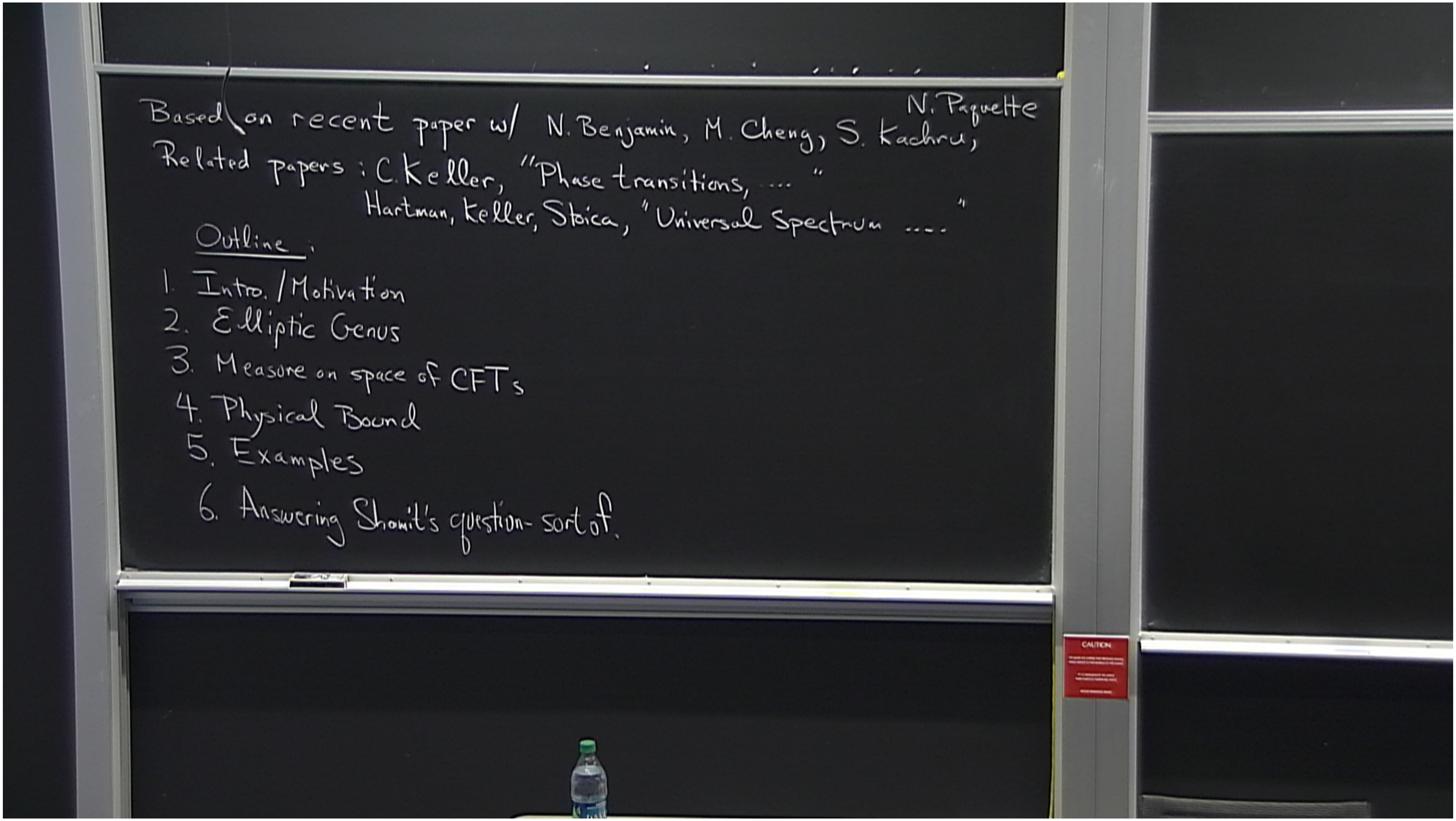
Date: Apr 17, 2015 10:00 AM

URL: http://pirsa.org/15040134

Abstract: This talk is based on the recent paper co-authored with N. Benjamin, M. Cheng, S. Kachru, and N. Paquette.


N. Paquette
Based on recent paper w/ N. Benjamin, M. Cheng, S. Kachru,
papers : C. Keller, "Phase transitions, ... "
Hartman, Keller, Stoica, "Universal Spectrum .... "
/Motivation


Based on recent paper w/ N. Benjamin, M. Cheng, S. Kachru, N. Paquette
Related papers: C. Keller, "Phase transitions, ..."
Hartman, Keller, Stoica, "Universal Spectrum ..."
Outline:
1. Intro/Motivation
2. Elliptic Genus
3. Measure on space of CFTs
4. Physical Bound
5. Examples
6. Answering Shamit's question - sort of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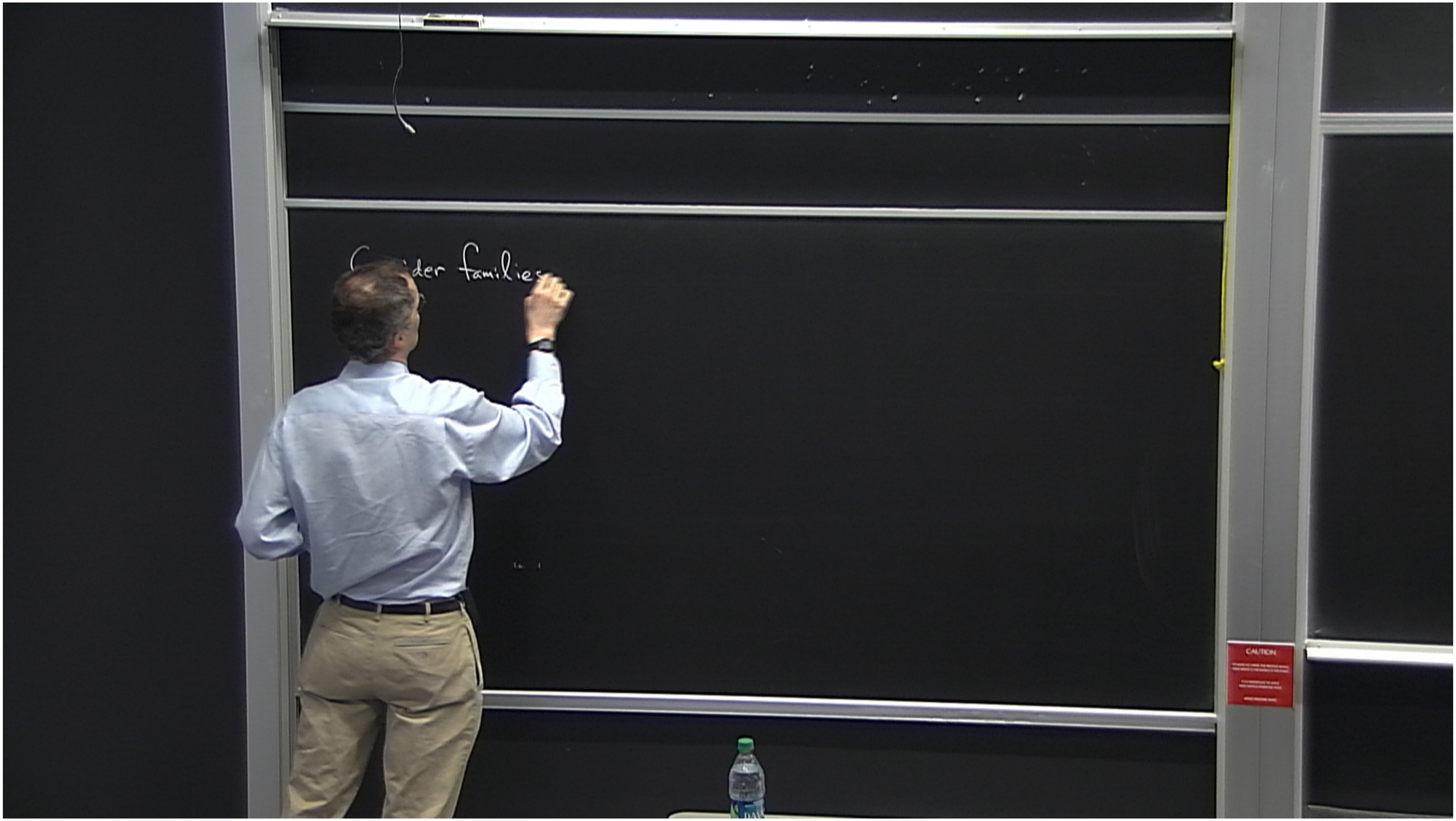
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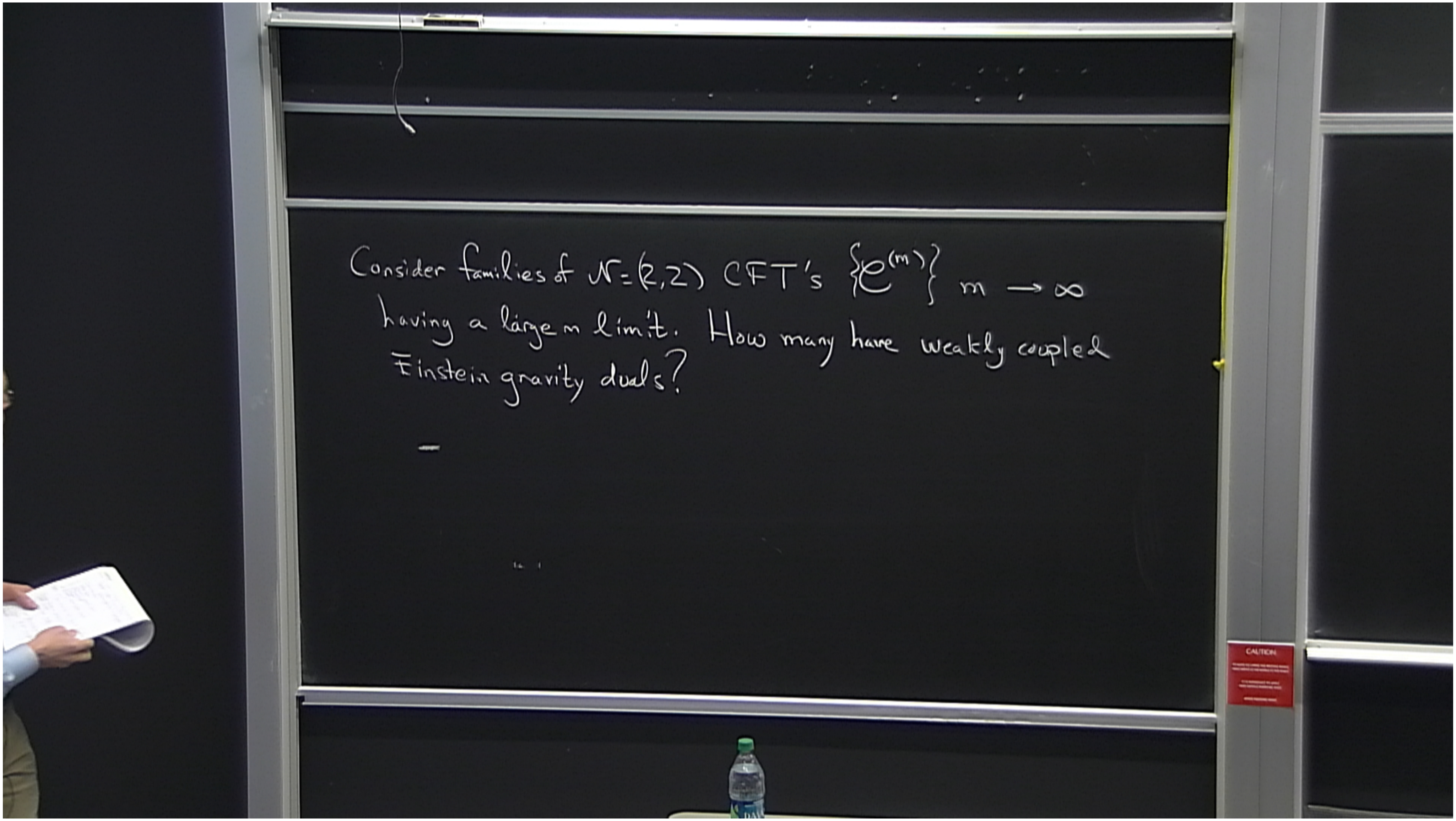
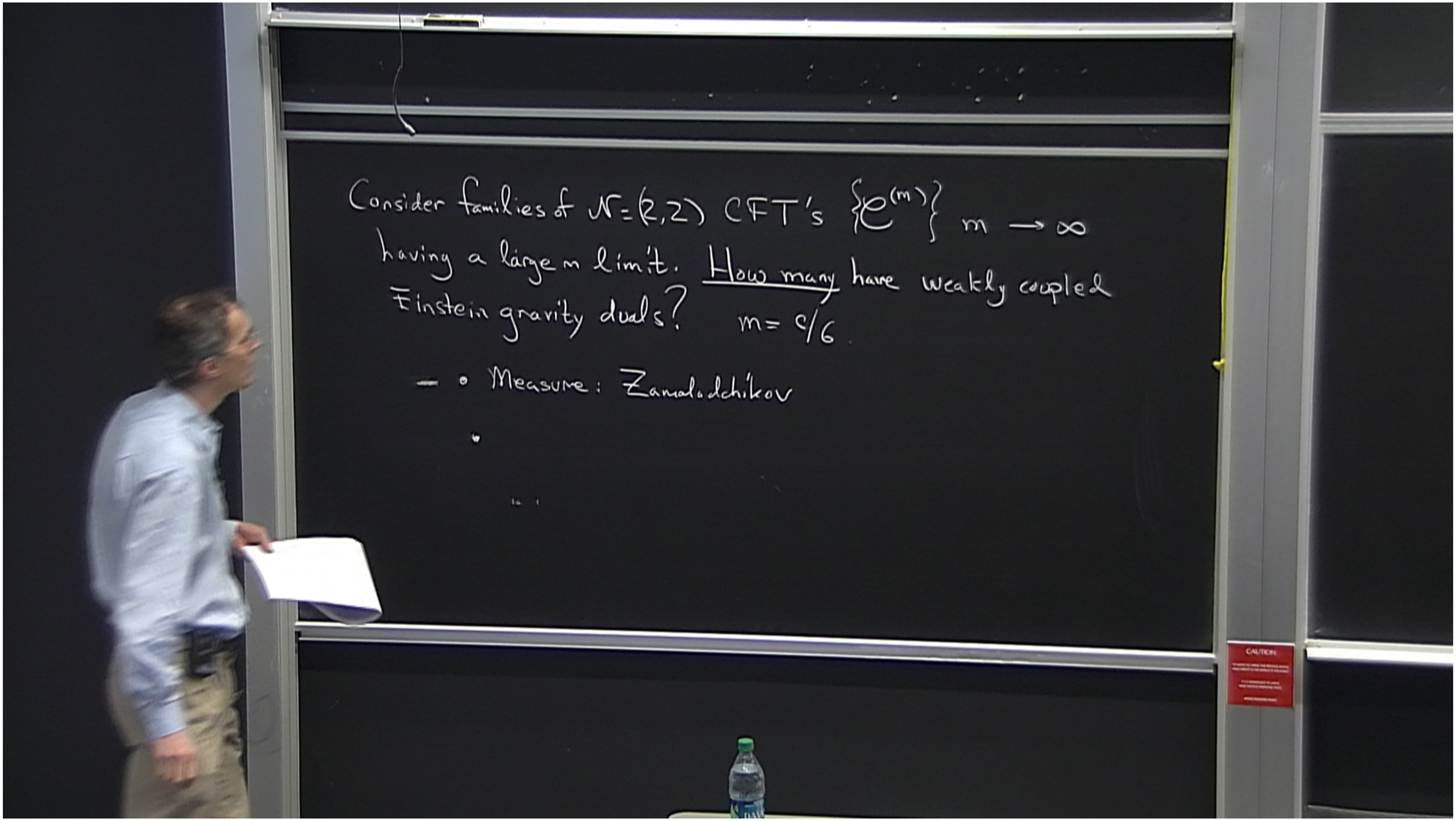
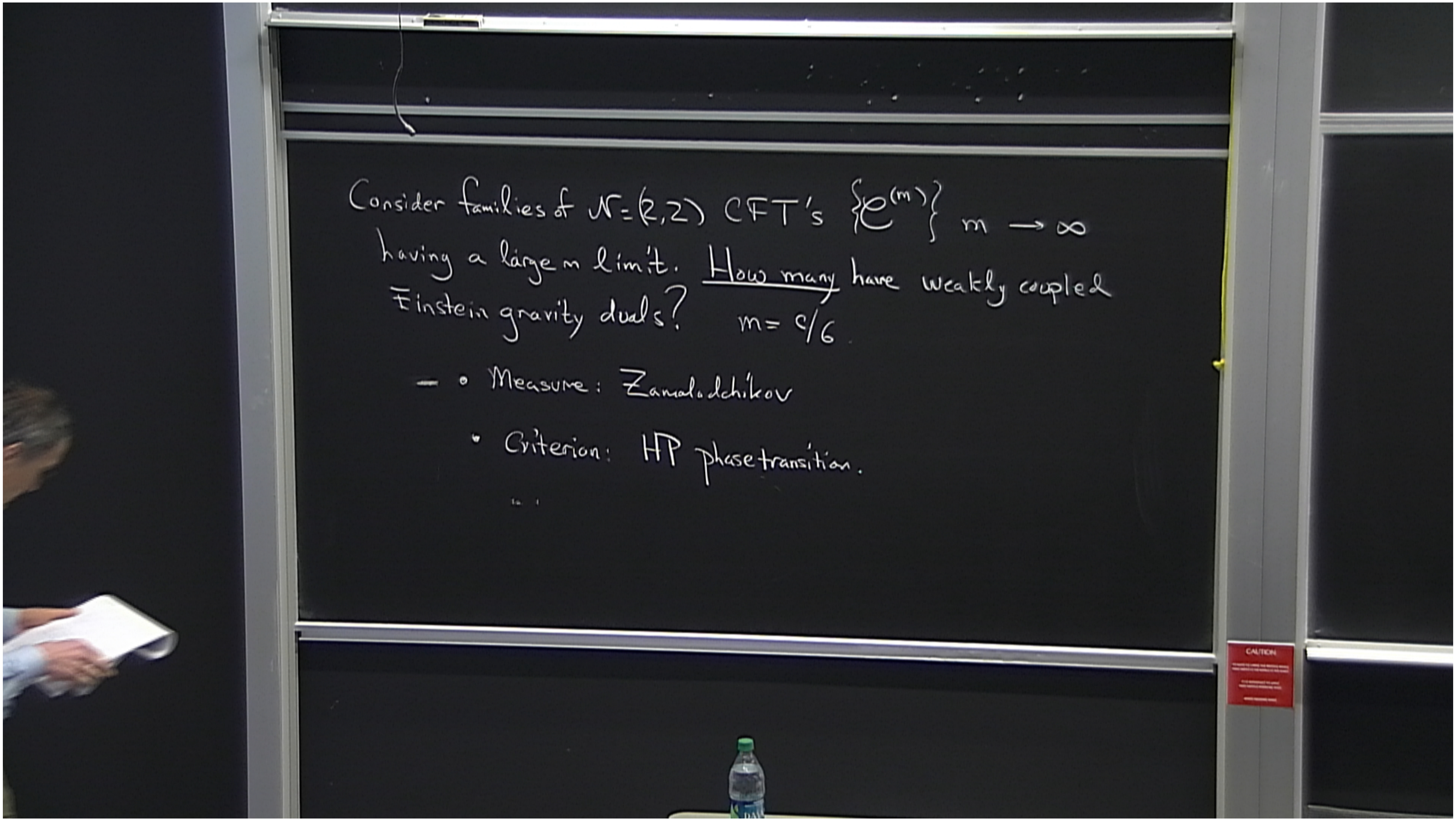

Consider families of $\mathcal{N}=(2,2)$ CFT's $\{\mathcal{C}^{(m)}\}$ $m \to \infty$
having a large m limit. How many have weakly coupled
Einstein gravity duals?


Consider families of N=(2,2) CFT's {C^(m)} m → ∞
having a large m limit. How many have weakly coupled
Einstein gravity duals? m= c/6
• Measure: Zamolodchikov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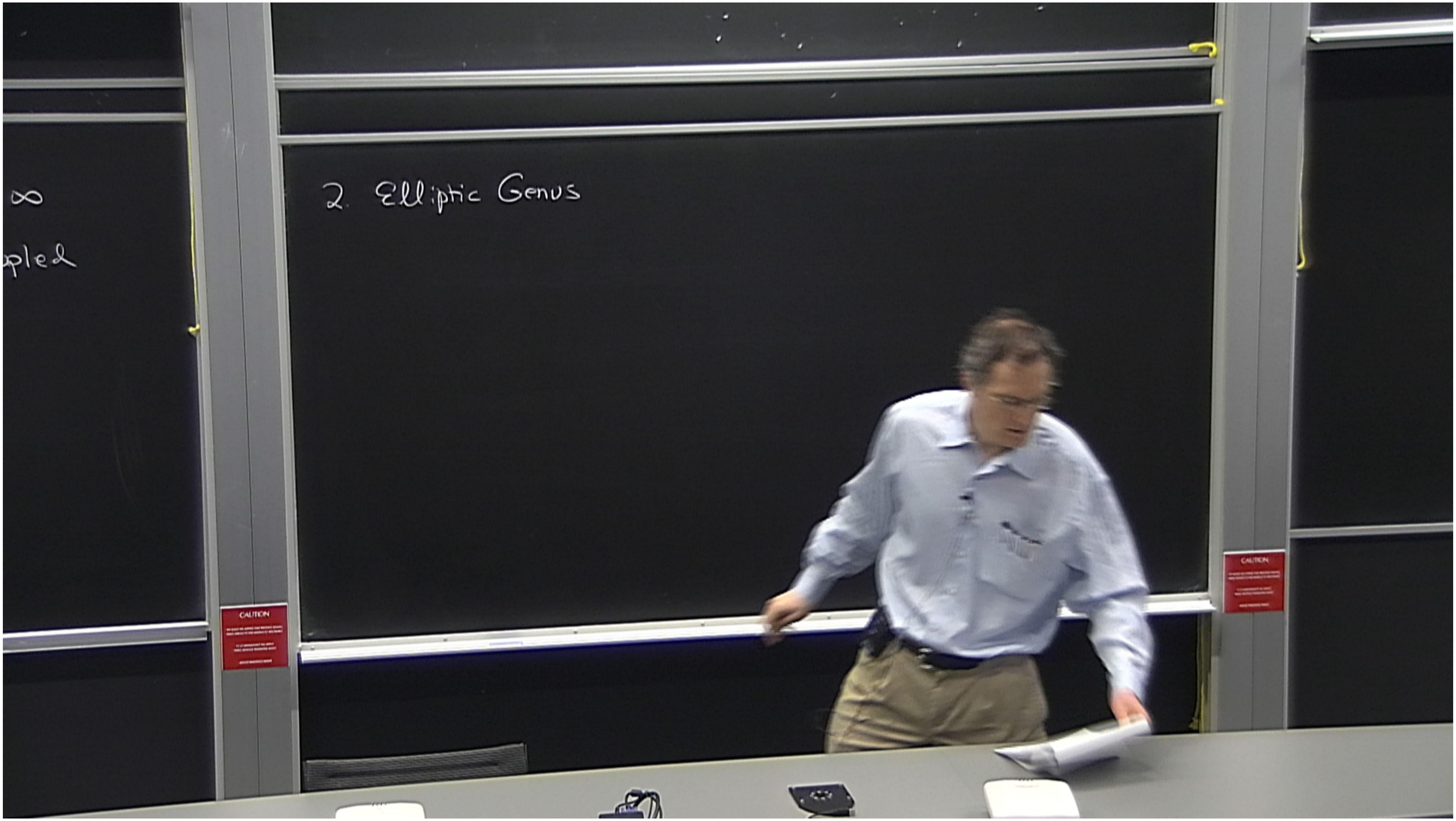
• Measure: Zamolodchikov
• Criterion: HP phase transition.


2. Elliptic Genus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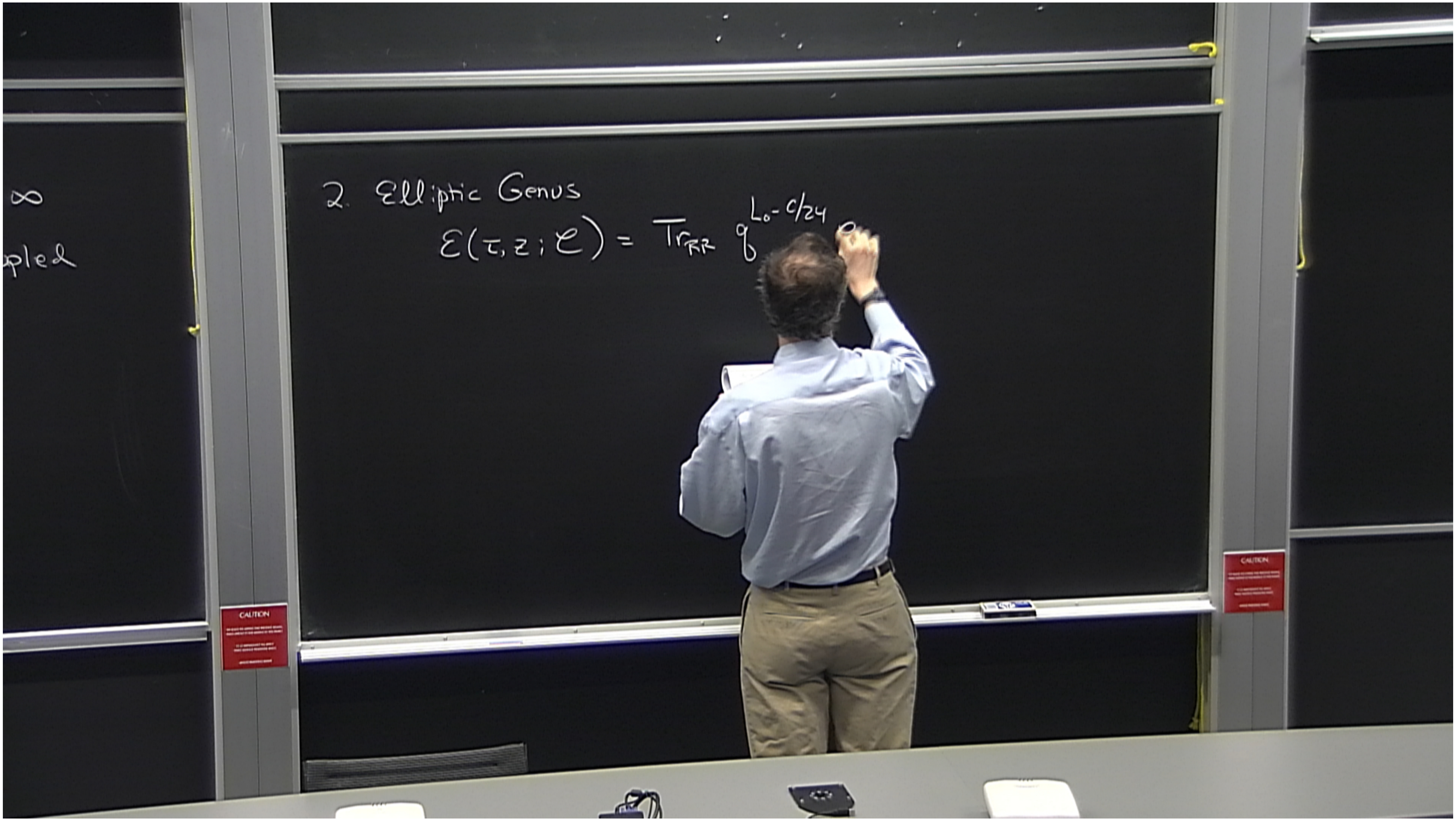
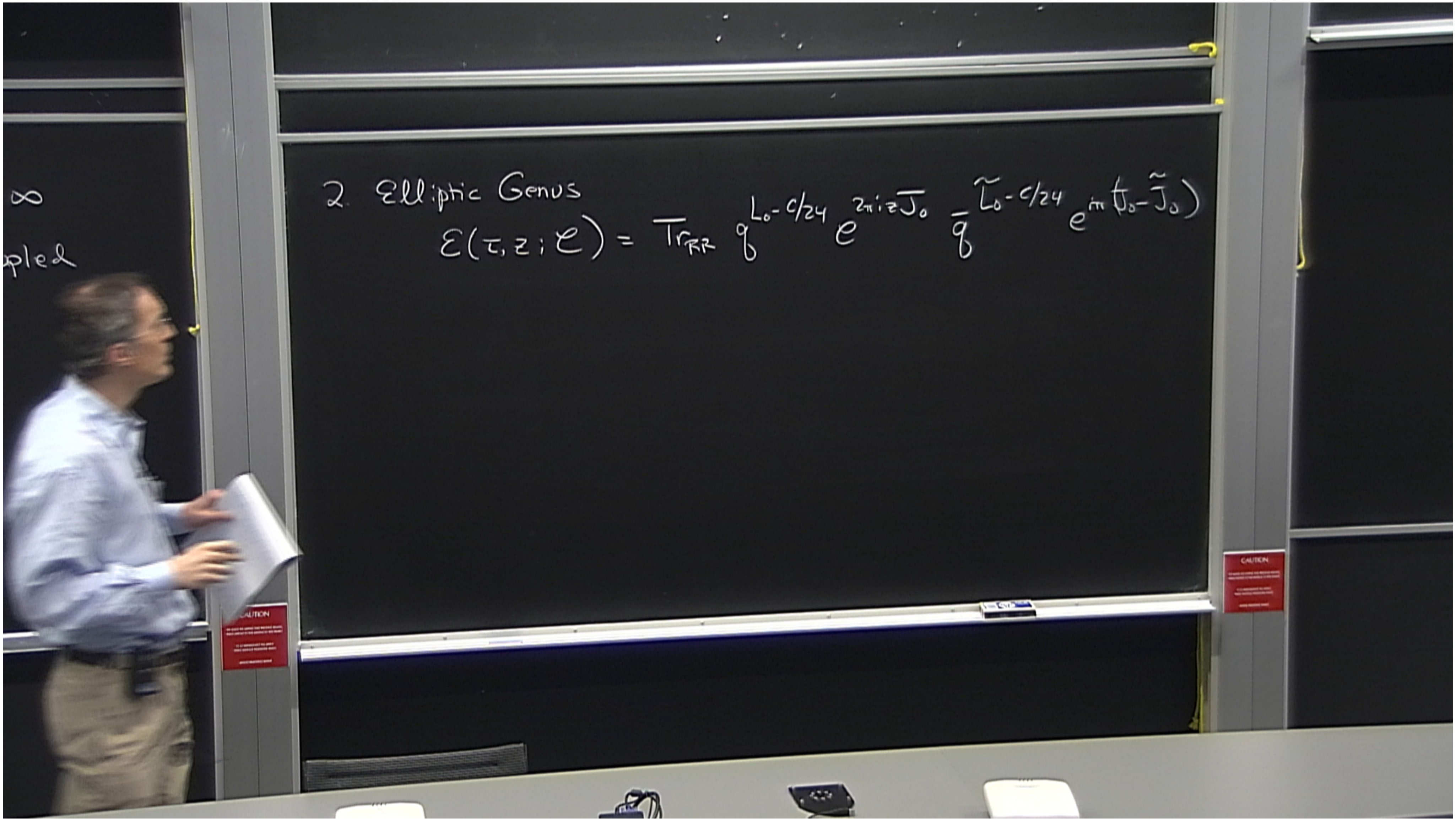
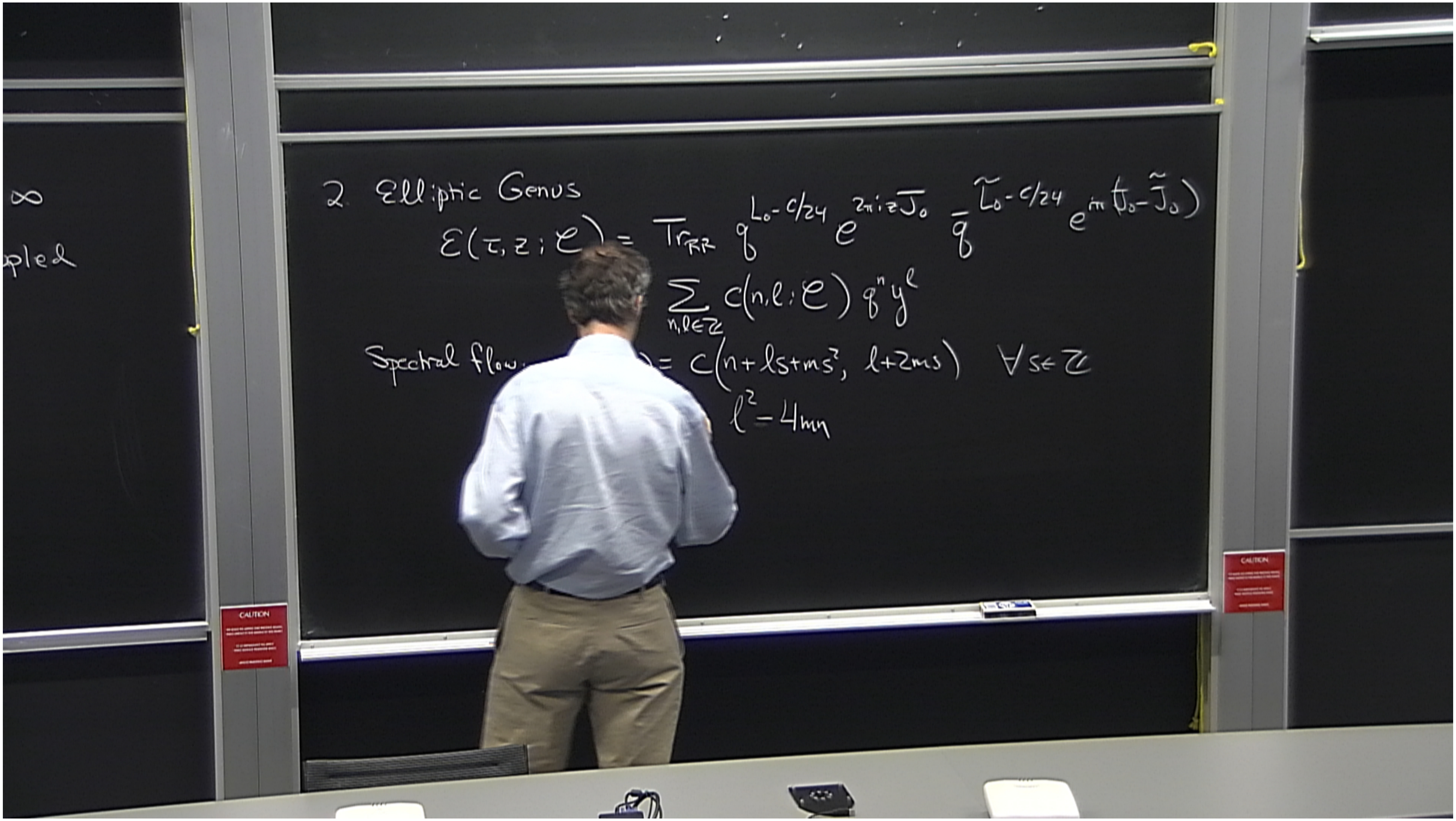
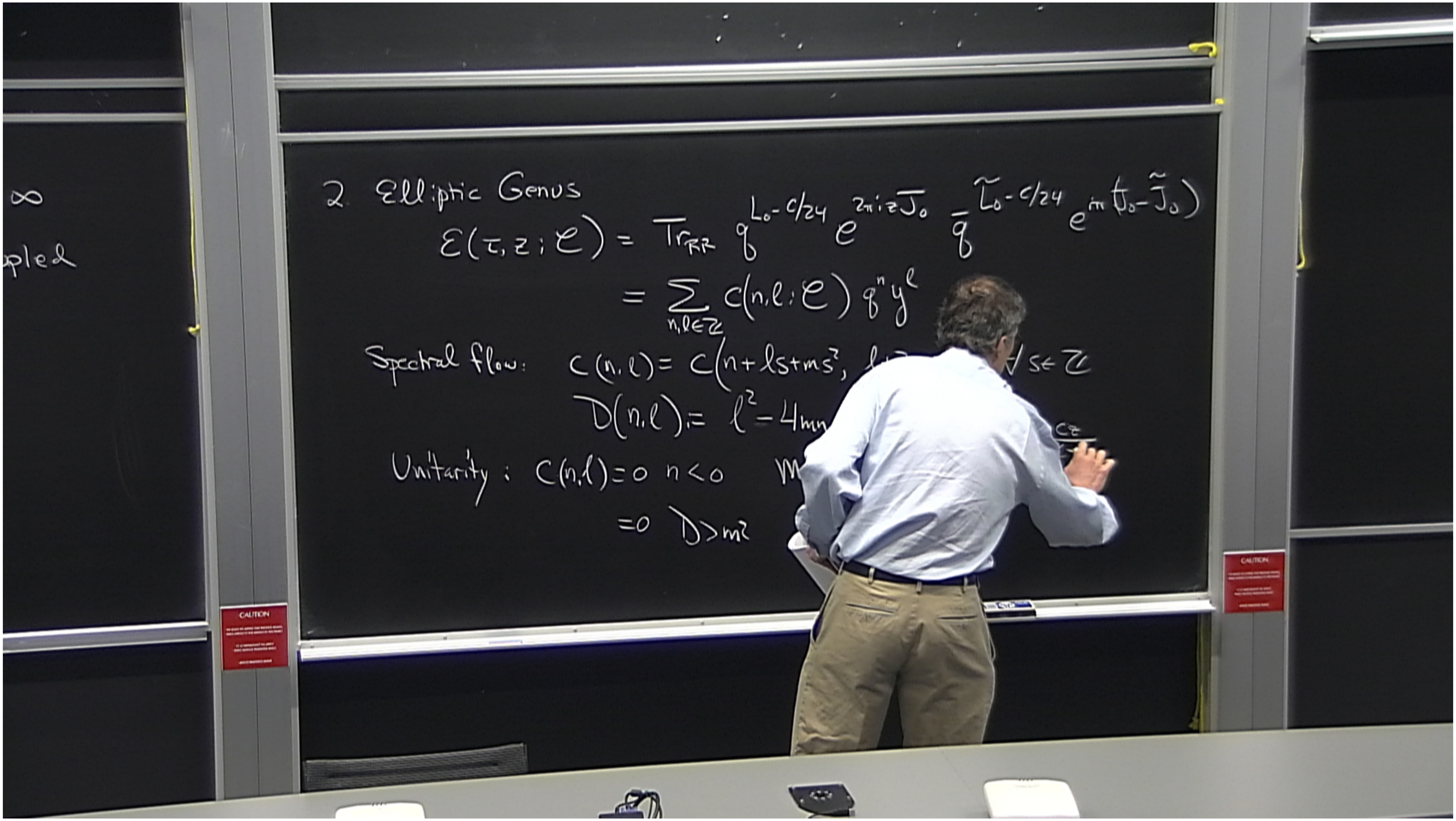

2. Elliptic Genus
$\mathcal{E}(\tau, z; \mathcal{C}) = \mathrm{Tr}_{RR}\, q^{L_0 - c/24}$
CAUTION
CAUTION


2. Elliptic Genus
$\mathcal{E}(\tau, z; \mathcal{C}) = \mathrm{Tr}_{RR}\, q^{L_0 - c/24} e^{2\pi i z J_0} \bar{q}^{\tilde{L}_0 - c/24} e^{i\pi(J_0 - \tilde{J}_0)}$


∞
pled
2. Elliptic Genus
E(τ, z ; C) = Tr_RR q^(L₀ - c/24) e^(2πizJ₀) q̄^(L̃₀ - c/24) e^(iπ(J₀ - J̃₀))
Σ_(n,l∈Z) c(n,l ; C) q^n y^l
Spectral flow:
= c(n + ls + ms², l + 2ms) ∀ s ∈ Z
l² - 4mn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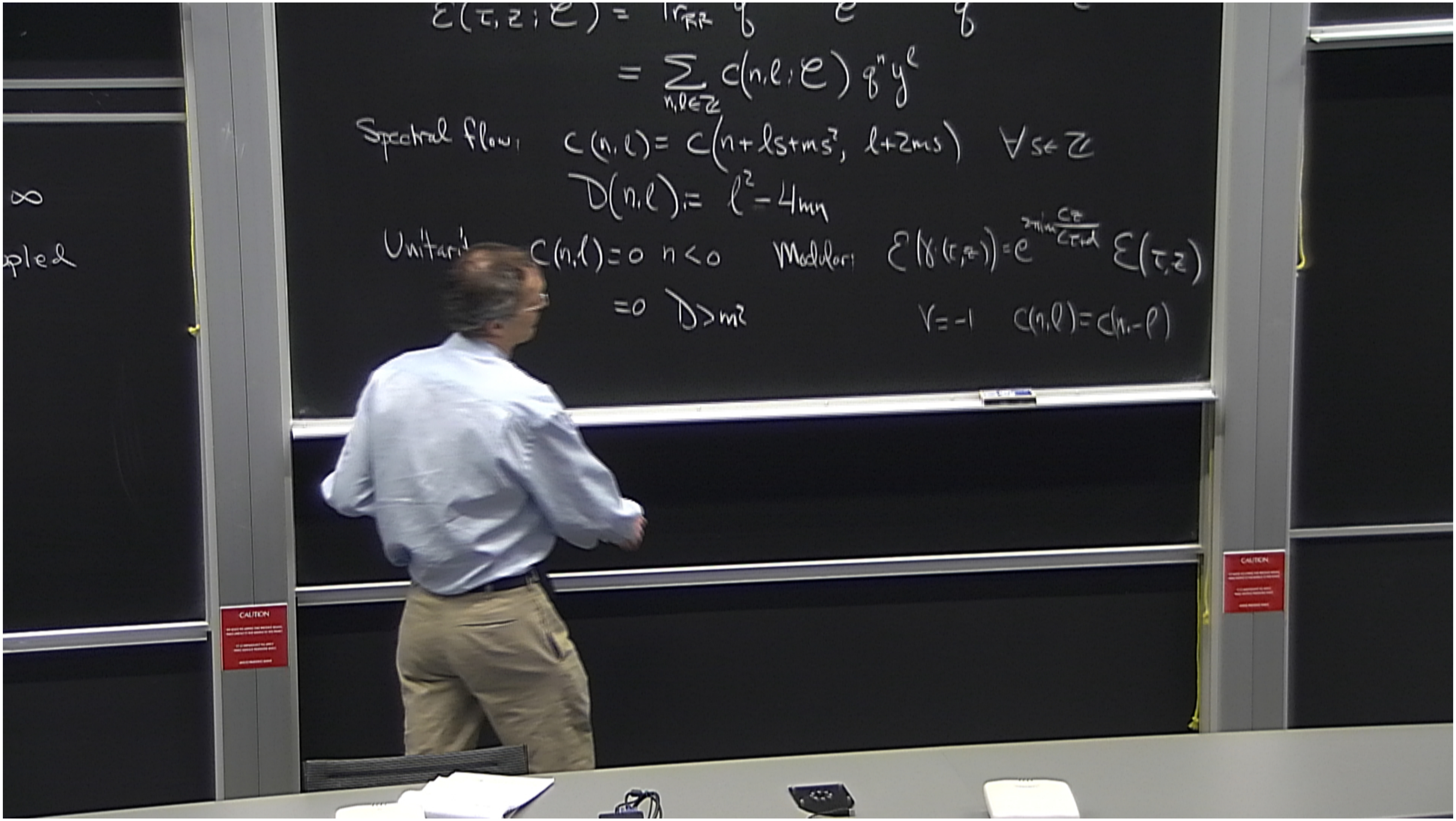
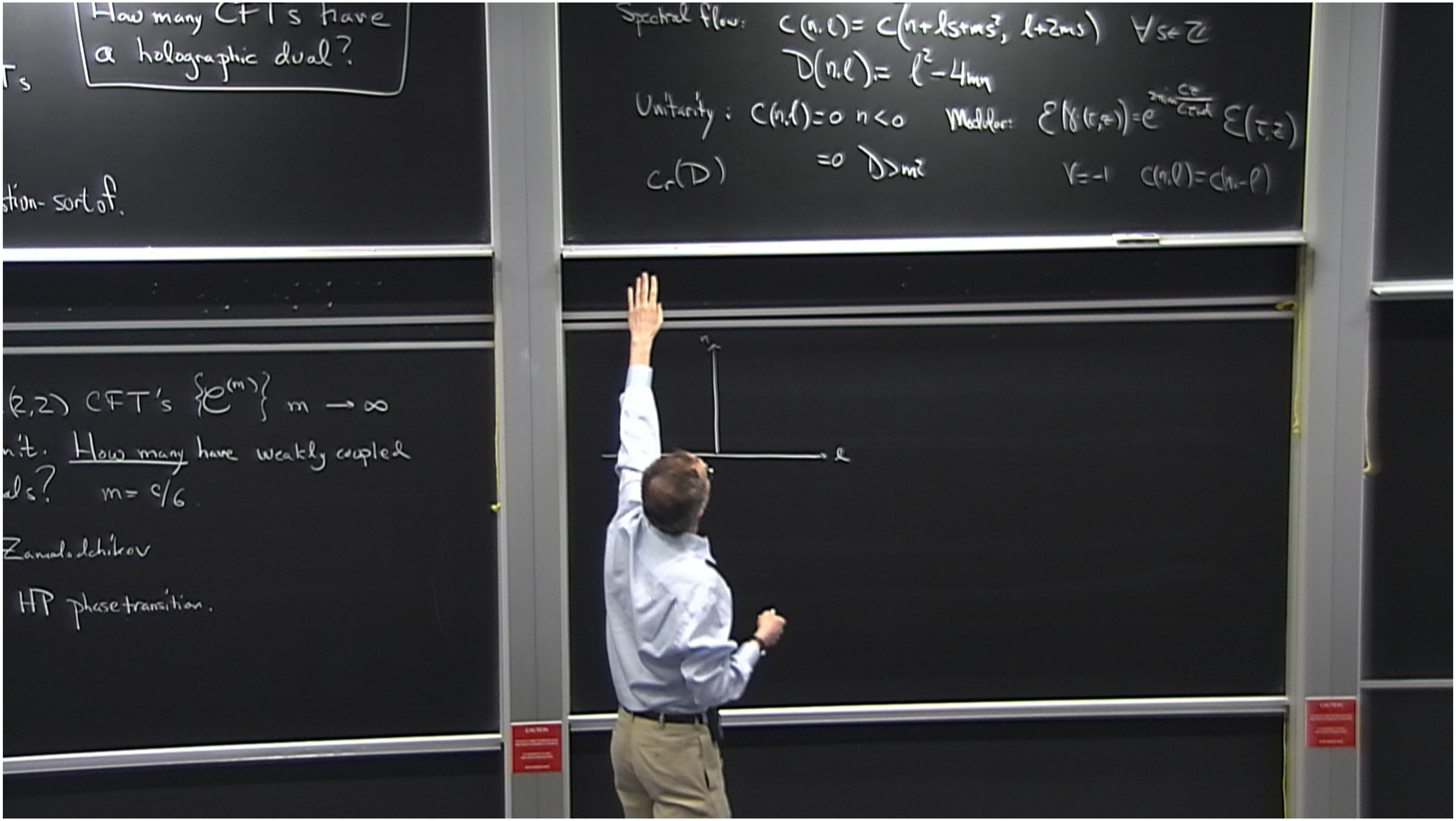
= Σ_(n,l∈Z) c(n,l;C) q^n y^l
Spectral flow: c(n,l) = c(n+ls+ms², ∀ s∈Z
D(n,l) := l² - 4mn
Unitarity: c(n,l) = 0 n<0
=0 D>m²


= Σ_{n,ℓ∈Z} c(n,ℓ;C) q^n y^ℓ
Spectral flow: c(n,ℓ) = c(n+ℓs+ms², ℓ+2ms) ∀ s∈Z
D(n,ℓ) := ℓ² − 4mn
Unitarity c(n,ℓ) = 0 n<0
=0 D>m²
Modular:
γ = −1 c(n,ℓ) = c(n,−ℓ)
CAUTION


