

Title: Measuring the Elliptic Genus

Date: Apr 17, 2015 10:00 AM

URL: <http://pirsa.org/15040134>

Abstract: This talk is based on the recent paper co-authored with N. Benjamin, M. Cheng, S. Kachru, and N. Paquette.

Based on recent paper w/ N. Benjamin, M. Cheng, S. Kachru, N. Paquette  
D papers: C. Keller, "Phase transitions, ..."  
Hartman, Keller, Stoica, "Universal Spectrum ..."

Outline:

/ Motivation

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CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD IS DAMAGED  
BY THE BOARD IS DAMAGED  
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BY THE BOARD IS DAMAGED

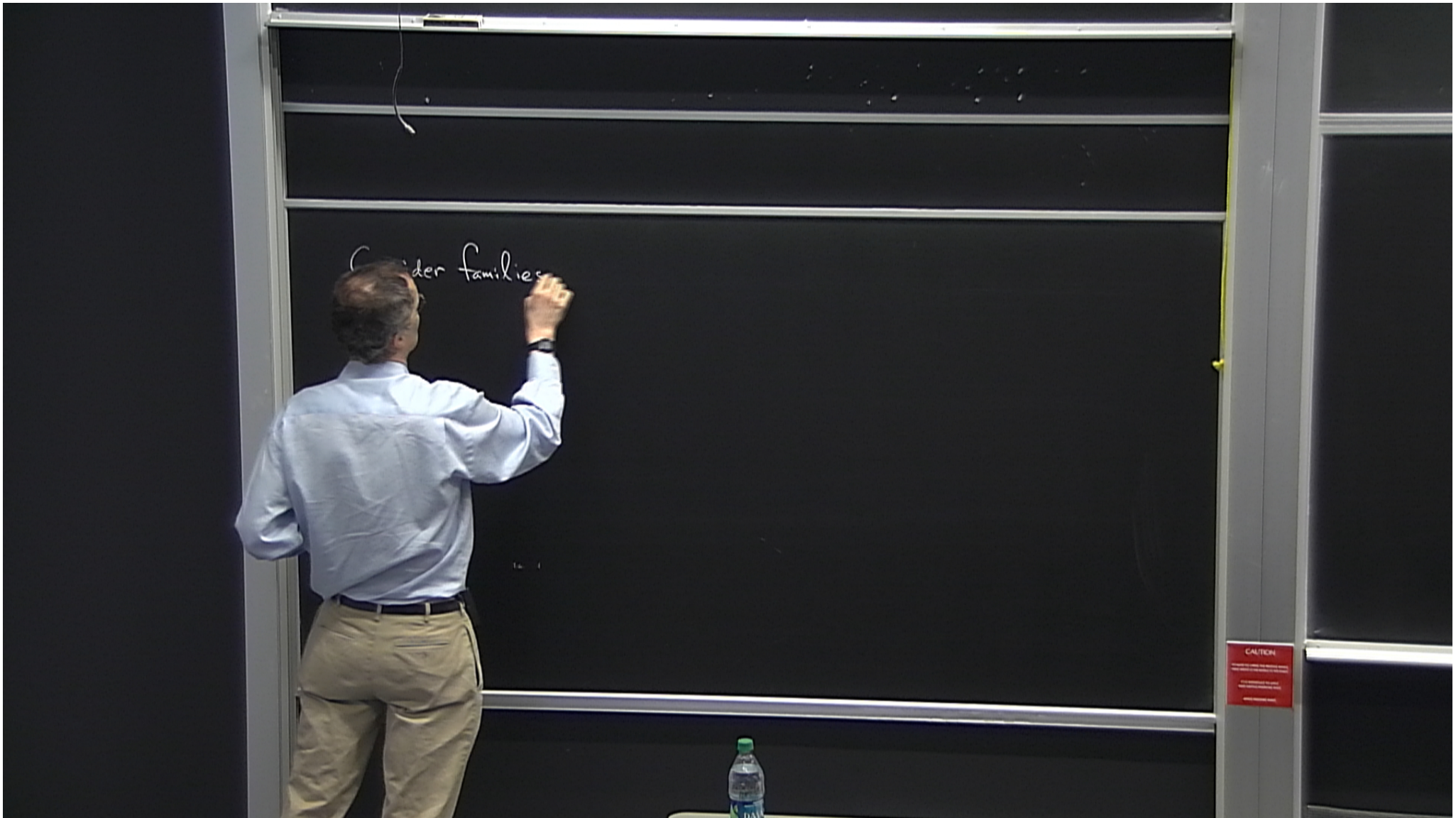


Based on recent paper w/ N. Benjamin, M. Cheng, S. Kachru, N. Pagnette

Related papers: C. Keller, "Phase transitions, ..."  
Hartman, Keller, Stoica, "Universal Spectrum ..."

Outline

1. Intro. / Motivation
2. Elliptic Genus
3. Measure on space of CFTs
4. Physical Bound
5. Examples
6. Answering Showit's question - sort of.



Consider families of  $\mathcal{N}=(2,2)$  CFT's  $\{\mathcal{C}^{(m)}\}$   $m \rightarrow \infty$   
having a large  $m$  limit. How many have weakly coupled  
Einstein gravity duals?

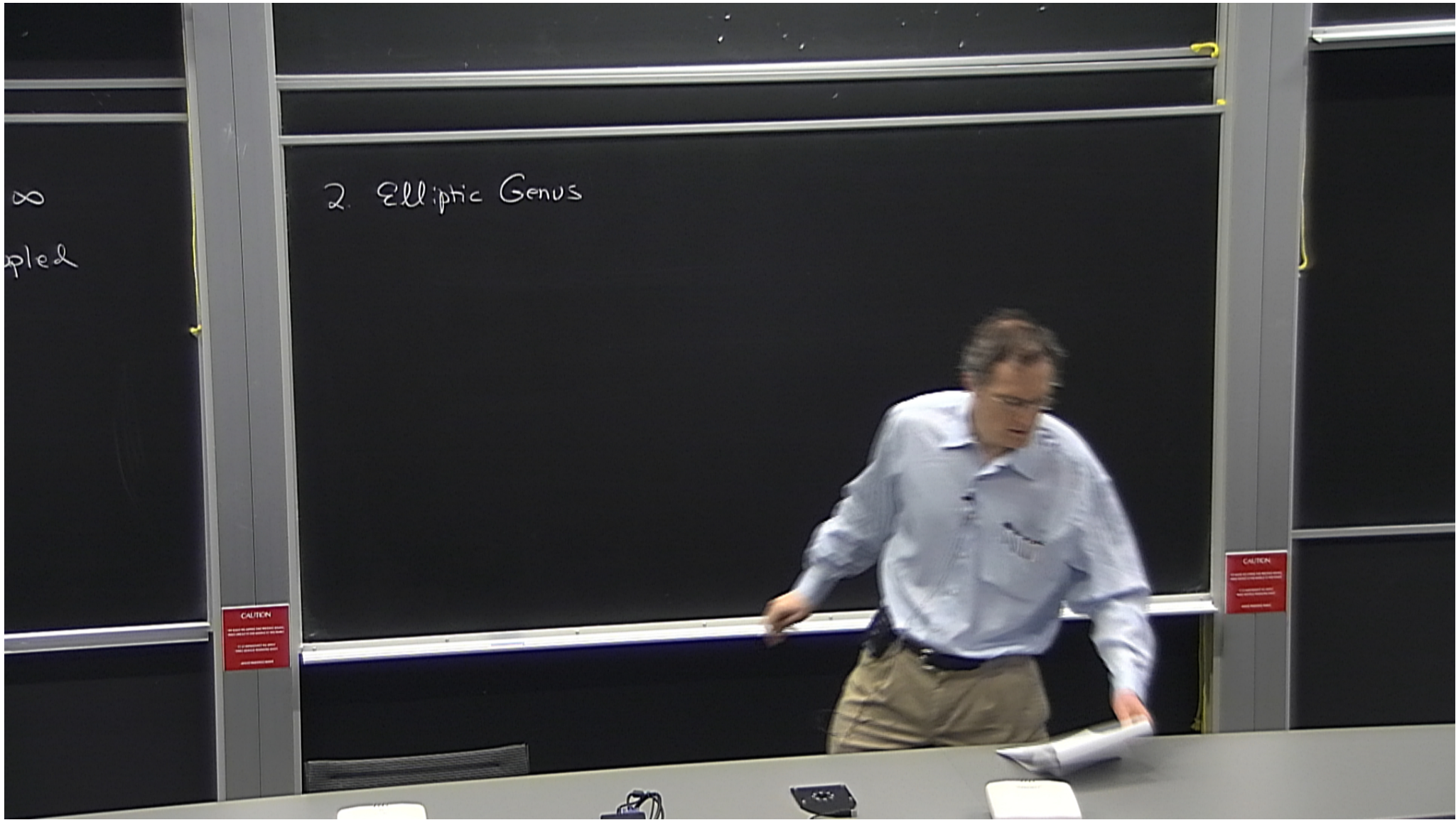
Consider families of  $\mathcal{N}=(2,2)$  CFT's  $\{\mathcal{C}^{(m)}\}$   $m \rightarrow \infty$   
having a large  $m$  limit. How many have weakly coupled  
Einstein gravity duals?  $m = c/6$

— • Measure: Zamolodchikov

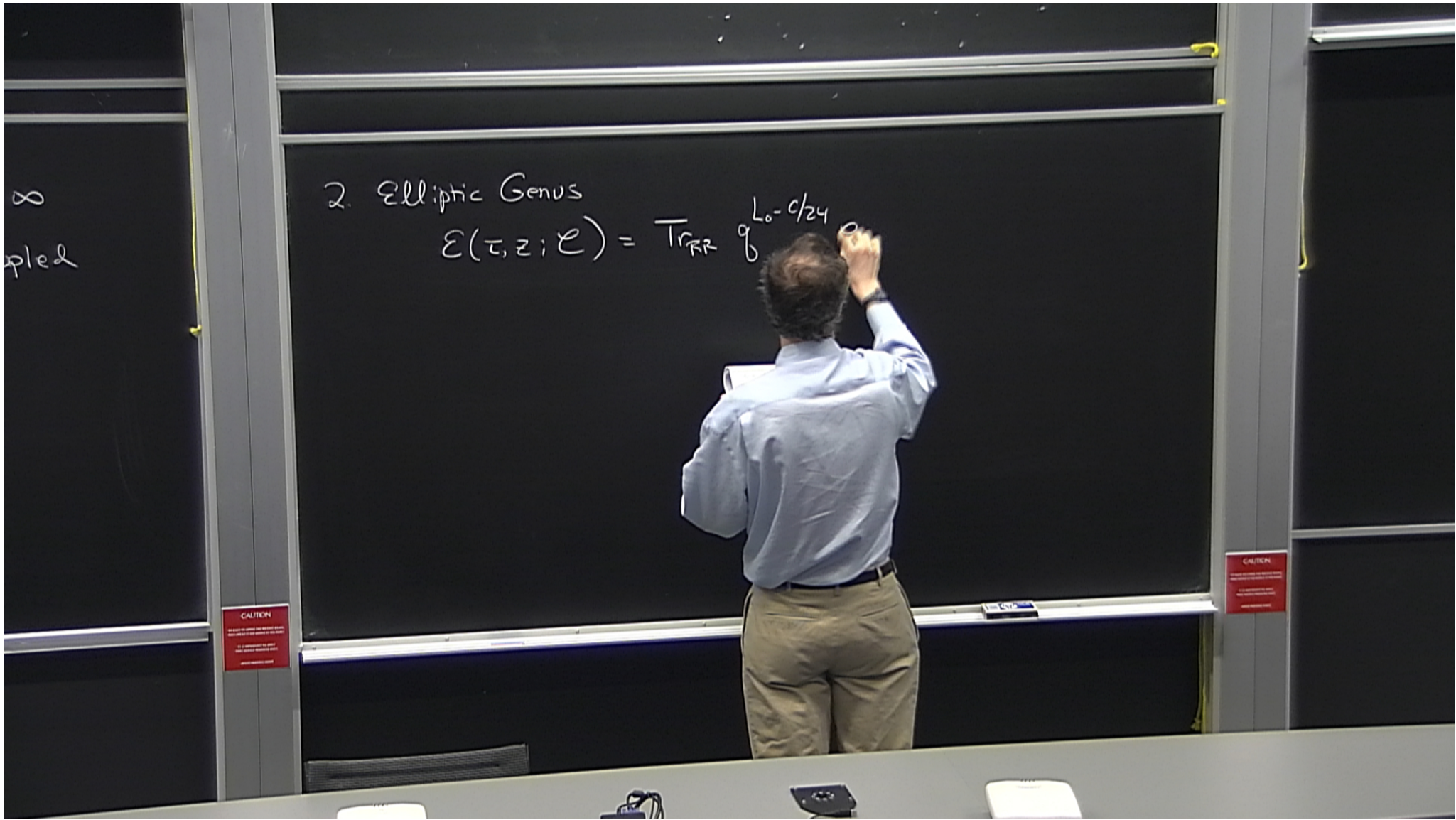
Consider families of  $\mathcal{N}=(2,2)$  CFT's  $\{\mathcal{E}^{(m)}\}$   $m \rightarrow \infty$   
having a large  $m$  limit. How many have weakly coupled  
Einstein gravity duals?  $m = c/6$

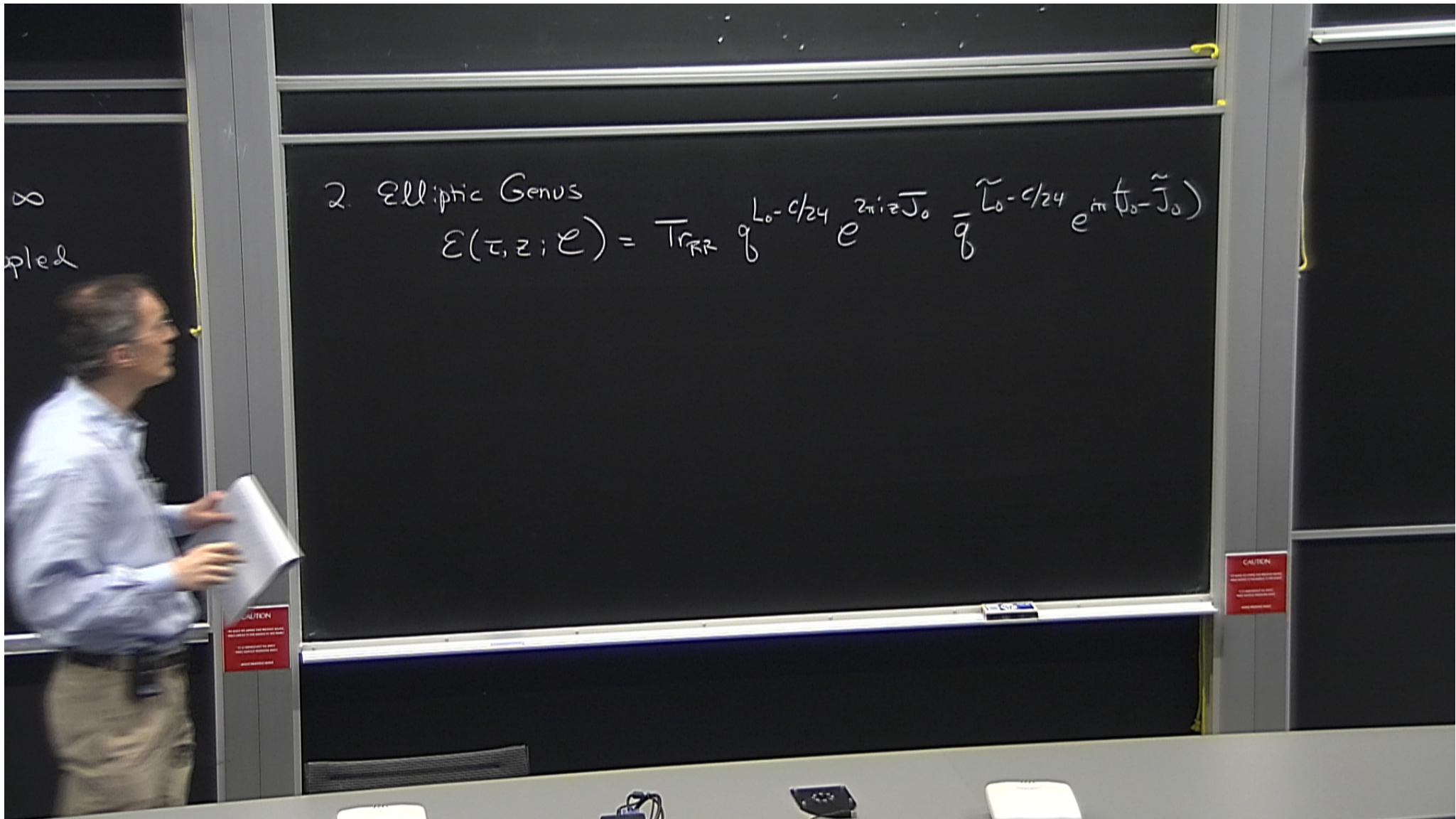
- Measure: Zamolodchikov
- Criterion: HP phase transition.











2 Elliptic Genus

$$E(\tau, z; \mathcal{L}) = \text{Tr}_{\mathbb{R}^2} q^{L_0 - c/24} e^{2\pi i z \bar{J}_0} \bar{q}^{\tilde{L}_0 - c/24} e^{i\pi(\tilde{J}_0 - \tilde{J}_0)}$$

$\infty$   
pled

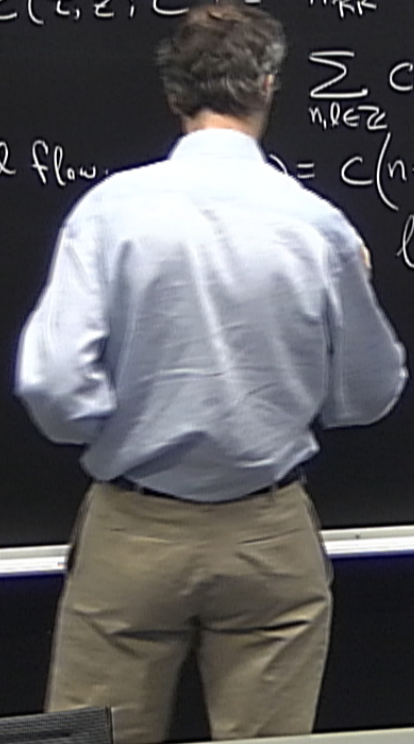
$\infty$   
pled

## 2 Elliptic Genus

$$\mathcal{E}(\tau, z; \mathcal{E}) = \text{Tr}_{RR} q^{L_0 - c/24} e^{2\pi i z \tilde{J}_0} \bar{q}^{\tilde{L}_0 - c/24} e^{i\pi (z_0 - \tilde{J}_0)}$$

$$\sum_{n, l \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l$$

Spectral flow:  $c(n + ls + ms^2, l + 2ms) \quad \forall s \in \mathbb{Z}$   
 $l^2 = 4mn$



CAUTION

CAUTION

$\infty$   
pled

## 2 Elliptic Genus

$$E(\tau, z; \mathcal{E}) = \text{Tr}_{RR} q^{L_0 - c/24} e^{2\pi i z \tilde{J}_0} \bar{q}^{\tilde{L}_0 - c/24} e^{i\pi m(\tilde{J}_0 - \tilde{J}_0)}$$

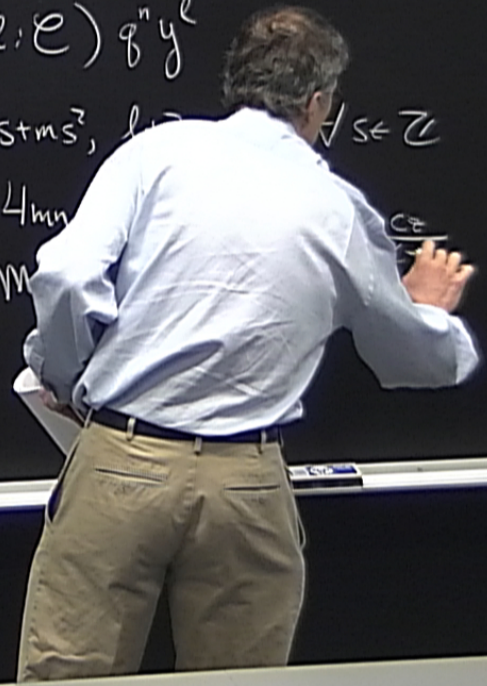
$$= \sum_{n, l \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l$$

Spectral flow:  $c(n, l) = c(n + ls + ms^2, l + 2ms)$   $\forall s \in \mathbb{Z}$

$$D(n, l) = l^2 - 4mn$$

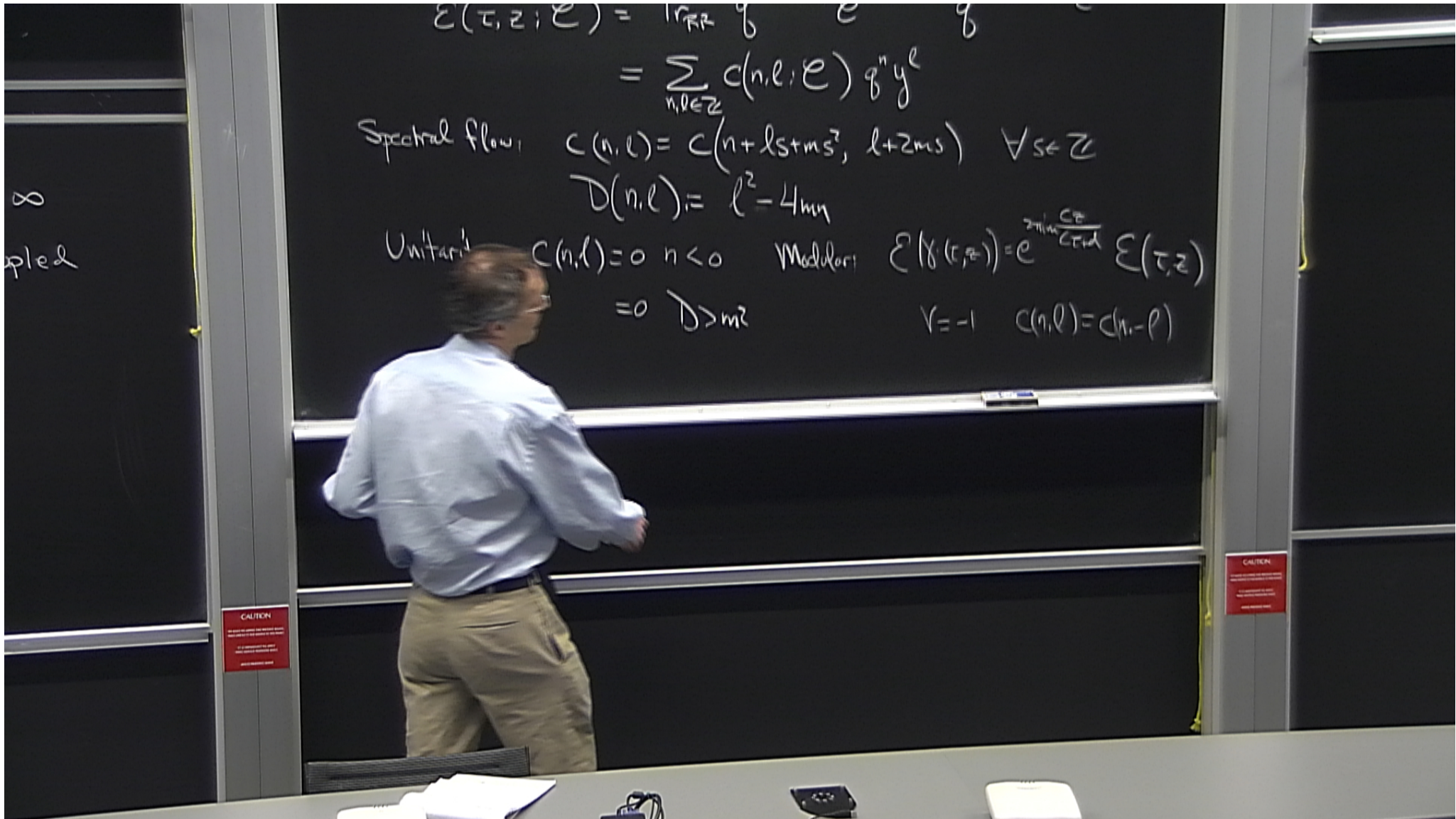
Unitarity:  $c(n, l) = 0 \quad n < 0$

$$= 0 \quad D > m^2$$



CAUTION

CAUTION



$$\begin{aligned} \mathcal{E}(\tau, z; \mathcal{E}) &= \sum_{n, l \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l \\ &= \sum_{n, l \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l \end{aligned}$$

Spectral flow:  $c(n, l) = c(n + l s + m s^2, l + 2 m s) \quad \forall s \in \mathbb{Z}$

$$D(n, l) = l^2 - 4 m n$$

Unitarity:  $c(n, l) = 0 \quad n < 0$     Modularity:  $\mathcal{E}(\gamma(\tau, z)) = e^{\frac{2\pi i c_{2,2}}{c_{1,1}} \tau} \mathcal{E}(\tau, z)$   
 $= 0 \quad D > m^2$      $\gamma = -1 \quad c(n, l) = c(n, -l)$

How many CFTs have  
 $\alpha$  holographic dual?

Ts

tion- sort of.

$(2,2)$  CFT's  $\{e^{(m)}\}$   $m \rightarrow \infty$   
 nit. How many have weakly coupled  
 als?  $m = c/6$

Zamolodchikov

HP phase transition.

Spectral flow:  $c(n,l) = c(n+lsm^2, l+2ms) \quad \forall s \in \mathbb{Z}$   
 $D(n,l) = l^2 - 4lm$

Unitarity:  $c(n,l) = 0 \quad n < 0$     Modulo:  $\mathcal{E}(g(\tau, z)) = e^{\frac{2\pi i c \tau}{6h}} \mathcal{E}(\tau, z)$

$c_r(D) = 0 \quad D > m^2$      $V = -1 \quad c(n,l) = c(n-p)$





Papers

Hartman, Keay, Shen, "Transitions, ..."  
Hartman, Keay, Shen, "Universal Spectrum ..."

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How many CFTs have a holographic dual?

Consider families of  $\mathcal{N}=(2,2)$  CFT's  $\{\mathcal{E}^{(m)}\}_{m \rightarrow \infty}$  having a large  $m$  limit. How many have weakly coupled Einstein gravity duals?  $m = c/g^2$

- • Measure: Zamolodchikov
- Criterion: HP phase transition.

$$\mathcal{E}(\tau, z; \mathcal{E}) = \frac{1}{\text{Vol}(\Gamma \backslash \mathbb{H}^2)} \sum_{n, l \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l$$

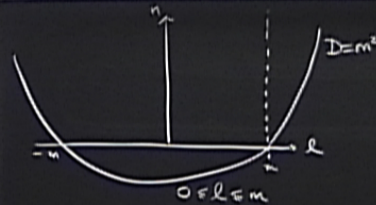
$$= \frac{1}{\text{Vol}(\Gamma \backslash \mathbb{H}^2)} e^{-\frac{L_0 - c/24}{g}} e^{m(l_0 - J_0)}$$

Spectral flow:  $c(n, l) = c(n + ks + ms^2, l + 2ms) \quad \forall s \in \mathbb{Z}$

$$D(n, l) = l^2 - 4mn$$

Unitarity:  $c(n, l) = 0 \quad n < 0$  Modulo:  $\mathcal{E}(\mathcal{E}(\tau, z)) = e^{\frac{c}{24} \frac{1}{\tau^2}} \mathcal{E}(\tau, z)$

$$c_r(D) = 0 \quad D > m^2 \quad V = -1 \quad c(n, l) = c(n, -l)$$



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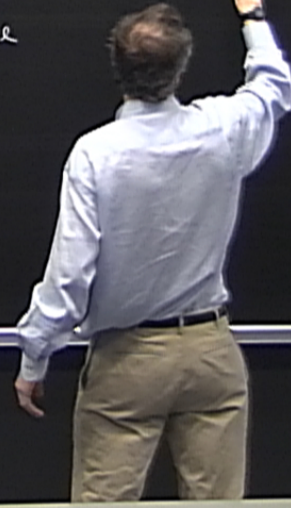
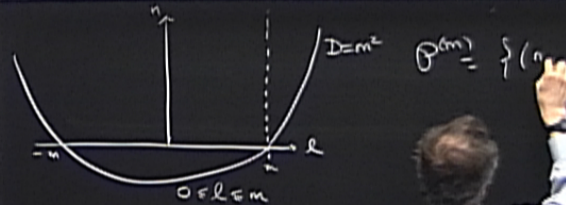
$$\mathcal{E}(\tau, z; \mathcal{E}) = \frac{1}{\sqrt{g}} e^{i(m\tau - J_0)} = \sum_{n \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l$$

Spectral flow:  $c(n, l) = c(n + ks + ms^2, l + 2ms) \quad \forall s \in \mathbb{Z}$

$$D(n, l) = l^2 - 4mn$$

Unitarity:  $c(n, l) = 0 \quad n < 0$  Modulo:  $\mathcal{E}(\tau, z) = e^{\frac{2\pi i c}{24\tau}} \mathcal{E}(\tau, z)$

$c_r(D) = 0 \quad D > m^2$   $V = -1 \quad c(n, l) = c(n, -l)$



Papers

transitions, ...  
Hartman, Keay, Shiu, "Universal Spectrum ..."

