Title: Quiver Quantum Mechanics and Wall-Crossing

Date: Apr 16, 2015 03:00 PM

URL: http://pirsa.org/15040133

Abstract: I will talk about computation of the Witten index of 1d N=4 gauged linear sigma model which describes wall-crossing of BPS states in 4d N=2 theories. In the phase where the gauge group is broken to a finite group, the index is expressed as the JK-residue integral. Using this result, I am going to examine large-rank behaviour of the Kronecker quivers which describes the most simplest wall-crossing phenomena. I will also talk about how the refined Witten indices of quivers are preserved under the mutation process.

Witten Index of Quiver Quantum Mechanics and Wall-Crossing

Heeyeon Kim
Perimeter Institute

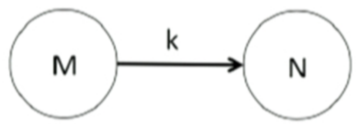
April 16th, 2015
(Mock) Modularity, Moonshine and Sting Theory

Based on,
arXiv:1407.2567 [hep-th] with Kentaro Hori, Piljin Yi,
arXiv:1503.02623 [hep-th], arXiv:1504.00068 [hep-th] with Piljin Yi and Seung-Joo Lee

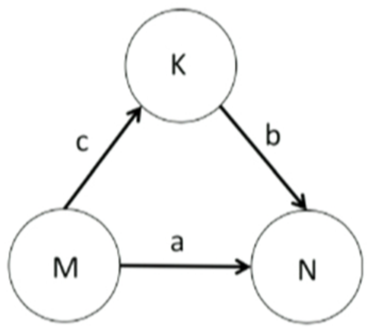
Contents

1. Quiver Quantum Mechanics and Wall-Crossings in $4d$ $\mathcal{N} = 2$ theories
2. Witten Index of 1d Gauged Linear Sigma Models
3. Large-Rank Behaviour and Mutation

   1) Kronecker Quivers

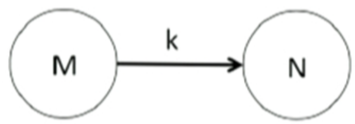


   2) Quiver with Loops

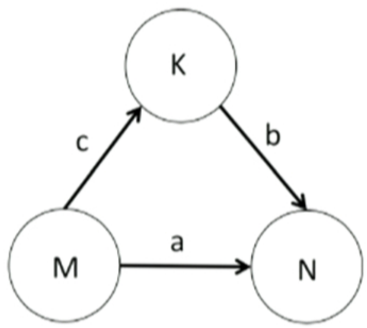


Contents

1. Quiver Quantum Mechanics and Wall-Crossings in $4d$ $\mathcal{N} = 2$ theories
2. Witten Index of 1d Gauged Linear Sigma Models
3. Large-Rank Behaviour and Mutation
   1) Kronecker Quivers

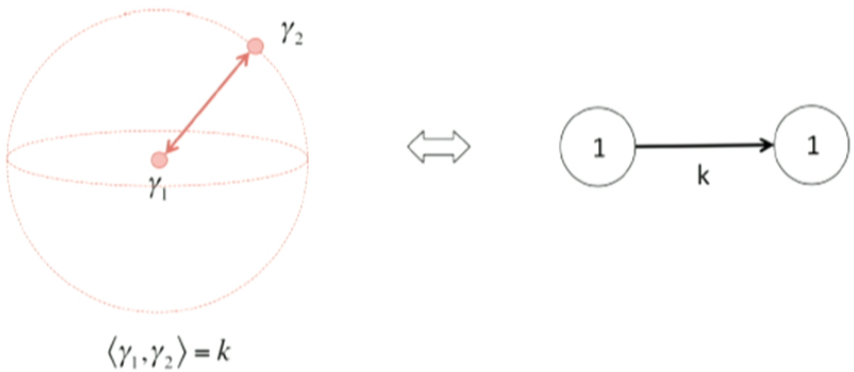


   2) Quiver with Loops



Quiver Quantum Mechanics and Wall-Crossing

- Consider $4d$ $\mathcal{N}=2$ SUSY theories obtained from type IIB string theory compactifed on CY3
- BPS particles come from D3-branes wrapping supersymmetric three-cycles
- Worldvolume theory of N BPS particles $\rightarrow$ $1d$ $\mathcal{N}=4$ $U(N)$ gauge theory
- $\langle\gamma_i,\gamma_j\rangle = k$ intersection $\rightarrow$ $1d$ $\mathcal{N}=4$ bifundamental hypermultiplet



[Denef, 2002][Denef-Moore, 2007]

Quiver Quantum Mechanics and Wall-Crossing

Quiver Quantum Mechanics (1d $\mathcal{N}=4$ GLSM)

- $\Sigma = (A_0, x_1, x_2, x_3, D, \lambda, \bar{\lambda})$ and $\Phi^a = (\phi, \bar{\phi}, \psi, \bar{\psi}, F, \bar{F})^a$
- Global symmetry : $SU(2)_L \times U(1)_R$ (J, I)
- It depends on real parameter ζ with

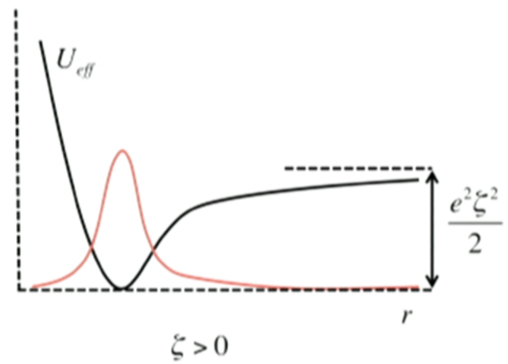
$$\zeta \int dt\ D$$

Wall-Crossing in "Coulomb picture"

- When $|\vec{x} - \vec{x}'| = r$ large,

$$U_{eff} = \frac{e^2}{2}\left(\frac{\langle\gamma_1, \gamma_2\rangle}{r} - \zeta\right)^2$$

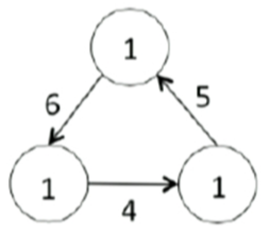
- At $\zeta \leq 0$, bound states become non-renormalizable.