How many CFTs have
a holographic dual?
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Spectral flow: c(n,l) = c(n+ls+ms², l+2ms) ∀s∈Z
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c_r(D)
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HP phase transition.
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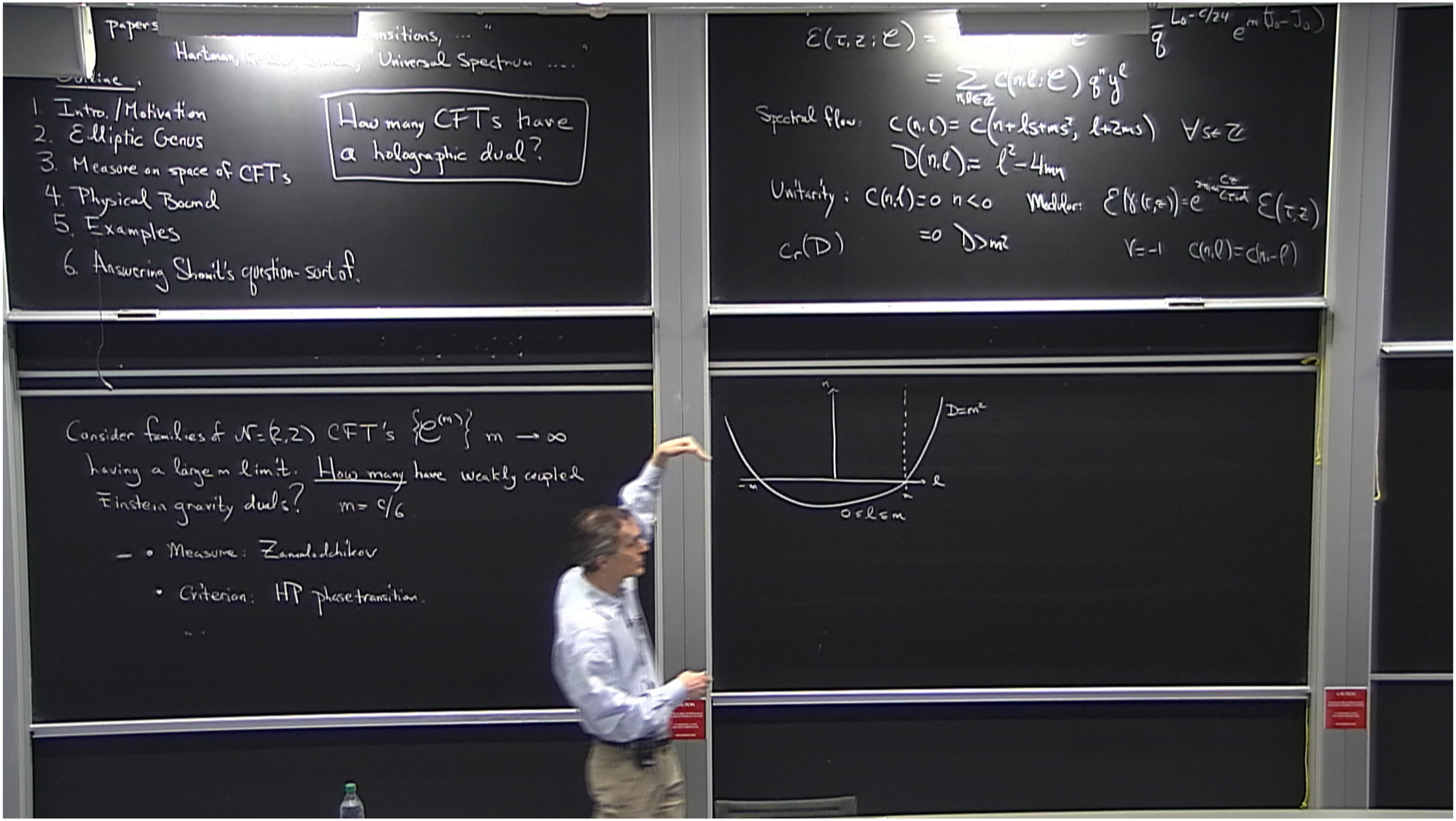
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Unitarity: c(n,l) = 0 n<0
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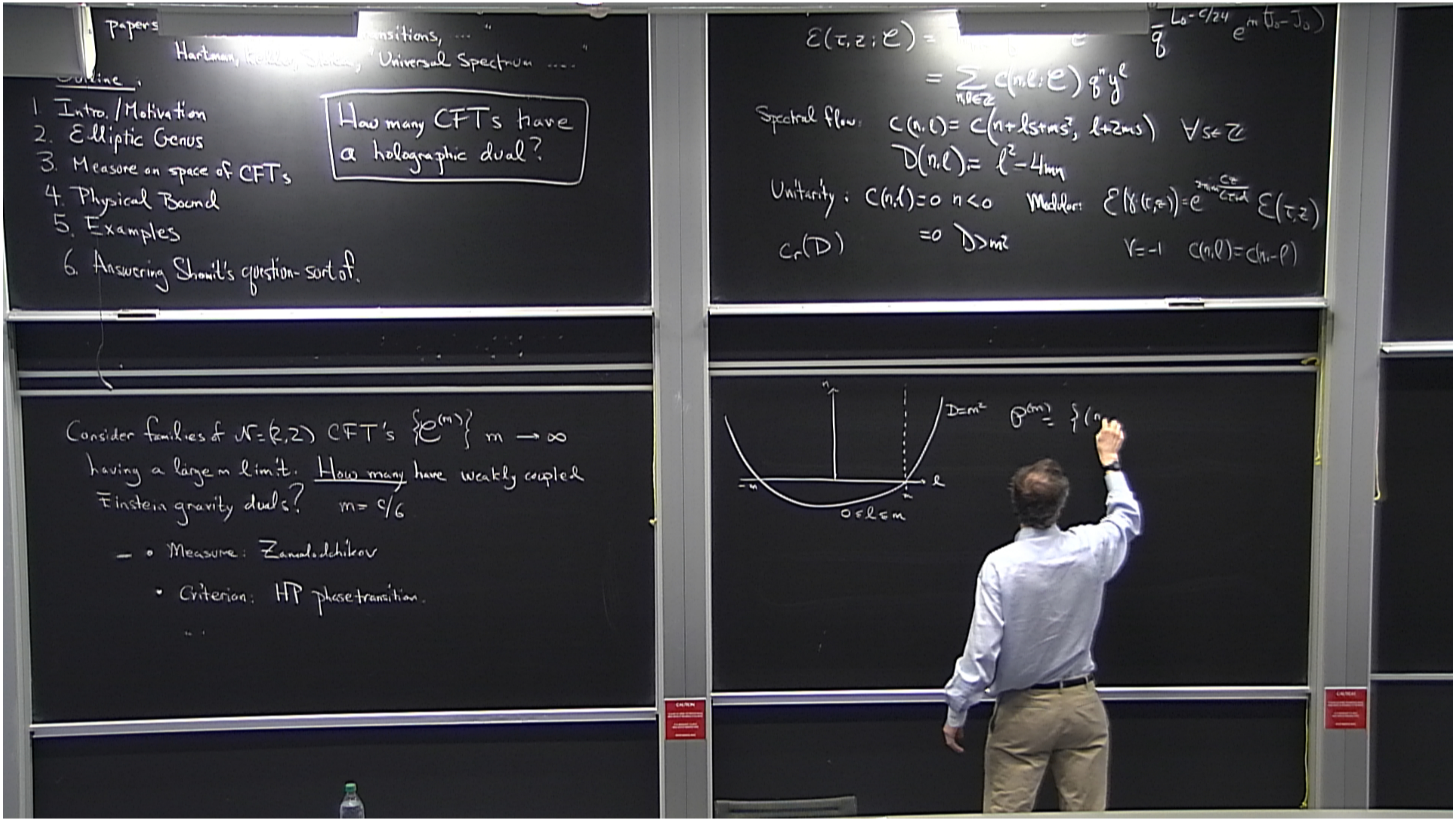
Modular:
γ = −1 c(n,l) = c(n,−l)
D=m²
l
0≤l≤m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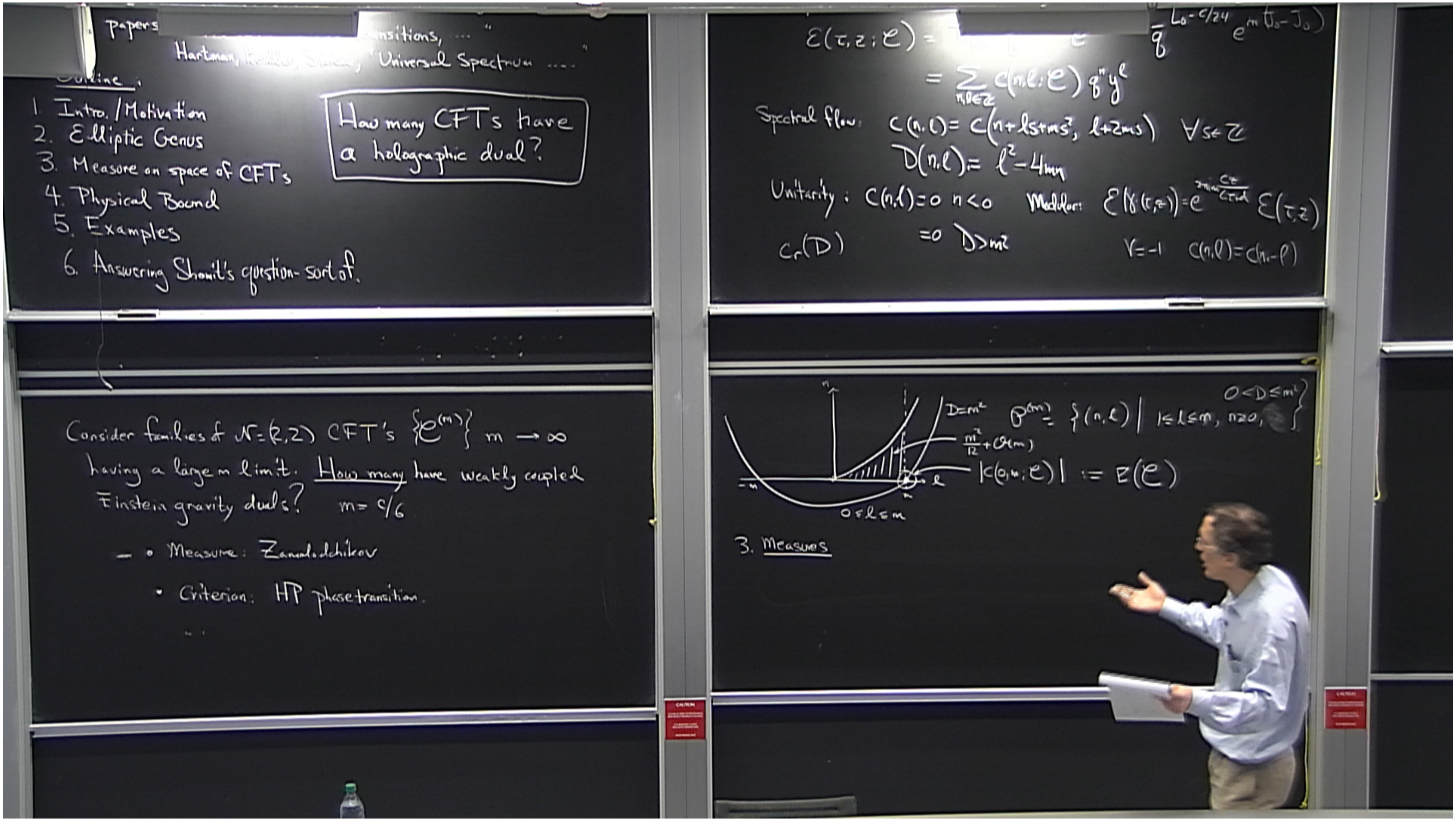

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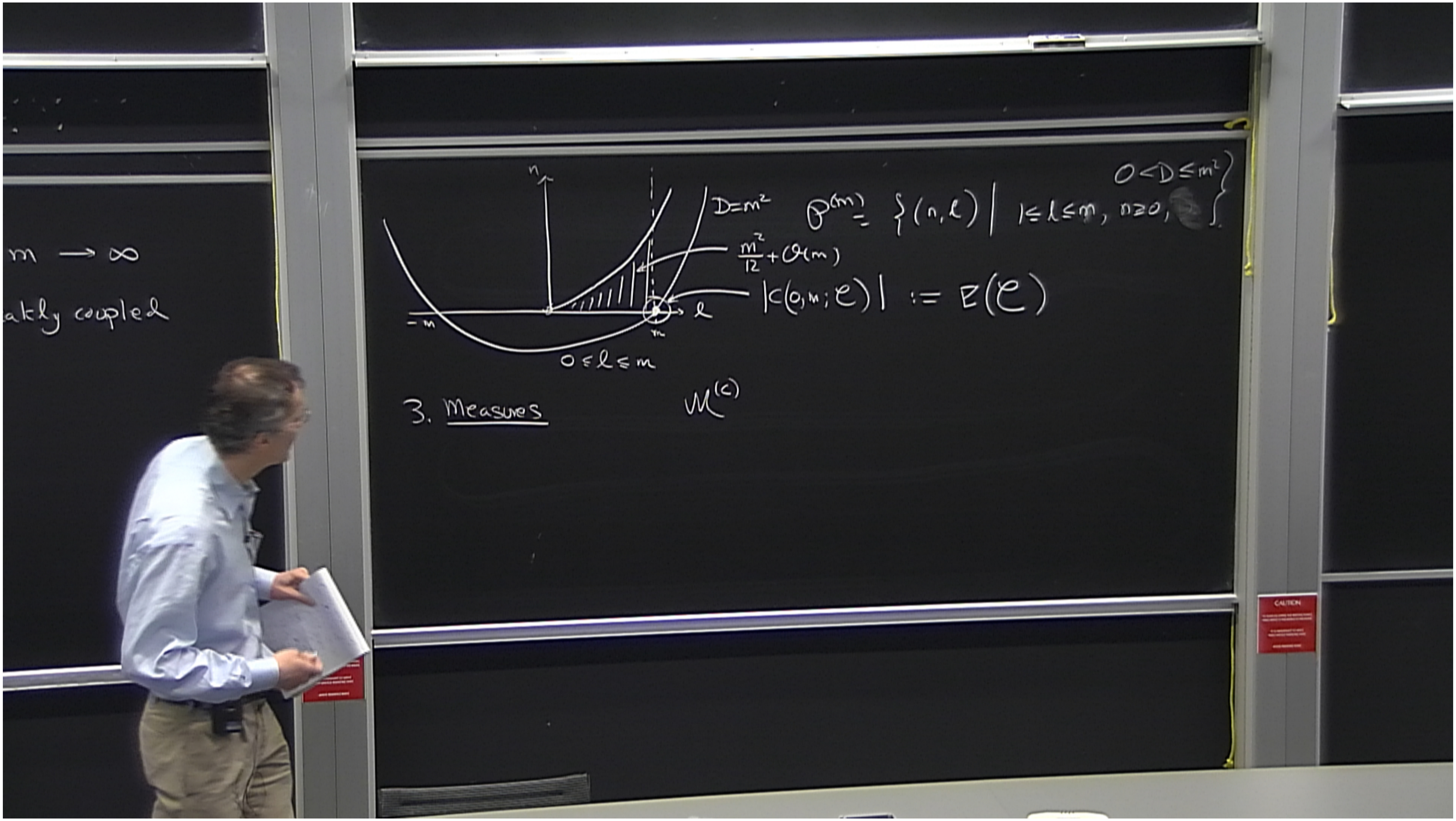
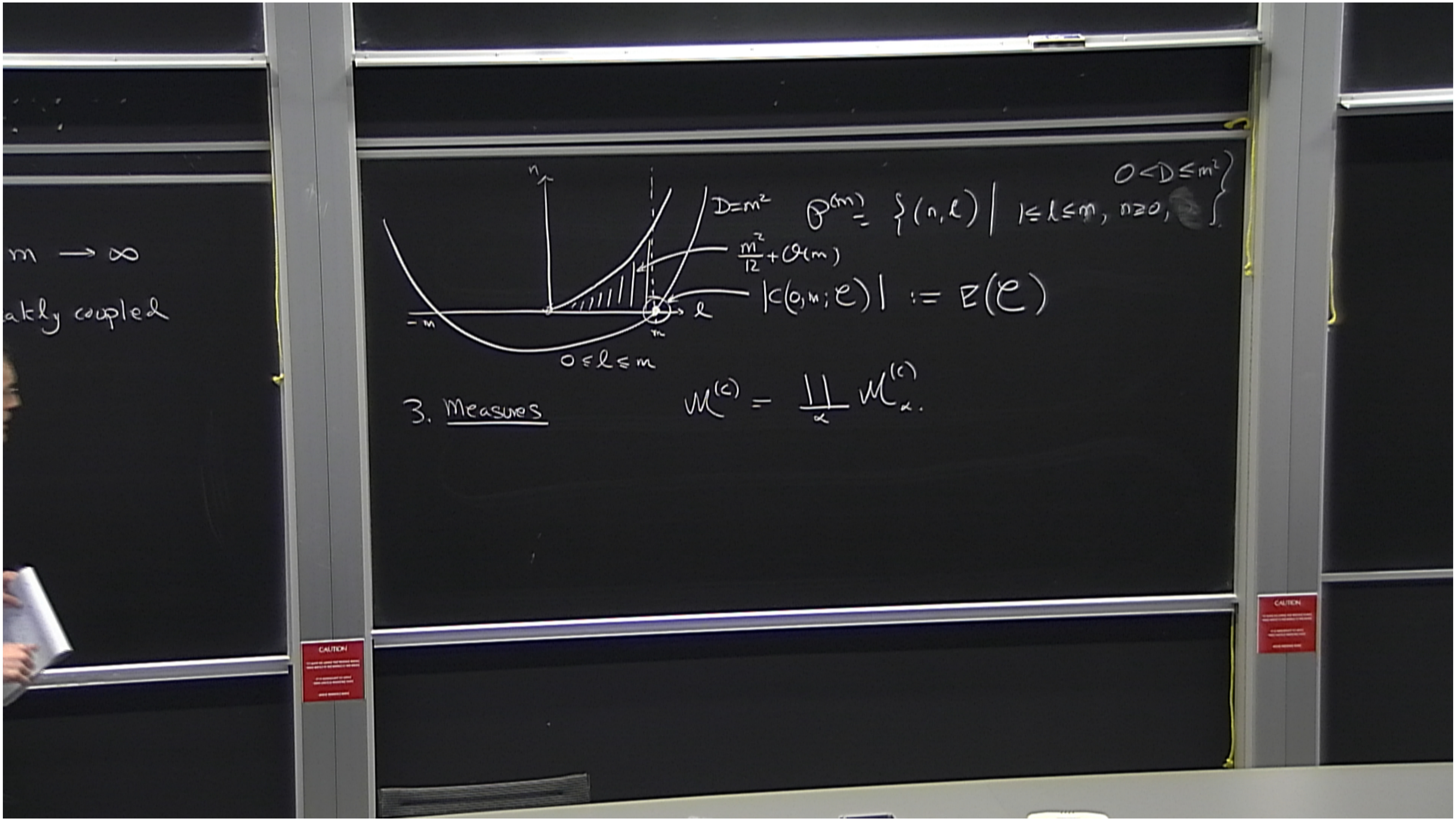
0 ≤ ℓ ≤ m
3. Measures


m → ∞
akly coupled
D=m²
0<D≤m²
ρ(m) = {(n,ℓ) | 1≤ℓ≤m, n≥0,
m²/12 + O(m)
|c(0,m;ℓ)| := e(ℓ)
-m
m
ℓ
n
0≤ℓ≤m
3. Measures
M(c)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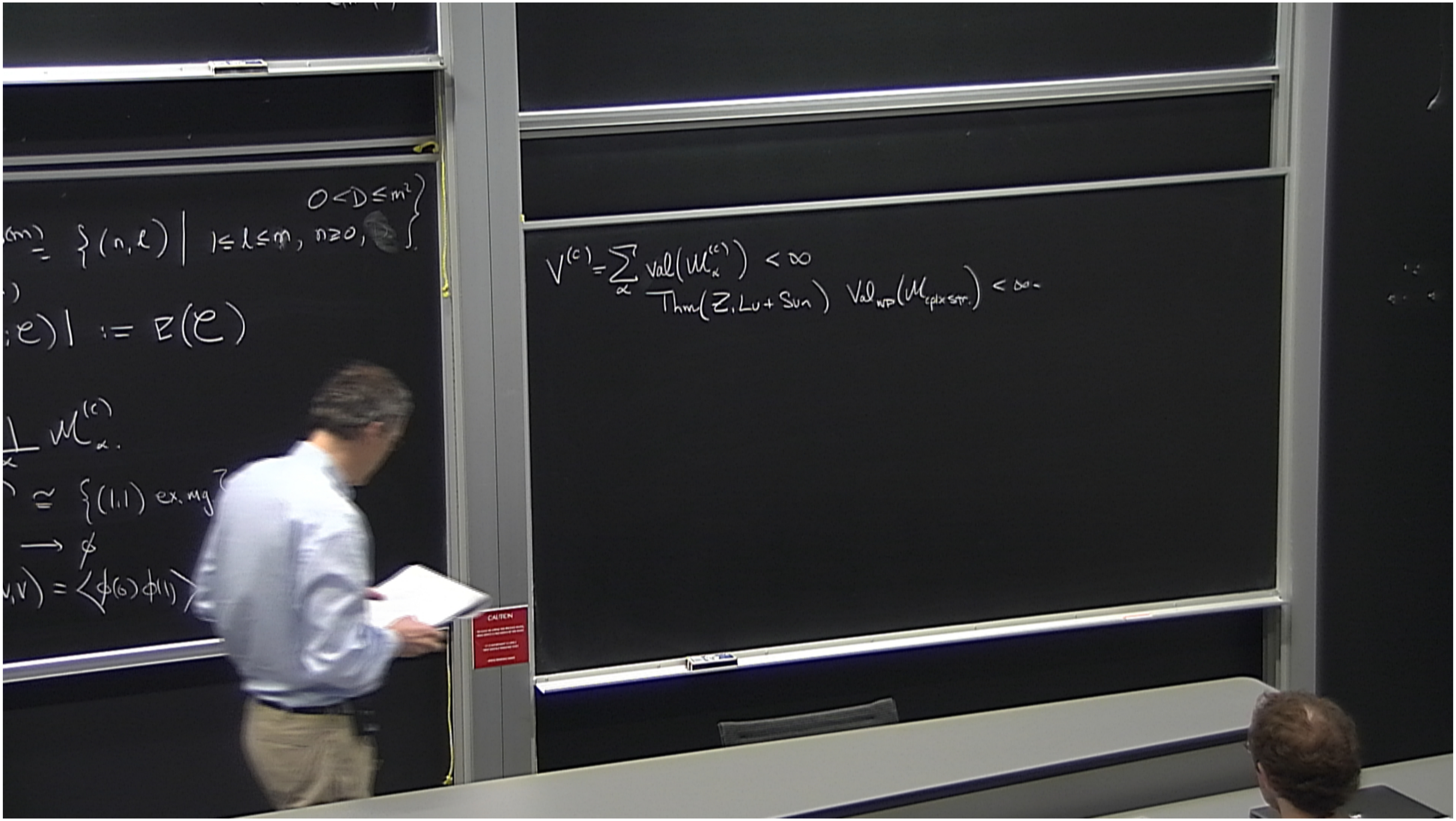
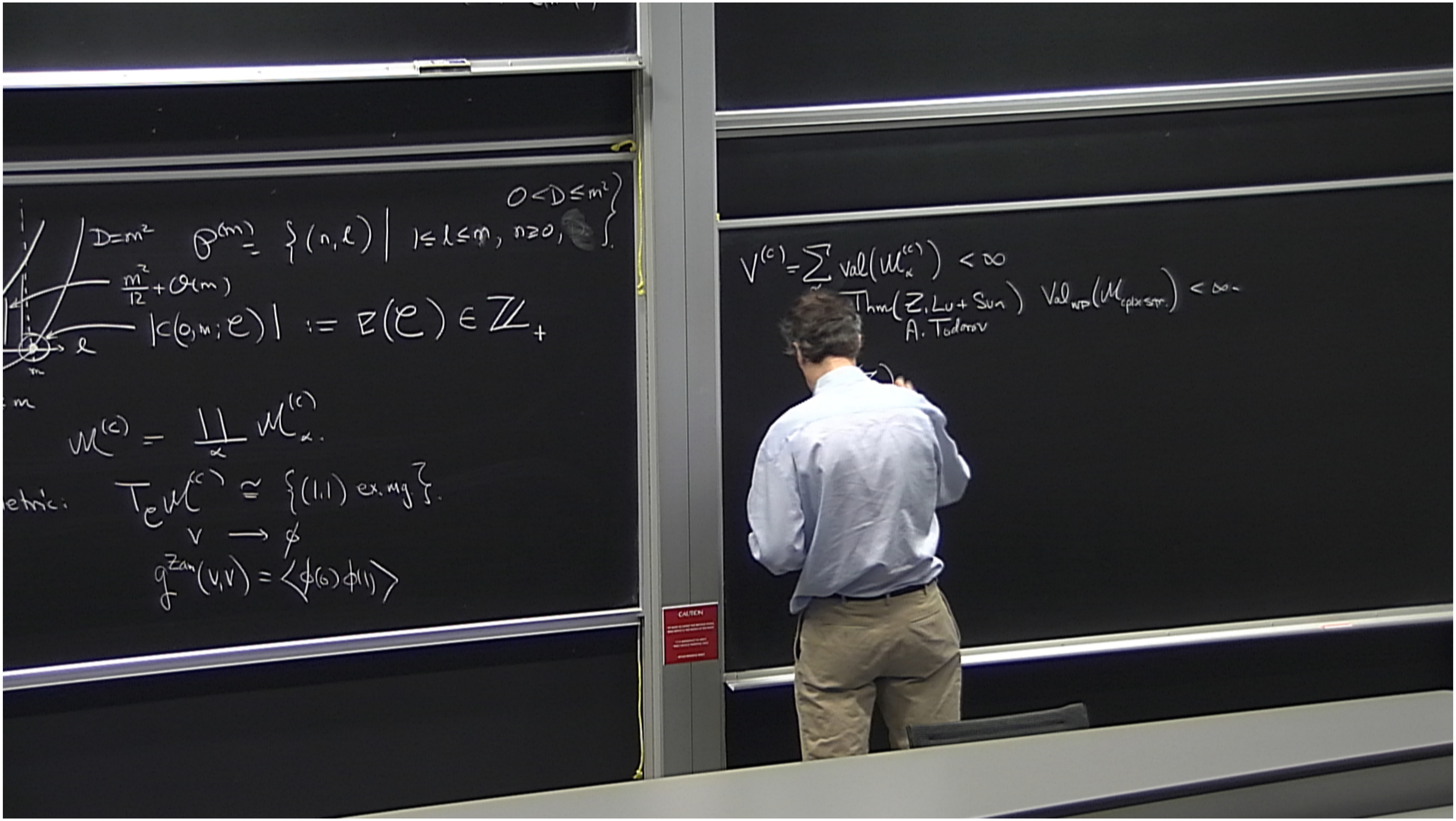
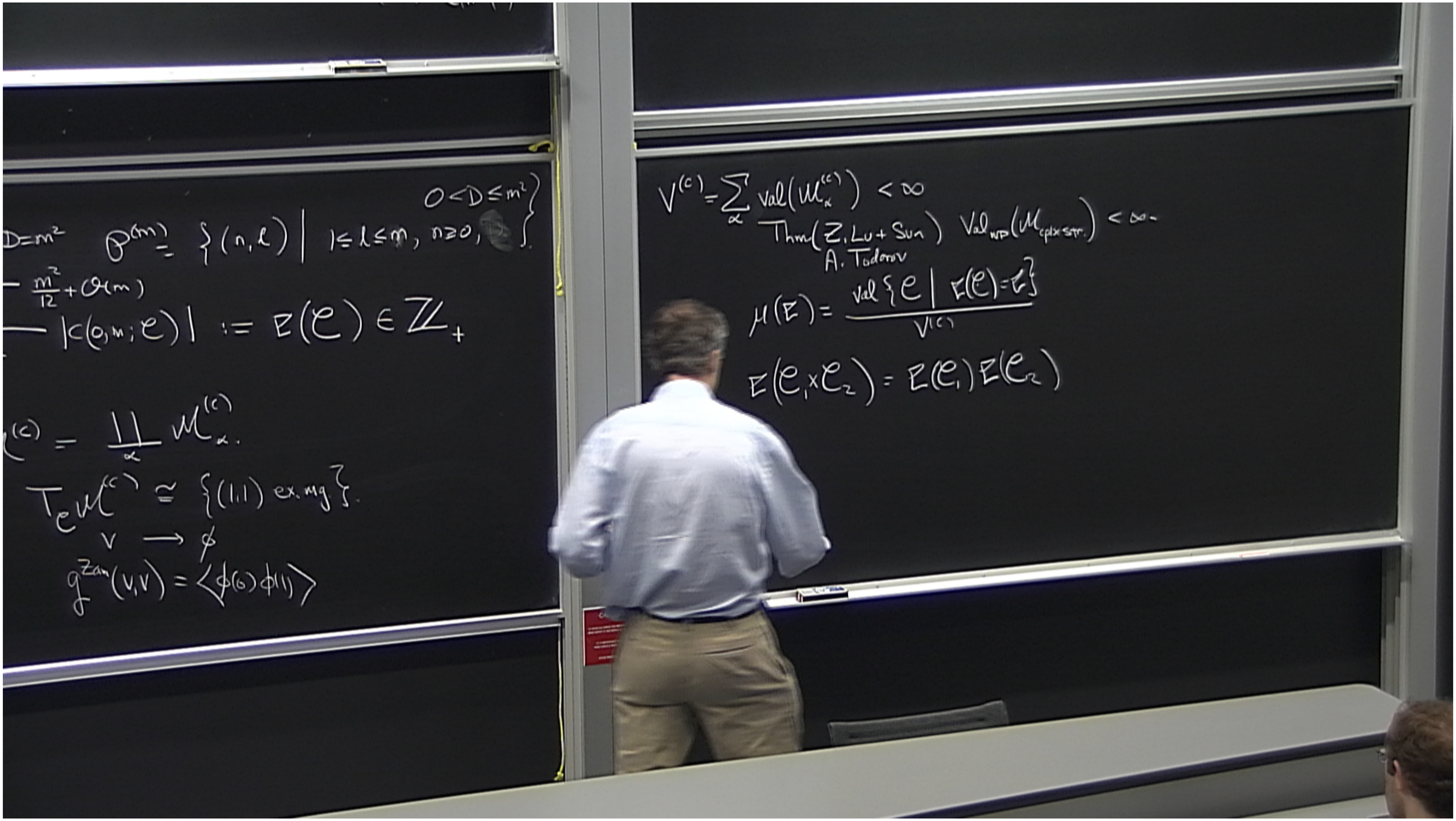
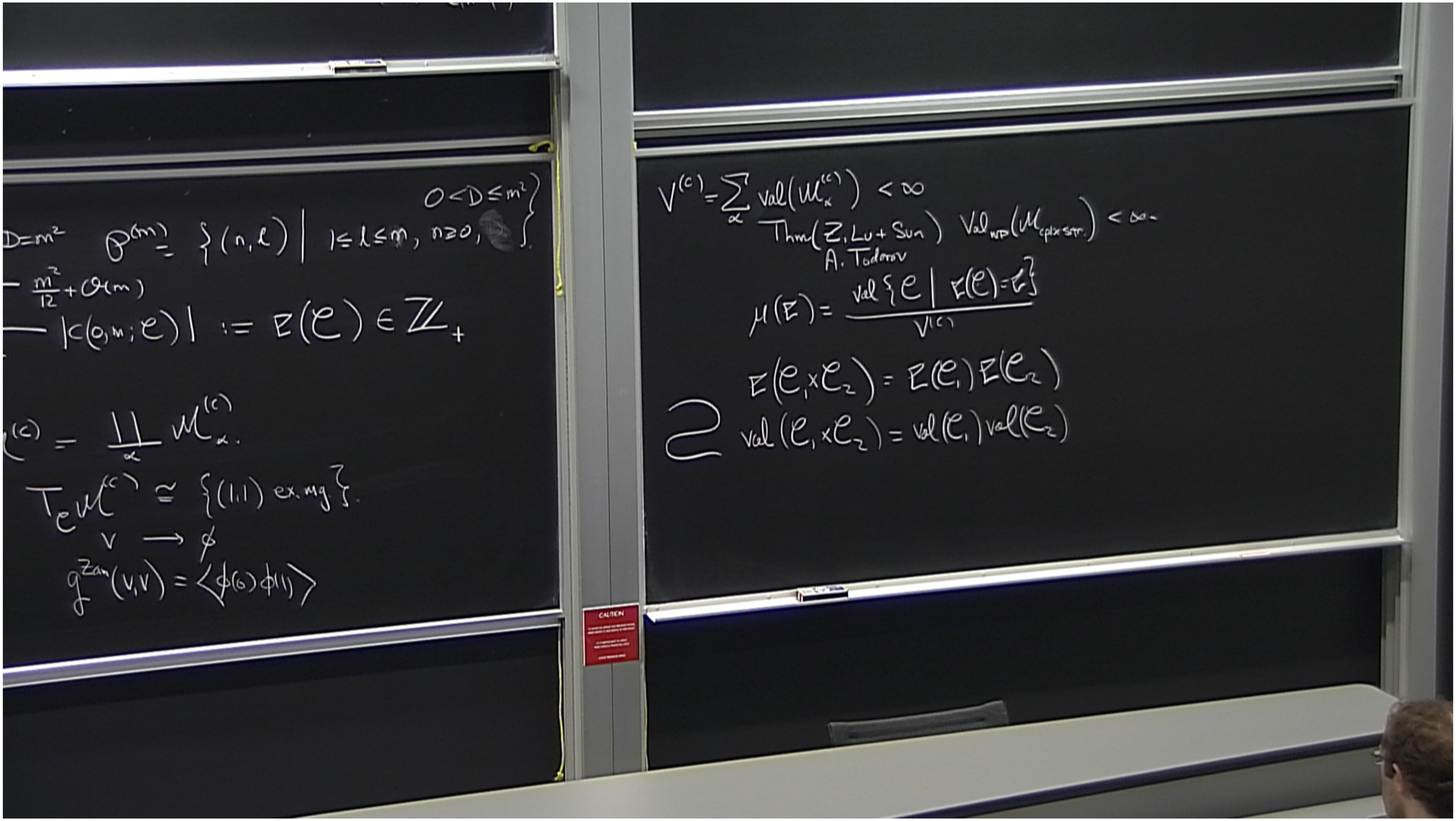
|c(0,m;ℓ)| := c(ℓ)
-m
m
ℓ
n
0≤ℓ≤m
3. Measures
M^(c) = ∐_α M_α^(c).

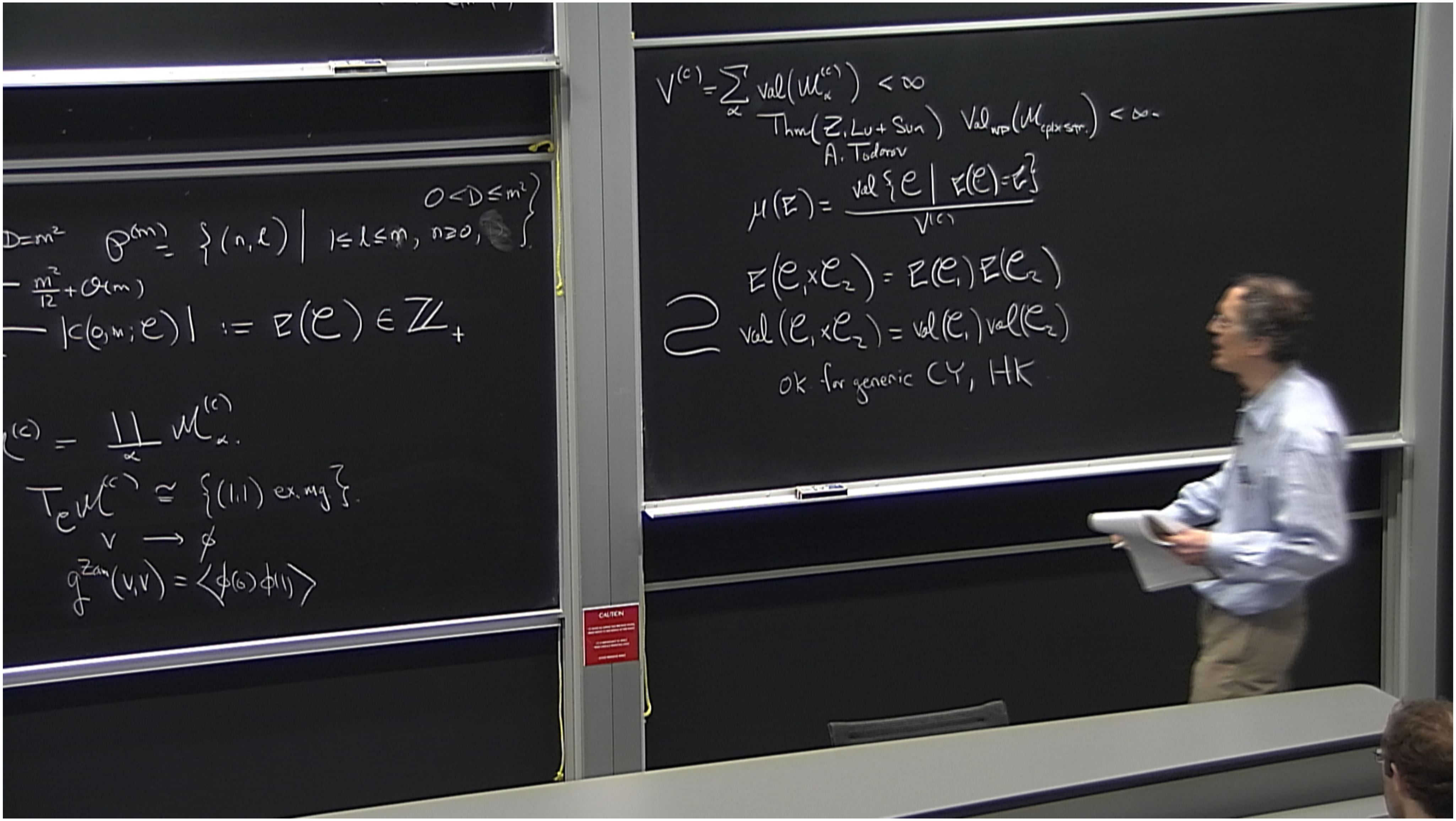

V(c) = Σα Val(Mα(c)) < ∞
Thm (Z. Lu + Sun) ValWP(Mcplx str.) < ∞.

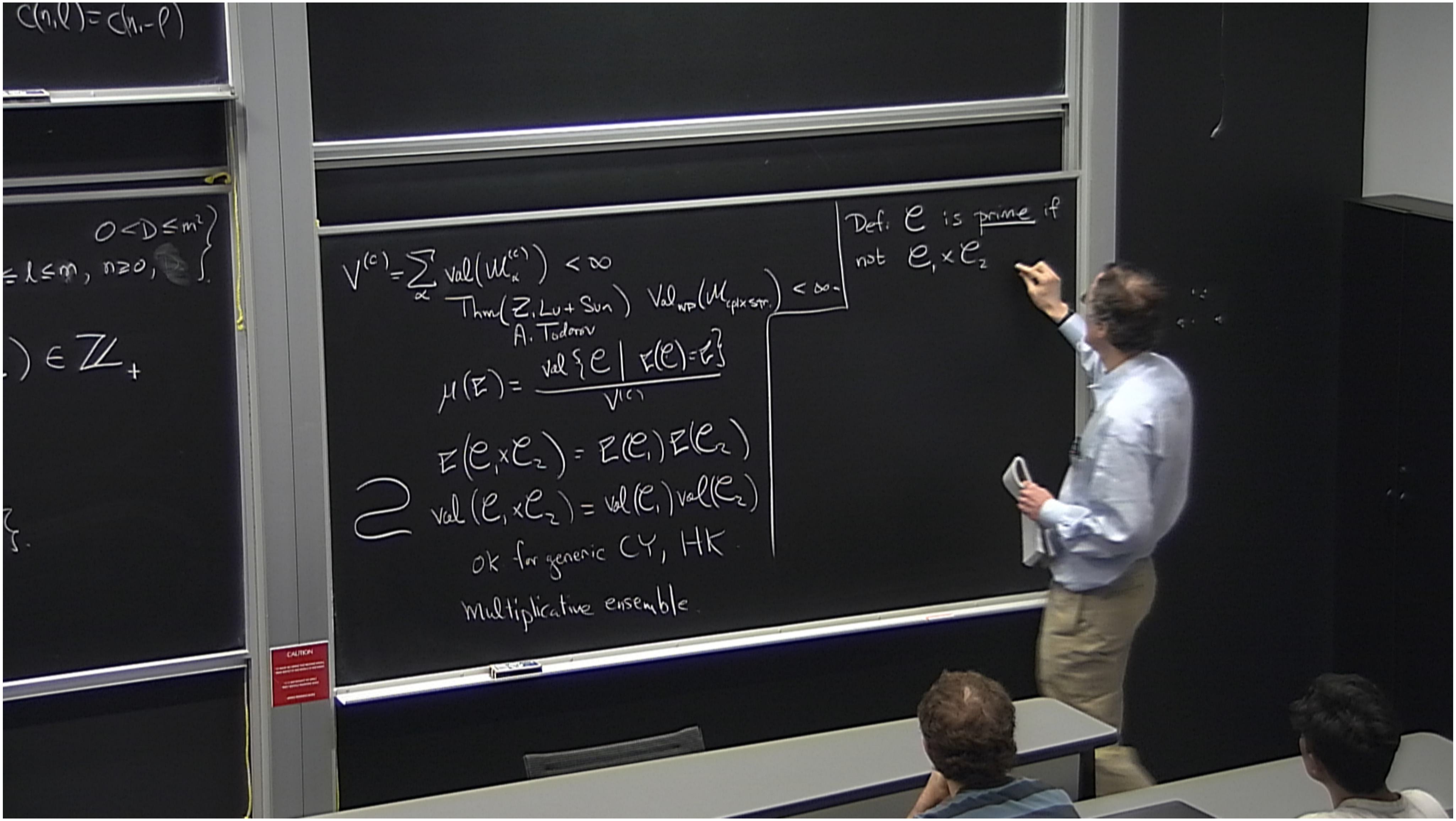

0<D≤m²
D=m² P^(m) = {(n,ℓ) | 1≤ℓ≤m, n≥0,
m²/12 + O(m)
|c(0,m;ℓ)| := e(ℓ) ∈ Z+
M^(c) = ∐α M^(c)α.
etric: TeM^(c) ≅ {(1,1) ex. mg}.
v → φ
g^Zam(v,v) = ⟨φ(0)φ(1)⟩
V^(c) = Σ Val(M^(c)α) < ∞
Thm (Z. Lu + Sun) ValWP(Mcplx str.) < ∞.
A. Todorov

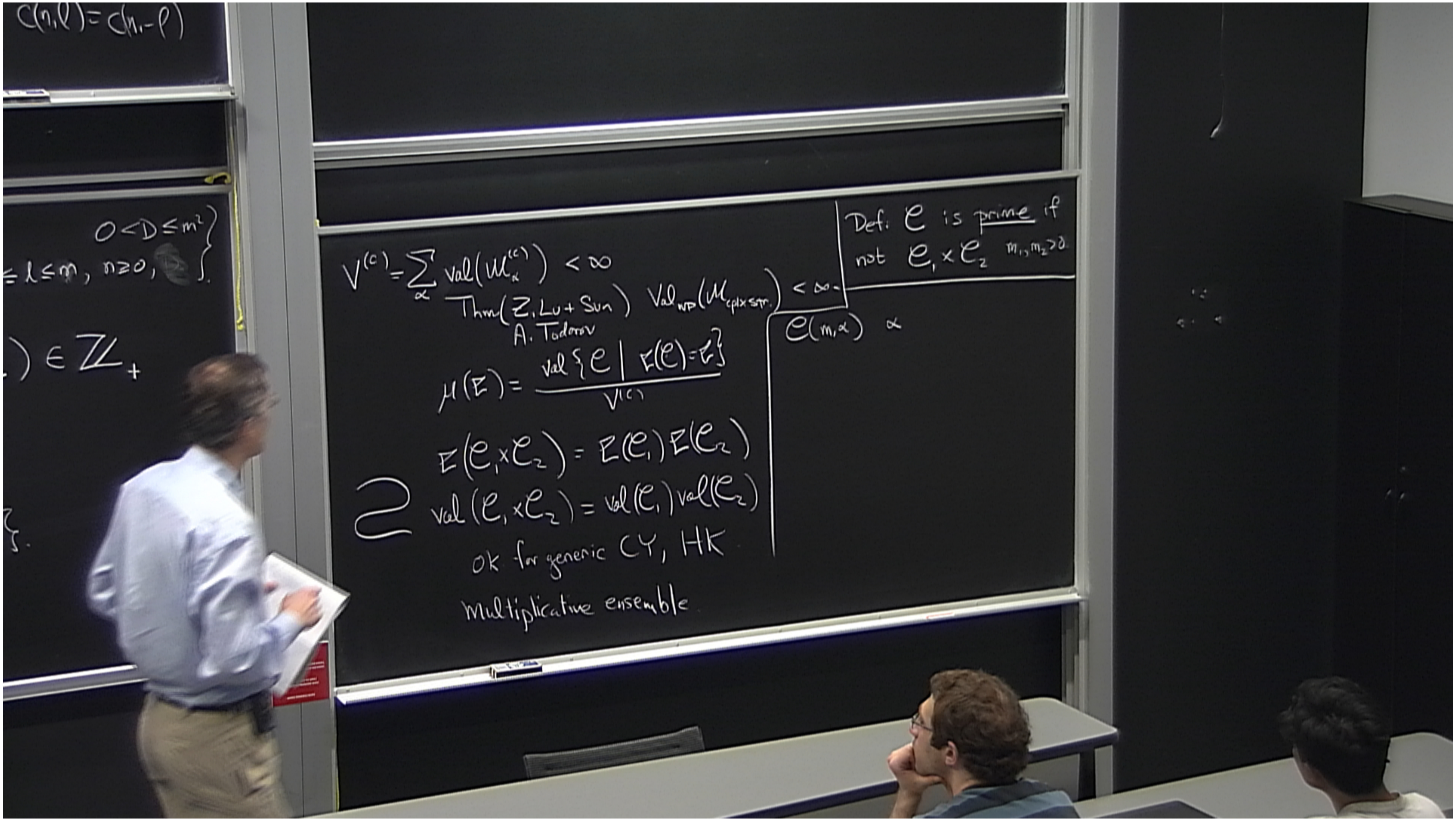

0<D≤m²
D=m² P(m) = {(n,l) | 1≤l≤m, n≥0,
m²/12 + O(m)
|c(0,m;C)| := E(C) ∈ Z+
= ∐α M(c)α
TCM(c) ≅ {(1,1) ex.mg}
v ⟶ φ
gZam(v,v) = ⟨φ(0)φ(1)⟩
V(c) = Σα val(M(c)α) < ∞
Thm(Z. Lu + Sun) ValWP(Mcplx str.) < ∞
A. Todorov
μ(E) = val{C | E(C)=E} / V(c)
E(C1×C2) = E(C1)E(C2)

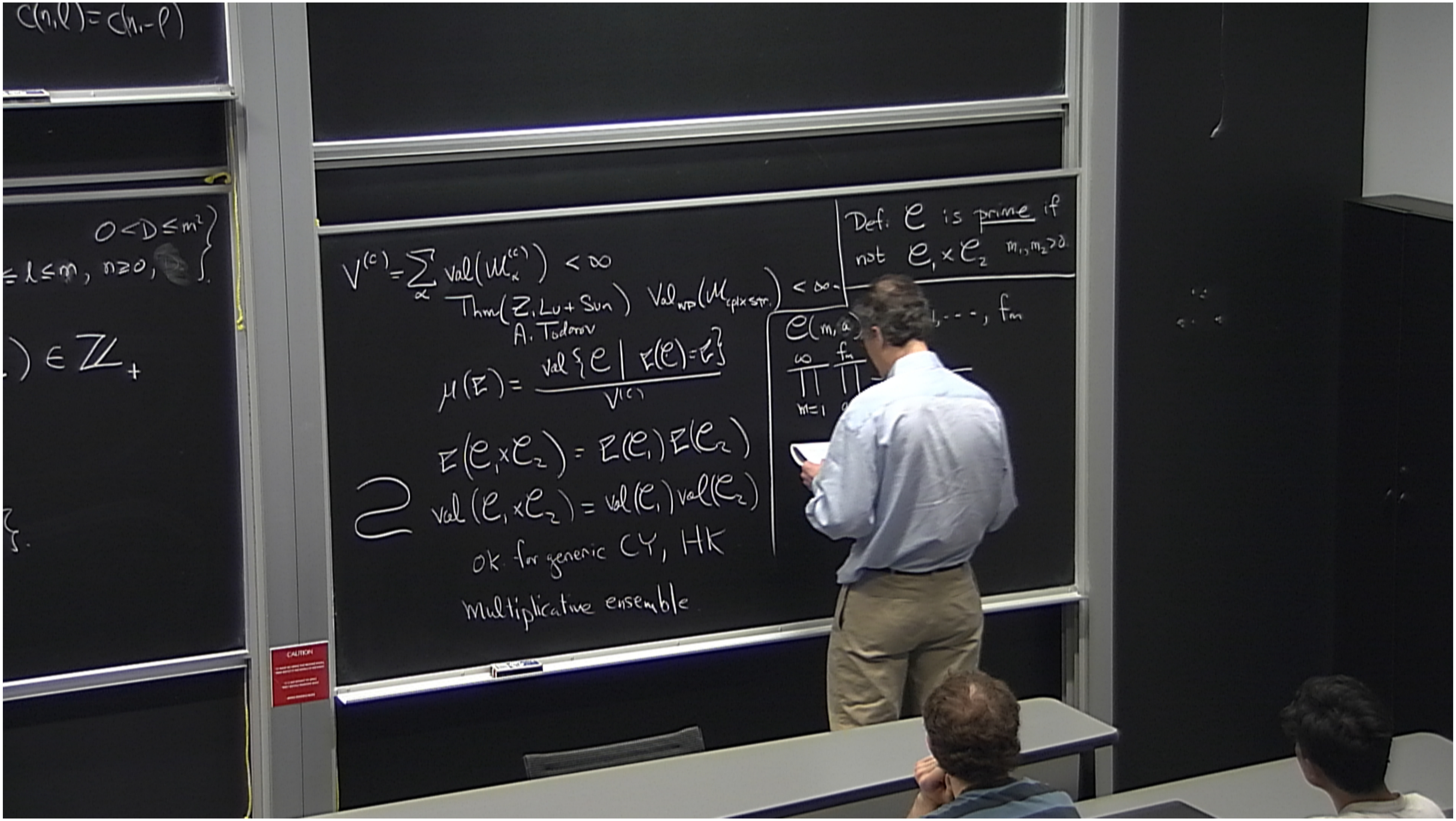


0<D≤m²
D=m² P(m) = {(n,ℓ) | 1≤ℓ≤m, n≥0,
m²/12 + O(m)
|c(0,m;C)| := E(C) ∈ Z+
= ∐α Mα(c)
TC M(c) ≅ {(1,1) ex. mg}
V ⟶ ϕ
gZam(v,v) = ⟨ϕ(0)ϕ(1)⟩
V(c) = Σα vol(Mα(c)) < ∞
Thm(Z. Lu + Sun) VolWP(Mcplx str.) < ∞.
A. Todorov
μ(E) = vol{C | E(C)=E} / V(c)
E(C1×C2) = E(C1)E(C2)
vol(C1×C2) = vol(C1)vol(C2)


Thm (Z. Lu + Sun)
A. Todorov
ok for generic CY, HK


c(n,ℓ) = c(n,−ℓ)
0 < D ≤ m²
≤ ℓ ≤ m, n ≥ 0,
) ∈ ℤ+
V^(c) = Σα val(M_α^(c)) < ∞
Thm (Z. Lu + Sun) A. Todorov
Val_WP(M_cplx str.) < ∞.
μ(Z) = val{C | Z(C) = Z} / V^(c)
Z(C1×C2) = Z(C1) Z(C2)
val(C1×C2) = val(C1) val(C2)
ok for generic CY, HK
multiplicative ensemble
Def: C is prime if
not C1×C2
CAUTION


c(n,l) = c(n,-l)
0 < D ≤ m²
≤ l ≤ m, n ≥ 0,
) ∈ Z+
V^(c) = Σ_α val(M_α^(c)) < ∞
Thm (Z. Lu + Sun) Val_WP(M_cplx str.) < ∞.
A. Todorov
μ(E) = val{C | E(C) = E} / V^(c)
E(C1 × C2) = E(C1) E(C2)
val(C1 × C2) = val(C1) val(C2)
ok for generic CY, HK
multiplicative ensemble
Def: C is prime if not C1 × C2 m1, m2 > 0.
C(m, α) α