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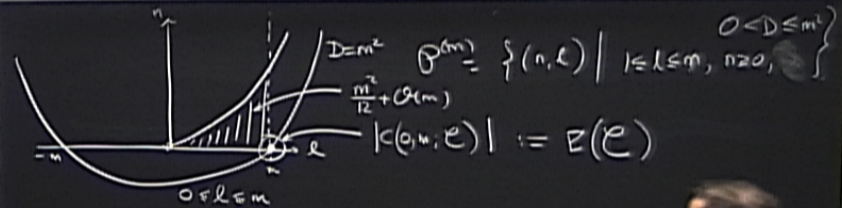
$$\mathcal{E}(\tau, z; \mathcal{E}) = \frac{1}{\eta(\tau)} \sum_{n \in \mathbb{Z}} c(n, l; \mathcal{E}) q^n y^l$$

Spectral flow:  $c(n, l) = c(n + l + ms^2, l + 2ms) \quad \forall s \in \mathbb{Z}$

$$D(n, l) = l^2 - 4mn$$

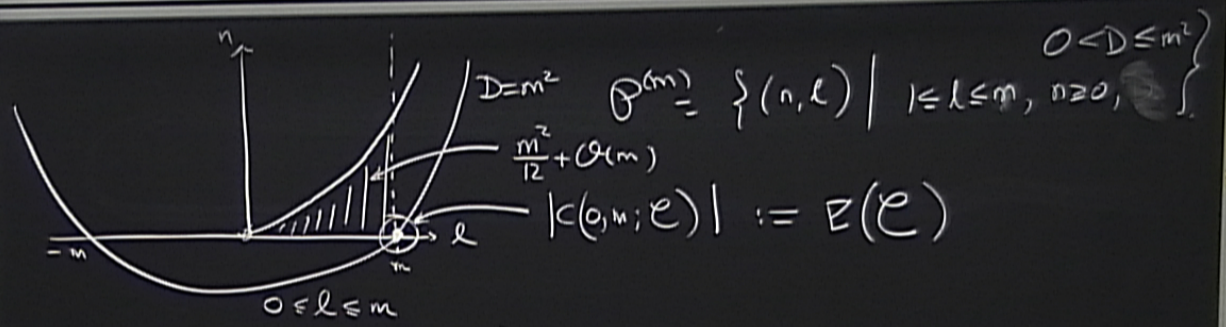
Unitarity:  $c(n, l) = 0 \quad n < 0$  Modulo:  $\mathcal{E}(\tau, z) = e^{\frac{2\pi i c_{\text{eff}}}{24\tau}} \mathcal{E}(\tau, z)$

$$c_r(D) = 0 \quad D > m^2 \quad \forall -1 \quad c(n, l) = c(n, -l)$$



3. Measures

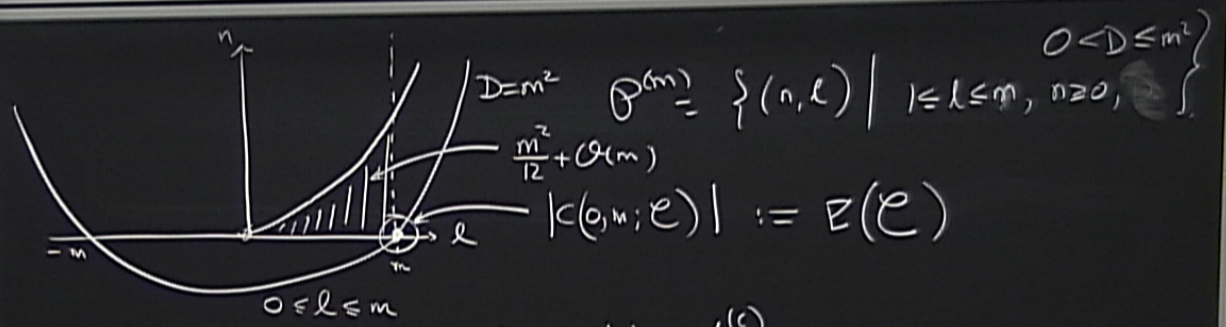
$m \rightarrow \infty$   
 akly coupled



3. Measures

$\mathcal{M}(\mathbb{R})$

$m \rightarrow \infty$   
 akly coupled



$$0 < D \leq m^2$$

$$P^{(m)} = \{(n, l) \mid 1 \leq l \leq m, n \geq 0\}$$

$$\frac{m^2}{2} + O(m)$$

$$|c(0, m; e)| := E(e)$$

$$0 \leq l \leq m$$

3. Measures

$$\mathcal{M}^{(c)} = \prod_{\alpha} \mathcal{M}_{\alpha}^{(c)}$$

CAUTION

CAUTION

$$M^{(m)} = \left\{ (n, l) \mid 1 \leq l \leq m, n \geq 0, \left. \begin{matrix} 0 < D \leq m^2 \\ \end{matrix} \right\} \right\}$$

$$|e\rangle := E(e)$$

$$\frac{1}{\alpha} M_{\alpha}^{(c)}$$

$$\cong \{(1,1) \text{ ex. mg?}\}$$

$$\rightarrow \phi$$

$$(u, v) = \langle \phi(6) \phi(1) \rangle$$

$$V^{(c)} = \sum_{\alpha} \frac{\text{val}(M_{\alpha}^{(c)})}{\text{Thm}(Z, Lu + Su)} < \infty \quad \text{Val}_{\text{WP}}(M_{\text{cplx str.}}) < \infty$$

CAUTION

$D = m^2$   
 $\mathcal{P}^{(m)} = \{(n, l) \mid 1 \leq l \leq m, n \geq 0, 0 < D \leq m^2\}$   
 $\frac{m^2}{12} + O(m)$   
 $|c_{(0, m); e}| := \mathbb{Z}(e) \in \mathbb{Z}_+$   
 $\mathcal{M}^{(c)} = \prod_{\alpha} \mathcal{M}_{\alpha}^{(c)}$   
 metric:  $T_e \mathcal{M}^{(c)} \cong \{(1, 1) \text{ ex. mg.}\}$   
 $v \rightarrow \phi$   
 $g_{Zam}^{(c)}(v, v) = \langle \phi(v) \phi(v) \rangle$

$V^{(c)} = \sum \text{val}(\mathcal{M}_{\alpha}^{(c)}) < \infty$   
 $\text{Thm}(\mathbb{Z}, Lu + Su)$   
 $A. Todorov$   
 $\text{Val}_{\text{WP}}(\mathcal{M}_{\text{cplx str.}}) < \infty$

$$D = m^2 \quad \mathcal{P}^{(m)} = \left\{ (n, \ell) \mid 1 \leq \ell \leq m, n \geq 0, \right\} \quad \left. \begin{array}{l} 0 < D \leq m^2 \\ \end{array} \right\}$$

$$- \frac{m^2}{12} + O(m)$$

$$- |c(0, m; \mathcal{P})| := \mathcal{E}(\mathcal{P}) \in \mathbb{Z}_+$$

$$V^{(c)} = \prod_{\alpha} \mathcal{M}_{\alpha}^{(c)}$$

$$T_{\mathcal{P}} \mathcal{M}^{(c)} \cong \{ (1,1) \text{ ex. mg} \}$$

$$v \rightarrow \phi$$

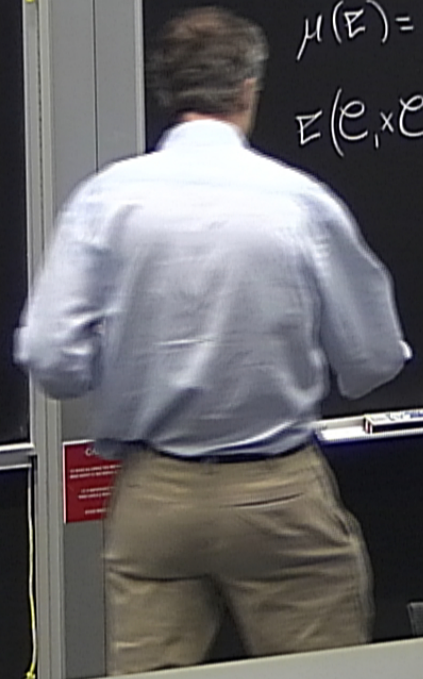
$$g^{Z_{\text{an}}}(v, v) = \langle \phi(v) \phi(1) \rangle$$

$$V^{(c)} = \sum_{\alpha} \text{val}(\mathcal{M}_{\alpha}^{(c)}) < \infty$$

$$\text{Thm}(\mathbb{Z}, Lu + Su) \quad \text{A. Todorov} \quad \text{Val}_{\text{WP}}(\mathcal{M}_{\text{cplx str.}}) < \infty$$

$$\mu(\mathcal{P}) = \frac{\text{val} \{ \mathcal{P} \mid \mathcal{E}(\mathcal{P}) = \mathcal{P} \}}{V^{(c)}}$$

$$\mathcal{E}(\mathcal{P}_1 \times \mathcal{P}_2) = \mathcal{E}(\mathcal{P}_1) \mathcal{E}(\mathcal{P}_2)$$





$$D = m^2 \quad \rho^{(m)} = \left\{ (n, l) \mid 1 \leq l \leq m, n \geq 0, \right. \\ \left. - \frac{m^2}{12} + O(m) \right\} \quad \left. \begin{array}{l} 0 < D \leq m^2 \\ \end{array} \right\}$$

$$|c(0, m; e)| := \#(e) \in \mathbb{Z}_+$$

$$V^{(c)} = \prod_{\alpha} \mathcal{W}_{\alpha}^{(c)}$$

$$T_e \mathcal{W}^{(c)} \cong \{ (1,1) \text{ ex. mg} \}$$

$$v \rightarrow \phi$$

$$g^{\text{Zam}}(v, v) = \langle \phi(v) \phi(v) \rangle$$

$$V^{(c)} = \sum_{\alpha} \text{val}(\mathcal{W}_{\alpha}^{(c)}) < \infty$$

$$\text{Thm}(\mathbb{Z}, Lu + Su_n) \quad \text{Val}_{\text{WP}}(\mathcal{W}_{\text{cplx str.}}) < \infty$$

A. Todorov

$$\mu(e) = \frac{\text{val} \{ e \mid \#(e) = e \}}{V^{(c)}}$$

$$\#(e_1 \times e_2) = \#(e_1) \#(e_2)$$

$$\supseteq \text{val}(e_1 \times e_2) = \text{val}(e_1) \text{val}(e_2)$$

$$D = m^2 \quad \mathcal{P}^{(m)} = \left\{ (n, \ell) \mid 1 \leq \ell \leq m, n \geq 0, \right\}$$

$$- \frac{m^2}{12} + O(m)$$

$$- |k(0, m; e)| := \nu(e) \in \mathbb{Z}_+$$

$$V^{(c)} = \prod_{\alpha} W_{\alpha}^{(c)}$$

$$T_e W^{(c)} \cong \{(1,1) \text{ ex. mg}\}$$

$$v \rightarrow \phi$$

$$g^{\text{Zar}}(v, v) = \langle \phi(v), \phi(v) \rangle$$

$$V^{(c)} = \sum_{\alpha} \text{val}(W_{\alpha}^{(c)}) < \infty$$

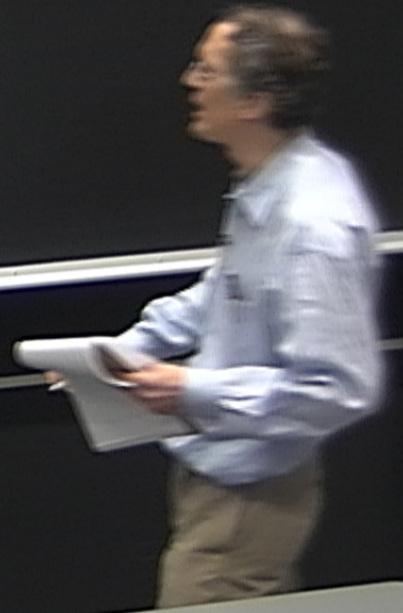
$$\text{Thm}(\mathbb{Z}, Lu + Su) \quad \text{A. Todorov} \quad \text{val}_{\text{WP}}(W_{\text{cptx str.}}) < \infty$$

$$\mu(E) = \frac{\text{val}\{e \mid \nu(e) = \nu\}}{\nu^{(c)}}$$

$$\nu(e_1 \times e_2) = \nu(e_1) \nu(e_2)$$

$$\nu(e_1, \times e_2) = \nu(e_1) \nu(e_2)$$

OK for generic CY, HK



$$c(n, l) = c(n, -l)$$

$$\left. \begin{array}{l} 0 < D \leq m^2 \\ \leq l \leq m, n \geq 0, \end{array} \right\}$$

$$) \in \mathbb{Z}_+$$

$$V^{(c)} = \sum_{\alpha} \text{val}(U_{\alpha}^{(c)}) < \infty$$

$$\text{Thm. (Z, Lu + Suu)} \quad \text{A. Todorov} \quad \text{Val}_{\text{WP}}(U_{\text{epix str.}}) < \infty$$

$$\mu(E) = \frac{\text{val} \{ e \mid \mathbb{E}(e) = E \}}{V^{(c)}}$$

$$\begin{aligned} \mathbb{E}(e_1 \times e_2) &= \mathbb{E}(e_1) \mathbb{E}(e_2) \\ \text{val}(e_1 \times e_2) &= \text{val}(e_1) \text{val}(e_2) \end{aligned}$$

OK for generic CY, HK

Multiplicative ensemble

Def:  $e$  is prime if  
not  $e_1 \times e_2$

$$c(n, l) = c(n, -l)$$

$$0 < D \leq m^2$$

$$l \leq m, n \geq 0$$

$$) \in \mathbb{Z}_+$$

$$V^{(c)} = \sum_{\alpha} \text{val}(U_{\alpha}^{(c)}) < \infty$$

Thm. (Z, Lu + Sun)  
A. Todorov

$$\text{Val}_{\text{WP}}(U_{\text{cpt} \times \text{str}}) < \infty$$

Def:  $e$  is prime if  
not  $e_1 \times e_2$   $m_1, m_2 > 0$

$$\mu(e) = \frac{\text{val} \{ e \mid \mathbb{Z}(e) = e \}}{V^{(c)}}$$

$$e(m, \alpha) \propto$$

$$\mathbb{Z}(e_1 \times e_2) = \mathbb{Z}(e_1) \mathbb{Z}(e_2)$$

$$\supseteq \text{val}(e_1 \times e_2) = \text{val}(e_1) \text{val}(e_2)$$

OK for generic CY, HK  
Multiplicative ensemble

$$c(n, l) = c(n, -l)$$

$$0 < D \leq m^2$$

$$l \leq m, n \geq 0$$

$$) \in \mathbb{Z}_+$$

$$V^{(c)} = \sum_x \text{val}(M_x^{(c)}) < \infty$$

Thm (Z, Lu + Sun)  
A. Todorov

$$\text{Val}_{\text{WP}}(M_{\text{epix str.}}) < \infty$$

$$\mu(E) = \frac{\text{val} \{ e \mid E(e) = E \}}{V^{(c)}}$$

$$E(e_1 \times e_2) = E(e_1) E(e_2)$$

$$\supseteq \text{val}(e_1 \times e_2) = \text{val}(e_1) \text{val}(e_2)$$

OK for generic CY, HK  
Multiplicative ensemble

Def:  $e$  is prime if  
not  $e_1 \times e_2$   $m_1, m_2 > 0$

$$e(m, a)$$

$\infty$	$f_m$
$\prod_{m=1}^{\infty}$	$\prod_{m=1}^{\infty}$

CAUTION

$$c(n, l) = c(n, -l)$$

$$\left. \begin{array}{l} 0 < D \leq m^2 \\ l \leq m, n \geq 0 \end{array} \right\}$$

$$) \in \mathbb{Z}_+$$

$$V^{(c)} = \sum_{\alpha} \text{val}(\mathcal{M}_{\alpha}^{(c)}) < \infty$$

Thm (Z, Lu + Sun) Val<sub>NP</sub>(M<sub>cpixstr</sub>) < ∞  
A. Todorov

$$\mu(E) = \frac{\text{val} \{ e \mid E(e) = E \}}{V^{(c)}}$$

$$\begin{aligned} E(e_1 \times e_2) &= E(e_1) E(e_2) \\ \Rightarrow \text{val}(e_1 \times e_2) &= \text{val}(e_1) \text{val}(e_2) \end{aligned}$$

OK for generic CY, HK  
Multiplicative ensemble

Def:  $e$  is prime if not  $e_1 \times e_2$   $m_1, m_2 > 0$

$$e(m, a) \quad a = 1, \dots, f_m$$

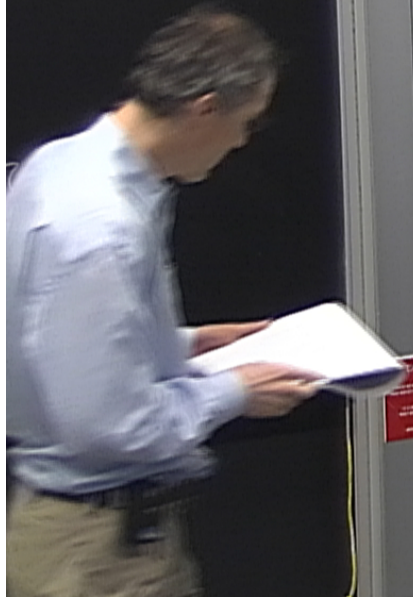
$$\prod_{m=1}^{\infty} \prod_{a=1}^{f_m} \frac{1}{1 - V(m, a) E(m, a)^s q^m}$$

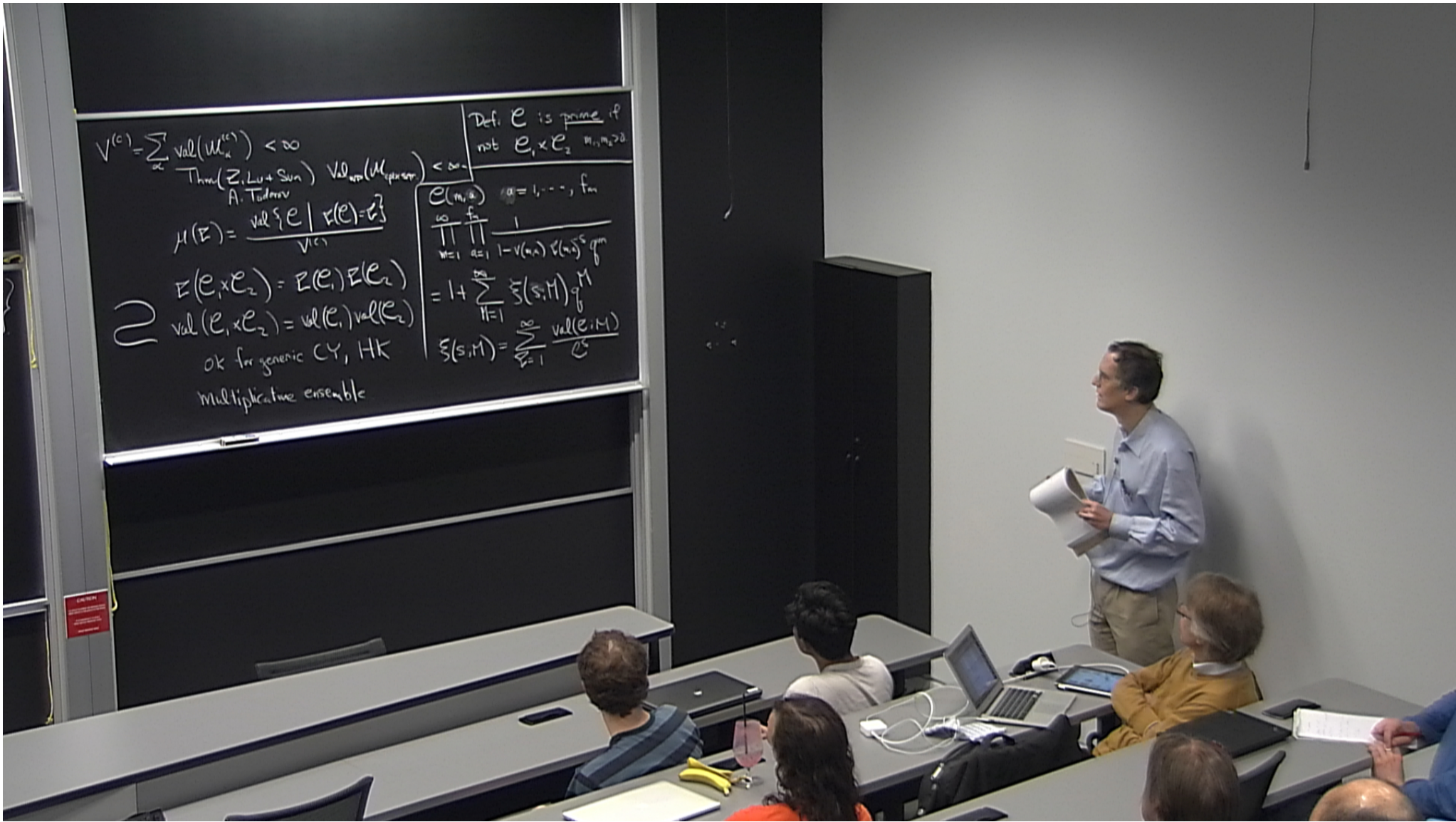


$c(n, l) = c(n, -l)$   
 $0 < D \leq m^2$   
 $l \leq m, n \geq 0$   
 $\dots) \in \mathbb{Z}_+$

$V^{(C)} = \sum_{\alpha} \text{val}(M_{\alpha}^{(C)}) < \infty$   
 Thm (Z, Lu + Sun)  $\text{val}_{\text{np}}(M_{\text{cplx str.}}) < \infty$   
 A. Todorov  
 $\mu(E) = \frac{\text{val}\{e \mid \chi(e) = \tau\}}{V^{(C)}}$   
 $\mathbb{Z} \ni \chi(e_1 \times e_2) = \chi(e_1) \chi(e_2)$   
 $\text{val}(e_1 \times e_2) = \text{val}(e_1) \text{val}(e_2)$   
 ok for generic CY, HK  
 Multiplicative ensemble

not  $e_1 \times e_2$   $m_1, m_2 > 0$   
 $e(m, a) \quad a = 1, \dots, f_m$   
 $\prod_{m=1}^{\infty} \prod_{a=1}^{f_m} \frac{1}{1 - v(m, a) e^{(m, a) s} q^m}$   
 $= 1 + \sum_{M=1}^{\infty} \xi(s, M) q^M$   
 $\xi(s, M) = \sum_{e \in M} \frac{\text{val}(e; M)}{e^s}$





$V^{(c)} = \sum_{\alpha} \text{val}(W_{\alpha}^{(c)}) < \infty$   
 Thm (Z. Lu + Sun)  $\text{Val}_{\text{gen}}(W_{\text{gen}}) < \infty$   
 A. Tudman  
 $\mu(E) = \frac{\text{val}\{e \mid v(e) = c\}}{V^{(c)}}$   
 $E(e_1, e_2) = E(e_1)E(e_2)$   
 $\text{val}(e_1, e_2) = \text{val}(e_1)\text{val}(e_2)$   
 ok for generic CY, HK  
 multiplicative ensemble

Def.  $e$  is prime if  
 not  $e_1 \times e_2$   $m_1, m_2 > 0$

$e(m, a) \quad a = 1, \dots, f_m$   
 $\prod_{m=1}^{\infty} \prod_{a=1}^{f_m} \frac{1}{1 - v(m, a) v^{(m)} q^m}$   
 $= 1 + \sum_{M=1}^{\infty} \xi(s, M) q^M$   
 $\xi(s, M) = \sum_{e=1}^{\infty} \frac{\text{val}(e, M)}{e^s}$

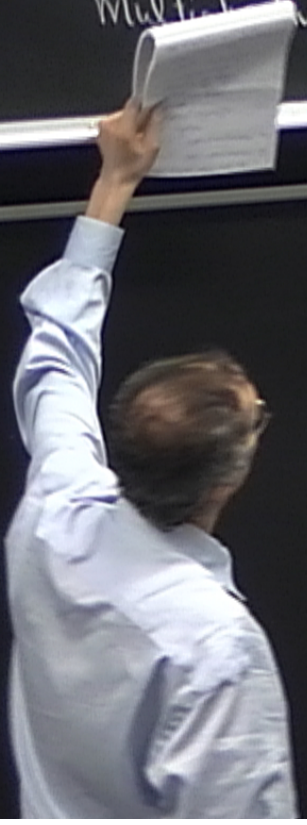


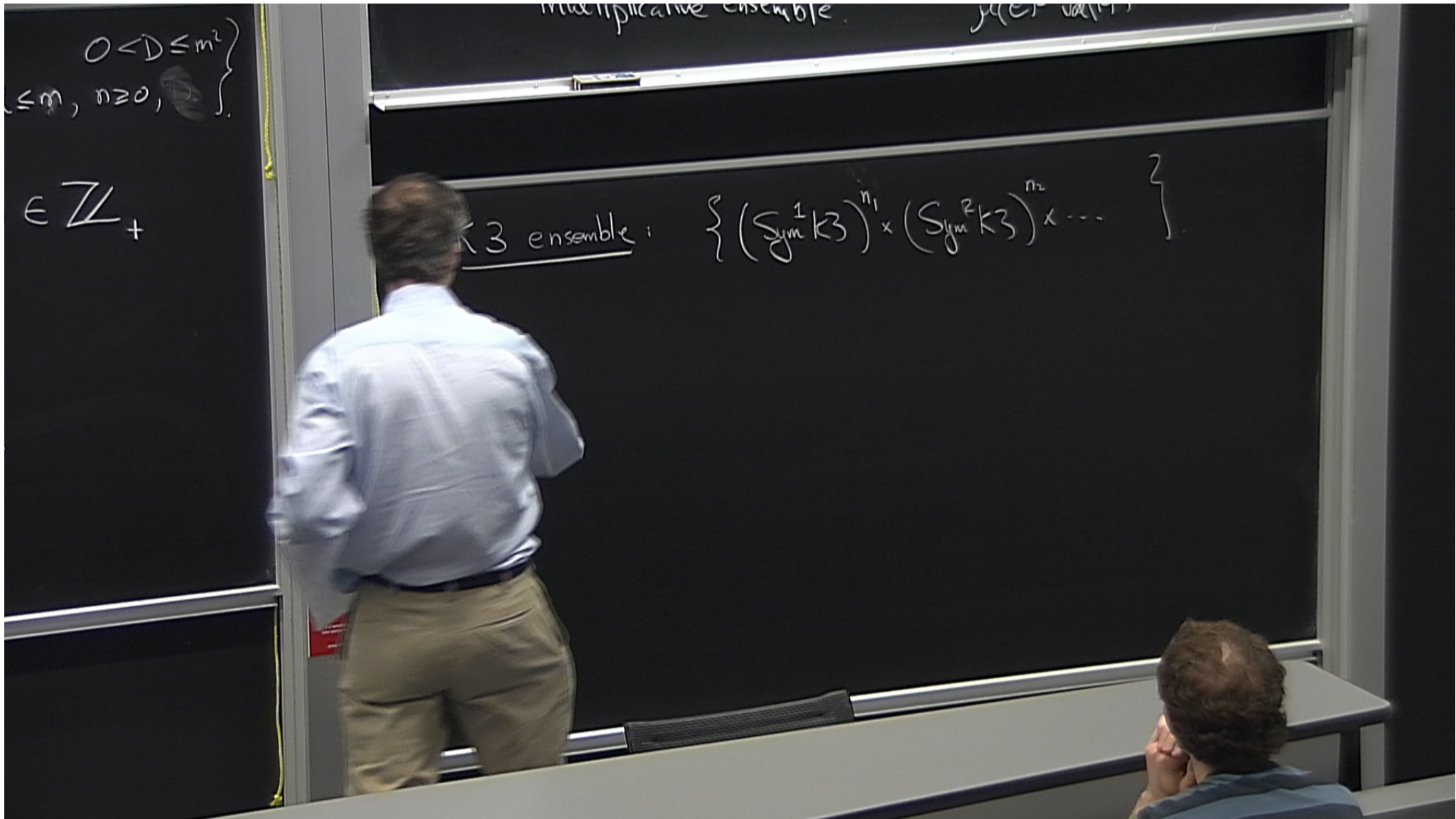
$E(\tau, z)$   
(n-p)

