Title: How does extended supersymmetry affect the elliptic genus?

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Abstract: The elliptic genus of K3 and its decomposition into characters of the N=4 superconformal algebra of associated conformal field theories can be viewed as the outset of Mathieu Moonshine. Thus, extended supersymmetry induces additional properties of the elliptic genus, which so far lack a satisfactory geometric interpretation. We investigate the implications of this decomposition on geometric structures that underlie the elliptic genus.

How does extended supersymmetry affect the elliptic genus?

(Mock) Modularity, Moonshine and String Theory

Perimeter Institute, Waterloo, Canada, April 13-17, 2015

Katrin Wendland
Albert-Ludwigs-Universität Freiburg

The elliptic genus under extended supersymmetry

Introduction
1. The elliptic genus of Calabi-Yau manifolds
2. Effects of extending the $N = (2,2)$ superconformal algebra
3. Interpretation and symmetry-surfing

[Creutzig/W15], [Carnahan/Grimm/W15] *work in progress*

[W15] *K3 en route from geometry to conformal field theory*; to appear in: Proceedings of the 2013 Summer School "Geometric, Algebraic and Topological Methods for Quantum Field Theory", Villa de Leyva, Colombia; arXiv:1503.08426 [math.DG]

[W14] *Snapshots of conformal field theory*; in "Mathematical Aspects of Quantum Field Theories", Mathematical Physics Studies, Springer 2015, pp. 89-129; arXiv:1404.3108 [hep-th]

[Taormina/W13] *A twist in the M_{24} moonshine story*, to appear in Confluentes Mathematici; arXiv:1303.3221 [hep-th]

[Taormina/W12] *Symmetry-surfing the moduli space of Kummer K3s*, to appear in: Proceedings of String-Math 2012; arXiv:1303.2931 [hep-th]

[Taormina/W11] *The overarching finite symmetry group of Kummer surfaces in the Mathieu group M_{24}*, JHEP **1308**:152 (2013); arXiv:1107.3834 [hep-th]

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1. The elliptic genus of Calabi-Yau manifolds

Let M denote a compact Calabi-Yau D-fold, $T := T^{1,0}M$,

$$\mathbb{E}_{q,-y} := y^{-\frac{D}{2}} \Lambda_{-y} T^* \otimes \bigotimes_{n=1}^{\infty} \left[\Lambda_{-yq^n} T^* \otimes \Lambda_{-y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T \right],$$

where for any bundle $E \to M$, $\Lambda_x E := \bigoplus_{m=0}^{\infty} x^m \Lambda^m E$, $S_x E := \bigoplus_{m=0}^{\infty} x^m S^m E$

Definition

With $q := e^{2\pi i \tau}$, $y := e^{2\pi i z}$ for $\tau, z \in \mathbb{C}$, $\mathrm{Im}(\tau) > 0$,

$$\mathcal{E}_M(\tau, z) := \chi(\mathbb{E}_{q,-y}) = \int_M \mathrm{Td}(M)\, \mathrm{ch}(\mathbb{E}_{q,-y})$$

is the ELLIPTIC GENUS of M.