Def: 𝒞 is prime if not 𝒞1 × 𝒞2
V(c) = Σα val(M(c)α) < ∞
Thm (Z, Lu + Sun) Val_WP(M_cplx str.) < ∞
A. Todorov
μ(Z) = val{𝒞 | Z(𝒞) = Z} / V(c)
Z(𝒞1 × 𝒞2) = Z(𝒞1) Z(𝒞2)
val(𝒞1 × 𝒞2) = val(𝒞1) val(𝒞2)
ok for generic CY, HK
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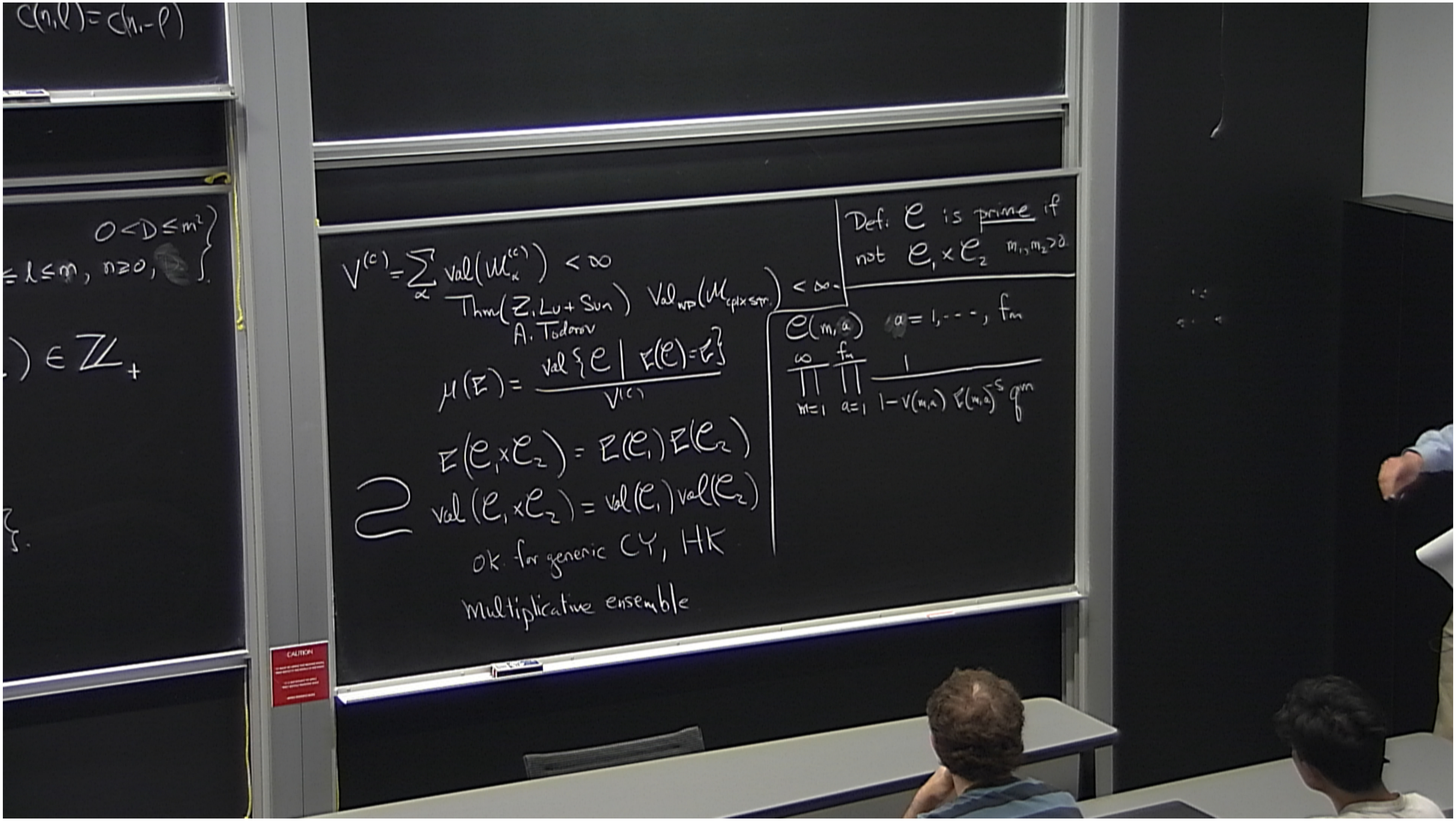
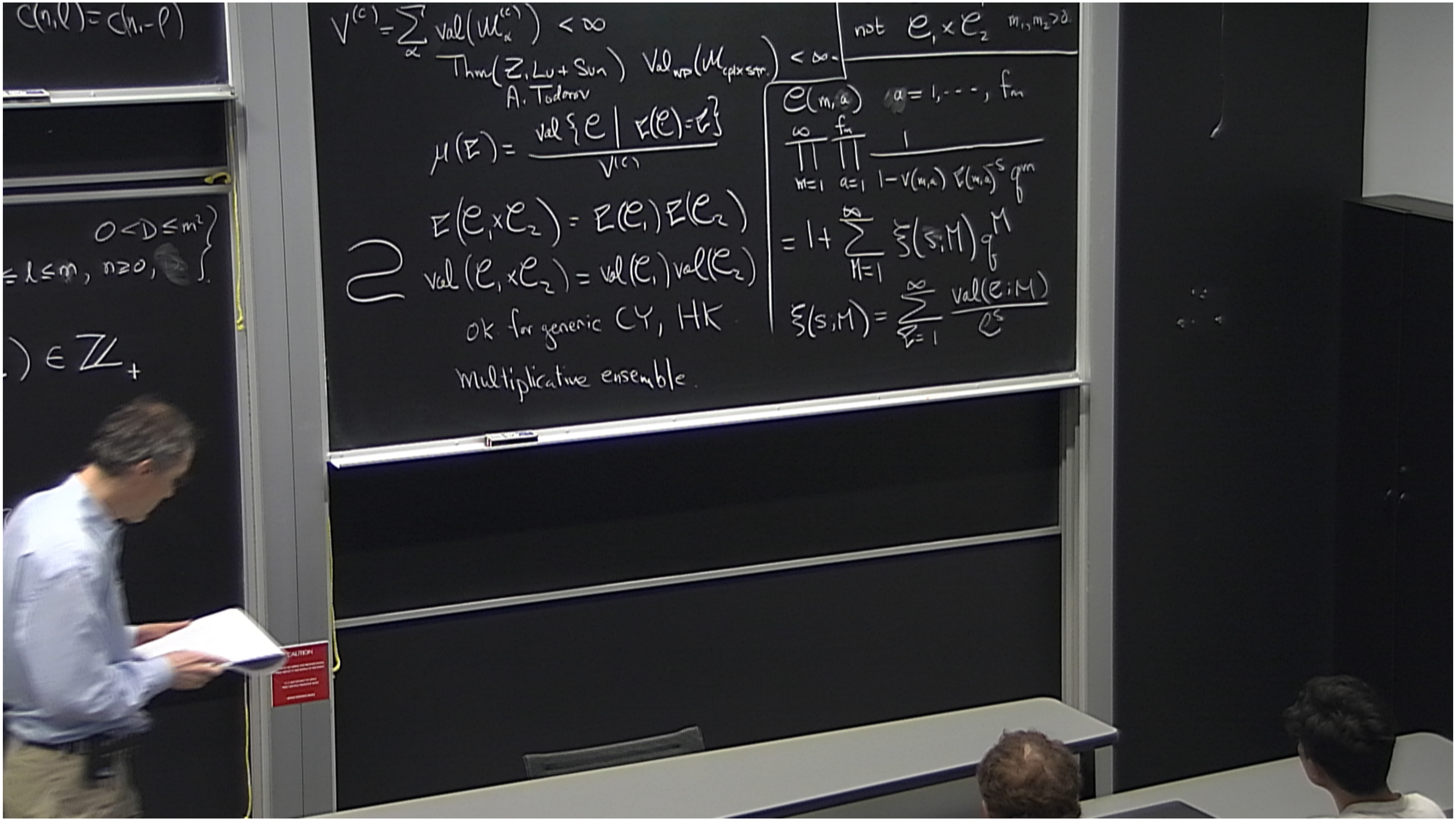
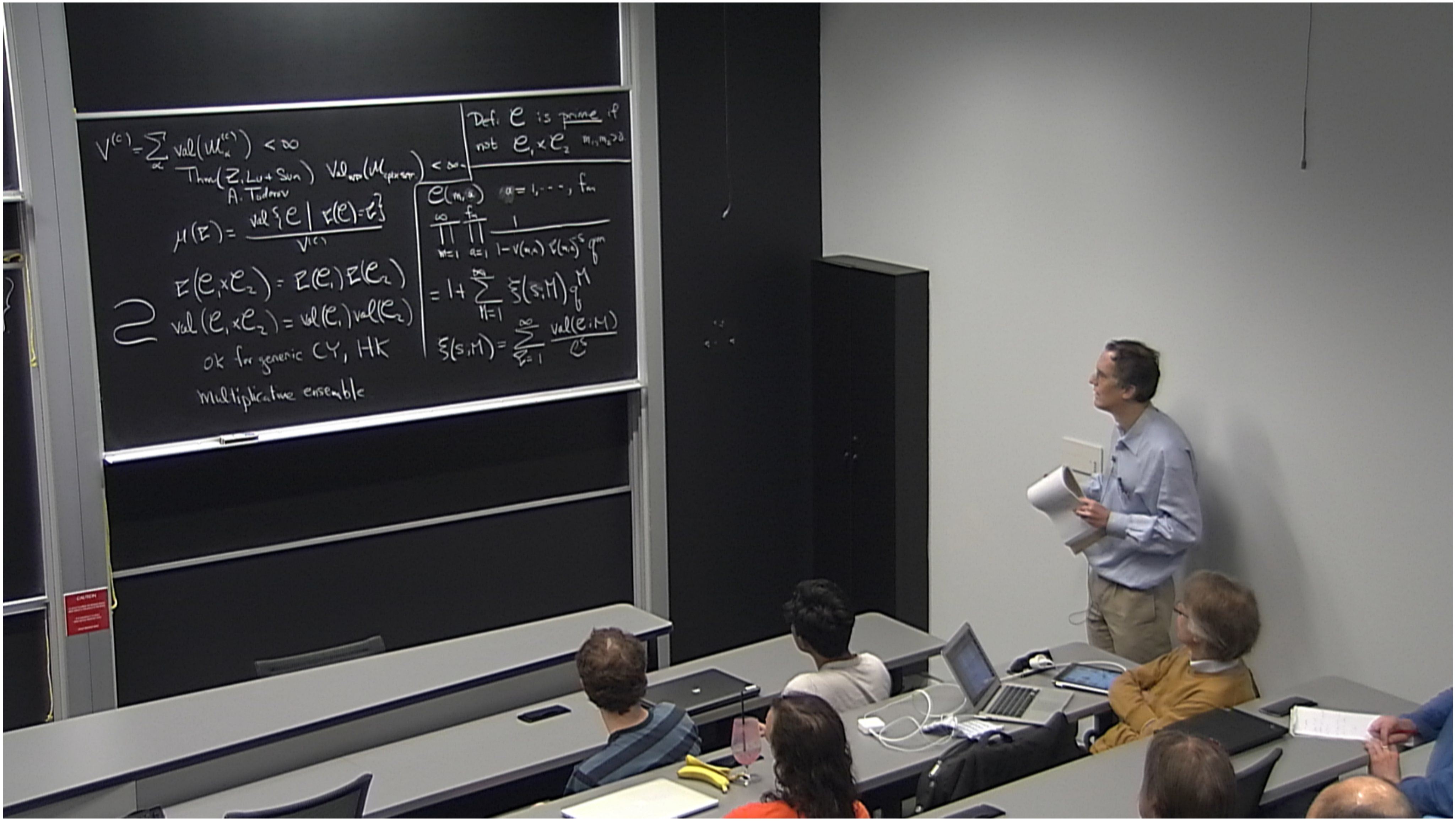


C(n,l) = C(n,-l)
0 < D ≤ m²
≤ l ≤ m, n ≥ 0,
) ∈ Z+
V^(c) = Σ_α val(M_α^(c)) < ∞
Thm (Z. Lu + Sun) Val_WP(M_cplx str.) < ∞.
A. Todorov
μ(E) = val{C | E(C) = E} / V^(c)
E(C1×C2) = E(C1)E(C2)
val(C1×C2) = val(C1)val(C2)
OK for generic CY, HK
multiplicative ensemble
Def: C is prime if not C1×C2 m1,m2>0.
C(m,a) a = 1,...,f_m
∏_{m=1}^∞ ∏_{a=1}^{f_m} 1 / (1 − V(m,a) E(m,a)^−s q^m)
CAUTION


not $\mathcal{C}_1 \times \mathcal{C}_2$ $m_1, m_2 > 0$
Thm (Z. Lu + Sun) $Val_{WP}(\mathcal{M}_{cplx\ str.}) < \infty$.
A. Todorov
ok for generic CY, HK
Multiplicative ensemble


Def: C is prime if
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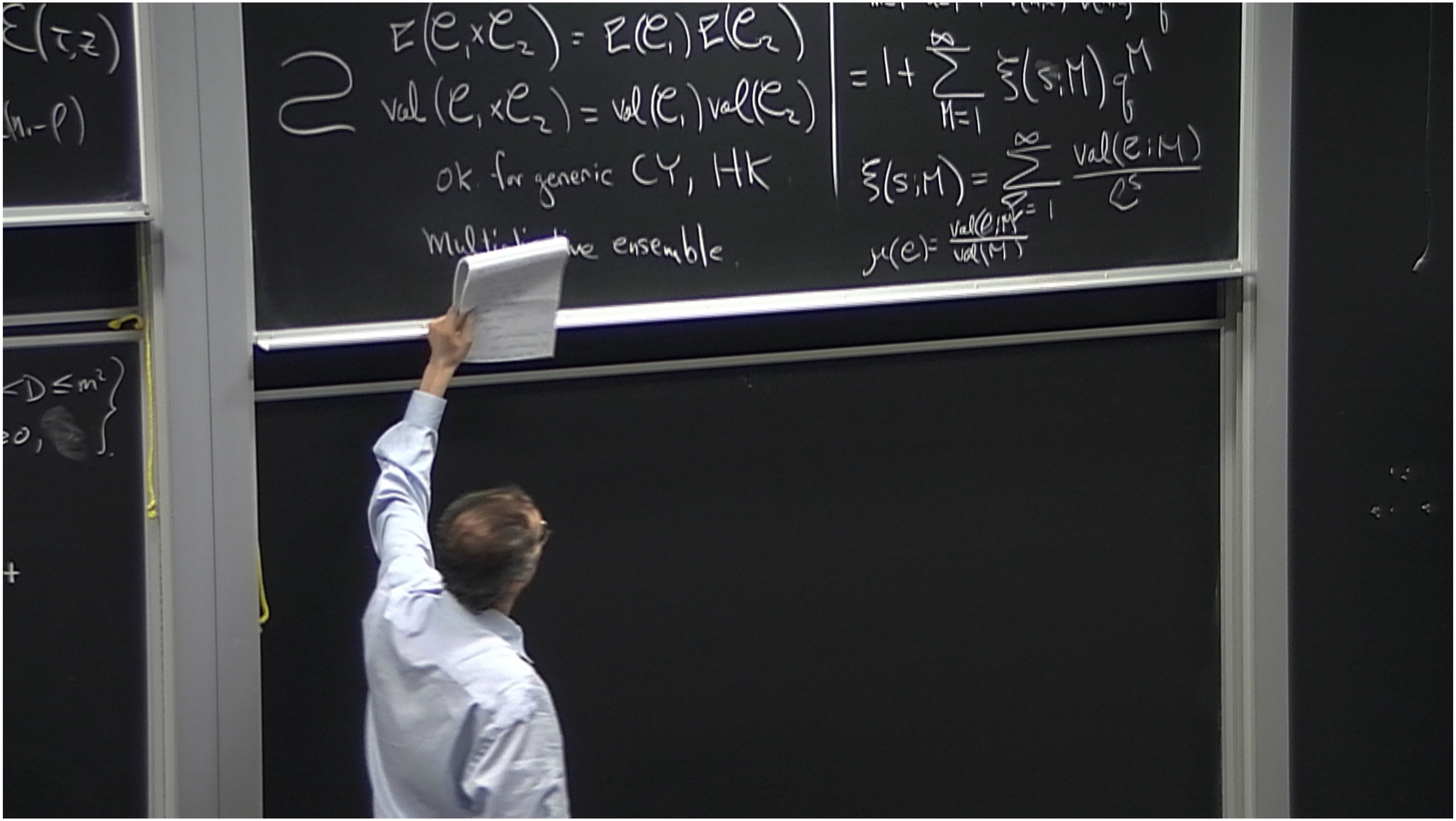
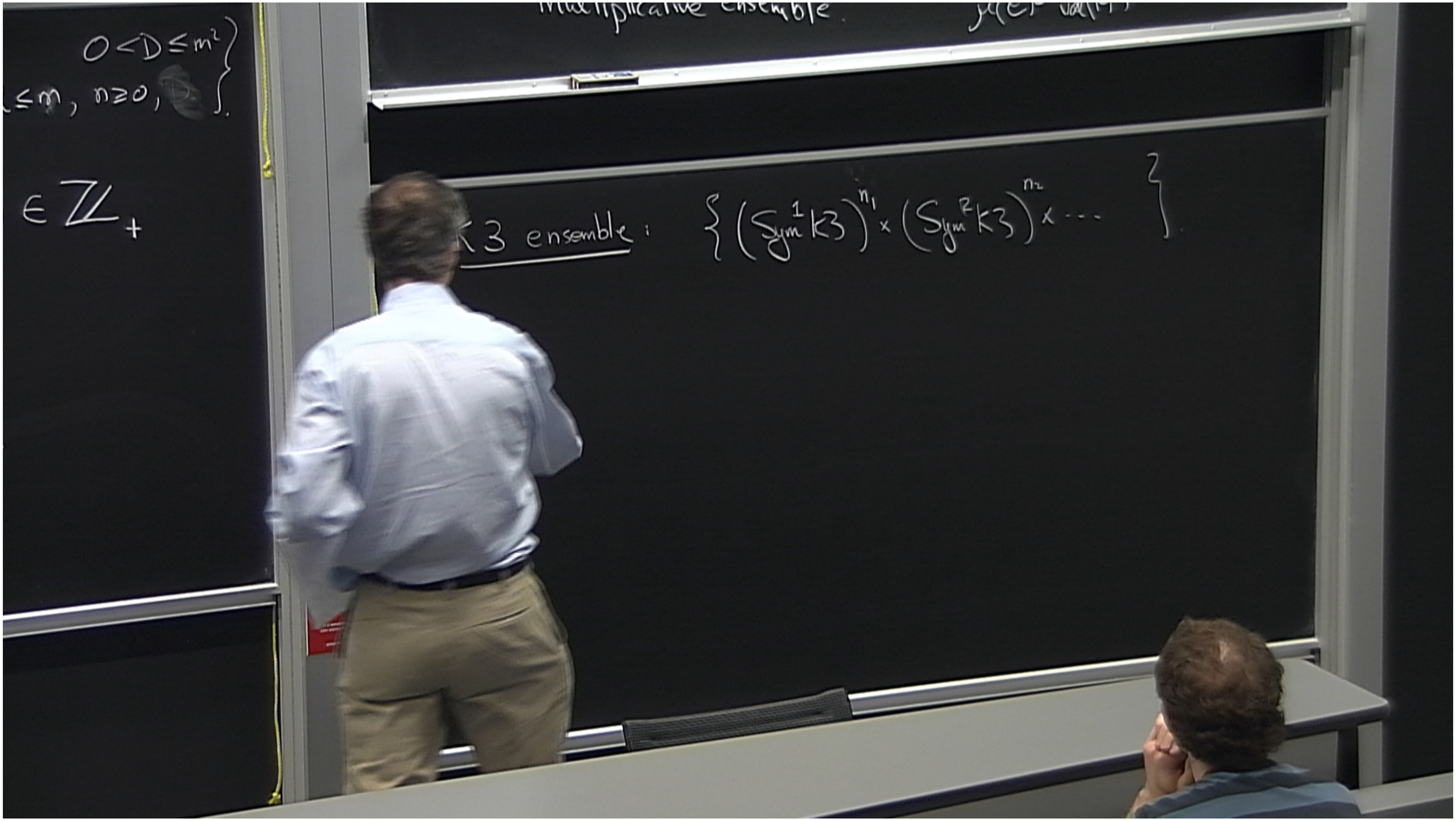
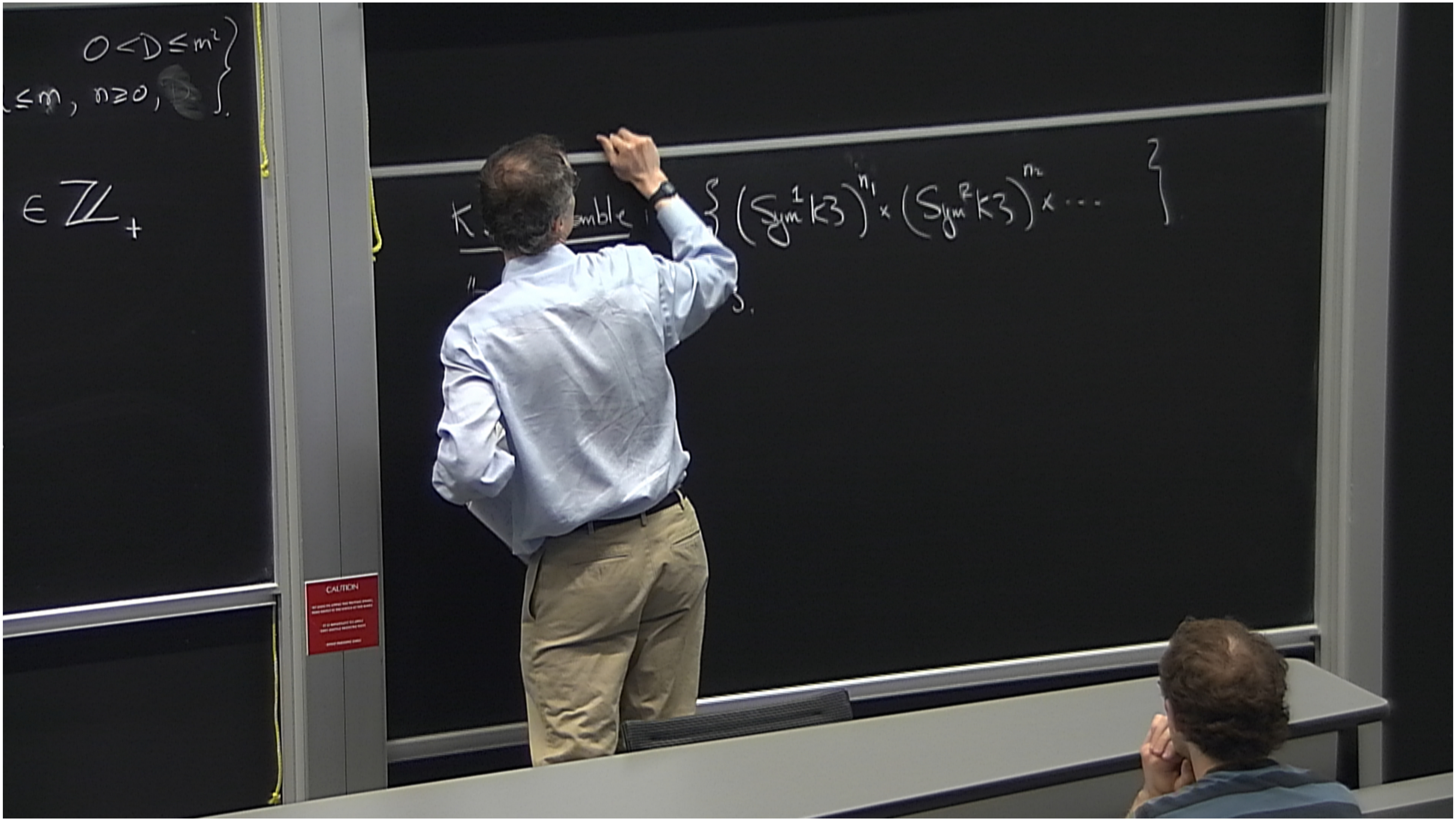
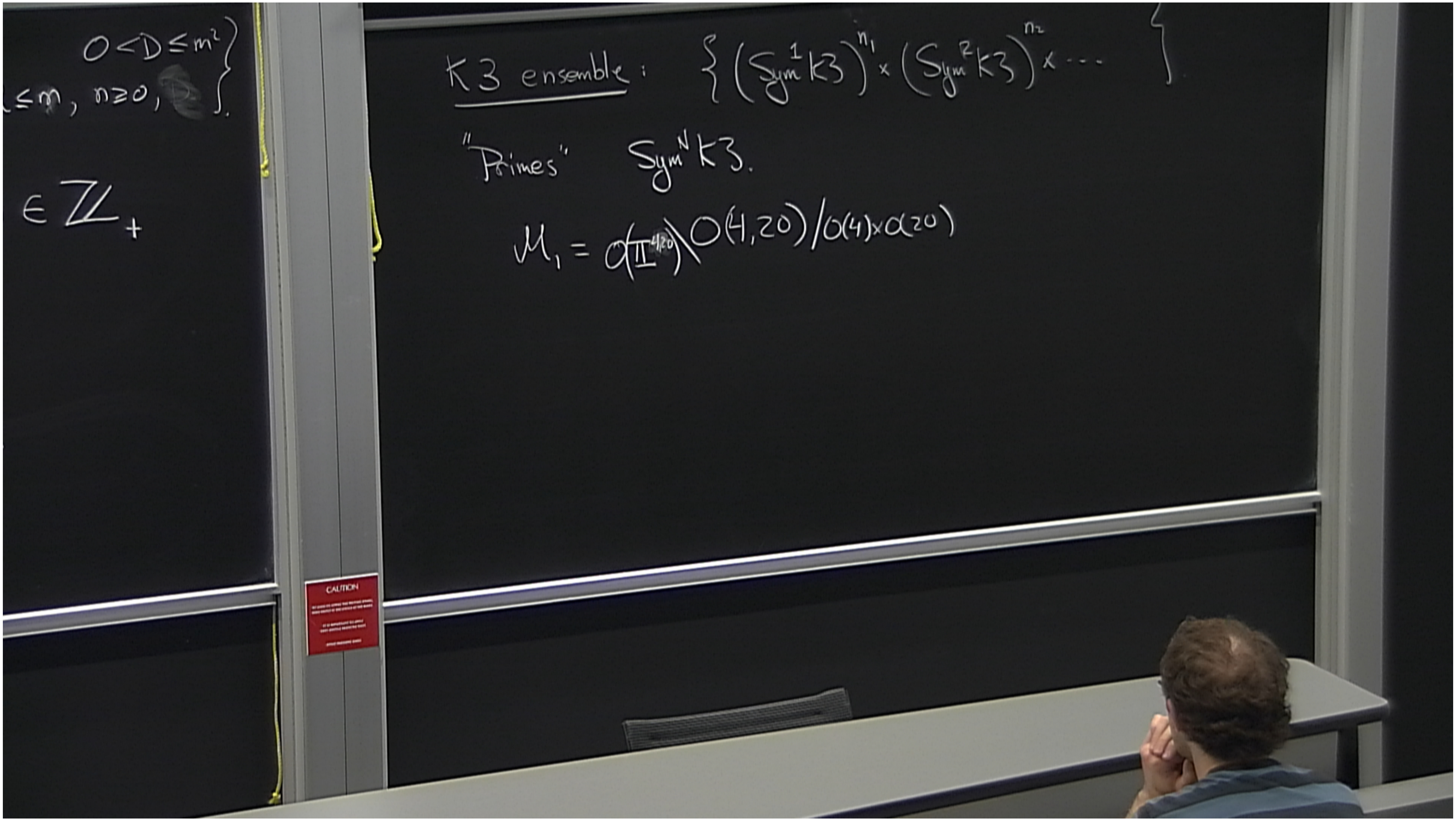
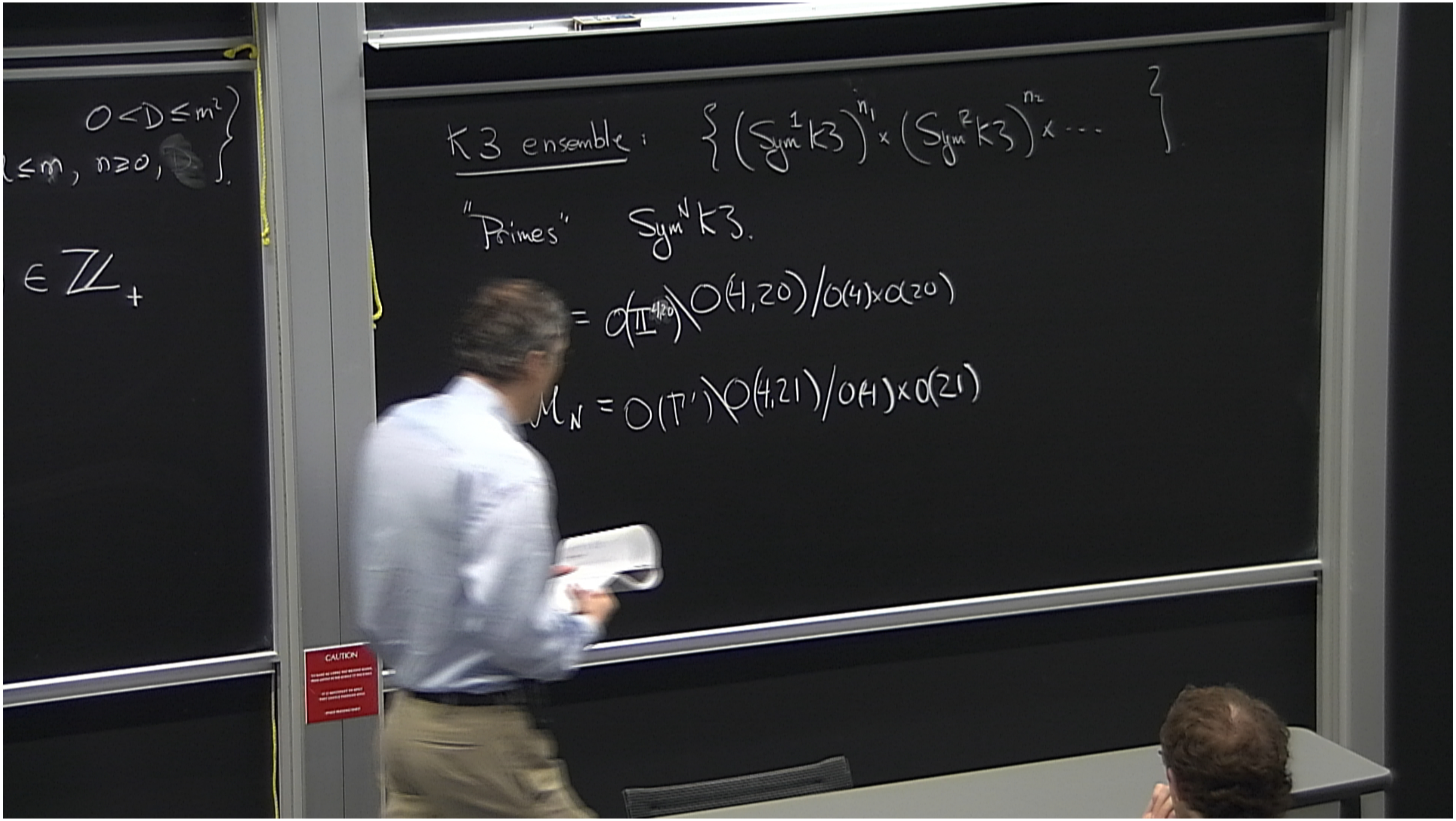

Z(C₁×C₂) = Z(C₁)Z(C₂)
val(C₁×C₂) = val(C₁)val(C₂)
ok for generic CY, HK
ensemble
= 1 + Σ_{M=1}^∞ ξ(s;M) q^M
ξ(s;M) = Σ_{ℓ=1}^∞ val(ℓ;M) / ℓ^s
μ(ℓ) = val(ℓ;M) / val(M)


0<D≤m²
n≥0,
∈ ℤ+
K3 ensemble: { (Sym¹K3)^n₁ × (Sym²K3)^n₂ × ... }


0 < D ≤ m²
≤ m, n ≥ 0,
∈ ℤ+
K3 … mble
{ (Sym¹ K3)^{n₁} × (Sym² K3)^{n₂} × … }


0 < D ≤ m²
≤ m, n ≥ 0,
∈ ℤ+
K3 ensemble: {(Sym¹K3)^n1 × (Sym²K3)^n2 × ... }
"Primes" Sym^N K3.
M1 = O(II^4,20)\O(4,20)/O(4)×O(20)
CAUTION


0<D≤m²
K3 ensemble:
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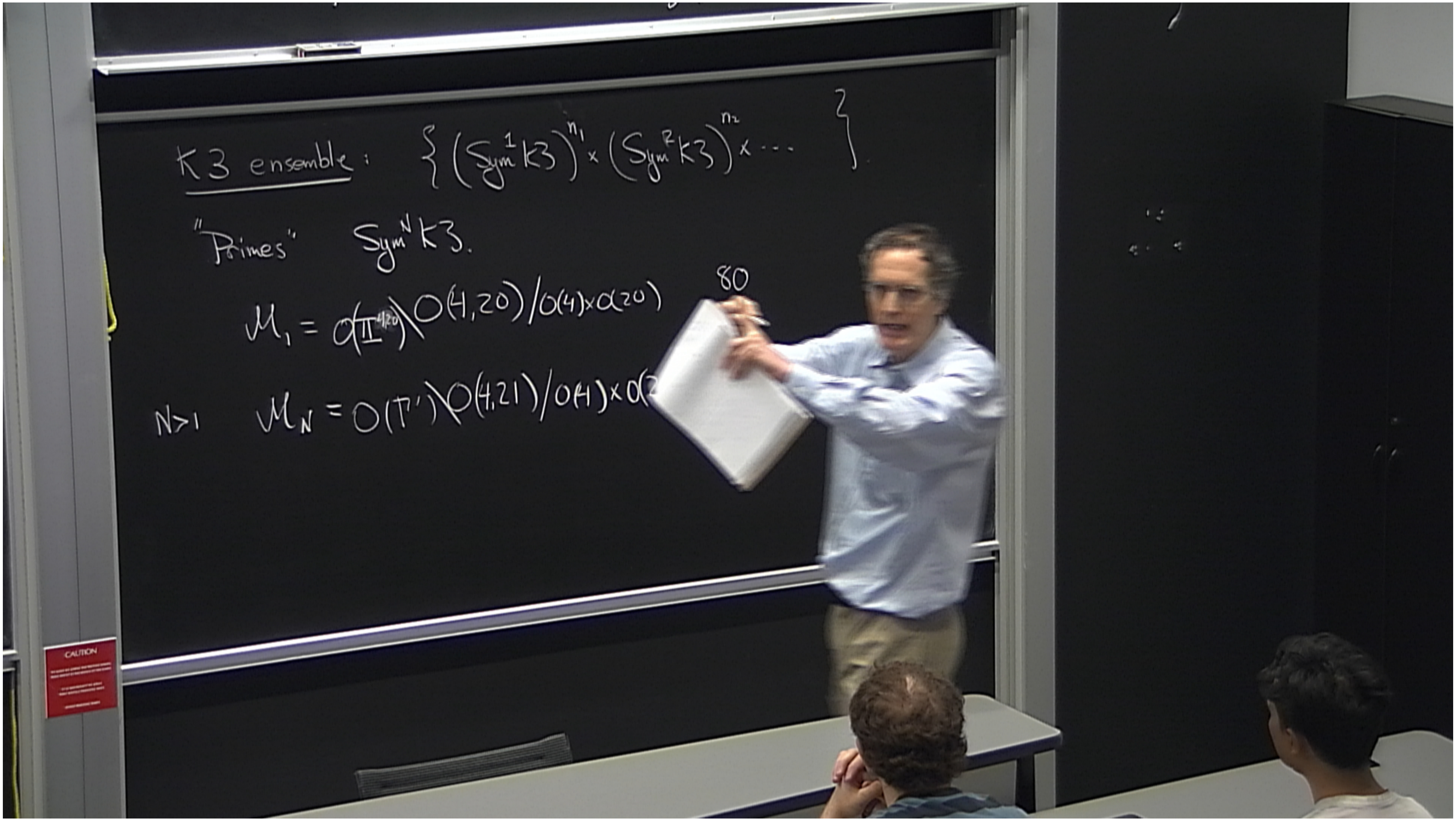
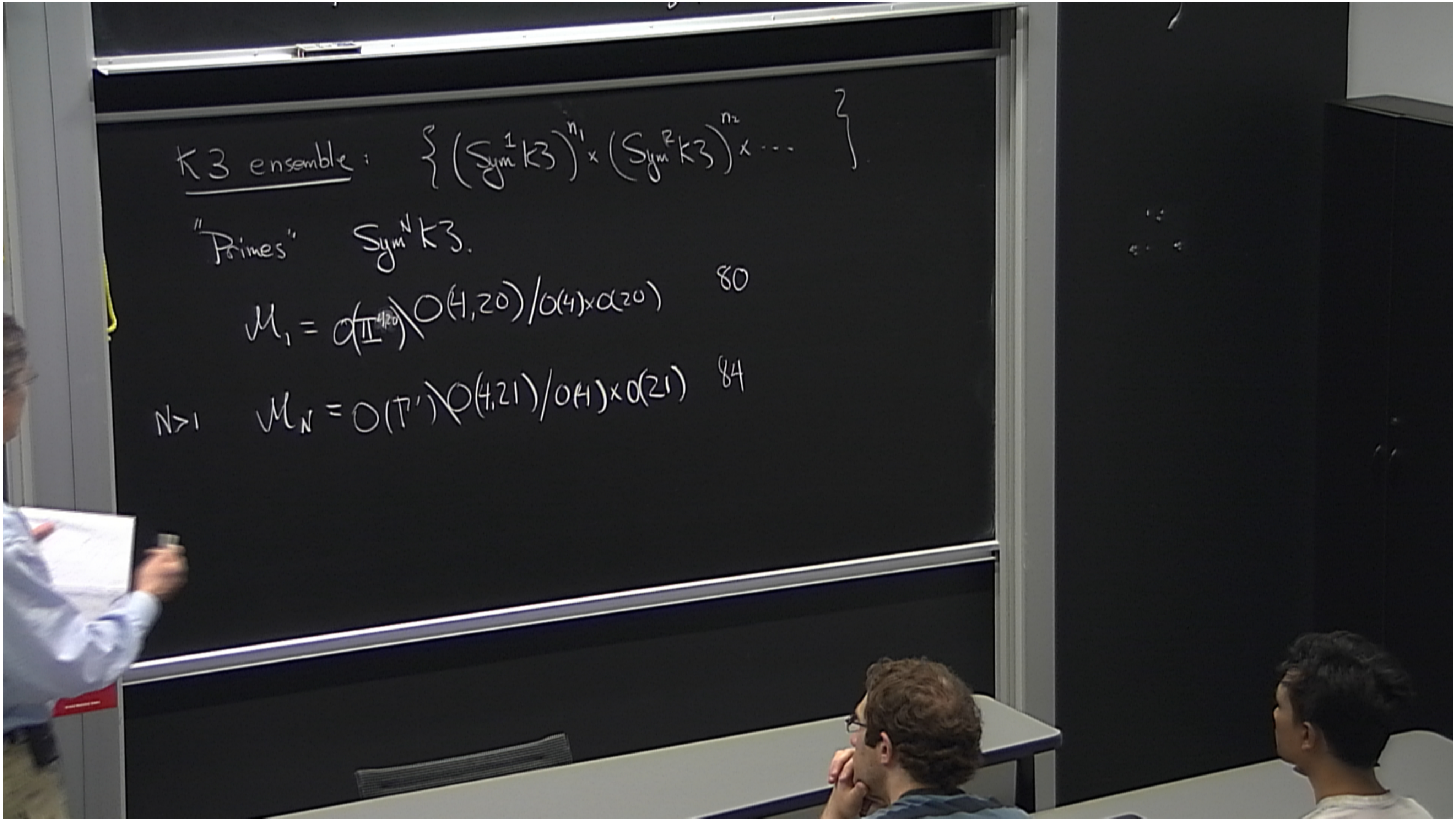
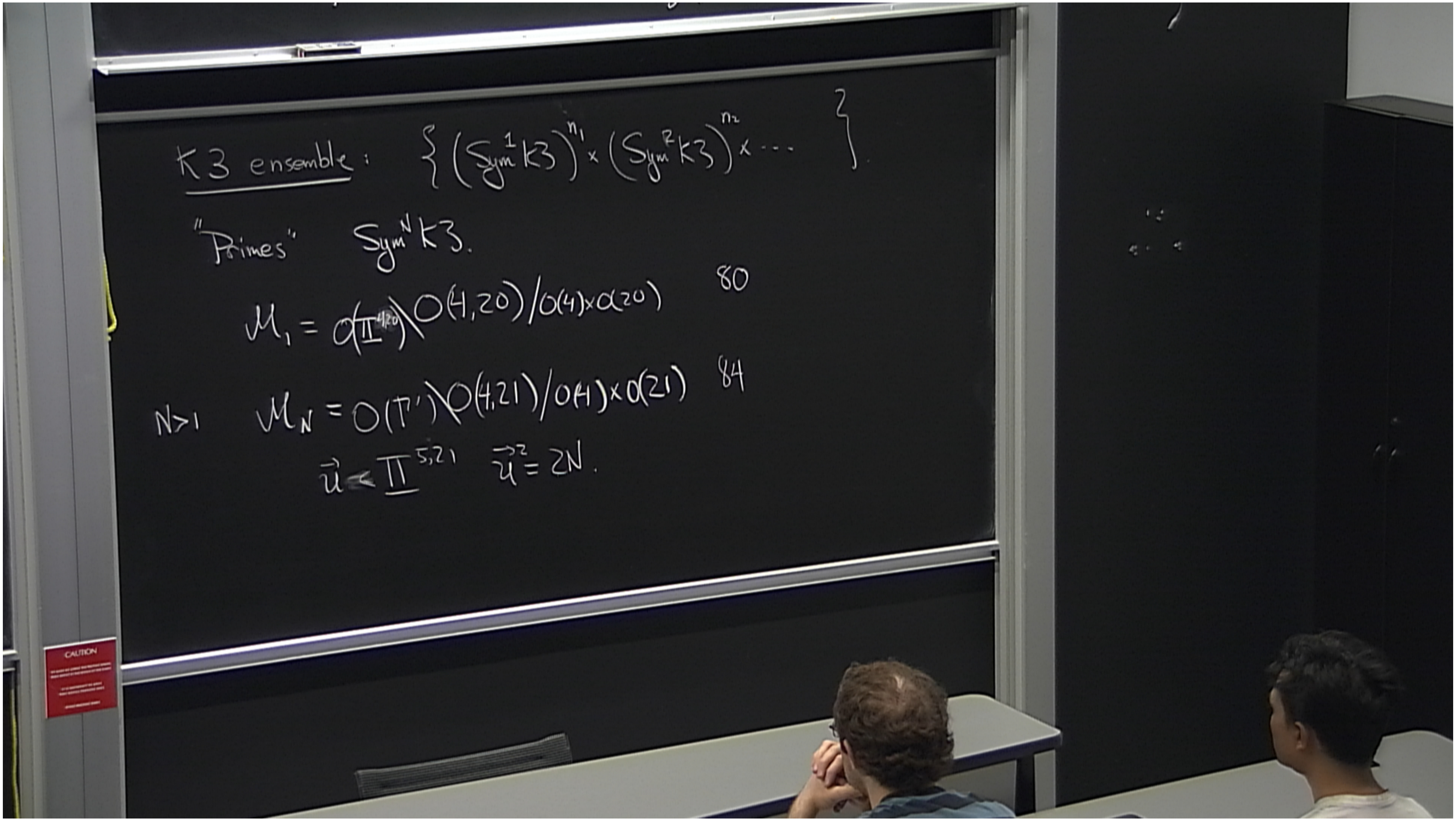
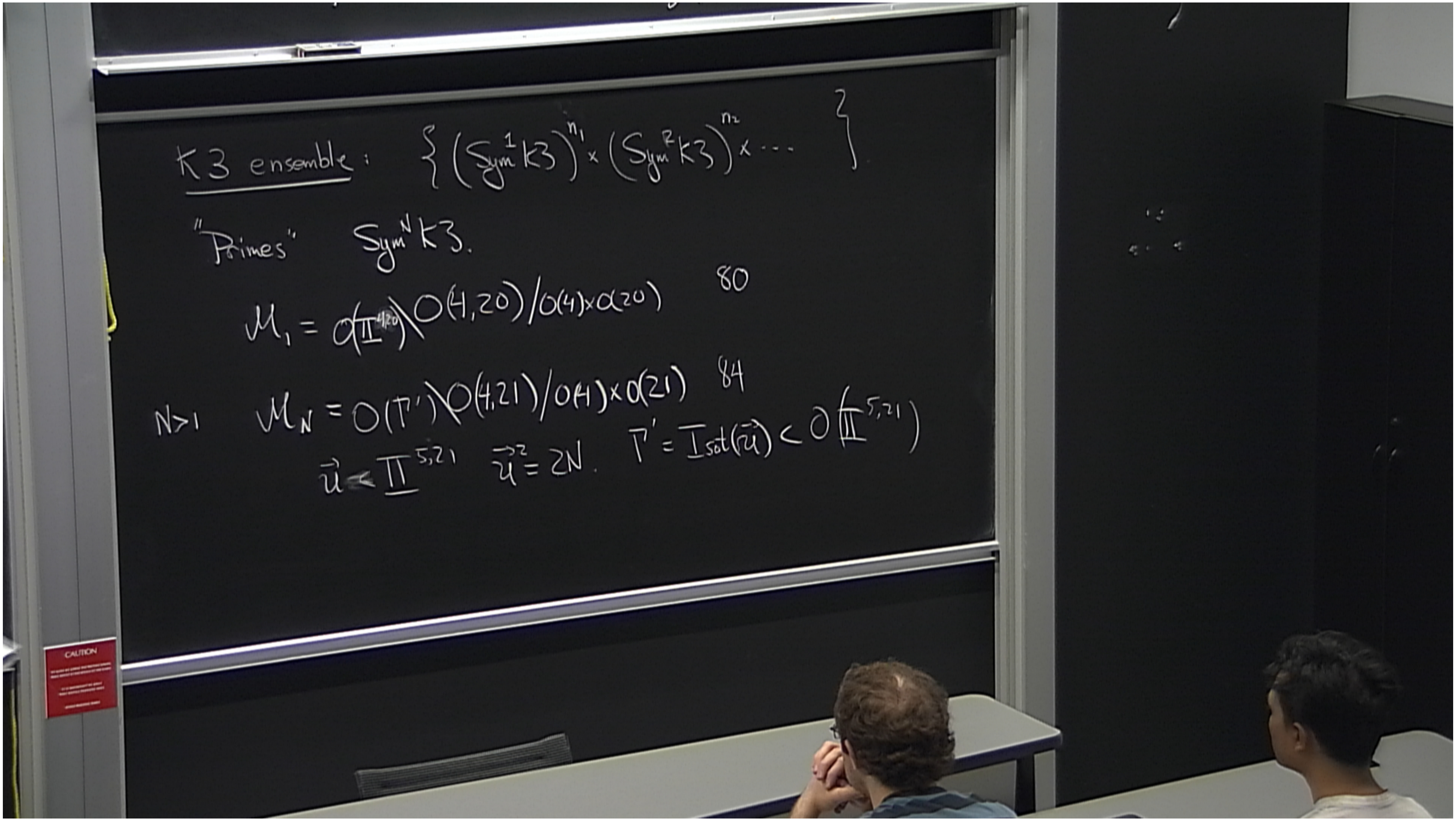
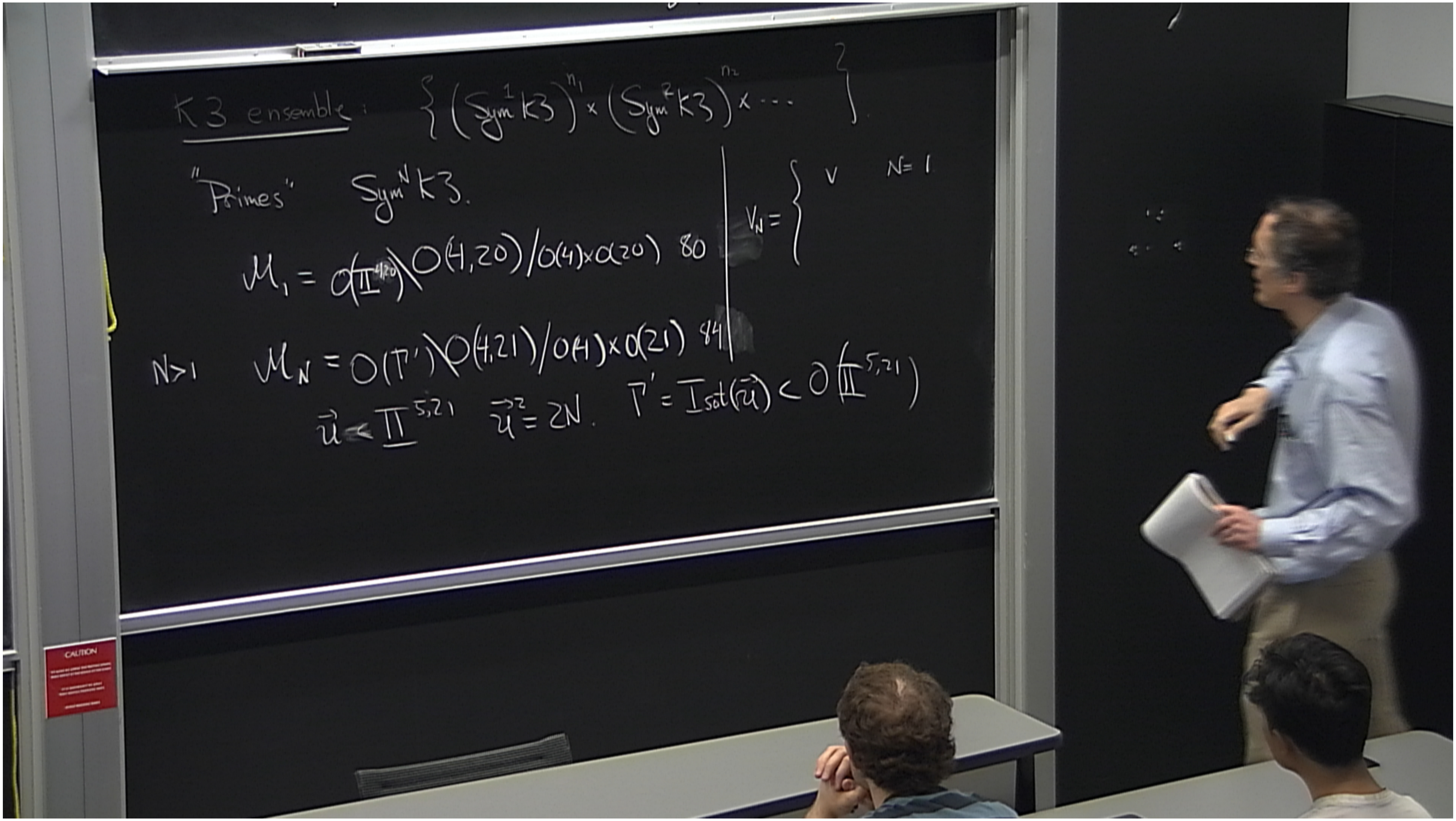
"Primes" Sym^N K3.
∈ Z+
= O(II^4,20)\O(4,20)/O(4)×O(20)
M_N = O(Γ')\O(4,21)/O(4)×O(21)
CAUTION