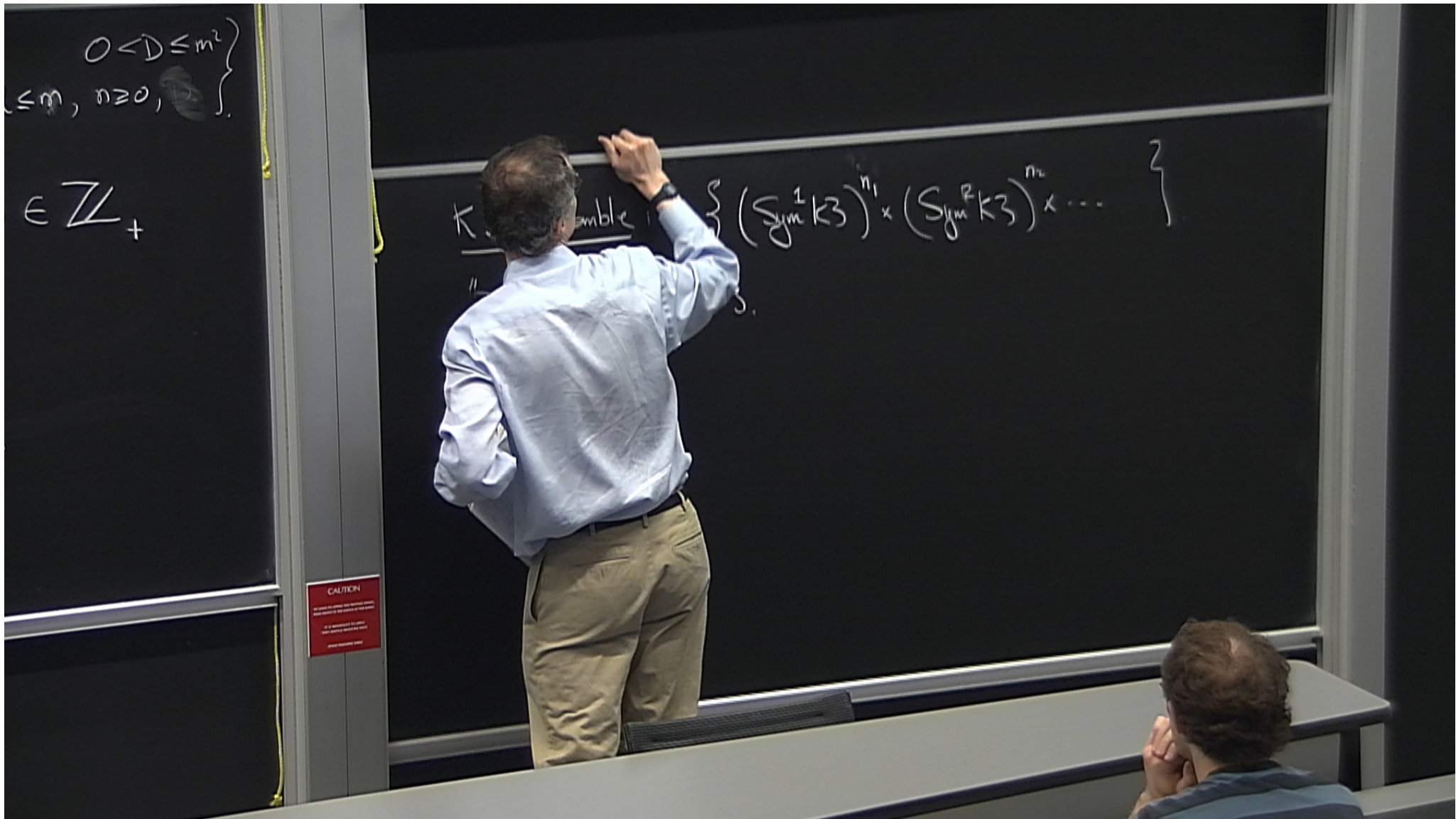
$Z(\mathcal{C}_1 \times \mathcal{C}_2) = Z(\mathcal{C}_1) Z(\mathcal{C}_2)$   
 $\text{val}(\mathcal{C}_1 \times \mathcal{C}_2) = \text{val}(\mathcal{C}_1) \text{val}(\mathcal{C}_2)$   
ok. for generic CY, HK  
multiplicative ensemble

$$= 1 + \sum_{H=1}^{\infty} \xi(s; M) q^H$$
$$\xi(s; M) = \sum_{\text{val}(\mathcal{C}; M)=1}^{\infty} \frac{\text{val}(\mathcal{C}; M)}{q^s}$$
$$\mu(\mathcal{C}) = \frac{\text{val}(\mathcal{C}; M)}{\text{val}(M)}$$

$\langle D \leq m^2 \rangle$   
 $\geq 0$   
+







$$\left. \begin{array}{l} 0 < D \leq m^2 \\ \leq m, n \geq 0, \dots \end{array} \right\} \\ \in \mathbb{Z}_+$$

ensemble

$$\left\{ (\text{Sym}^1 K^3)^{n_1} \times (\text{Sym}^2 K^3)^{n_2} \times \dots \right\}$$

CAUTION  
NE PAS TOUCHER LES ÉCRANS  
ÉLECTRONIQUES DE LA SALLE  
DE LEÇURES  
NE PAS TOUCHER LES ÉCRANS  
ÉLECTRONIQUES DE LA SALLE  
DE LEÇURES

$$\left. \begin{array}{l} 0 < D \leq m^2 \\ \leq m, n \geq 0, \end{array} \right\}$$

$$\in \mathbb{Z}_+$$

$$\underline{K3 \text{ ensemble}}: \left\{ (\text{Sym}^1 K3)^{n_1} \times (\text{Sym}^2 K3)^{n_2} \times \dots \right\}$$

"Primes"  $\text{Sym}^N K3$ .

$$\mathcal{M}_1 = \alpha(\mathbb{H}^{4,20}) \backslash O(4,20) / O(4) \times O(20)$$

CAUTION  
 BE CAREFUL OF THE HOT SURFACE  
 WHEN OPERATING OR MAINTAINING THE BOARD  
 ALL SURFACES ARE HOT  
 PLEASE REPORT ANY DAMAGE

$$\left. \begin{array}{l} 0 < D \leq m^2 \\ l \leq m, n \geq 0, \dots \end{array} \right\}$$

$$\in \mathbb{Z}_+$$

$$\underline{K3 \text{ ensemble}}: \left\{ (\text{Sym}^1 K3)^{n_1} \times (\text{Sym}^2 K3)^{n_2} \times \dots \right\}$$

"Primes"  $\text{Sym}^N K3$ .

$$= d(\mathbb{H}^{4,20}) \backslash O(4,20) / O(4) \times O(20)$$

$$M_N = O(\mathbb{P}^1) \backslash O(4,21) / O(4) \times O(21)$$

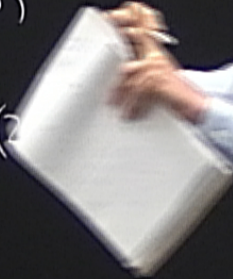
CAUTION  
DO NOT TOUCH THE BOARD OR THE MARKERS  
IF YOU NEED TO USE THE BOARD  
PLEASE ASK THE LECTURER

K3 ensemble:  $\left\{ (\text{Sym}^1 k3)^{n_1} \times (\text{Sym}^2 k3)^{n_2} \times \dots \right\}$

"Primes"  $\text{Sym}^n k3$ .

$$\mathcal{M}_1 = \frac{O(\mathbb{I}^{4,20}) \backslash O(4,20) / O(4) \times O(20)}{80}$$

$$N > 1 \quad \mathcal{M}_N = \frac{O(\mathbb{I}^{4,21}) \backslash O(4,21) / O(4) \times O(21)}{80}$$



CAUTION

K3 ensemble:  $\left\{ (\text{Sym}^1 k3)^{n_1} \times (\text{Sym}^2 k3)^{n_2} \times \dots \right\}$

"Primes"  $\text{Sym}^n k3$ .

$$\mathcal{M}_1 = \frac{O(\mathbb{P}^1) \times O(4, 20)}{O(4) \times O(20)} \quad 80$$

$$N > 1 \quad \mathcal{M}_N = \frac{O(\mathbb{P}^1) \times O(4, 21)}{O(4) \times O(21)} \quad 84$$

K3 ensemble:  $\left\{ (\text{Sym}^1 k3)^{n_1} \times (\text{Sym}^2 k3)^{n_2} \times \dots \right\}$

"Primes"  $\text{Sym}^N k3$ .

$$\mathcal{M}_1 = \alpha(\mathbb{P}^1) \backslash \text{O}(4, 20) / \text{O}(4) \times \alpha(20) \quad 80$$

$$N > 1 \quad \mathcal{M}_N = \text{O}(\mathbb{P}^1) \backslash \text{O}(4, 21) / \text{O}(4) \times \text{O}(21) \quad 84$$

$$\vec{u} \in \mathbb{P}^{5, 21} \quad \vec{u}^2 = 2N.$$

CAUTION



K3 ensemble:  $\left\{ (\text{Sym}^1 k_3)^{n_1} \times (\text{Sym}^2 k_3)^{n_2} \times \dots \right\}$

"Primes"  $\text{Sym}^N k_3$ .

$$\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^2) \setminus \mathcal{O}(4, 20) / \mathcal{O}(4) \times \mathcal{O}(20) \quad 80$$

$$N > 1 \quad \mathcal{M}_N = \mathcal{O}(\mathbb{P}^2) \setminus \mathcal{O}(4, 21) / \mathcal{O}(4) \times \mathcal{O}(21) \quad 84$$

$$\vec{u} \in \mathbb{P}^{5, 21} \quad \vec{u}^2 = 2N. \quad \mathbb{P}' = \text{I}_{\text{sat}}(\vec{u}) \subset \mathcal{O}(\mathbb{P}^{5, 21})$$

CAUTION

K3 ensemble:  $\left\{ (\text{Sym}^1 K3)^{n_1} \times (\text{Sym}^2 K3)^{n_2} \times \dots \right\}$

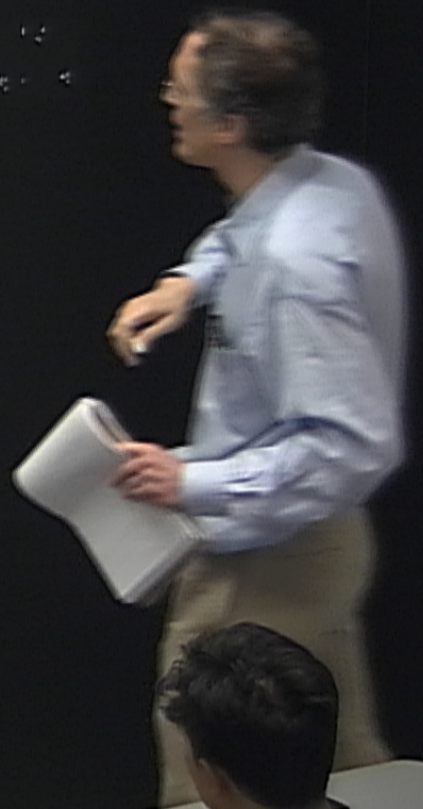
"Primes"  $\text{Sym}^N K3$ .

$$\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^1) \times \mathcal{O}(4, 20) / \mathcal{O}(4) \times \mathcal{O}(20) \quad 80$$

$$N > 1 \quad \mathcal{M}_N = \mathcal{O}(\mathbb{P}^1) \times \mathcal{O}(4, 21) / \mathcal{O}(4) \times \mathcal{O}(21) \quad 94$$

$$\vec{u} \in \mathbb{P}^{5, 21} \quad \vec{u}^2 = 2N. \quad \mathbb{P}' = \text{I}_{\text{sat}}(\vec{u}) \subset \mathcal{O}(\mathbb{P}^{5, 21})$$

$$V_N = \left\{ \begin{array}{l} v \\ N=1 \end{array} \right.$$



CAUTION

K3 ensemble:  $\left\{ (\text{Sym}^1 K3)^{n_1} \times (\text{Sym}^2 K3)^{n_2} \times \dots \right\}$

"Primes"  $\text{Sym}^N K3$ .

$$\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^1) \times \mathcal{O}(4, 20) / \mathcal{O}(4) \times \mathcal{O}(20) \quad 80$$

$$N > 1 \quad \mathcal{M}_N = \mathcal{O}(\mathbb{P}^1) \times \mathcal{O}(4, 21) \times \mathcal{O}(21) \quad 94$$

$$\vec{u} \in \mathbb{P}^{5, 21} \quad \vec{u}^2 = 2 \quad \text{I}_{\text{sat}}(\vec{u}) \subset \mathcal{O}(\mathbb{P}^{5, 21})$$

$$V_N = \begin{cases} V & N=1 \\ N^d W & N>1 \end{cases}$$

$$d = \frac{1}{2} \dim(\mathcal{M}_N) = 42$$

CAUTION

K3 ensemble:  $\left\{ (\text{Sym}^1 K3)^{n_1} \times (\text{Sym}^2 K3)^{n_2} \times \dots \right\}$

"Primes"  $\text{Sym}^N K3$ .

$$\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^2) \backslash \mathcal{O}(4, 20) / \mathcal{O}(4) \times \mathcal{O}(20) \quad 80$$

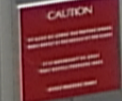
$$N > 1 \quad \mathcal{M}_N = \mathcal{O}(\mathbb{P}^2) \backslash \mathcal{O}(4, 21) / \mathcal{O}(4) \times \mathcal{O}(21) \quad 84$$

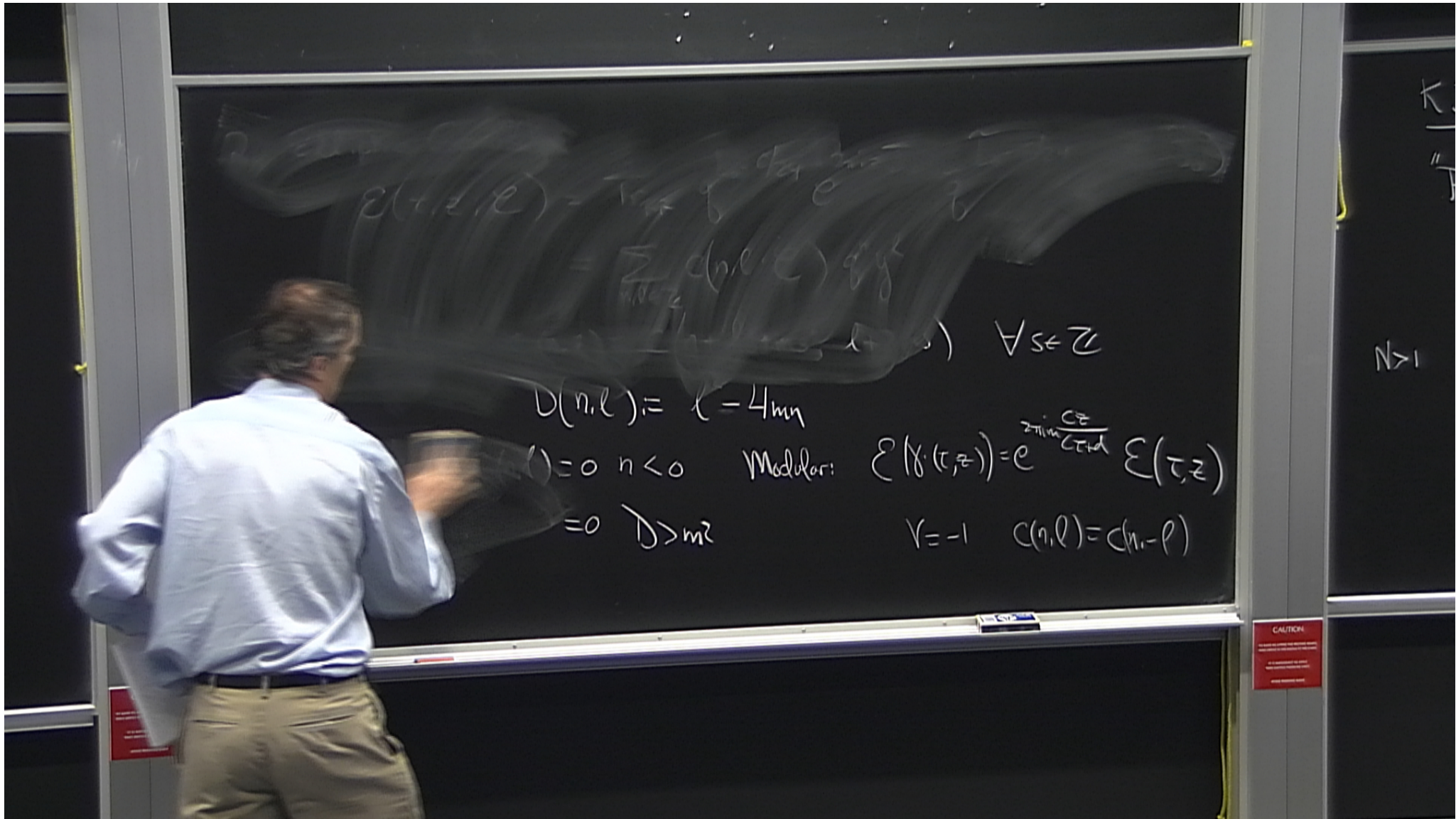
$$\vec{u} \in \mathbb{P}^{5, 21} \quad \vec{u}^2 = 2N. \quad \mathbb{P}' = \text{I}_{\text{sat}}(\vec{u}) \subset \mathcal{O}(\mathbb{P}^{5, 21})$$

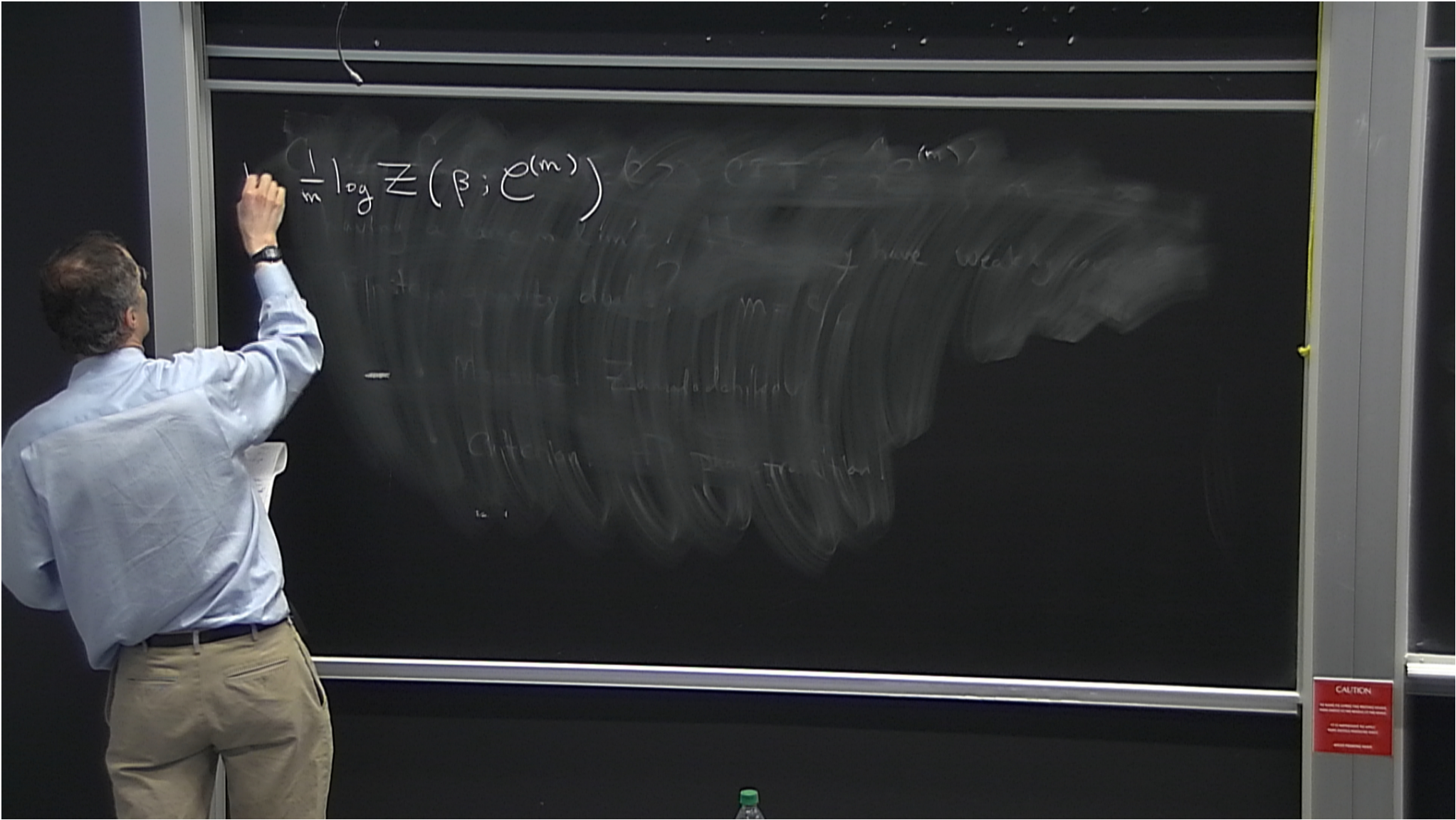
$$\mathcal{O} \rightarrow \mathcal{O}^{(1)} + \dots + \mathcal{O}^{(N)}$$

$$V_N = \begin{cases} V & N=1 \\ N^d W & N>1 \end{cases}$$

$$d = \frac{1}{2} \dim(\mathcal{M}_N) = 42$$

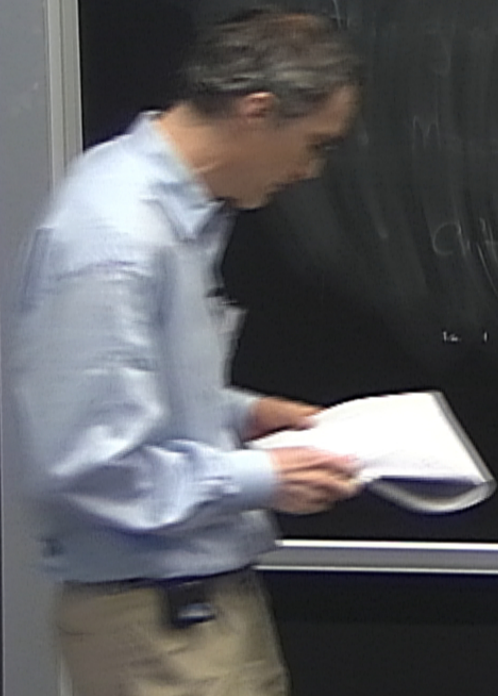
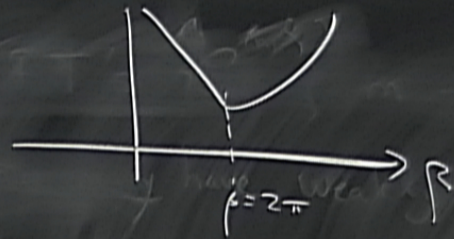






$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \rho^{(m)})$$

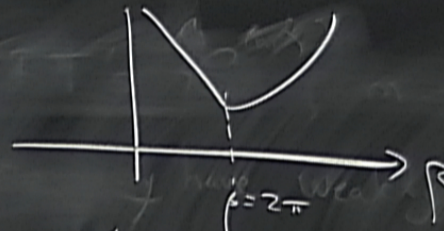
$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$



CAUTION  
Do not touch the chalkboard surface.  
Use the eraser for cleaning.  
Do not use sharp objects.



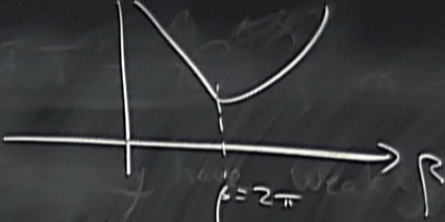
$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$



$$Z(\beta; \mathcal{C}^{(m)}) \sim e^{m F(\beta) + \mathcal{O}(\log m)}$$



CAUTION  
The board is heavy and may become loose.  
Please do not touch the board.  
If you need to touch the board,  
please use the board eraser.

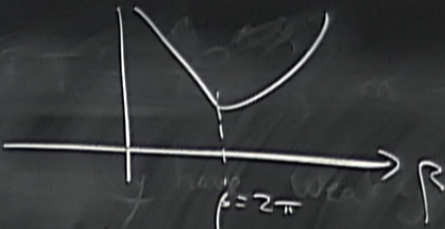
$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$


$$\log Z(\beta; \mathcal{C}^{(m)}) \underset{m \rightarrow \infty}{\sim} m F(\beta) + \mathcal{O}(\log m)$$

DMMV:

$$Z_{NS,R}(\tau; \mathcal{C}) = q^{m/4} \mathcal{E}(\tau, z = \tau/2; \mathcal{C})$$

$\mathcal{C}^{(m)} = S$

$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$


$$\log Z(\beta; \mathcal{C}^{(m)}) \underset{m \rightarrow \infty}{\sim} m F(\beta) + \mathcal{O}(\log m)$$

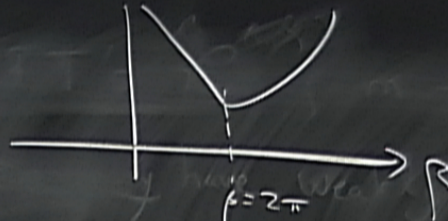
DMMV:

$$Z_{NS,R}(\tau; \mathcal{C}) = q^{m/4} \mathcal{E}(\tau, z = \tau/2; \mathcal{C})$$

$$\mathcal{C}^{(m)} = \text{Sym}^m K3$$

$$F(\beta) = \begin{cases} \beta/4 & \beta \geq 2\pi \end{cases}$$

$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$

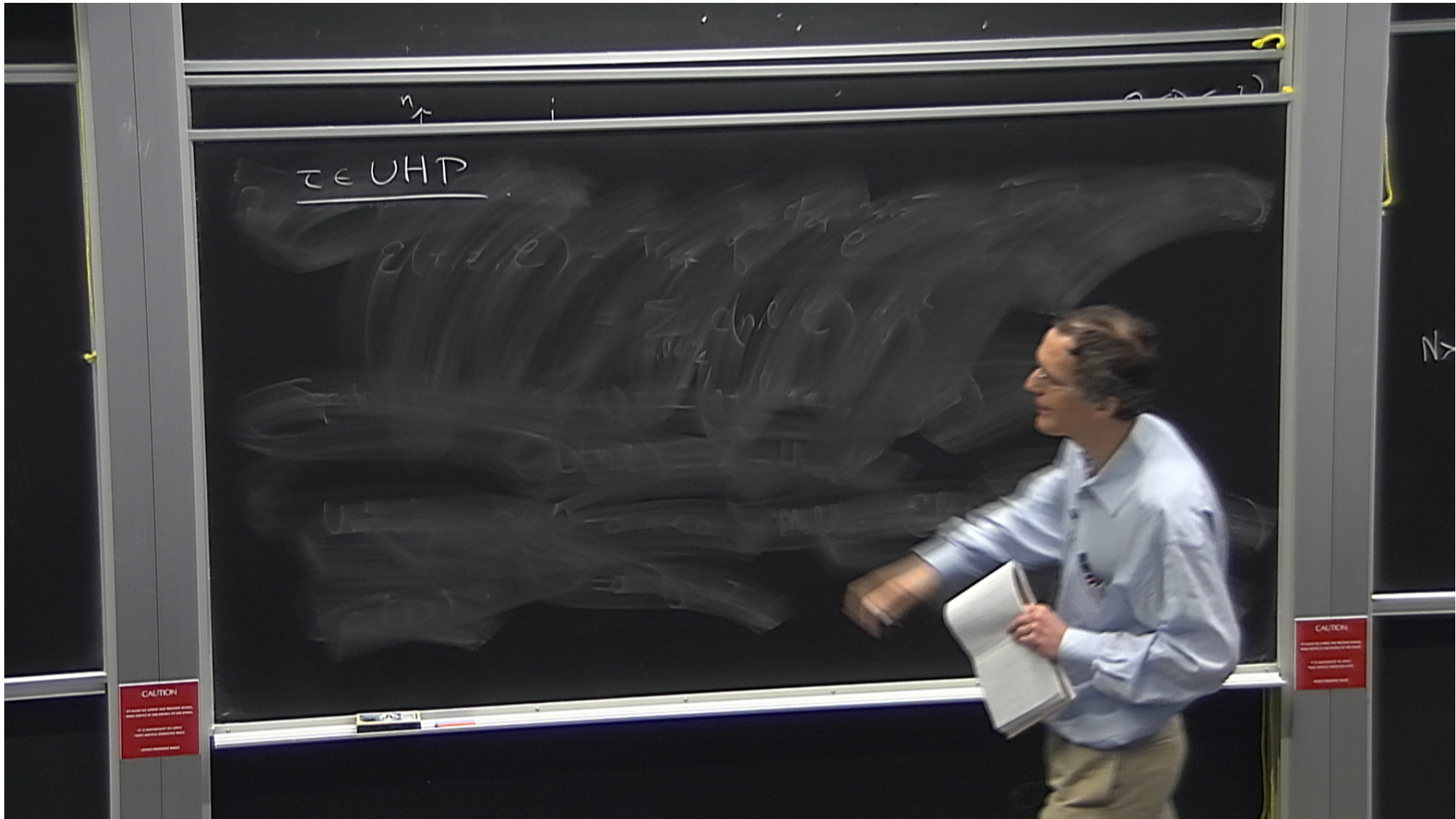


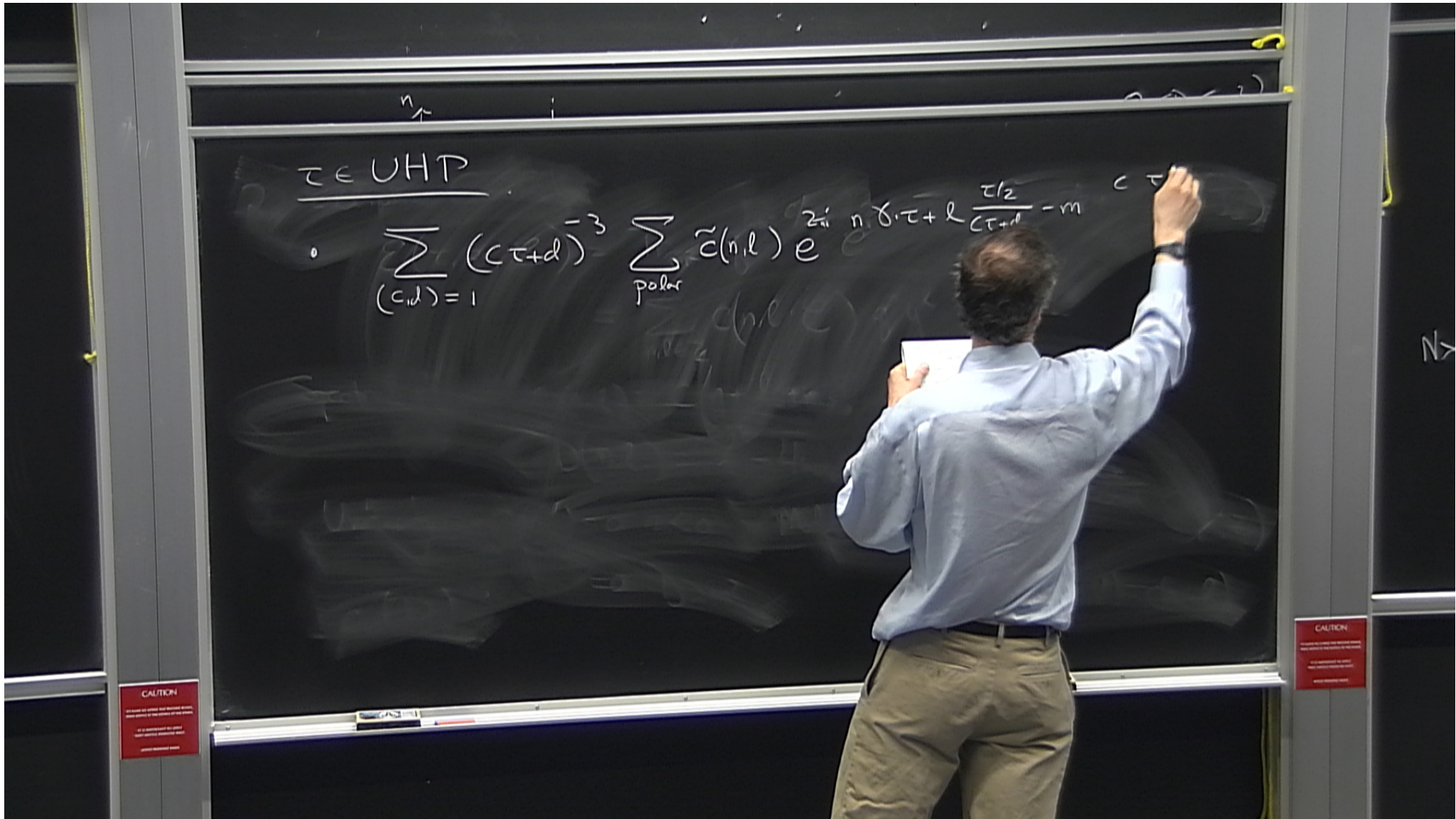
$$\log Z(\beta; \mathcal{C}^{(m)}) \underset{m \rightarrow \infty}{\sim} m F(\beta) + \mathcal{O}(\log m)$$

