Title: Signatures of Mathieu Moonshine in Z_2-orbifolds of Conformal Field Theories

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Abstract:

Signatures of Mathieu Moonshine in $\mathbb{Z}_2$ – orbifold CFTs on K3

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Workshop on `Mock modularity, Moonshine and String Theory'

Waterloo, 16 April 2015

Based on work with Katrin Wendland (Freiburg)

`A twist in the M24 moonshine story'; arXiv:1303.3221
`Symmetry-surfing the moduli space of Kummer K3s'; arXiv:1303.2931
`The overarching finite symmetry group of Kummer surfaces in M24'; arXiv:1107.3834

Outline

1. Margolin's V_{45} and the $\mathbb{Z}_2^4 \rtimes A_8$ action on it
2. Three slides to motivate our interest in $\mathbb{Z}_2^4 \rtimes A_8$
3. Why we restrict ourselves to geometric symmetries in $\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$
4. Identification of the generic massive states in $\tilde{R}\tilde{R}$ sector of $\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$ at $(h, Q; \bar{h}, \bar{Q}) = (\frac{5}{4}, 1; \frac{1}{4}, \bar{Q})$ using free fields
5. Action of geometric symmetries on the massive states obtained in (4) and emergence of V_{45}^{CFT}
6. The $\mathbb{Z}_2^4 \rtimes A_8$ action on V_{45}^{CFT}
7. Conclusions

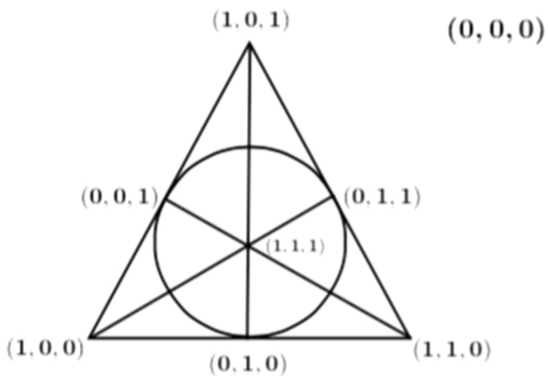
1. Margolin's 45-dimensional space

$$V_{45} = \mathcal{V}_A \oplus \mathcal{V}_B \oplus \ldots \oplus \mathcal{V}_O$$

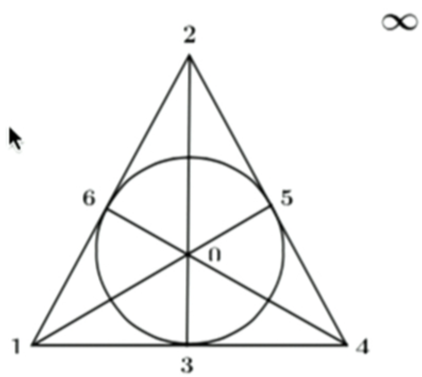
carries an irreducible 45d representation of M_{24}

$\mathcal{V}_X \cong \mathbb{C}^3$ designed to carry a 3d irrep of $GL_3(\mathbb{F}_2)$, $\quad X = A, B, \ldots, O$

$\mathrm{Aff}(V) = \mathbb{Z}_2^3 \rtimes GL_3(\mathbb{F}_2)$, V 3d vector space over $\mathbb{F}_2$



$\mathbb{P}(\mathbb{F}_2^3)$



Fano plane

$\mathbb{Z}_2^4 \rtimes A_8$ action on Margolin's 45-dimensional space

$$V_{45} = \mathcal{V}_A \oplus \mathcal{V}_B \oplus \ldots \oplus \mathcal{V}_O$$