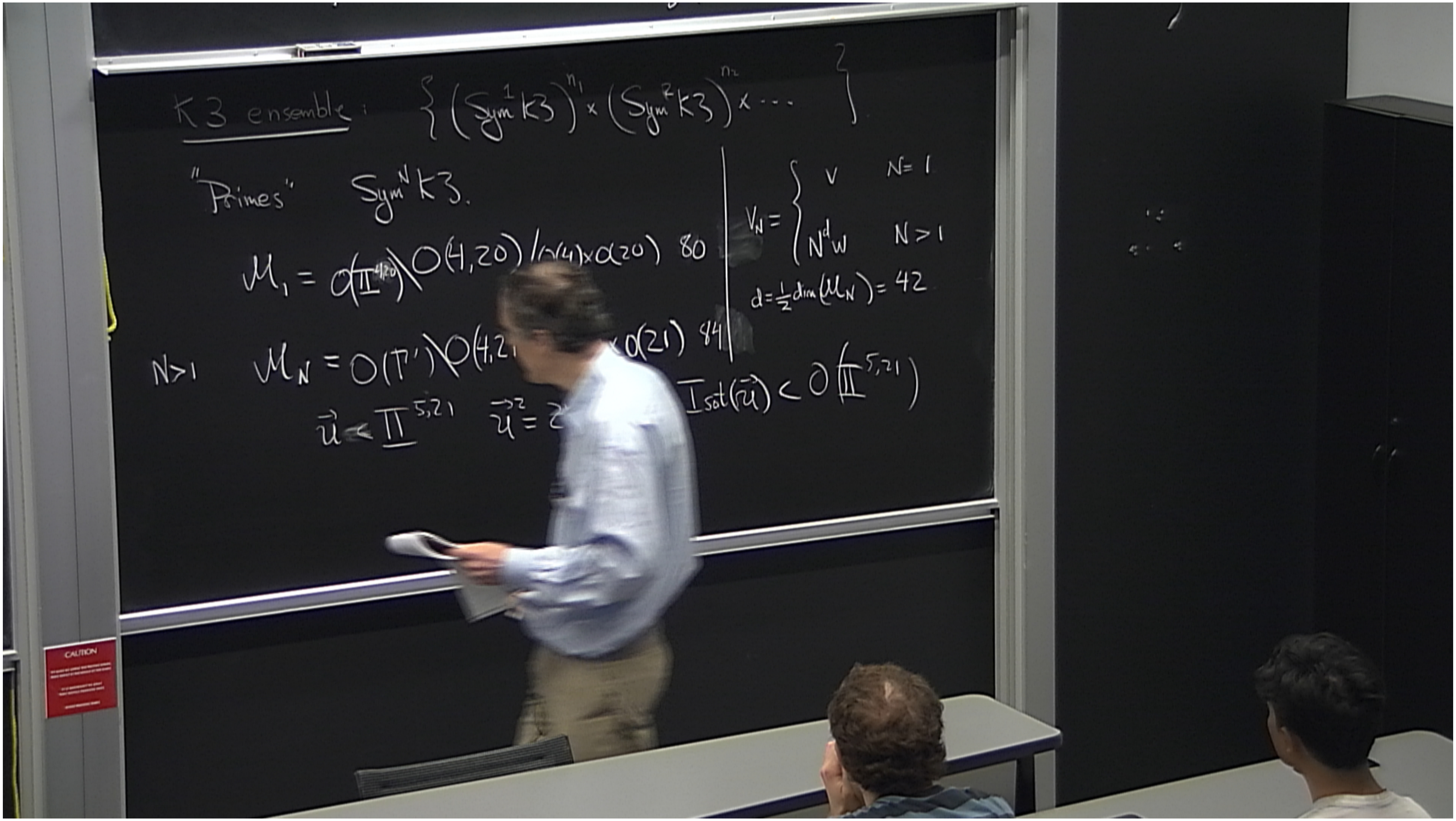

K3 ensemble: { (Sym^1 K3)^n1 × (Sym^2 K3)^n2 × ··· }
"Primes" Sym^N K3.
80
M1 = O(II^4,20)\O(4,20)/O(4)×O(20)
N>1 MN = O(Γ')\O(4,21)/O(4)×O(2
CAUTION

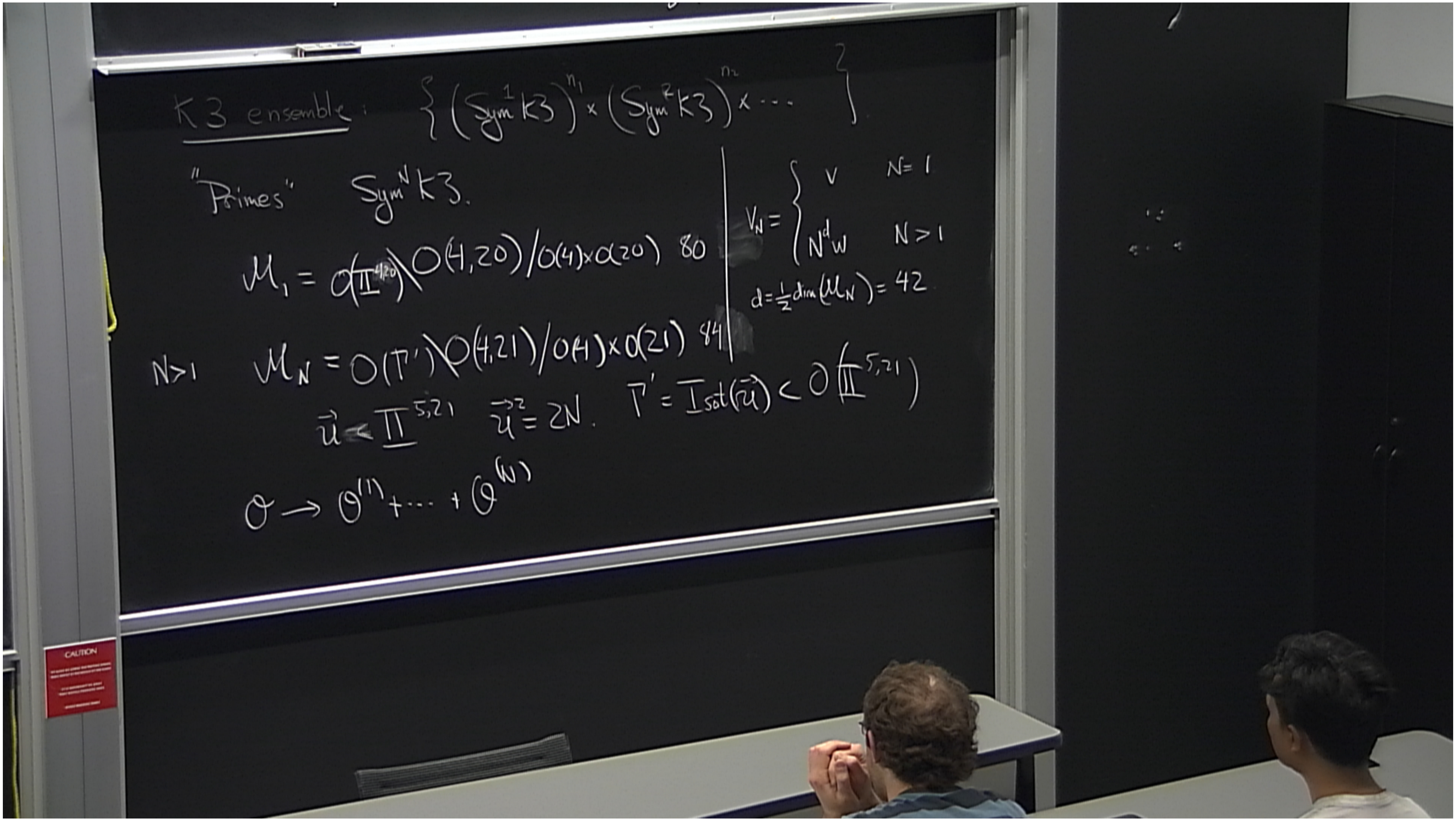

K3 ensemble: { (Sym^1 K3)^n1 × (Sym^2 K3)^n2 × ... }
"Primes" Sym^N K3.
M1 = O(II^4,20)\O(4,20)/O(4)×O(20)
80
N>1
MN = O(Γ')\O(4,21)/O(4)×O(21)
84

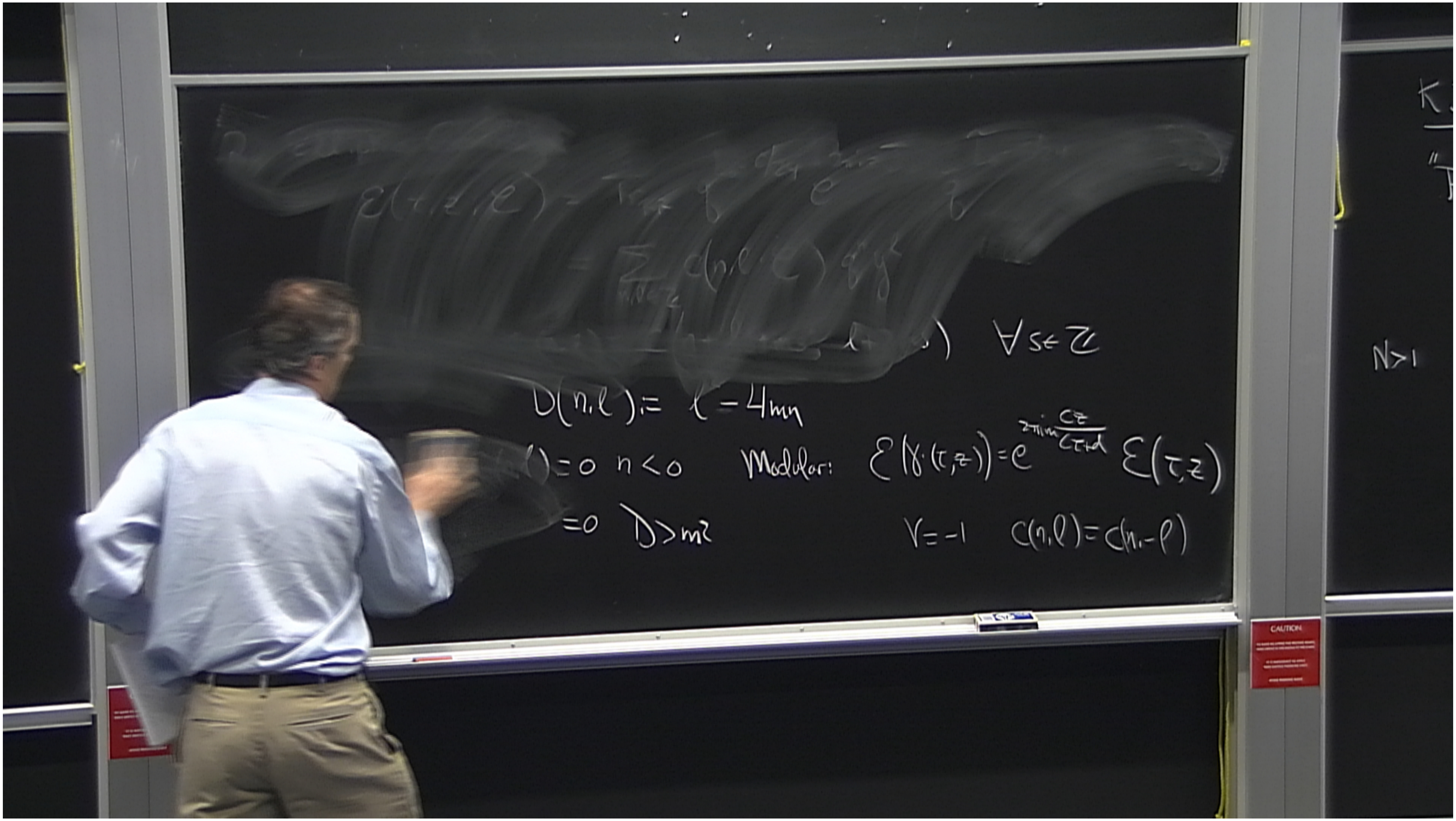

K3 ensemble: $\{ (\mathrm{Sym}^1 K3)^{n_1} \times (\mathrm{Sym}^2 K3)^{n_2} \times \cdots \}$
"Primes" $\mathrm{Sym}^N K3$.
$\mathcal{M}_1 = O(\Pi^{4,20}) \backslash O(4,20) / O(4) \times O(20)$ 80
$N>1$ $\mathcal{M}_N = O(\Gamma') \backslash O(4,21) / O(4) \times O(21)$ 84
$\vec{u} \in \Pi^{5,21}$ $\vec{u}^2 = 2N$.
CAUTION

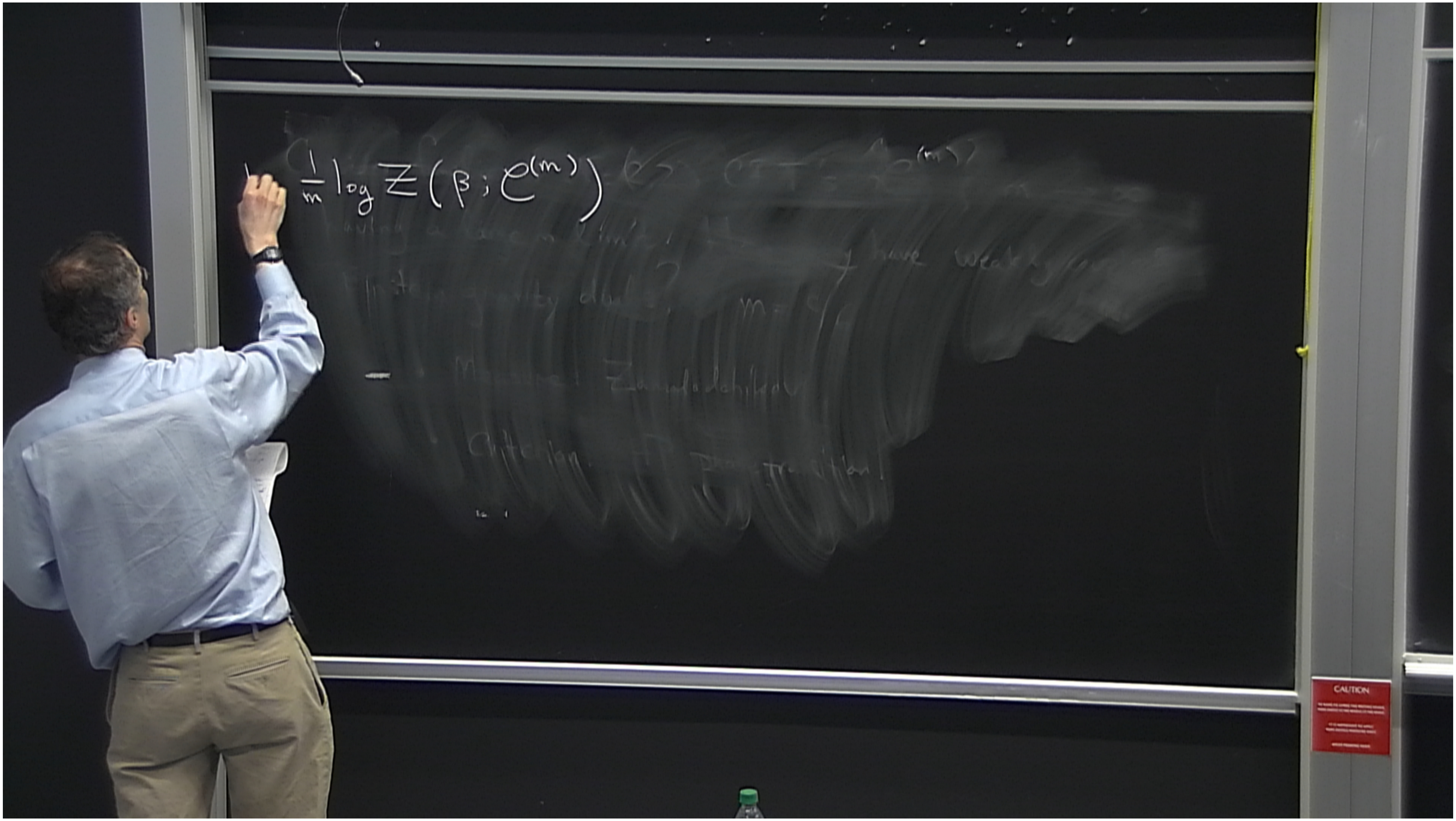

K3 ensemble: $\{ (\mathrm{Sym}^1 K3)^{n_1} \times (\mathrm{Sym}^2 K3)^{n_2} \times \cdots \}$
"Primes" $\mathrm{Sym}^N K3$.
$\mathcal{M}_1 = O(II^{4,20}) \backslash O(4,20)/O(4)\times O(20)$ 80
$N>1$ $\mathcal{M}_N = O(\Gamma') \backslash O(4,21)/O(4)\times O(21)$ 84
$\vec{u} \in II^{5,21}$ $\vec{u}^2 = 2N$. $\Gamma' = \mathrm{Isot}(\vec{u}) \subset O(II^{5,21})$

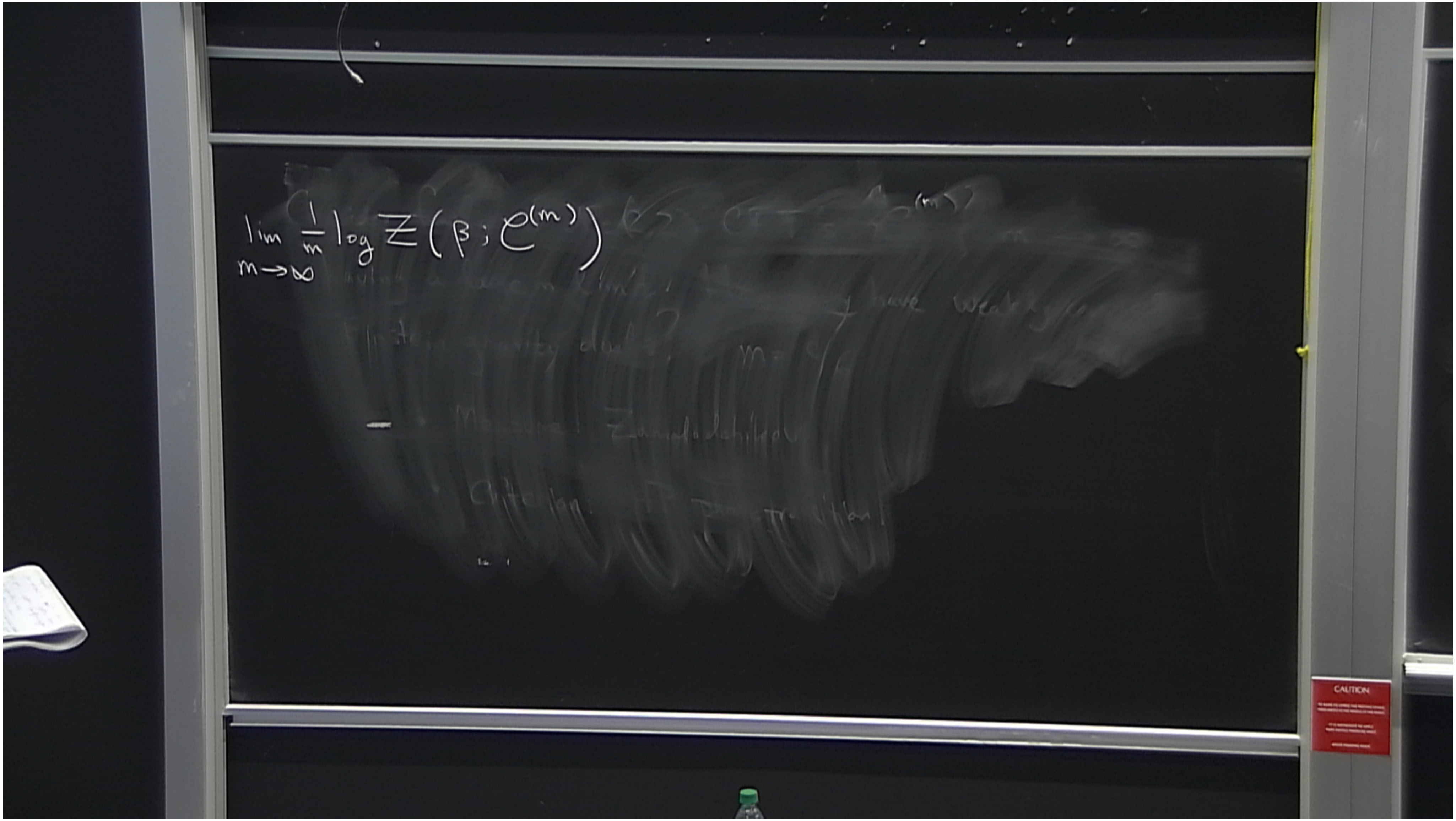

K3 ensemble
"Primes"
N>1
CAUTION

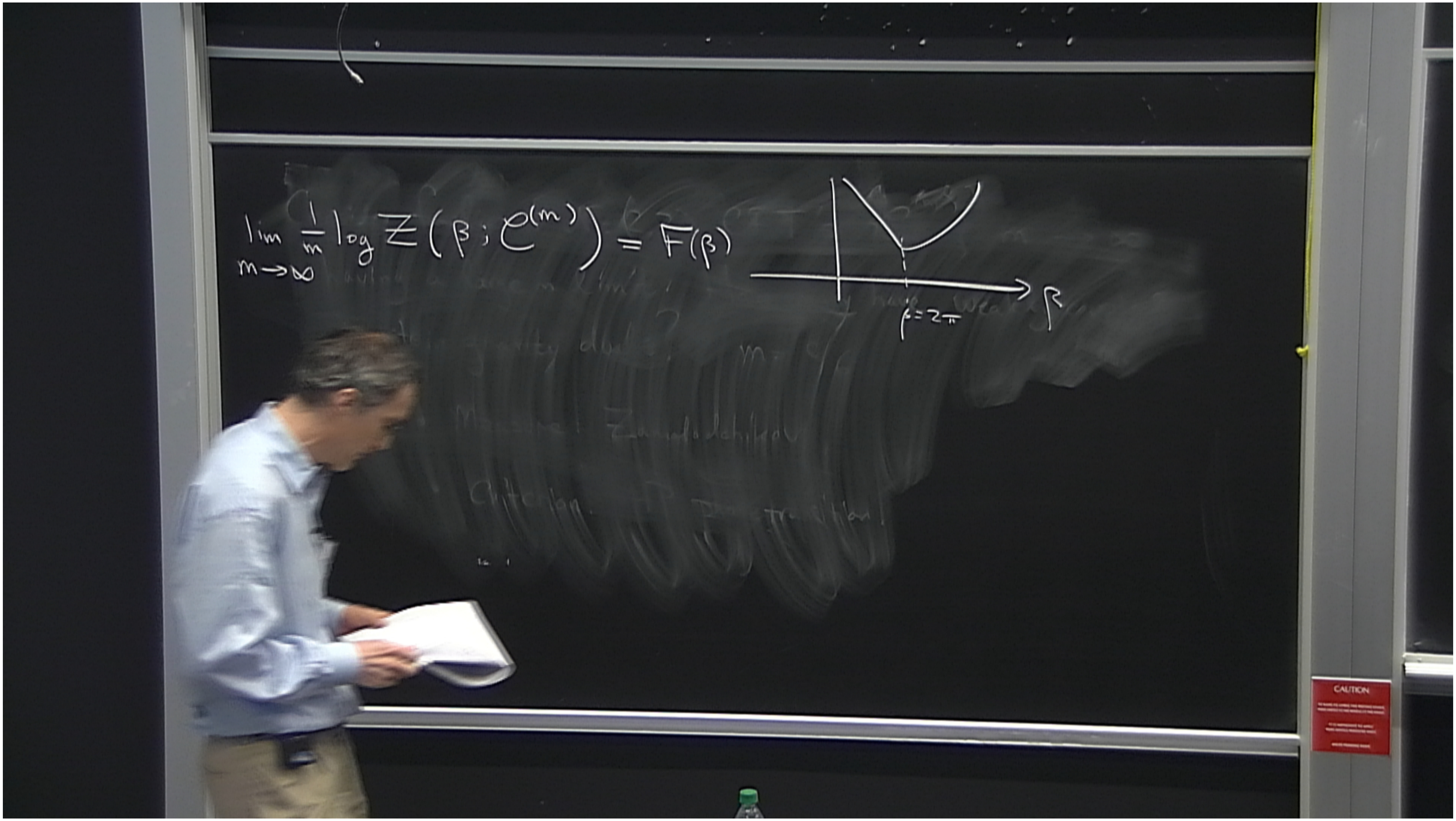

K3 ensemble : { (Sym^1 K3)^n1 × (Sym^2 K3)^n2 × · · · }
"Primes" Sym^N K3.
V_N = { V N=1 ; N^d W N>1
d = 1/2 dim(M_N) = 42
M_1 = O(II^4,20)\O(4,20)/O(4)×O(20) 80
N>1 M_N = O(Γ')\O(4,2
×O(21) 84
Isot(u) ⊂ O(II^5,21)
u ∈ II^5,21 u^2 = 2

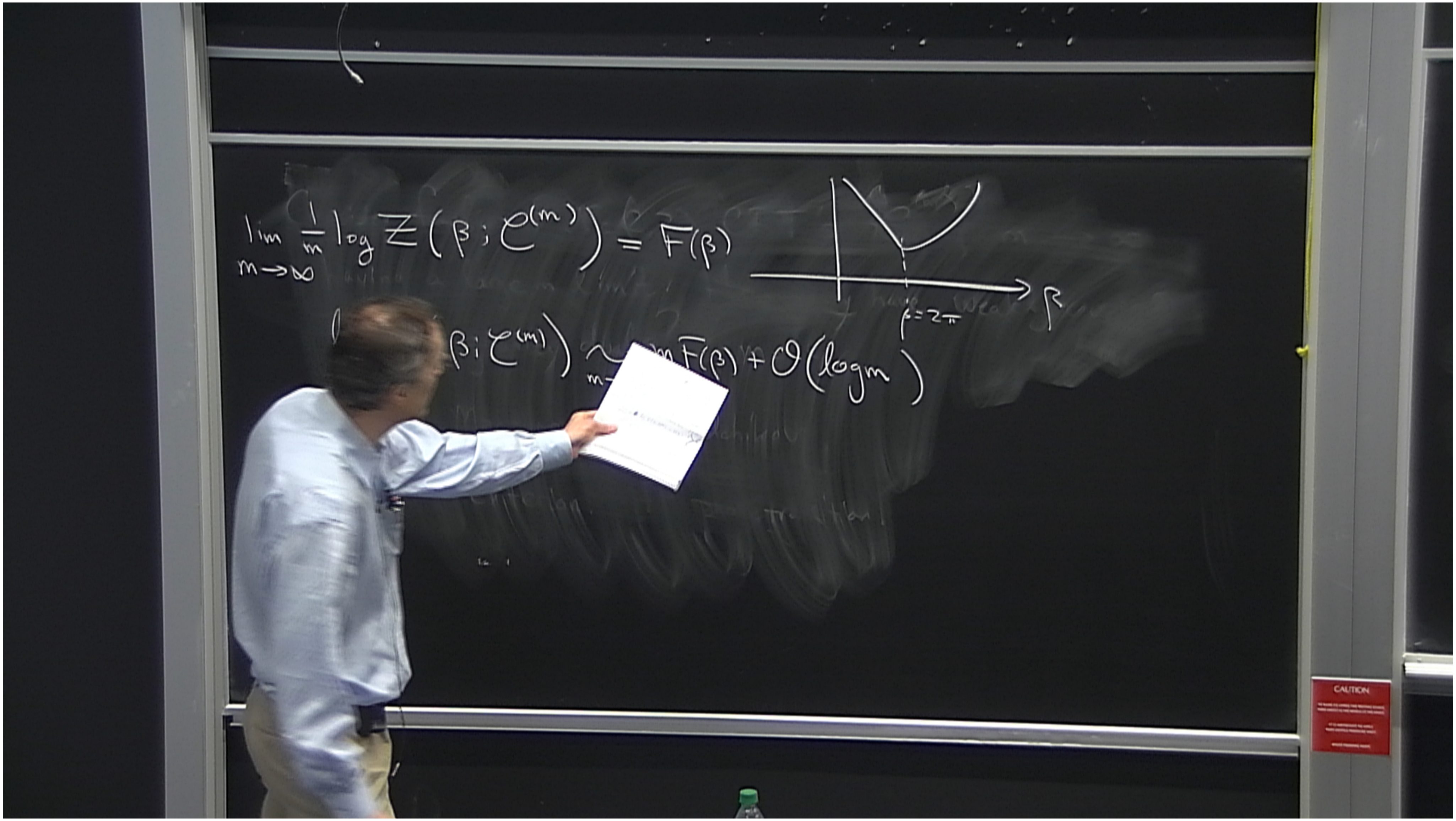

K3 ensemble : { (Sym^1 K3)^n1 × (Sym^2 K3)^n2 × · · · }
"Primes" Sym^N K3.
M1 = O(II^4,20)\O(4,20)/O(4)×O(20) 80
N>1 MN = O(Γ')\O(4,21)/O(4)×O(21) 84
u ∈ II^5,21 u^2 = 2N. Γ' = Isot(u) ⊂ O(II^5,21)
θ → θ^(1) + · · · + θ^(N)
VN = { V N=1 ; N^d W N>1
d = 1/2 dim(MN) = 42

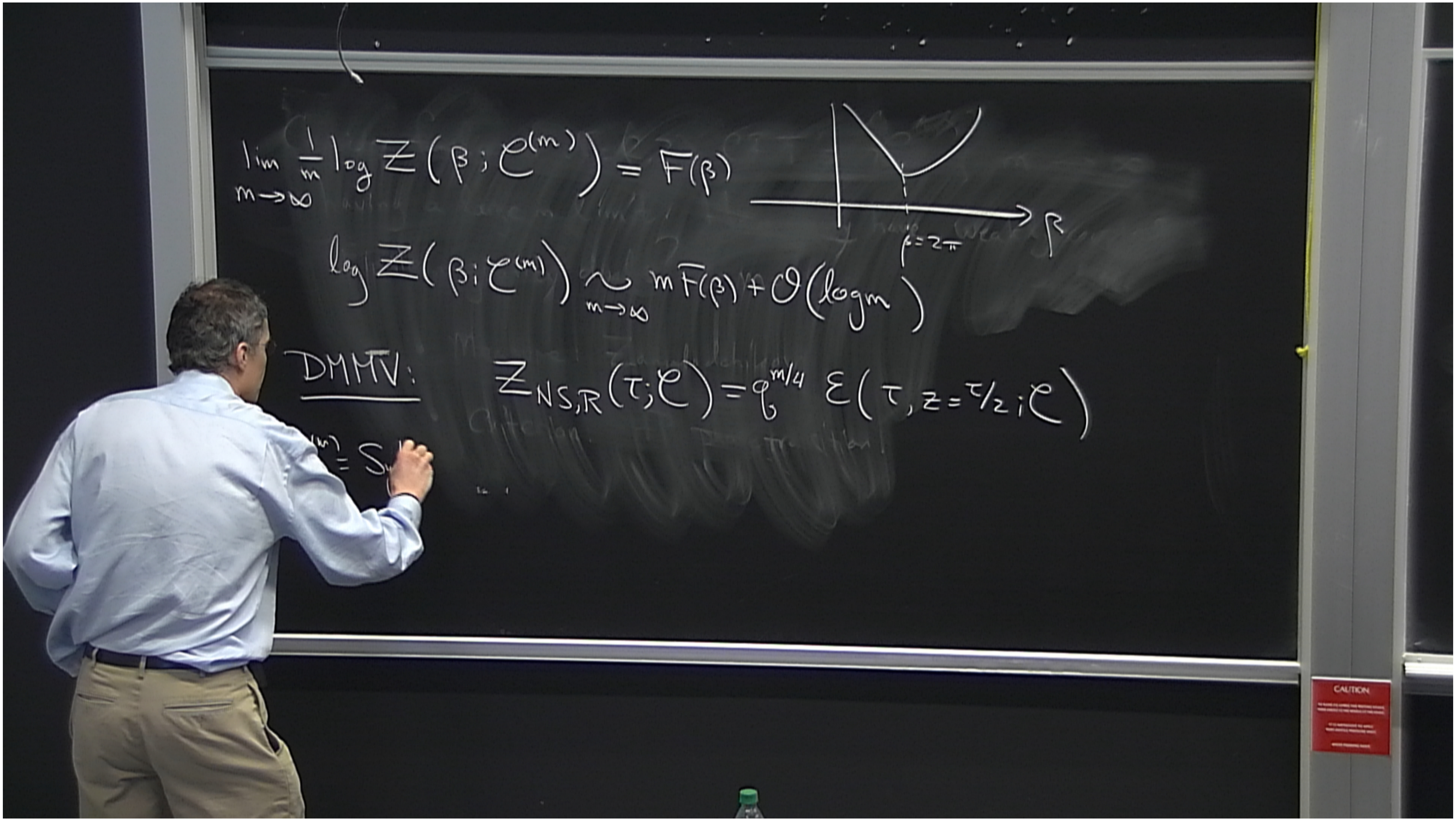

∀ s ∈ Z
D(n,ℓ) := ℓ − 4mn
Modular:
N>1

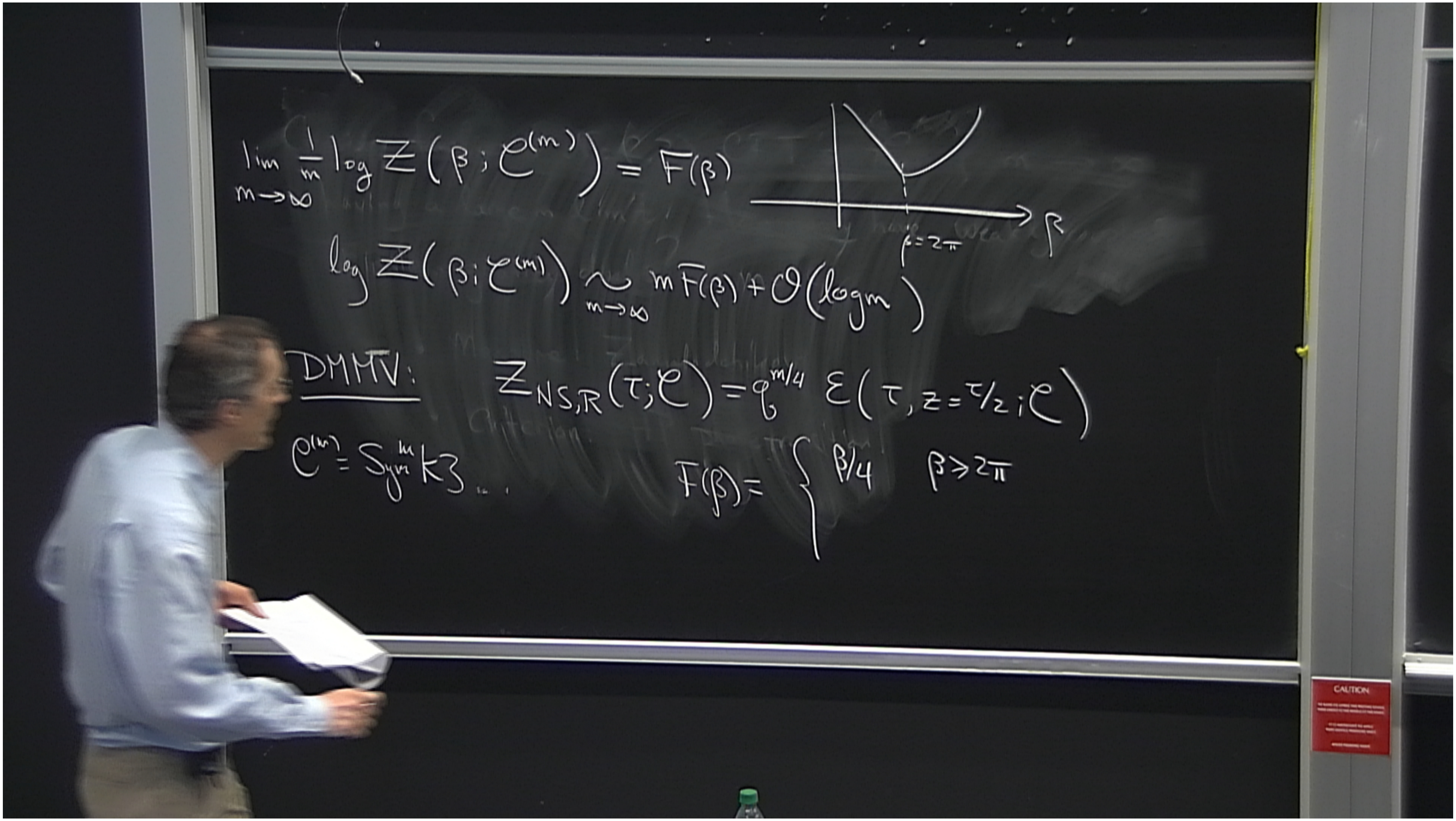

1/m log Z(β; e^(m))

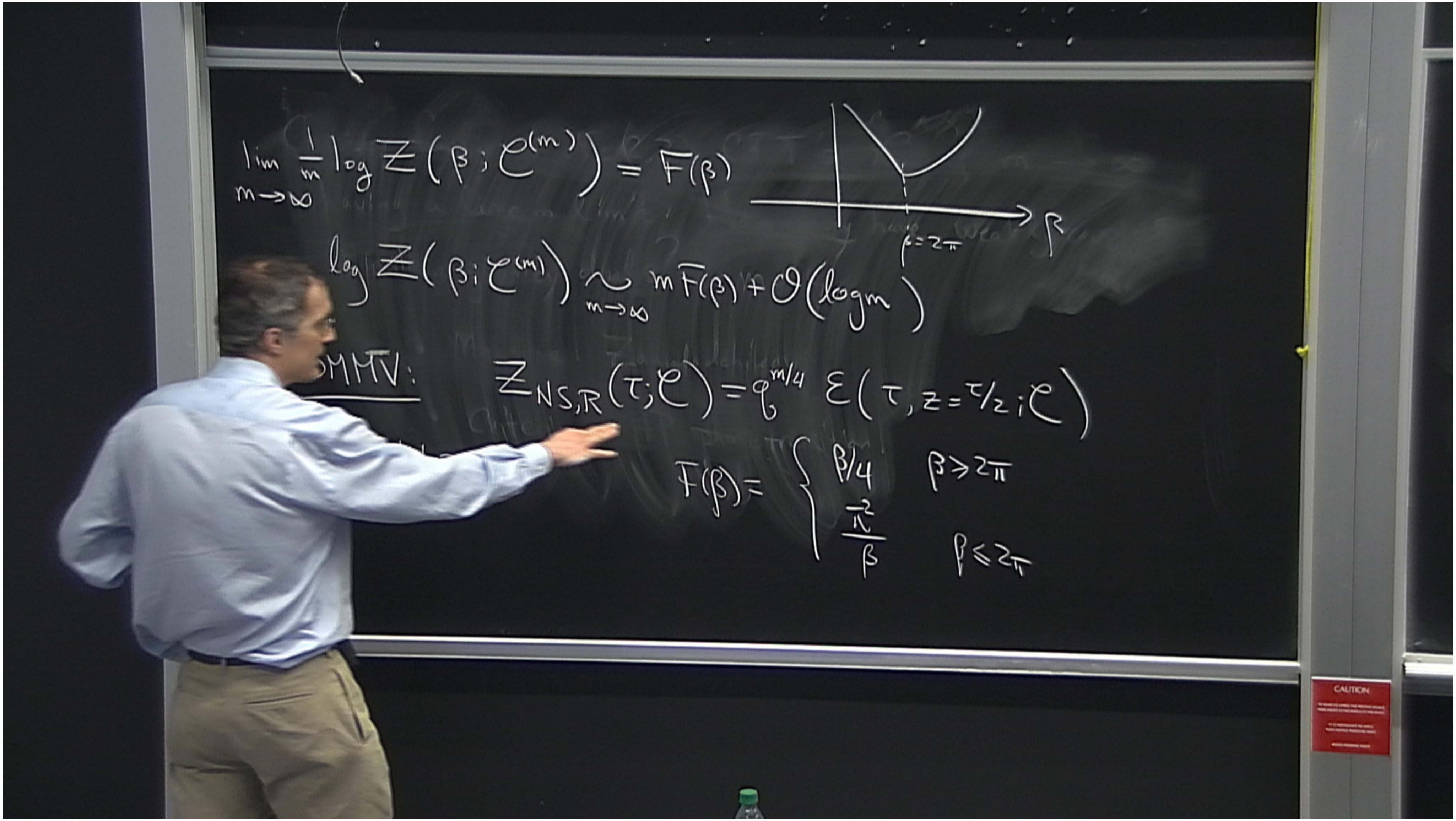

lim 1/m log Z(β; C^(m))
m→∞

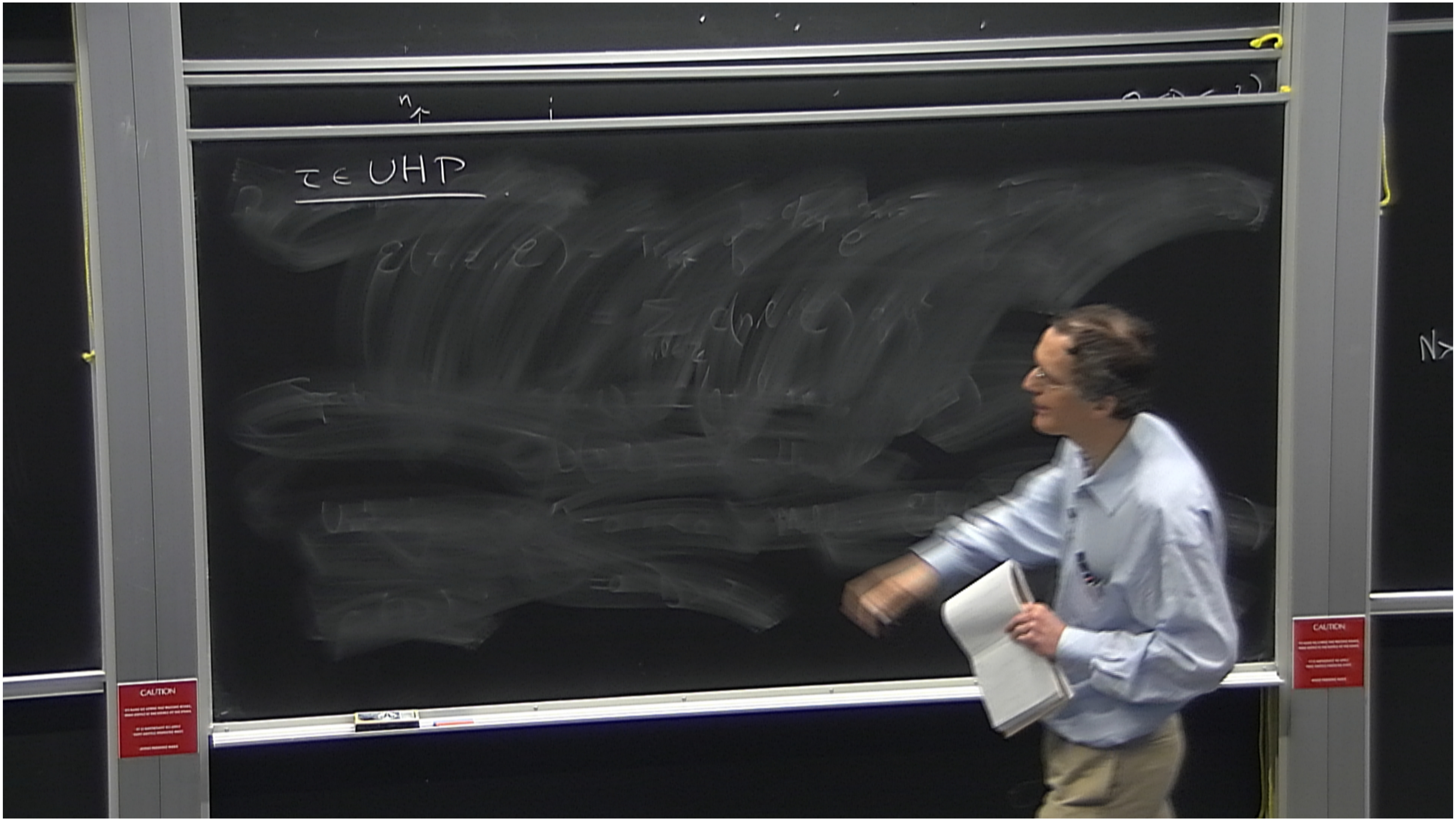

$\lim_{m\to\infty} \frac{1}{m} \log Z(\beta; C^{(m)}) = F(\beta)$
$\beta = 2\pi$
β

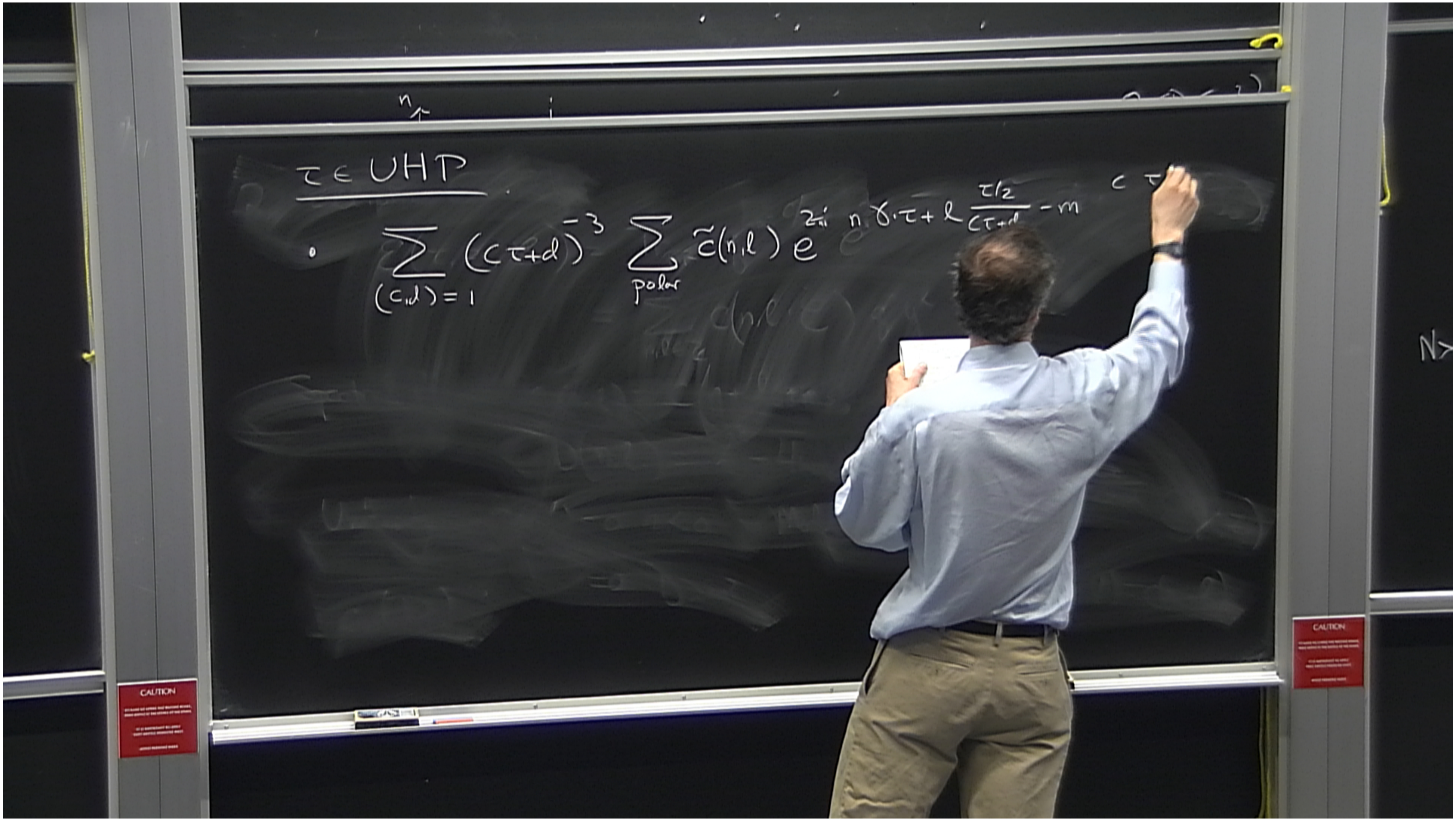

lim 1/m log Z(β; C^(m)) = F(β)
m→∞
β; C^(m)) ~ m F(β) + O(log m)
β
β=2π
CAUTION

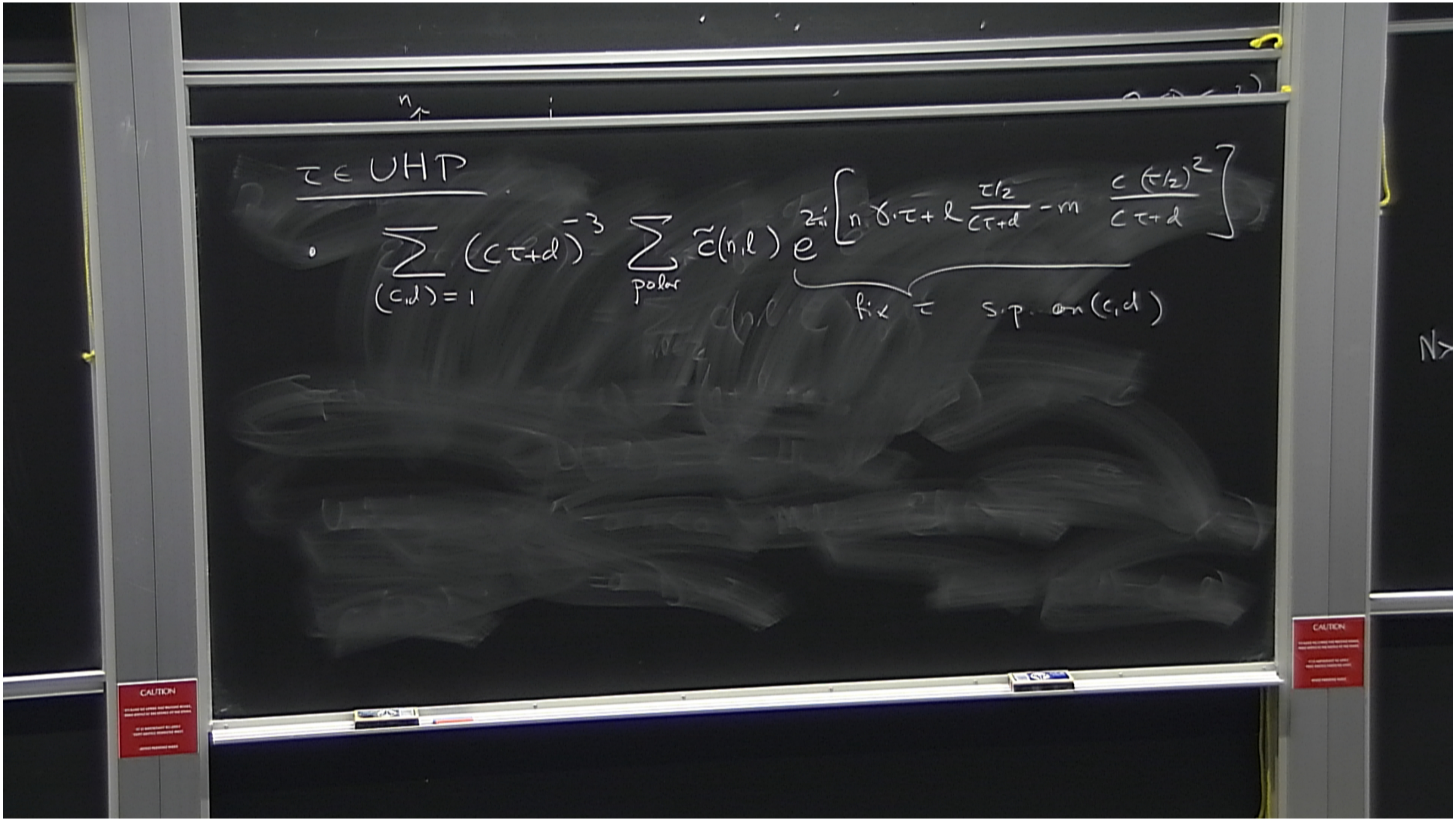

$\lim_{m\to\infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$
$\beta = 2\pi$
β
$\log Z(\beta; \mathcal{C}^{(m)}) \underset{m\to\infty}{\sim} m F(\beta) + \mathcal{O}(\log m)$
DMMV:
$Z_{NS,R}(\tau; \mathcal{C}) = q^{m/4}\, \mathcal{E}(\tau, z = \tau/2; \mathcal{C})$
CAUTION

