Title: Modular invariance and holographic CFTs

Date: Apr 15, 2015 11:00 AM

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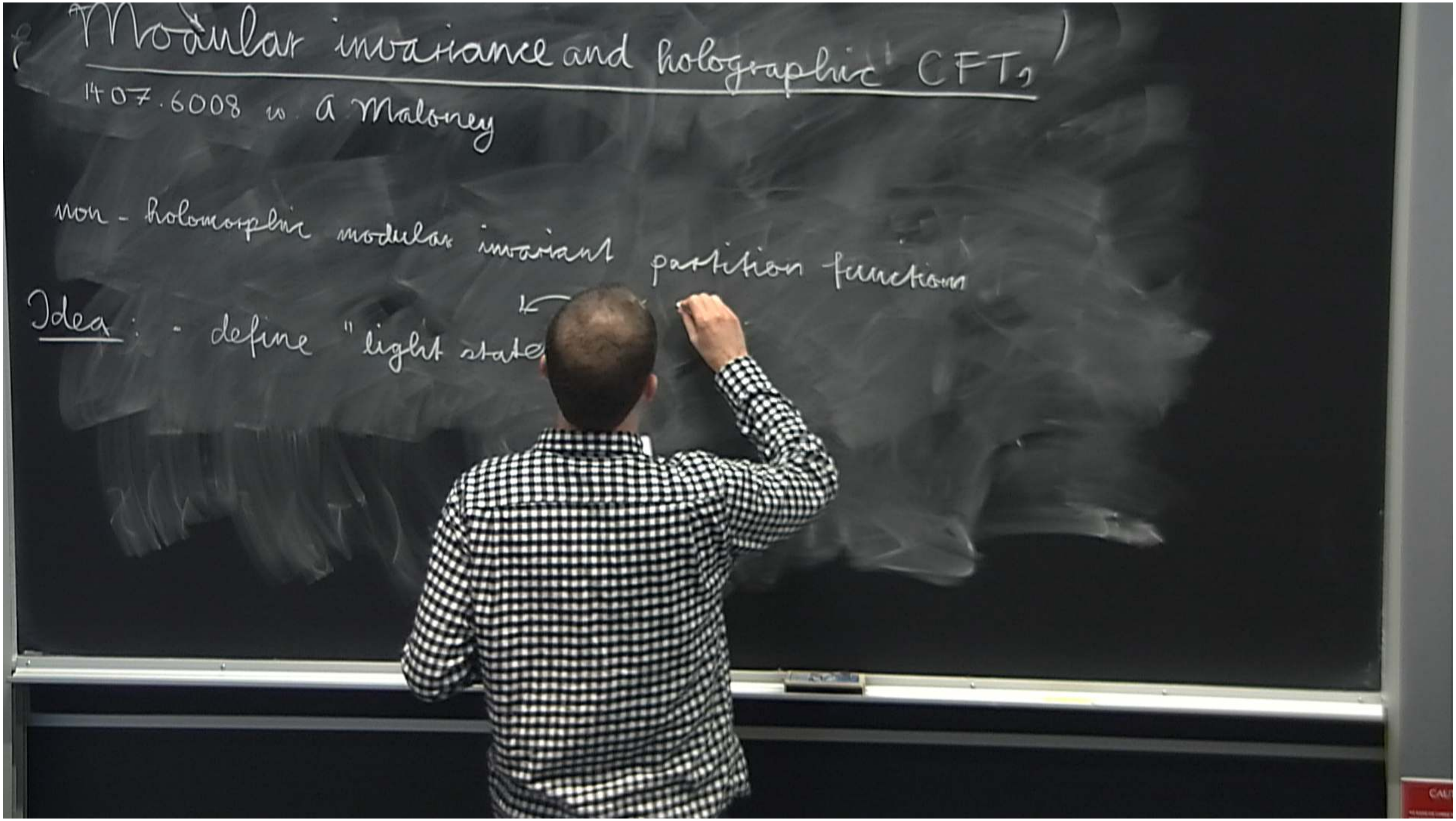
Abstract: Modular invariance plays an important role in AdS3/CFT2 holography. I discuss the structure of non-holomorphic CFT partition functions, namely in what sense the light spectrum determines the heavy spectrum and how to construct example partition functions using Poincare series. This yields necessary conditions on the spectrum of holographic CFTs. Finally I will discuss permutation orbifolds as examples of such theories.

Modular invariance and holographic CFT_2

1407.6008 w. A Maloney

non-holomorphic modular invariant partition functions


Modular invariance and holographic CFT$_2$
1407.6008 w. A. Maloney
non-holomorphic modular invariant partition functions
Idea: - define "light state

(Modular invariance and holographic CFT$_2$)

1407.6008 w. A Maloney

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· existence: $\exists$ modular inv $Z(\tau)$: with exactly the light spectrum

· uniqueness: are the heavy states fixed?

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holomorphic:

$$Z(q) = \underbrace{q^{-k} + a_{-k+1} q^{-k+1} \cdots + a_0}_{\text{light}} + a_1 q^1 \cdots$$