Fano plane

	c_0	c_1	c_2	c_3	c_4	c_5	c_6
A	$\infty\mathbf{0.15.23.46^{0}}$	$\infty\mathbf{1.05.26.34^{1}}$	$\infty\mathbf{2.03.16.45^{2}}$	$\infty\mathbf{3.02.14.56^{3}}$	$\infty\mathbf{4.06.13.25^{4}}$	$\infty\mathbf{5.01.24.36^{5}}$	$\infty\mathbf{6.04.12.35^{6}}$
B	$\infty\mathbf{2.03.14.56^{2}}$	$\infty 1.06.24.35^{3}$	$\infty\mathbf{3.02.15.46^{4}}$	$\infty\mathbf{0.16.23.45^{5}}$	$\infty 4.05.12.36^{0}$	$\infty 5.04.13.26^{1}$	$\infty 6.01.25.34^{6}$
C	$\infty\mathbf{3.02.16.45^{4}}$	$\infty 1.04.25.36^{1}$	$\infty\mathbf{0.14.23.56^{5}}$	$\infty\mathbf{2.03.15.46^{2}}$	$\infty 4.01.26.35^{0}$	$\infty 5.06.12.34^{6}$	$\infty 6.05.13.24^{3}$
D	$\infty 0.12.36.45^{5}$	$\infty\mathbf{3.06.14.25^{0}}$	$\infty 2.01.35.46^{3}$	$\infty\mathbf{4.05.13.26^{4}}$	$\infty\mathbf{1.02.34.56^{6}}$	$\infty 5.04.16.23^{2}$	$\infty 6.03.15.24^{1}$
E	$\infty 0.16.24.35^{4}$	$\infty\mathbf{4.02.13.56^{5}}$	$\infty 2.04.15.36^{2}$	$\infty\mathbf{1.06.25.34^{0}}$	$\infty\mathbf{3.05.14.26^{1}}$	$\infty 5.03.12.46^{3}$	$\infty 6.01.23.45^{6}$
F	$\infty 0.14.26.35^{1}$	$\infty 1.04.23.56^{0}$	$\infty\mathbf{4.01.25.36^{5}}$	$\infty 3.05.12.46^{6}$	$\infty\mathbf{5.03.16.24^{2}}$	$\infty\mathbf{2.06.13.45^{4}}$	$\infty 6.02.15.34^{3}$
G	$\infty 0.12.34.56^{0}$	$\infty 1.02.35.46^{1}$	$\infty\mathbf{5.06.13.24^{4}}$	$\infty 3.04.15.26^{3}$	$\infty\mathbf{2.01.36.45^{5}}$	$\infty\mathbf{4.03.16.25^{2}}$	$\infty 6.05.14.23^{6}$
H	$\infty 0.13.26.45^{0}$	$\infty 1.03.25.46^{1}$	$\infty 2.06.15.34^{2}$	$\infty\mathbf{5.04.12.36^{6}}$	$\infty 4.05.16.23^{4}$	$\infty\mathbf{6.02.14.35^{3}}$	$\infty\mathbf{3.01.24.56^{5}}$
I	$\infty 0.16.25.34^{2}$	$\infty 1.06.23.45^{1}$	$\infty 2.05.13.46^{4}$	$\infty\mathbf{6.01.24.35^{6}}$	$\infty 4.03.15.26^{0}$	$\infty\mathbf{3.04.12.56^{3}}$	$\infty\mathbf{5.02.14.36^{5}}$
J	$\infty\mathbf{4.06.12.35^{6}}$	$\infty 1.03.24.56^{3}$	$\infty 2.05.14.36^{5}$	$\infty 3.01.26.45^{1}$	$\infty\mathbf{6.04.15.23^{0}}$	$\infty 5.02.16.34^{2}$	$\infty\mathbf{0.13.25.46^{4}}$
K	$\infty\mathbf{6.04.13.25^{6}}$	$\infty 1.02.36.45^{3}$	$\infty 2.01.34.56^{2}$	$\infty 3.05.16.24^{5}$	$\infty\mathbf{0.12.35.46^{0}}$	$\infty 5.03.14.26^{1}$	$\infty\mathbf{4.06.15.23^{4}}$
L	$\infty\mathbf{1.05.24.36^{5}}$	$\infty\mathbf{5.01.23.46^{3}}$	$\infty 2.06.14.35^{0}$	$\infty 3.04.16.25^{2}$	$\infty 4.03.12.56^{1}$	$\infty\mathbf{0.15.26.34^{6}}$	$\infty 6.02.13.45^{4}$
M	$\infty\mathbf{5.01.26.34^{3}}$	$\infty\mathbf{0.15.24.36^{4}}$	$\infty 2.04.13.56^{2}$	$\infty 3.06.12.45^{6}$	$\infty 4.02.16.35^{5}$	$\infty\mathbf{1.05.23.46^{1}}$	$\infty 6.03.14.25^{0}$
N	$\infty 0.13.24.56^{4}$	$\infty\mathbf{2.04.16.35^{2}}$	$\infty\mathbf{6.05.12.34^{6}}$	$\infty 3.01.25.46^{3}$	$\infty 4.02.15.36^{5}$	$\infty 5.06.14.23^{0}$	$\infty\mathbf{1.03.26.45^{1}}$
O	$\infty 0.14.25.36^{5}$	$\infty\mathbf{6.03.12.45^{6}}$	$\infty\mathbf{1.04.26.35^{1}}$	$\infty 3.06.15.24^{3}$	$\infty 4.01.23.56^{0}$	$\infty 5.02.13.46^{4}$	$\infty\mathbf{2.05.16.34^{2}}$

$$\mathcal{B} = \langle P_A, P_B, ..., P_O \rangle \qquad V_{45} = \mathcal{V} \otimes \mathcal{B}$$