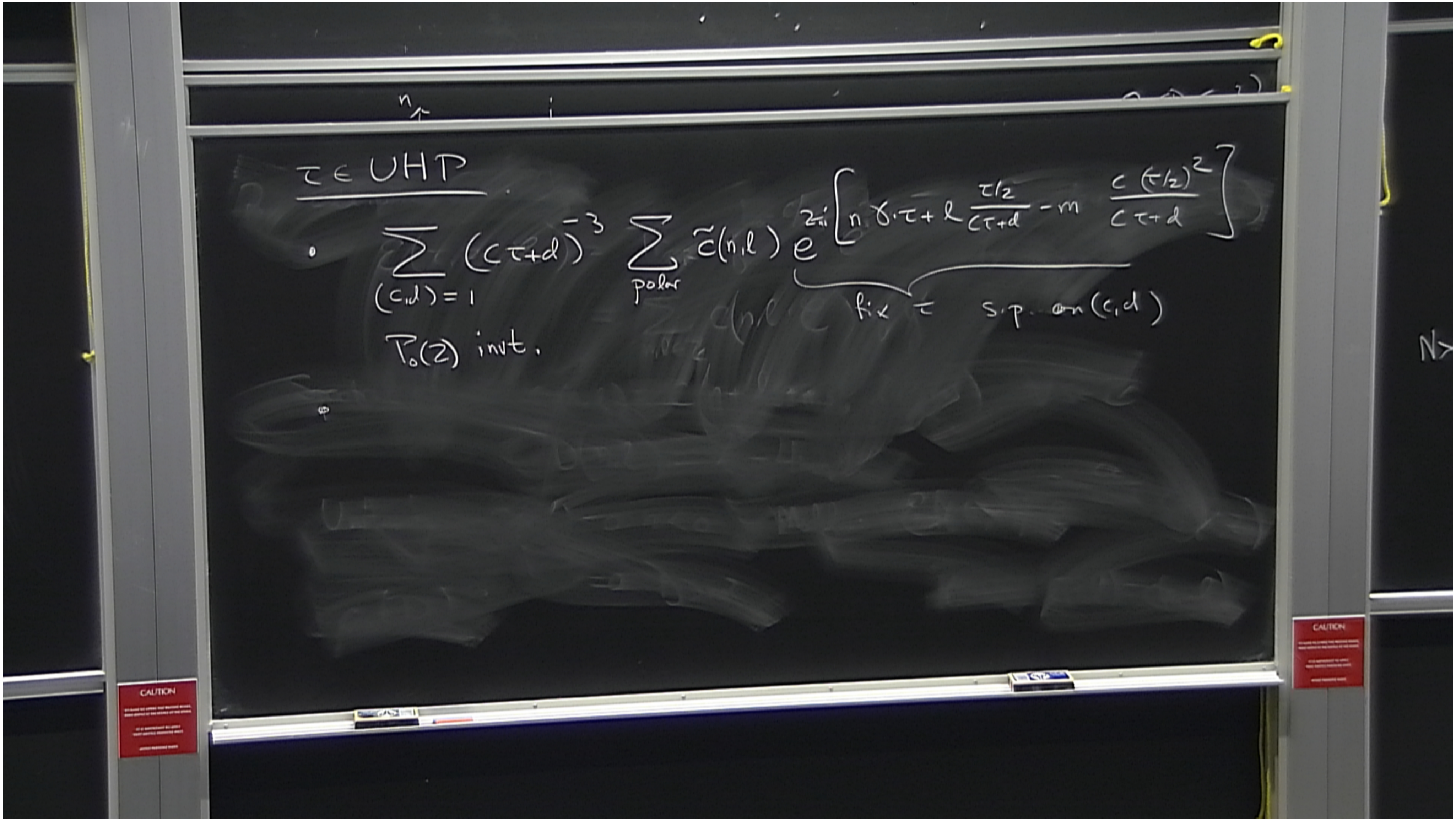

lim m→∞ 1/m log Z(β; C^(m)) = F(β)
β = 2π
β
log Z(β; C^(m)) ~ m→∞ m F(β) + O(log m)
DMMV:
Z_NS,R(τ; C) = q^(m/4) E(τ, z = τ/2; C)
C^(m) = Sym^m K3 ...
F(β) = { β/4 β ≥ 2π
CAUTION

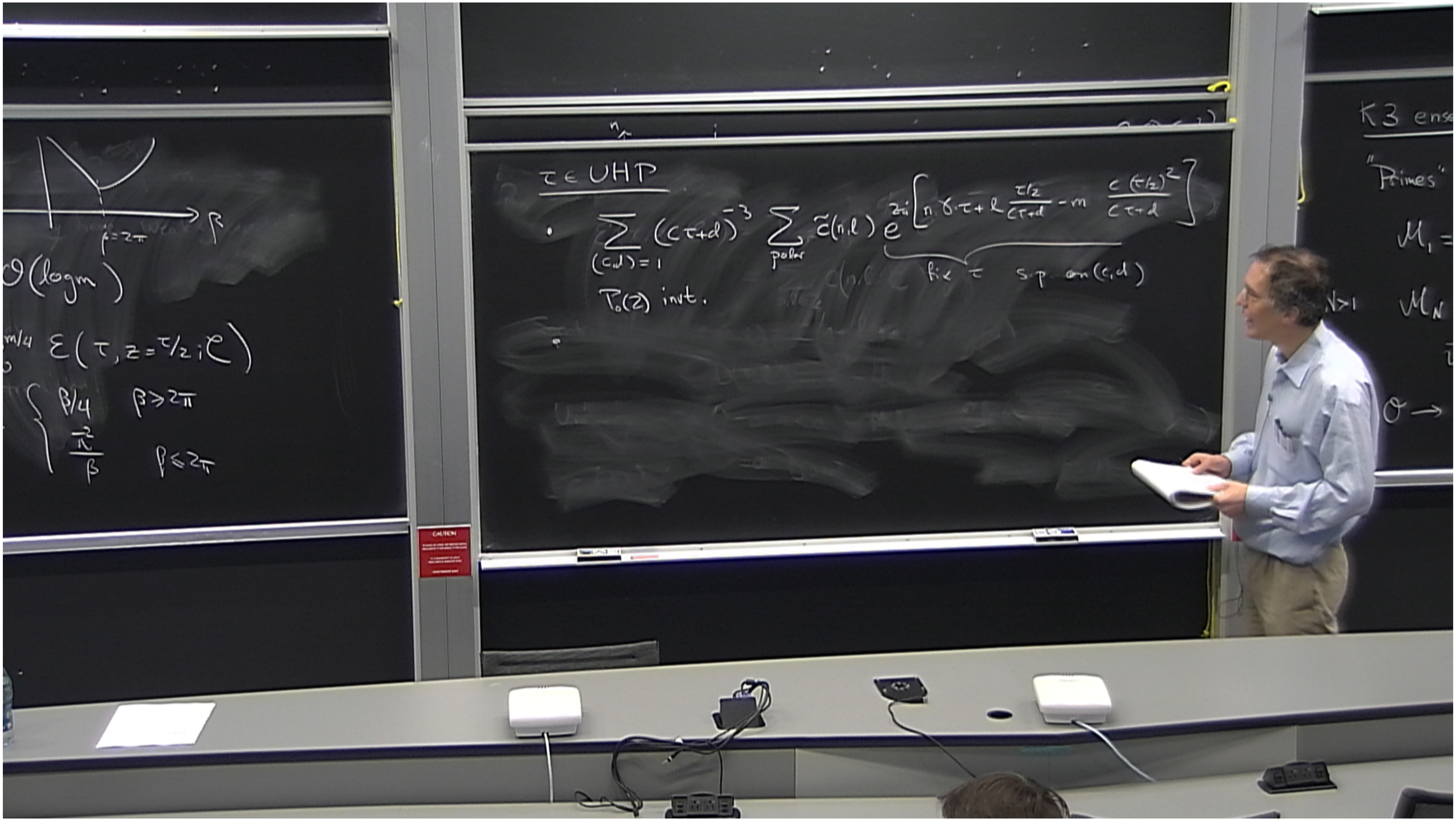

lim m→∞ 1/m log Z(β; C^(m)) = F(β)
β
β=2π
log Z(β; C^(m)) ~ m→∞ m F(β) + O(log m)
MMV:
Z_NS,R(τ; C) = q^(m/4) E(τ, z = τ/2; C)
F(β) = { β/4 β ≥ 2π ; π²/β β ≤ 2π

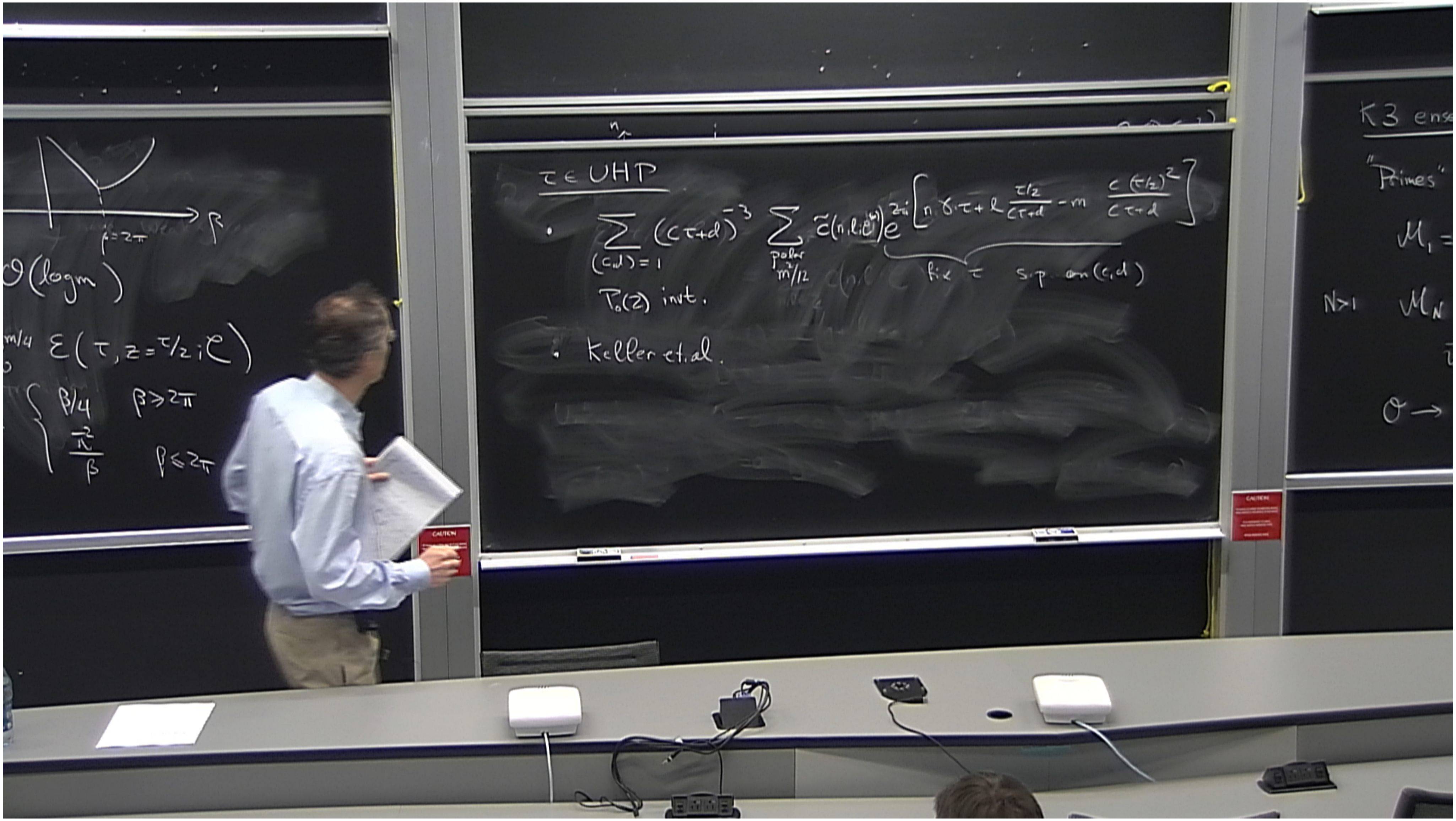

τ ∈ UHP


τ ∈ UHP
(c,d) = 1
polar
CAUTION

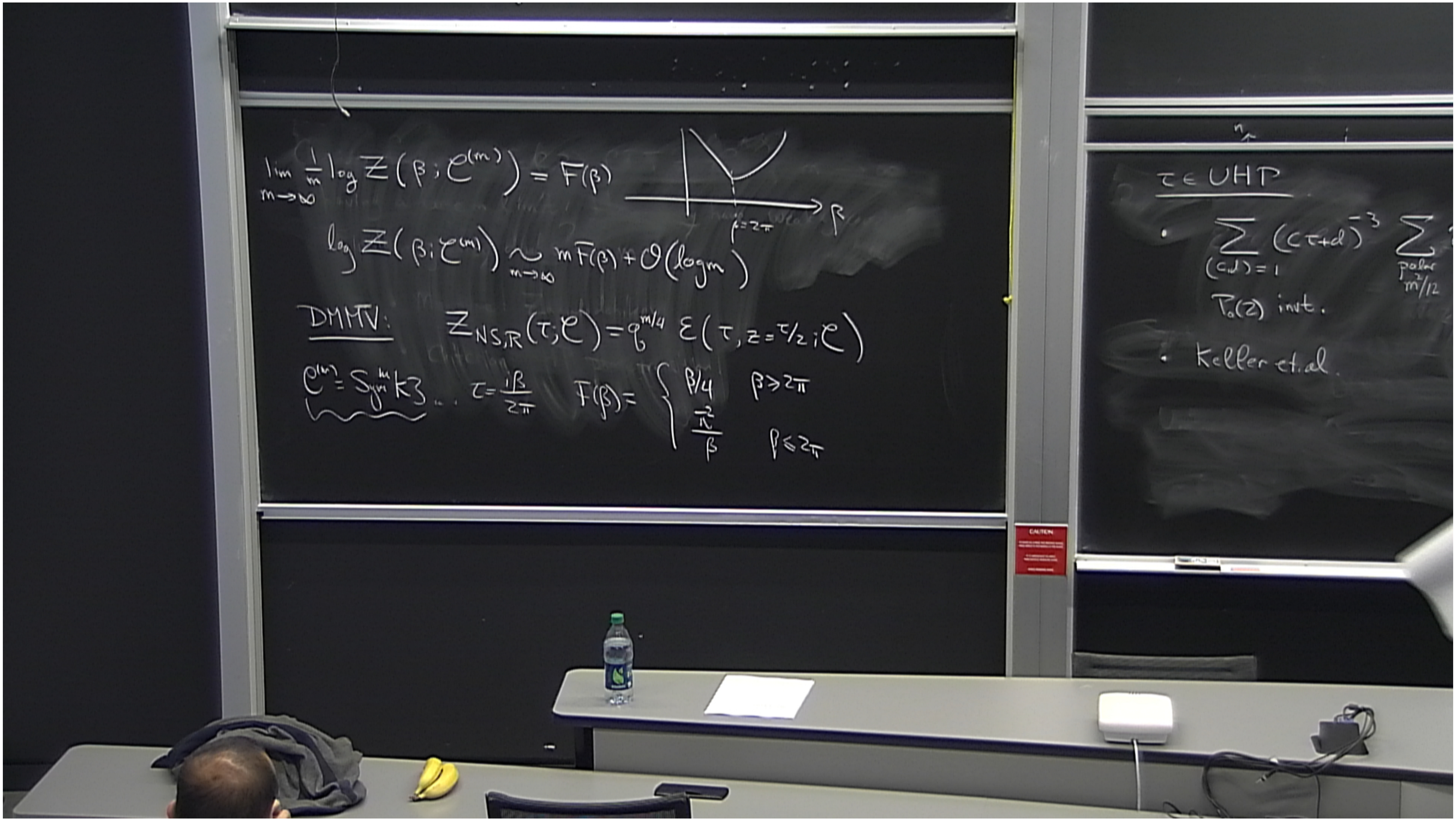

τ ∈ UHP
• Σ_{(c,d)=1} (cτ+d)^{-3} Σ_{polar} c̃(n,l) e^{2πi[n γ·τ + l (τ/2)/(cτ+d) − m c(τ/2)^2/(cτ+d)]}
fix τ s.p. on (c,d)

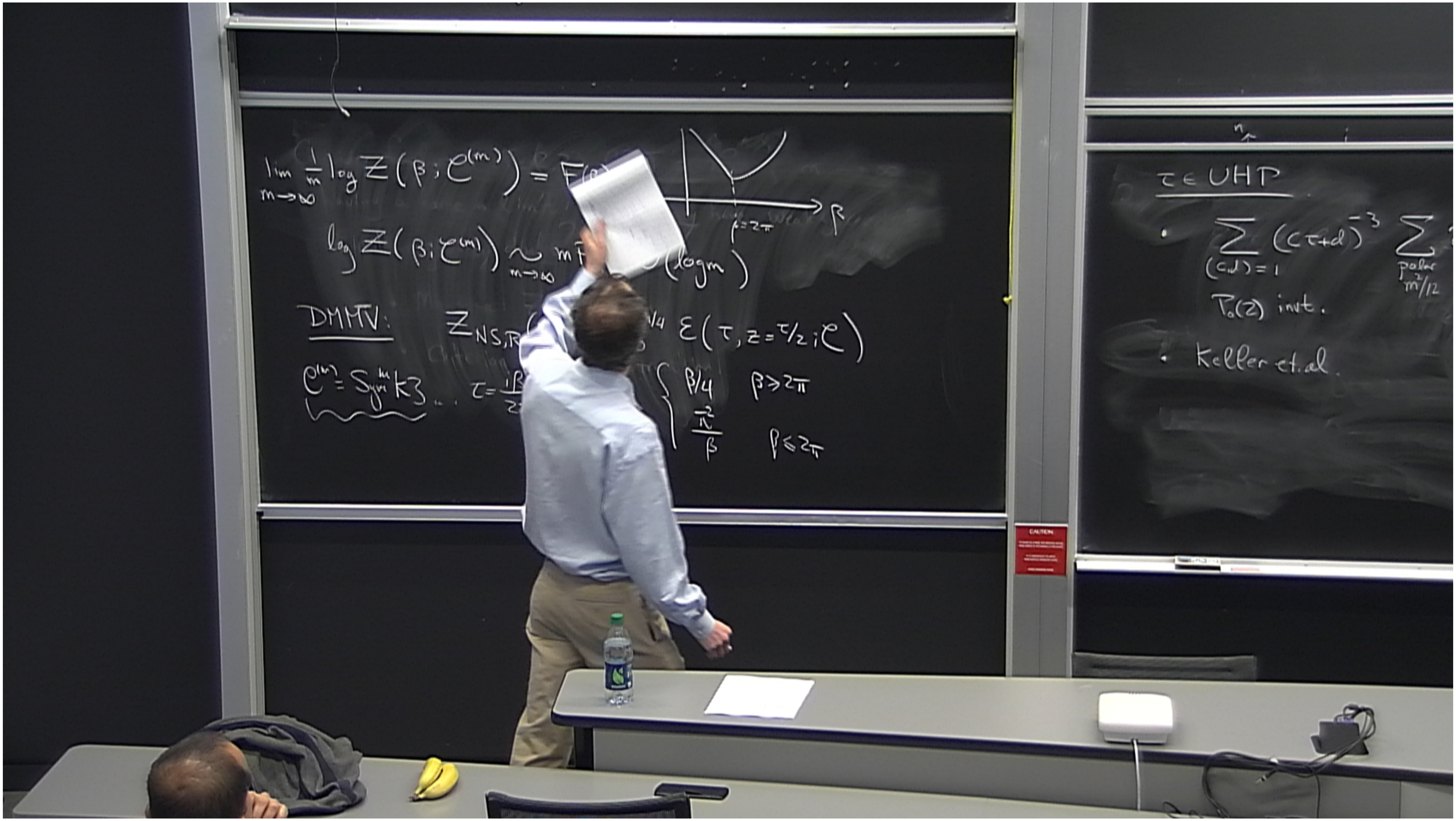

$\tau \in UHP$
$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{polar} \tilde{c}(n,\ell)\, e^{2\pi i\left[n\, \gamma\cdot\tau + \ell \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$
fix τ s.p. on (c,d)
$\Gamma_0(2)$ invt.

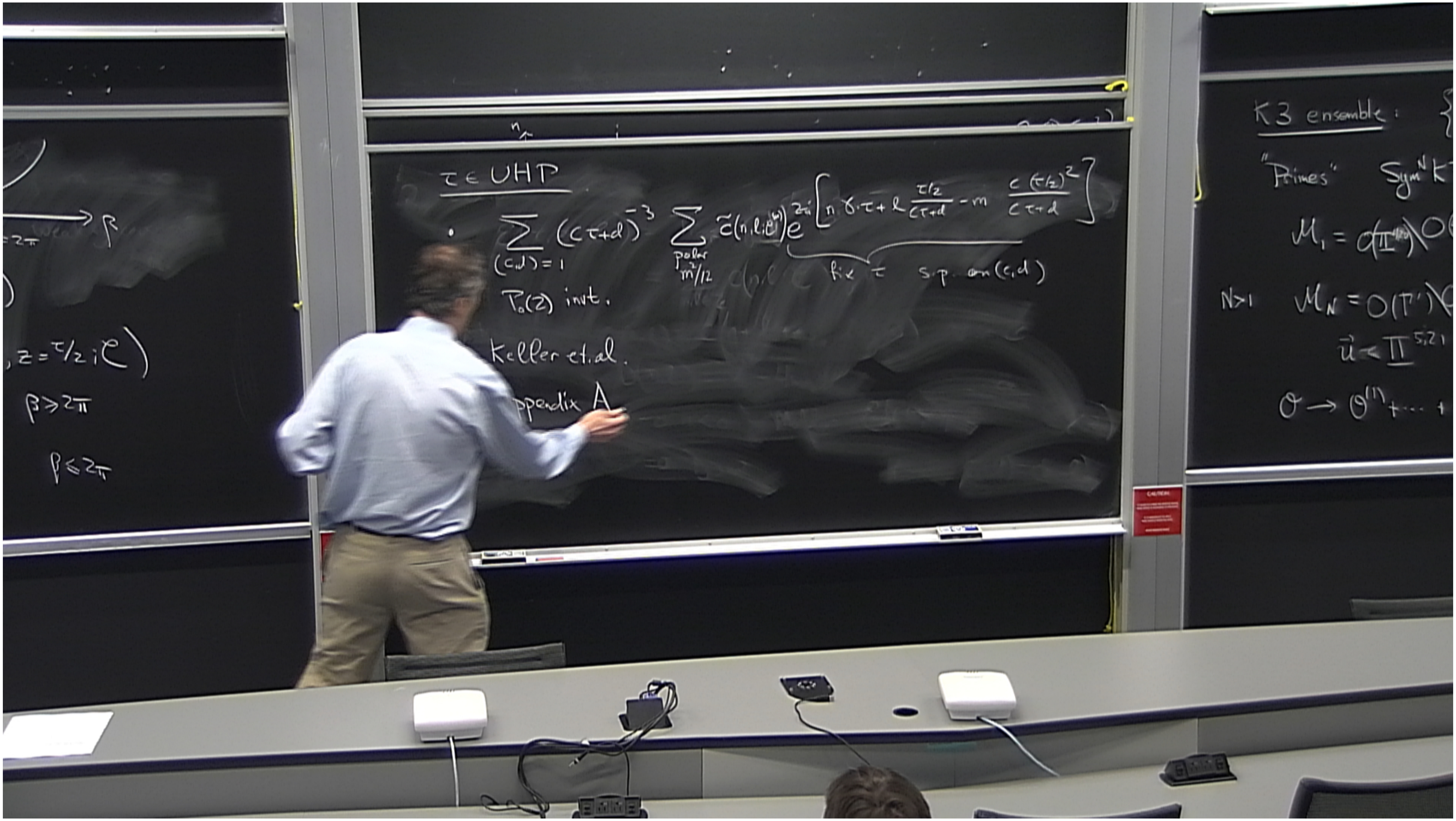

τ ∈ UHP
(c,d) = 1
polar
Γ0(2) invt.
fix τ s.p. on (c,d)
O(log m)
β ≥ 2π
β ≤ 2π
K3 ens
"Primes"

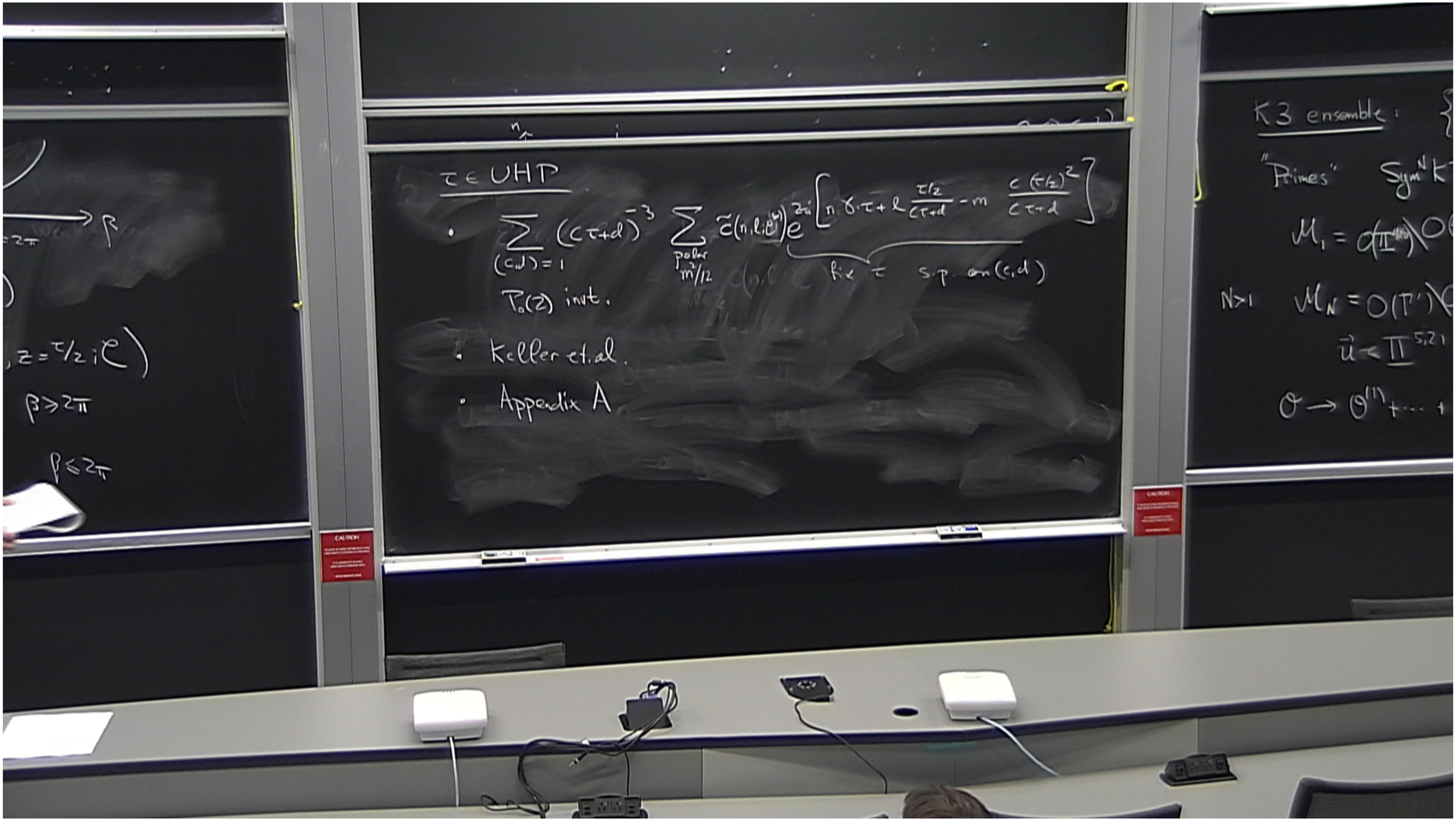


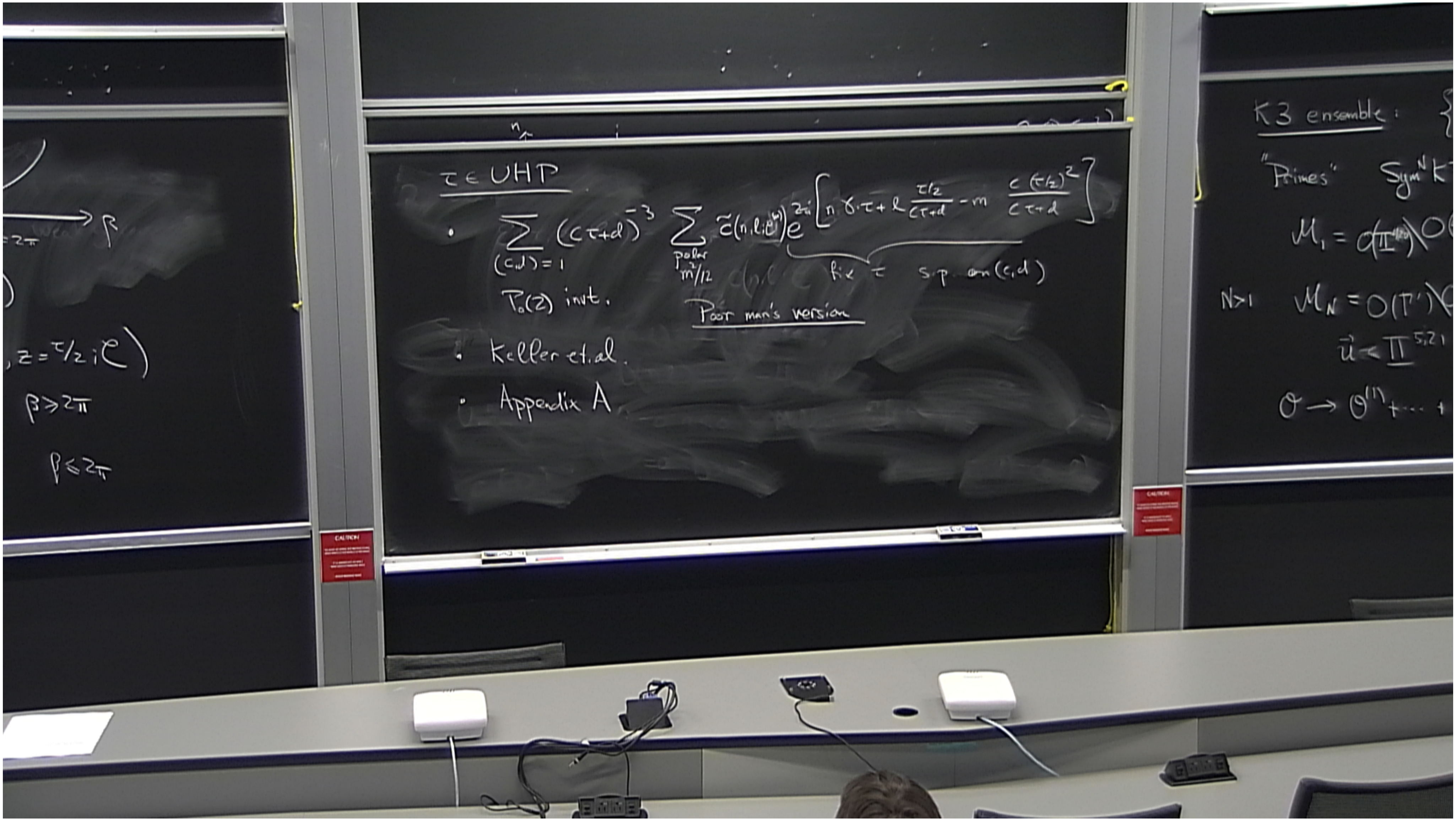
τ ∈ UHP
Γ₀(2) invt.
fix τ
s.p. on (c,d)
Keller et. al.
K3 ens
"Primes"
N>1

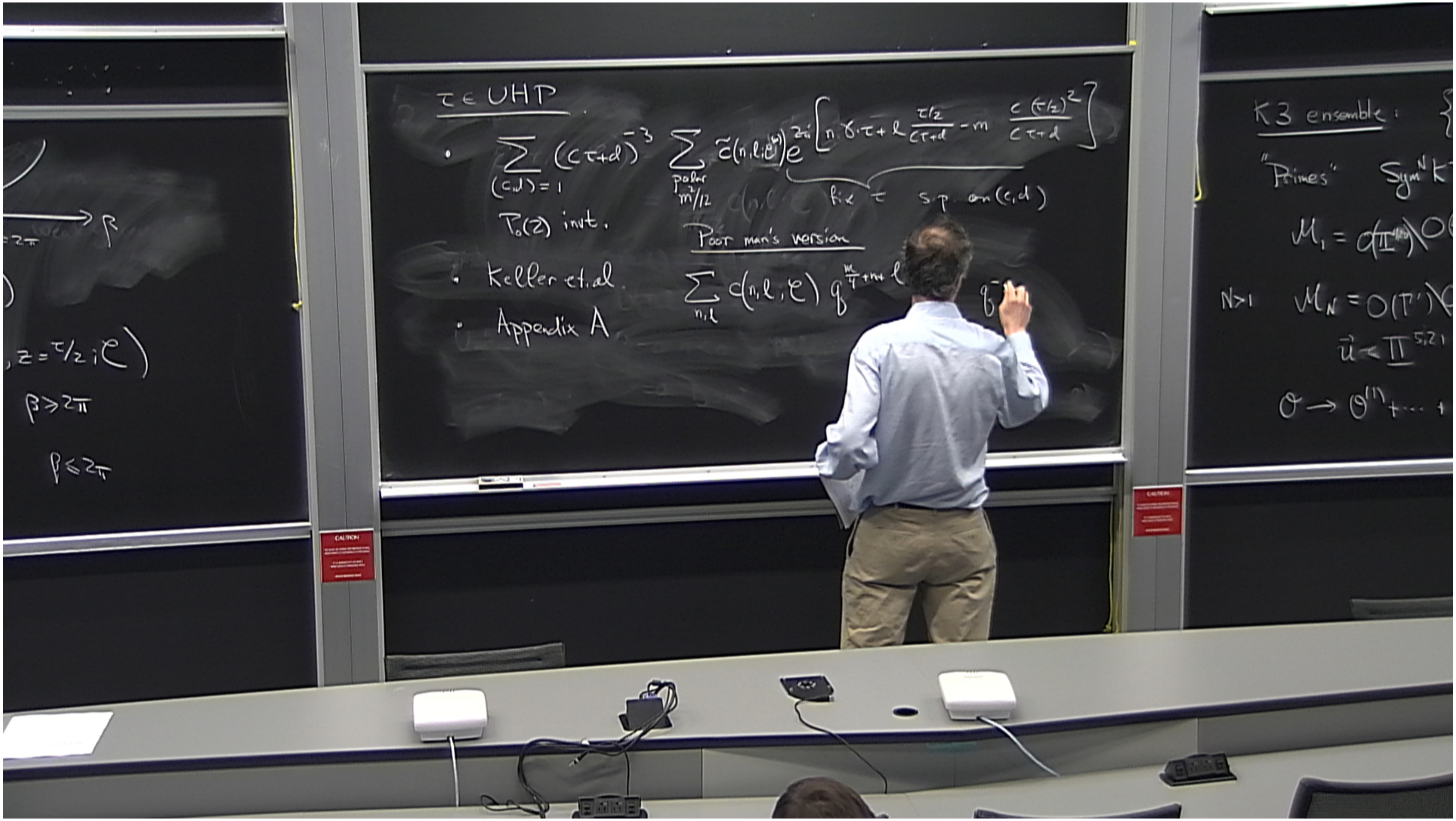

lim 1/m log Z(β; C^(m)) = F(β)
m→∞
log Z(β; C^(m)) ~ mF(β) + O(log m)
DMMV:
Z_NS,R(τ; C) = q^(m/4) E(τ, z = τ/2; C)
C^(m) = Sym^m K3
τ = iβ/2π
F(β) = β/4 β ≥ 2π
π²/β β ≤ 2π
β = 2π
τ ∈ UHP
Σ (cτ+d)^(-3)
(c,d) = 1
P_0(z) invt.
Keller et.al

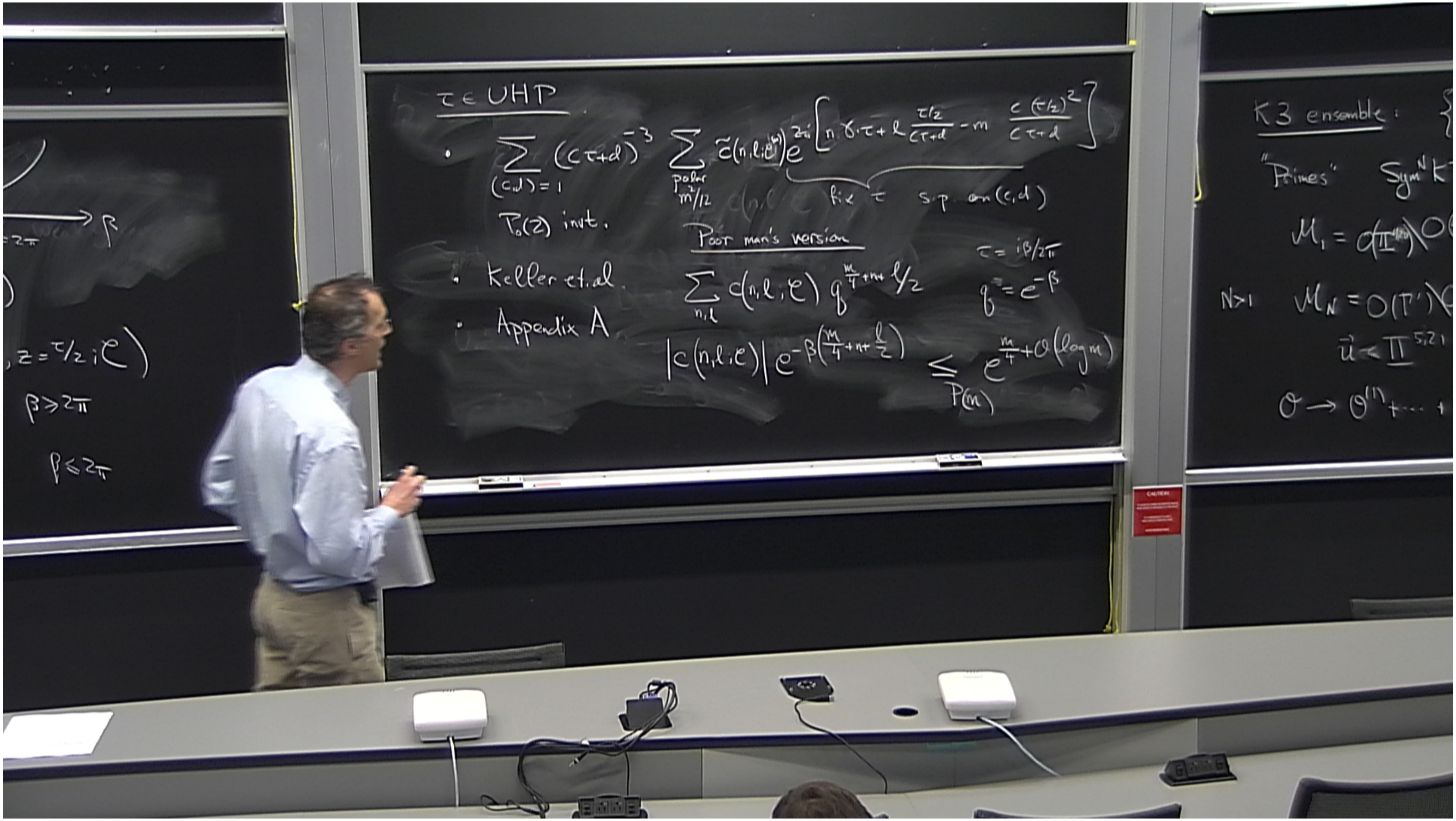


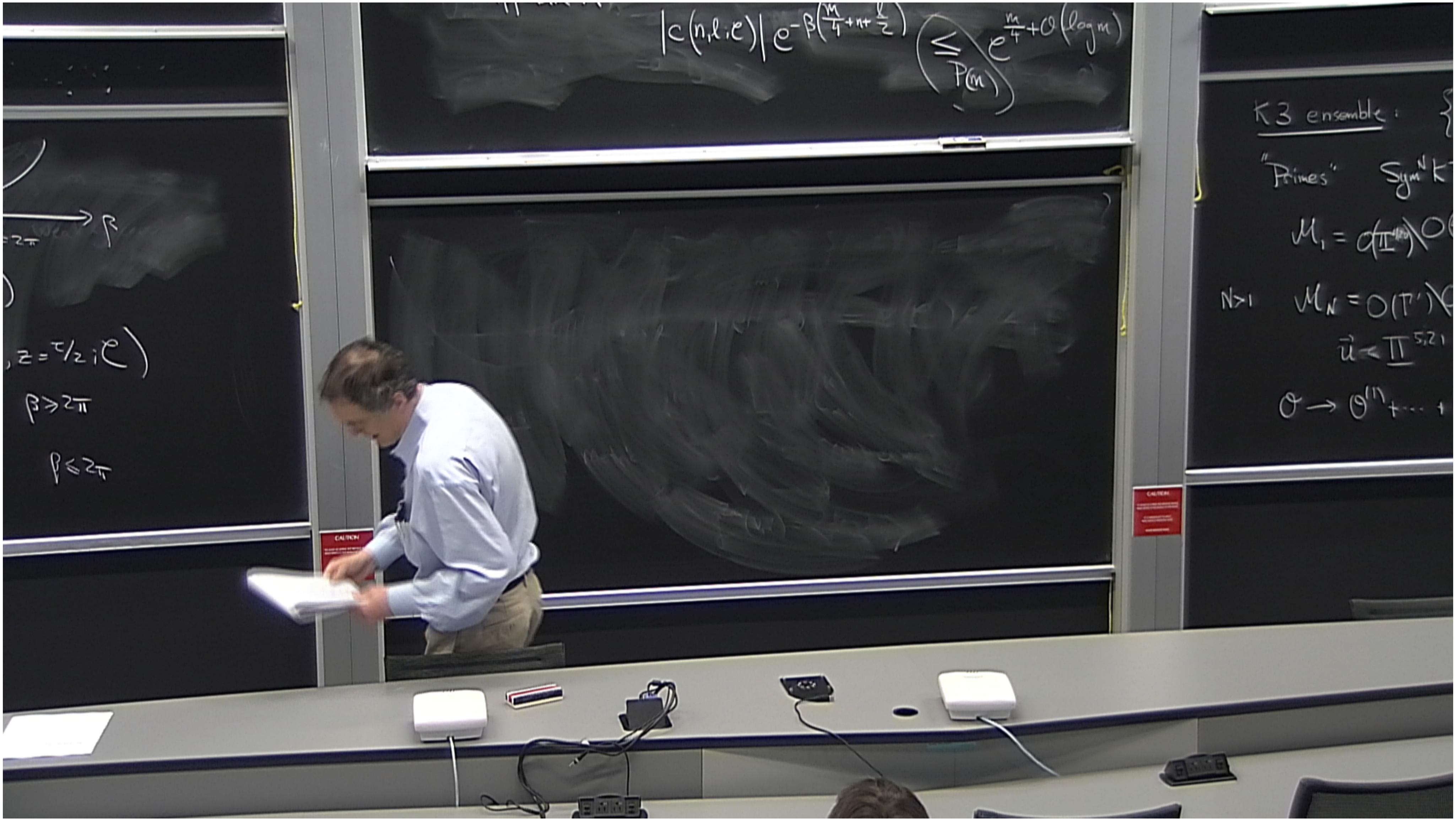

τ ∈ UHP
P₀(Z) invt.
Keller et.al.
ppendix A
fix τ
s.p. on (c,d)
K3 ensemble:
"Primes"
N>1
β≫2π
β≤2π

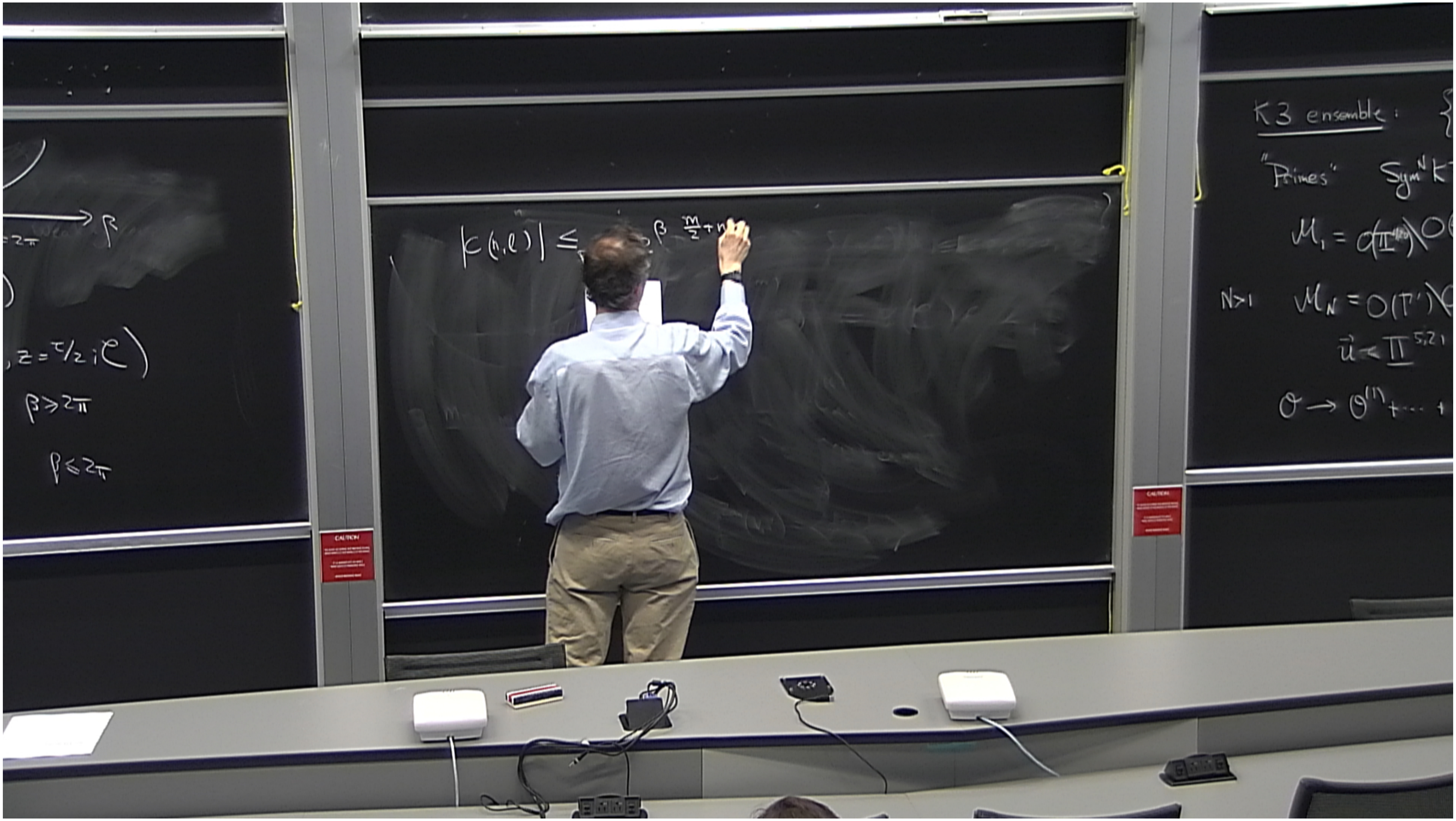

τ ∈ UHP
Γ₀(2) invt.
Keller et.al.
Appendix A
fix τ
s.p. on (c,d)
K3 ensemble :
"Primes"
N>1
β>>2π
β≤2π

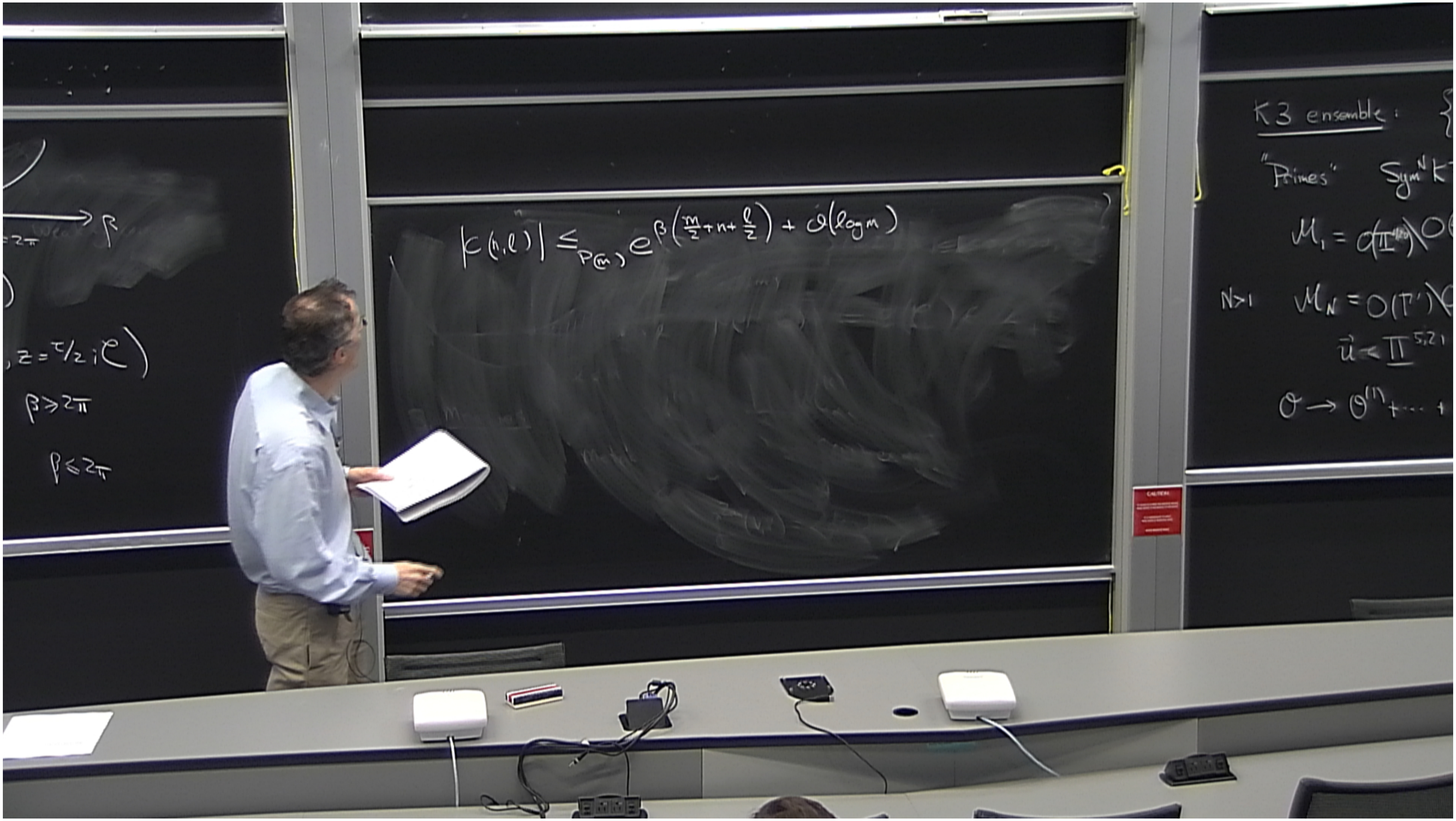

τ ∈ UHP
(c,d) = 1
polar
P₀(z) invt.
fix τ
s.p. on (c,d)
Poor man's version
Keller et. al.
Appendix A
β ≫ 2π
β ≤ 2π
K3 ensemble :
"Primes"
N>1