- existence ✓
- uniqueness ✓

light

- existence ✓
- uniqueness ✓

2) elliptic genus : light = polar

- existence ✗
- uniqueness ✓

light

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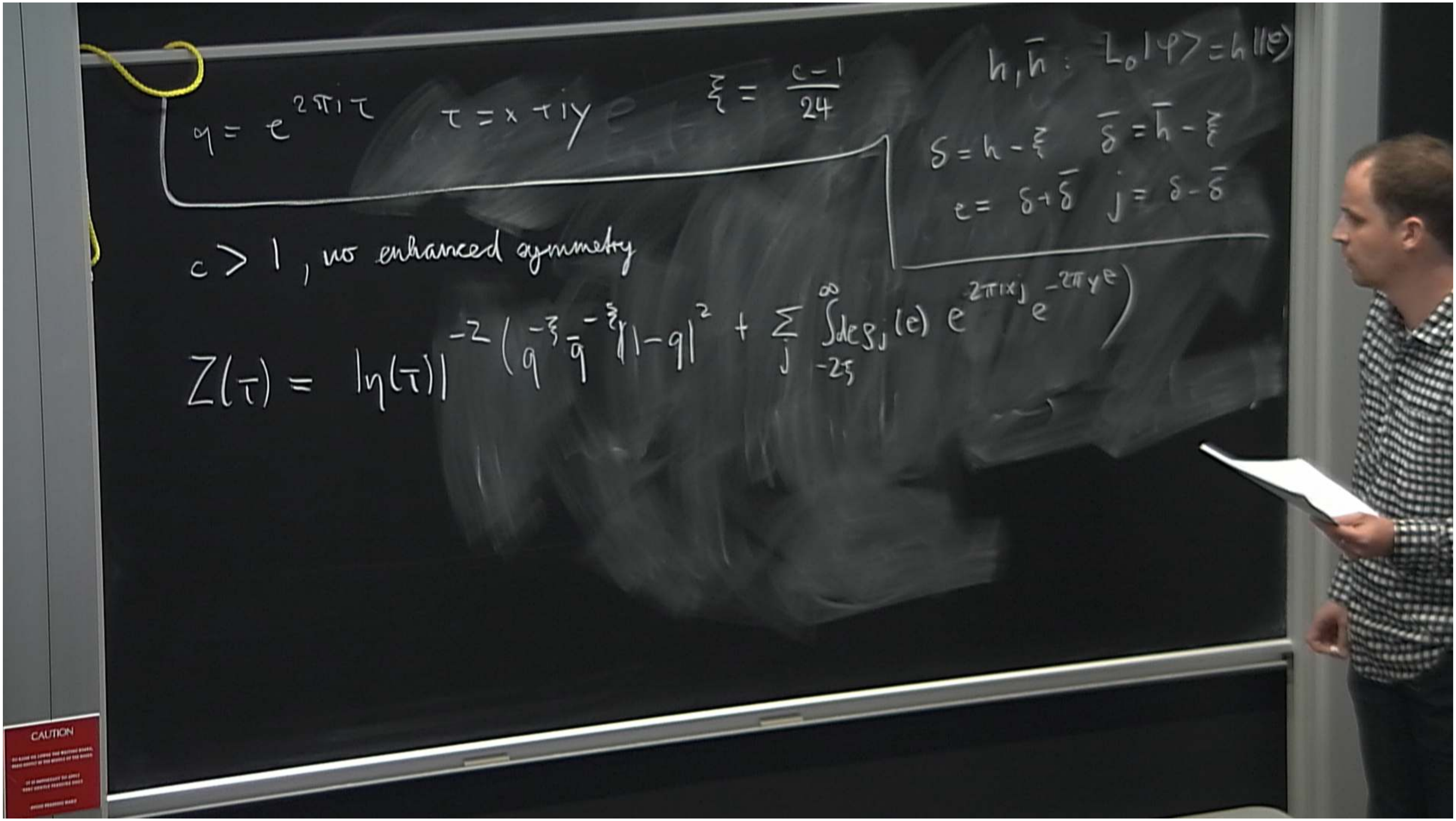
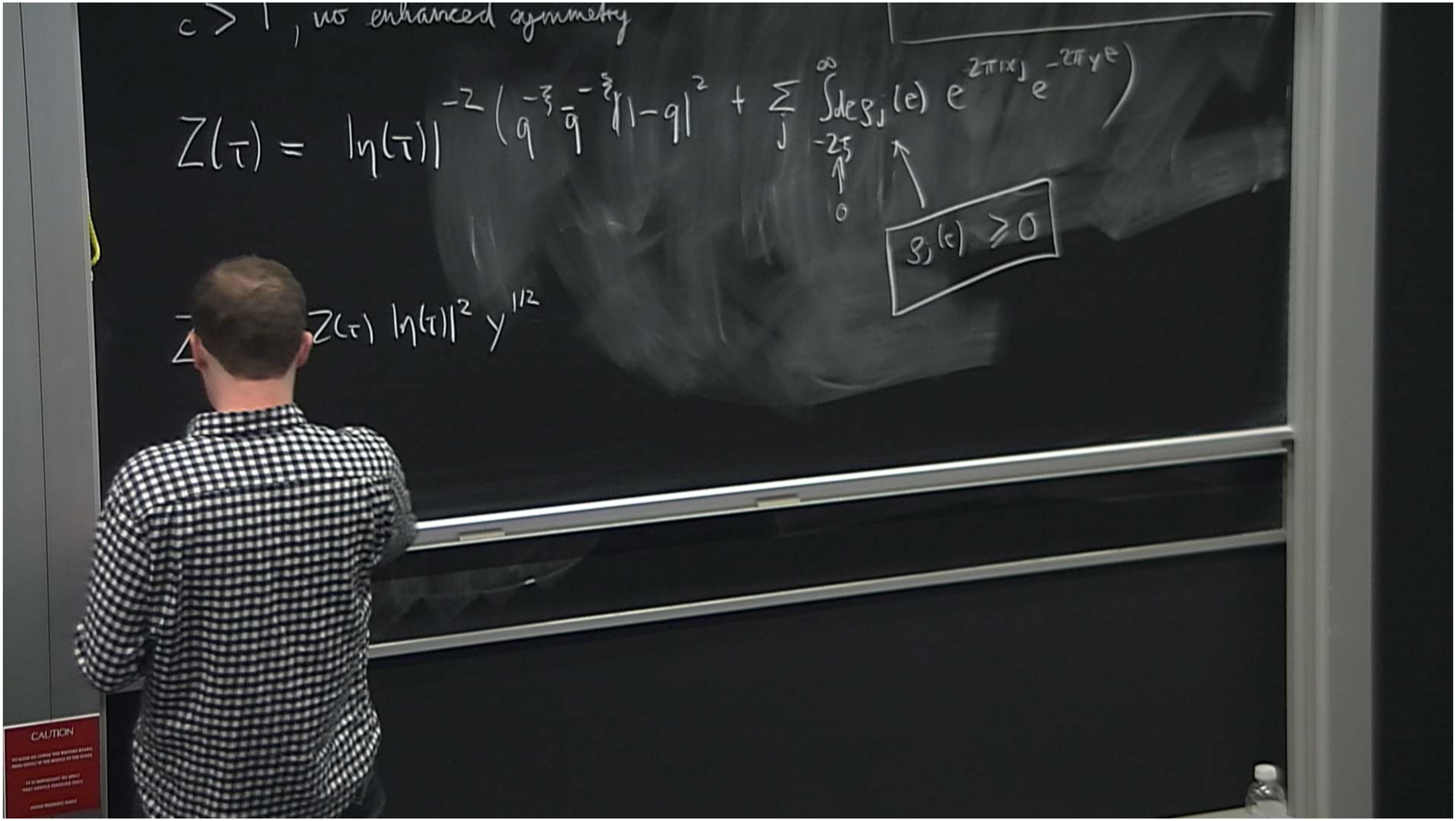
2) elliptic genus : light = polar

- existence ✗
- uniqueness ✓

3) non-holomorphic case?


$q = e^{2\pi i \tau}$ $\tau = x + iy$ $\xi = \frac{c-1}{24}$
$h, \bar{h} : L_0|\varphi\rangle = h|\varphi\rangle$
$\delta = h - \xi$ $\bar{\delta} = \bar{h} - \xi$
$e = \delta + \bar{\delta}$ $j = \delta - \bar{\delta}$
$c > 1$, no enhanced symmetry
$Z(\tau) = |\eta(\tau)|^{-2}\left(q^{-\xi}\bar{q}^{-\xi}|1-q|^2 + \sum_j \int_{-2\xi}^{\infty} de\, \rho_j(e)\, e^{2\pi i x j} e^{-2\pi y e}\right)$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0
$\boxed{\rho_j(E) \geq 0}$
$Z(\tau) |\eta(\tau)|^2 y^{1/2}$
CAUTION

$$Z^{p}_{\Delta,\bar\Delta}(\tau) := F^{p}_{\Delta\bar\Delta}(\tau) + F^{p}_{\Delta\bar\Delta}\left(-\frac{1}{\tau}\right)$$

$$= F^{p}_{\Delta\bar\Delta}(\tau) + \int d\delta\, d\bar\delta\; S_{\Delta\bar\Delta}(\delta,\bar\delta)\, F^{p}_{\delta\bar\delta}(\tau)$$

$$\Rightarrow (u_1, u_2) = (\sqrt{2\delta}, \sqrt{2\bar\delta})$$

$$\Rightarrow \quad f_{-1/\tau} = \hat f_\tau$$

$$F^{p}_{\delta,\bar\delta}(\tau) = y^{1/2}\, e^{\pi i \tau u_1^2}\, e^{-\pi i \bar\tau u_2^2} = f_\tau(\vec u)$$