A_8 action on array by conjugation on rows $\Rightarrow$ permutation of labels A, B,...O $\Rightarrow$ permutation of $P_A, P_B, ..., P_O$

$\mathbb{Z}_2^4 \rtimes A_8$ action on Margolin's 45-dimensional space

$V_{45} = \mathcal{V}_A \oplus \mathcal{V}_B \oplus \ldots \oplus \mathcal{V}_O$

3d irrep of $GL_3(\mathbb{F}_2)$ constructed from a rank 3 lattice Λ_3^{b7}

$b_7 := \frac{1}{2}(-1 + \sqrt{-7})$

Fano plane

root vectors of Λ_3^{b7}

	0			1			2			3			4			5			6	
2	0	0	-1	$\overline{b7}$	1	$-\overline{b7}$	1	-1	$\overline{b7}$	1	1	$b7$	$b7$	0	$-b7$	0	$b7$	-1	-1	$\overline{b7}$
0	2	0	-1	$-\overline{b7}$	1	0	$b7$	$b7$	$\overline{b7}$	-1	-1	1	-1	$\overline{b7}$	1	$\overline{b7}$	1	$b7$	$-b7$	0
0	0	2	$b7$	0	$b7$	$\overline{b7}$	1	-1	0	$b7$	$-b7$	1	-1	$-\overline{b7}$	-1	$\overline{b7}$	-1	1	1	$\overline{b7}$

point frames

	023			134			245			356			460			501			612	
2	0	0	-1	$\overline{b7}$	1	$-\overline{b7}$	1	-1	$\overline{b7}$	1	1	$b7$	$b7$	0	$-b7$	0	$b7$	-1	-1	$\overline{b7}$
0	$b7$	$b7$	$\overline{b7}$	-1	-1	1	-1	$\overline{b7}$	1	$\overline{b7}$	1	$b7$	$-b7$	0	0	2	0	-1	$-\overline{b7}$	1
0	$b7$	$-b7$	1	-1	$-\overline{b7}$	-1	$\overline{b7}$	-1	1	1	$\overline{b7}$	0	0	2	$b7$	0	$b7$	$\overline{b7}$	1	-1

line frames

- induced A_8 action on fibres: $\tau \in A_8$, $\tau X \tau^{-1} = Y \Rightarrow \mathcal{V}_X \to \mathcal{V}_Y$
- TWIST: when induced maps between Fano planes associated with permuted rows are not all identical (τ fixed)
- For any fixed labelling of the array compatible with the linear structure of the Fano planes $\mathbb{P}_X(\mathbb{F}_2^3)$, the group A_8 acts with a twist on V_{45}, i.e. the A_8 action does NOT factorize according to $V_{45} = \mathcal{V} \otimes \mathcal{B}$