Wall-Crossing in Coulomb branch : MPS formula

Coulomb branch analysis counts the BPS states with multi-centered nature.

[Manschot-Pioline-Sen, 2010,2011]



$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 0 & & 0 & & & & \\
 & & & 0 & & 2 & & 0 & & & \\
 & & 0 & & 0 & & 0 & & 0 & & \\
 & 0 & & 0 & & 3 & & 0 & & 0 & \\
0 & & 0 & & 26 & & 26 & & 0 & & 0 \\
 & 0 & & 0 & & 3 & & 0 & & 0 & \\
 & & 0 & & 0 & & 0 & & 0 & & \\
 & & & 0 & & 2 & & 0 & & & \\
 & & & & 0 & & 0 & & & & \\
 & & & & & 1 & & & & &
\end{array}
,\qquad
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 2 & & 0 & \\
0 & & 26 & & 26 & & 0 \\
 & 0 & & 2 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
,\qquad
\begin{array}{ccc}
 & 1 & \\
26 & & 26 \\
 & 1 &
\end{array}
.
$$

Quiver Invariants

- Appears when quiver has a non-trivial loop
- Invariant under wall-crossing
- Singlet under $SU(2)_L$
- Exponentially large degeneracy

Is there any systematic way to find a BPS index which include all the BPS states?

Quiver Invariants

- Appears when quiver has a non-trivial loop
- Invariant under wall-crossing
- Singlet under $SU(2)_L$
- Exponentially large degeneracy

Is there any systematic way to find a BPS index which include all the BPS states?

Quiver Invariants

- Appears when quiver has a non-trivial loop
- Invariant under wall-crossing
- Singlet under $SU(2)_L$
- Exponentially large degeneracy

Is there any systematic way to find a BPS index which include all the BPS states?

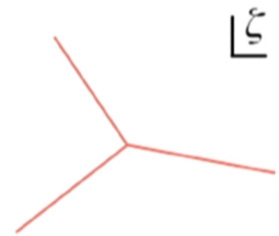
Witten Index of 1d Gauged Linear Sigma Model

Direct path integral of quiver quantum mechanics on S^1

$$\Omega(y,\zeta) = \mathrm{Tr}\ (-1)^{2J_3} y^{2(J_3 - I)}$$

- Invariant under small deformation
- Integer valued

  $\rightarrow$ fails when there exist non-compact direction



When $e \rightarrow 0$, path integral localizes to Cartan zero mode integral $u = x_3 + iA_o$.

1-loop determinant reads

$$g_{1-loop}^{vector} = \left(\frac{1}{2\sinh[z/2]}\right)^r \prod_{\alpha\in\Delta} \frac{\sinh[\alpha(u)/2]}{\sinh[(\alpha(u)-z)/2]} \ ,$$

$$g_{1-loop}^{chiral} = \prod_i \frac{-\sinh[(Q_i(u) + (R_i/2 - 1)z + f_i a)/2]}{\sinh[(Q_i(u) + R_i z/2 + f_i a)/2]}$$

where $e^{z/2} = y$.

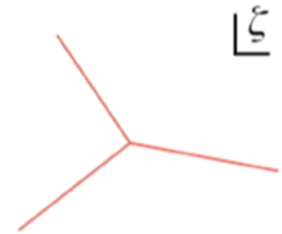
Witten Index of 1d Gauged Linear Sigma Model

Direct path integral of quiver quantum mechanics on S^1

$$\Omega(y,\zeta) = \text{Tr}\ (-1)^{2J_3} y^{2(J_3 - I)}$$

- Invariant under small deformation
- Integer valued

  $\rightarrow$ fails when there exist non-compact direction



When $e \rightarrow 0$, path integral localizes to Cartan zero mode integral $u = x_3 + iA_o$.

1-loop determinant reads

$$g_{1-loop}^{vector} = \left(\frac{1}{2\sinh[z/2]}\right)^r \prod_{\alpha\in\Delta} \frac{\sinh[\alpha(u)/2]}{\sinh[(\alpha(u)-z)/2]}\ ,$$

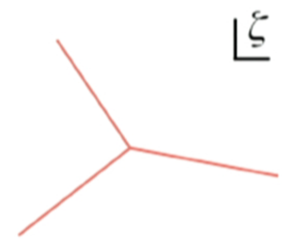
$$g_{1-loop}^{chiral} = \prod_i \frac{-\sinh[(Q_i(u) + (R_i/2 - 1)z + f_i a)/2]}{\sinh[(Q_i(u) + R_i z/2 + f_i a)/2]}$$

where $e^{z/2} = y$.

Witten Index of 1d Gauged Linear Sigma Model

Direct path integral of quiver quantum mechanics on S^1

$$\Omega(y,\zeta) = \mathrm{Tr}\,(-1)^{2J_3} y^{2(J_3-I)}$$



- Invariant under small deformation
- Integer valued

  $\rightarrow$ fails when there exist non-compact direction

When $e \rightarrow 0$, path integral localizes to Cartan zero mode integral $u = x_3 + iA_o$.

1-loop determinant reads

$$g_{1-loop}^{vector} = \left(\frac{1}{2\sinh[z/2]}\right)^r \prod_{\alpha\in\Delta} \frac{\sinh[\alpha(u)/2]}{\sinh[(\alpha(u)-z)/2]} \;,$$

$$g_{1-loop}^{chiral} = \prod_i \frac{-\sinh[(Q_i(u)+(R_i/2-1)z+f_i a)/2]}{\sinh[(Q_i(u)+R_i z/2+f_i a)/2]}$$

where $e^{z/2} = y$.