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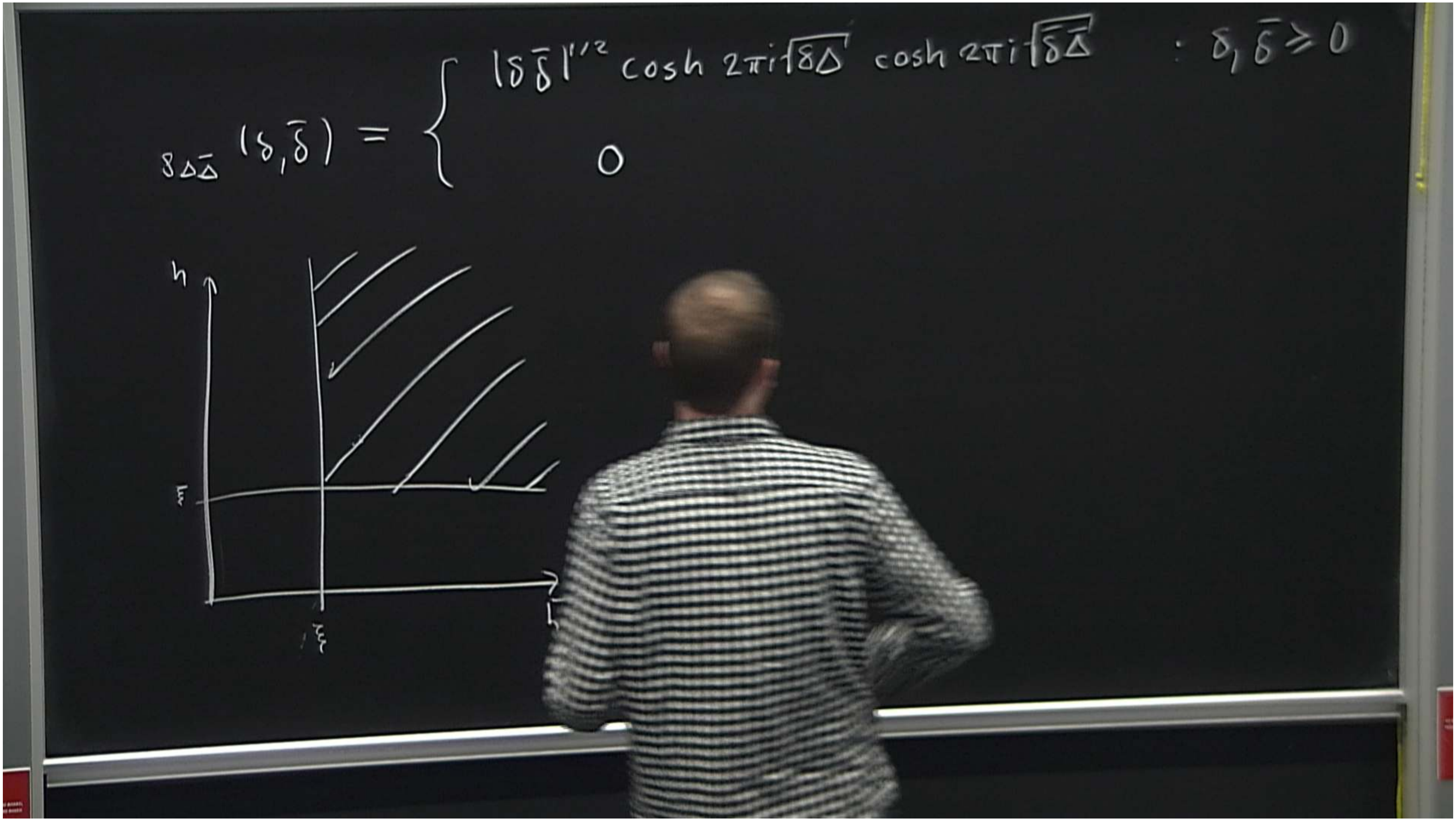
$g_{\Delta\bar{\Delta}}(\delta,\bar{\delta}) = \begin{cases} |\delta\bar{\delta}|^{1/2} \cosh 2\pi i\sqrt{\delta\Delta} \cosh 2\pi i\sqrt{\bar{\delta}\bar{\Delta}} & : \delta, \bar{\delta} \geq 0 \\ 0 & \end{cases}$
h
ξ
ξ