$\mathbb{Z}_2^4 \rtimes A_8$ action on Margolin's 45-dimensional space

$\mathbb{Z}_2^4$ action on $\mathcal{B}$: faithful

$\mathbb{Z}_2^4$ action on $\mathcal{V}$: trivial

conjugate by $\sigma \in S_8 \setminus A_8$

	c_0	c_1	c_2	c_3	c_4	c_5	c_6
A	$\infty\mathbf{0.15.23.46}^{\mathbf{0}}$	$\infty\mathbf{1.05.26.34}^{\mathbf{1}}$	$\infty\mathbf{2.03.16.45}^{\mathbf{2}}$	$\infty\mathbf{3.02.14.56}^{\mathbf{3}}$	$\infty\mathbf{4.06.13.25}^{\mathbf{4}}$	$\infty\mathbf{5.01.24.36}^{\mathbf{5}}$	$\infty\mathbf{6.04.12.35}^{\mathbf{6}}$
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C	$\infty\mathbf{3.02.16.45}^{\mathbf{4}}$	$\infty 1.04.25.36^1$	$\infty\mathbf{0.14.23.56}^{\mathbf{5}}$	$\infty\mathbf{2.03.15.46}^{\mathbf{2}}$	$\infty 4.01.26.35^0$	$\infty 5.06.12.34^6$	$\infty 6.05.13.24^3$
D	$\infty 0.12.36.45^5$	$\infty\mathbf{3.06.14.25}^{\mathbf{0}}$	$\infty 2.01.35.46^3$	$\infty\mathbf{4.05.13.26}^{\mathbf{4}}$	$\infty\mathbf{1.02.34.56}^{\mathbf{6}}$	$\infty 5.04.16.23^2$	$\infty 6.03.15.24^1$
E	$\infty 0.16.24.35^4$	$\infty\mathbf{4.02.13.56}^{\mathbf{5}}$	$\infty 2.04.15.36^2$	$\infty\mathbf{1.06.25.34}^{\mathbf{0}}$	$\infty\mathbf{3.05.14.26}^{\mathbf{1}}$	$\infty 5.03.12.46^3$	$\infty 6.01.23.45^6$
F	$\infty 0.14.26.35^1$	$\infty 1.04.23.56^0$	$\infty\mathbf{4.01.25.36}^{\mathbf{5}}$	$\infty 3.05.12.46^6$	$\infty\mathbf{5.03.16.24}^{\mathbf{2}}$	$\infty\mathbf{2.06.13.45}^{\mathbf{4}}$	$\infty 6.02.15.34^3$
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J	$\infty\mathbf{4.06.12.35}^{\mathbf{6}}$	$\infty 1.03.24.56^3$	$\infty 2.05.14.36^5$	$\infty 3.01.26.45^1$	$\infty\mathbf{6.04.15.23}^{\mathbf{0}}$	$\infty 5.02.16.34^2$	$\infty\mathbf{0.13.25.46}^{\mathbf{4}}$
K	$\infty\mathbf{6.04.13.25}^{\mathbf{6}}$	$\infty 1.02.36.45^3$	$\infty 2.01.34.56^2$	$\infty 3.05.16.24^5$	$\infty\mathbf{0.12.35.46}^{\mathbf{0}}$	$\infty 5.03.14.26^1$	$\infty\mathbf{4.06.15.23}^{\mathbf{4}}$
L	$\infty\mathbf{1.05.24.36}^{\mathbf{5}}$	$\infty\mathbf{5.01.23.46}^{\mathbf{3}}$	$\infty 2.06.14.35^0$	$\infty 3.04.16.25^2$	$\infty 4.03.12.56^1$	$\infty\mathbf{0.15.26.34}^{\mathbf{6}}$	$\infty 6.02.13.45^4$
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O	$\infty 0.14.25.36^5$	$\infty\mathbf{6.03.12.45}^{\mathbf{6}}$	$\infty\mathbf{1.04.26.35}^{\mathbf{1}}$	$\infty 3.06.15.24^3$	$\infty 4.01.23.56^0$	$\infty 5.02.13.46^4$	$\infty\mathbf{2.05.16.34}^{\mathbf{2}}$

$\mathbb{Z}_2^4 \rtimes A_8$ maximal in M_{24} and acts with a twist on V_{45}

2. Why is $\mathbb{Z}_2^4 \rtimes A_8$ of interest to us ?

$\mathbb{Z}_2^4 \rtimes A_8$ is the overarching group obtained by collecting the symmetries of all finite symplectic automorphism groups of Kummer surfaces

Let G_X be the symmetry group of a Kummer surface X equipped with a complex structure and a dual Kähler form induced by those of the underlying torus $T(L)$. Then,

- $G_X \cong G_t \rtimes G_T$ with $G_t \cong \mathbb{Z}_2^4$ and $G_T \cong G'_T/\mathbb{Z}_2$ with G'_T a group of non-translational holomorphic symplectic automorphisms of $T(L)$. G'_T classified by Fujiki, $G'_T \subset SU(2)$.
- The maximal groups G_X are

(i) $G_0 := G_X \cong \mathbb{Z}_2^4 \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$	square lattice
(ii) $G_1 := G_X \cong \mathbb{Z}_2^4 \rtimes A_4$	tetrahedral lattice
(iii) $G_2 := G_X \cong \mathbb{Z}_2^4 \rtimes S_3$	triangular lattice

Generic symmetry group on Kummer surfaces

16 singular points : $\vec{F}_{\vec{a}} = \frac{1}{2}\sum_{i=1}^{4} a_i \vec{\lambda}_i,$ $\quad \vec{\lambda}_i$ generators of lattice Λ; $\vec{a} \in \mathbb{F}_2^4$

shifts by half lattice vectors generate $G_t \simeq \mathbb{Z}_2^4$

G_t acts by permutation of labels $\vec{a} \in \mathbb{F}_2^4$, hence acts on the Kummer lattice Π by permutations of $E_{\vec{a}}$