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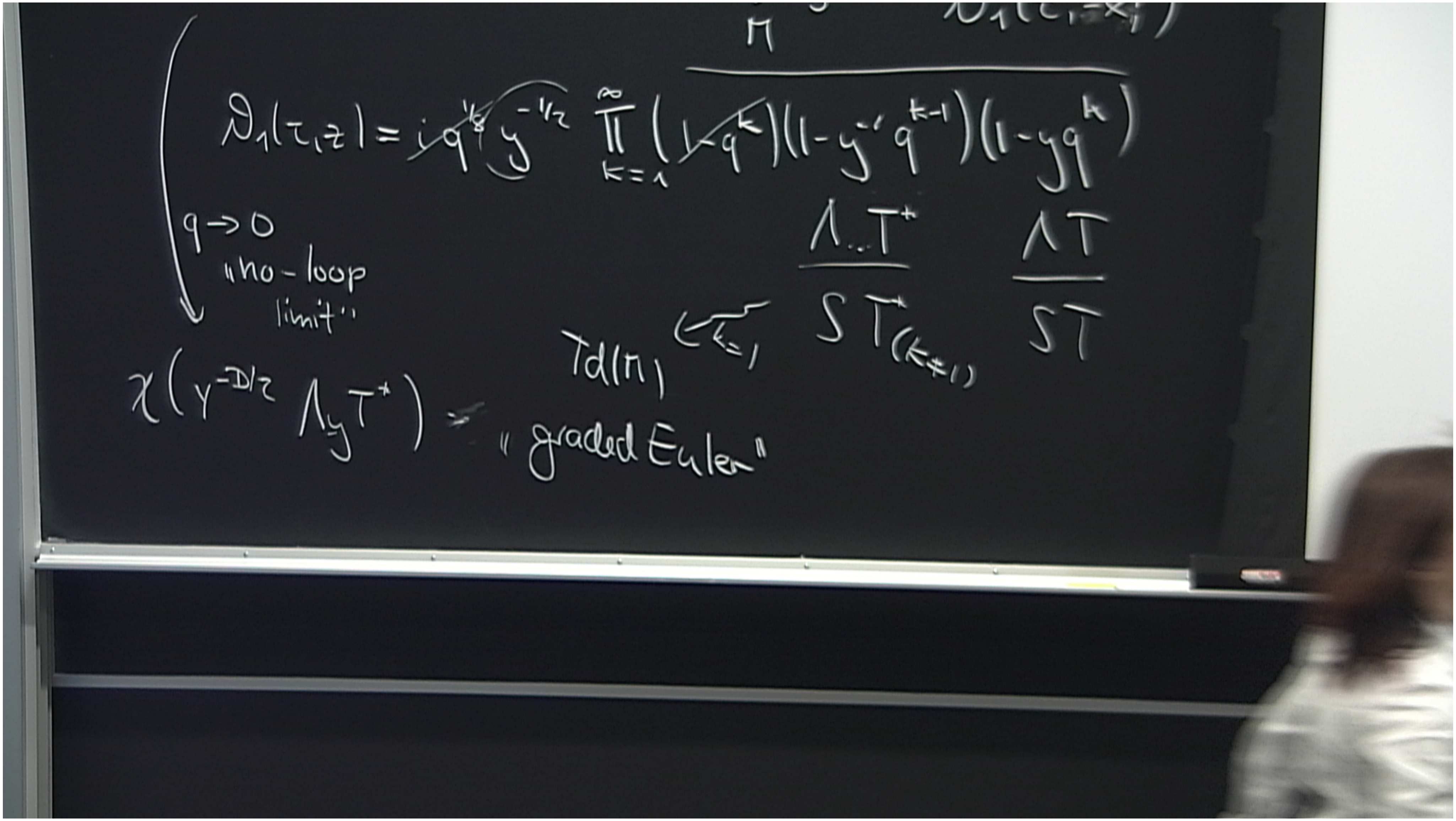
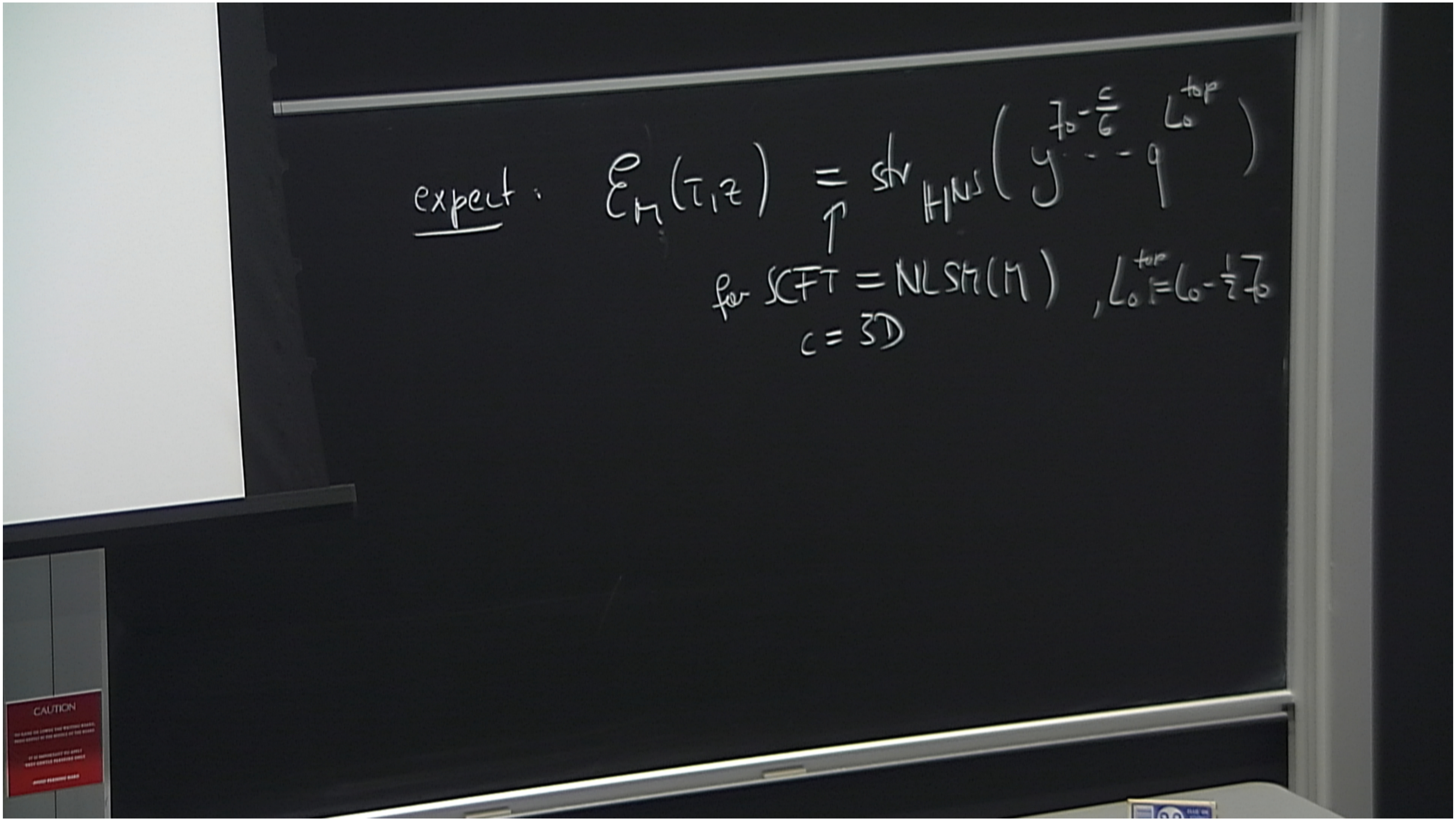
$$c(T) = \prod_j (1 + x_j)$$