SMMV:

$$Z_{NS,R}(\tau; \mathcal{C}) = q^{m/4} \mathcal{E}(\tau, z = \tau/2; \mathcal{C})$$

$$F(\beta) = \begin{cases} \beta/4 & \beta > 2\pi \\ \frac{\pi^2}{\beta} & \beta \leq 2\pi \end{cases}$$





$\tau \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\text{polar}} \tilde{c}(n,l) e^{2i n \delta \cdot \tau + l \left( \frac{\tau/2}{c\tau+d} - m \right)}$$

CAUTION  
 Do not touch the board when it is in use.  
 Do not touch the board when it is in use.  
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$\tau \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\text{polar}} \tilde{c}(n,l) e^{2\pi i \left[ n \delta \cdot \tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$$

$\text{fix} = \text{span}(c,d)$

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD FRAME  
OR THE BOARD MOUNTING BRACKET

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
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$\tau \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\text{polar}} \tilde{c}(n,l) e^{2\pi i \left[ n \delta \cdot \tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$$

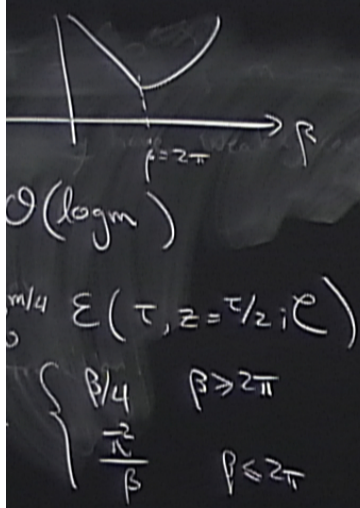
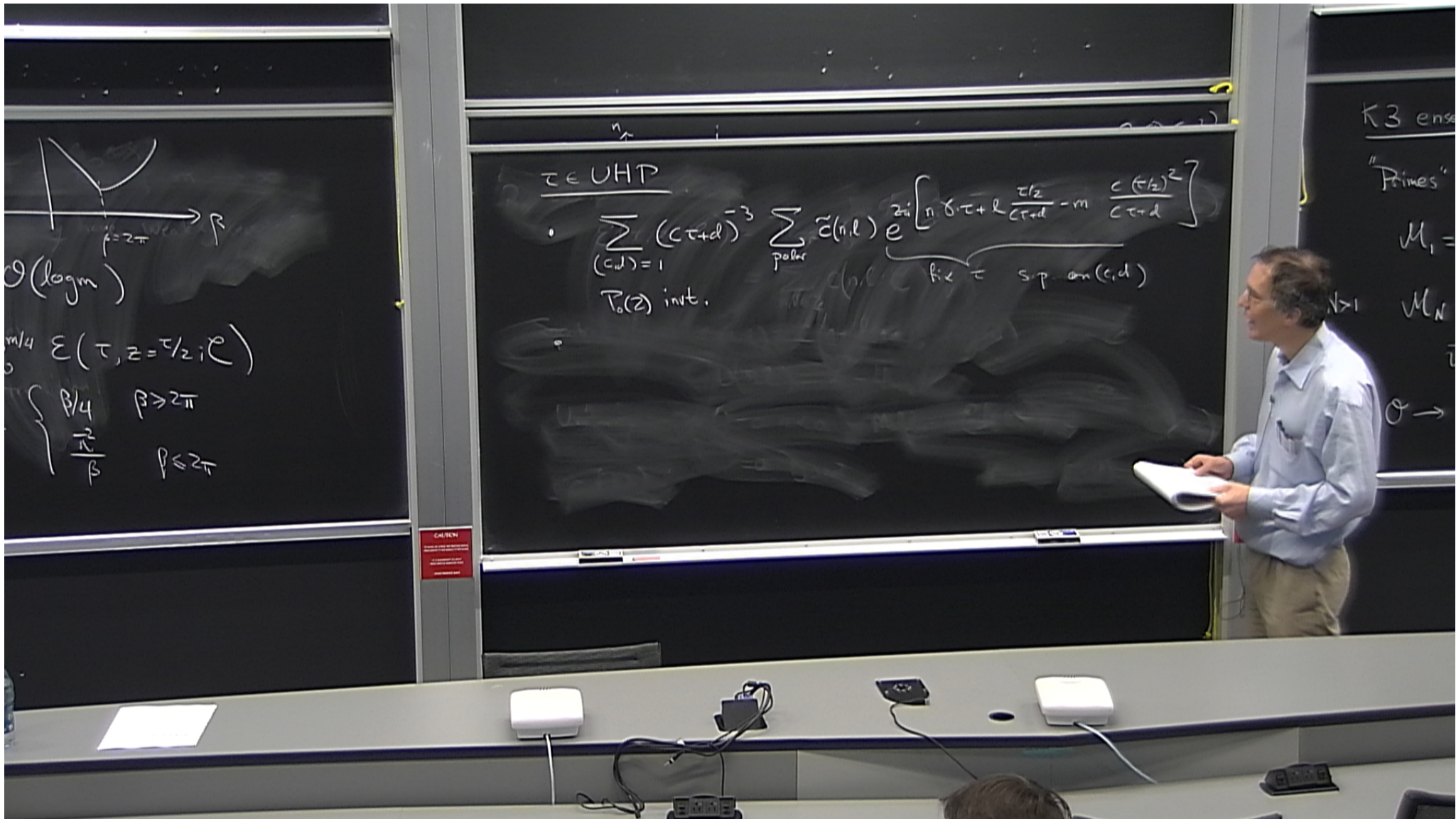
$P_0(z)$  invt.

fix = sp. on (c,d)

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD OR THE BOARD OR THE BOARD

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD OR THE BOARD OR THE BOARD





$z \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\text{poles}} \tilde{z}(n,l) e^{zi \left[ n\delta\tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$$

fix = s.p. on (c,d)

$T_0(z)$  int.

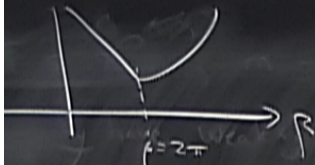
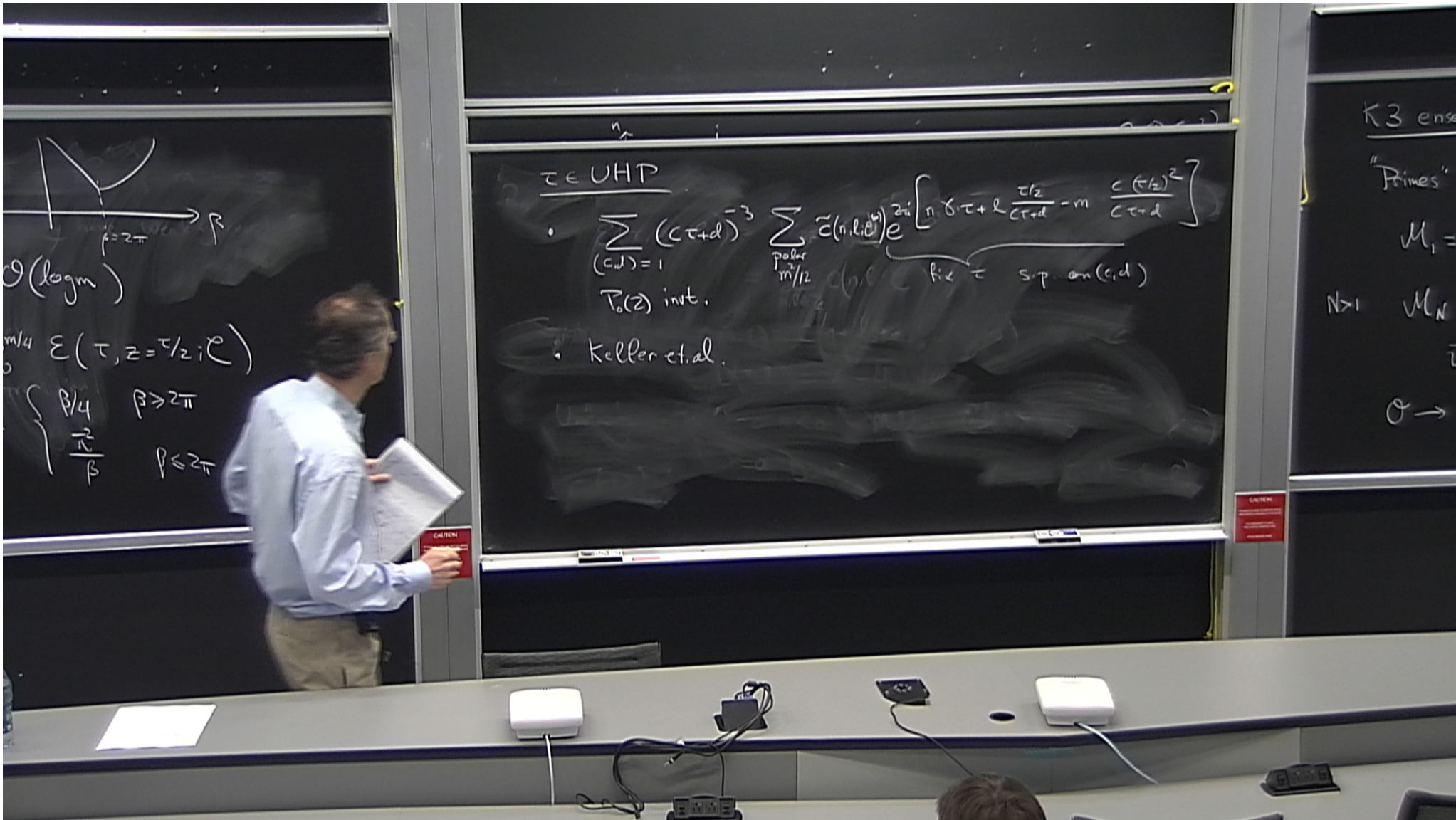
K3 ens

"Primes"

$M_1 =$

$M_N$

$\sigma \rightarrow$



$O(\log m)$

$\frac{m/4}{\beta} \varepsilon(\tau, z = \tau/2 i^{\beta})$

$\left\{ \begin{array}{ll} \beta/4 & \beta \geq 2\pi \\ \frac{2}{\beta} & \beta \leq 2\pi \end{array} \right.$

$z \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/2}} \tilde{c}(n, \tau) z^n e^{\left[ n \delta \tau + k \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$$

fix = s.p. on (c,d)

$T_0(z)$  int.

Keller et al.

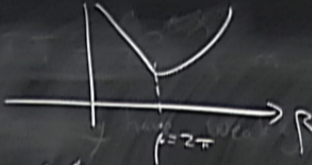
$K3$  ens

"Primes"

$\mu_1 =$

$N > 1 \quad \mu_N$

$\sigma \rightarrow$

$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$


$$\log Z(\beta; \mathcal{C}^{(m)}) \underset{m \rightarrow \infty}{\sim} m F(\beta) + \mathcal{O}(\log m)$$

DMTV:  $Z_{NS,R}(\tau; \mathcal{C}) = q^{m/4} \mathcal{E}(\tau, z = \tau/2; \mathcal{C})$

$$\mathcal{C}^{(m)} = \sum_{\gamma \in \mathcal{C}} k_{\gamma} \dots \quad \tau = \frac{i\beta}{2\pi} \quad F(\beta) = \begin{cases} \beta/4 & \beta \geq 2\pi \\ \frac{\pi^2}{\beta} & \beta \leq 2\pi \end{cases}$$

$z \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/2}}$$

$T_0(z)$  inst.

Keller et al.

$$\lim_{m \rightarrow \infty} \frac{1}{m} \log Z(\beta; \mathcal{C}^{(m)}) = F(\beta)$$

$$\log Z(\beta; \mathcal{C}^{(m)}) \sim m F(\beta) + o(m) \quad (\log m)$$



DMFT:

$$Z_{NS, \mathbb{R}} \sim \int \mathcal{E}(\tau, z = \tau/2i) \mathcal{C}$$

$$\mathcal{C}^{(m)} = \sum_{\text{spin}} k_3$$

$$\tau = \frac{i\beta}{2}$$

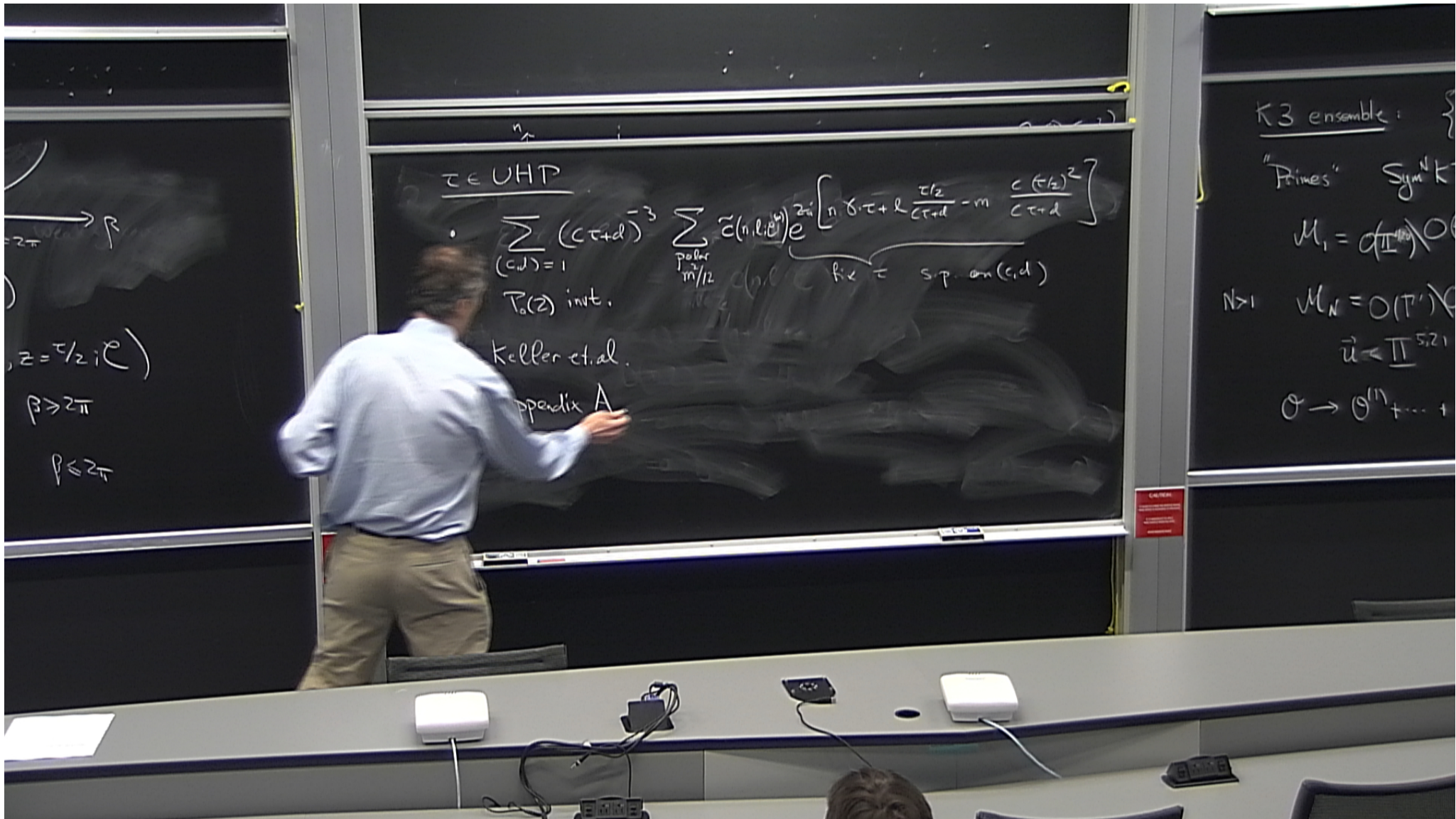
$$\left. \begin{array}{l} \beta/4 \quad \beta \geq 2\pi \\ \frac{\pi^2}{\beta} \quad \beta \leq 2\pi \end{array} \right\}$$

$z \in \text{UHP}$

$$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\text{poles}} \frac{1}{m^{3/2}}$$

$T_0(z)$  invt.

Keller et al.



$\beta > 2\pi$   
 $z = \tau/2 i c$   
 $\beta > 2\pi$   
 $\beta < 2\pi$

$\tau \in \text{UHP}$   
 $\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/12}} \tilde{c}(n, l, \tau) e^{2\pi i \left[ n \delta \cdot \tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$   
 $\tau_0(z)$  invt.  
 Keller et al.  
 Appendix A

K3 ensemble:  
 "Primes" Sym<sup>N</sup> K  
 $\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(1)$   
 $N > 1$   $\mathcal{M}_N = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(N)$   
 $\vec{u} \in \mathbb{P}^{5,2,1}$   
 $\mathcal{O} \rightarrow \mathcal{O}^{\oplus 11} + \dots$

$\beta > 2\pi$   
 $z = \tau/2 i$   
 $\beta > 2\pi$   
 $\beta < 2\pi$

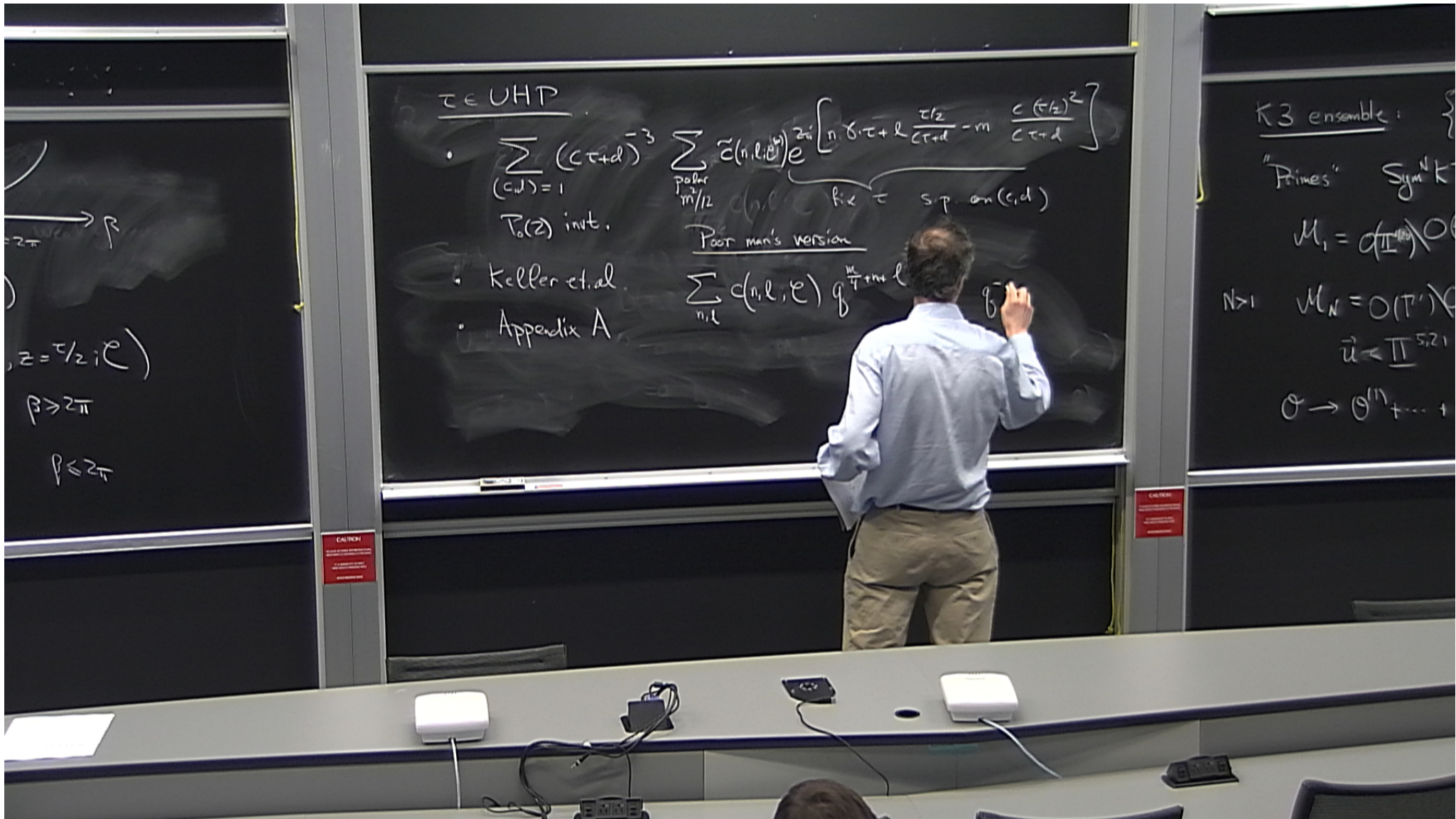
$\tau \in \text{UHP}$   
 $\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/12}} \tilde{c}(n, \tau) e^{2\pi i \left[ n \delta \cdot \tau + \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$   
 $T_0(z)$  invt.  
 • Keller et al.  
 • Appendix A

K3 ensemble:  
 "Primes" Sym<sup>k</sup>  
 $\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(1)$   
 $N > 1$   $\mathcal{M}_N = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(N)$   
 $\vec{u} \in \mathbb{T}^{5,21}$   
 $\mathcal{O} \rightarrow \mathcal{O}^{\oplus 11} + \dots$

$\beta > 2\pi$   
 $z = \tau/2 i \mathcal{C}$   
 $\beta > 2\pi$   
 $\beta < 2\pi$

$\tau \in \text{UHP}$   
 $\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/12}} \tilde{c}(n, l; \tau) e^{2\pi i \left[ n \delta \cdot \tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$   
 $\tau_0(z)$  invt.  
Port man's version  
 $f_z = \text{sp. an}(c, d)$   
 • Keller et al.  
 • Appendix A

K3 ensemble:  
 "Primes" Sym<sup>n</sup> k  
 $\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(1)$   
 $N > 1$   $\mathcal{M}_N = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(N)$   
 $\vec{u} \in \mathbb{P}^{5,2,1}$   
 $\mathcal{O} \rightarrow \mathcal{O}^{\oplus 11} + \dots$



$z \in \text{UHP}$

- $$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/2}} \tilde{c}(n,l,e) e^{2\pi i \left[ n\delta\tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$$

$\text{fix } \tau = \text{sp. on } (c,d)$   
Poor man's version

- Keller et al.
- Appendix A

K3 ensemble:

"Primes"  $\text{Sym}^k$

$\mathcal{M}_1 = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(1)$

$N > 1$   $\mathcal{M}_N = \mathcal{O}(\mathbb{P}^1) \otimes \mathcal{O}(N)$

$\vec{u} \in \mathbb{P}^{5,2,1}$

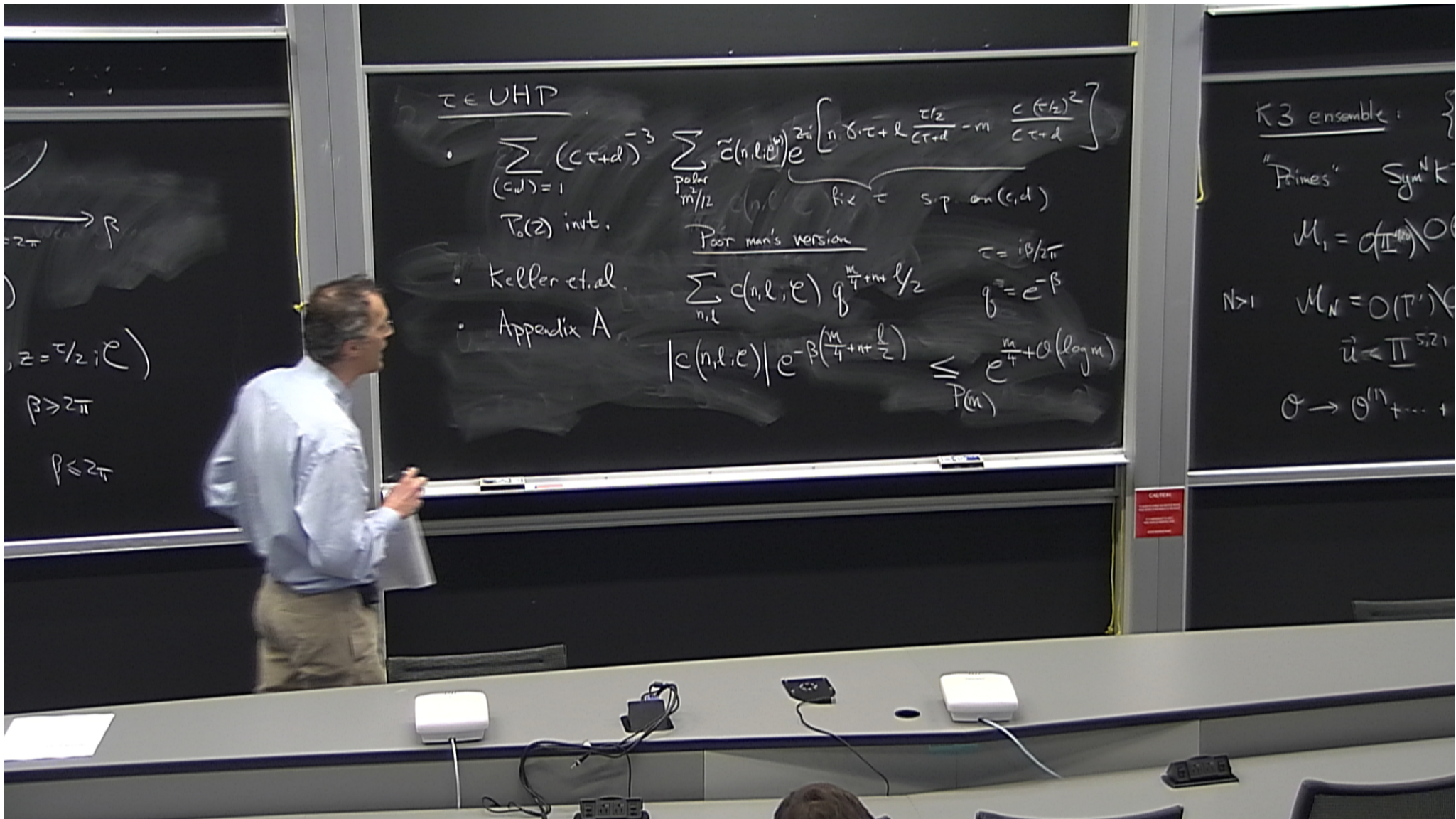
$\sigma \rightarrow \sigma^{(1)} + \dots$

$\beta > 2\pi$

$\beta < 2\pi$

$z = \tau/2 + i\epsilon$





$z \in \text{UHP}$

- $$\sum_{(c,d)=1} (c\tau+d)^{-3} \sum_{\substack{\text{poles} \\ m/2}} \tilde{c}(n,l,e) e^{2\pi i \left[ n\delta\tau + l \frac{\tau/2}{c\tau+d} - m \frac{c(\tau/2)^2}{c\tau+d} \right]}$$

$T_0(z)$  invt.       $\text{poles } m/2$        $\text{fix } \tau$  s.p. on  $(c,d)$

POOR MAN'S VERSION
- Keller et al.       $\sum_{n,l} c(n,l,e) q^{\frac{m}{4} + n + \frac{l}{2}}$        $\tau = i\beta/2\pi$
- Appendix A       $|c(n,l,e)| e^{-\beta(\frac{m}{4} + n + \frac{l}{2})} \leq \frac{e^{\frac{m}{4} + O(\log m)}}{P(m)}$        $\beta = e^{-\beta}$

K3 ensemble:

"Primes" Sym<sup>k</sup>

$\mathcal{M}_1 = O(\pi^{1/2})$

$N > 1$        $\mathcal{M}_N = O(\pi^{N/2})$

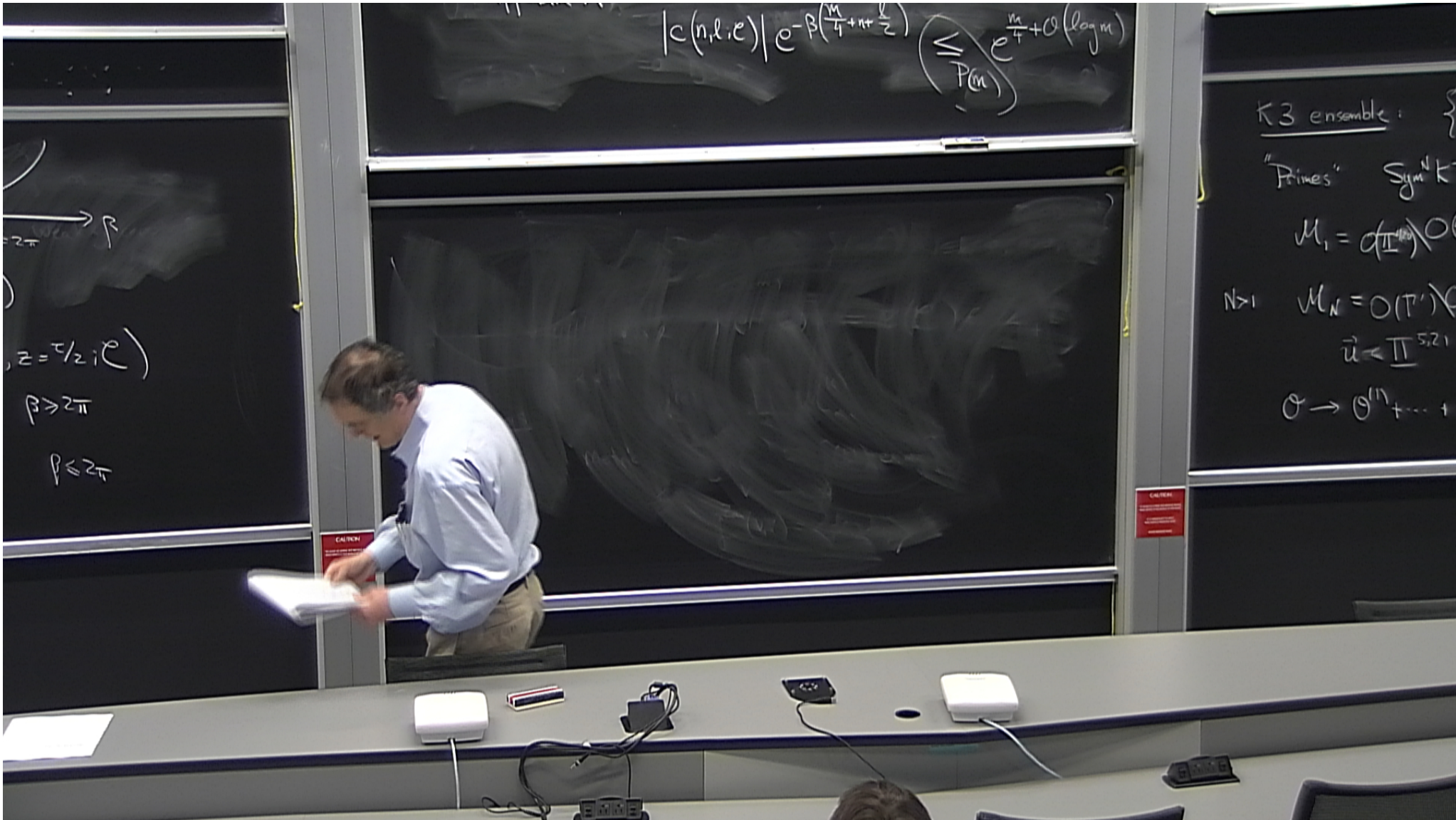
$\vec{u} \ll \pi^{5/2}$

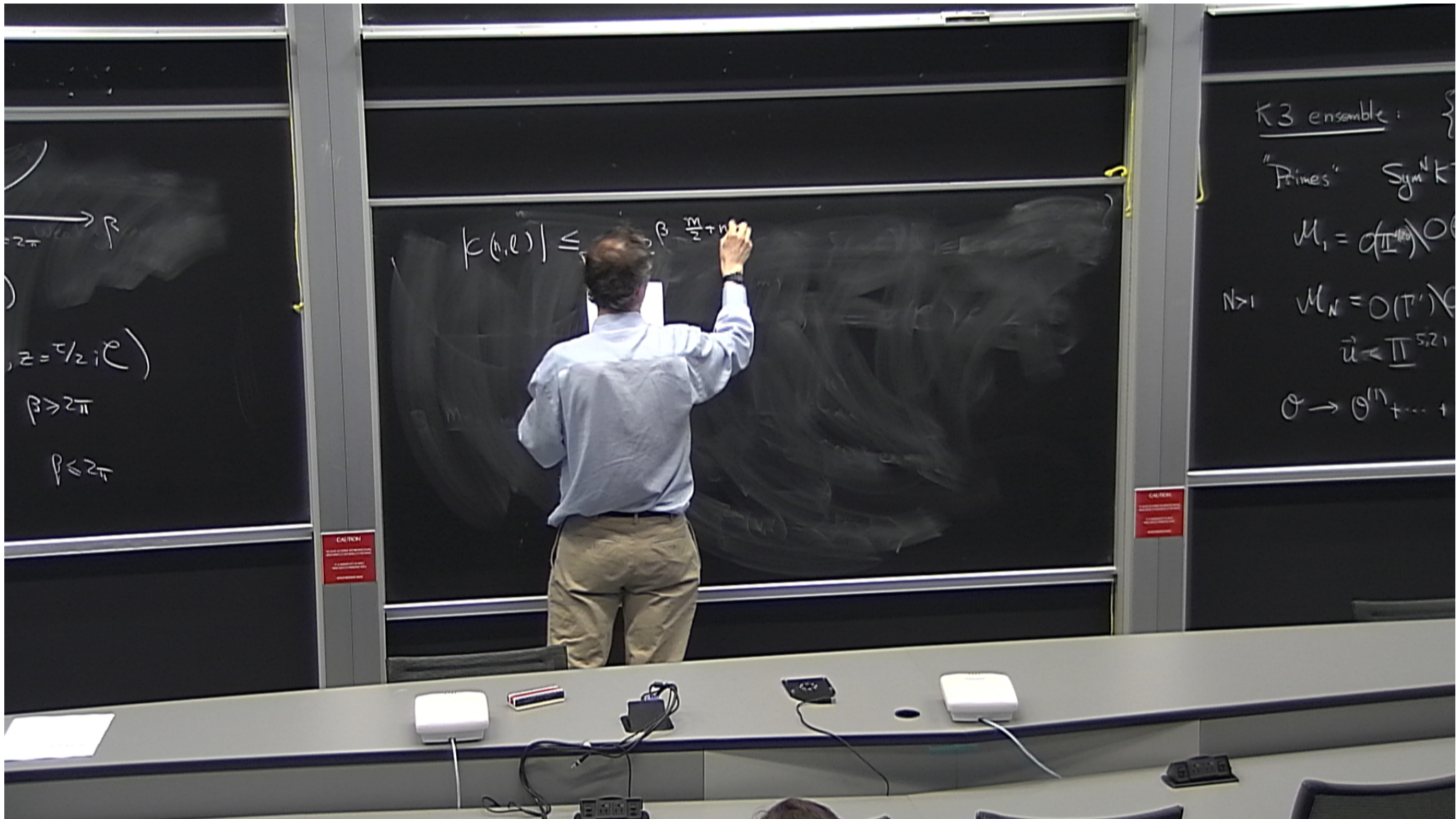
$\sigma \rightarrow O^{(1)} + \dots$

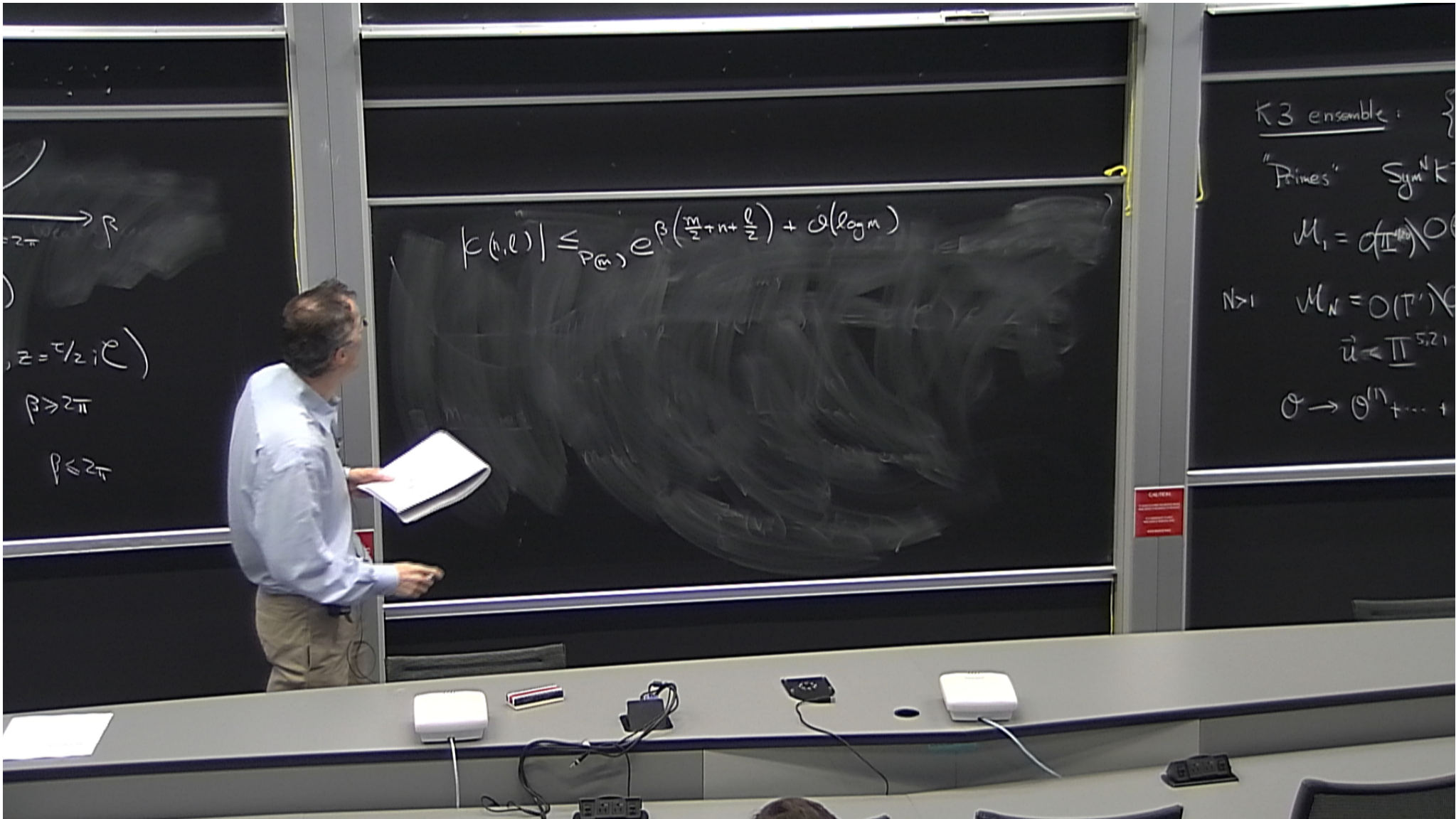
$\beta > 2\pi$

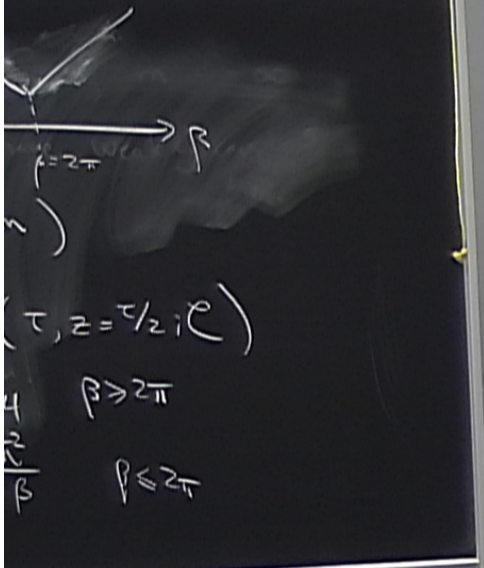
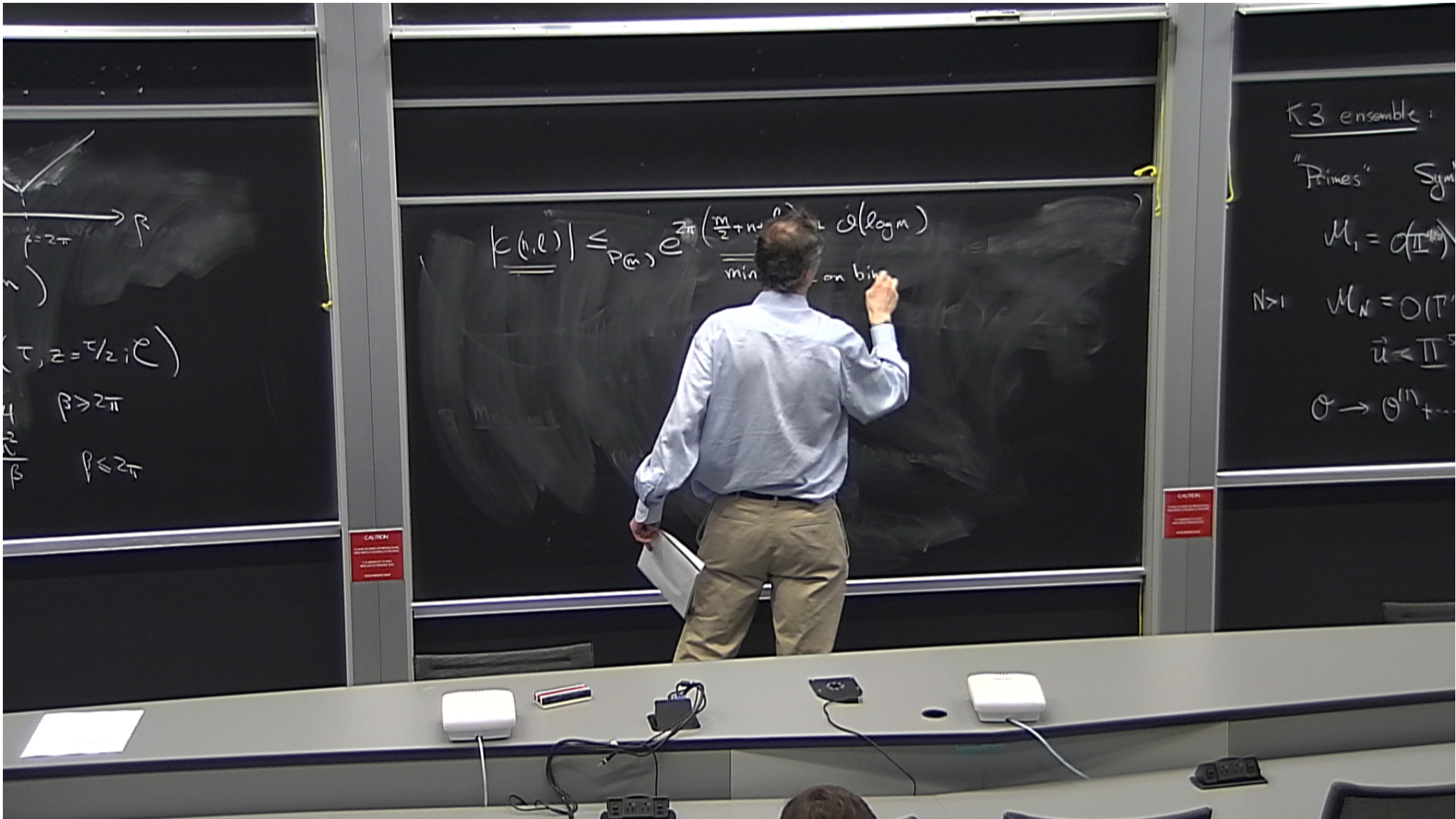
$\beta < 2\pi$

$z = \tau/2 + i\epsilon$





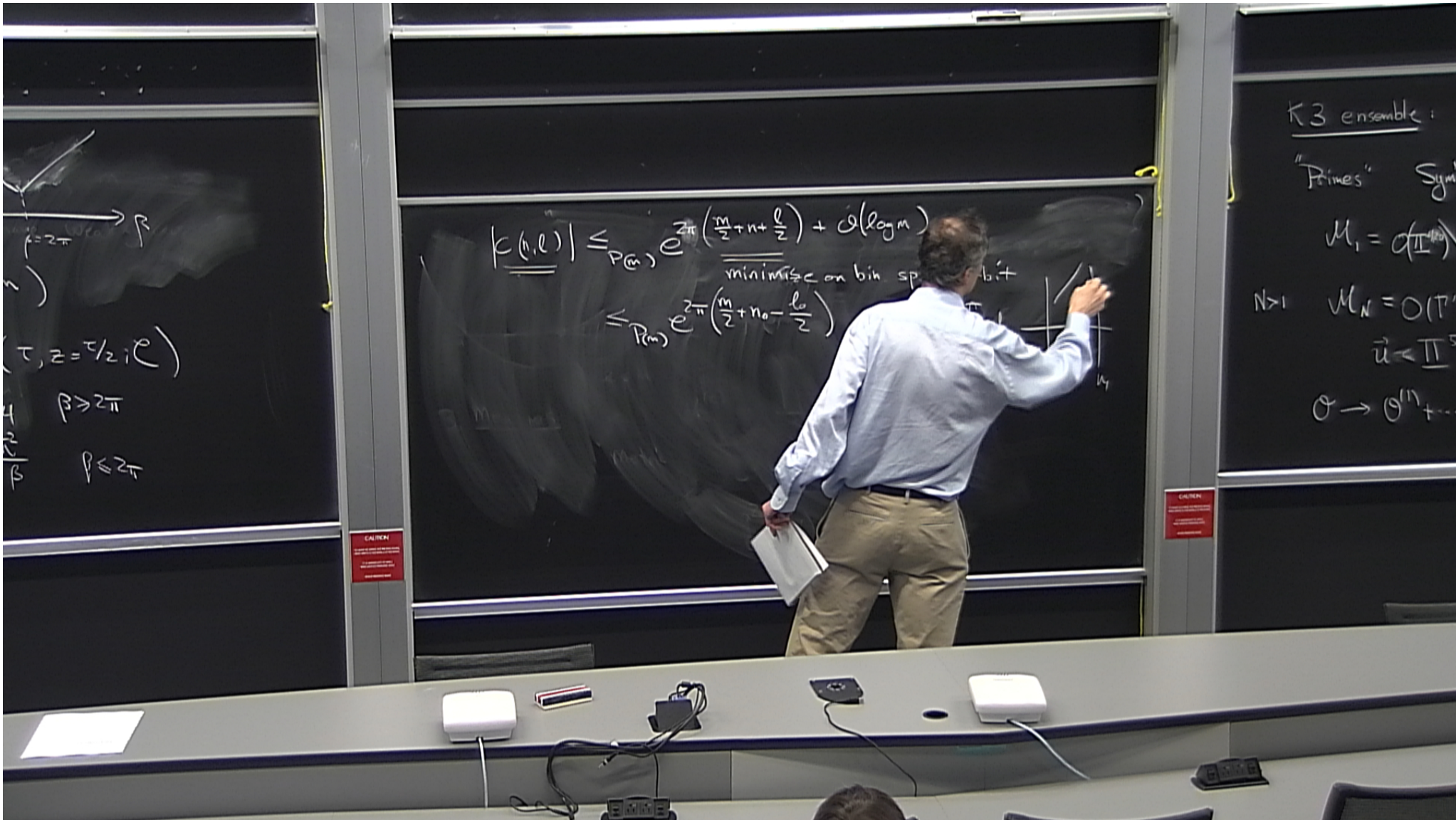




$$|k(m, l)| \leq P(m) e^{2\pi \left( \frac{m}{2} + n \right) + O(\log m)}$$

min on branch cut

K3 ensemble:  
 "Primes" Sym  
 $M_1 = O(\pi^{1/2})$   
 $N > 1 \quad M_N = O(\pi^N)$   
 $\tilde{u} < \pi^5$   
 $\sigma \rightarrow \sigma^{11} + \dots$



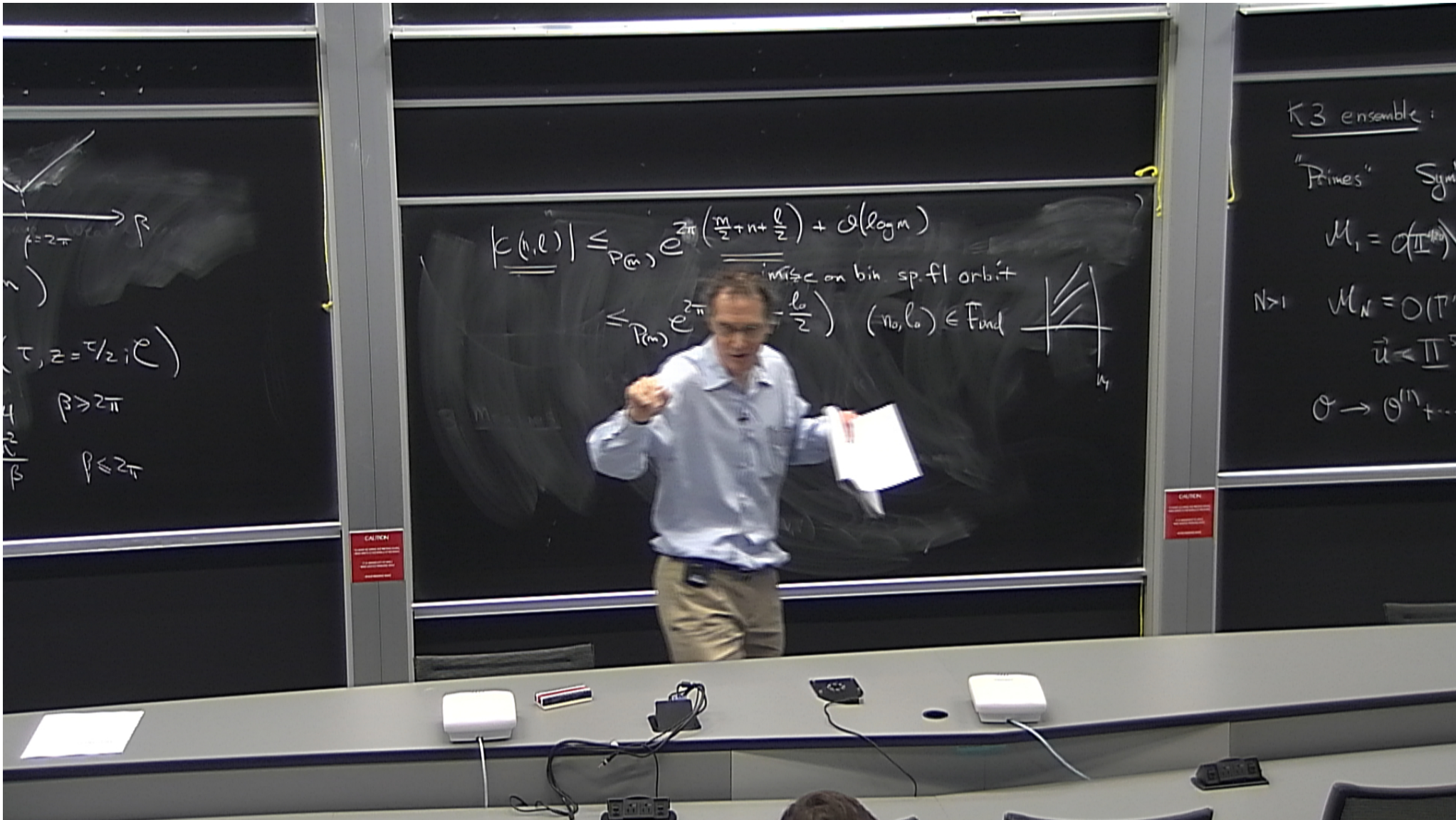
$\beta = 2\pi$   
 $(\tau, z = r/2, e)$   
 $\beta > 2\pi$   
 $\beta < 2\pi$

$$|c(n, l)| \leq \binom{m}{n} e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + \mathcal{O}(\log m)}$$

minimise on bin sp bit

$$\leq \binom{m}{n} e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)}$$

K3 ensemble:  
 "Primes" Sym  
 $\mathcal{M}_1 = \mathcal{O}(\Pi)$   
 $N > 1 \quad \mathcal{M}_N = \mathcal{O}(\Pi^N)$   
 $\vec{u} < \Pi^5$   
 $\sigma \rightarrow \sigma^{11} + \dots$



$\beta = 2\pi$   
 $(\tau, z = \tau/2, e)$   
 $\beta > 2\pi$   
 $\beta < 2\pi$

$$|k(m, l)| \leq P(m) e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + \mathcal{O}(\log m)}$$

m is on bin sp. fl orbit

$$\leq P(m) e^{2\pi \left( -\frac{l_0}{2} \right)} \quad (n_0, l_0) \in \Gamma_{\text{fund}}$$



K3 ensemble:  
 "Primes" Sym  
 $\mathcal{M}_1 = \mathcal{O}(\mathbb{I}^{(1)})$   
 $N > 1 \quad \mathcal{M}_N = \mathcal{O}(\mathbb{I}^{(N)})$   
 $\vec{u} < \mathbb{I}^5$   
 $\sigma \rightarrow \sigma^{11} + \dots$

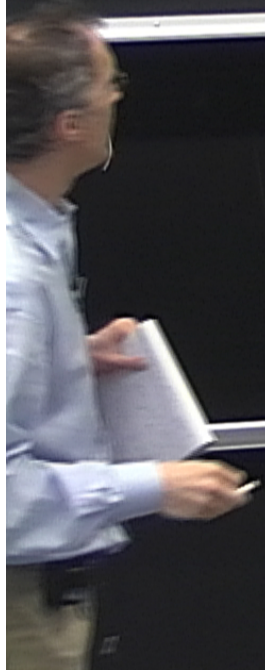
have  
al?

$$|c(n, l)| \leq P_m e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimize on bin. sp. fl. orbit

$$\leq P_m e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \text{Fund}$$

$$P(\log m) \leq e^{O(\log m)}$$



CAUTION

CAUTION



have  
l?

$$|c(n, l)| \leq P(m) e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin sp. fl. orbit

$$\leq P(m) e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \mathbb{F}_{ind}$$



$$|L(\mathcal{L}^m)| \leq e^{O(\log m)}$$



CAUTION

have  
l?

$$|c(n, l)| \leq P(m) e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin sp. fl. orbit

$$\leq P(m) e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \mathbb{F}_{ind}$$



$$|L(\mathcal{L}^m)| \leq e^{O(\log m)}$$

Examples:

$K_3$   
Prime  
 $N > 1$   
 $\mathcal{O}$

are  
?

$$|k(n, l)| \leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin. sp. fl orbit

$$\leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \text{Fund}$$



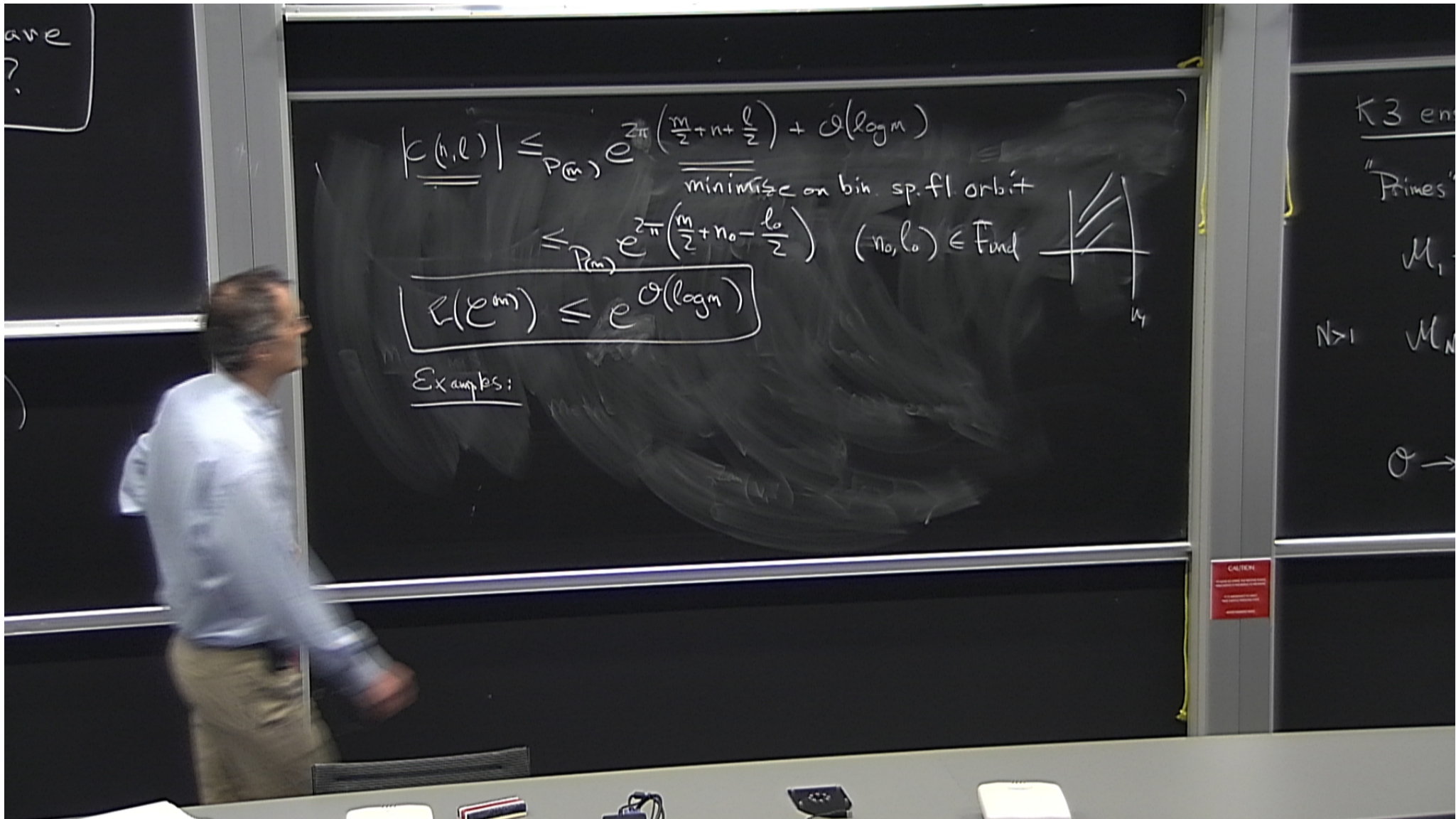
$$|L(\mathcal{L}^m)| \leq e^{O(\log m)}$$