7

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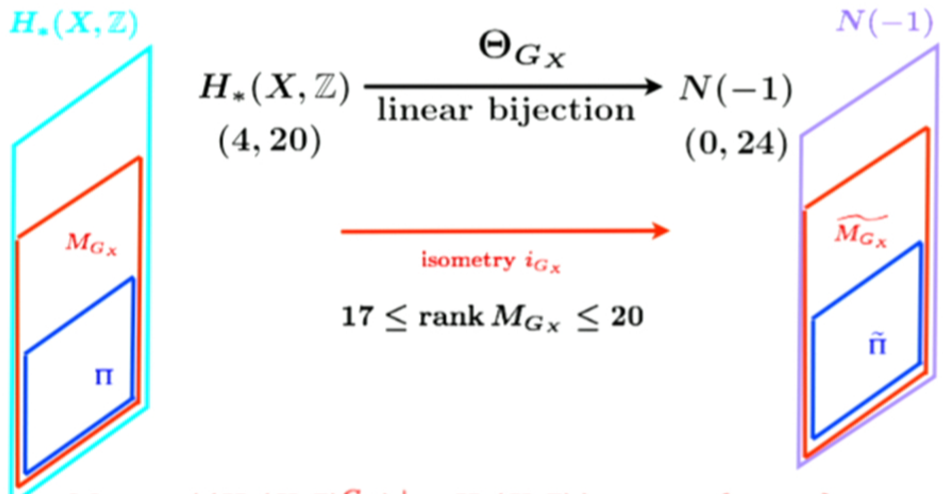
Surfing the moduli space of Kummer surfaces



$$M_{G_X} := \underbrace{\left(\left(H_*(X,\mathbb{Z})^{G_X}\right)^\perp \cap H_*(X,\mathbb{Z})\right) \oplus \text{span}_\mathbb{Z}\{v_0 - v\}}_{L_{G_X} \oplus \text{span}_\mathbb{Z}\{-2\},\ \text{see Kondo ('98)}} \oplus \text{span}_\mathbb{Z}\{e\}$$

Usually, Θ_{G_X} depends on G_X. However

A Kummer tetrahedral: $G_A \simeq (\mathbb{Z}_2)^4 \rtimes A_4$ acts on M_A of rank 20

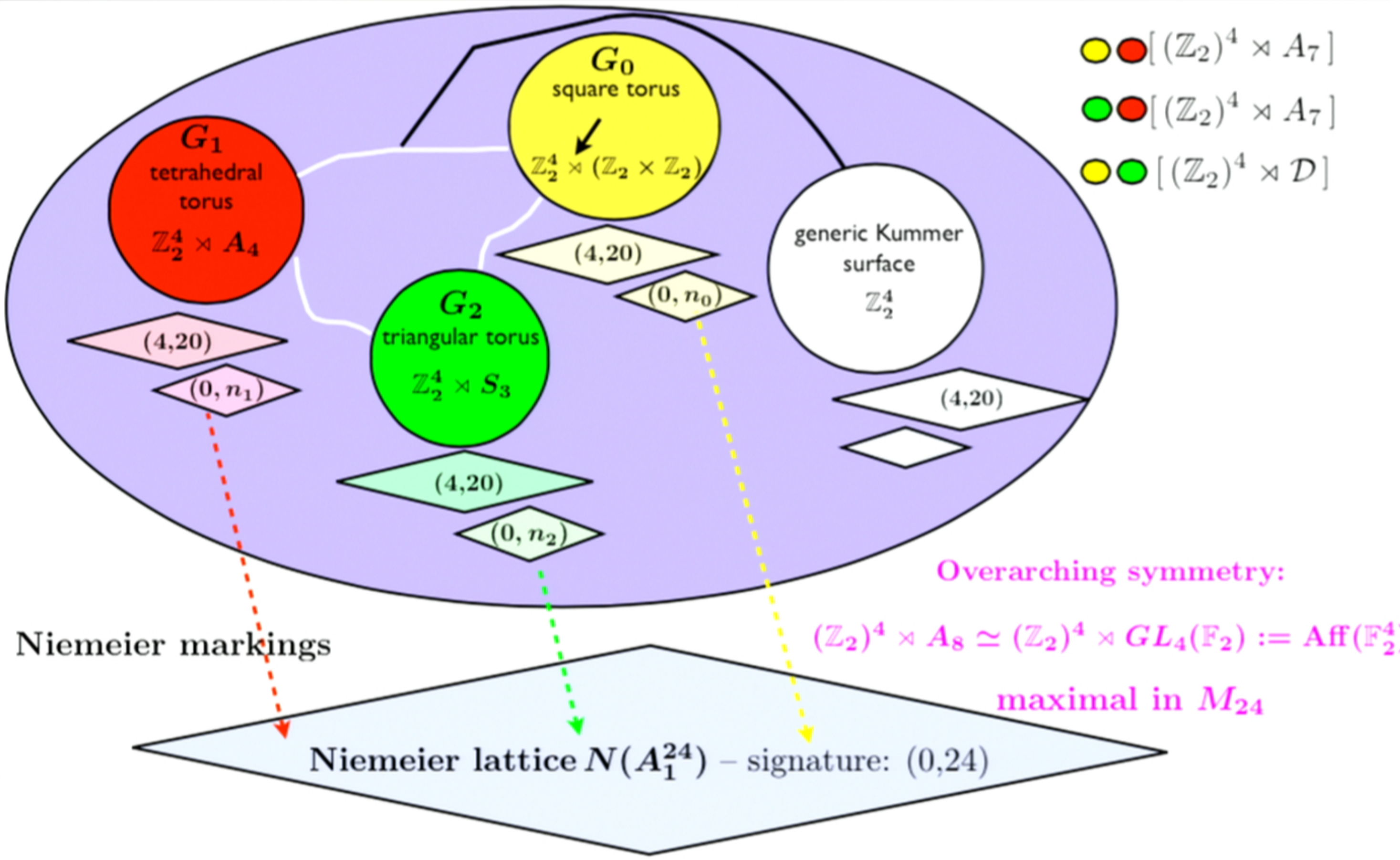
B Kummer square: $G_B \simeq (\mathbb{Z}_2)^4 \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$ acts on M_B of rank 20