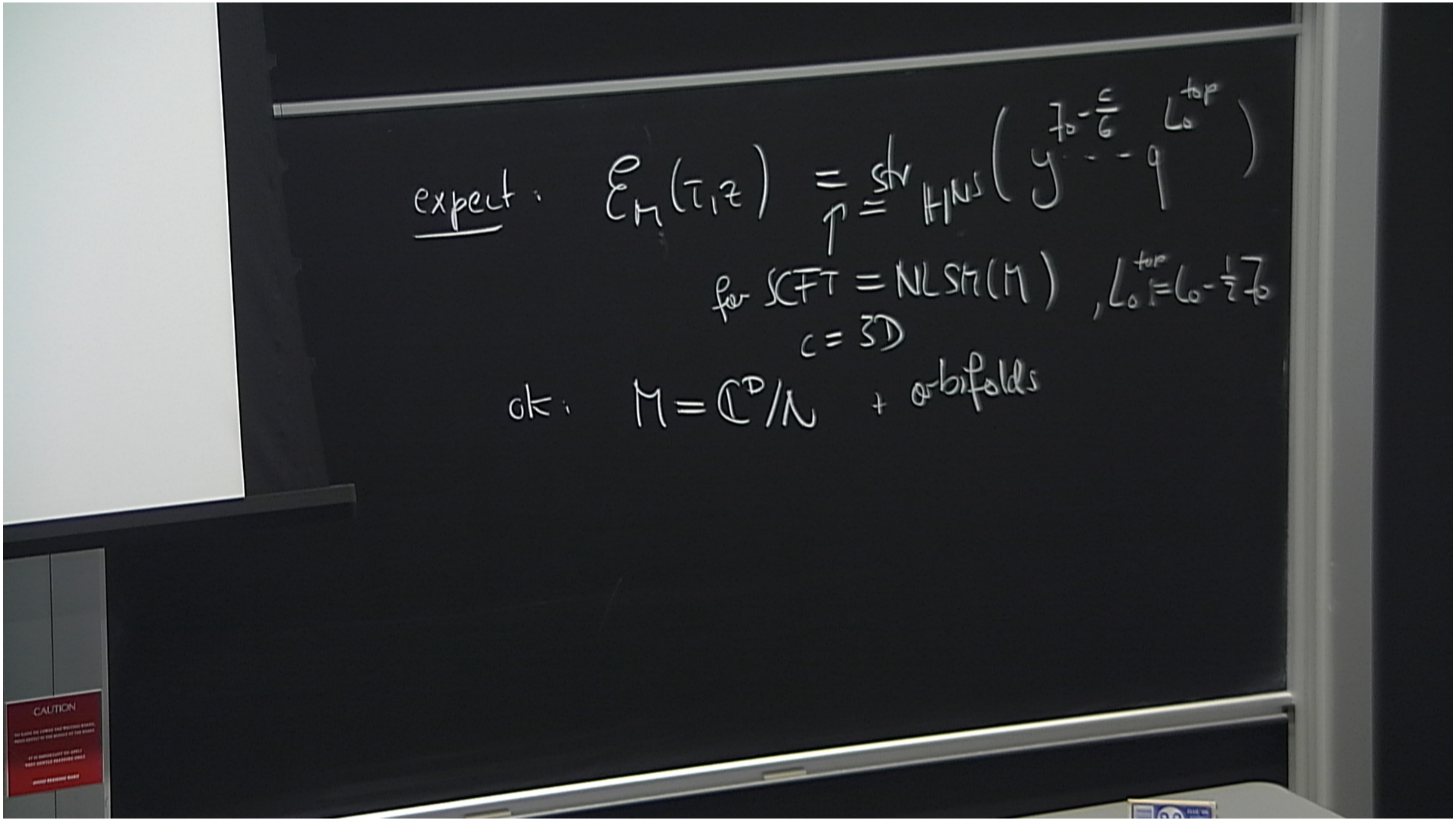
$$\Downarrow$$

$$\mathcal{E}_M = \chi(\mathbb{E}_{q,-y}) \qquad \int_M \prod_j x_j \, \frac{\vartheta_1(\tau, z, -x_j)}{\vartheta_1(\tau, z)}$$

$$\vartheta_1(\tau, z) = i\, q^{1/8} y^{-1/2} \prod_{k=1}^{\infty} (1 - q^k)(1 - y^{-1} q^{k-1})(1 - y q^k)$$


M
ϑ1(τ,z) = i q^1/8 y^-1/2 ∏ k=1 ∞ (1-q^k)(1-y^-1 q^k-1)(1-yq^k)
q → 0
"no-loop limit"
Λ..T*
ST*
ΛT
ST
Td(M)
k=1
(k≠1)
χ(y^-D/2 Λ_y T*) =
"graded Euler"


expect: $\mathcal{E}_M(\tau, z) = \mathrm{str}_{\mathbb{H}^{NS}}\left(y^{J_0 - \frac{c}{6}} \cdots q^{L_0^{top}} \right)$
↑
for SCFT = NLSM(M), $L_0^{top} := L_0 - \frac{1}{2} J_0$
$c = 3D$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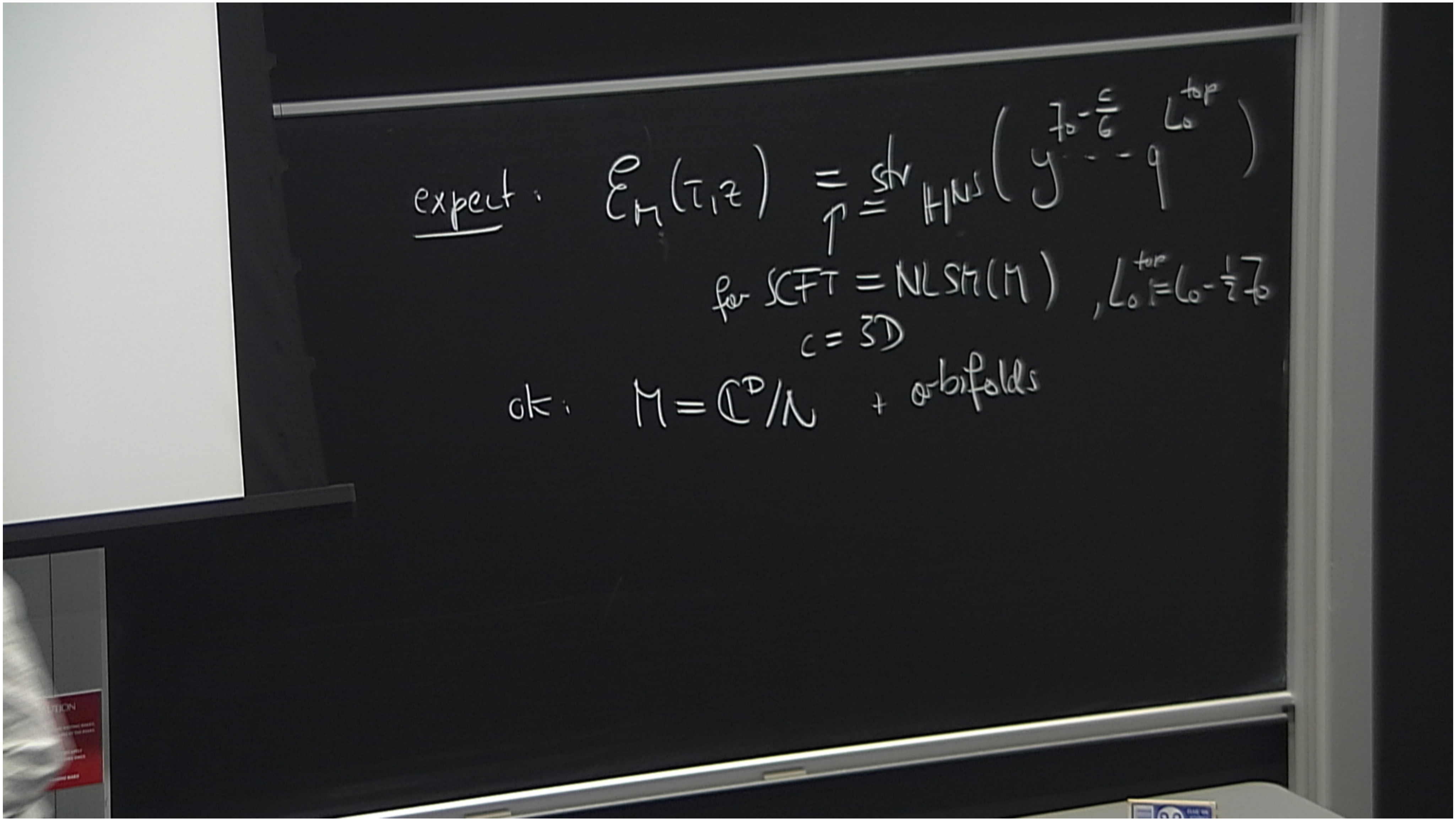
ok: $M = \mathbb{C}^D/\Lambda$ + orbifolds
CAUTION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Chiral de Rham complex

Definition [Malikov/Schechtman/Vaintrob99]
For open $U \subset M$ with local holomorphic coordinates $z^1, \ldots, z^D$:
$\Omega^{ch}_M(U)$:= Fock space for the fields ϕ^j, p_j, ψ^j, ρ_j, $j \in \{1, \ldots, D\}$,
(D copies of a $(bc - \beta\gamma)$-system)
where $\phi^j \leftrightarrow z^j$, $p_j \leftrightarrow \frac{\partial}{\partial z^j}$, $\psi^j \leftrightarrow dz^j$, $\rho_j \leftrightarrow \frac{\partial}{\partial(dz^j)}$.
This yields a sheaf of vertex algebras over M,
the CHIRAL DE RHAM COMPLEX Ω^{ch}_M.