Examps:

$K3$  em  
 "Primes"  
 $\mathcal{M}_1$   
 $N > 1$   $\mathcal{M}_N$   
 $\sigma \rightarrow$

CAUTION

CAUTION



$$|k(n, l)| \leq_{P(m)} e^{2\pi i \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin. sp. fl orbit

$$\leq_{P(m)} e^{2\pi i \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \text{Fund}$$

$$L(x^m) \leq e^{O(\log m)}$$

Examps:



K3 em  
 "Primes"  
 M,  
 N > 1  
 M  
 σ →

are  
?

$$|c(n, l)| \leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin sp. fl orbit

$$\leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \text{Fund}$$



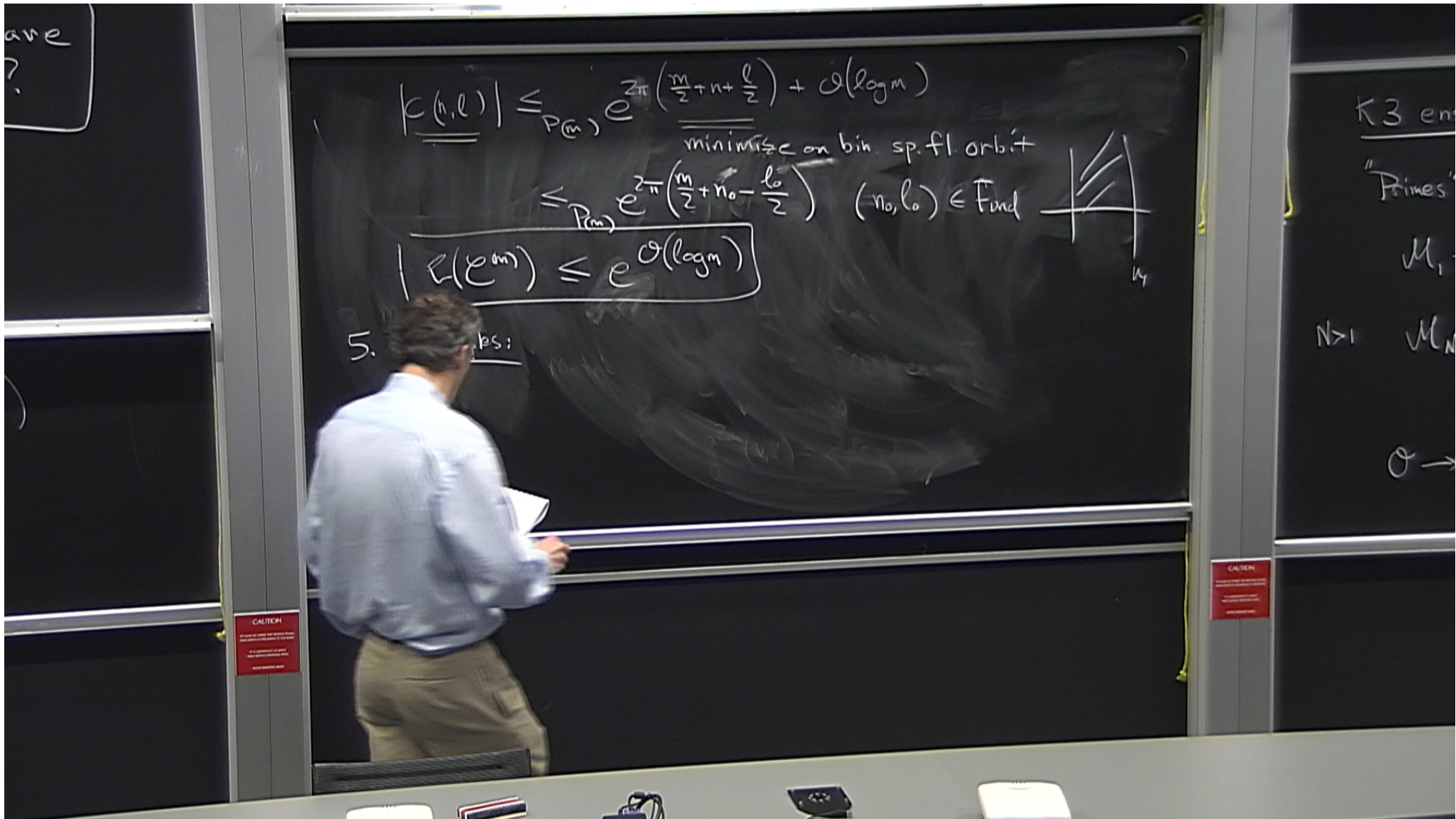
$$L(\mathcal{L}^m) \leq e^{O(\log m)}$$

Exampks:

$K3$  em  
 "Primes"  
 $\mathcal{M}_1$   
 $N > 1$   $\mathcal{M}_N$   
 $\sigma \rightarrow$

CAUTION

CAUTION



$$|c(n, l)| \leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin sp. fl orbit

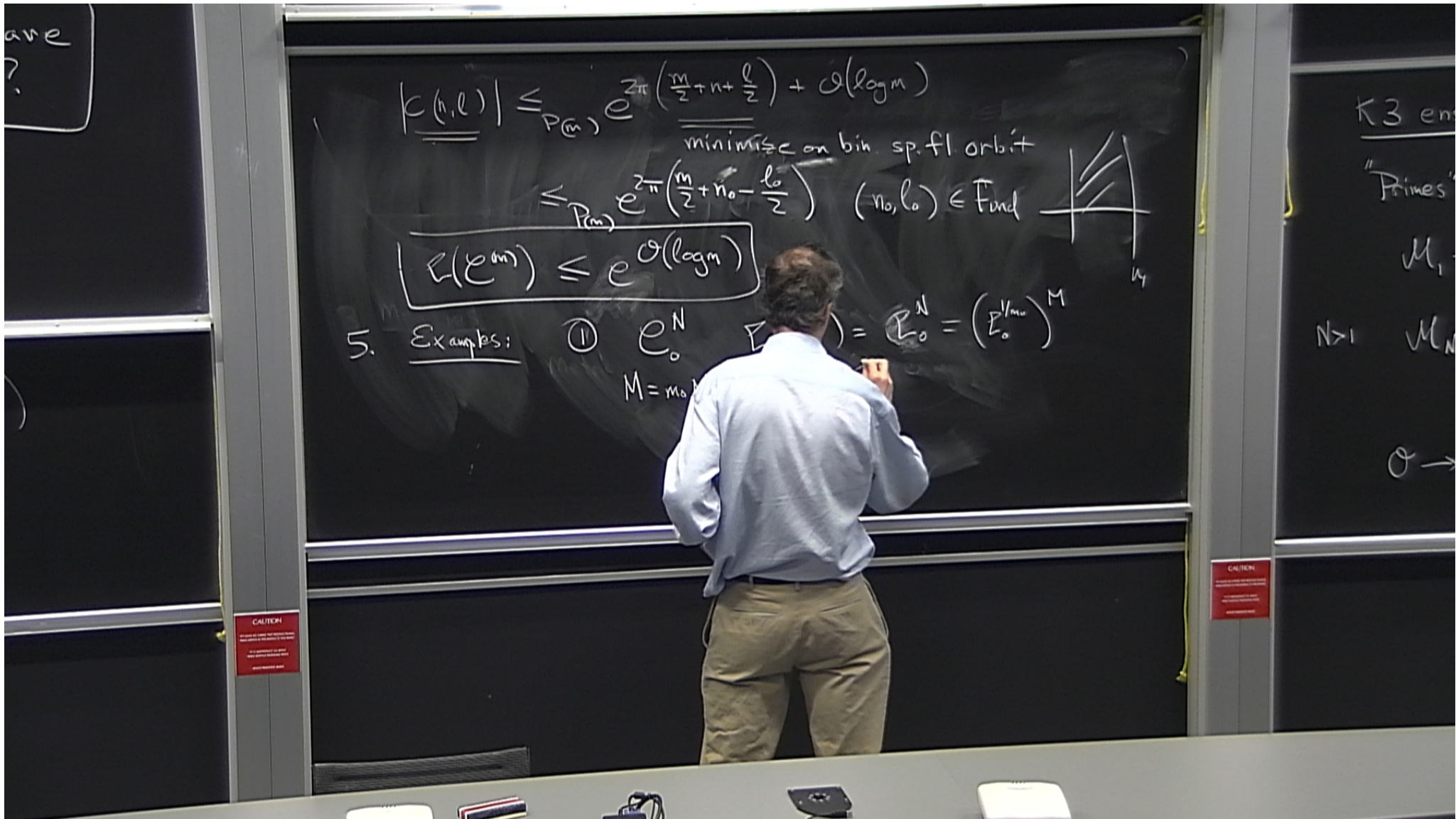
$$\leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \Gamma_{ind}$$



$$|R(\rho^m)| \leq e^{O(\log m)}$$

5. ps:

K3 em  
 "Primes"  
 $\mathcal{M}_1$   
 $N > 1$   $\mathcal{M}_N$   
 $\sigma \rightarrow$



$$|c(n, l)| \leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin. sp. fl orbit

$$\leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \text{Fund}$$



$$\boxed{L(\mathcal{L}^m) \leq e^{O(\log m)}}$$

5. Examples: ①  $e^N$   $\sum_{M=m_0}^N \dots = \sum_0^N = \left( \sum_0^{1/m_0} \right)^M$

K3 em  
"Primes"  
 $\mathcal{M}_1$   
 $N > 1$   $\mathcal{M}_N$   
 $\sigma \rightarrow$

are  
?

$$|c(n, l)| \leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n + \frac{l}{2} \right) + O(\log m)}$$

minimise on bin sp. fl orbit

$$\leq_{P(m)} e^{2\pi \left( \frac{m}{2} + n_0 - \frac{l_0}{2} \right)} \quad (n_0, l_0) \in \mathbb{F}_{ind}$$



$$\boxed{\Sigma(\mathcal{E}^m) \leq e^{O(\log m)}}$$

5. Examples: ①  $e_0^N$      $\Sigma(\mathcal{E}_0^N) = \mathcal{E}_0^N = \left( \mathcal{E}_0^{1/m_0} \right)^M$   
 $M = m_0 N$      $\mathcal{E}_0 > 1$

K3 em

"Primes"

$\mathcal{M}_1$

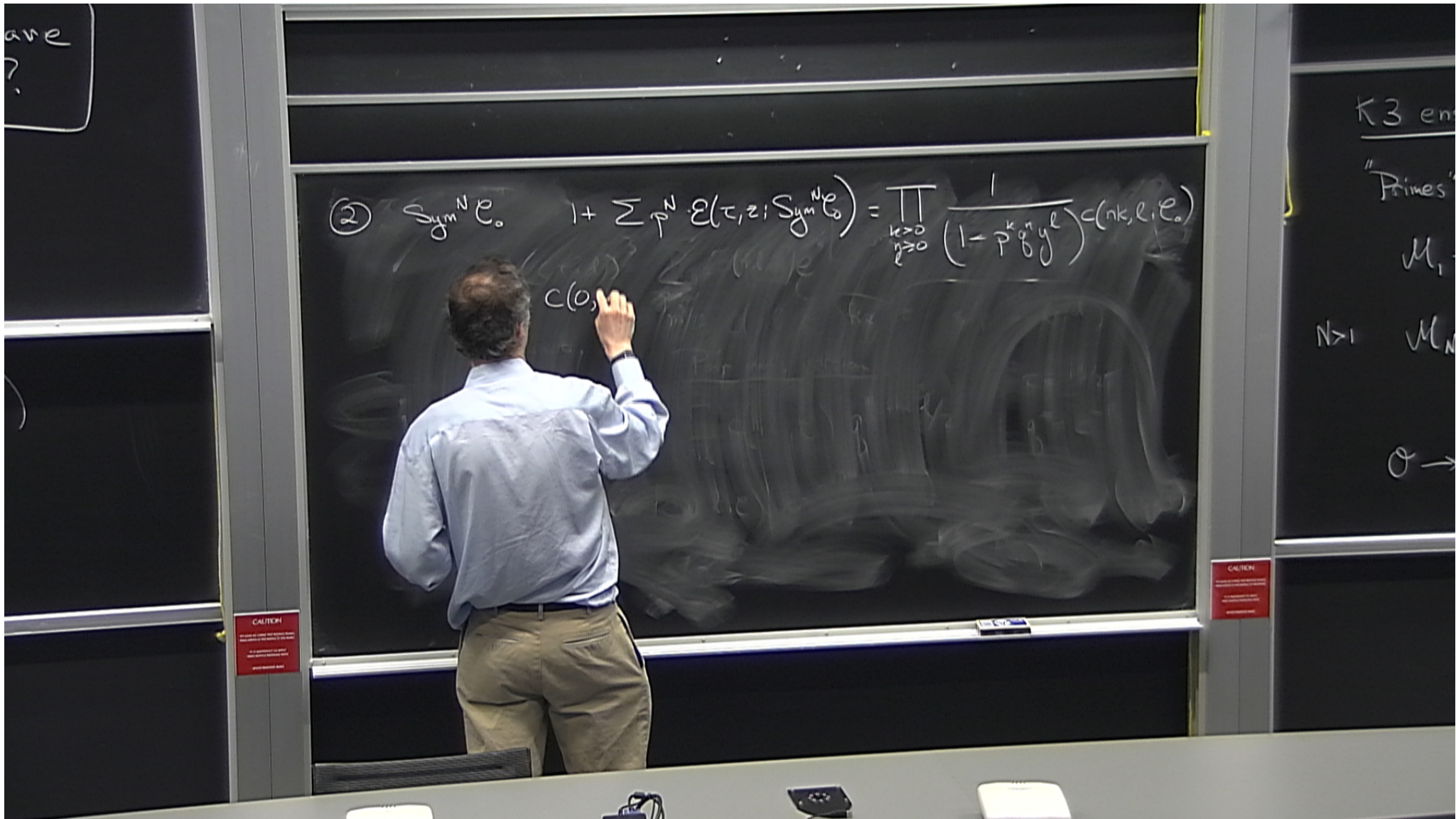
$N > 1$      $\mathcal{M}_N$

$\sigma \rightarrow$

CAUTION

CAUTION





$$\textcircled{2} \text{Sym}^N \mathcal{L}_0 \quad 1 + \sum_{p \in \mathcal{P}} \mathcal{E}(p, z; \text{Sym}^N \mathcal{L}_0) = \prod_{\substack{k \geq 0 \\ p \geq 0}} \frac{1}{(1 - p^{k+1} y^k)^{c(nk, l, \mathcal{L}_0)}}$$

$c(0)$

$K3$  em  
 "Primes"  
 $\mathcal{M}_1$   
 $N > 1$   $\mathcal{M}_N$   
 $\mathcal{O} \rightarrow$

$$\textcircled{2} \text{Sym}^N \mathcal{L}_0 \quad 1 + \sum_{p \in \mathcal{P}} \mathcal{E}(\tau, z; \text{Sym}^N \mathcal{L}_0) = \prod_{\substack{k \geq 0 \\ p \geq 0}} \frac{1}{(1 - p^{k+1} y^k)} c(nk, l, \mathcal{L}_0)$$

$$c(0, m; \mathcal{L}_0) > \frac{1}{1 - p^m}$$

$K3$  em

"Primes"

$\mathcal{M}_1$

$N > 1$   $\mathcal{M}_N$

$\sigma \rightarrow$

$$(2) \text{Sym}^N \mathcal{C}_0 \quad 1 + \sum p^N \mathcal{E}(\tau, z; \text{Sym}^N \mathcal{C}_0) = \prod_{\substack{k \geq 0 \\ p \geq 0}} \frac{1}{(1 - p^{k+1} y^k)^{c(nk, l, \mathcal{C}_0)}}$$

$$c(0, m; \mathcal{C}_0) > 0 \quad \frac{1}{(1 - p y^m)^{\mathcal{E}_0}} \Rightarrow$$

$$\mathcal{E}(\text{Sym}^N \mathcal{C}_0) = \binom{N + \mathcal{E}_0 - 1}{\mathcal{E}_0 - 1}$$

$$N = M / m_0$$

$$\textcircled{2} \quad \text{Sym}^N \mathcal{L}_0 \quad 1 + \sum p^N \mathcal{E}(\tau, z; \text{Sym}^N \mathcal{L}_0) = \prod_{\substack{k \geq 0 \\ l \geq 0}} \frac{1}{(1 - p^k y^l)^{c(nk, l, \mathcal{L}_0)}}$$

$$c(0, m; \mathcal{L}_0) > 0 \quad \frac{1}{(1 - p y^m)^{\mathcal{E}_0}} \Rightarrow$$

$$\mathcal{E}(\text{Sym}^N \mathcal{L}_0) = \binom{N + \mathcal{E}_0 - 1}{\mathcal{E}_0 - 1} = \text{polynomial in } M$$

$$N = M / m_0$$

$$(2) \text{Sym}^N \mathcal{L}_0 \quad 1 + \sum p^N \mathcal{E}(\tau, z; \text{Sym}^N \mathcal{L}_0) = \prod_{\substack{k \geq 0 \\ p \geq 0}} \frac{1}{(1 - p^{k+1} y^k)^{c(nk, l, \mathcal{L}_0)}}$$

$$c(0, m; \mathcal{L}_0) > 0 \quad \frac{1}{(1 - p y^m)^{\mathcal{E}_0}} \Rightarrow$$

$$\mathcal{E}(\text{Sym}^N \mathcal{L}_0) = \binom{N + \mathcal{E}_0 - 1}{\mathcal{E}_0 - 1} = \text{polynomial in } M$$

$$N = M/m_0 \quad \mathcal{E}_0(k_3) = 2 \quad \mathcal{E}(\text{Sym}^N k_3) = N + 1$$

$$\textcircled{2} \text{Sym}^N \mathcal{E}_0 \quad 1 + \sum_{\substack{k \geq 0 \\ l \geq 0}} p^N \mathcal{E}(\tau, z; \text{Sym}^N \mathcal{E}_0) = \prod_{k \geq 0} \frac{1}{(1 - p^k y^l)} c(nk, l; \mathcal{E}_0)$$

$$c(0, m; \mathcal{E}_0) > 0 \quad \frac{1}{(1 - p y^m)^{\mathcal{E}_0}} \Rightarrow$$

$$\mathcal{E}(\text{Sym}^N \mathcal{E}_0) = \binom{N + \mathcal{E}_0 - 1}{\mathcal{E}_0 - 1} = \text{polynomial in } M$$

$$N = M / m_0 \quad \mathcal{E}_0(k_3) = 2 \quad \mathcal{E}(\text{Sym}^N k_3) = N + 1$$

$$\textcircled{3} \text{Sym}^{N_1}(\text{Sym}^{N_2} \mathcal{E}_0)$$

$$\textcircled{2} \quad \text{Sym}^N \mathcal{C}_0 \quad 1 + \sum_{\substack{k \geq 0 \\ l \geq 0}} p^{N-k-l} \mathcal{E}(\tau, z; \text{Sym}^N \mathcal{C}_0) = \prod_{k \geq 0, l \geq 0} \frac{1}{(1 - p^k q^l)} c(nk, li; \mathcal{C}_0)$$

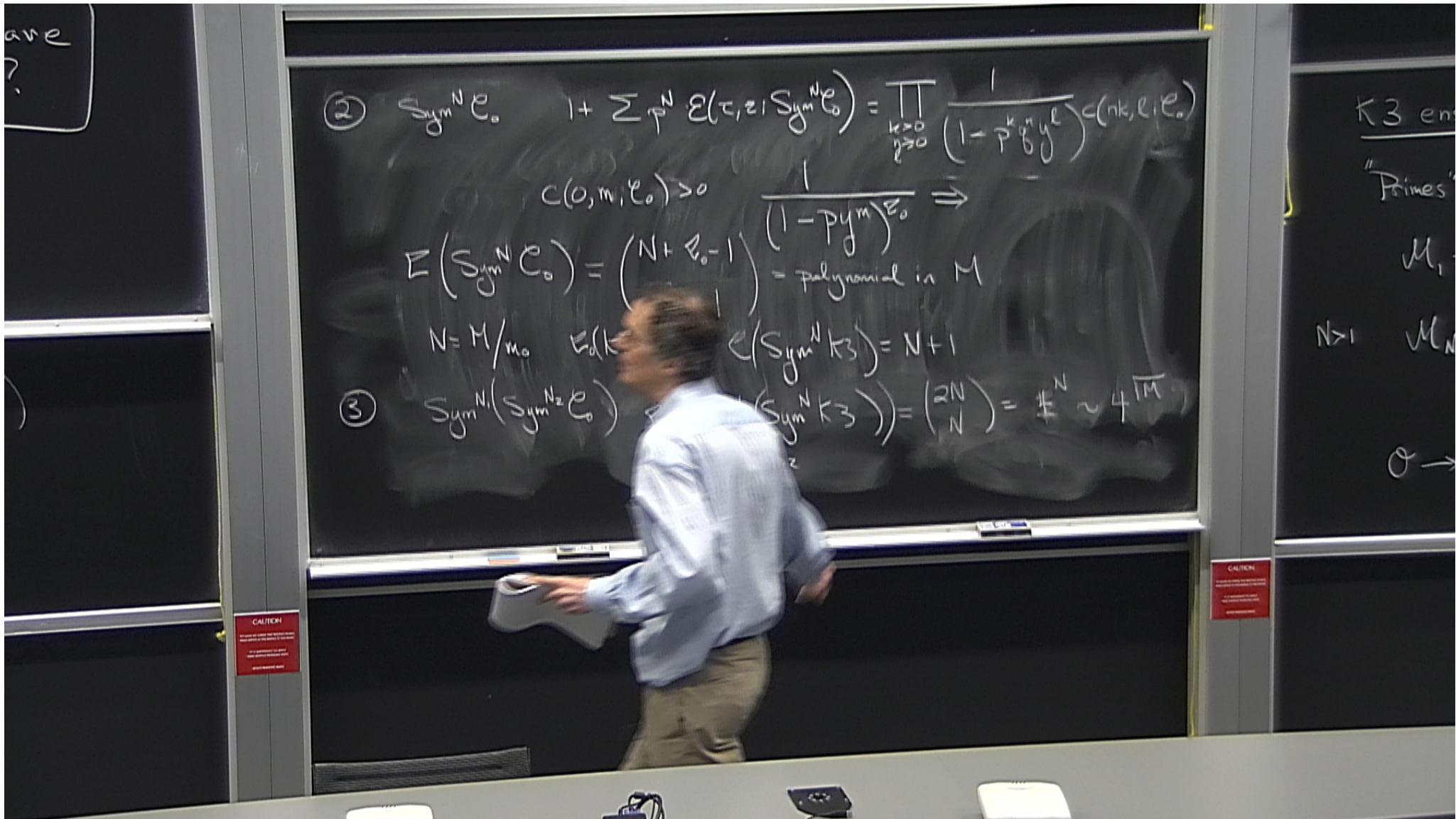
$$c(0, m; \mathcal{C}_0) > 0 \quad \frac{1}{(1 - p y^m)^{E_0}} \Rightarrow$$

$$\mathcal{E}(\text{Sym}^N \mathcal{C}_0) = \binom{N + E_0 - 1}{E_0 - 1} = \text{polynomial in } M$$

$$N = M/m_0 \quad E_0(k_3) = 2 \quad \mathcal{E}(\text{Sym}^N k_3) = N + 1$$

$$\textcircled{3} \quad \text{Sym}^{N_2} \mathcal{C}_0 \quad \mathcal{E}(\text{Sym}^N (\text{Sym}^N k_3)) = \binom{2N}{N} = \#^N \sim 4\sqrt{M}$$

$$M = N^2$$



$$(2) \quad \sum_{N \geq 0} \text{Sym}^N E_0 \cdot 1 + \sum_{p \geq 1} p^N \mathcal{E}(\tau, z, \text{Sym}^N E_0) = \prod_{\substack{k \geq 0 \\ p \geq 0}} \frac{1}{(1 - p^k y^k)^{c(nk, l; E_0)}}$$

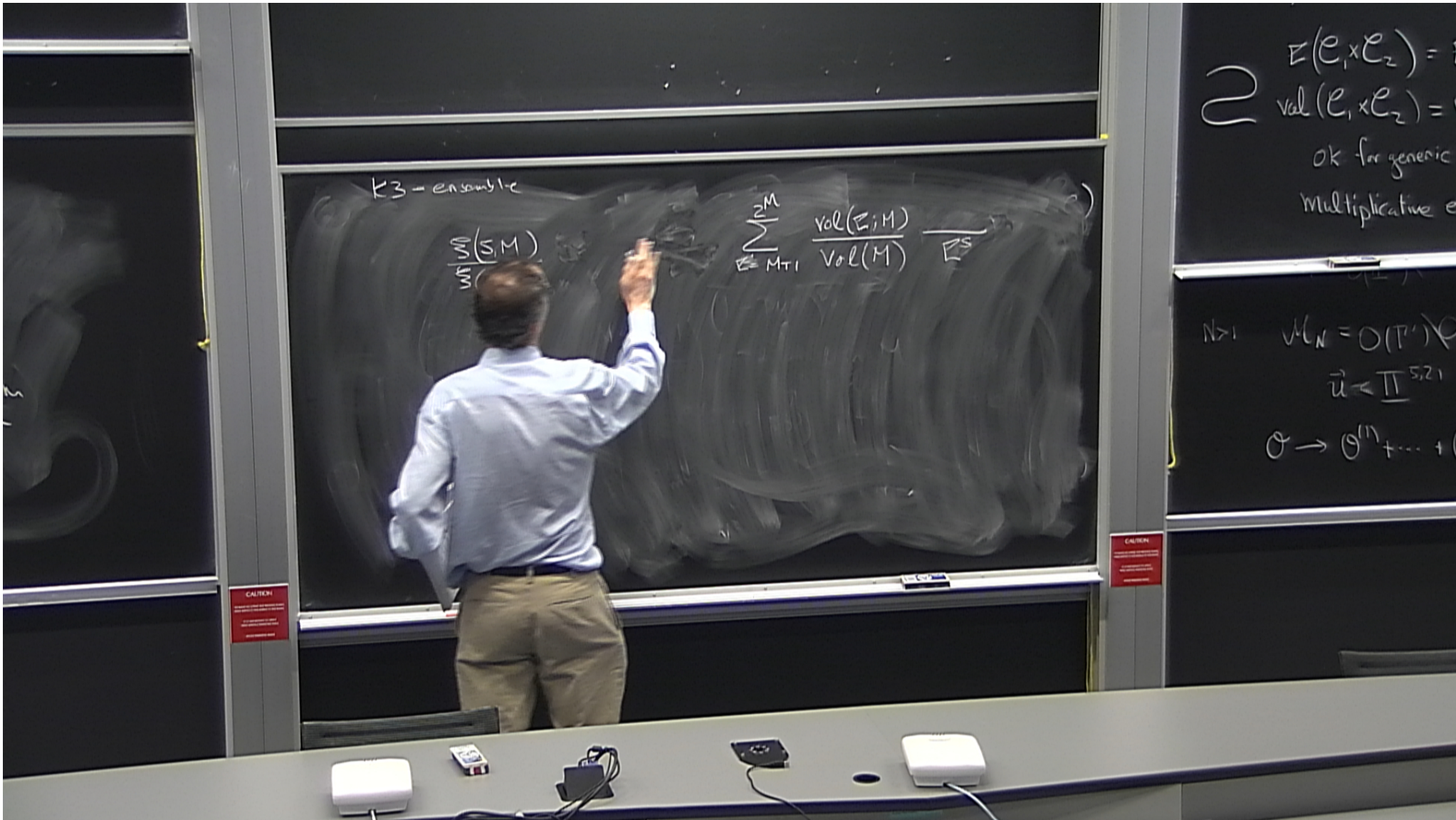
$$c(0, m; E_0) > 0 \quad \frac{1}{(1 - py^m)^{E_0}} \Rightarrow \mathcal{E}(\text{Sym}^N E_0) = \binom{N + E_0 - 1}{N} = \text{polynomial in } M$$

$$N = M/m_0 \quad \mathcal{E}(\text{Sym}^N K_3) = N + 1$$

$$(3) \quad \sum_{N \geq 0} \text{Sym}^N(\text{Sym}^{N_2} E_0) \cdot \mathcal{E}(\text{Sym}^N K_3) = \binom{2N}{N} = \#^N \sim 4^{\sqrt{N}}$$