combined symmetry: $\mathbb{Z}_2^4 \rtimes A_7$

with $\Theta_{G_A} = \Theta_{G_B} := \Theta_{AB}$ combining map for i_{G_A} and i_{G_B}

The Niemeier lattice N is a device carrying both actions simultaneously, i.e. allowing a combined symmetry on N

$\mathbb{Z}_2^4 \rtimes A_8$ as overarching symmetry



Kummer surface $X = \widetilde{T/\mathbb{Z}_2}$, $T = \mathbb{C}^2/\Lambda$ 9

Interestingly...

The action of G_0, G_1 and G_2 on V_{45} factorizes according to $V_{45} = \mathcal{V} \otimes \mathcal{B}$, i.e. the maximal geometric symmetry groups of Kummer surfaces act without a twist on V_{45} (unlike $\mathbb{Z}_2^4 \rtimes A_8$)

3. Why focus on geometric symmetries in $\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$?

$\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$ on K3: $\mathbb{Z}_2$-orbifold construction induced by the Kummer construction for K3 surfaces

- Fix a preferred N=(2,2) within N=(4,4) - fix a $U(1)_L$ and a $U(1)_R$ current
- Require the symmetries to fix the N=(4,4) SCA pointwise
- Restrict to symmetries compatible with taking a large volume limit

The symmetry groups G are then those induced geometrically in the underlying toroidal theory in a fixed geometric interpretation

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4. Identification of generic massive states in $\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$ on K3

Identify states in the $\tilde{R}\tilde{R}$ sector of partition function with $(h, Q; \bar{h}, \bar{Q}) = (\frac{5}{4}, 1; \frac{1}{4}, \bar{Q})$ using free fields

- 2 complex currents and conjugates $\quad j^1_\pm \sim j^1 \pm i j^2,\ j^2_\pm \sim j^3 \pm i j^4$

  odd under $\mathbb{Z}_2$

- 2 Dirac fermions and conjugates $\quad \chi^1_\pm,\ \chi^2_\pm$

  odd under $\mathbb{Z}_2$

- N=4, c=6 SCA: $T \sim j\,j + \partial\chi\,\chi,\ G^\pm, G'^\pm \sim \chi\, j,\ J^\pm, J^3 \sim \chi\,\chi,$