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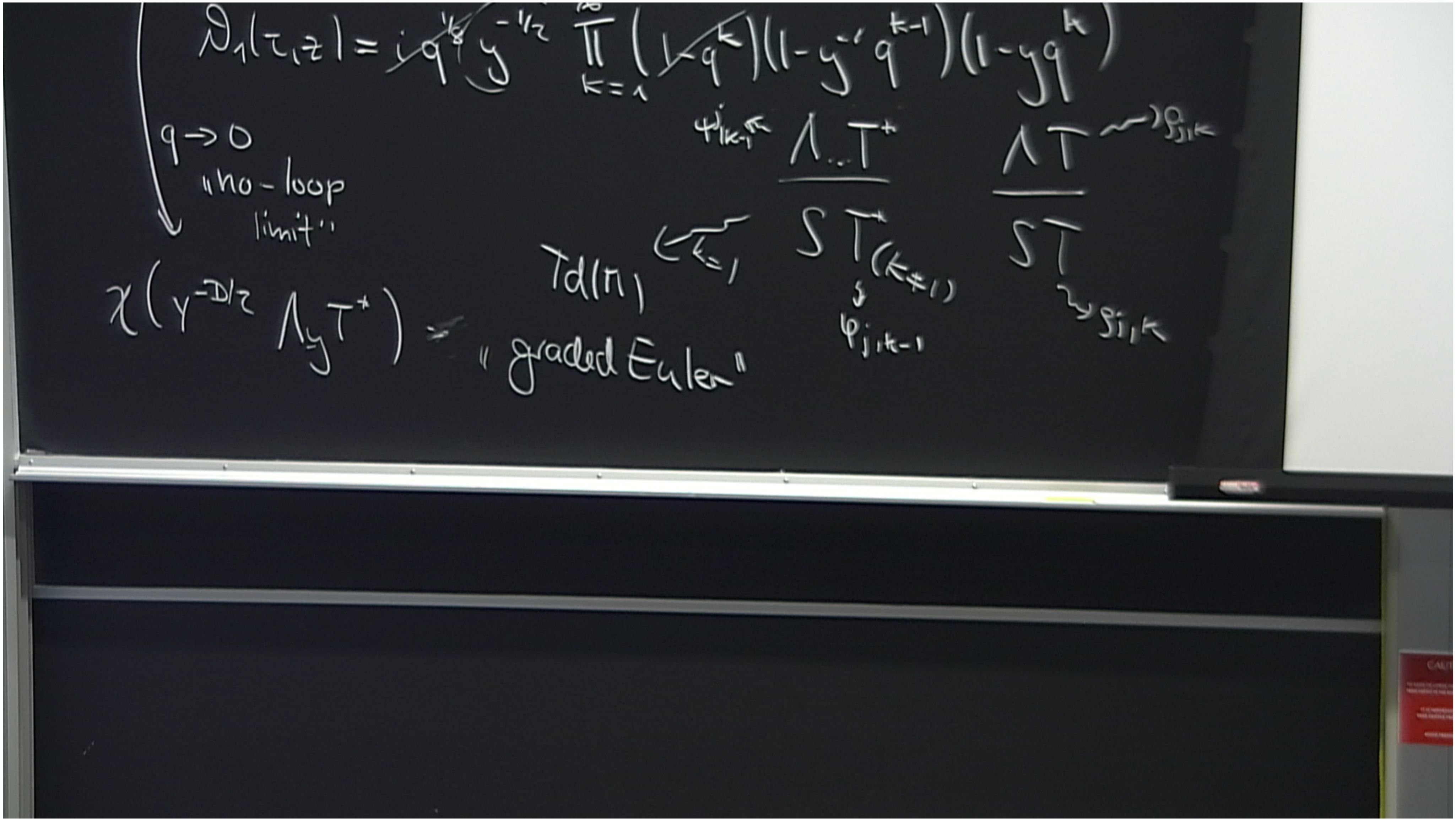
Theorem [Malikov/Schechtman/Vaintrob99;Borisov/Libgober00]
$H^*(M, \Omega^{ch}_M)$ carries the structure of a topological $N=2$ superconformal vertex algebra, induced from globally well-defined fields on M,

$$L^{top} = -:p_j \partial\phi^j: - :\rho_j \partial\psi^j:, \; J = :\rho_j \psi^j:, \; Q = -:\psi^j p_j:, \; G = :\rho_j \partial \phi^j:.$$