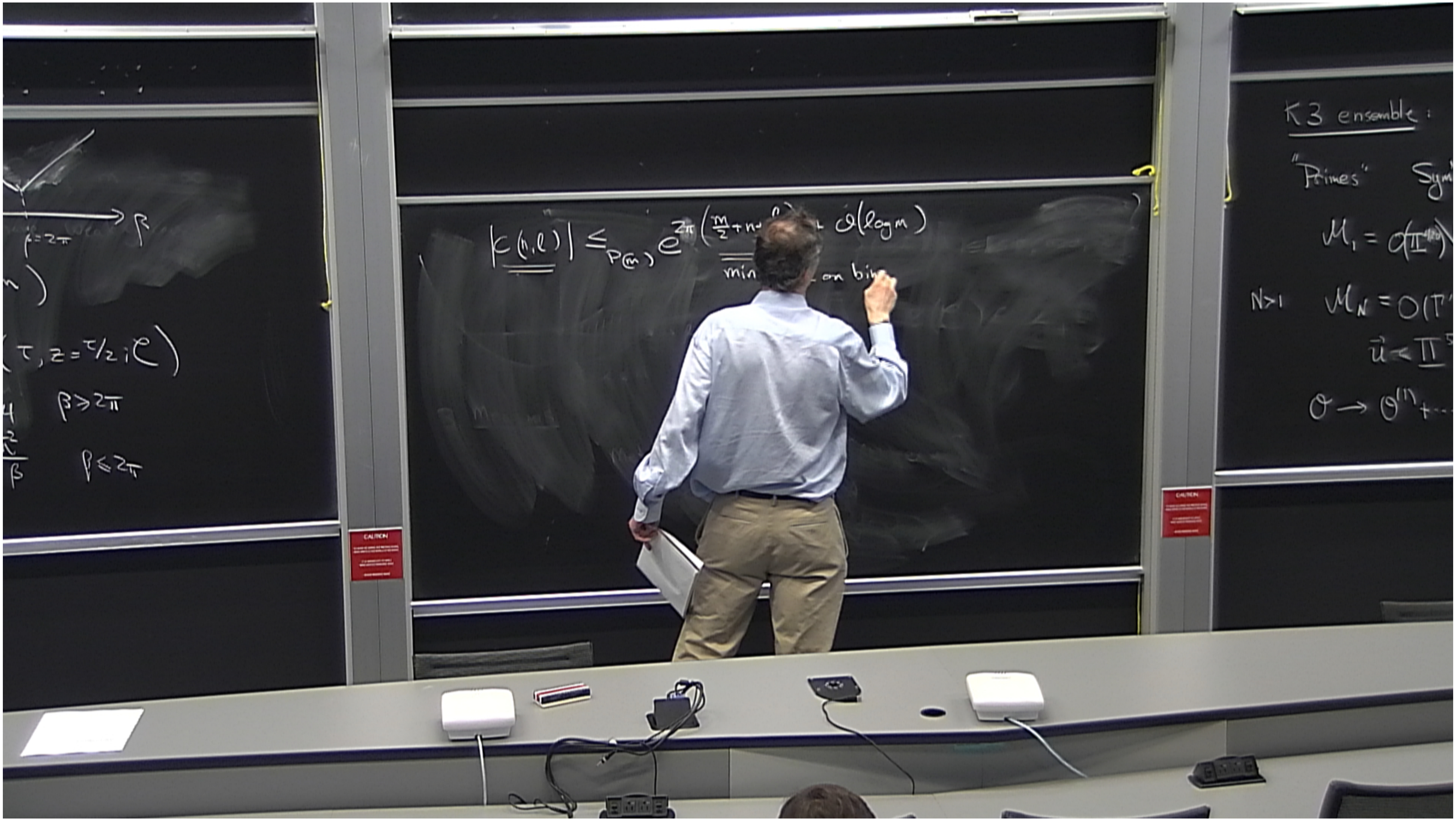

τ ∈ UHP
Poor man's version
Keller et.al
Appendix A
K3 ensemble
"Primes"

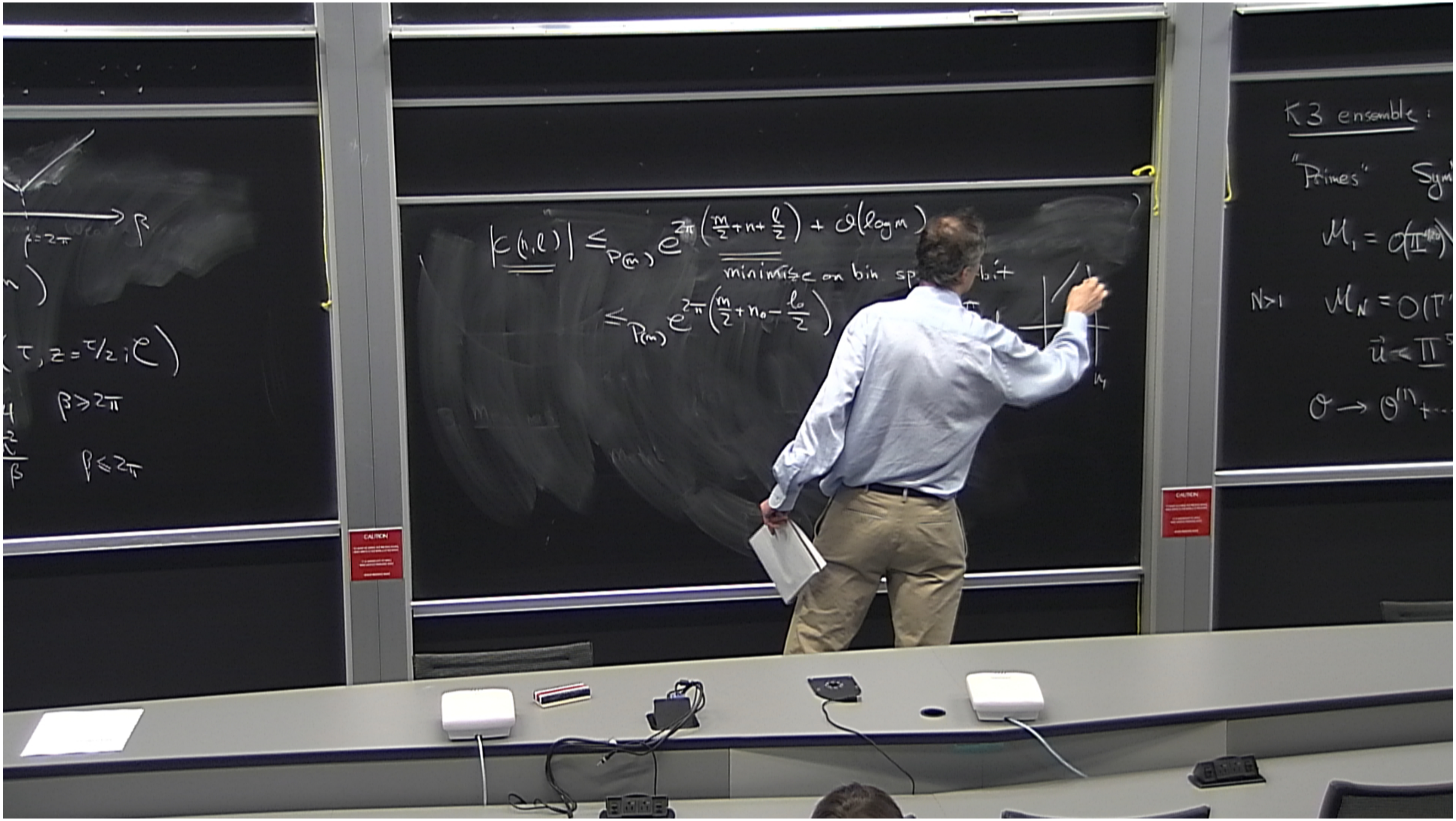

τ ∈ UHP
Poor man's version
Keller et.al
Appendix A
K3 ensemble
"Primes"
N>1

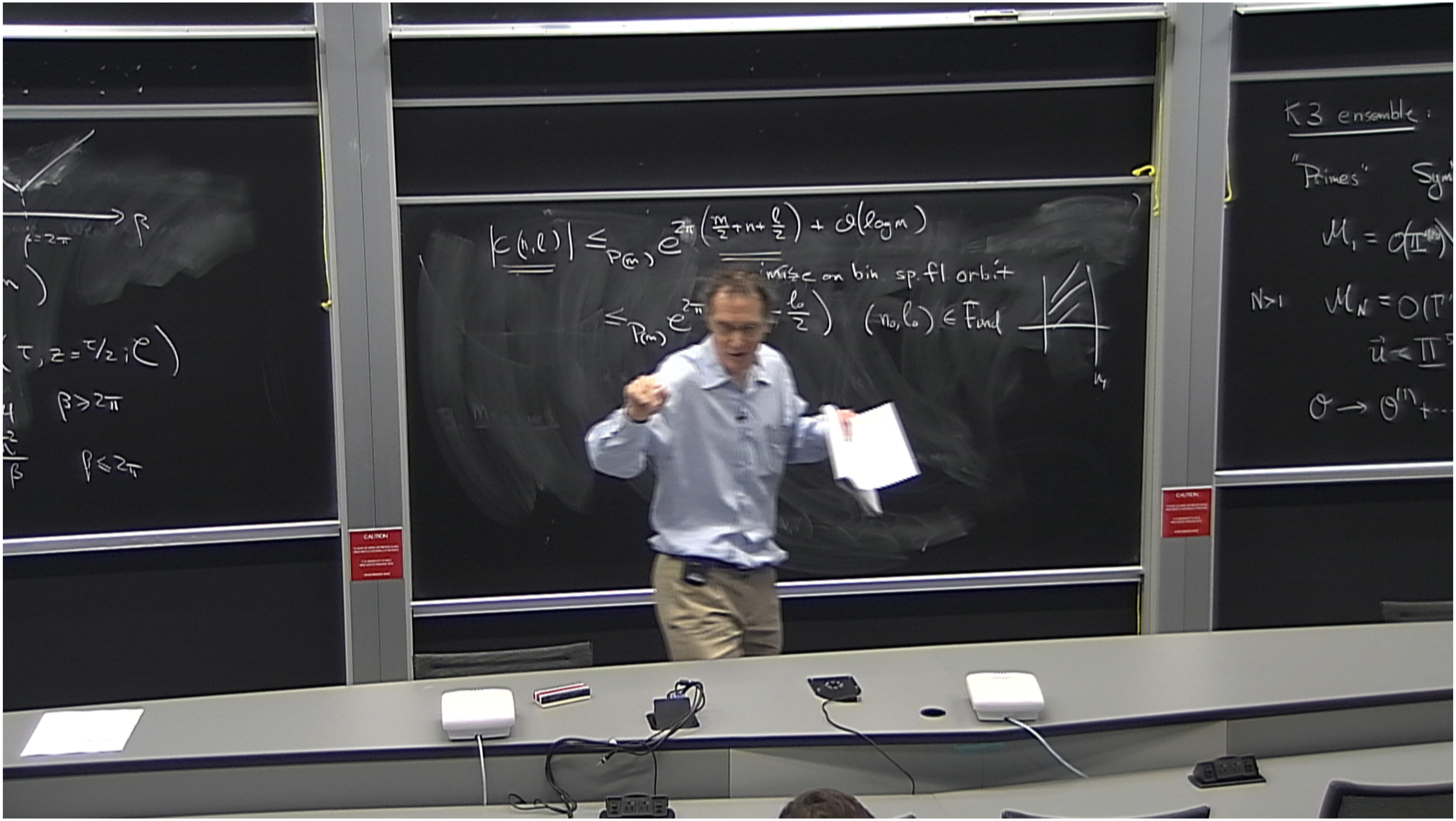

K3 ensemble :
"Primes"
N>1
β >> 2π
β ≤ 2π
CAUTION

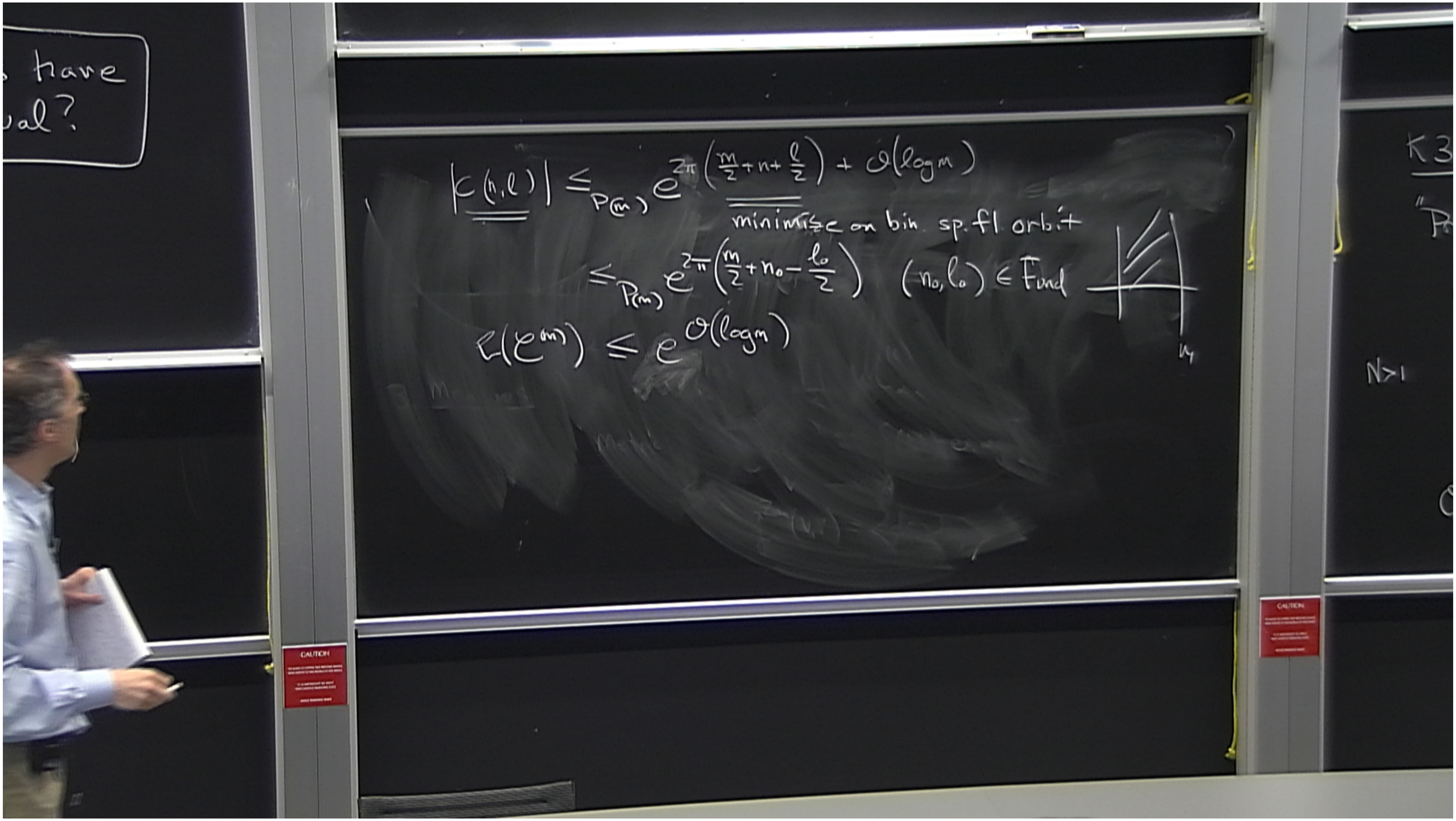

K3 ensemble :
"Primes"
N>1
β>>2π
β≤2π


|c(n,l)| ≤_{P(m)} e^{β(m/2+n+l/2)} + O(log m)
β>>2π
β≤2π
K3 ensemble:
"Primes"
N>1

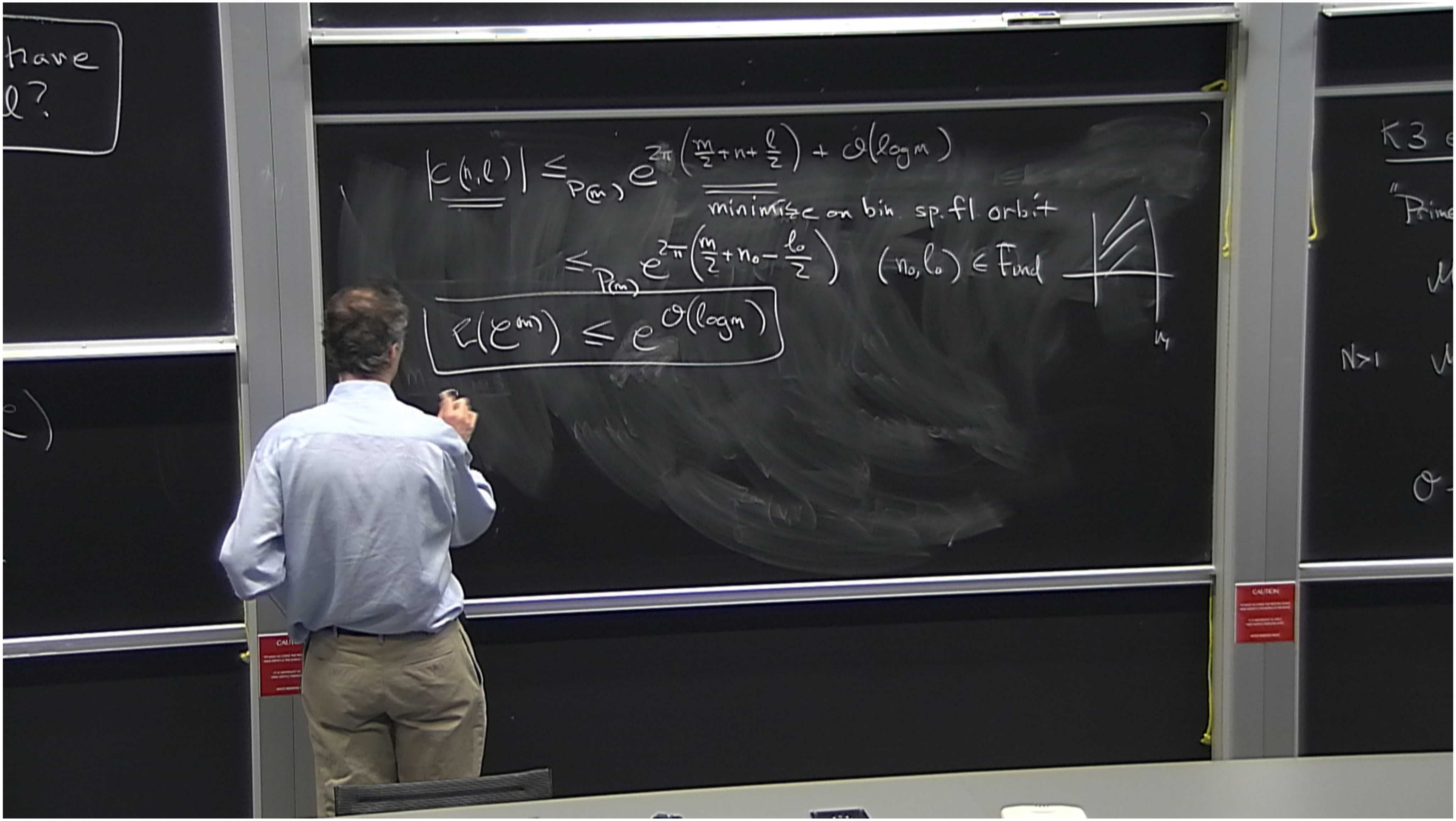


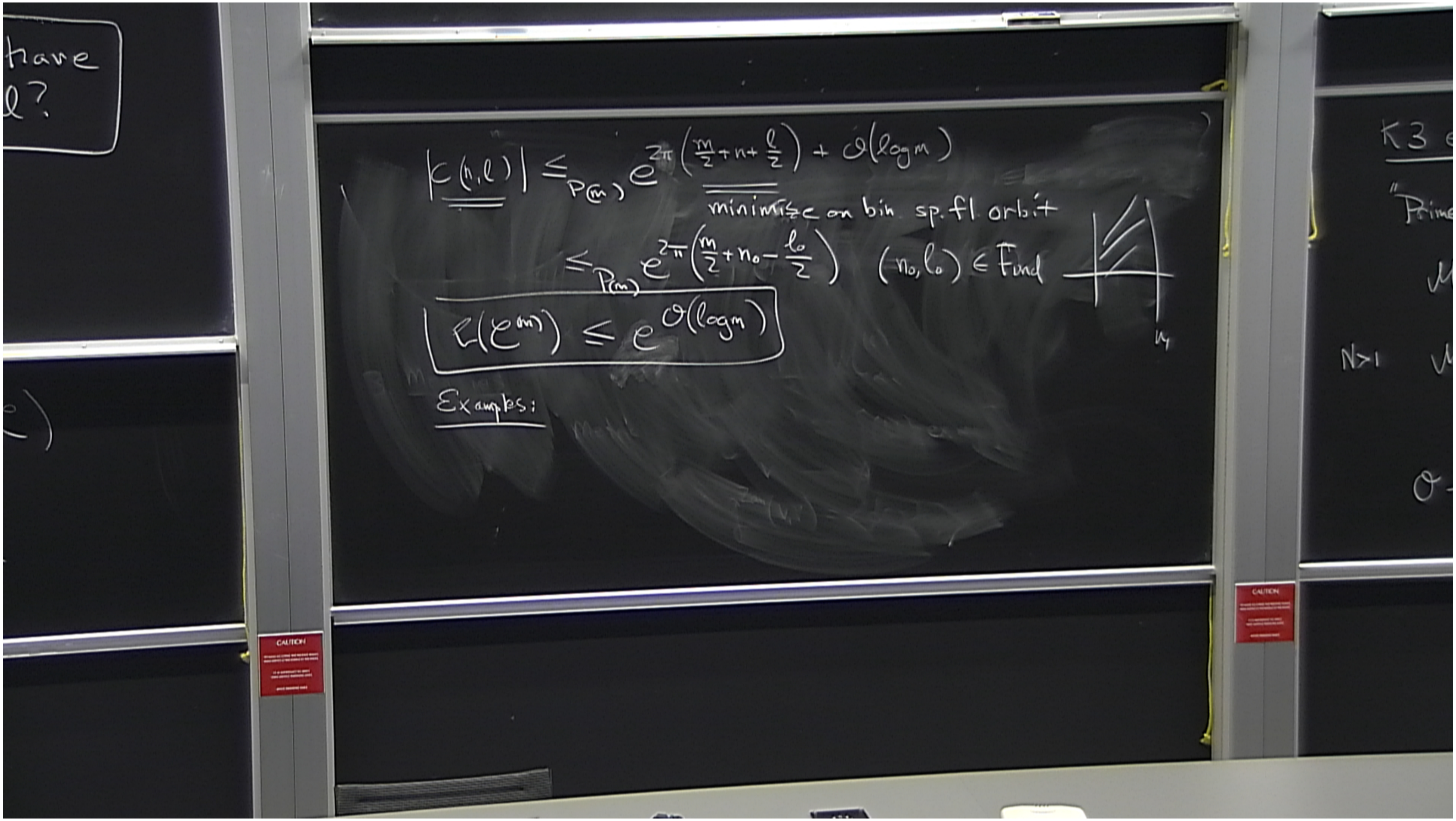

K3 ensemble :
"Primes"
minimise on bin sp
β ≥ 2π
β ≤ 2π

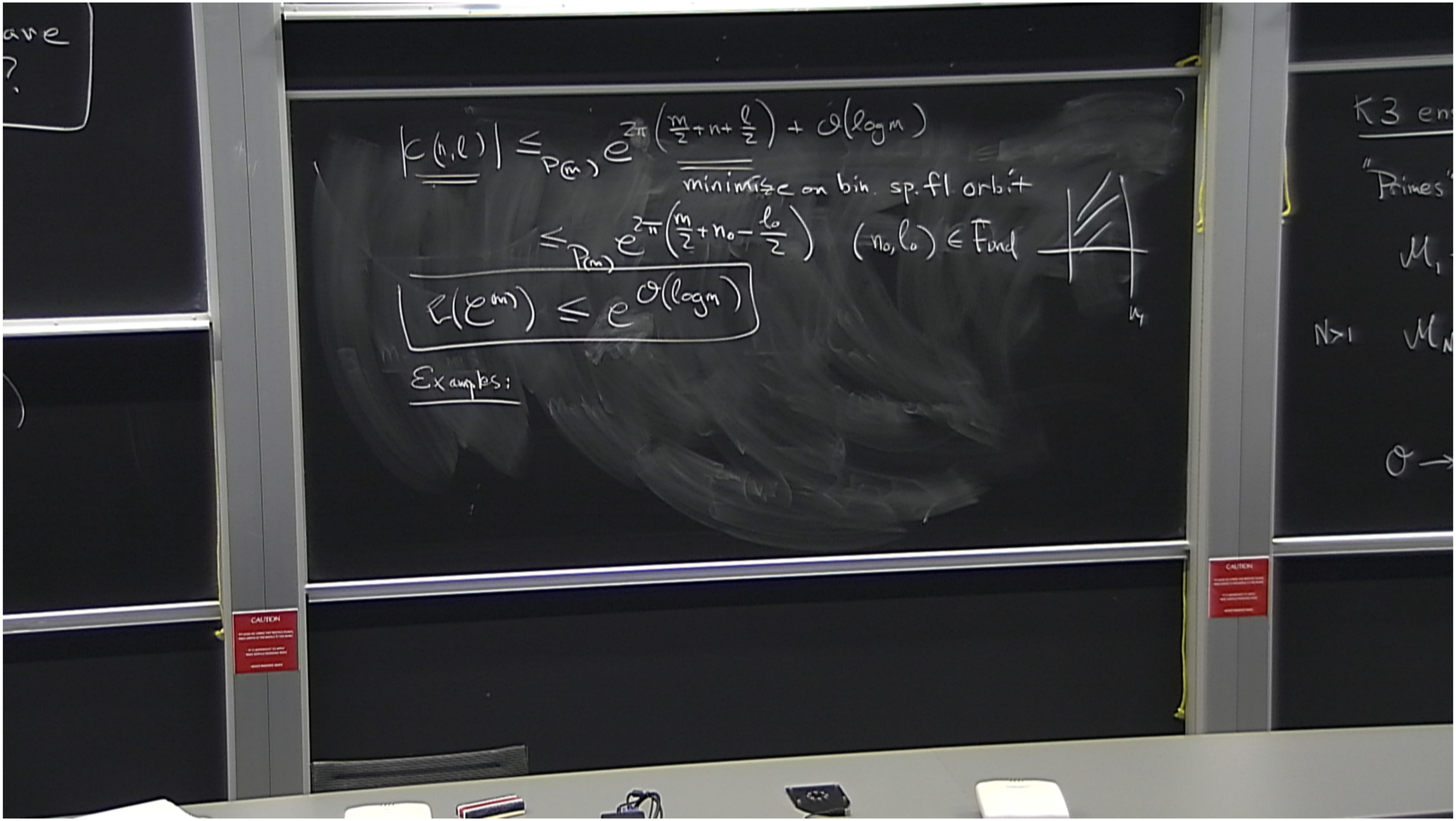

K3 ensemble :
"Primes"
(n₀, l₀) ∈ Fund
β > 2π
β ≤ 2π

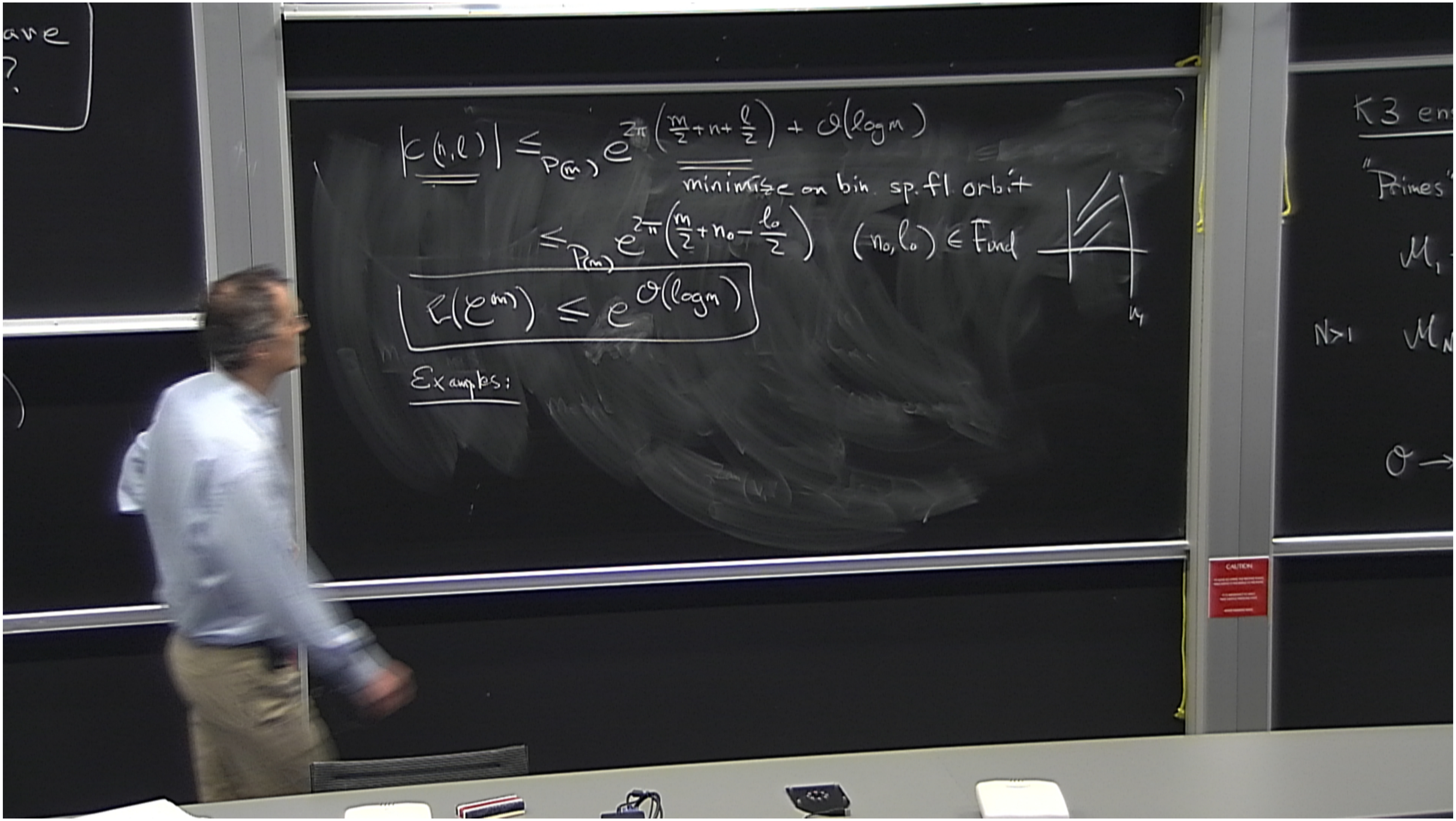


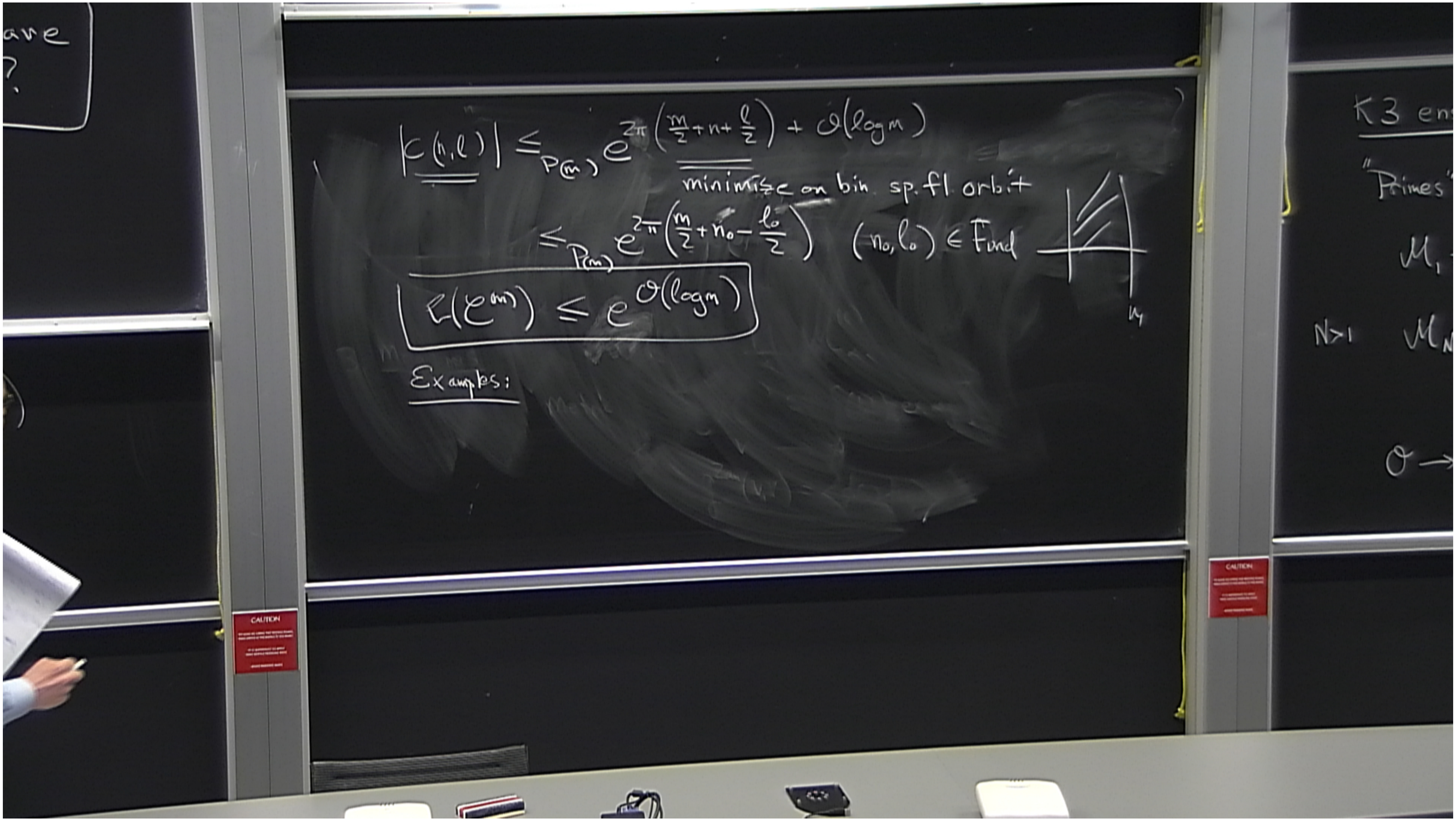
have
al?
|c(n,l)| ≤P(m) e^2π(m/2+n+l/2) + O(log m)
minimise on bin. sp. fl orbit
≤P(m) e^2π(m/2+n0−l0/2) (n0,l0) ∈ Fund
ℓ(C(m)) ≤ e^O(log m)
K3
N>1
CAUTION
CAUTION

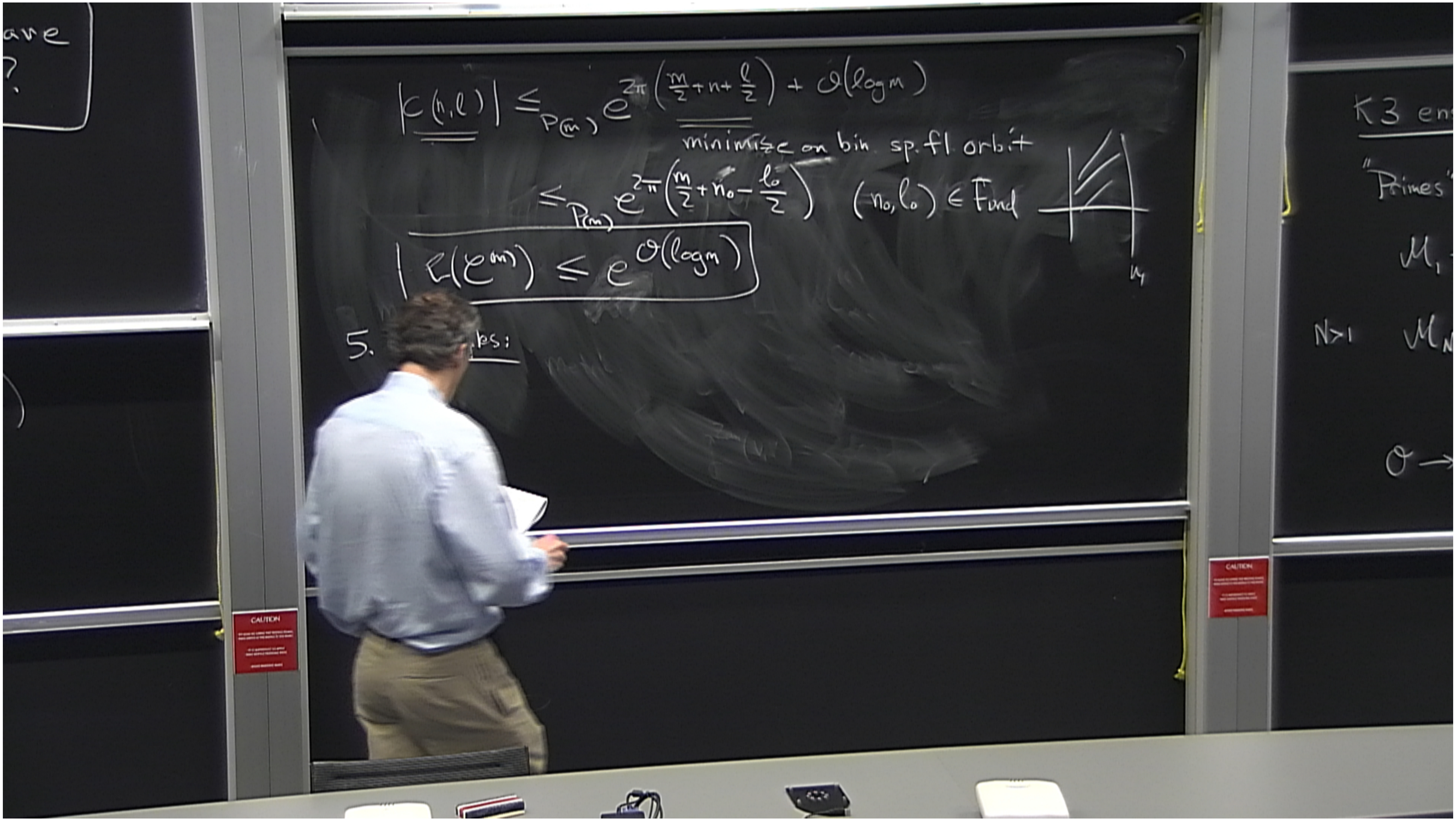

|c(n,l)| ≤P(m) e^2π(m/2 + n + l/2) + O(log m)
minimise on bin. sp. fl. orbit
≤P(m) e^2π(m/2 + n0 − l0/2)
(n0, l0) ∈ Fund
L(ℓ(m)) ≤ e^O(log m)
K3
N>1

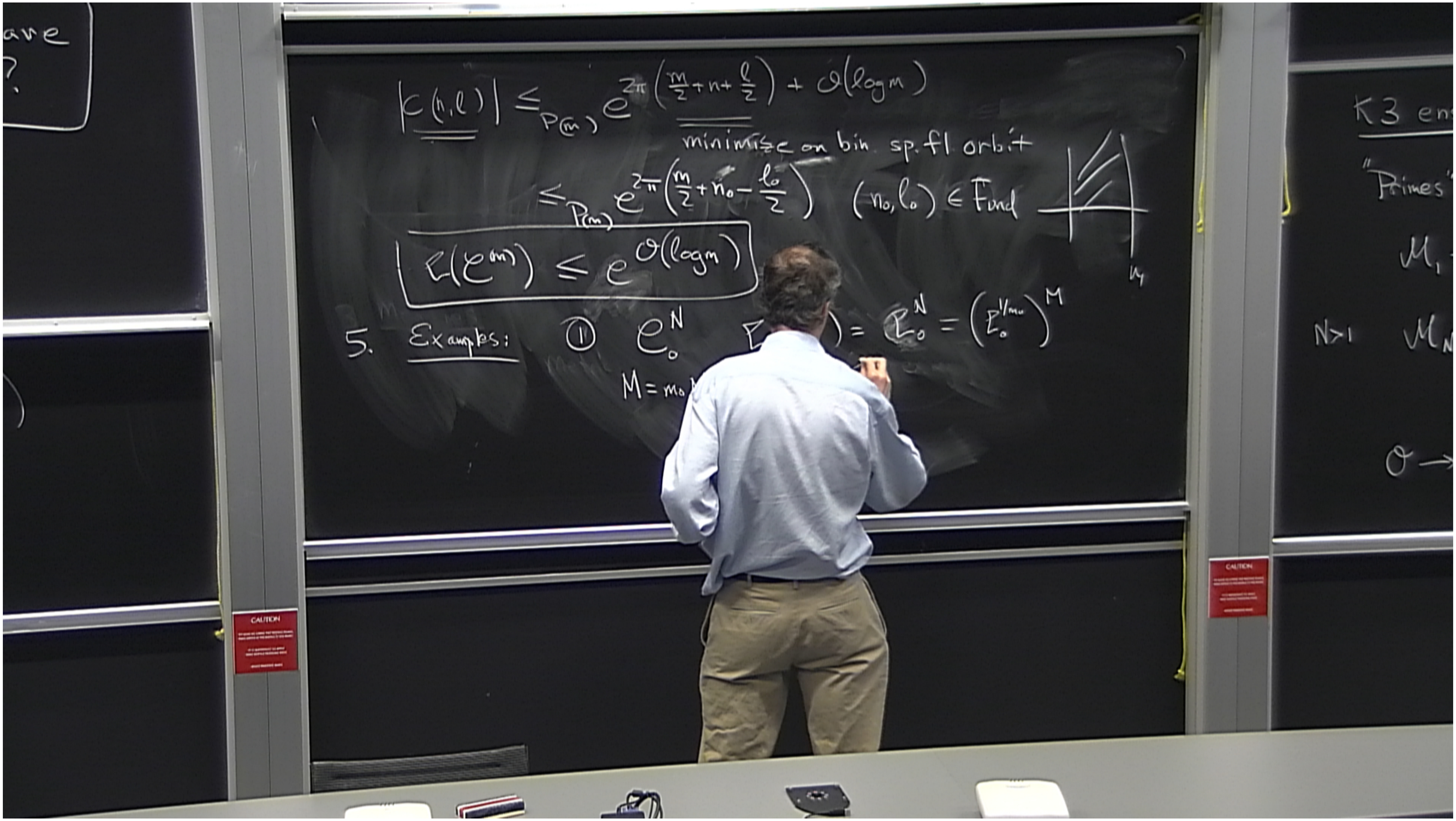

have
l?
$|c(n,\ell)| \leq_{P(m)} e^{2\pi\left(\frac{m}{2}+n+\frac{\ell}{2}\right) + O(\log m)}$
minimise on bin. sp. fl. orbit
$\leq_{P(m)} e^{2\pi\left(\frac{m}{2}+n_0-\frac{\ell_0}{2}\right)}$ $(n_0,\ell_0) \in$ Fund
$c(\varphi^{(m)}) \leq e^{O(\log m)}$
Examples:
K3
"Prime
N>1

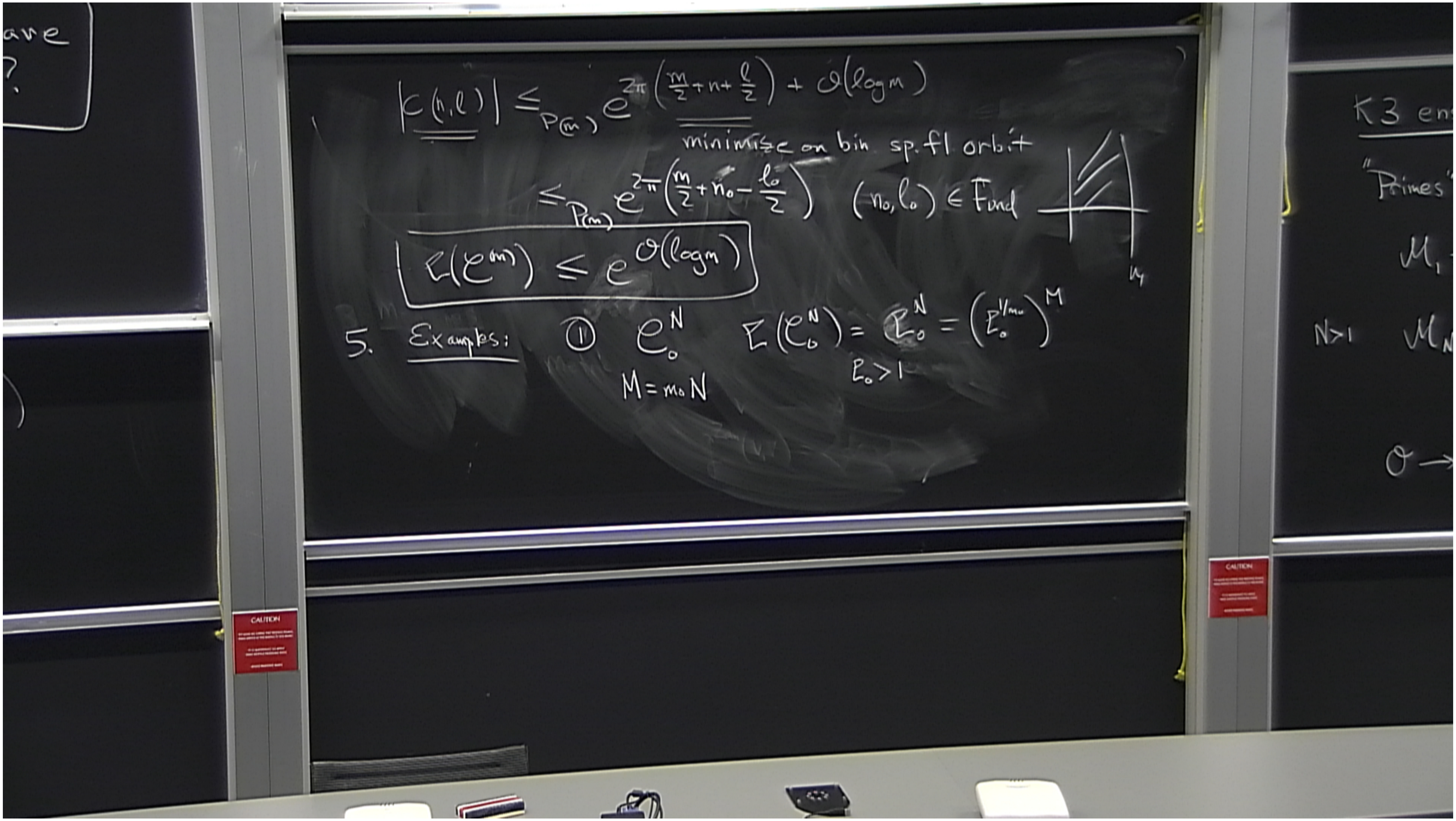

are
?
|c(n,ℓ)| ≤_{P(m)} e^{2π(m/2 + n + ℓ/2) + O(log m)}
minimise on bin. sp. fl. orbit
≤_{P(m)} e^{2π(m/2 + n₀ − ℓ₀/2)} (n₀, ℓ₀) ∈ Fund
ℓ(C^{(m)}) ≤ e^{O(log m)}
Examples:
K3 en
"Primes"
N>1
CAUTION