↑ conformal weight h

↑ charge Q

- $\mathbb{Z}_2$-orbifold partition function:

$$Z^{\tilde{R}\tilde{R}}_{\text{untwisted}}(\tau,z;\bar\tau,\bar z) = \frac{1}{2|\eta(\tau)|^8}\,(1+\sum_{\substack{(p_L;p_R)\in\Gamma,\\(p_L;p_R)\neq(0;0)}} q^{\frac{p_L^2}{2}}\bar q^{\frac{p_R^2}{2}})\left|\frac{\vartheta_1(\tau,z)}{\eta(\tau)}\right|^4 + 8\left|\frac{\vartheta_2(\tau,z)}{\vartheta_2(\tau)}\right|^4$$

$$Z^{\tilde{R}\tilde{R}}_{\text{twisted}}(\tau,z;\bar\tau,\bar z) = 8\left|\frac{\vartheta_3(\tau,z)}{\vartheta_3(\tau)}\right|^4 + 8\left|\frac{\vartheta_4(\tau,z)}{\vartheta_4(\tau)}\right|^4$$

$$Z^{\tilde{R}\tilde{R}}_{\text{orb}}(\tau,z;\bar\tau,\bar z) = Z^{\tilde{R}\tilde{R}}_{\text{untwisted}}(\tau,z;\bar\tau,\bar z) + Z^{\tilde{R}\tilde{R}}_{\text{twisted}}(\tau,z;\bar\tau,\bar z)$$

- K3 elliptic genus:

$$Z_{K3}(\tau,z) := Z^{\tilde{R}\tilde{R}}_{orb}(\tau,z;\bar\tau,\bar z=0)$$

$$q = e^{2i\pi\tau},\ y := e^{2i\pi z}$$

$$= 24\,ch_0^{\tilde{R}}(q,y) + 2\{(-1+45q+231q^2+\dots\}\,\widetilde{ch}^{\tilde{R}}(q,y)$$

↑ ↑

$-y-y^{-1}+2+q(\dots)+\dots$

counting the states with $(h, Q; \bar{h}, \bar{Q}) = (\frac{5}{4}, 1; \frac{1}{4}, \bar{Q})$

from orbifold partition function

- untwisted sector:

$a_1^K \; \chi_0^1 \chi_0^2 \; \bar{\chi}_0^\ell \, \sigma$	$\chi_1^k \; \chi_0^\ell \; \; \sigma$	$\chi_1^k \; \chi_0^\ell \; \bar{\chi}_0^1 \bar{\chi}_0^2 \; \sigma$
$K = 1, .., 4, \; \ell = 1, 2$	$k, \ell = 1, 2$	
8 massless fermions	1 massless boson 3 massive bosons	1 massless boson 3 massive bosons

- twisted sector: $\boxed{a_{\frac{1}{2}}^K \, \chi_{\frac{1}{2}}^\ell \, T_{\vec{a}},}$ $K = 1, .., 4, \ell = 1, 2, \; \vec{a} \in \mathbb{F}_2^4, \quad 4 \times 2 \times 16 = 128$ states

96 massive fermionic twisted states

$\left\{ W T_{\vec{a}} \mid W \in \mathbf{3} \cup \bar{\mathbf{3}}, \; \vec{a} \in \mathbb{F}_2^4 \right\}$ where

$\mathbf{3} := \{\chi^1 j_+^2 + \chi^2 j_+^1, \; \chi^1 j_+^1, \; \chi^2 j_+^2 \}, \quad \bar{\mathbf{3}} := \{\chi^1 j_-^1 - \chi^2 j_-^2, \; \chi^1 j_-^2, \; \chi^2 j_-^1 \},$

net number of massive states: $96 - 6 = 90$

4. Action of geometric symmetries on states

$$G_X = \mathbb{Z}_2^4 \rtimes G_T,\ \ G_T \subset SO(3), \text{X Kummer}$$

- G_X acts on $T_{\vec{a}}$ as a permutation group through affine linear maps on $\mathbb{F}_2^4$; $R_{G_X} : G_X \to Aff(\mathbb{F}_2^4) := \mathbb{Z}_2^4 \rtimes GL_4(\mathbb{F}_2)$
- G_X acts linearly as subgroup of $SO(3)$ on $\mathbf{3}$ and $\bar{\mathbf{3}}$

$$\widehat{\mathbf{96}} = (\langle \mathbf{3} \rangle \oplus \langle \bar{\mathbf{3}} \rangle) \otimes \langle T_{\vec{a}} \rangle := (\langle \mathbf{3} \rangle \oplus \langle \bar{\mathbf{3}} \rangle) \otimes \langle \mathbf{16} \rangle$$

tensor product of representation spaces $\mathbf{3} \oplus \bar{\mathbf{3}}$ of $SO(3)$ and $\mathbf{16}$ of $\text{Aff}(\mathbb{F}_2^4)$

decoupling of 6 dimensions: unique choice

$$\widehat{96} = (\langle 3\rangle \oplus \langle \bar{3}\rangle) \otimes \langle T_{\vec{a}}\rangle := (\langle 3\rangle \oplus \langle \bar{3}\rangle) \otimes \langle 16\rangle$$