The elliptic genus $\mathcal{E}_M(\tau, z)$ is the bigraded Euler characteristic of Ω^{ch}_M.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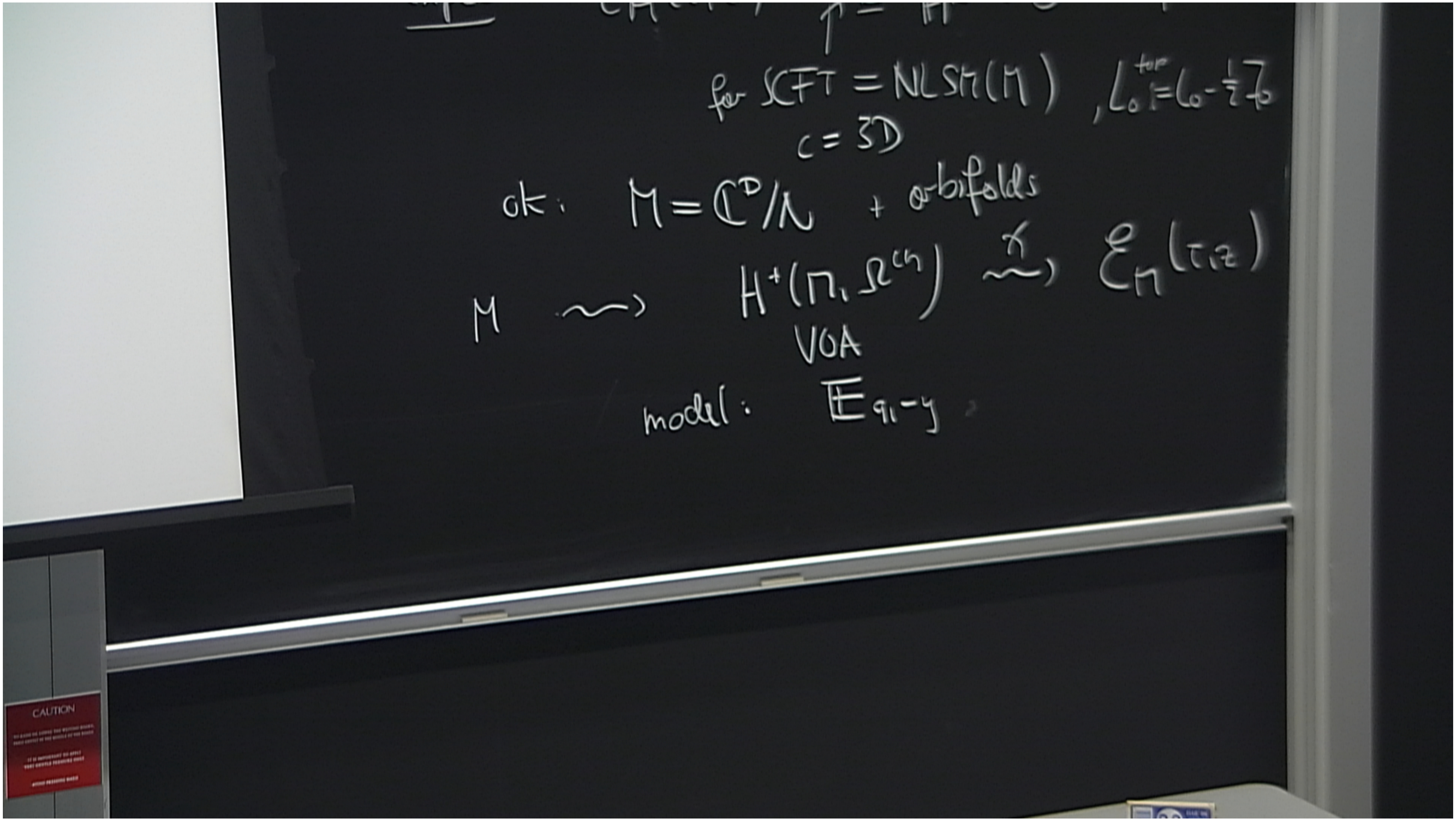
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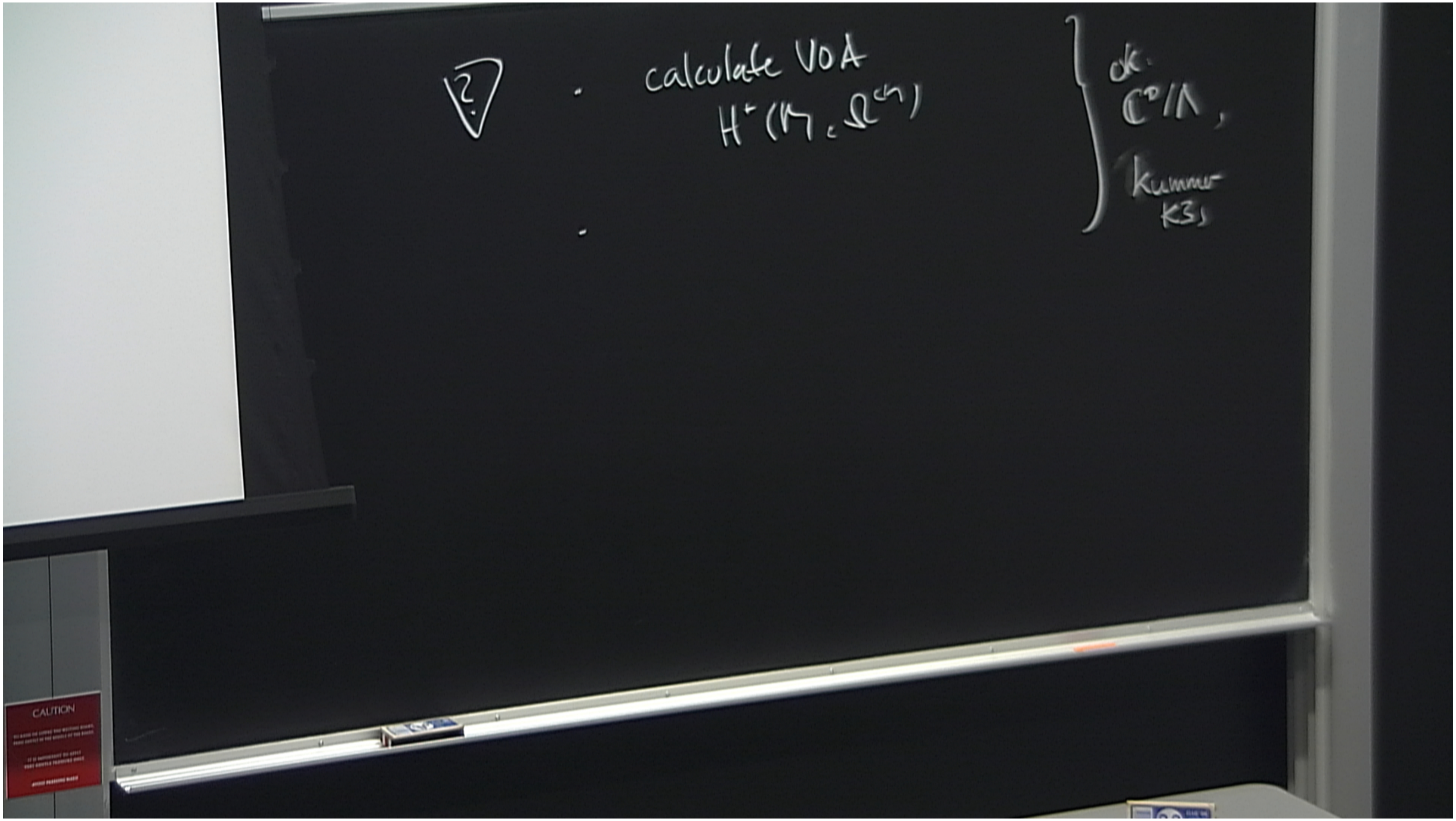
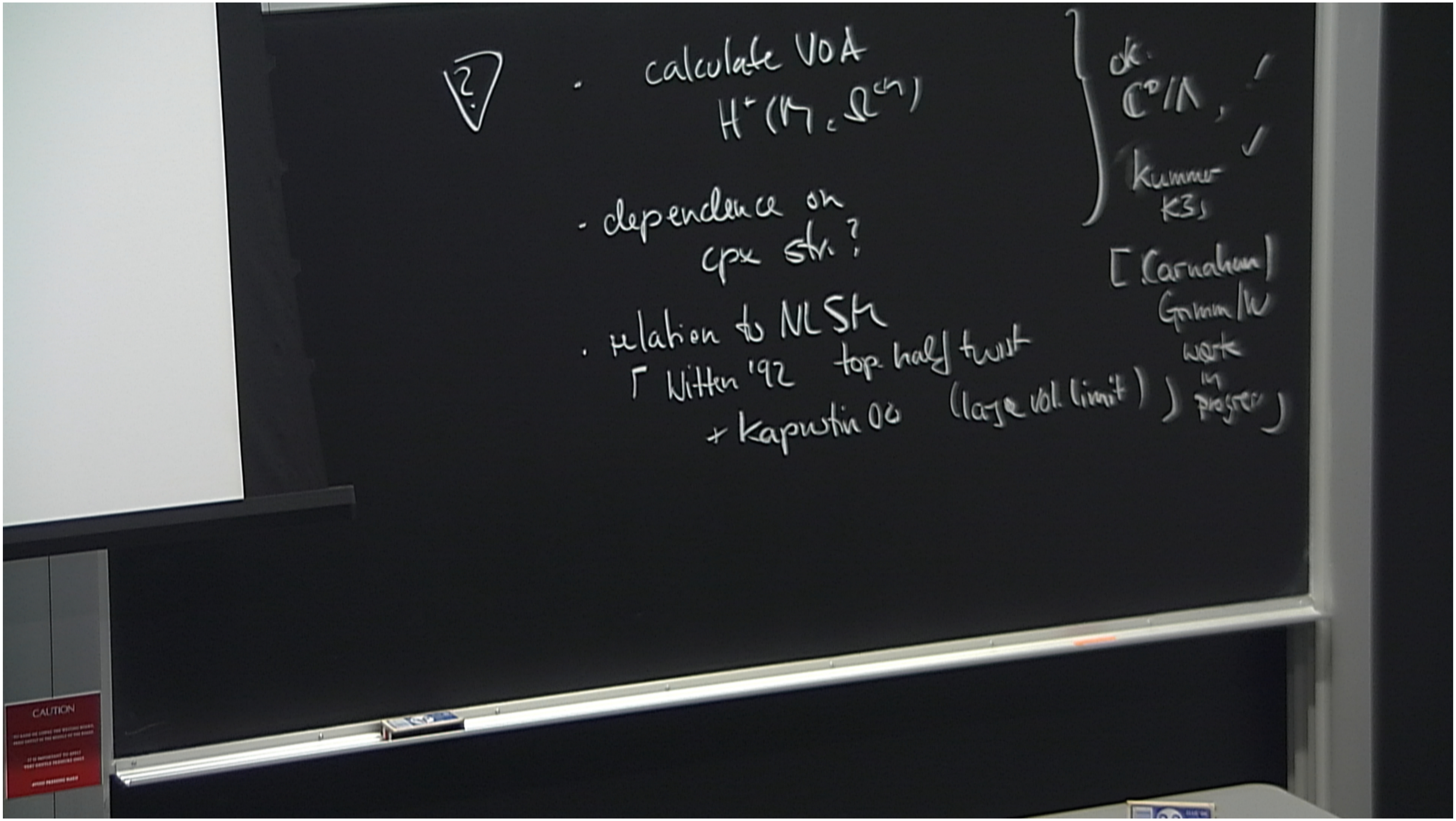
$M \rightsquigarrow H^*(M, \Omega^{ch}) \overset{\chi}{\rightsquigarrow} \mathcal{E}_M(\tau, z)$
VOA
model: $\mathbb{E}_{q,-y}$
CAUTION