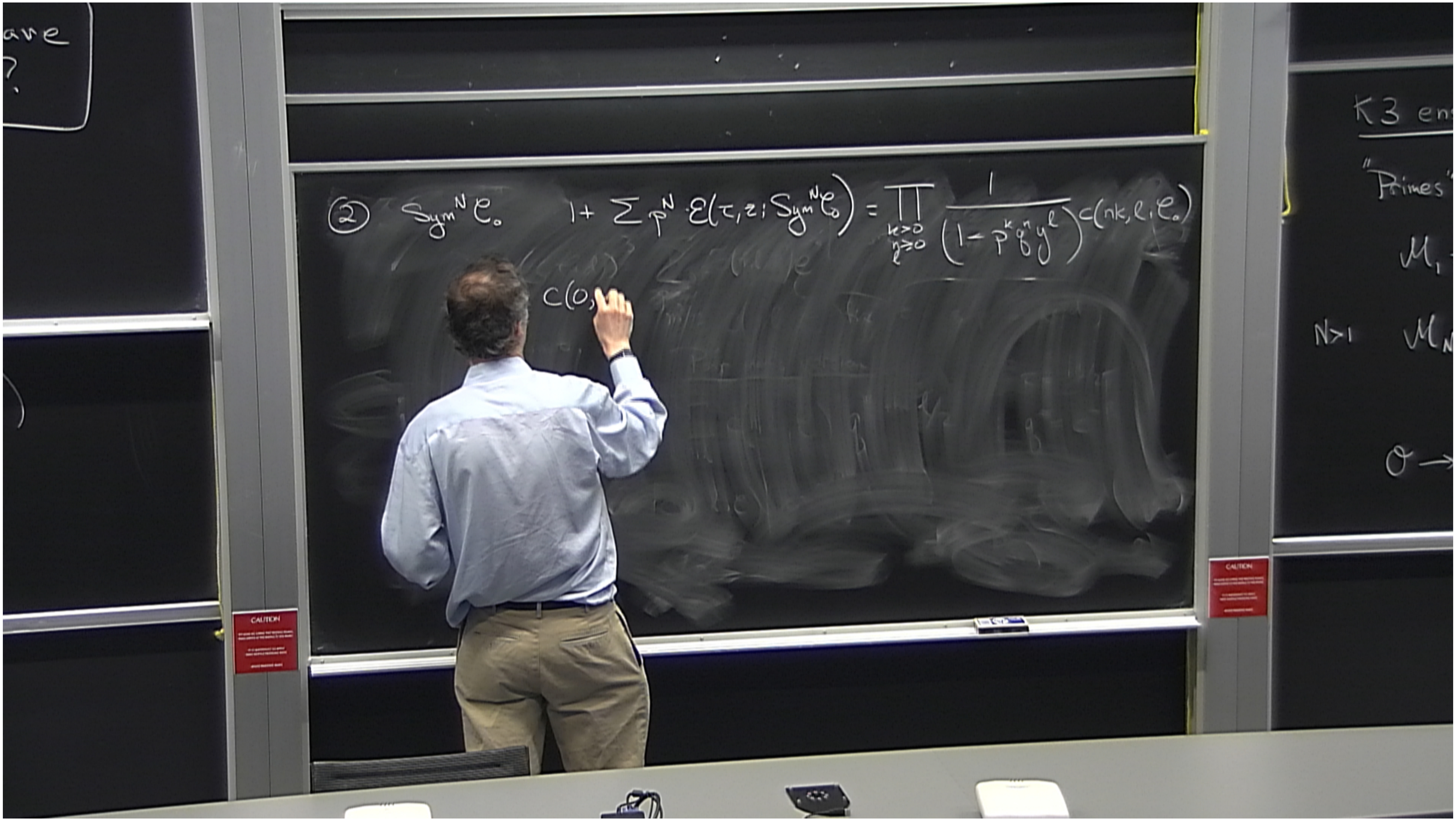

$|c(n,\ell)| \leq_{P(m)} e^{2\pi\left(\frac{m}{2}+n+\frac{\ell}{2}\right)+\mathcal{O}(\log m)}$
minimise on bin. sp. fl. orbit
$\leq_{P(m)} e^{2\pi\left(\frac{m}{2}+n_0-\frac{\ell_0}{2}\right)}$ $(n_0,\ell_0) \in$ Fund
$c(\mathcal{Z}^{(m)}) \leq e^{\mathcal{O}(\log m)}$
Examples:
K3 en
"Primes"
N>1

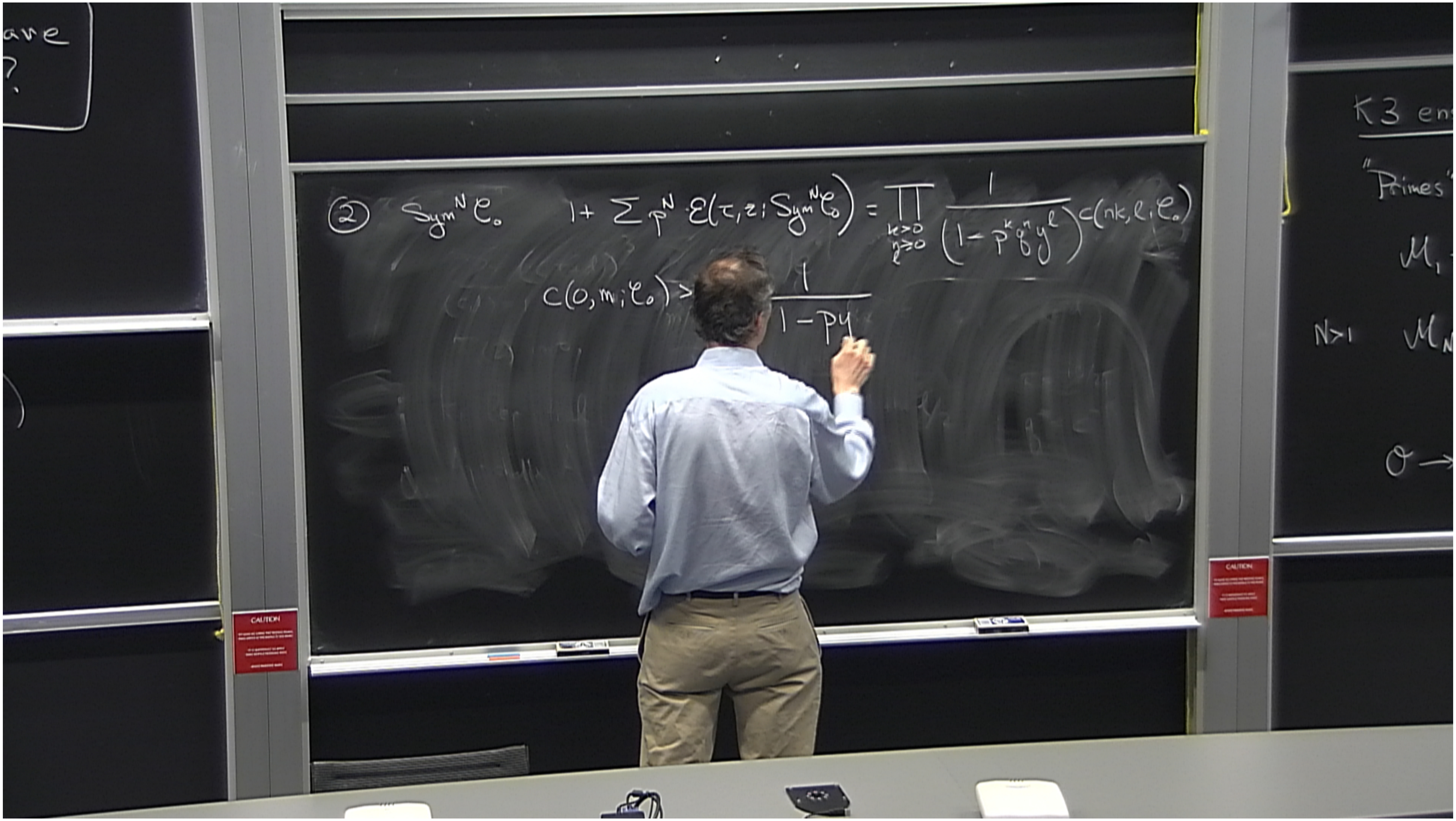

$|c(n,\ell)| \leq_{P(m)} e^{2\pi\left(\frac{m}{2}+n+\frac{\ell}{2}\right) + O(\log m)}$
minimise on bin. sp. fl. orbit
$\leq_{P(m)} e^{2\pi\left(\frac{m}{2}+n_0-\frac{\ell_0}{2}\right)}$ $(n_0,\ell_0) \in$ Fund
$\ell(C^{(m)}) \leq e^{O(\log m)}$
Examples:
K3
"Primes"
$N>1$

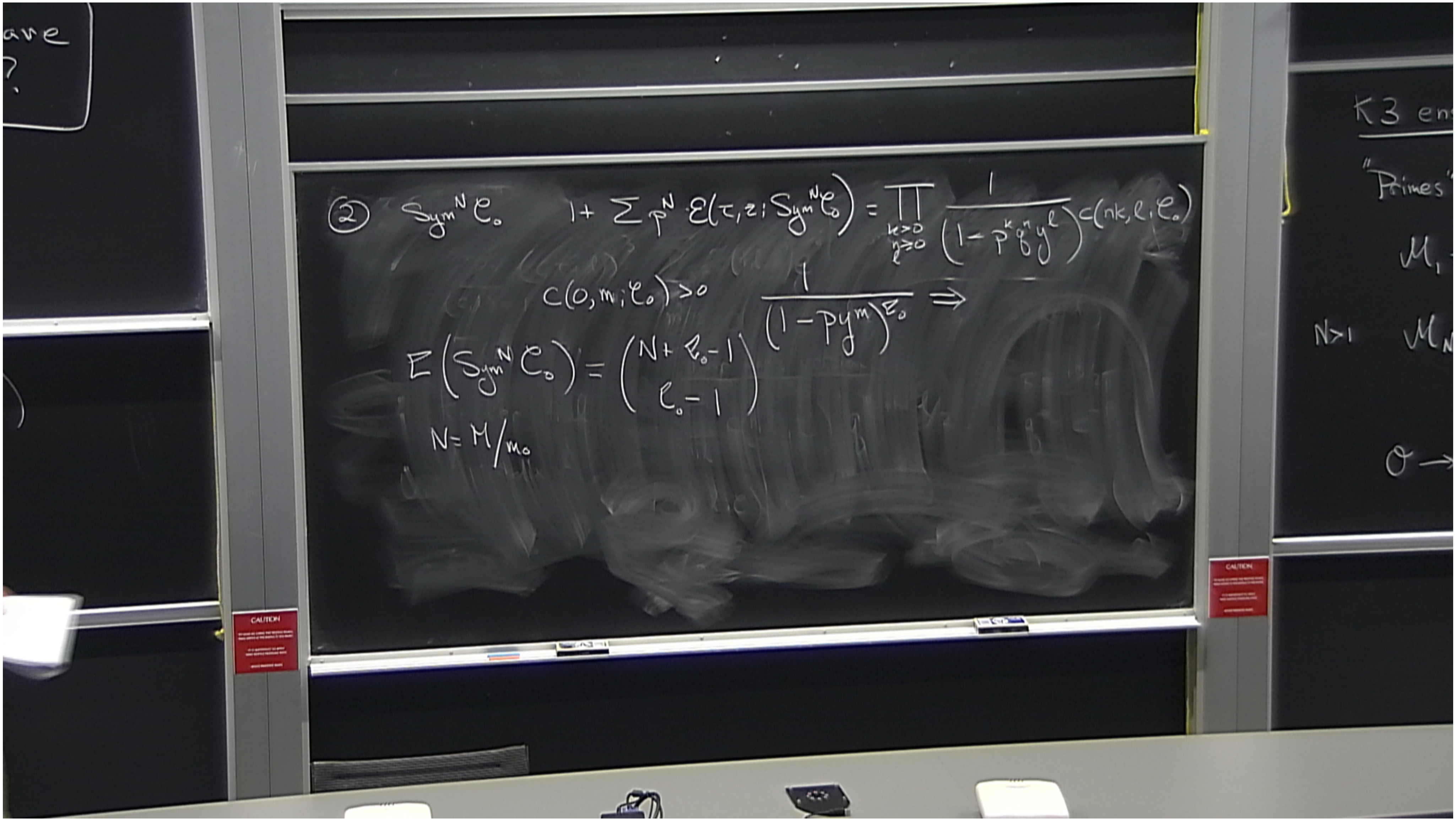

|c(n,l)| ≤_{P(m)} e^{2π(m/2+n+l/2)} + O(log m)
minimize on bin sp. fl orbit
≤_{P(m)} e^{2π(m/2+n_0−l_0/2)}   (n_0, l_0) ∈ Fund
c(χ^m) ≤ e^{O(log m)}
5.
K3 en
"Primes"
N>1


minimize on bin. sp. fl orbit
5. Examples:
"Primes"
N>1

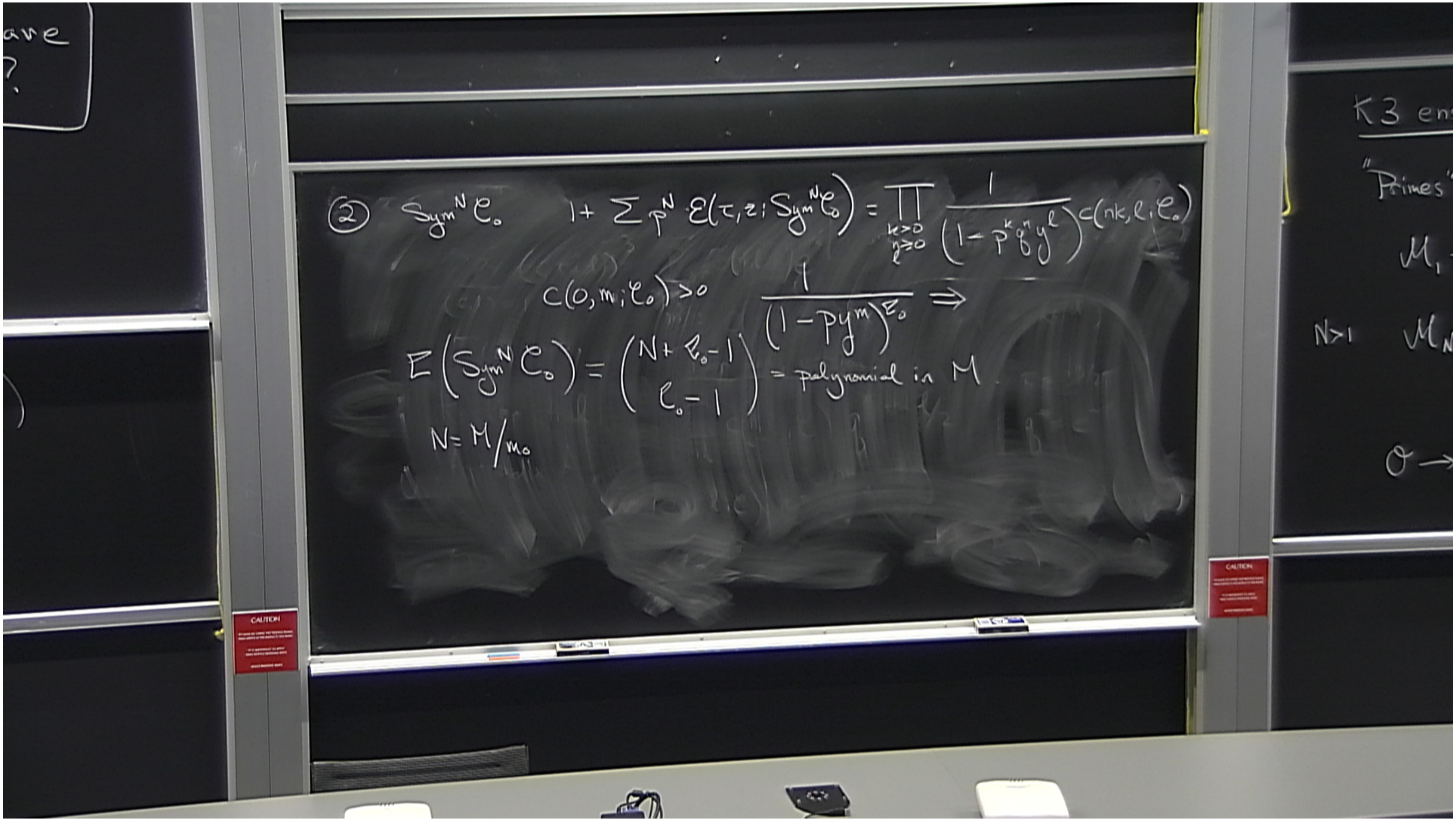

minimise on bin sp.fl orbit
$(n_0, l_0) \in$ Fund
5. Examples:
$M = m_0 N$
$E_0 > 1$
K3 en
"Primes"
N>1

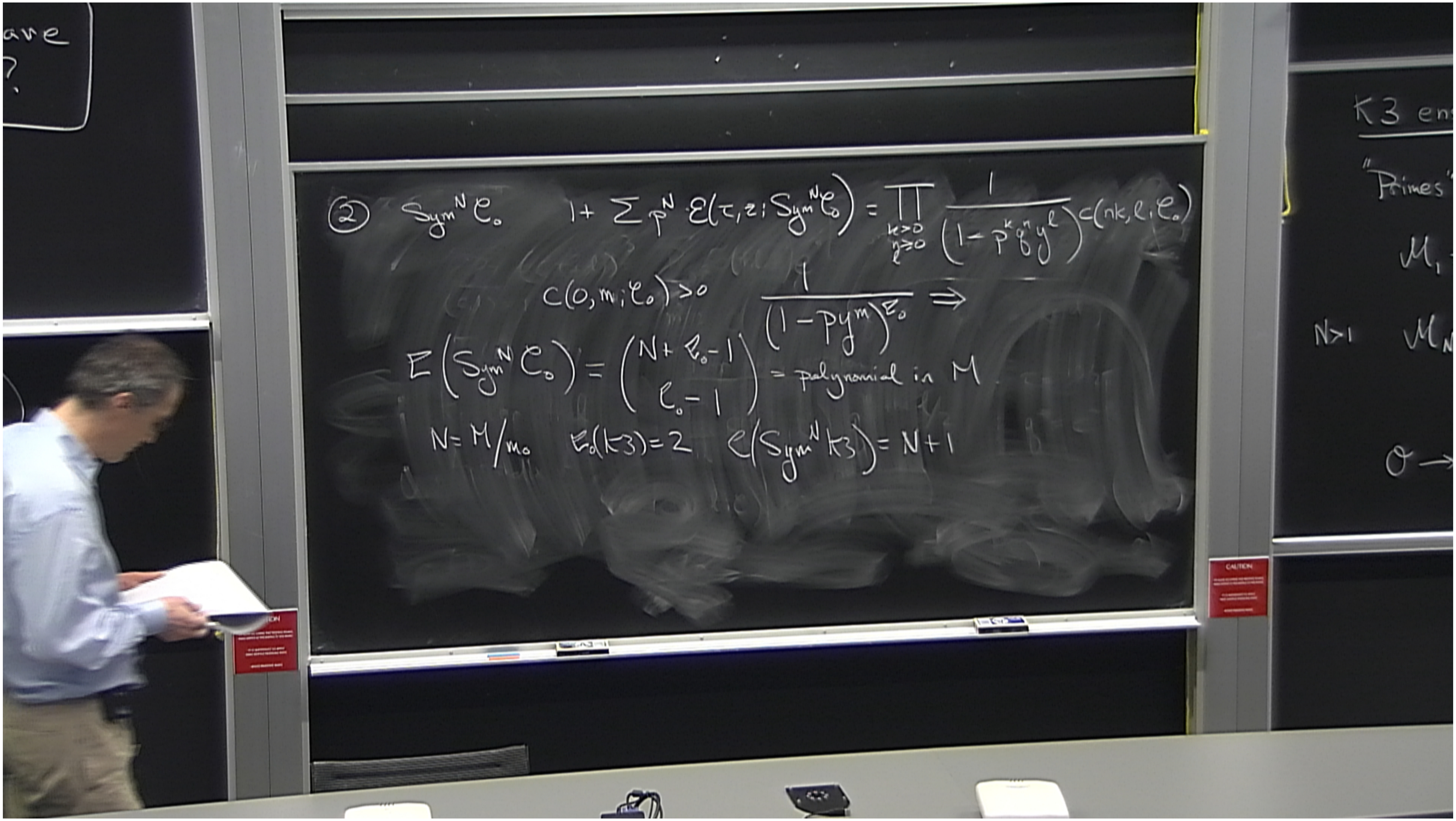

(2) SymN C0
1 + Σ pN E(τ, z; SymN C0) = Π 1 / (1 − pk qn yl)c(nk, l, C0)
k>0
n≥0
c(0,
K3 en
"Primes"
N>1

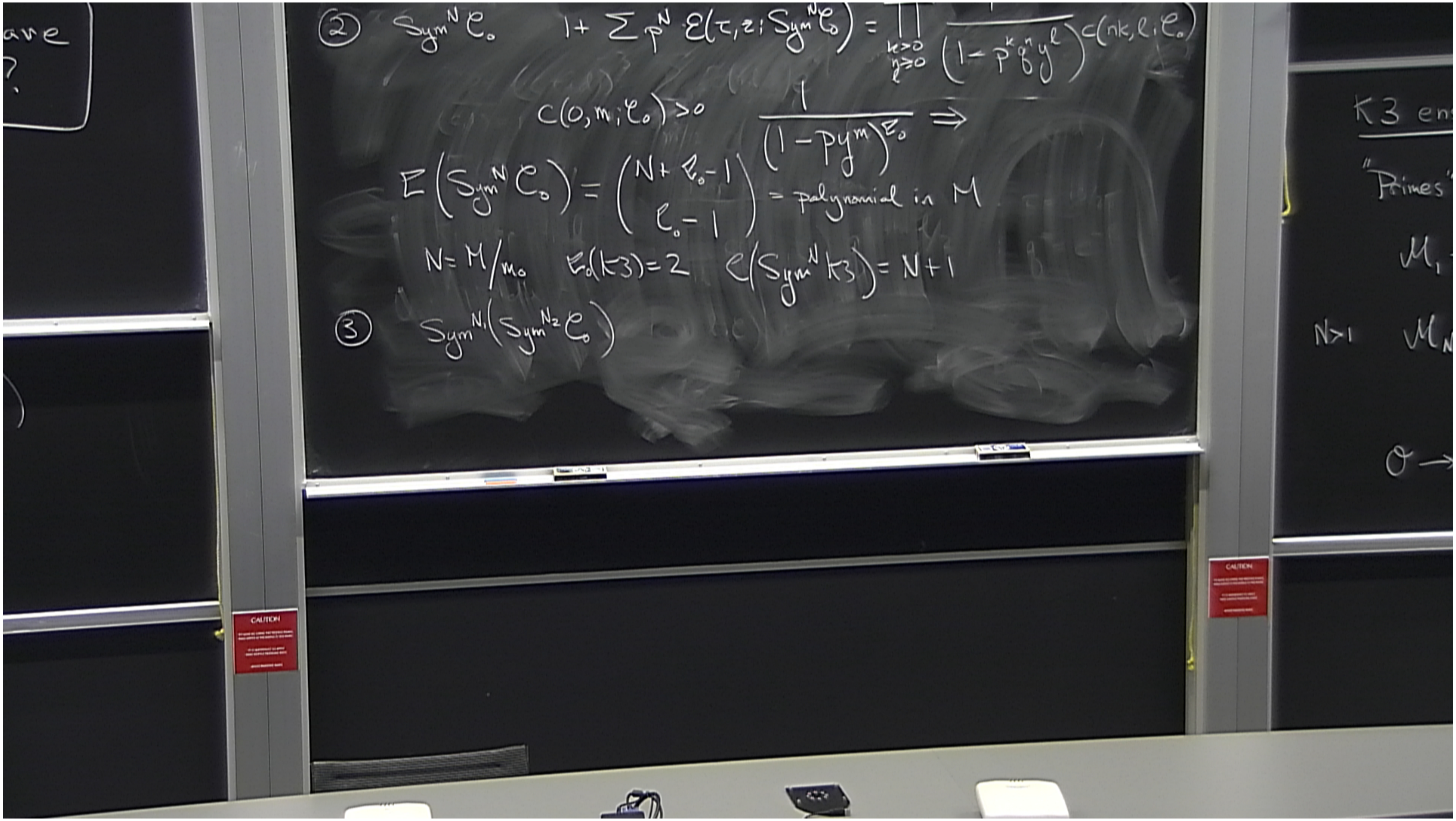

② Sym^N C₀   1 + Σ p^N ℰ(τ, z; Sym^N C₀) = ∏ 1 / (1 − p^k q^n y^ℓ)^c(nk, ℓ; C₀)
k>0
n≥0
ℓ
c(0, m; C₀) >
1 / (1 − py
K3 en
"Primes"
N>1

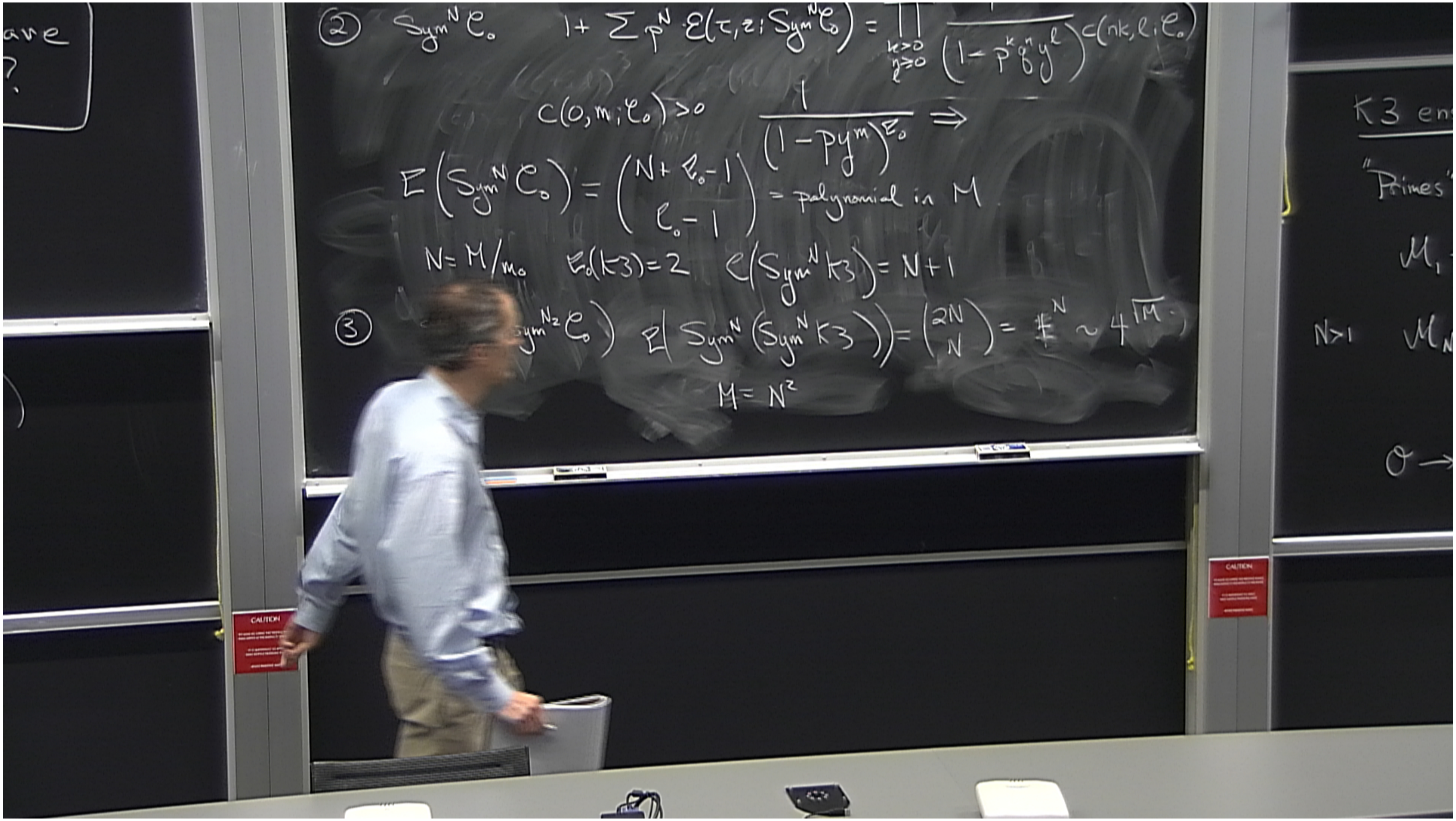


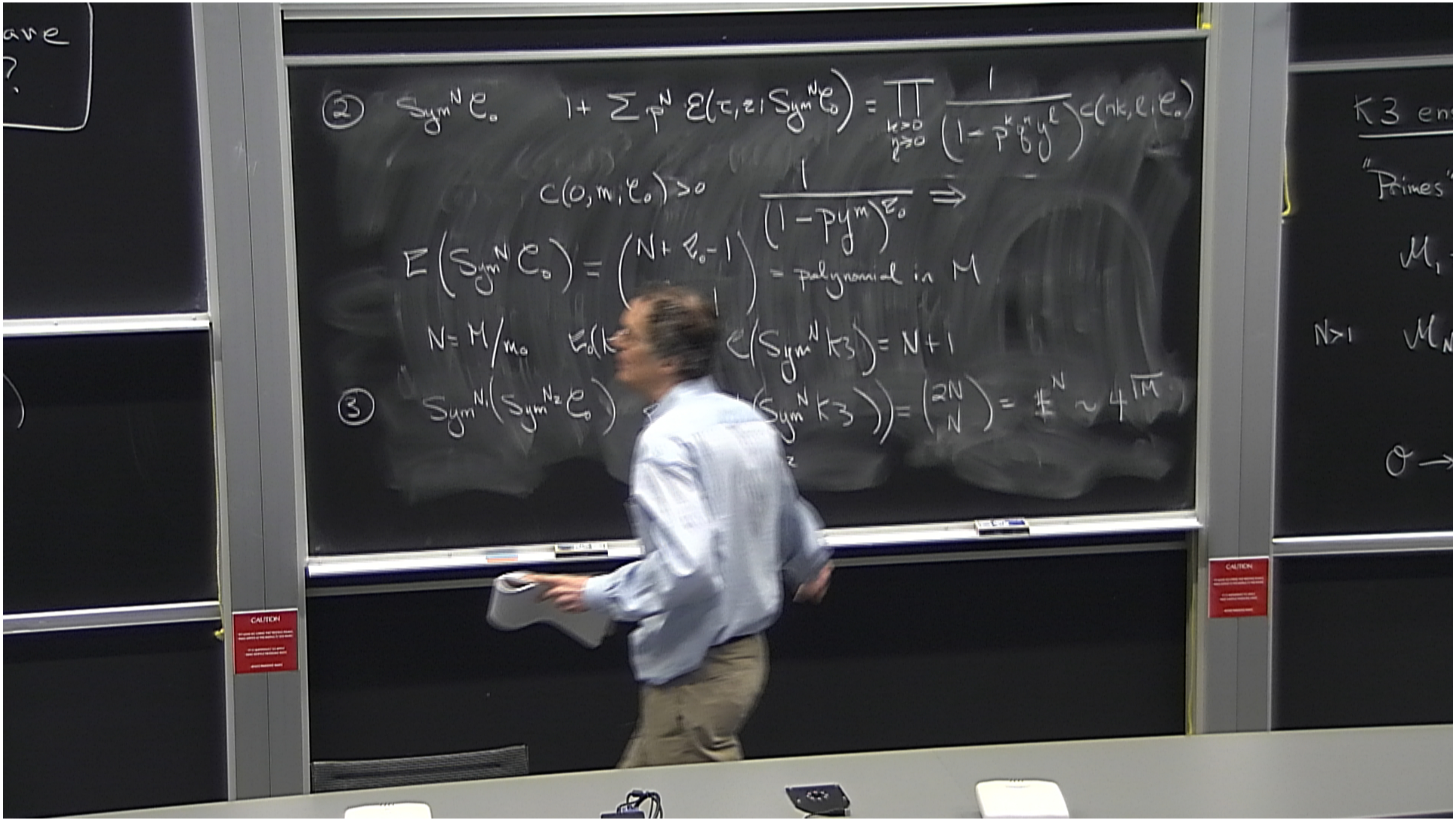
② Sym^N 𝓔₀   1 + Σ p^N 𝓔(τ, z; Sym^N 𝓔₀) = ∏_{k>0, n≥0, ℓ} 1/(1 − p^k q^n y^ℓ)^{c(nk, ℓ, 𝓔₀)}
c(0, m; 𝓔₀) > 0   1/(1 − p y^m)^{𝓔₀} ⇒
𝓔(Sym^N 𝓔₀) = (N + 𝓔₀ − 1 choose 𝓔₀ − 1)
N = M/m₀
K3 en
"Primes"
N>1
CAUTION

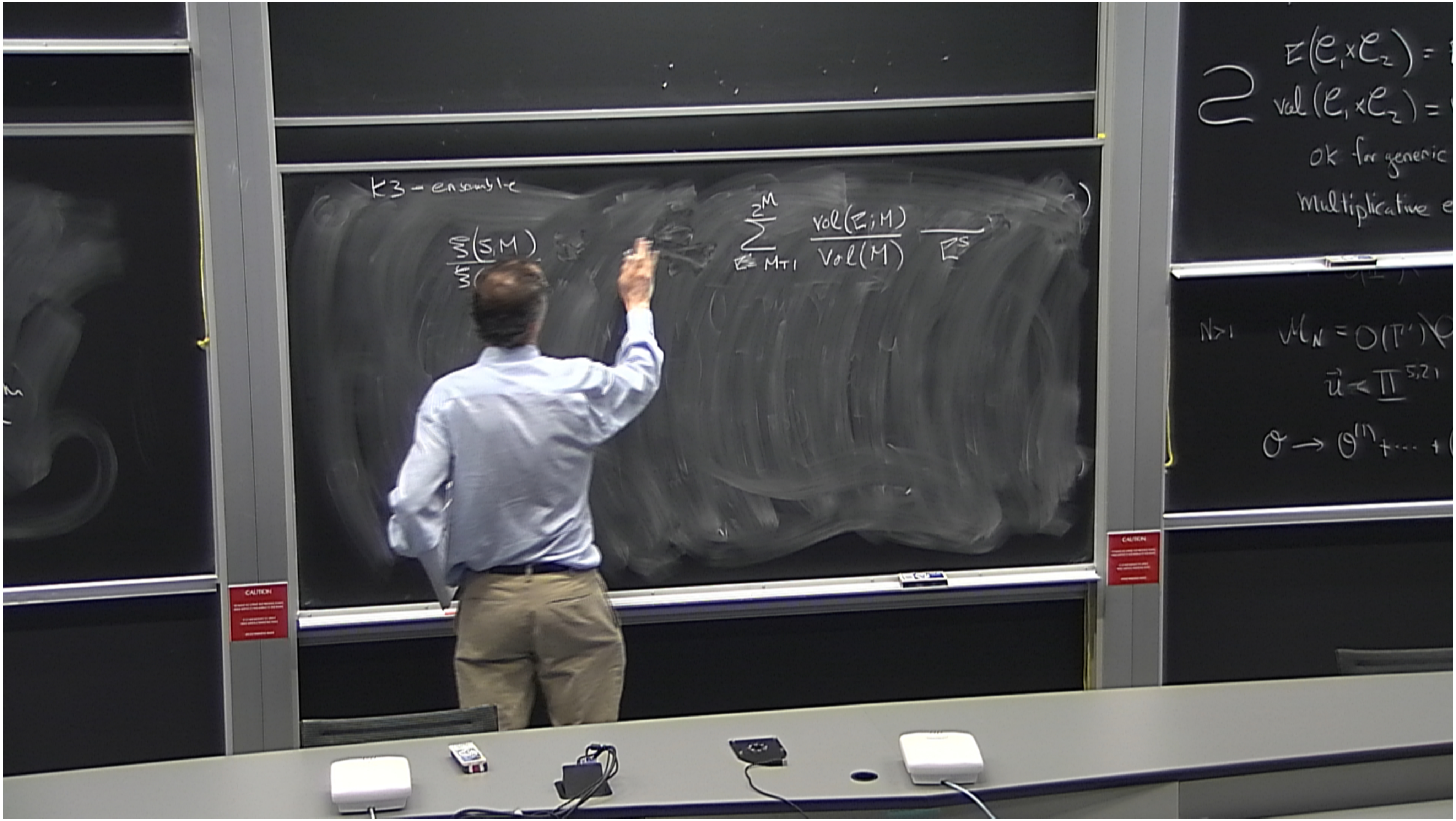

(2) $\mathrm{Sym}^N \mathcal{C}_0$
$1 + \sum p^N \mathcal{E}(\tau, z; \mathrm{Sym}^N \mathcal{C}_0) = \prod_{k>0, n\geq 0, \ell} \frac{1}{(1 - p^k q^n y^\ell)^{c(nk,\ell;\mathcal{C}_0)}}$
$c(0, m; \mathcal{C}_0) > 0$ $\frac{1}{(1 - p y^m)^{\mathcal{C}_0}} \Rightarrow$
$\mathcal{E}(\mathrm{Sym}^N \mathcal{C}_0) = \binom{N + \mathcal{C}_0 - 1}{\mathcal{C}_0 - 1}$ = polynomial in M
$N = M / m_0$
K3 ens
"Primes"
$N > 1$

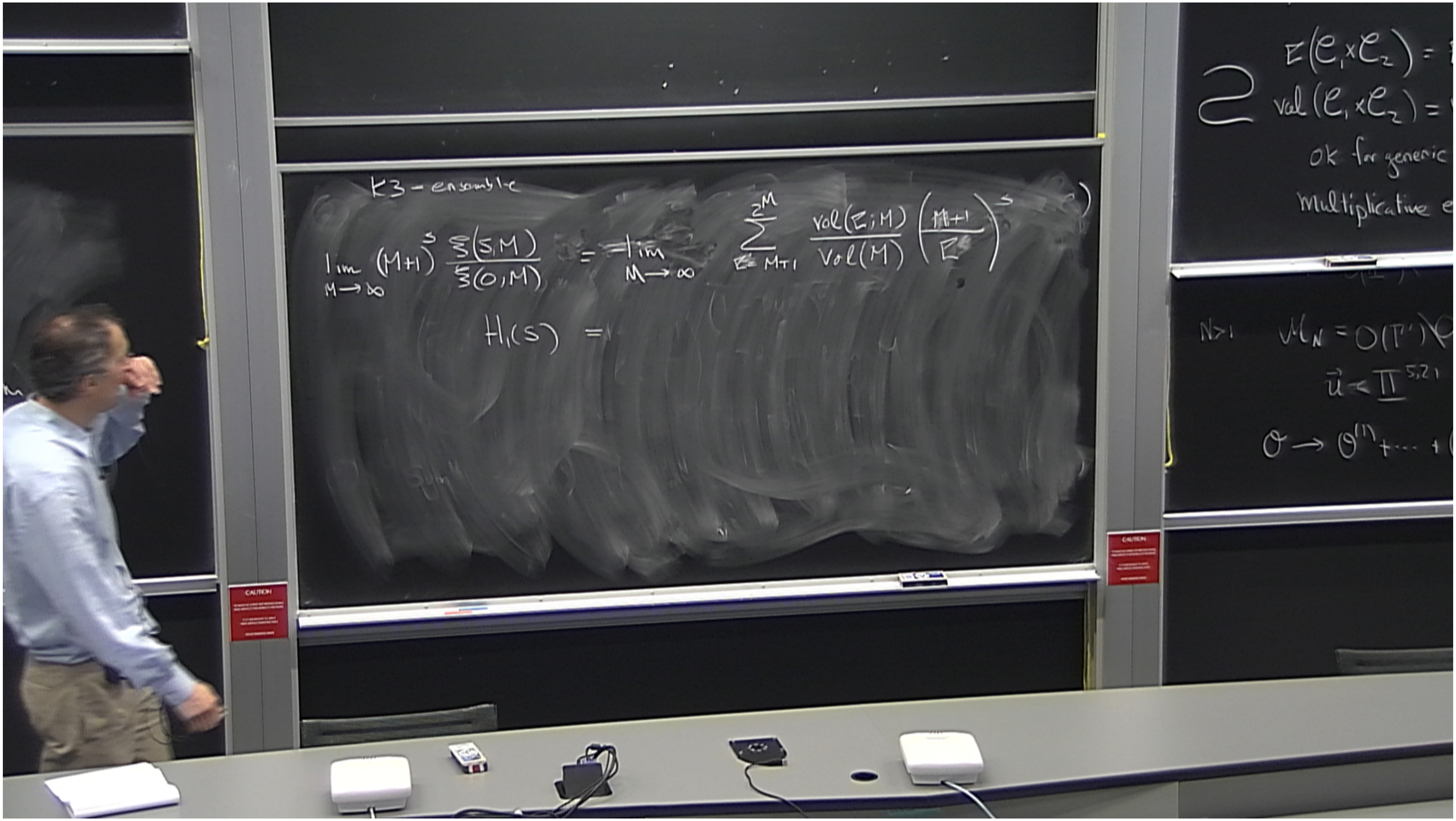

② $\mathrm{Sym}^N \mathcal{E}_0$ $\quad 1 + \sum p^N \mathcal{E}(\tau, z; \mathrm{Sym}^N \mathcal{E}_0) = \prod_{k>0,\, n\geq 0,\, \ell} \frac{1}{(1-p^k q^n y^\ell)^{c(nk,\ell,\mathcal{E}_0)}}$
$c(0,m;\mathcal{E}_0) > 0 \quad \frac{1}{(1-py^m)^{\mathcal{E}_0}} \Rightarrow$
$\mathcal{E}(\mathrm{Sym}^N \mathcal{E}_0) = \binom{N+\mathcal{E}_0-1}{\mathcal{E}_0-1}$ = polynomial in M
$N = M/m_0 \quad \mathcal{E}_0(K3) = 2 \quad \mathcal{E}(\mathrm{Sym}^N K3) = N+1$

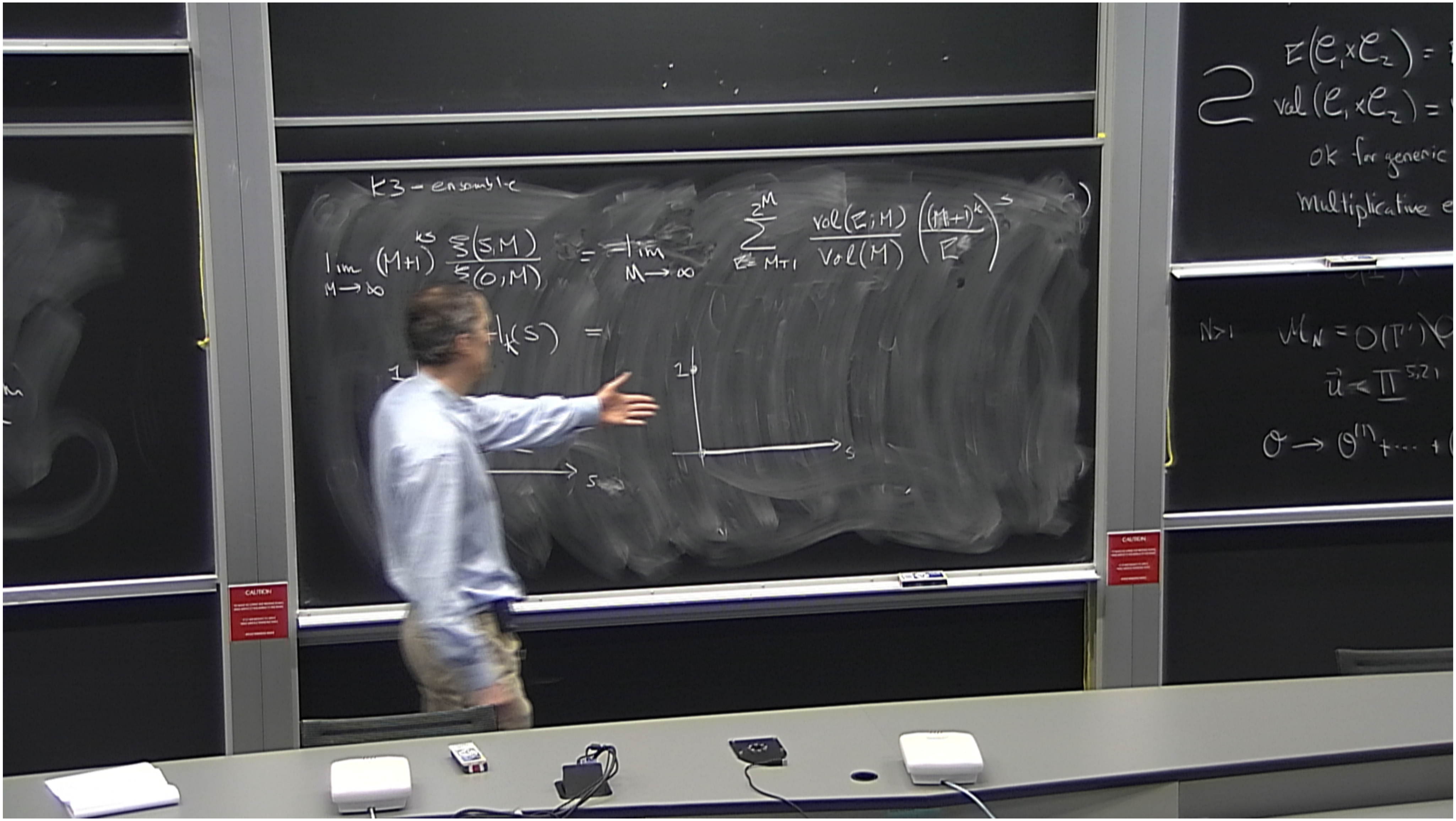

② Sym^N C₀   1 + Σ p^N E(τ, z; Sym^N C₀) = ∏ 1/(1 − p^k q^n y^ℓ)^{c(nk, ℓ; C₀)}
k>0, n≥0, ℓ
c(0, m; C₀) > 0   1/(1 − p y^m)^{C₀} ⇒
E(Sym^N C₀) = (N + C₀ − 1 choose C₀ − 1) = polynomial in M
N = M/m₀   E₀(K3) = 2   E(Sym^N K3) = N + 1
③ Sym^{N₁}(Sym^{N₂} C₀)
K3 en
"Primes"
N>1

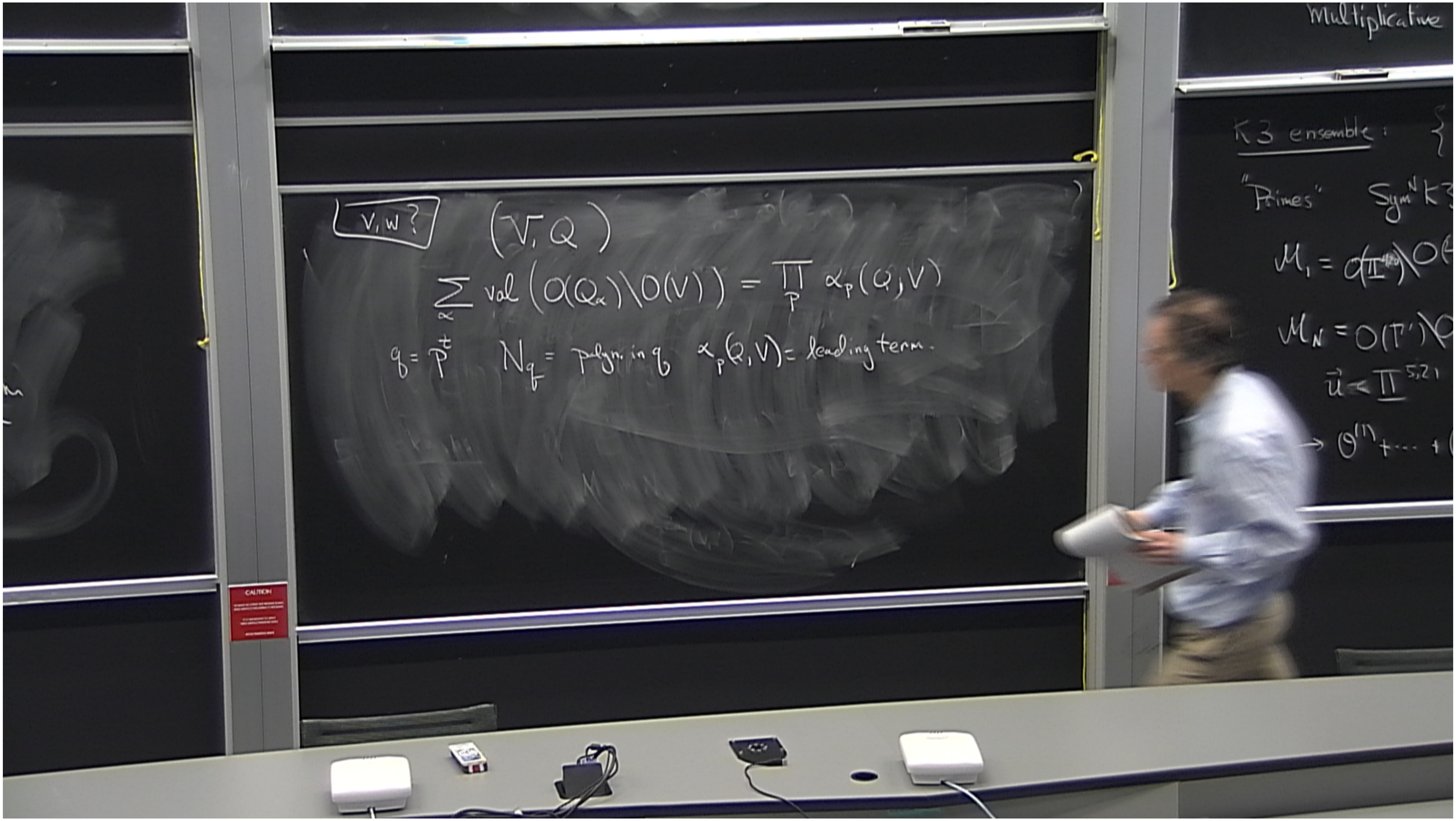

② Sym^N 𝒞₀   1 + Σ p^N ℰ(τ, z; Sym^N 𝒞₀) = ∏ 1/(1 − p^k q^n y^ℓ)^c(nk, ℓ; 𝒞₀)
c(0, m; 𝒞₀) > 0   1/(1 − py^m)^𝒞₀ ⇒
ℰ(Sym^N 𝒞₀) = (N + 𝒞₀ − 1 choose 𝒞₀ − 1) = polynomial in M
N = M/m₀   𝒞₀(K3) = 2   𝒞(Sym^N K3) = N + 1
③ ℰ(Sym^N (Sym^N K3)) = (2N choose N) = 4^N ~ 4^√M
M = N²
K3 en
"Primes"
N>1
CAUTION

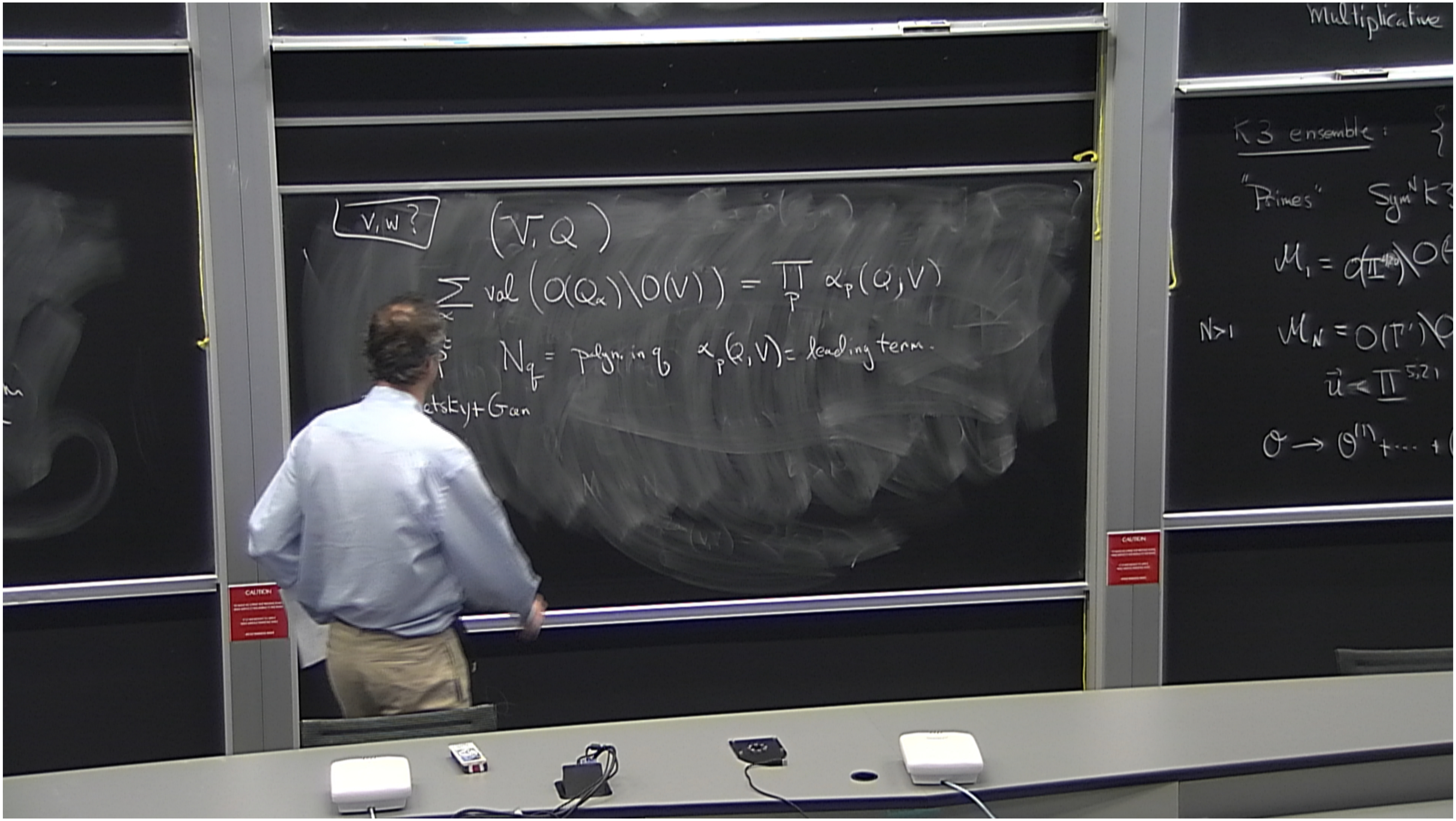

② Sym^N C₀   1+ Σ p^N E(τ, z; Sym^N C₀) = ∏ 1/(1 − p^k q^n y^ℓ)^c(nk, ℓ; C₀)
c(0, m; C₀) > 0   1/(1 − py^m)^C₀ ⇒
E(Sym^N C₀) = (N + C₀ − 1) = polynomial in M
N = M/m₀
E(Sym^N K3) = N + 1
③ Sym^N₁(Sym^N₂ C₀)
(Sym^N K3)) = (2N N) = 4^N ~ 4^√M
K3 en
"Primes"
N>1