Witten Index of 1d Gauged Linear Sigma Model

$$\Omega(y,\zeta) = \frac{1}{|W_G|}\int d^r D \int d^r\lambda_+^0 d^r\bar{\lambda}_+^0 \int d^r u d^r \bar{u}\; g_{\text{1-loop}}(u,D)\; e^{\lambda_0\bar{\psi}\phi + \text{c.c}} e^{-\frac{1}{e^2}D^2 - i\zeta D}$$

After careful treatment of gaugino zero modes and D-integrals we are left with residue integral over selected poles. (cf. [Benini-Eager-Hori-Tachikawa, 2013])

$$\Omega(y,\zeta) = \frac{1}{|W_G|}\sum_{Q_*} \text{JK-Res}_{\eta=\zeta_{FI}}(Q_*)\; g_{1-loop}(u,0)\; d^r u$$

where

$$\text{JK-Res}_\eta(Q_*)\frac{d^r u}{Q_1(u)\cdots Q_r(u)} = \begin{cases} \frac{1}{|DetQ|}, & \text{If } \eta \in \text{Cone}\{Q\} \\ 0, & \textit{otherwise} \end{cases}$$

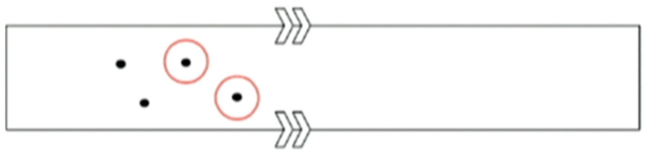
for singularities where r hyperplane meet. [Jeffrey-Kirwan, 2003]

[Hori-Kim-Yi, 2014][Cordova, 2014][Hwang-Kim-Kim-Park, 2014]

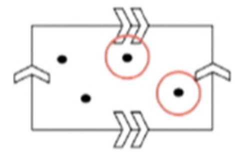
Witten Index of 1d Gauged Linear Sigma Model

$$\Omega(y,\zeta) = \frac{1}{|W_G|}\int d^r D \int d^r \lambda_+^0 d^r \bar{\lambda}_+^0 \int d^r u d^r \bar{u}\; g_{\text{1-loop}}(u,D)\; e^{\lambda_0 \bar{\psi}\phi + \text{c.c}} e^{-\frac{1}{e^2}D^2 - i\zeta D}$$

After careful treatment of gaugino zero modes and D-integrals we are left with residue integral over selected poles. (cf. [Benini-Eager-Hori-Tachikawa, 2013])



cf. 2d elliptic genus



$$\Omega(y,\zeta) = \frac{1}{|W_G|}\sum_{Q_*} \text{JK-Res}_{\eta=\zeta_{FI}}(Q_*)\; g_{1-loop}(u,0)\; d^r u$$

where

$$\text{JK-Res}_\eta(Q_*)\frac{d^r u}{Q_1(u)\cdots Q_r(u)} = \begin{cases} \frac{1}{|DetQ|}\,, & \text{If } \eta \in \text{Cone}\{Q\} \\ 0\,, & \textit{otherwise} \end{cases}$$

for singularities where r hyperplane meet. [Jeffrey-Kirwan, 2003]

[Hori-Kim-Yi, 2014][Cordova, 2014][Hwang-Kim-Kim-Park, 2014]

Quiver Quantum Mechanics with Large Rank

- Scaling behaviour?
  → For quiver invariants, we expect blackhole like degeneracy ($\log \Omega \sim N^2$). What about multi-centered states?

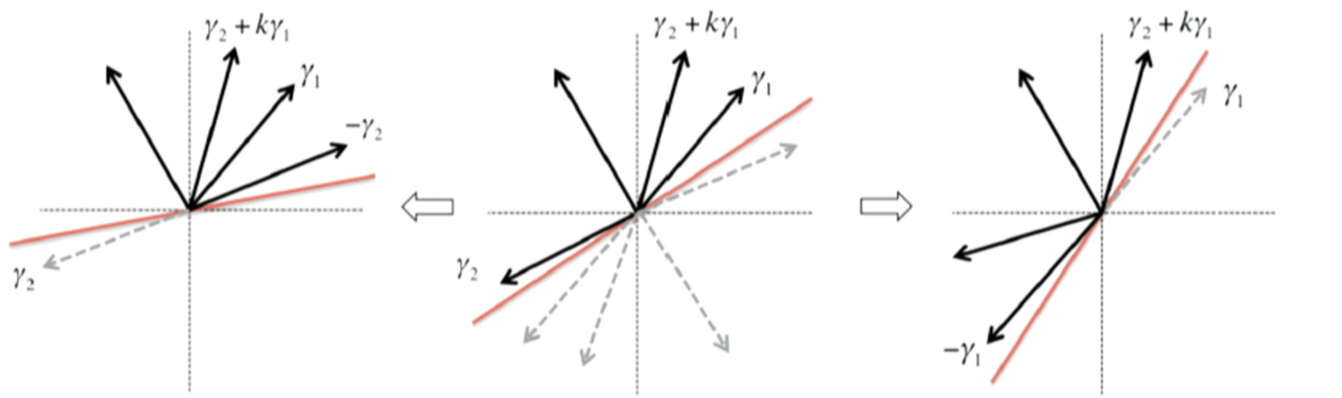
- Mutation equivalence?

$$\mu_k^L(\gamma_i) = \begin{pmatrix} -\gamma_k & i = k \\ \gamma_i + [b_{ki}]_+ \gamma_k & \text{otherwise} \end{pmatrix}$$

$$\mu_k^R(\gamma_i) = \begin{pmatrix} -\gamma_k & i = k \\ \gamma_i + [b_{ik}]_+ \gamma_k & \text{otherwise} \end{pmatrix}$$

Basis change of charge vectors
→ Nontrivial equivalence between moduli space of two different quivers?
→ How does "quiver invariants" transform under mutation?