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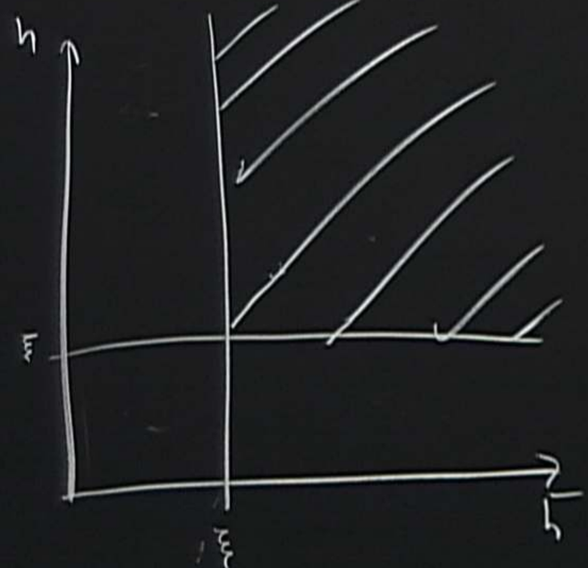
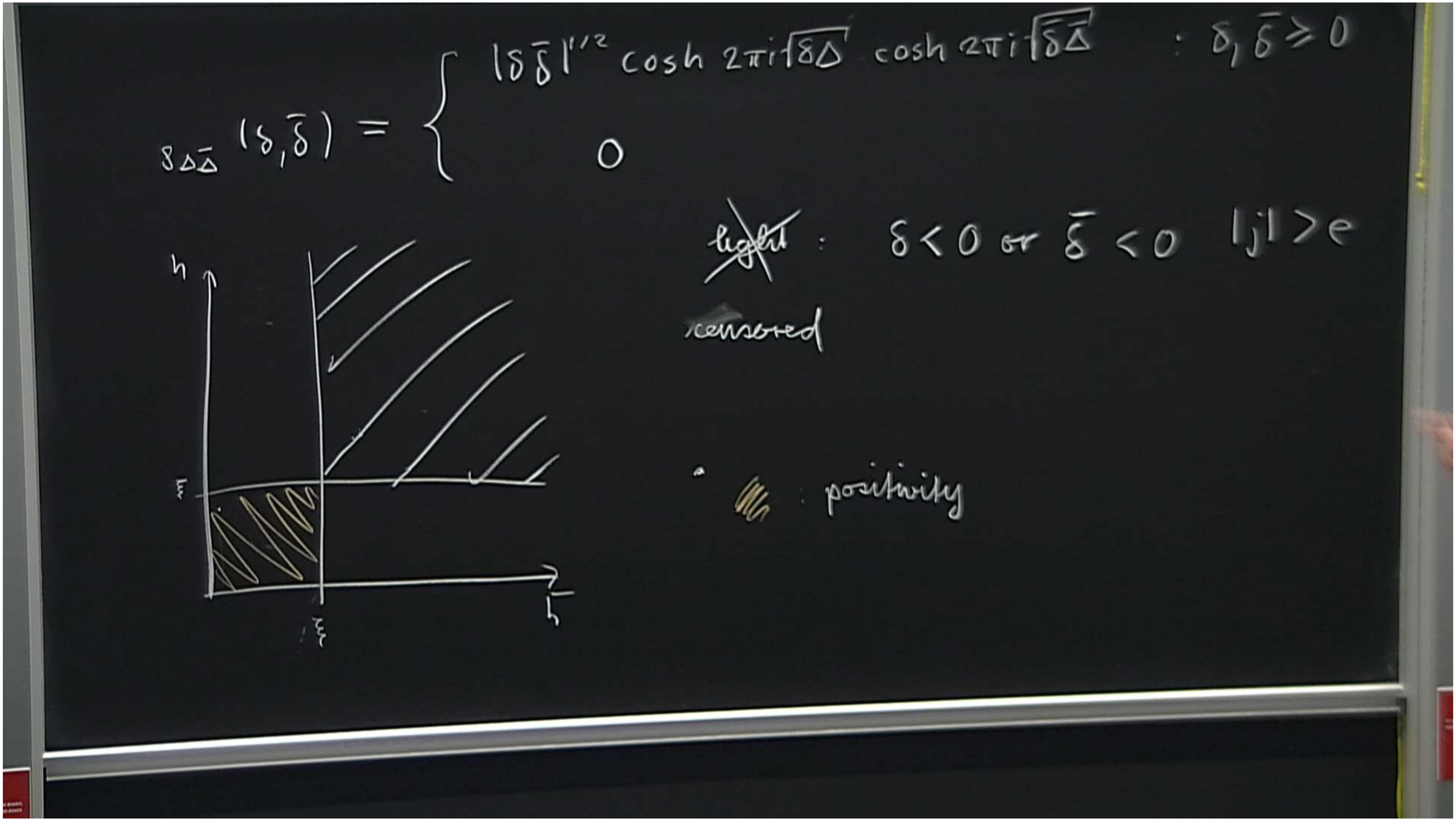


~~light~~ : $\delta < 0$ or $\bar{\delta} < 0$ $\quad |j| > e$

censored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censored
: positivity
h
ξ
ξ
$\bar{h}$

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: positivity

$$g_{\Delta\bar{\Delta}}(\delta,\bar{\delta}) = \begin{cases} |\delta\bar{\delta}|^{1/2}\cosh 2\pi i\sqrt{\delta\Delta}\,\cosh 2\pi i\sqrt{\bar{\delta}\bar{\Delta}} & : \delta,\bar{\delta}\geq 0 \\ 0 & \end{cases}$$

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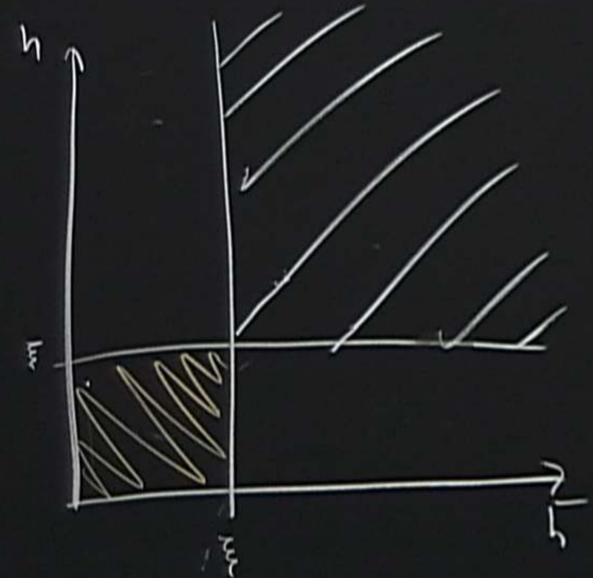
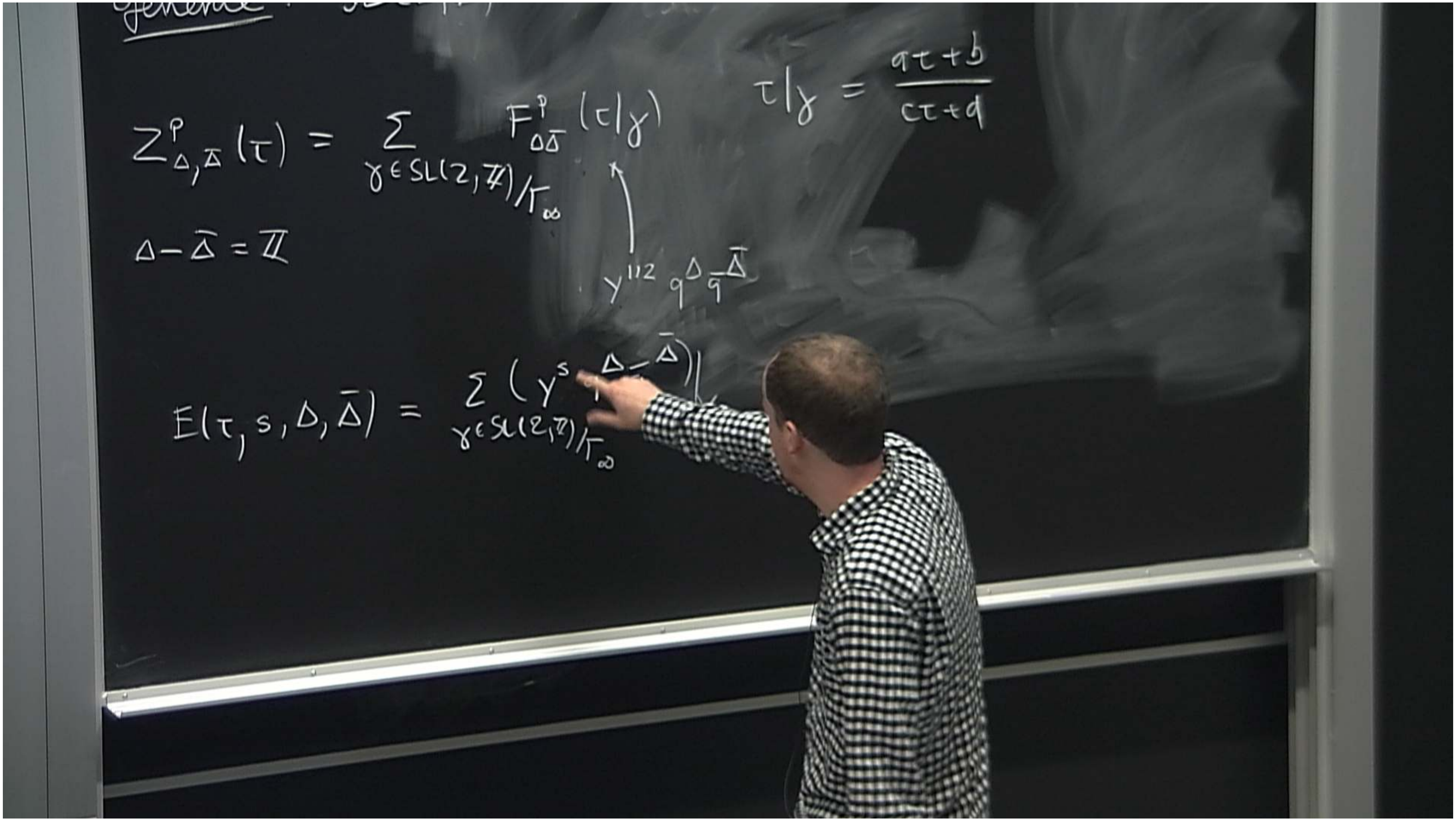