?
calculate VOA
H*(M, Ω^ch)
ok:
ℂ^D/Λ,
Kummer
K3s
CAUTION



calculate VOA
$H^*(M, \Omega^{ch})$
dependence on cpx str. ?
relation to NLSM
[Witten '92 top. half twist
+ Kapustin 05 (large vol. limit)]
ok.
$\mathbb{C}^D/\Lambda$,
Kummer K3s
[Carnahan / Gamm / W work in progress]

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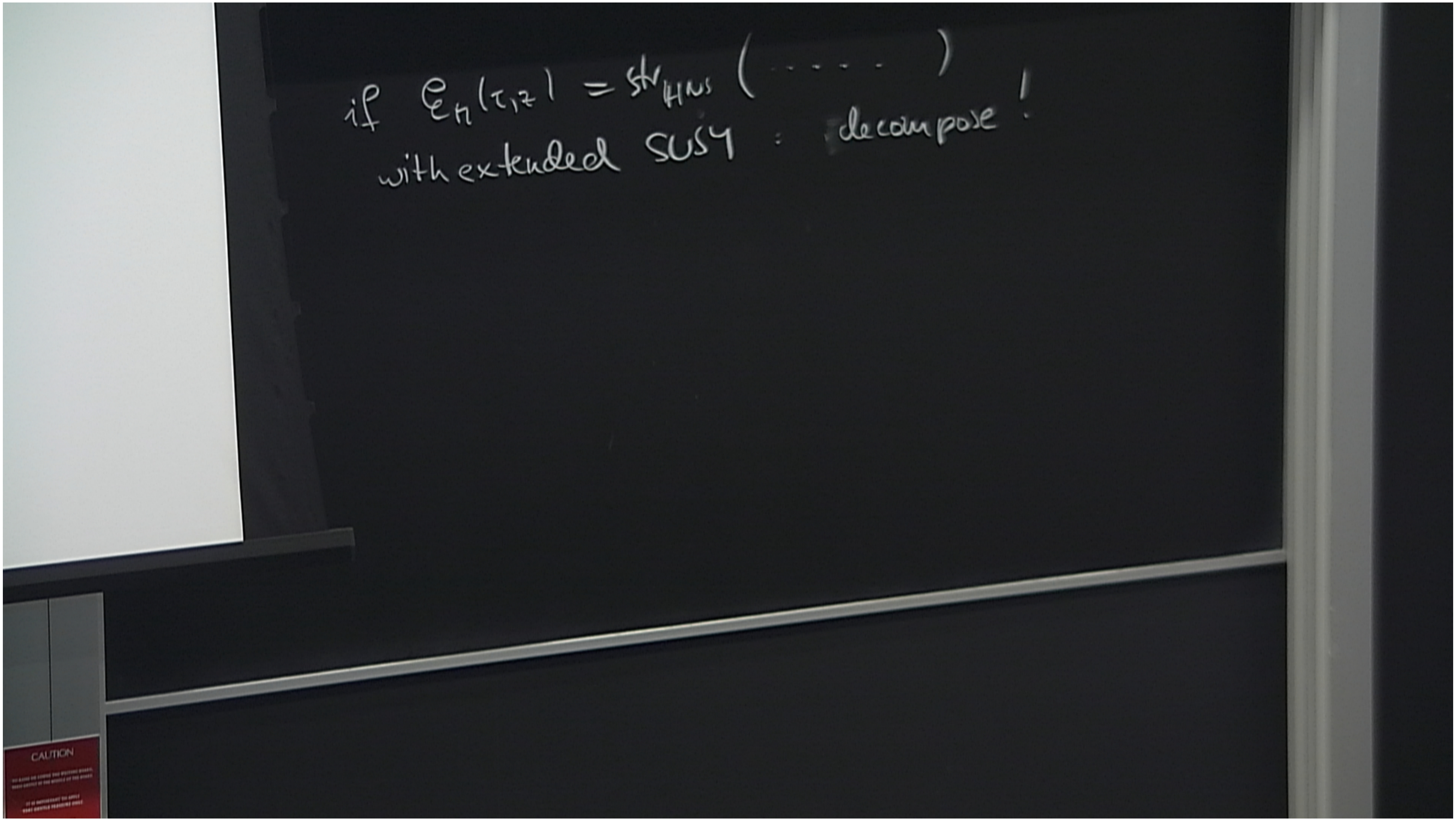
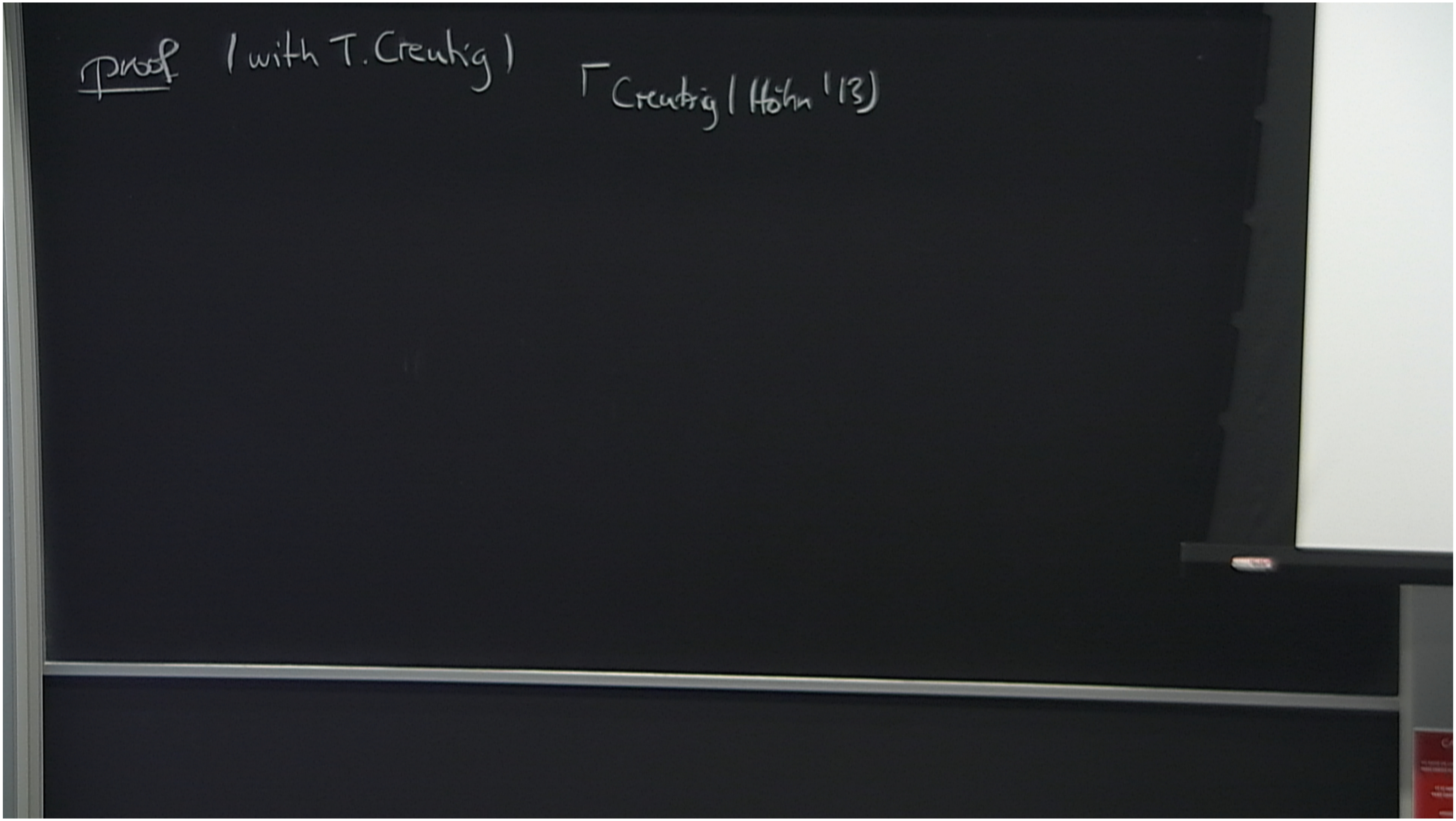