Mutation transformation



[Gaiotto-Moore-Neitzke, 2010][Alim et al., 2011]

Quiver Quantum Mechanics with Large Rank

- Scaling behaviour?
  → For quiver invariants, we expect blackhole like degeneracy ($\log \Omega \sim N^2$). What about multi-centered states?
- Mutation equivalence?

$$\mu_k^L(\gamma_i) = \begin{pmatrix} -\gamma_k & i = k \\ \gamma_i + [b_{ki}]_+\gamma_k & \text{otherwise} \end{pmatrix}$$

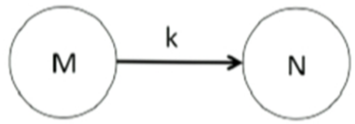
$$\mu_k^R(\gamma_i) = \begin{pmatrix} -\gamma_k & i = k \\ \gamma_i + [b_{ik}]_+\gamma_k & \text{otherwise} \end{pmatrix}$$

Basis change of charge vectors

→ Nontrivial equivalence between moduli space of two different quivers?

→ How does "quiver invariants" transform under mutation?

Kronecker Quivers



- Bound state of $\gamma = M\gamma_1 + N\gamma_2$ with $\langle \gamma_1, \gamma_2 \rangle = k$
- Exhibits simplest wall-crossing with two chambers $\zeta_1 > 0$, $\zeta_1 < 0$.
- Compact moduli space when M, N are mutually co-prime

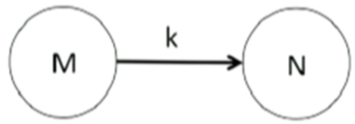
$$\{\Phi^{1,2,\cdots,k} \in \mathbb{C}^M \times \mathbb{C}^N | \Phi^\dagger \cdot \Phi = \zeta_1\}/S(U(M) \times U(N))$$

- Dimension of classical moduli space

$$\mathrm{dim}\mathcal{M}_{(M,N)_k} = k \cdot M \cdot N - (M^2 + N^2 - 1)$$

- Reineke formula [Reineke, 2003]

Kronecker Quivers



- Bound state of $\gamma = M\gamma_1 + N\gamma_2$ with $\langle\gamma_1, \gamma_2\rangle = k$
- Exhibits simplest wall-crossing with two chambers $\zeta_1 > 0$, $\zeta_1 < 0$.
- Compact moduli space when M, N are mutually co-prime

$$\{\Phi^{1,2,\cdots,k} \in \mathbb{C}^M \times \mathbb{C}^N | \Phi^\dagger \cdot \Phi = \zeta_1\}/S(U(M) \times U(N))$$

- Dimension of classical moduli space

$$\mathrm{dim}\mathcal{M}_{(M,N)_k} = k \cdot M \cdot N - (M^2 + N^2 - 1)$$

- Reineke formula [Reineke, 2003]

Witten index of Kronecker Quivers

There exists a class of quivers that only rank $r = M + N - 1$ poles contributes

Each residue integral of non-degenerate pole contributes to the Witten index

$$\prod_{p\neq q} \frac{\sinh[(f_p(a_i) - f_q(a_i))/2]}{\sinh[(f_p(a_i) - f_q(a_i) - z)/2]} \prod_{k\neq l} \frac{\sinh[(g_k(a_i) - g_l(a_i))/2]}{\sinh[(g_k(a_i) - g_l(a_i) - z)/2]}$$
$$\times \prod_{f_p\neq g_k} \frac{\sinh[(f_p(a_i) - g_k(a_i) - z)/2]}{\sinh[(f_p(a_i) - g_k(a_i))/2]}$$

$\rightarrow$ 1 in the Witten index limit $z \rightarrow 0$

Witten index can be computed by counting number of contributing poles recursively.

[H. Kim, 2015]

Counting Witten Index of Kronecker Quivers

Such iterative counting procedure is encoded in the recursion relation

$$f^k_{n+1} = \binom{k-1}{1} \sum_{\substack{a_1,\cdots,a_{k-1} \\ \sum a_i = n}} f^k_{a_1} f^k_{a_2} \cdots f^k_{a_{k-1}} + \binom{k-1}{2} \sum_{\substack{a_1,\cdots,a_{2(k-1)} \\ \sum a_i = n}} f^k_{a_1} f^k_{a_2} \cdots f^k_{a_{2(k-1)}}$$

$$+ \cdots + \binom{k-1}{k-1} \sum_{\substack{a_1,\cdots,a_{(k-1)^2} \\ \sum a_i = n}} f^k_{a_1} f^k_{a_2} \cdots f^k_{a_{(k-1)^2}} .$$

Define a generating function $f_k(x) = \sum_{n=1}^{\infty} f^k_n x^n$, it satisfies the algebraic equation

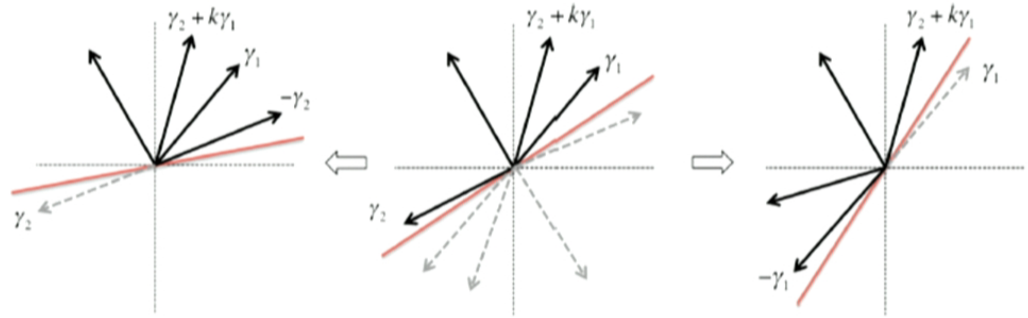
$$f_k(x) = x(1 + f_k(x)^{(k-1)})^{(k-1)} .$$

[Weist, 2009]