: positivity


$Z^{\rho}_{\Delta,\bar{\Delta}}(\tau) = \sum_{\gamma \in SL(2,\mathbb{Z})/\Gamma_\infty} F^{\rho}_{\Delta\bar{\Delta}}(\tau|\gamma)$
$\tau|\gamma = \frac{a\tau+b}{c\tau+d}$
$\Delta - \bar{\Delta} = \mathbb{Z}$
$y^{1/2} q^{\Delta} \bar{q}^{\bar{\Delta}}$
$E(\tau, s, \Delta, \bar{\Delta}) = \sum_{\gamma \in SL(2,\mathbb{Z})/\Gamma_\infty}$

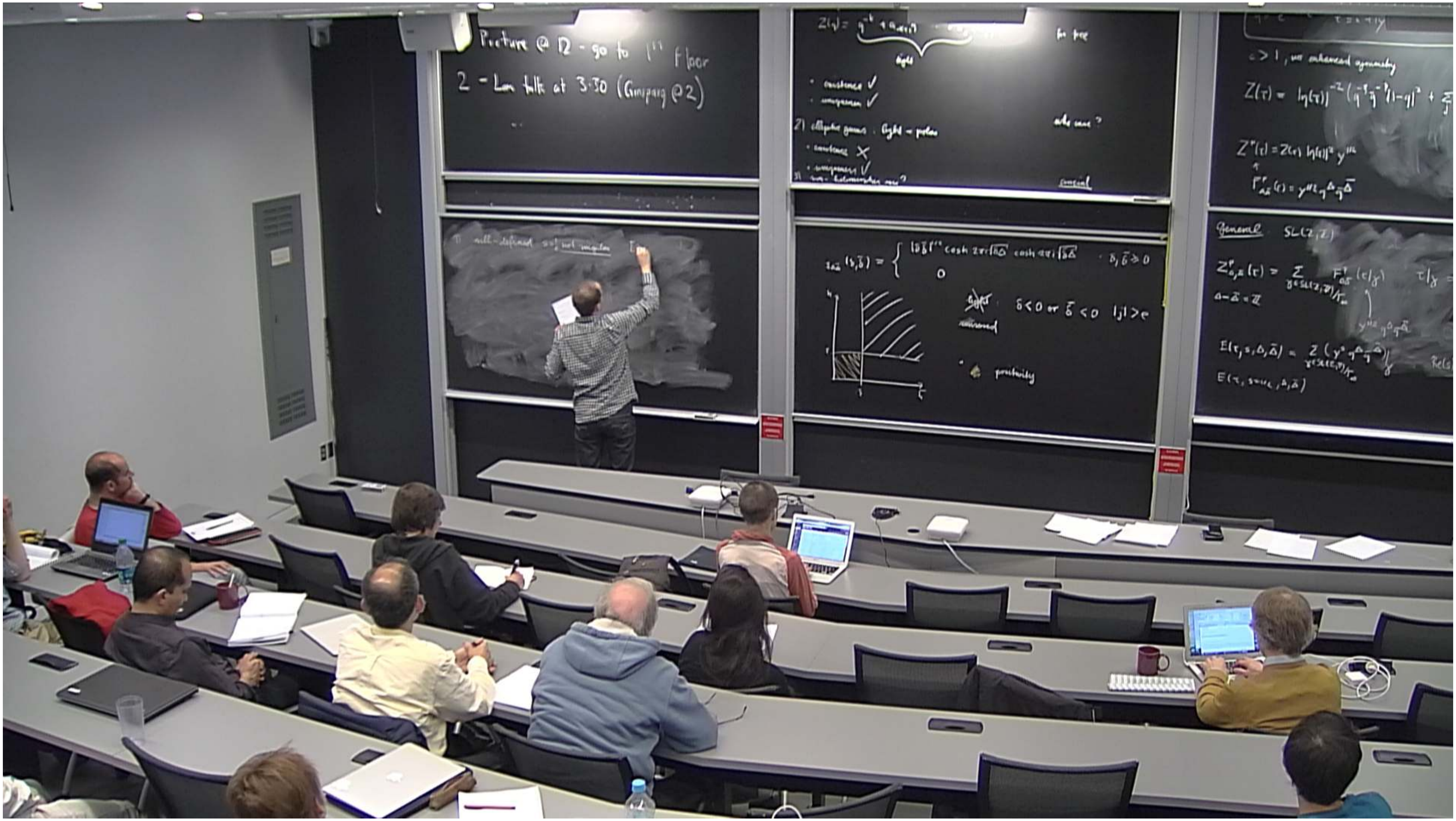
$$Z^{p}_{\Delta,\bar{\Delta}}(\tau) = \sum_{\gamma \in SL(2,\mathbb{Z})/\Gamma_\infty} F^{p}_{\Delta\bar{\Delta}}(\tau|\gamma) \qquad \tau|\gamma = \frac{a\tau+b}{c\tau+d}$$