$K_3$  em  
"Primes"  
 $M_1$   
 $N > 1$   $M_N$   
 $\sigma \rightarrow$





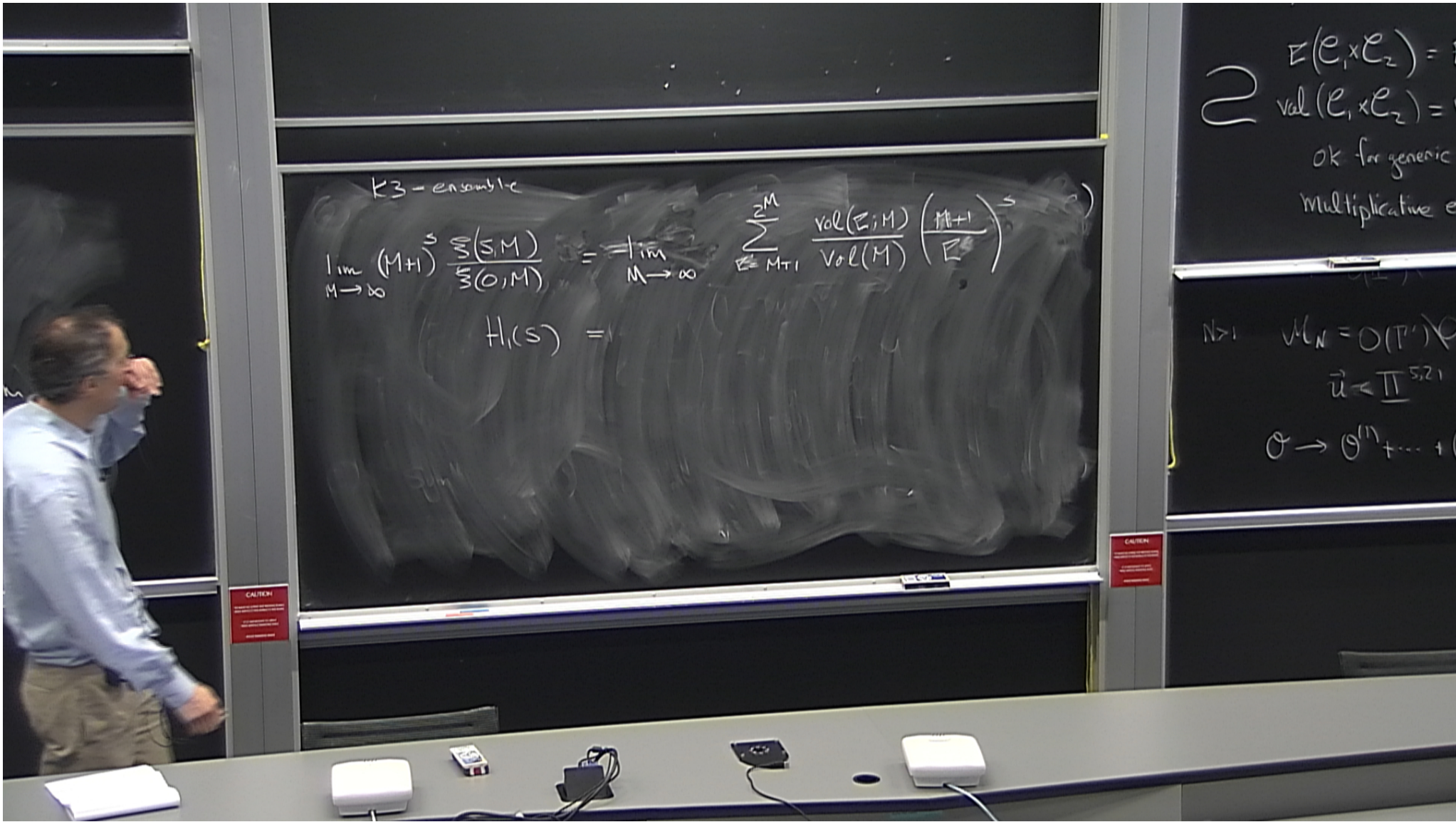
$K3$ -ensemble

$$\frac{\text{Vol}(S, M)}{\text{Vol}(M)}$$

$$\sum_{E \in M_T} \frac{\text{Vol}(E_i, M)}{\text{Vol}(M)}$$

$$\begin{aligned} E(e_1, e_2) &= \\ \sum \text{vol}(e_1, e_2) &= \\ &\text{ok for generic} \\ &\text{Multiplicative } e \end{aligned}$$

$$\begin{aligned} N > 1 \quad \text{Vol}_N &= O(\pi^N) \\ \vec{u} &\in \pi^{5,21} \\ \sigma &\rightarrow \sigma^N + \dots + \end{aligned}$$



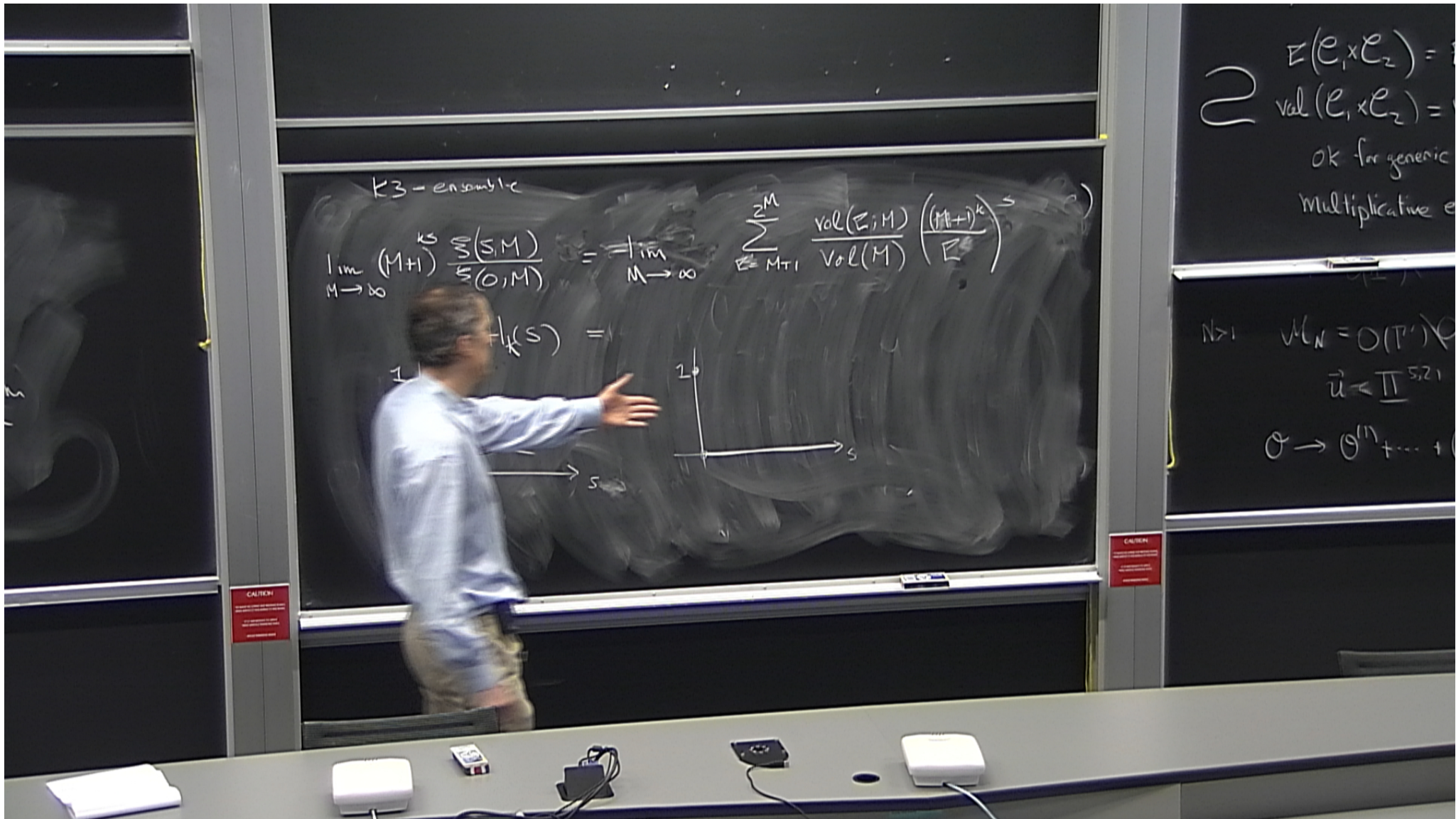
K3-ensemble

$$\lim_{M \rightarrow \infty} (M+1)^s \frac{\sum_{\mathbb{Z}^M} \text{Vol}(S, M)}{\text{Vol}(O, M)} = \lim_{M \rightarrow \infty} \sum_{E \in \mathbb{Z}^M} \frac{\text{Vol}(E, M)}{\text{Vol}(M)} \left( \frac{M+1}{|E|} \right)^s$$

$$H_1(s) =$$

$\sum \mathbb{E}(e_1, e_2) =$   
 $\sum \text{Vol}(e_1, e_2) =$   
 ok for generic  
 Multiplicative e

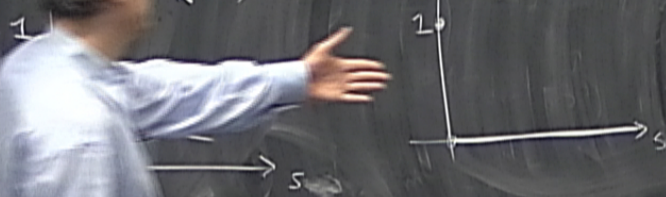
$N > 1$   
 $\mu_N = O(\pi^N)$   
 $\vec{u} \in \pi^{5,21}$   
 $\sigma \rightarrow \sigma^N + \dots + 1$



K3-ensemble

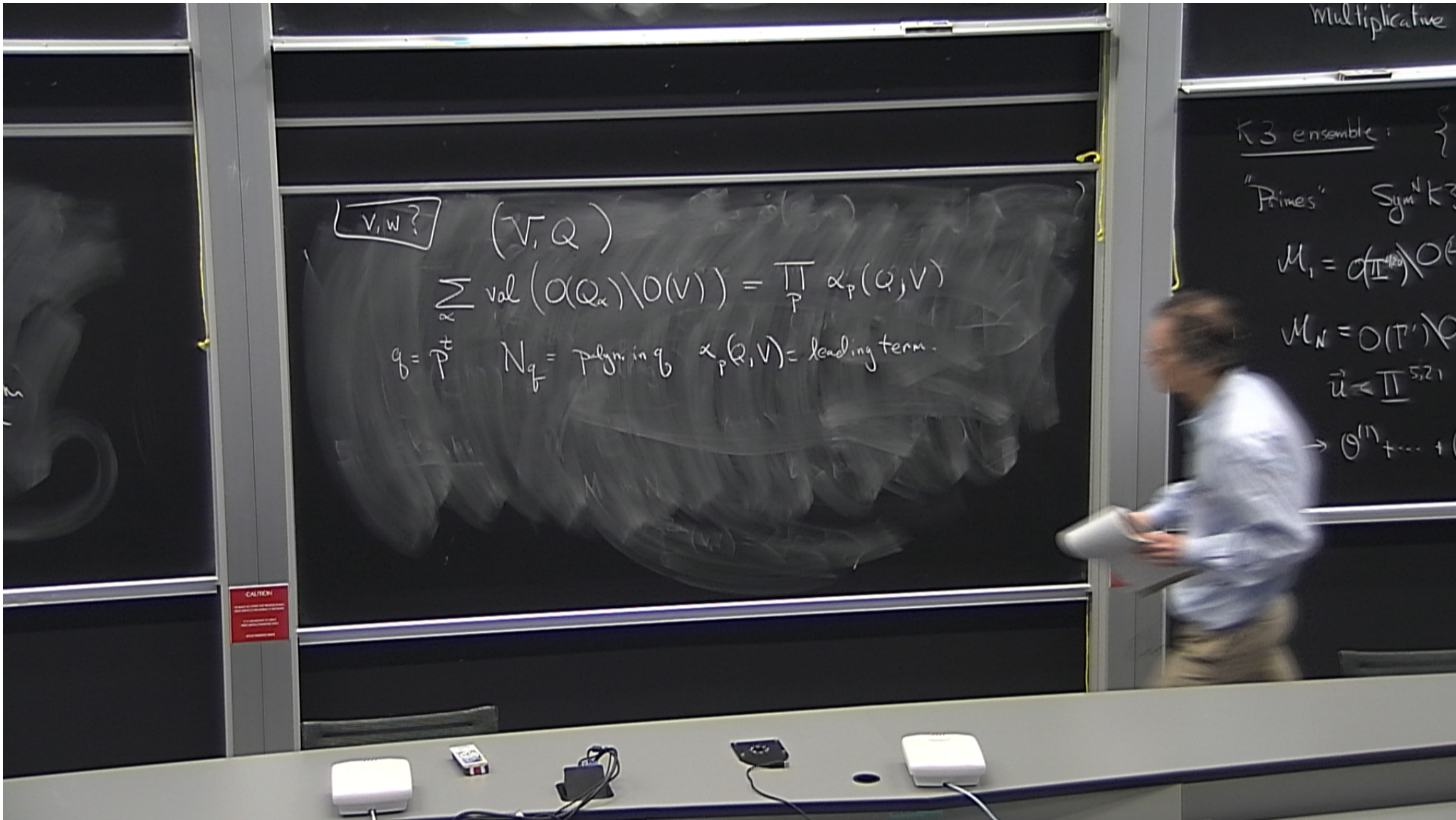
$$\lim_{M \rightarrow \infty} (M+1)^k \frac{\sum_{S \in \mathcal{S}(M)} \zeta(S, M)}{\zeta(0, M)} = \lim_{M \rightarrow \infty} \sum_{E \in \mathcal{M}_1^M} \frac{\text{Vol}(E; M)}{\text{Vol}(M)} \left( \frac{(M+1)^k}{|E|} \right)^s$$

$$h_k(s) =$$



$\sum E(e_1, e_2) =$   
 $\sum \text{Vol}(E_1, E_2) =$   
 ok for generic  
 Multiplicative e

$N > 1$   $\mathcal{M}_N = O(\mathbb{P}^1)$   
 $\vec{u} \in \mathbb{T}^{5,21}$   
 $\sigma \rightarrow \sigma^m + \dots +$



$v, w?$

$(V, Q)$

$$\sum_{\alpha} \text{val}(\alpha(Q_{\alpha}) \setminus O(V)) = \prod_P \alpha_P(Q, V)$$

$q = P^{\pm}$   $N_q = \text{Polym. in } q$   $\alpha_P(Q, V) = \text{leading term.}$

Multiplicative

$K3$  ensemble:  $\{$

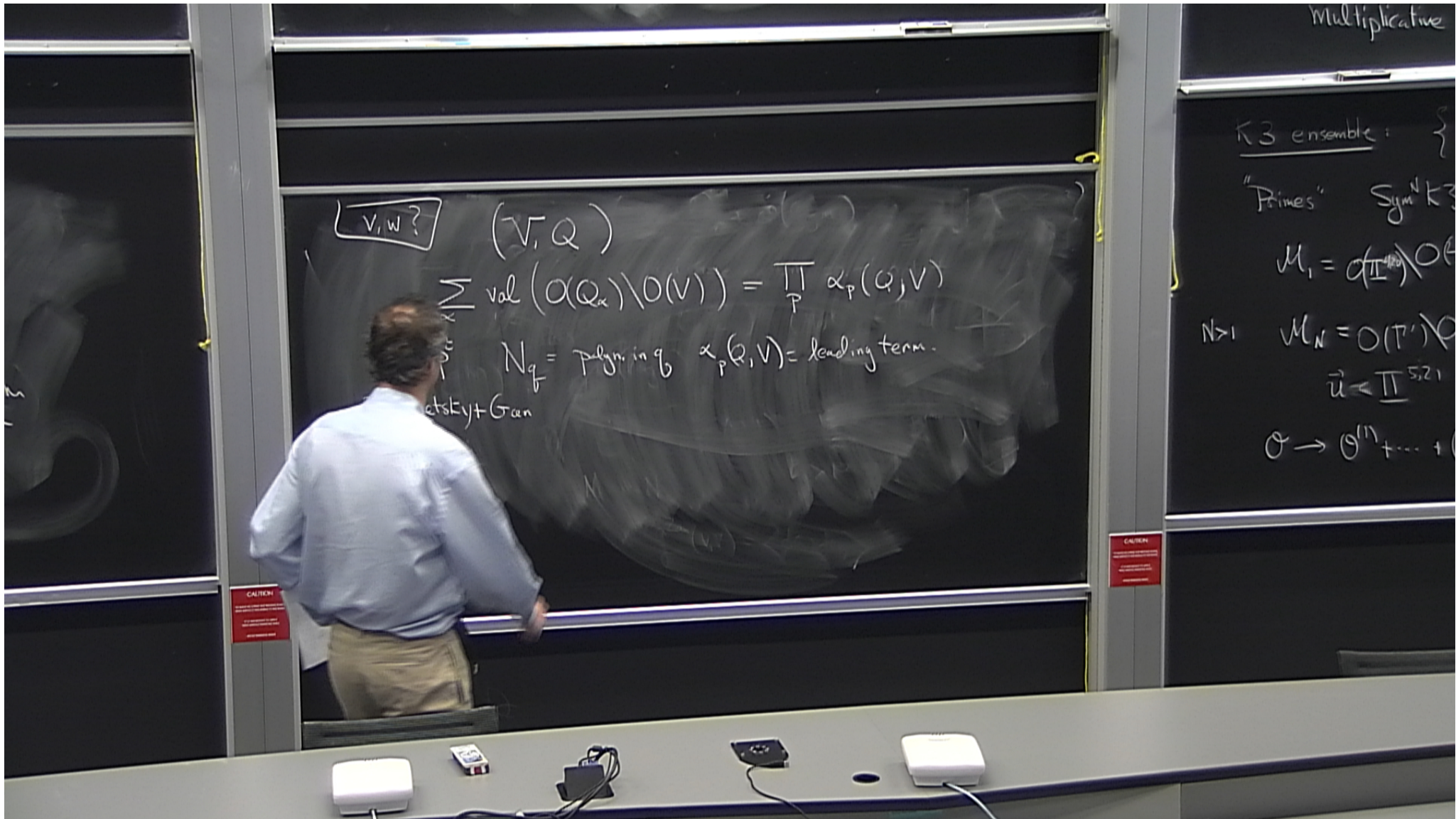
"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = \alpha(\mathbb{P}^1) \setminus O(\mathbb{P}^1)$$

$$\mathcal{M}_N = O(\mathbb{P}^1) \setminus O(\mathbb{P}^1)$$

$$\vec{u} \ll \mathbb{P}^{5,2,1}$$

$$\rightarrow O^{(1)} + \dots + O^{(N)}$$



$v, w?$

$(V, Q)$

$$\sum_x \text{val}(\alpha(Q_x) \setminus O(V)) = \prod_P \alpha_P(Q, V)$$

$N_q = \text{polym. in } q$      $\alpha_P(Q, V) = \text{leading term.}$

etstky+Gan

Multiplicative

$K3$  ensemble:  $\{$

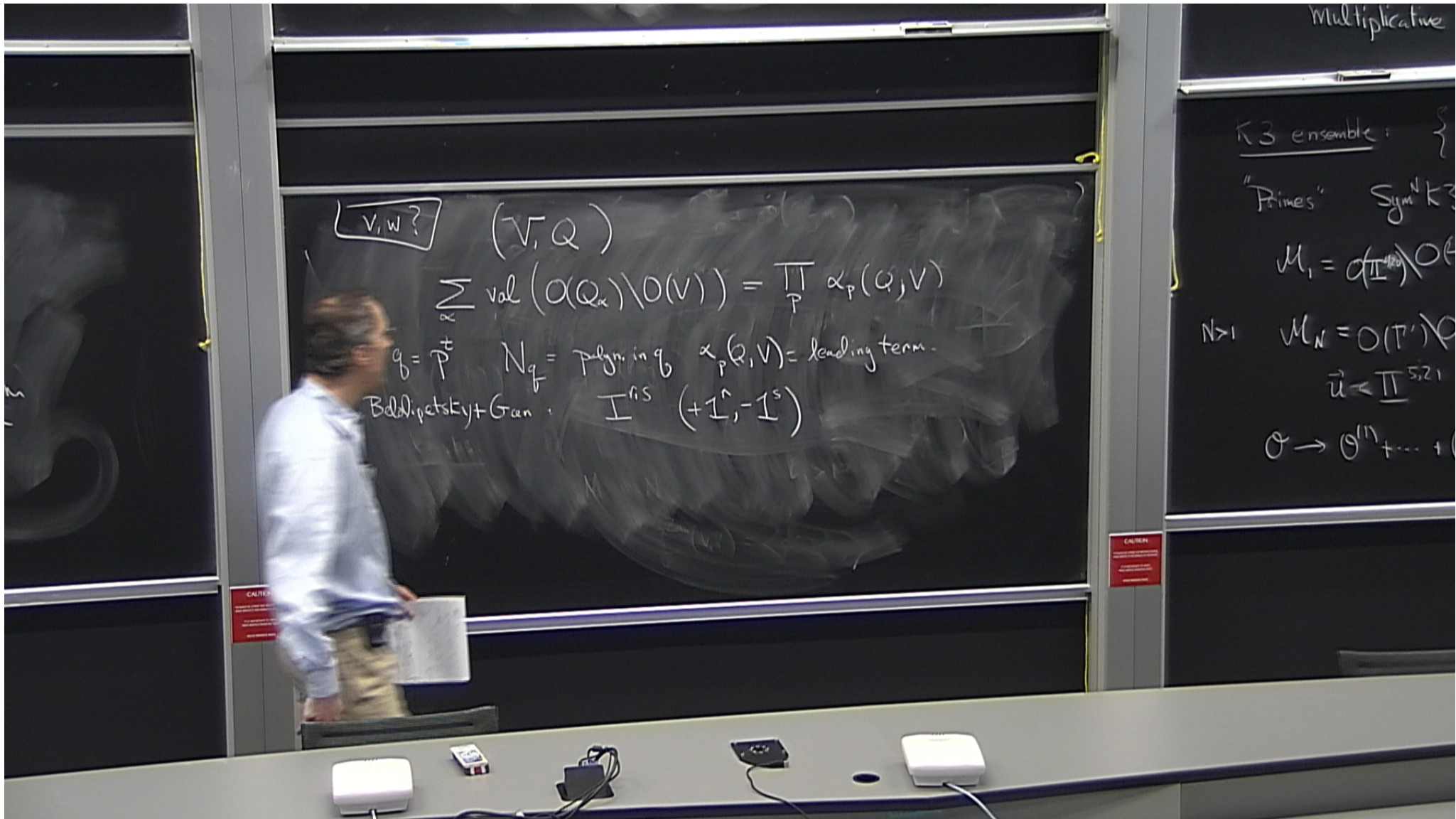
"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = \alpha(\mathbb{P}^1) \setminus O$$

$$N > 1 \quad \mathcal{M}_N = O(\mathbb{P}^1) \setminus O$$

$$\vec{u} \ll \mathbb{P}^{5,2,1}$$

$$\sigma \rightarrow \sigma^{11} + \dots +$$



$v, w?$

$(V, Q)$

$$\sum_{\alpha} \text{val}(\alpha(Q_{\alpha}) \setminus O(V)) = \prod_P \alpha_P(Q, V)$$

$q = \mathbb{F}^{\pm}$   $N_q = \text{Polym. in } q$   $\alpha_P(Q, V) = \text{leading term.}$   
 Bdd, pectky + Gen.  $\mathbb{I}^{ns}$   $(+I^s, -I^s)$

Multiplicative

$K3$  ensemble:  $\{$

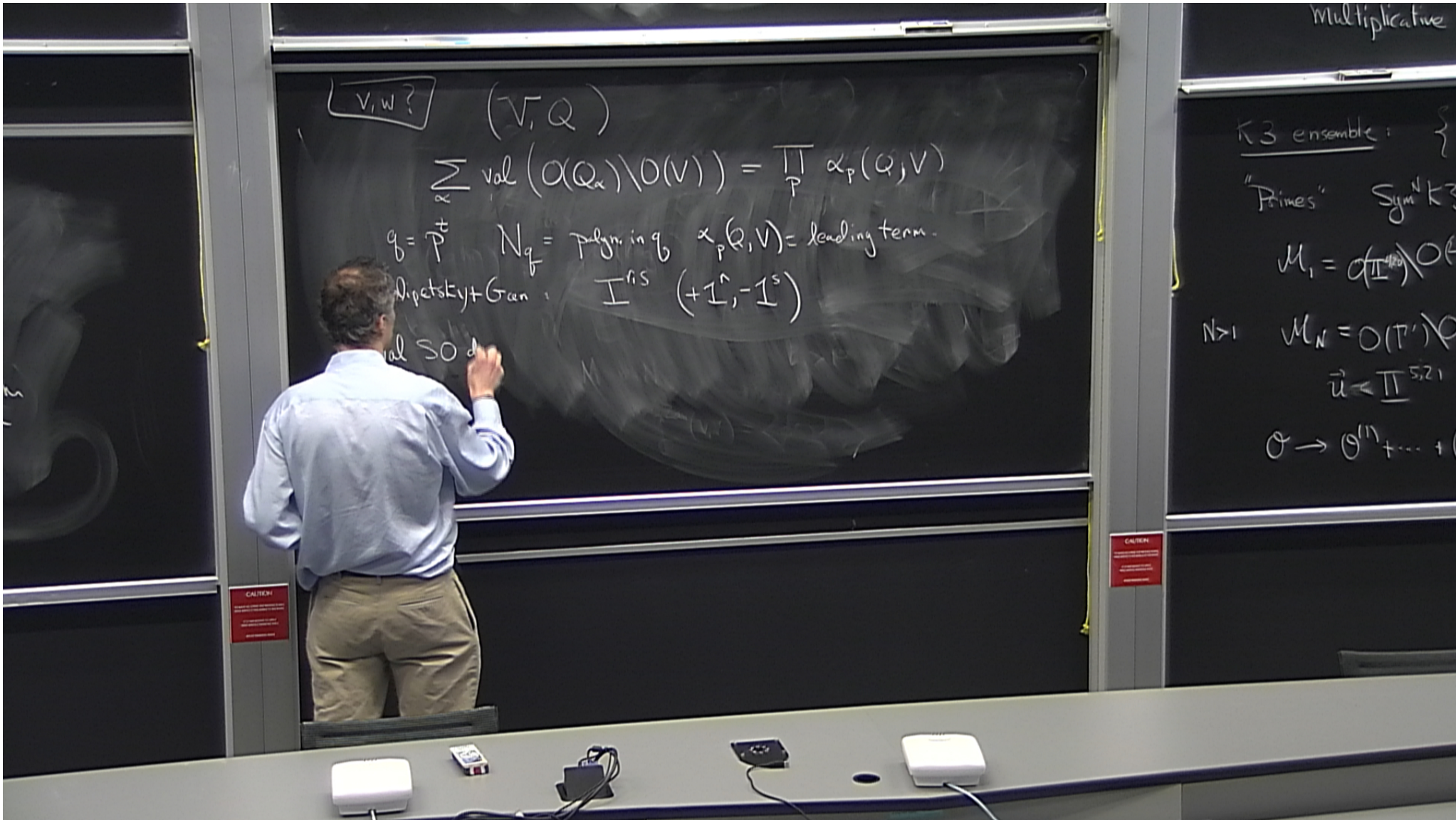
"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = \alpha(\mathbb{I}^{ns}) \setminus O(V)$$

$$N > 1 \quad \mathcal{M}_N = O(\mathbb{I}^{ns}) \setminus O(V)$$

$$\vec{u} \leftarrow \mathbb{I}^{5,21}$$

$$\sigma \rightarrow \sigma^{11} + \dots +$$



$v, w?$

$(V, Q)$

$$\sum_{\alpha} \text{val}(\alpha(Q_{\alpha}) \backslash O(V)) = \prod_P \alpha_P(Q, V)$$

$q = P^{\pm}$   $N_q = \text{polym. in } q$   $\alpha_P(Q, V) = \text{leading term.}$

Dirichlet + Gen:  $I^{ns} (+I^s, -I^s)$

val so d

Multiplicative

$K3$  ensemble:  $\{$

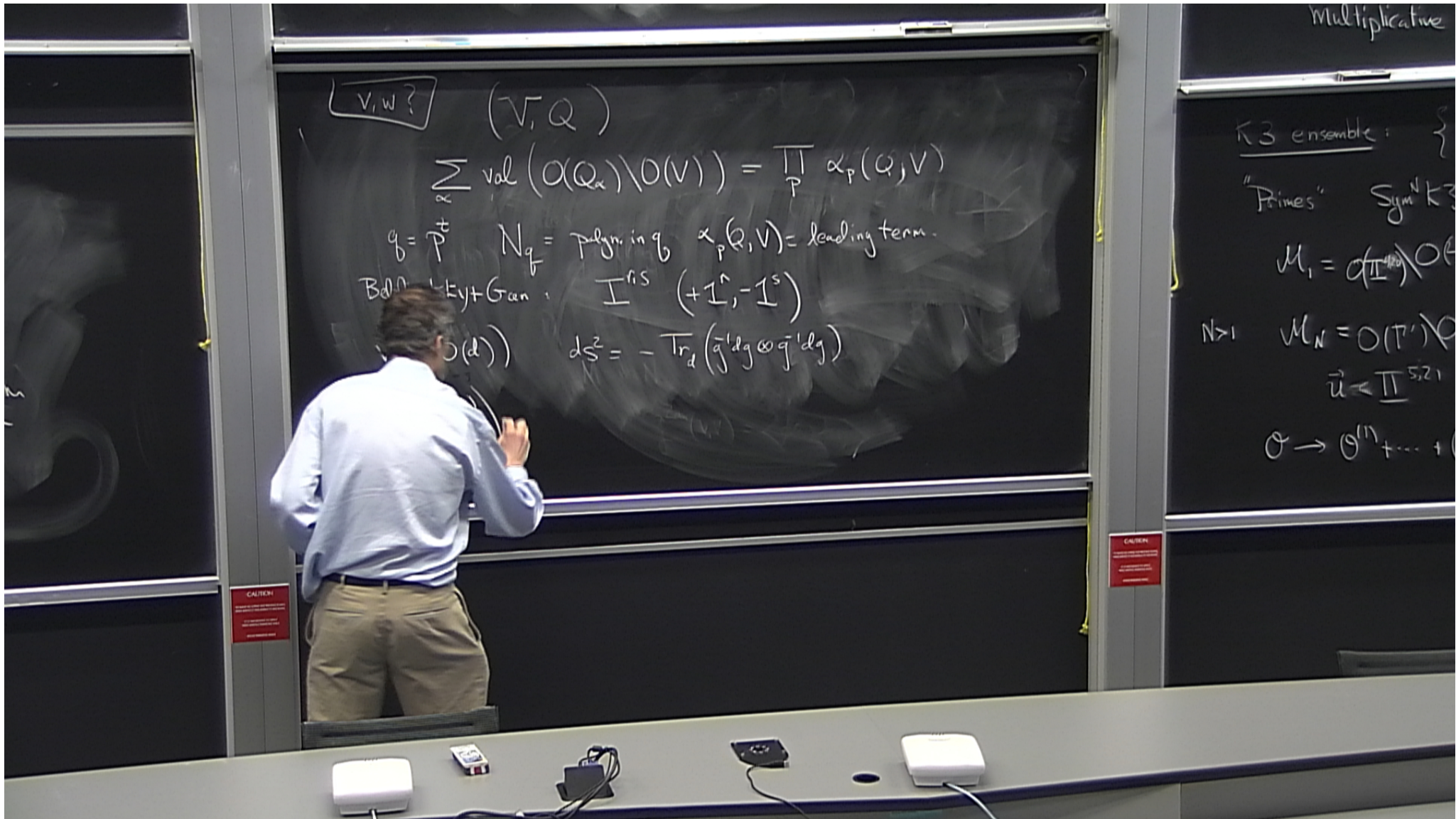
"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = \alpha(\mathbb{I}^{ns}) \backslash O(V)$$

$$N > 1 \quad \mathcal{M}_N = O(\mathbb{I}^s) \backslash O(V)$$

$$\vec{u} \leftarrow \mathbb{I}^{s, 2, 1}$$

$$\sigma \rightarrow \sigma^{11} + \dots + \sigma$$



$V, W?$   $(V, Q)$

$$\sum_{\alpha} \text{val}(\alpha(Q_{\alpha}) \setminus O(V)) = \prod_P \alpha_P(Q, V)$$