From this,

$$\Omega_{(d,(k-1)d+1)_k} = \Omega_{(d-1,d)_k} = \frac{k}{d(k-1)d+1)} \binom{(k-1)^2 d + (k-1)}{d-1}$$

Mutation and Quiver Quantum Mechanics



- 1d Seiberg-like duality
- Cluster algebra of $4d$ $\mathcal{N} = 2$ theories

Mutation and Quiver Quantum Mechanics

Definitions

$$\mu_k^L(\gamma_i) = \begin{cases} -\gamma_k & i = k \\ \gamma_i + [b_{ki}]_+ \gamma_k & \text{otherwise} \end{cases}$$

$$\mu_k^R(\gamma_i) = \begin{cases} -\gamma_k & i = k \\ \gamma_i + [b_{ik}]_+ \gamma_k & \text{otherwise} \end{cases}$$

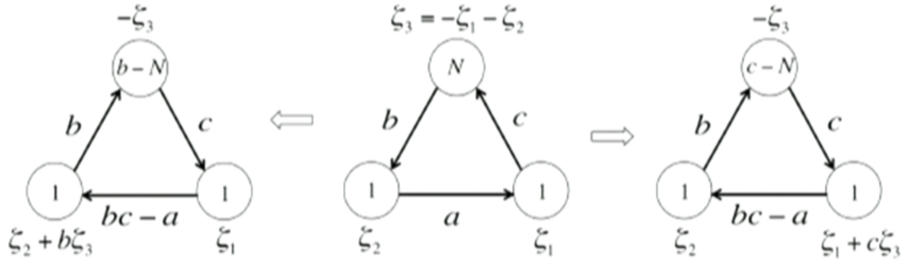
$$\mu_k(b_{ij}) = \begin{cases} -b_{ij} & \text{if } i = k \text{ or } j = k \\ b_{ij} + \mathrm{sgn}(b_{ik})[b_{ik} b_{kj}]_+ & \text{otherwise} \end{cases}$$

$$\mu_k^L(N_i) = \begin{cases} -N_k + \sum_j [b_{kj}]_+ N_j & i = k \\ N_i & \text{otherwise} \end{cases}$$

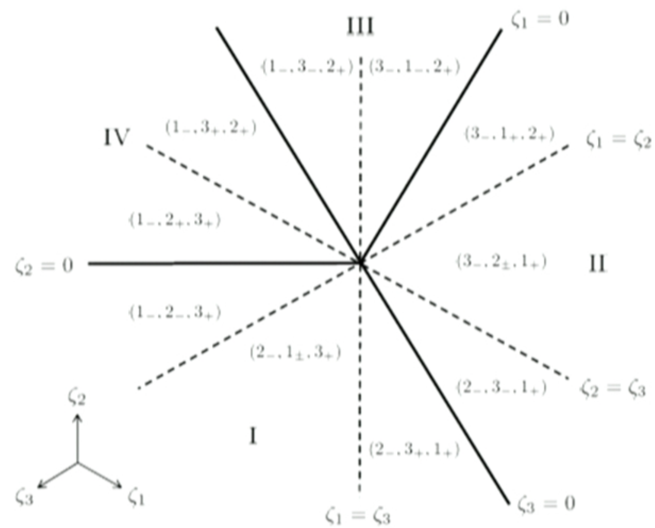
$$\mu_k^R(N_i) = \begin{cases} -N_k + \sum_j [b_{jk}]_+ N_j & i = k \\ N_i & \text{otherwise} \end{cases}$$

Test of Mutation Equivalence

Example : (11N) quivers

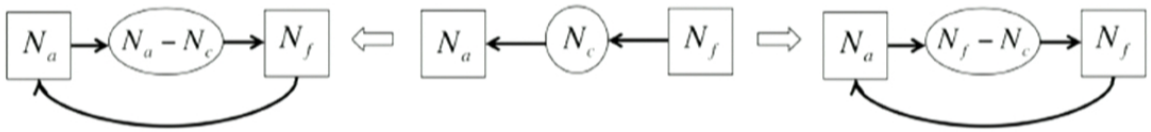


Chamber structure of (11N) quiver



Test of Mutation Equivalence

Prototype of left/right mutation : 1d SQCD



$$\text{JK-res}_{\zeta>0}\, g(u,z) = \sum_{A\in C(N_f,N_c)} \prod_{\substack{i\in A\\ j\in A'}} \frac{-\sinh[(a_i-a_j-z)/2]}{\sinh[(a_i-a_j)/2]}$$

$$\times \prod_{i\in A}\prod_{\beta=1}^{N_a} \frac{-\sinh[(-a_i+b_\beta+(R_f+R_a)z/2-z)/2]}{\sinh[(-a_i+b_\beta+(R_f+R_a)z/2)/2]}$$

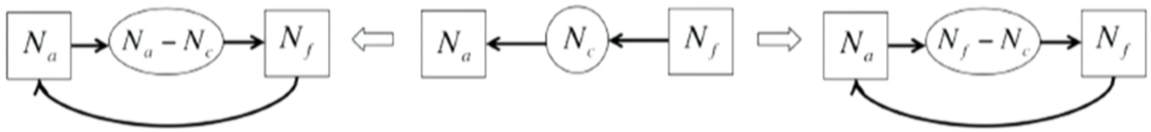
$$= \sum_{A'\in C(N_f,N_f-N_c)} \prod_{\substack{i\in A\\ j\in A'}} \frac{-\sinh[(-a_j+a_i-z)/2]}{\sinh[(-a_j+a_i)/2]}$$

$$\prod_{j\in A'}\prod_{\beta=1}^{N_a} \frac{-\sinh[(a_j-b_\beta-(R_a+R_f)z/2)/2]}{\sinh[(a_j-b_\beta+(2-R_a-R_f)z/2)/2]}$$

$$\prod_{\alpha\in A\cup A'}\prod_{\beta=1}^{N_a} \frac{-\sinh[(-a_\alpha+b_\beta+(R_a+R_f)z/2-z)/2]}{\sinh[(-a_\alpha+b_\beta+(R_a+R_f)z/2)/2]}$$