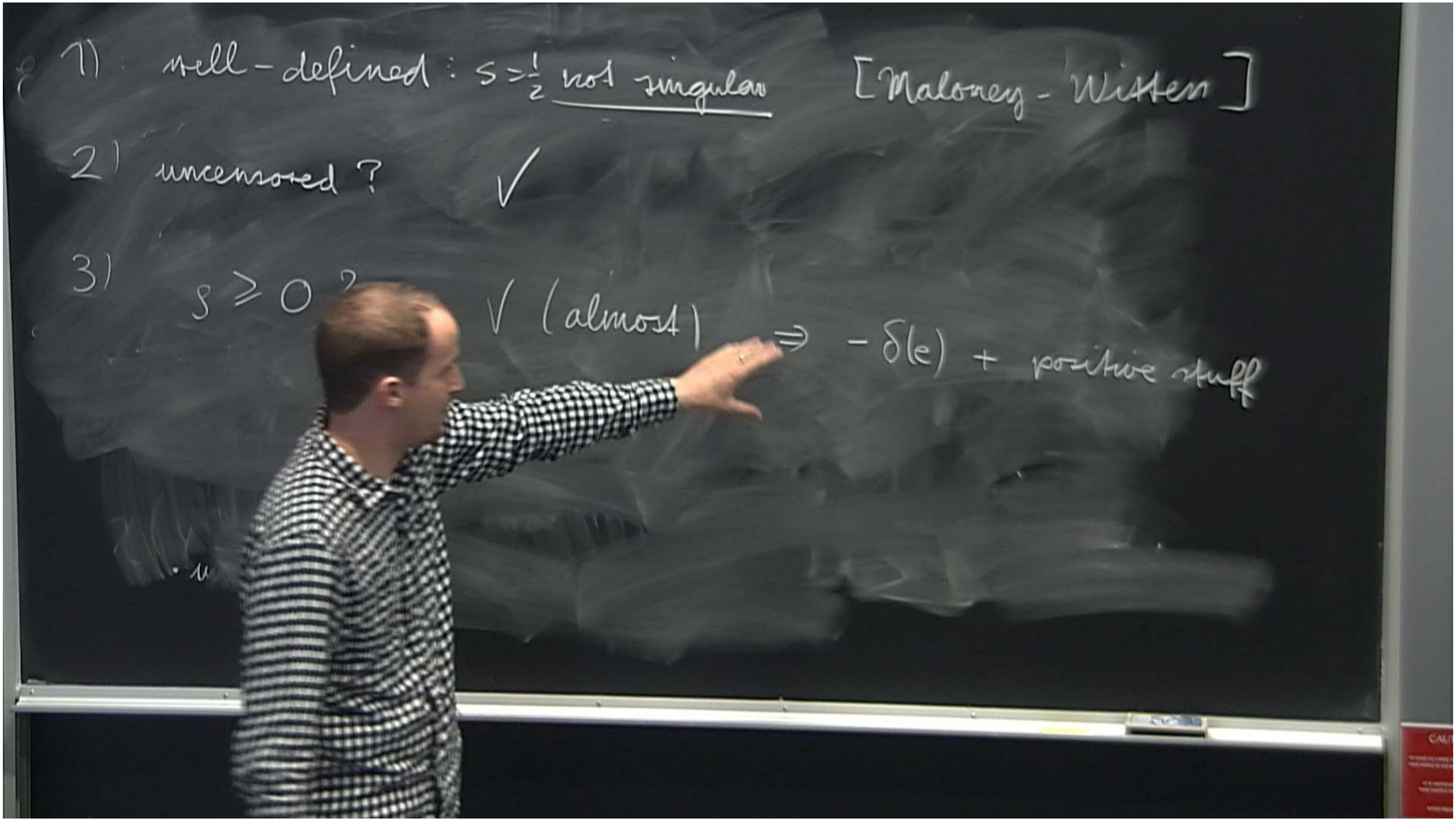
$$\Delta - \bar{\Delta} = \mathbb{Z}$$

$$y^{1/2}\, q^{\Delta}\bar{q}^{\bar{\Delta}}$$

$$E(\tau, s, \Delta, \bar{\Delta}) = \sum_{\gamma \in SL(2,\mathbb{Z})/\Gamma_\infty} \left(y^{s} q^{\Delta} \bar{q}^{\bar{\Delta}} \right)\Big|\gamma \qquad \mathrm{Re}(s) > 2$$