$q = P^{\pm}$   $N_q = \text{Polym. in } q$   $\alpha_P(Q, V) = \text{leading term.}$

BdV + Kyt + G. an.  $I^{r,s} (+I^r, -I^s)$

$$dS^2 = -\text{Tr}_q(\dot{q}^i dg \otimes \dot{q}^j dg)$$

Multiplicative

$K3$  ensemble:  $\{$

"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = \alpha(\mathbb{I}^{1,2}) \setminus O(4)$$

$$N > 1 \quad \mathcal{M}_N = O(\mathbb{I}^{1,2}) \setminus O(4)$$

$$\vec{u} \in \mathbb{I}^{5,2,1}$$

$$\mathcal{O} \rightarrow \mathcal{O}^{11} + \dots + \mathcal{O}^1$$



$V, W?$   $(V, Q)$

$$\sum_{\alpha} \text{val}(\alpha(Q_{\alpha}) \setminus O(V)) = \prod_P \alpha_P(Q, V)$$

$g = P^{\pm}$   $N_g = \text{polym. in } g$   $\alpha_P(Q, V) = \text{leading term.}$   
Bddipetskiy + Gran  $I^{r,s} (+I^r, -I^s)$

$$\text{val}(SO(d)) \quad dS^2 = -\text{Tr}_g(\dot{g}^i dg^j \otimes g^i dg^j) \quad R^2 = 2$$

$$\text{val}\left(\frac{SO(d)}{SO(d-1)}\right) \text{val}\left(\frac{SO(d-1)}{SO(d-2)}\right) \times \dots \times \text{val}\left(\frac{SO(3)}{SO(2)}\right) \text{val}(SO(2))$$

Multiplicative

$K3$  ensemble:  $\{$

"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = O(\mathbb{P}^1) \setminus O(\mathbb{P}^1)$$

$$N > 1 \quad \mathcal{M}_N = O(\mathbb{P}^1) \setminus O(\mathbb{P}^1)$$

$$\vec{u} \in \mathbb{P}^{5,2,1}$$

$$\sigma \rightarrow \sigma^{11} + \dots + \dots$$

$$V_{r,s} = \text{val} \left( \text{SO}(\mathbb{I}^{n,s}) \setminus \text{SO}(r,s) / \text{SO}(r) \times \text{SO}(s) \right)$$

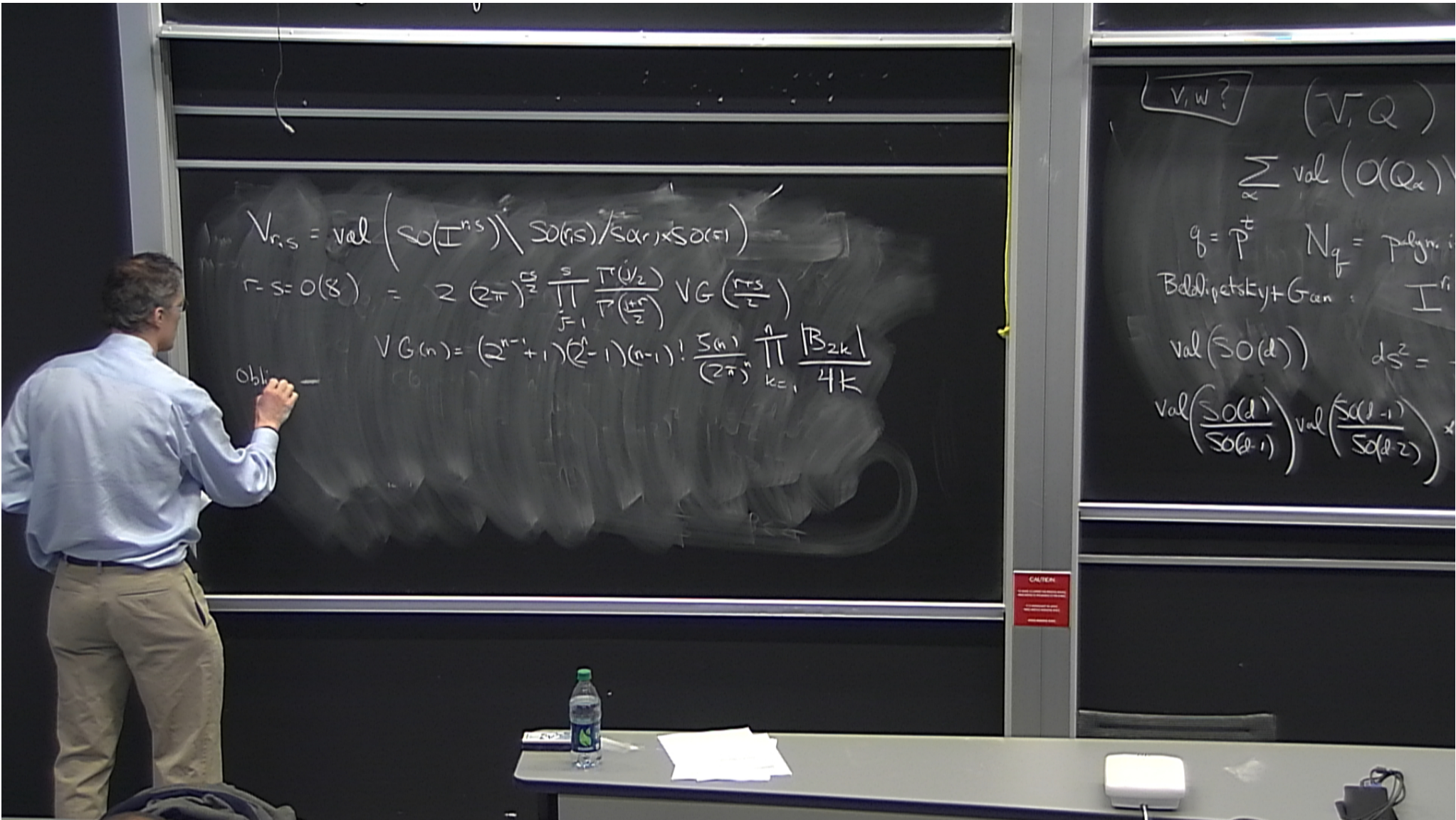
$V, W?$   $(-V, Q)$

$$\sum_{\alpha} \text{val} (O(Q_{\alpha}))$$

$g = P^{\pm}$   $N_g = \text{polynom.}$   
 Bddipetsky + Gan :  $\mathbb{I}^n$

$\text{val}(\text{SO}(d))$   $ds^2 =$

$$\text{val} \left( \frac{\text{SO}(d)}{\text{SO}(d-1)} \right) \text{val} \left( \frac{\text{SO}(d-1)}{\text{SO}(d-2)} \right) \times$$



$$V_{r,s} = \text{val} \left( \text{SO}(\mathbb{I}^{n,s}) \setminus \text{SO}(r,s) / \text{SO}(r) \times \text{SO}(s) \right)$$

$$r-s=0(8) = 2 (2\pi)^{\frac{rs}{2}} \prod_{j=1}^s \frac{\Gamma(j/2)}{\Gamma(\frac{r+s}{2})} \text{VG} \left( \frac{r+s}{2} \right)$$

$$\text{VG}(n) = (2^{n-1} + 1)(2^n - 1)(n-1)! \frac{\zeta(n)}{(2\pi)^n} \prod_{k=1}^n \frac{|B_{2k}|}{4k}$$

Obli -

$V, W?$

$(-V, Q)$

$$\sum_{\alpha} \text{val} (O(Q_{\alpha}))$$

$g = P^{\pm}$   $N_g = \text{polynom}$   
 Bddipetsky + Gan  $\mathbb{I}^n$

$$\text{val}(\text{SO}(d)) \quad dS^2 =$$

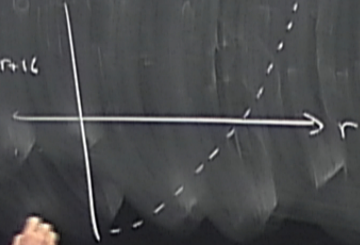
$$\text{val} \left( \frac{\text{SO}(d)}{\text{SO}(d-1)} \right) \text{val} \left( \frac{\text{SO}(d-1)}{\text{SO}(d-2)} \right) \times$$

$$V_{r,s} = \text{val} \left( \text{SO}(\mathbb{I}^{n,s}) \setminus \text{SO}(r,s) / \text{SO}(r) \times \text{SO}(s-1) \right)$$

$$r-s=0(s) = 2 \left( \frac{2\pi}{2} \right)^{\frac{r-s}{2}} \prod_{j=1}^s \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1+r}{2})} \text{VG} \left( \frac{r+s}{2} \right)$$

$$\text{VG}(n) = (2^{n-1} + 1) (2^n - 1) (n-1)! \frac{\zeta(n)}{(2\pi)^n} \prod_{k=1}^n \frac{|B_{2k}|}{4k}$$

Obligatory Morse  
by Voronoi



$$\boxed{V, W?} \quad (-\sqrt{V}, Q)$$

$$\sum_{\alpha} \text{val}(O(Q_{\alpha}))$$

$g = \mathbb{P}^n$   $N_g = \text{poly}$   
Beldjitsky + Gan  $\mathbb{I}^n$

$$\text{val}(\text{SO}(d)) \quad dS^2 =$$

$$\text{val} \left( \frac{\text{SO}(d)}{\text{SO}(d-1)} \right) \text{val} \left( \frac{\text{SO}(d-1)}{\text{SO}(d-2)} \right) \times$$

$$V_{r,s} = \text{val} \left( \text{SO}(\mathbb{I}^{n,s}) \setminus \text{SO}(r,s) / \text{SO}(r) \times \text{SO}(s-1) \right)$$

$$r-s=0(8) = 2 (2\pi)^{\frac{r-s}{2}} \prod_{j=1}^s \frac{\Gamma(\frac{j}{2})}{\Gamma(\frac{j+r}{2})} \text{VG} \left( \frac{r+s}{2} \right)$$

$$\text{VG}(n) = (2^{n-1} + 1) (2^n - 1) (n-1)! \frac{\zeta(n)}{(2\pi)^n} \prod_{k=1}^n \frac{|B_{2k}|}{4k}$$

Obligatory Measure 100  
log V\_{r,s+16}



$$V, W? \quad (\sqrt{V}, Q)$$

$$\sum_{\alpha} \text{val} (O(Q_{\alpha}))$$

$g = \mathbb{P}^{\pm}$   $N_g = \text{poly}$   
Baldigetsky + Gan  $\mathbb{I}^n$

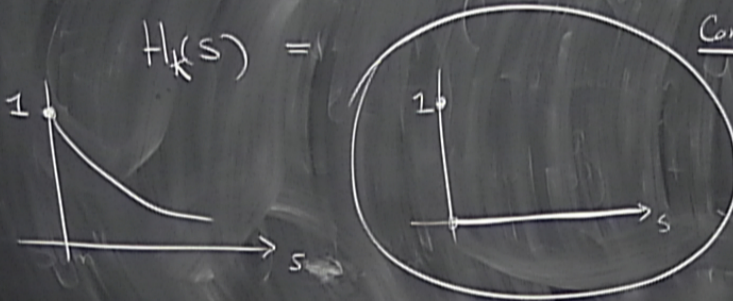
$$\text{val}(\text{SO}(d)) \quad dS^2 =$$

$$\text{val} \left( \frac{\text{SO}(d)}{\text{SO}(d-1)} \right) \text{val} \left( \frac{\text{SO}(d-1)}{\text{SO}(d-2)} \right) \times$$

$K3$ -ensemble

$$\lim_{M \rightarrow \infty} (M+1)^{K3} \frac{S(S, M)}{S(0, M)} = \lim_{M \rightarrow \infty} \sum_{E=M+1}^{2M} \frac{\text{Vol}(E; M)}{\text{Vol}(M)} \left( \frac{(M+1)^{K3}}{E} \right)^S$$

$$H_K(s) =$$



Conj.  $H_K(s) = \chi(s)$   
 $= \begin{cases} 1 & s=0 \\ 0 & s>0 \end{cases}$

OR Targeted  
 Multiplicative e

$K3$  ensemble:  $\{ \dots \}$

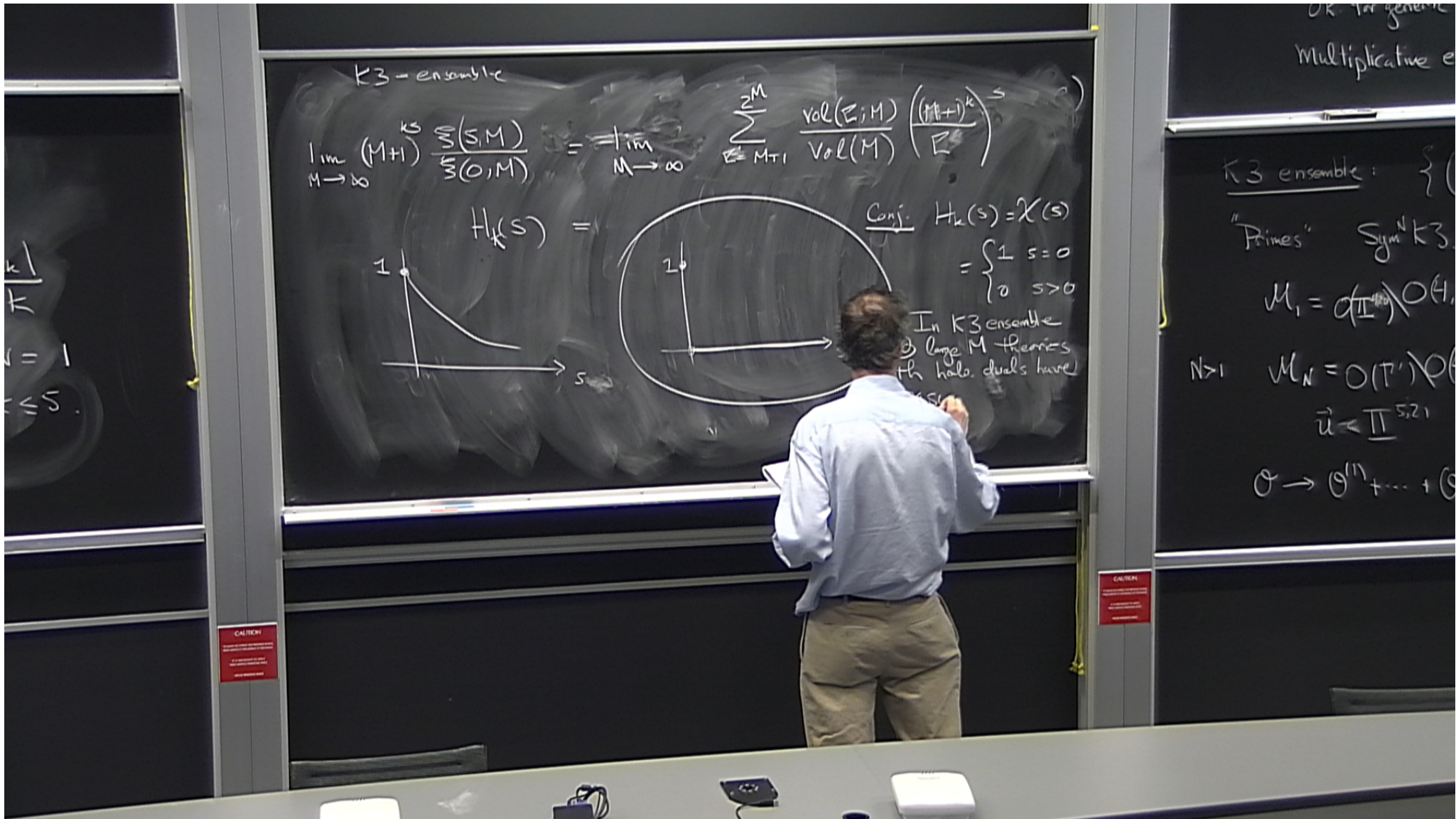
"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = O(\pi^{1/2}) \setminus O(4)$$

$$N > 1 \quad \mathcal{M}_N = O(\pi^{1/2}) \setminus O(4)$$

$$\vec{u} < \pi^{5/2}$$

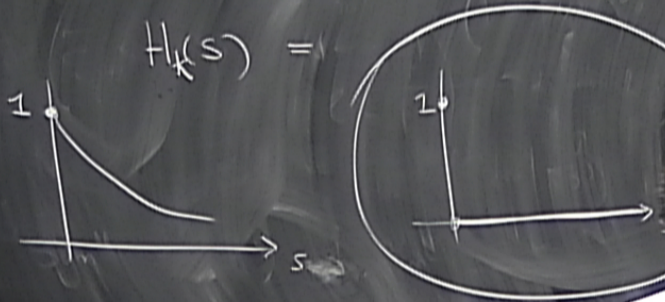
$$\sigma \rightarrow \sigma^{(1)} + \dots + \sigma^{(N)}$$



K3-ensemble

$$\lim_{M \rightarrow \infty} (M+1)^K \frac{Z(s, M)}{Z(0, M)} = \lim_{M \rightarrow \infty} \sum_{E \in \mathcal{E}_{M+1}} \frac{\text{Vol}(E; M)}{\text{Vol}(M)} \left( \frac{(M+1)^K}{|E|} \right)^s$$

$$H_K(s) =$$



Conj.  $H_K(s) = \chi(s)$   
 $= \begin{cases} 1 & s=0 \\ 0 & s>0 \end{cases}$

In K3 ensemble  
 @ large M theories  
 th. halo duals have

K3 ensemble: {

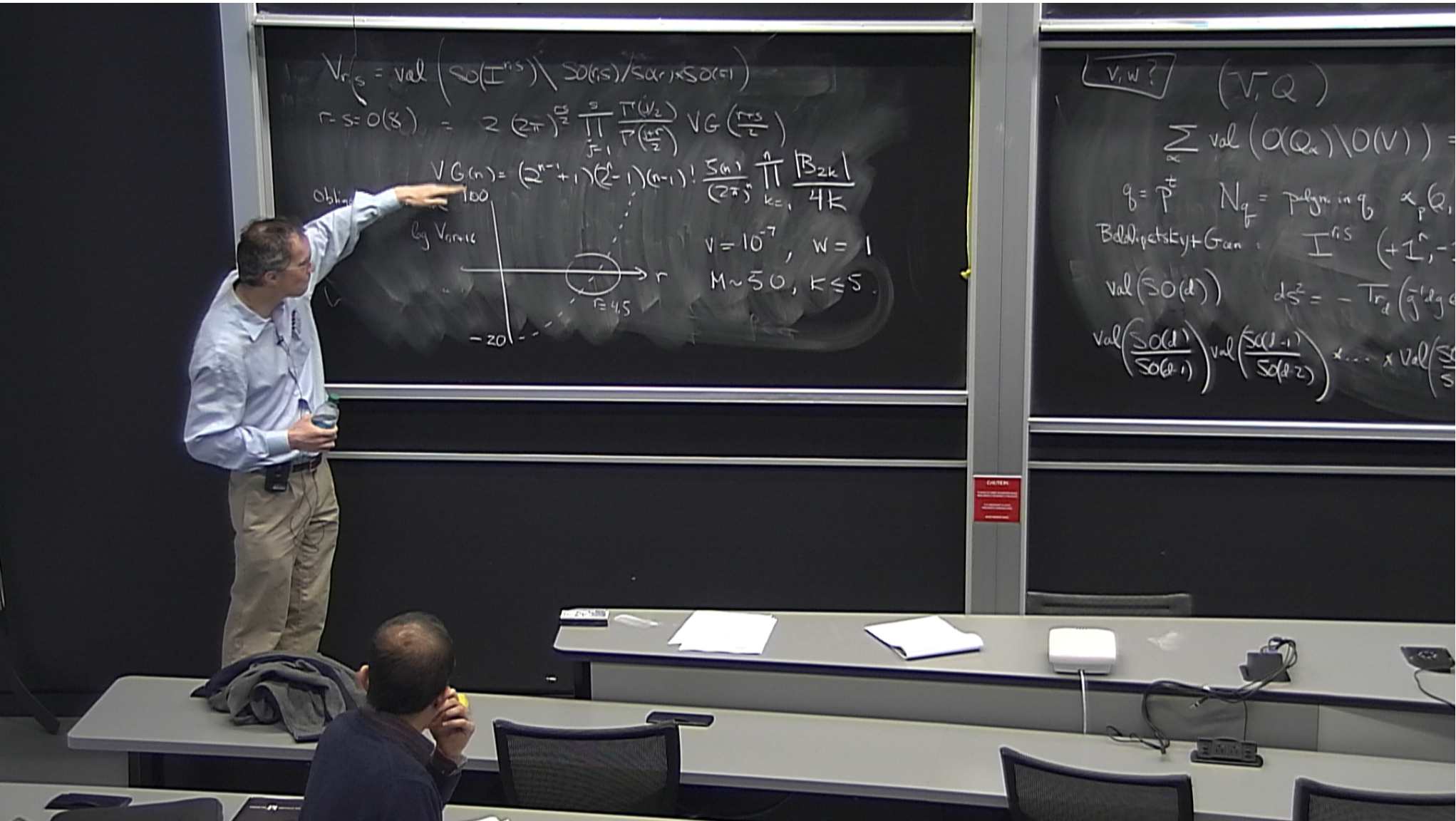
"Primes"  $\text{Sym}^N K3$

$$\mathcal{M}_1 = O(\pi^{4/2}) \times O(4)$$

$$N > 1 \quad \mathcal{M}_N = O(\pi^N) \times O(4)$$

$$\vec{u} < \pi^{5/2}$$

$$\sigma \rightarrow \sigma^{(1)} + \dots + \sigma^{(N)}$$

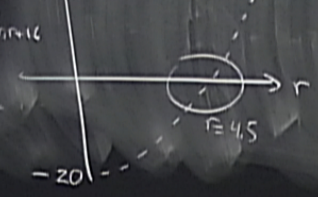


$$V_{r,s} = \text{val} \left( SO(I^{r,s}) \setminus SO(r,s) / SO(r) \times SO(s) \right)$$

$$r-s=0(s) = 2 \left( \frac{r}{2} \right)^{\frac{r}{2}} \prod_{j=1}^{\frac{r}{2}} \frac{\Gamma(V_j)}{\Gamma\left(\frac{r-j}{2}\right)} VG\left(\frac{r+s}{2}\right)$$

$$VG(n) = (2^{n-1} + 1)(2-1)(n-1)! \frac{5(n)}{(2\pi)^n} \prod_{k=1}^n \frac{|B_{2k}|}{4k}$$

Obl. = 100  
by Varin



$$V = 10^{-7}, W = 1$$

$$M \sim 50, K \leq 5$$

$V, W?$   $(V, Q)$

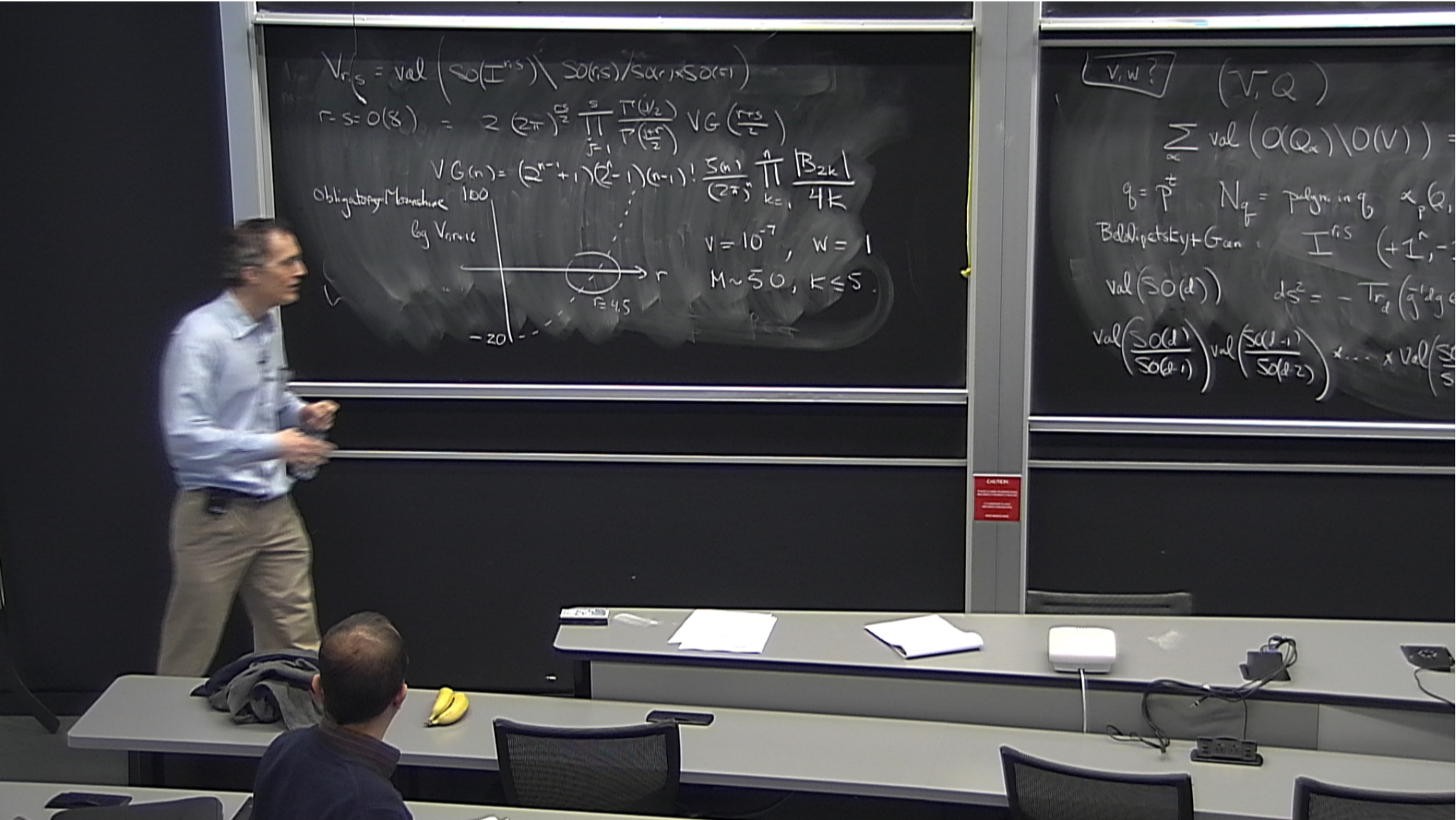
$$\sum_{\alpha} \text{val}(\alpha Q_{\alpha}) \setminus O(V) =$$

$g = P^{\dagger}$   $N_g = \text{p.l.m. in } g, \alpha_p Q_p,$   
 Baldegetsky + Gan.  $I^{r,s} (+I^r, -I^s)$

$\text{val}(SO(d)) \quad dS^2 = -Tr_d(g^{\dagger} g)$

$$\text{val}\left(\frac{SO(d)}{SO(d-1)}\right) \text{val}\left(\frac{SO(1-1)}{SO(1-2)}\right) \times \dots \times \text{val}\left(\frac{SO(1)}{SO(1)}\right)$$





$$V_{r,s} = \text{val} \left( SO(I^{r,s}) \setminus SO(r,s) / SO(r) \times SO(s) \right)$$

$$r-s=0(8) = 2 \left( \frac{r+s}{2} \right)^{\frac{r+s}{2}} \prod_{j=1}^{\frac{r+s}{2}} \frac{\Gamma(V_j)}{\Gamma\left(\frac{r+s}{2}\right)} VG\left(\frac{r+s}{2}\right)$$

$$VG(n) = (2^{n-1} + 1)(2^{n-1} - 1)(n-1)! \frac{5(n)}{(2^n)^n} \prod_{k=1}^n \frac{|B_{2k}|}{4k}$$

Obligatory Monochrome 100  
by V\_{r,s}

$v = 10^{-7}, w = 1$   
 $M \sim 50, K \leq 5$

Graph: A coordinate system with a horizontal axis labeled 'r' and a vertical axis labeled '-20'. A dashed circle is drawn around the origin, with the number '4.5' written below it.

$V, W?$   $(V, Q)$

$$\sum_{\alpha} \text{val}(\alpha Q_{\alpha}) \setminus O(V) =$$

$g = P^{\pm} \quad N_g = \text{p.l.g.m. in } g \quad \alpha_p Q_p$   
Baldpatsky + Gan  $I^{r,s} \quad (+I^r, -I^s)$

$\text{val}(SO(d)) \quad dS^2 = -Tr_d(g^{\pm} dg)$

$$\text{val}\left(\frac{SO(d)}{SO(d-1)}\right) \text{val}\left(\frac{SO(1-1)}{SO(1-2)}\right) \times \dots \times \text{val}\left(\frac{SO(1)}{SO(0)}\right)$$