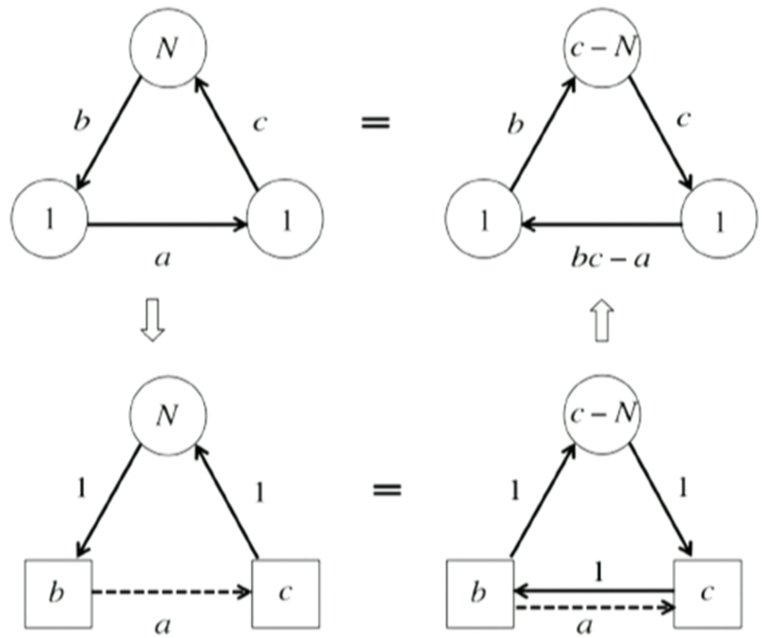
Test of Mutation Equivalence

Prototype of left/right mutation : 1d SQCD

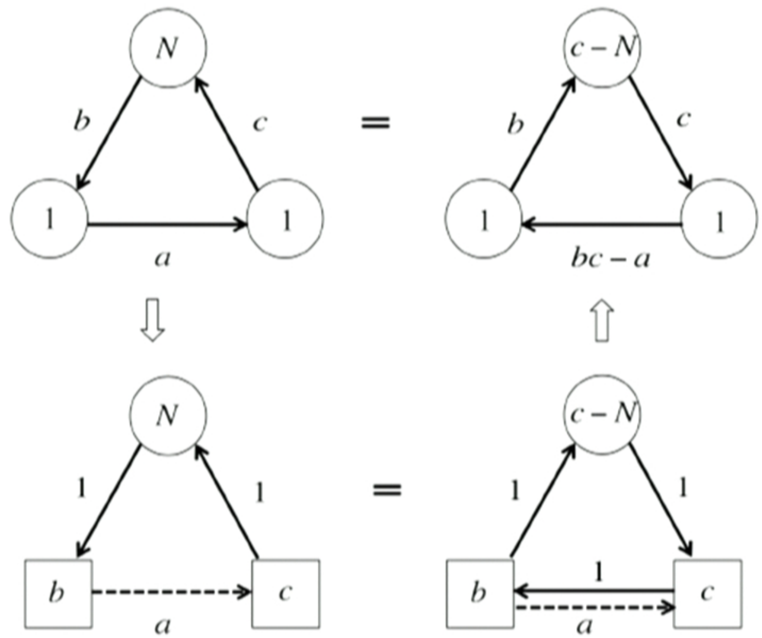


$$
\begin{aligned}
\text{JK-res}_{\zeta>0}\, g(u,z) \;=\; & \sum_{A\in C(N_f,N_c)} \prod_{\substack{i\in A\\ j\in A'}} \frac{-\sinh[(a_i-a_j-z)/2]}{\sinh[(a_i-a_j)/2]} \\
& \times \prod_{i\in A}\prod_{\beta=1}^{N_a} \frac{-\sinh[(-a_i+b_\beta+(R_f+R_a)z/2-z)/2]}{\sinh[(-a_i+b_\beta+(R_f+R_a)z/2)/2]} \\
= \; & \sum_{A'\in C(N_f,N_f-N_c)} \prod_{\substack{i\in A\\ j\in A'}} \frac{-\sinh[(-a_j+a_i-z)/2]}{\sinh[(-a_j+a_i)/2]} \\
& \prod_{j\in A'}\prod_{\beta=1}^{N_a} \frac{-\sinh[(a_j-b_\beta-(R_a+R_f)z/2)/2]}{\sinh[(a_j-b_\beta+(2-R_a-R_f)z/2)/2]} \\
& \prod_{\alpha\in A\cup A'}\prod_{\beta=1}^{N_a} \frac{-\sinh[(-a_\alpha+b_\beta+(R_a+R_f)z/2-z)/2]}{\sinh[(-a_\alpha+b_\beta+(R_a+R_f)z/2)/2]}
\end{aligned}
$$

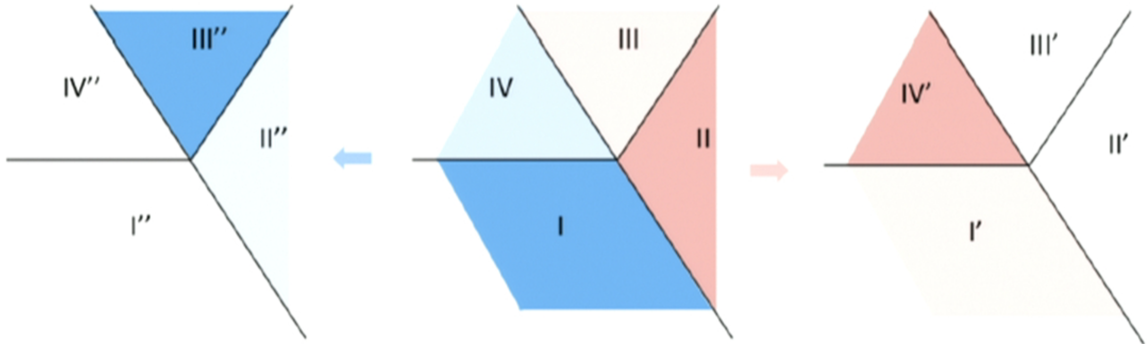
Test of Mutation Equivalence



Test of Mutation Equivalence



Mutation of (11N) Quivers



Right mutation : $\Omega(\text{II}) = \Omega(\text{IV}')$ and $\Omega(\text{III}) = \Omega(\text{I}')$