if $\mathcal{E}_H(\tau,z) = \mathrm{str}_{HNS}(\cdots)$
with extended SUSY : decompose !
CAUTION

2. Extending to $N=(4,4)$ supersymmetry

Assume: $D=2$, i.e. $c=6$ and $N=(4,4)$ supersymmetry.

3 types of $N=4$ irreps $\mathcal{H}_\bullet$ with $\chi_\bullet(\tau,z)=\mathrm{sTr}_{\mathcal{H}_\bullet}\left(y^{J_0}q^{L_0-1/4}\right)$:
VACUUM $\mathcal{H}_0$, MASSLESS MATTER $\mathcal{H}_{\text{m.m.}}$, MASSIVE MATTER $\mathcal{H}_{h>0}$.

$$\mathcal{E}_{K3}(\tau,z) = -2\chi_0(\tau,z)+20\chi_{m.m.}(\tau,z)+\sum_{n=1}^{\infty}A_n\chi_n(\tau,z)$$


proof (with T. Creutzig)
[Creutzig / Höhn '13)

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T : $SU(2)$ principal bdl

$\mathbb{E}_{q_1-y}$ — " —

holonomy / dist. $\Rightarrow$ $\mathbb{E}_{q_1-y} = \bigoplus_{n=1}^{\infty} W_n \otimes M_n$ ← multipl.

W_n: n-dim irrep. of $SU(2)$

$S^{n-1}(T)$

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M_n: multipl.; $N=4$ action

$$= \bigoplus_{n=1}^{\infty} S^{n-1}(T)\, \chi_n(\tau,z)$$

(since $SU(2)$-inv.)

χ_n: ($N=4$ char.)

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Conjecture [W13]

Let M=K3. There are polynomials p_n for every $n\in\mathbb{N}$, such that

$$\mathbb{E}_{q,-y}=-\mathcal{O}_{K3}\chi_0(\tau,z)-T\chi_{\mathrm{m.m.}}(\tau,z)+\sum_{n=1}^{\infty}p_n(T)\chi_n(\tau,z),$$

where $A_n=\int_{K3}\mathrm{Td}(K3)p_n(T)=\chi(p_n(T))$ for all $n\in\mathbb{N}$.