$$E(\tau, s = 1/2, \Delta, \bar{\Delta})$$

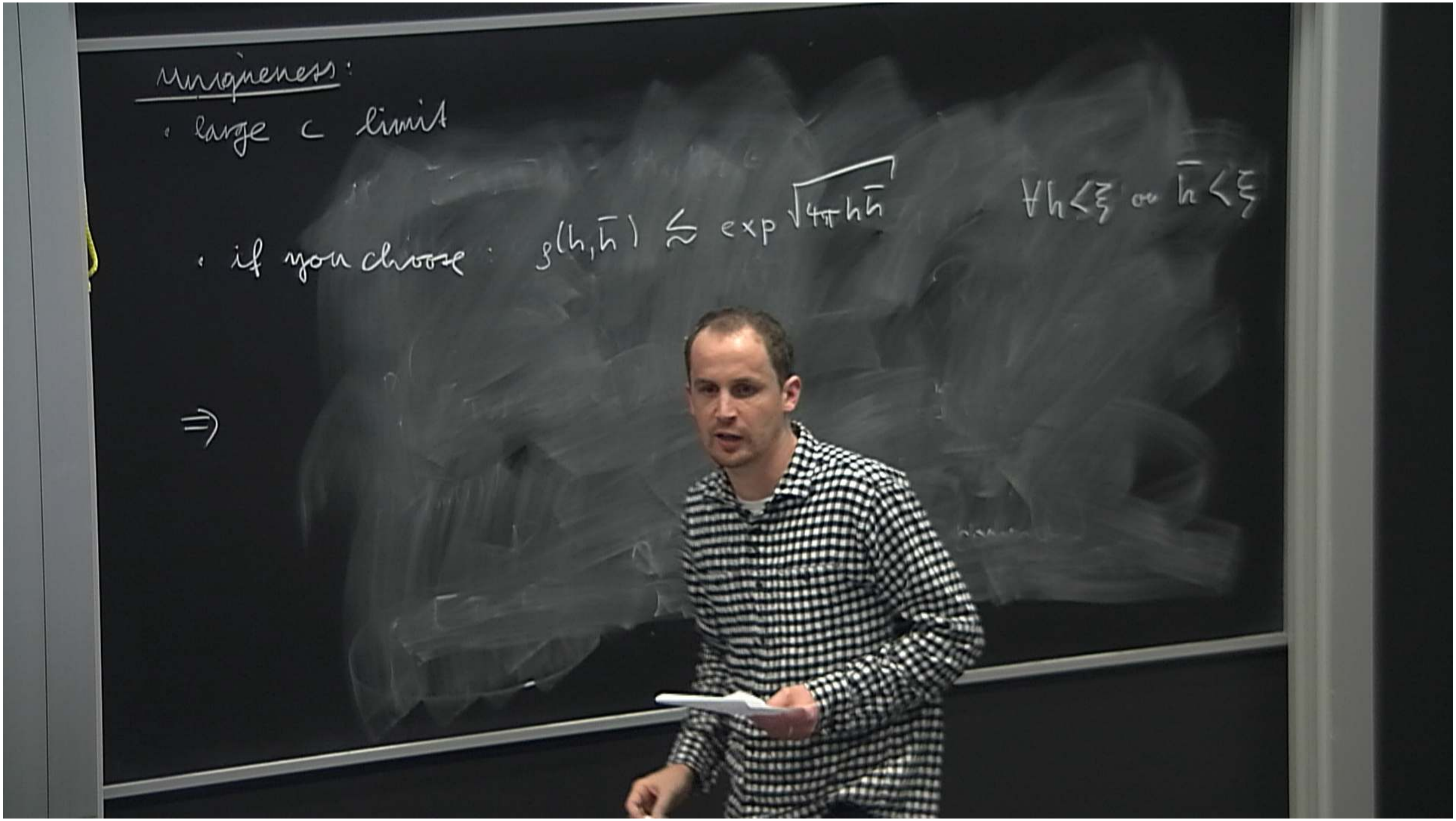


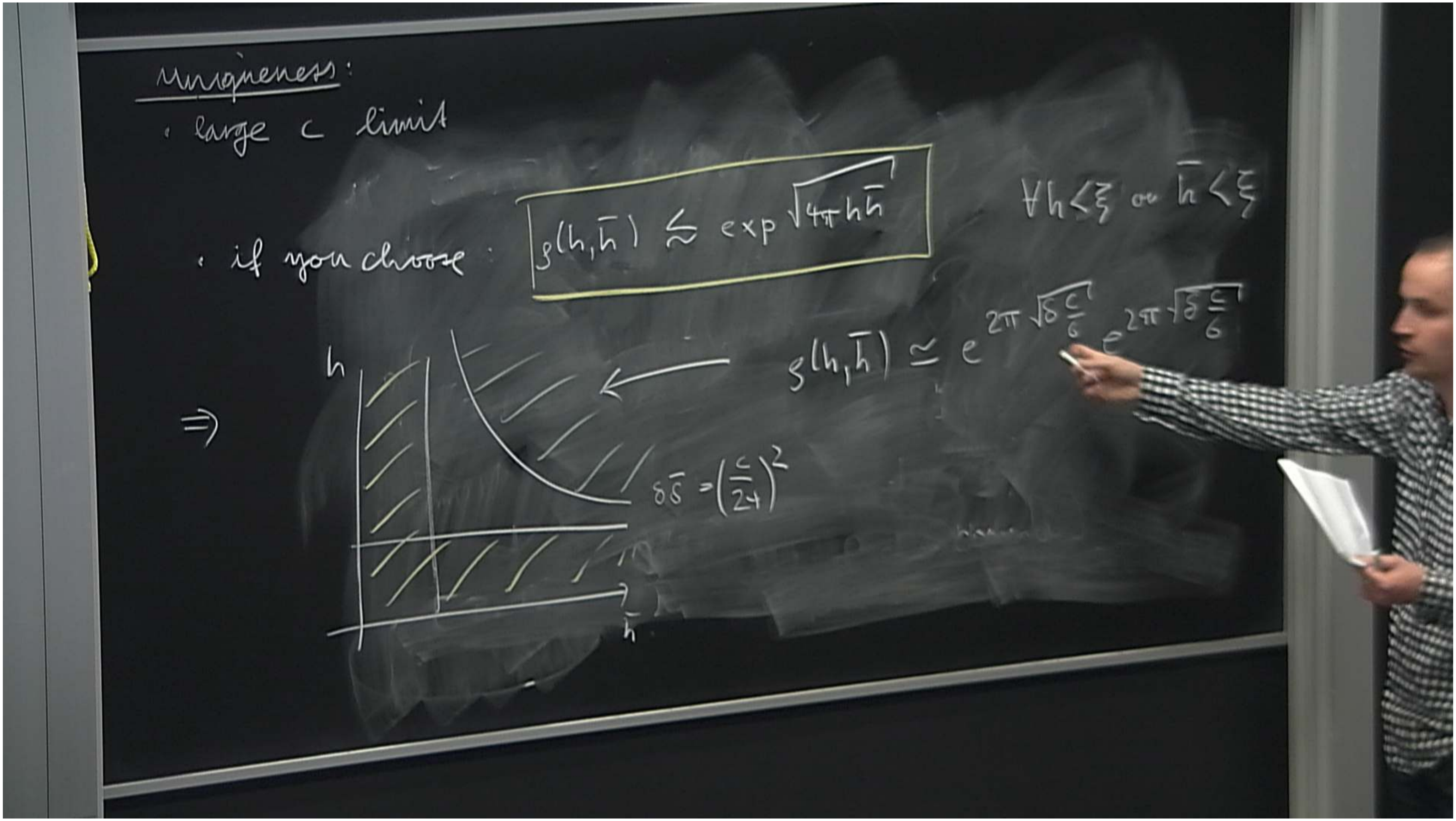

1) well-defined : s = 1/2 not singular [Maloney - Witten]
2) uncensored ? ✓
3) ρ ≥ 0 ? ✓ (almost) ⇒ − δ(e) + positive stuff