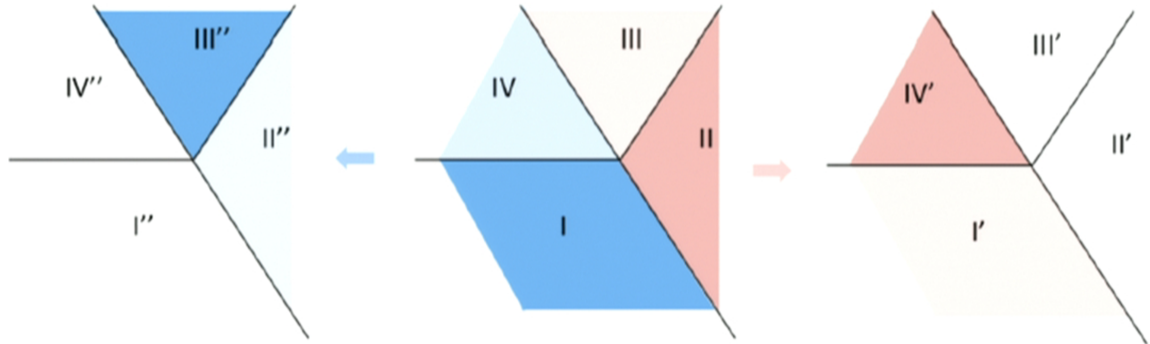
Left mutation : $\Omega(\text{I}) = \Omega(\text{III}'')$ and $\Omega(\text{IV}) = \Omega(\text{II}'')$

Under the assumption (Superpotential is generic.
There is no 1-cycle nor 2-cycle.

Mutation map is well defined for quiver with loops and can be done indefinitely.

[Kim-Lee-Yi, 2015]

Mutation of (11N) Quivers



Right mutation : $\Omega(\text{II}) = \Omega(\text{IV}')$ and $\Omega(\text{III}) = \Omega(\text{I}')$

Left mutation : $\Omega(\text{I}) = \Omega(\text{III}'')$ and $\Omega(\text{IV}) = \Omega(\text{II}'')$

Under the assumption (Superpotential is generic.
There is no 1-cycle nor 2-cycle.

Mutation map is well defined for quiver with loops and can be done indefinitely.

[Kim-Lee-Yi, 2015]

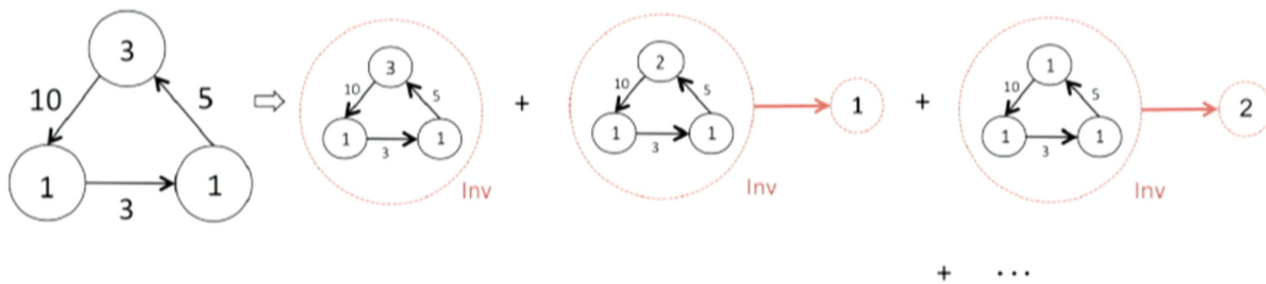
Mutation Map for Quiver Invariants

MPS formula [MPS, 2012]

$$\Omega_Q(\zeta) \sim \sum_{Q=\sum_p Q_p} \Omega^{\text{Coulomb}}_{Q/\{Q_p\}}(\zeta_p) \times \left(\prod_p \Omega_{Q_p}|_{\text{Invariant}} \right)$$

Example :

$$\Omega^{1,1,3}_{3,5,10} = \left(\Omega^{1,1}_{2}\right)^{\text{Coulomb}} \times \Omega^{1,1,2}_{3,5,10}\Big|_{\text{Inv}} \times 1 + \left(\Omega^{1,1,3}_{3,5,10}\right)^{\text{Coulomb}} \times 1^5$$



Mutation map for quiver invariants?

$$\Omega_{\mathrm{Q}}\Big|_{\text{Inv}} = \lim_{\beta\to\infty} \mathrm{Tr}_{L^2} (-1)^{2J_3} \mathbf{y}^{2I+2J_3} e^{-\beta H_{\mathrm{Q}}(\zeta=0)}$$