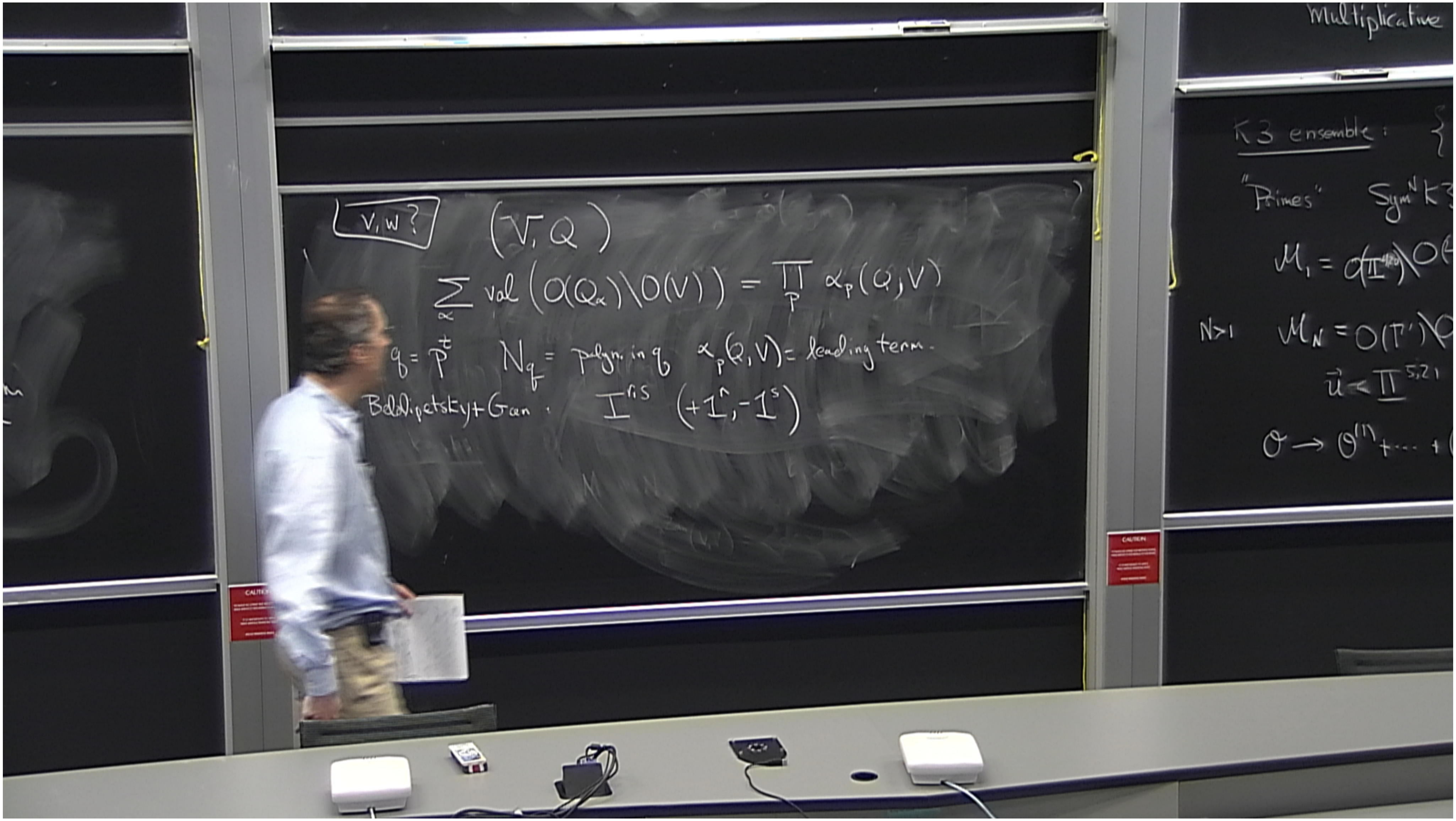

K3 - ensemble
Ok for generic
Multiplicative
N>1

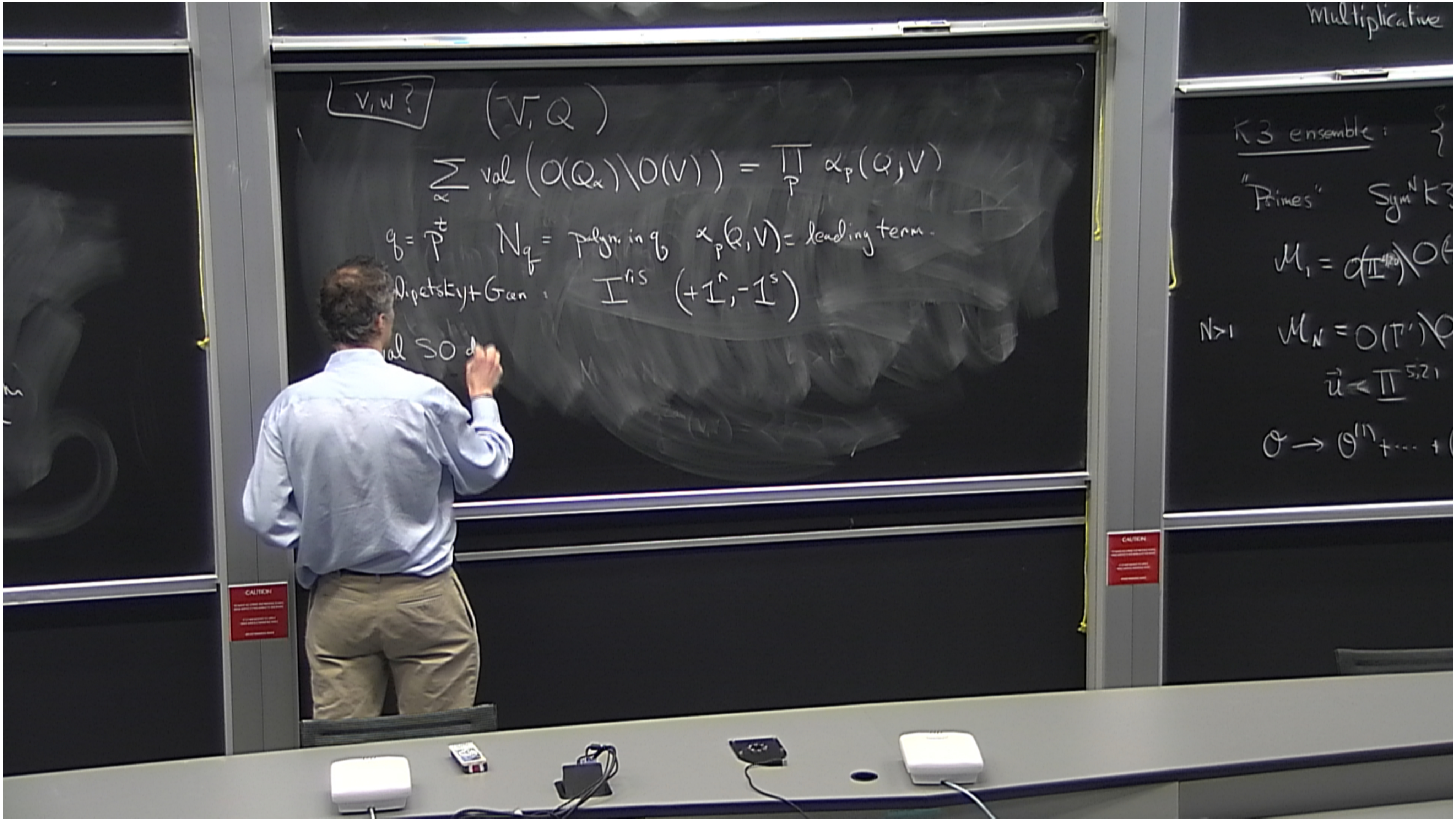

K3 - ensemble
lim (M+1)^s ξ(s,M) / ξ(0,M) = lim Σ vol(E;M) / Vol(M) ((M+1)/E)^s
M→∞
H1(s) =
vol(e1×e2) =
ok for generic
Multiplicative
N>1


K3 - ensemble
ok for generic

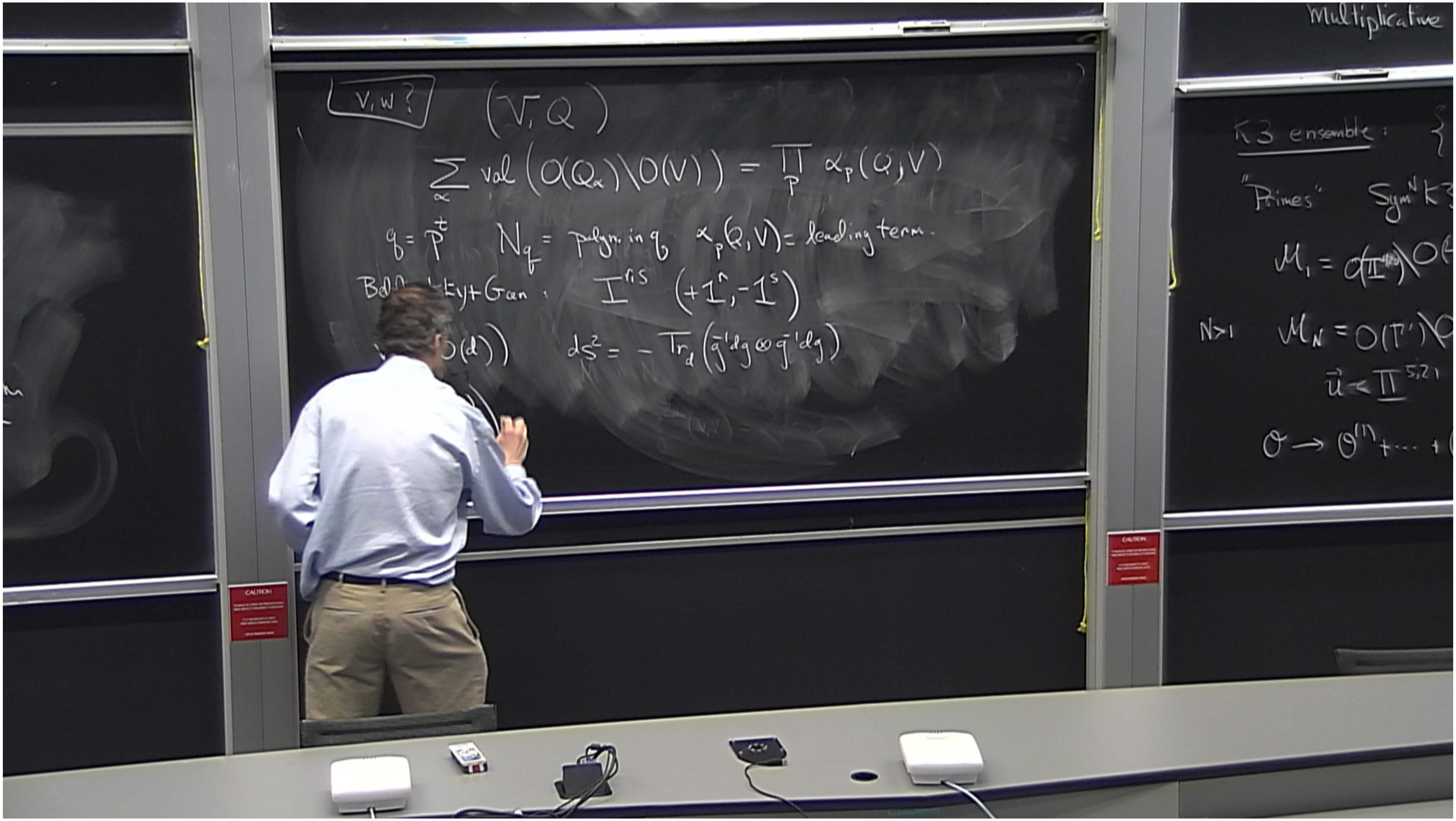

V, W?
(V, Q)
leading term.
Multiplicative
K3 ensemble
"Primes"

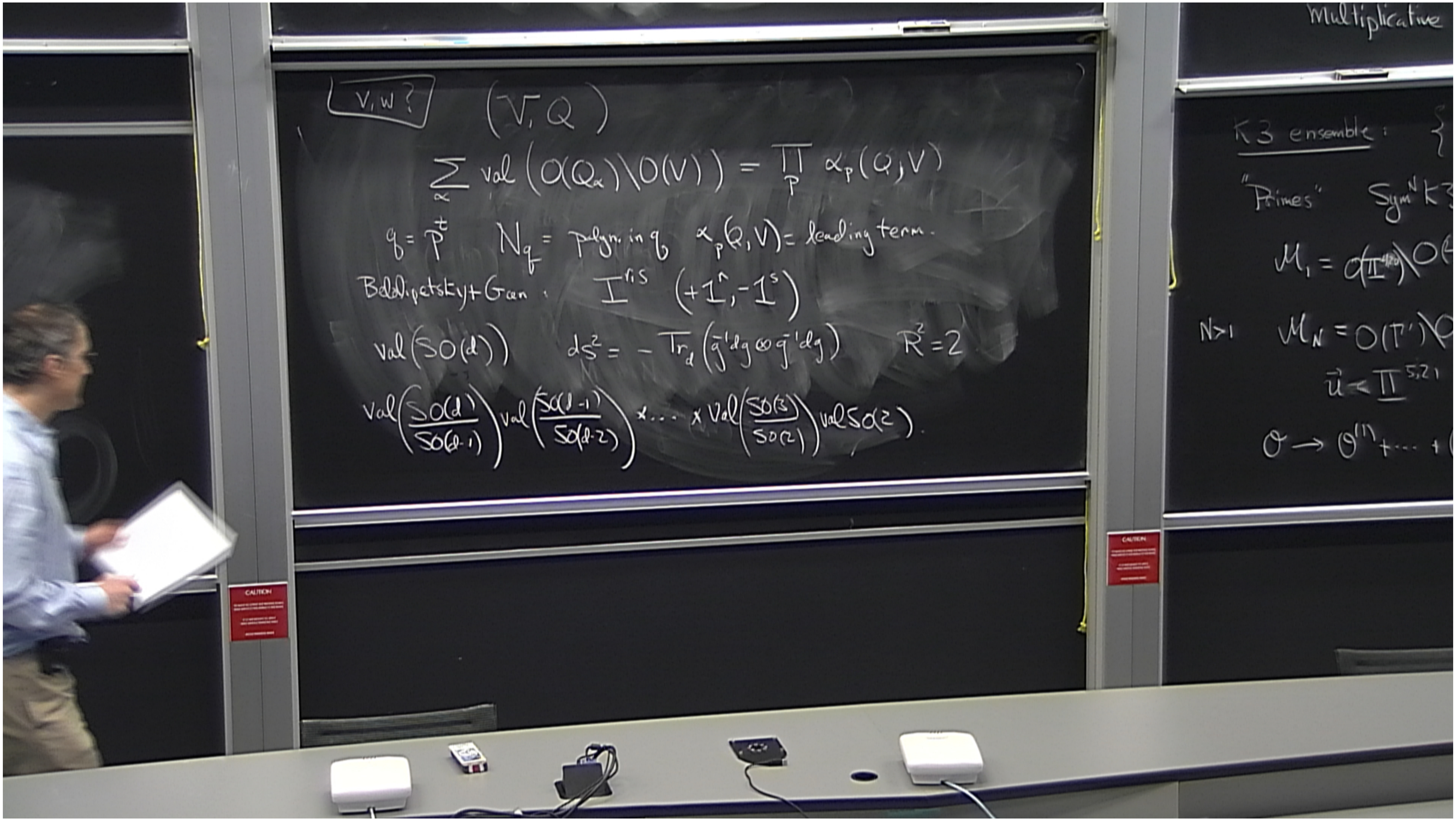

V, W ?
(V, Q)
α_p(Q,V) = leading term.
Multiplicative
K3 ensemble:
"Primes"
N>1

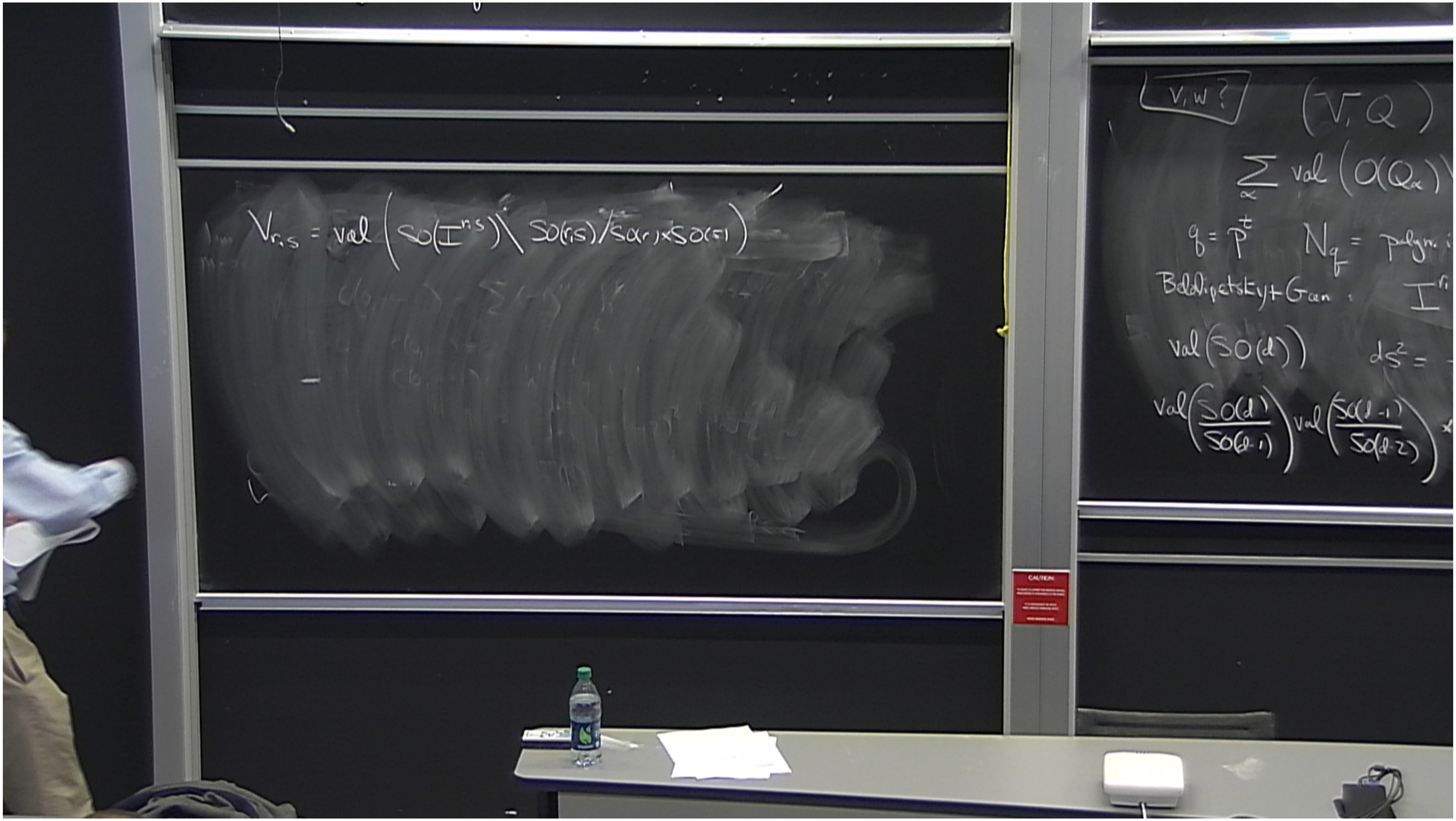

V, W ?
(V, Q)
leading term.
Multiplicative
K3 ensemble :
"Primes"
N>1

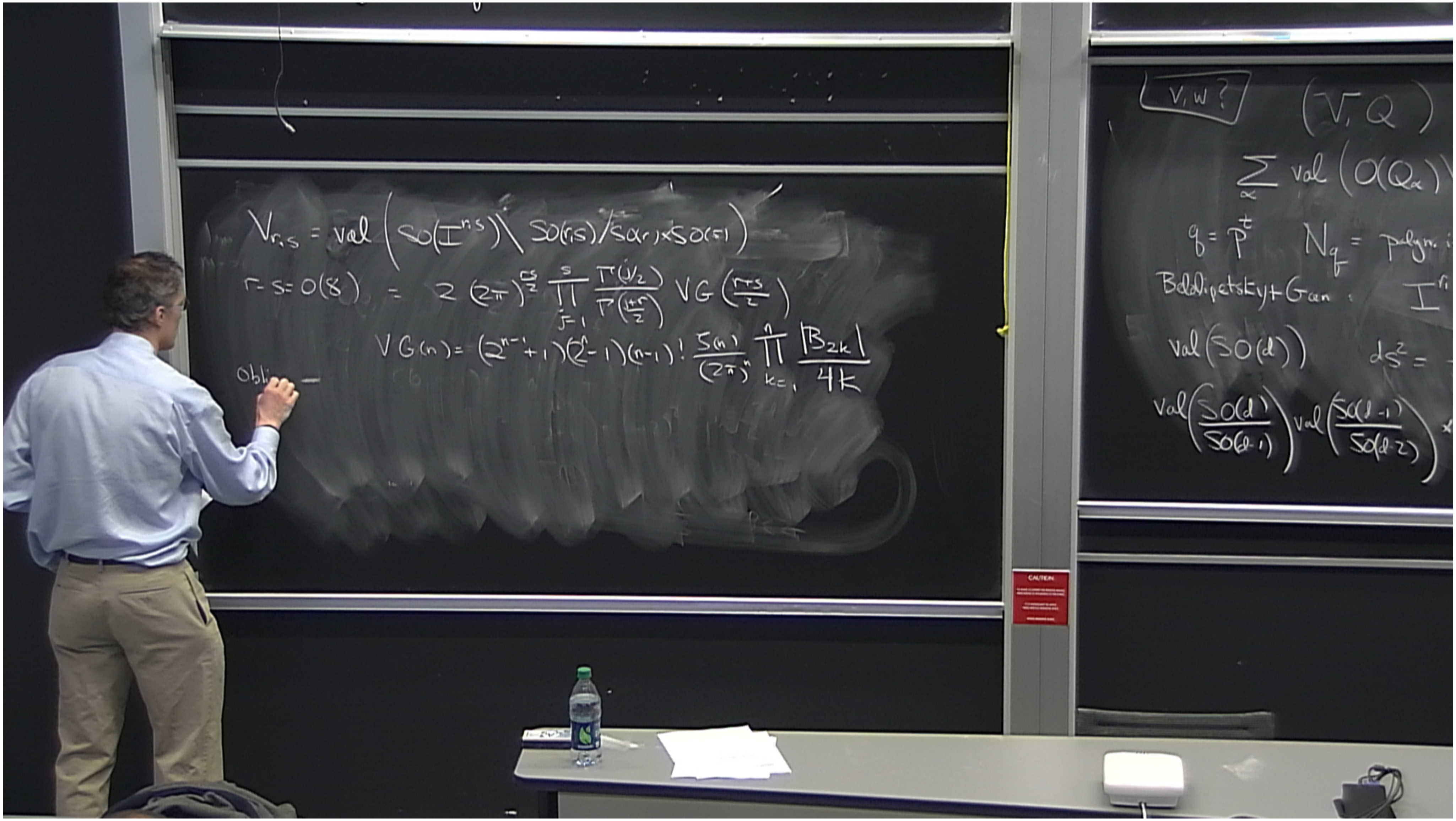


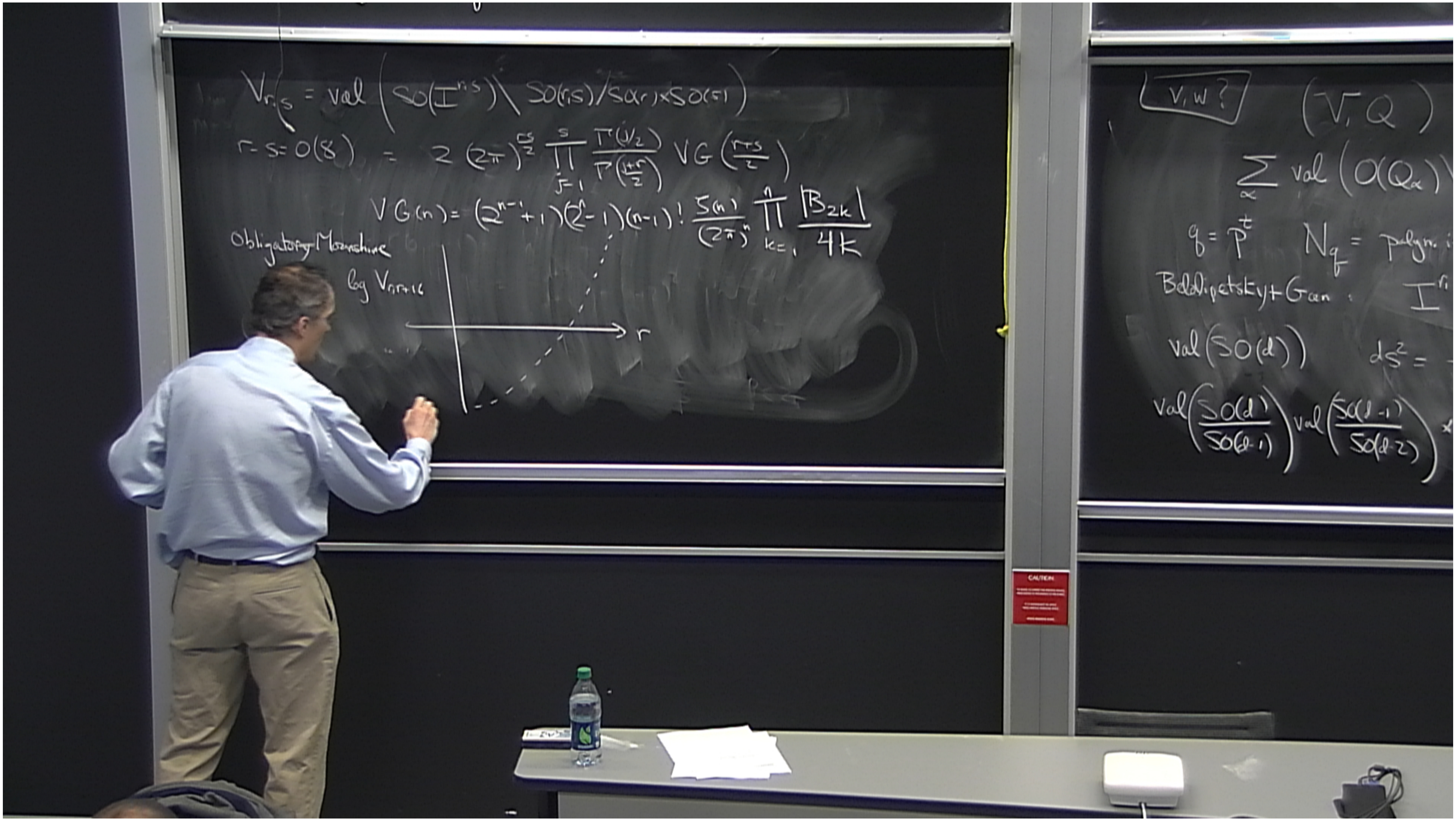
Multiplicative
V, W ?
(V, Q)
∑ val (O(Q_α)\O(V)) = ∏ α_p(Q,V)
q = p^t   N_q = polyn. in q   α_p(Q,V) = leading term.
Dipetsky + Gan :   I^{r,s} (+1^r, -1^s)
K3 ensemble :
"Primes"   Sym^N K3
M_1 = O(II^{4,20})\O(
N>1   M_N = O(Γ')\O
O → O^{(1)} + ··· +

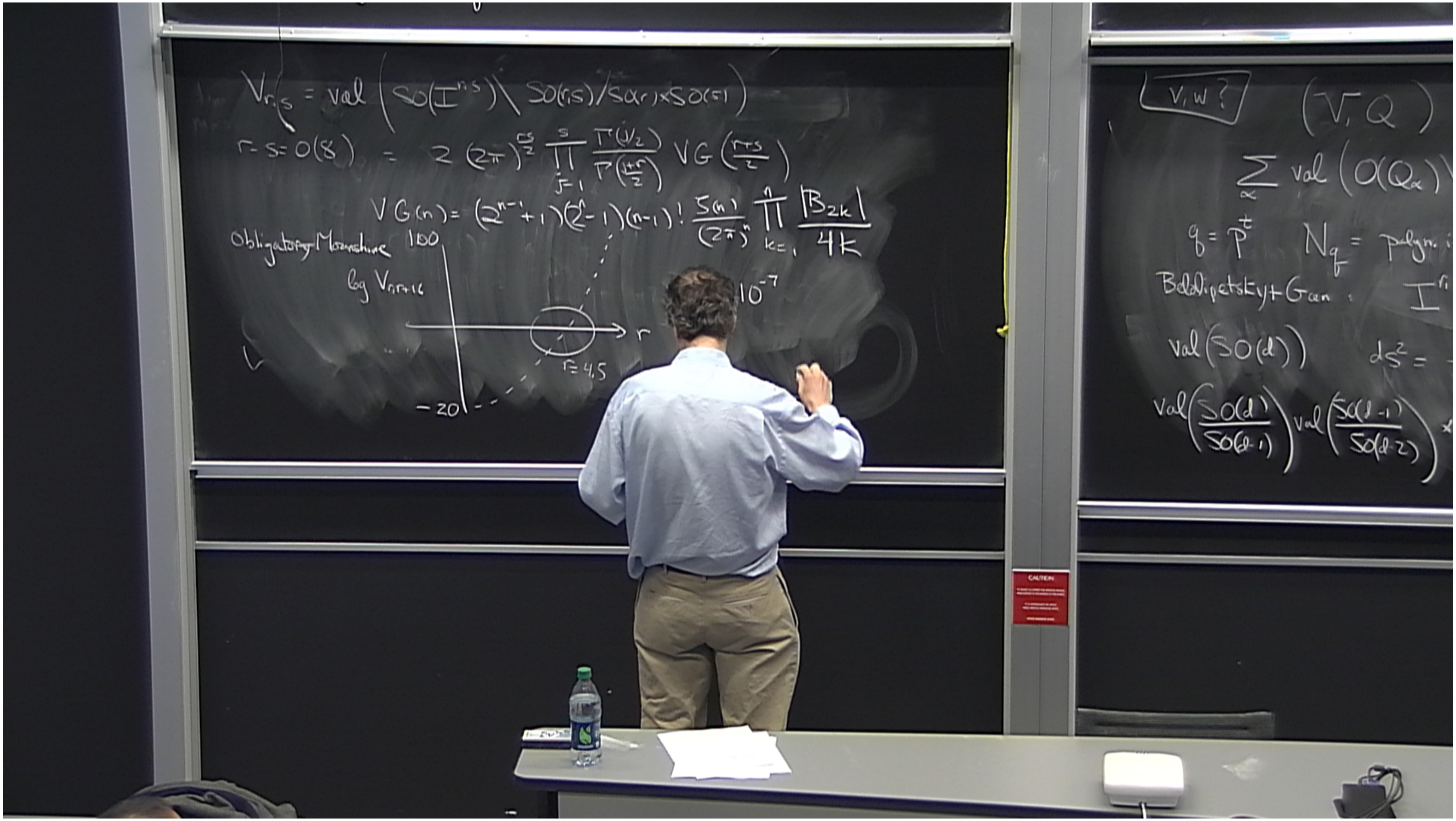

Multiplicative
V, W ?
(V, Q)
∑ val (O(Q_α)\O(V)) = ∏ α_p(Q,V)
q = p^t   N_q = polyn. in q   α_p(Q,V) = leading term.
I^{r,s}   (+1^r, -1^s)
ds² = − Tr_d (g⁻¹dg ⊗ g⁻¹dg)
K3 ensemble :
"Primes"   Sym^N K3
N>1
CAUTION


V, W ?
(V, Q)
leading term.
Multiplicative
K3 ensemble :
"Primes"
CAUTION
CAUTION

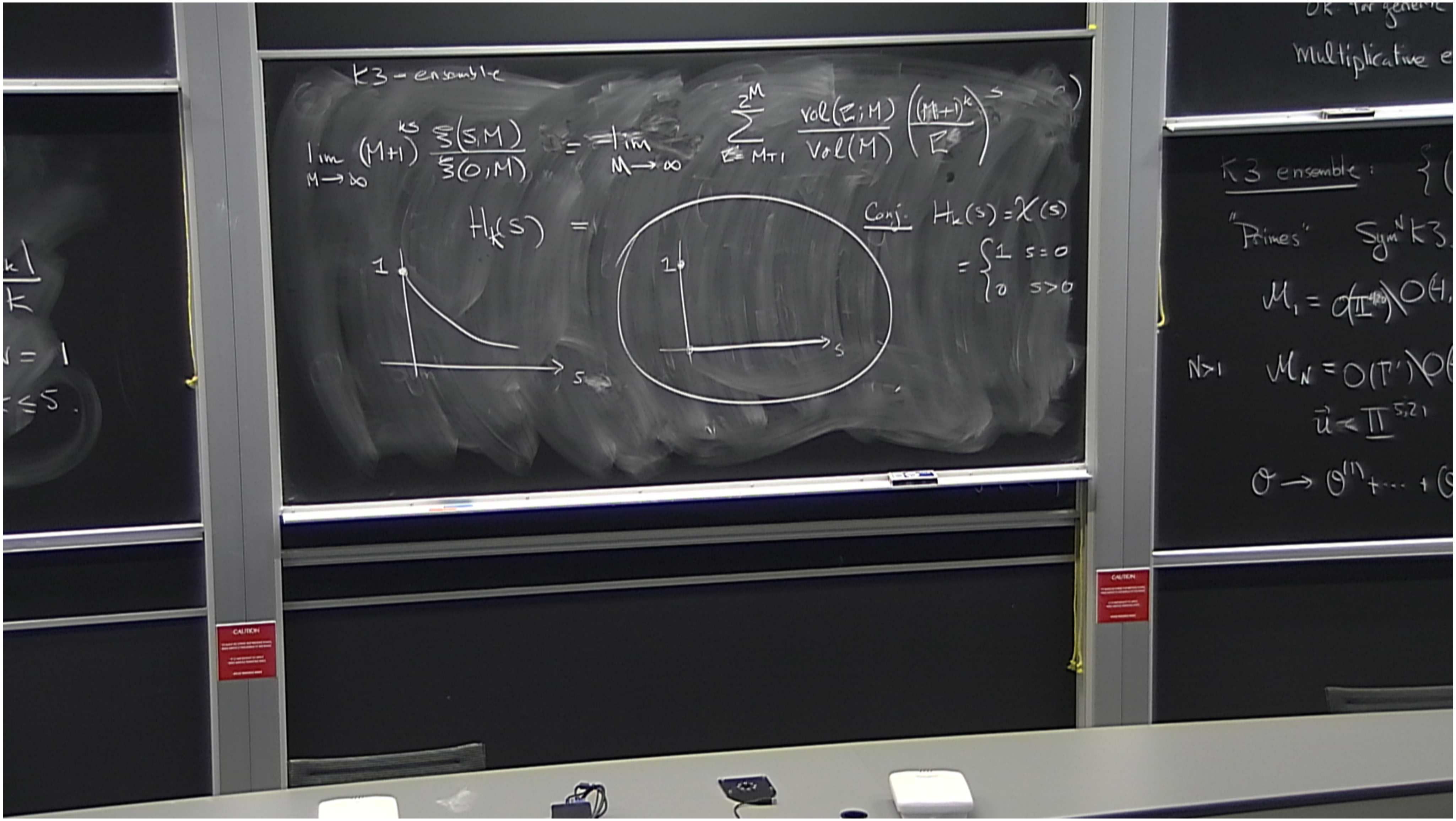

V, W ?
(V, Q)
Σ vol (O(Q_α))
q = p^t
N_q = polyn.
Belolipetsky + Gan
vol (SO(d))
ds² =
CAUTION

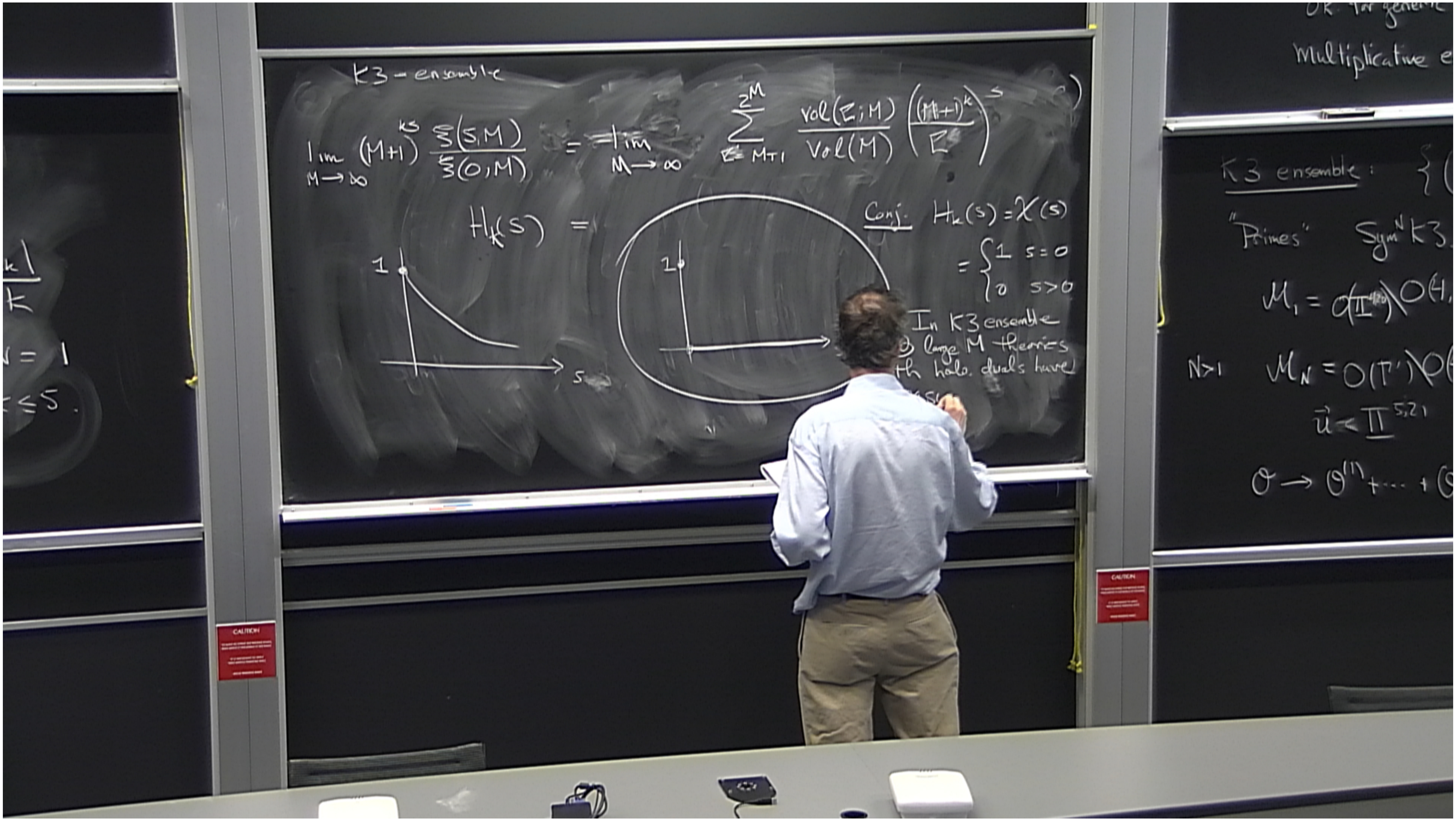

V_{r,s} = vol (SO(I^{r,s}) \ SO(r,s) / SO(r) × SO(s))
r − s = 0 (8)
VG(n) = (2^{n−1} + 1)(2^n − 1)(n−1)! ζ(n)/(2π)^n ∏_{k=1}^{n} |B_{2k}| / 4k
Obli
V, W ?
Σ vol (O(Q_α))
q = p^t
N_q = polyn.
Belolipetsky + Gan
vol (SO(d))
ds² =
vol (SO(d)/SO(d−1)) vol (SO(d−1)/SO(d−2))
CAUTION

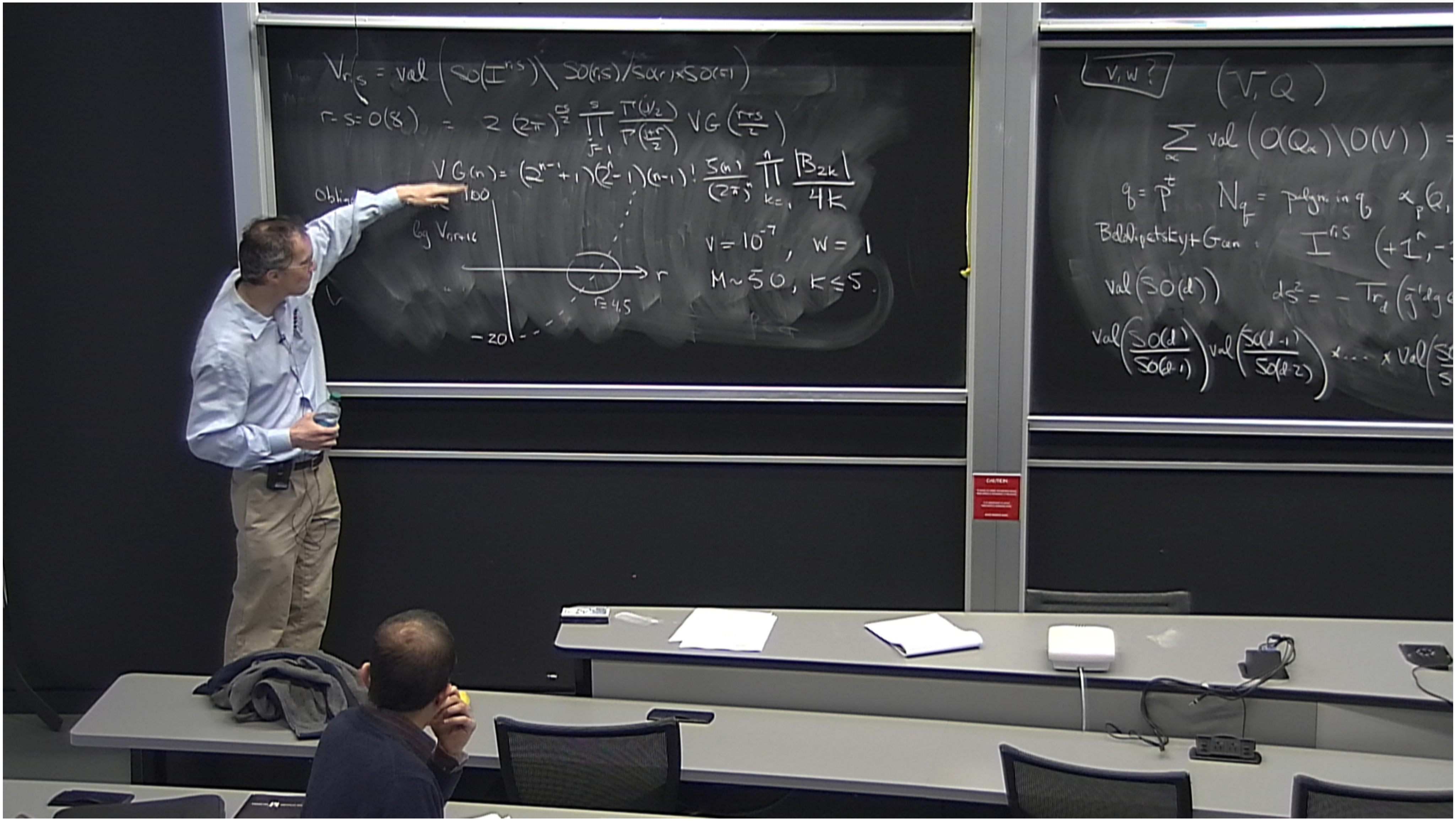

V_{r,s} = vol (SO(I^{r,s}) \ SO(r,s)/SO(r)×SO(s))
r−s = 0(8) = 2 (2π)^{rs/2} ∏_{j=1}^{s} Γ(j/2)/Γ((j+r)/2) VG((r+s)/2)
VG(n) = (2^{n−1}+1)(2^n−1)(n−1)! ζ(n)/(2π)^n ∏_{k=1}^{n} |B_{2k}|/4k
Obligatory Moonshine
log V_{r,r+16}
r
V,W ?
(V,Q)
∑_α vol (O(Q_α)
q = p^t
N_q = polyn.
Belolipetsky + Gan :
val(SO(d))
ds^2 =
val(SO(d)/SO(d−1)) val(SO(d−1)/SO(d−2))
CAUTION

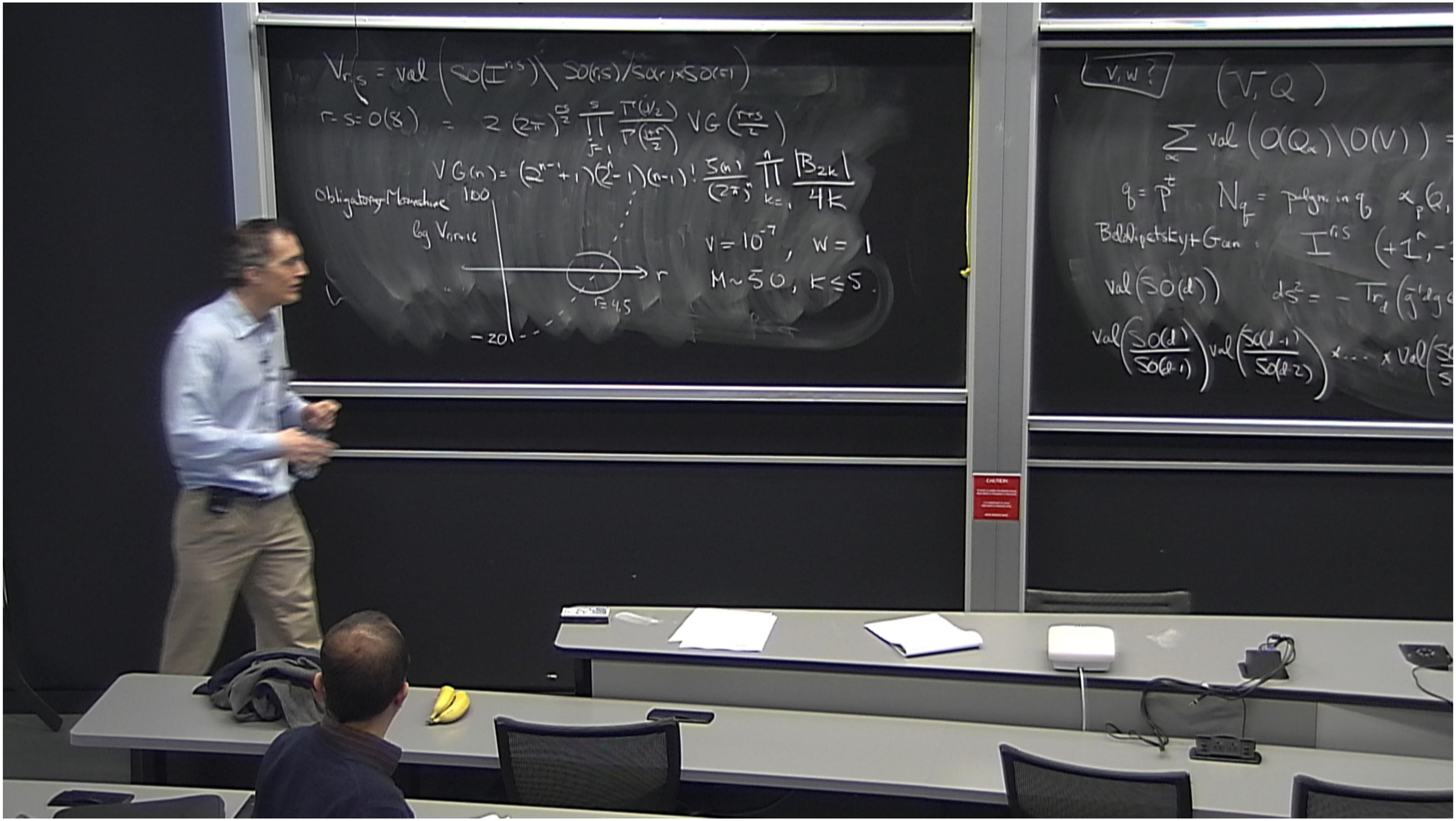


Obligatory Moonshine
log V
r=4.5
10^-7
v, w ?
Bellolipetsky+Gan
CAUTION


K3 - ensemble
Conj. $H_k(s) = \chi(s)$
K3 ensemble :
"Primes" $Sym^N K3$
Multiplicative


K3 - ensemble
Conj. $H_k(s) = \chi(s)$
In K3 ensemble
large M theories
th holo. duals have
K3 ensemble :
"Primes" $Sym^N K3$
Multiplicative e


V = 10^-7 , W = 1
M ~ 50 , k ≤ 5
-20
r ≅ 4.5


V, W ?
V = 10^-7 , W = 1
M ~ 50, k ≤ 5
r ≈ 4.5
100
-20
Obligatory Moonshine
log V