$$\downarrow$$

$$\widehat{96} = (\langle 3\rangle \oplus \langle \bar{3}\rangle) \otimes (\langle \mathcal{A}\rangle \oplus \langle N_{\vec{0}}\rangle) := (\langle 3\rangle \oplus \langle \bar{3}\rangle) \otimes (\langle 15\rangle \oplus \langle 1\rangle)$$

with $\mathcal{A} := \{A = \sum_{\vec{b}\in\mathbb{F}_2^4} c_{\vec{b}} T_{\vec{b}} \mid c_{\vec{b}} \in \mathbb{C},\ A \perp N_{\vec{0}},\ N_{\vec{0}} := \frac{1}{4}\sum_{\vec{b}\in\mathbb{F}_2^4} T_{\vec{b}}\}$

90-dimensional vector space of massive states accounting for the leading massive contribution in $\mathcal{E}_{K3}$:

$$\widehat{90} = [\langle 3\rangle \otimes \langle \mathcal{A}\rangle] \oplus [\langle \bar{3}\rangle \otimes \langle \mathcal{A}\rangle] := V_{45}^{CFT} \oplus \bar{V}_{45}^{CFT}$$

with

$$V_{45}^{CFT} = \mathcal{W} \otimes \mathcal{A},\ \mathcal{W} \cong \left\{ W N_{\vec{0}} | W \in \{\chi^1 j_+^2 + \chi^2 j_+^1,\ \chi^1 j_+^1,\ \chi^2 j_+^2\}\right\}$$

3 of $SO(3)$ 15 of $Aff(\mathbb{F}_2^4) := \mathbb{Z}_2^4 \rtimes GL_4(\mathbb{F}_2)$

5. $\mathbb{Z}_2^4 \rtimes A_8$ action on V_{45}^{CFT}

1. V_{45}^{CFT} is a vector space that 'collects' the actions of geometric symmetry groups when surfing. The combined action of those groups generates the action of $\mathrm{Aff}(\mathbb{F}_2^4)$

2. $\mathrm{Aff}(\mathbb{F}_2^4) := \mathbb{Z}_2^4 \rtimes GL_4(\mathbb{F}_2) \cong \mathbb{Z}_2^4 \rtimes A_8$

3. The representation of $\mathrm{Aff}(\mathbb{F}_2^4)$ on the *base* $\mathcal{A}$ of $V_{45}^{CFT} = \mathcal{W} \otimes \mathcal{A}$ is equivalent to Margolin's representation of $\mathbb{Z}_2^4 \rtimes A_8$ on the *base* $\mathcal{B}$ of $V_{45} = \mathcal{V} \otimes \mathcal{B}$

4. $\mathbb{Z}_2^4 \rtimes A_8$ acts with a twist on V_{45}

5. Each maximal geometric symmetry groups of Kummer surfaces $G_k = \mathbb{Z}_2^4 \rtimes (G_T)_k, k = 1, 2, 3$ is such that the non-translational part $(G_T)_0$ acts without a twist on V_{45}

6. The action of G_k on V_{45}^{CFT}, which is induced by the symmetries of the states in $\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$, is equivalent to the action of G_k viewed as a subgroup of $\mathbb{Z}_2^4 \rtimes A_8$ in Margolin's representation $M : G_k \to End_{\mathbb{C}}(V_{45})$

The twist in Margolin's representation is a manifestation of the geometric obstruction to accommodate larger geometric groups, in the framework of $\mathcal{C} = \mathcal{T}/\mathbb{Z}_2$ theories on K3. It is a manifestation of monodromy in the moduli space of SCFTs on K3.

6. Conclusions

1. We have provided a piece of evidence for Mathieu Moonshine explicitly from SCFTs on K3: the generic states accounting for the first order term in the massive sector of the elliptic genus of K3 in every $\mathbb{Z}_2$-orbifold CFT on K3 are uniquely characterized by the fact that the action of every geometric symmetry group of a $\mathbb{Z}_2$-orbifold CFT yields a well-defined faithful representation on them.

   Each such representation is obtained by restriction of the 45-dimensional representation of $\boldsymbol{M_{24}}$ constructed by Margolin.

2. In our quest to understand the action of $\boldsymbol{M_{24}}$ in the context of Mathieu Moonshine, we must 'move away' from Kummer surfaces, as our surfing methods, which were designed for dealing with that special class of K3 surfaces, provide the group $\mathbb{Z}_2^4 \rtimes \boldsymbol{A_8}$ as the largest possible overarching group by construction.

THANK YOU