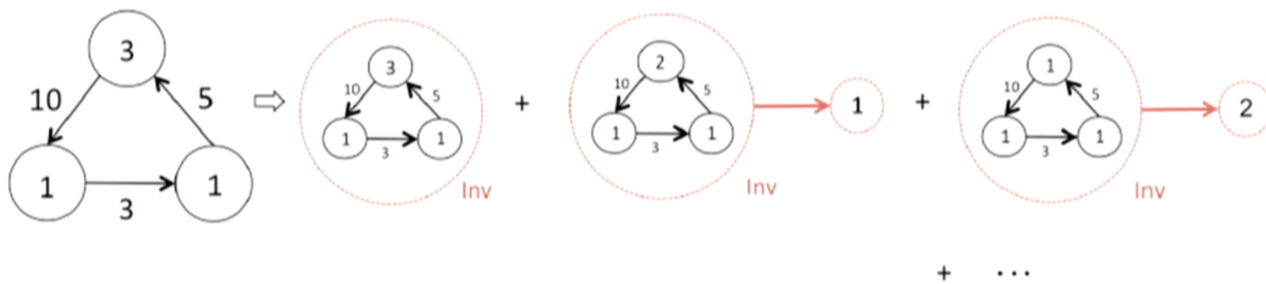
Mutation Map for Quiver Invariants

MPS formula [MPS, 2012]

$$\Omega_Q(\zeta) \sim \sum_{Q=\sum_p Q_p} \Omega^{\text{Coulomb}}_{Q/\{Q_p\}}(\zeta_p) \times \left(\prod_p \Omega_{Q_p}|_{\text{Invariant}} \right)$$

Example :

$$\Omega^{1,1,3}_{3,5,10} = \left(\Omega^{1,1}_{2}\right)^{\text{Coulomb}} \times \Omega^{1,1,2}_{3,5,10}\Big|_{\text{Inv}} \times 1 + \left(\Omega^{1,1,3}_{3,5,10}\right)^{\text{Coulomb}} \times 1^5$$



Mutation map for quiver invariants?

$$\Omega_Q\Big|_{\text{Inv}} = \lim_{\beta\to\infty} \text{Tr}_{L^2}\, (-1)^{2J_3} \mathbf{y}^{2I+2J_3} e^{-\beta H_Q(\zeta=0)}$$

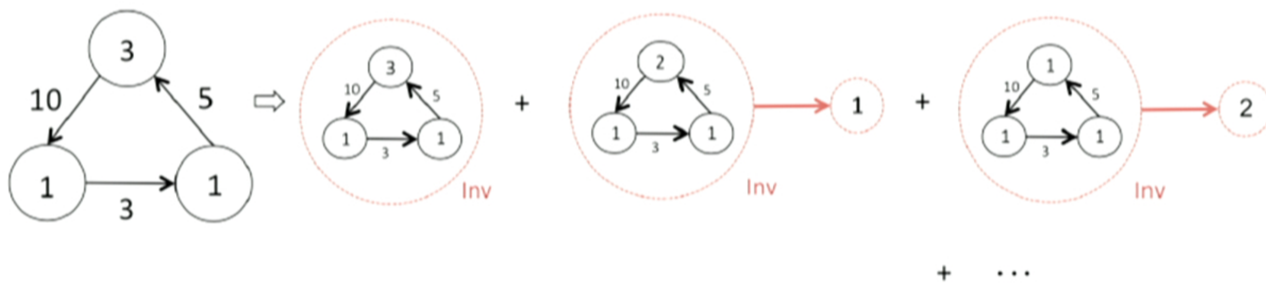
Mutation Map for Quiver Invariants

MPS formula [MPS, 2012]

$$\Omega_Q(\zeta) \sim \sum_{Q=\sum_p Q_p} \Omega^{\text{Coulomb}}_{Q/\{Q_p\}}(\zeta_p) \times \left(\prod_p \Omega_{Q_p}\big|_{\text{Invariant}} \right)$$

Example :

$$\Omega^{1,1,3}_{3,5,10} = \left(\Omega^{1,1}_{2}\right)^{\text{Coulomb}} \times \Omega^{1,1,2}_{3,5,10}\Big|_{\text{Inv}} \times 1 + \left(\Omega^{1,1,3}_{3,5,10}\right)^{\text{Coulomb}} \times 1^5$$



Mutation map for quiver invariants?

$$\Omega_Q\Big|_{\text{Inv}} = \lim_{\beta\to\infty} \text{Tr}_{L^2} (-1)^{2J_3} \mathbf{y}^{2I+2J_3} e^{-\beta H_Q(\zeta=0)}$$

Mutation Map for Quiver Invariants

For given mutation rule of γ_i, there is another natural choice of mutation map for dimension vector. [MPS, 2013] For $\Gamma = \sum_i N_i \gamma_i$, we can impose

$$\tilde{\mu}_k^L(\Gamma) = \begin{pmatrix} -\Gamma & \Gamma = n\gamma_k\,, \quad n \in \mathbf{Z}_+ \\ \Gamma + [\langle \gamma_k, \Gamma \rangle]_+ \gamma_k & \text{otherwise} \end{pmatrix}$$

$$\tilde{\mu}_k^R(\Gamma) = \begin{pmatrix} -\Gamma & \Gamma = n\gamma_k\,, \quad n \in \mathbf{Z}_+ \\ \Gamma + [\langle \Gamma, \gamma_k \rangle]_+ \gamma_k & \text{otherwise} \end{pmatrix}$$

They reduces to

$$\tilde{\mu}_k^{L,R}(N_i) = \begin{pmatrix} -N_k + \min\left(\sum_j [b_{jk}]_+ N_j, \sum_j [b_{kj}]_+ N_j\right) & i = k \\ N_i & \text{otherwise} \end{pmatrix}$$

We checked that "quiver invariant" is invariant under $\tilde{\mu}$, which maps a quiver to a quiver.

Summary

- We derived an index formula for 1d GLSM with at least $\mathcal{N} = 2$.
- Using this method, we have shown that the index of Kronecker quiver grows with $\sim e^{c(k)d}$ at large rank.
- Mutation equivalence has been checked with quivers with loop.
- We extracted the quiver invarints using MPS formula, and checked its transformation rule under mutation map.


Summary
• We derived an index formula for 1d GLSM with at least $N = 2$.
• Using this method, we have shown that the index of Kronecker quiver grows with $\sim e^{c(k)d}$ at large rank.
• Mutation equivalence has been checked with quivers with loop.
• We extracted the quiver invarints using MPS formula, and checked its transformation rule under mutation map.