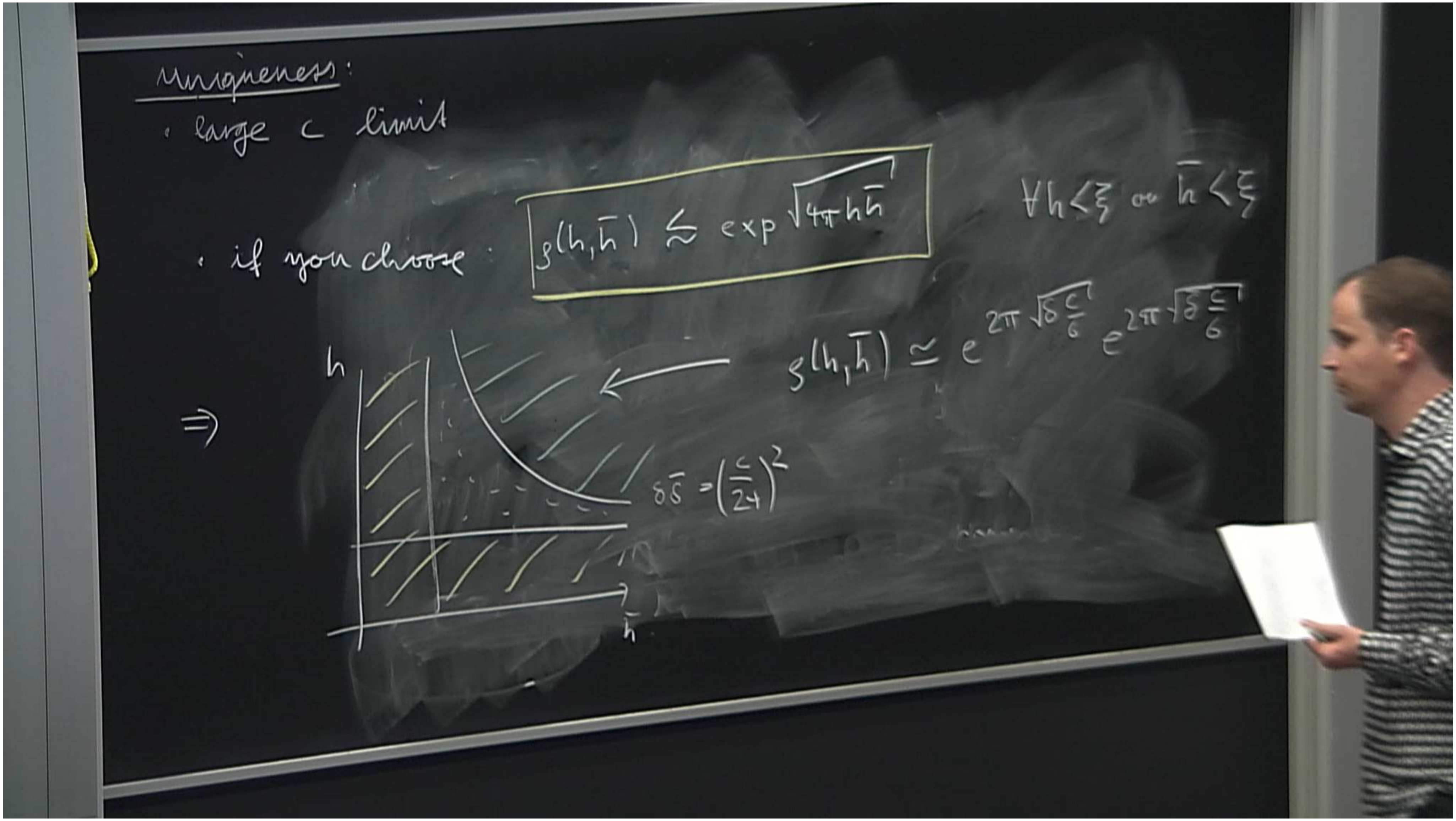
2) uncensored? ✓

3) $\rho \geq 0$? ✓ (almost) $\Rightarrow -\delta(e)$ + positive stuff

vacuum $\Rightarrow -6\,\delta(e) + \dots$


Uniqueness:
· large c limit
ρ(h, h̄) ≲ exp √(4π h h̄)   ∀ h < ξ or h̄ < ξ
· if you choose :
⇒


Uniqueness:
· large c limit
· if you choose: $\rho(h,\bar{h}) \lesssim \exp\sqrt{4\pi h\bar{h}}$
$\forall h<\xi$ or $\bar{h}<\xi$
$\Rightarrow$
$\rho(h,\bar{h}) \simeq e^{2\pi\sqrt{\delta\frac{c}{6}}}\, e^{2\pi\sqrt{\bar{\delta}\frac{c}{6}}}$
$\delta\bar{\delta} = \left(\frac{c}{24}\right)^2$
h
$\bar{h}$


Uniqueness:
· large c limit
· if you choose: $\boxed{g(h,\bar{h}) \lesssim \exp\sqrt{4\pi h\bar{h}}}$
$\forall h < \xi$ or $\bar{h} < \xi$
$g(h,\bar{h}) \simeq e^{2\pi\sqrt{\delta\frac{c}{6}}}\, e^{2\pi\sqrt{\bar{\delta}\frac{c}{6}}}$
$\Rightarrow$
h
$\delta\bar{\delta} = \left(\frac{c}{24}\right)^2$
$\bar{h}$

Uniqueness:

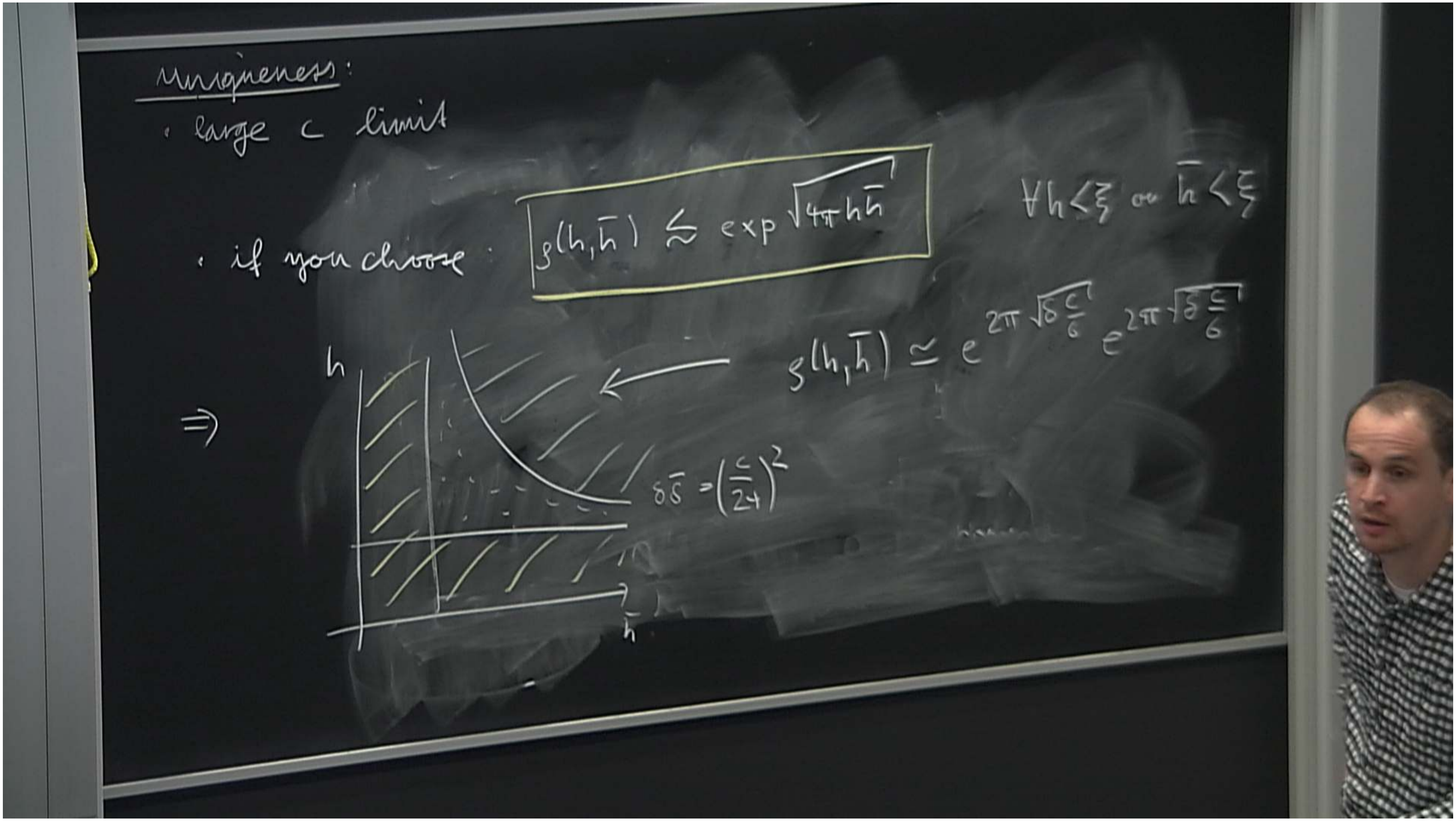
- large c limit

- if you choose: $\boxed{\rho(h,\bar h) \lesssim \exp\sqrt{4\pi h\bar h}}$ $\quad \forall h<\xi$ or $\bar h<\xi$

$$\rho(h,\bar h) \simeq e^{2\pi\sqrt{\delta\frac{c}{6}}}\, e^{2\pi\sqrt{\bar\delta\frac{c}{6}}}$$

$\Rightarrow$

$$\delta\bar\delta = \left(\frac{c}{24}\right)^2$$


Uniqueness:
· large c limit
· if you choose
ρ(h, h̄) ≲ exp √(4π h h̄)
∀ h < ξ or h̄ < ξ
⇒
ρ(h, h̄) ≃ e^{2π √(δ c/6)} e^{2π √(δ̄ c/6)}
δδ̄ = (c/24)²
h
h̄