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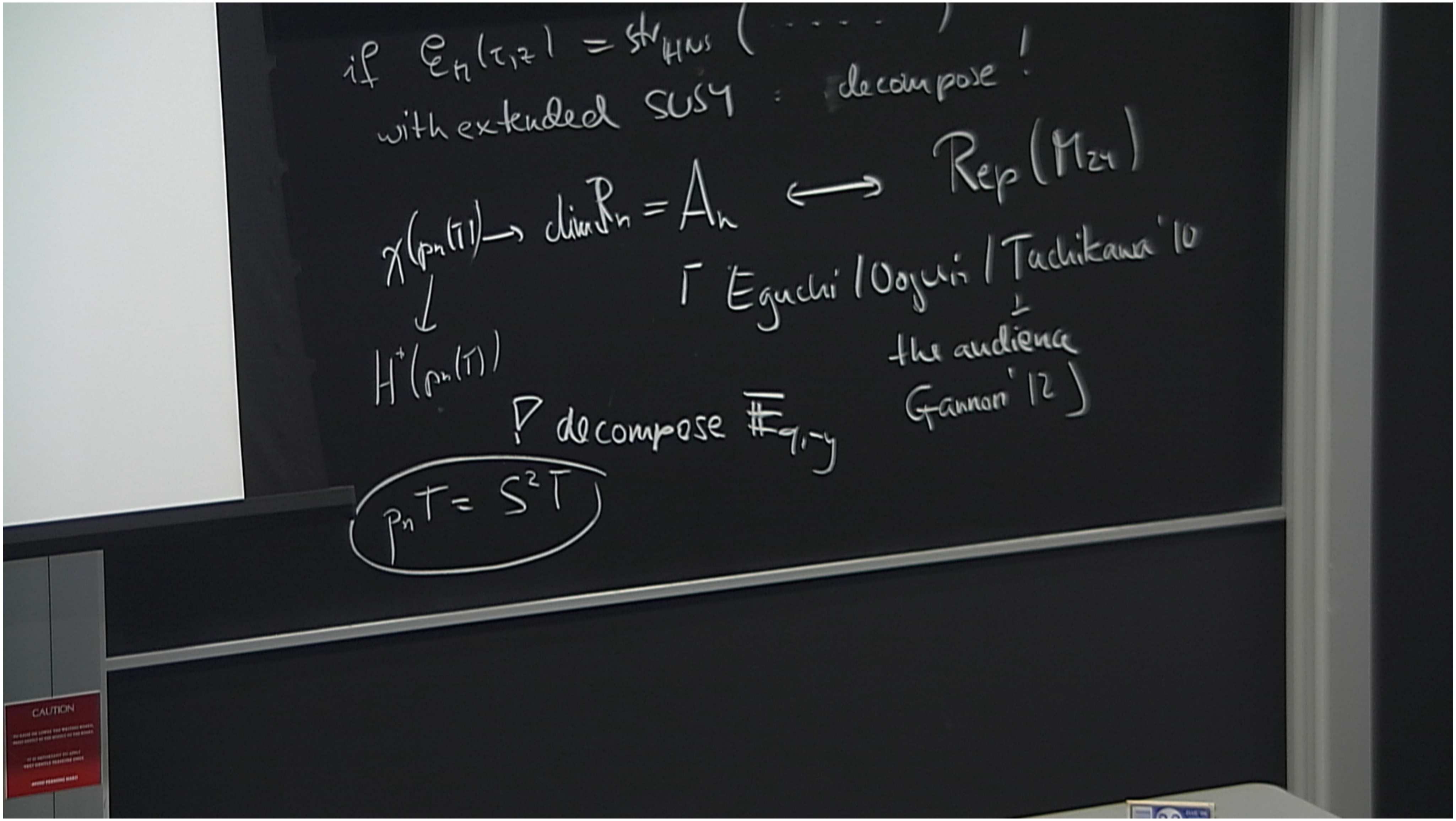
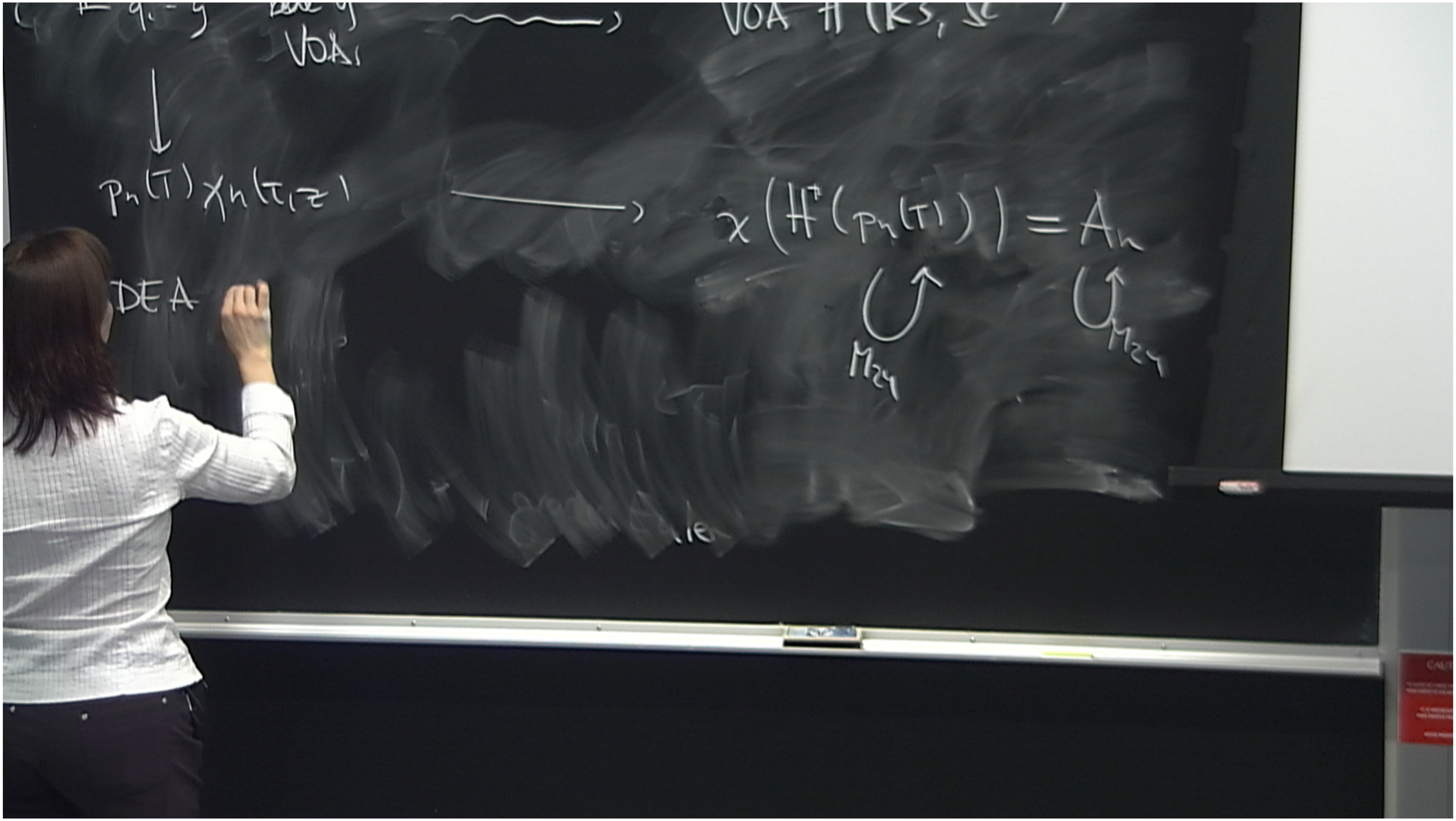
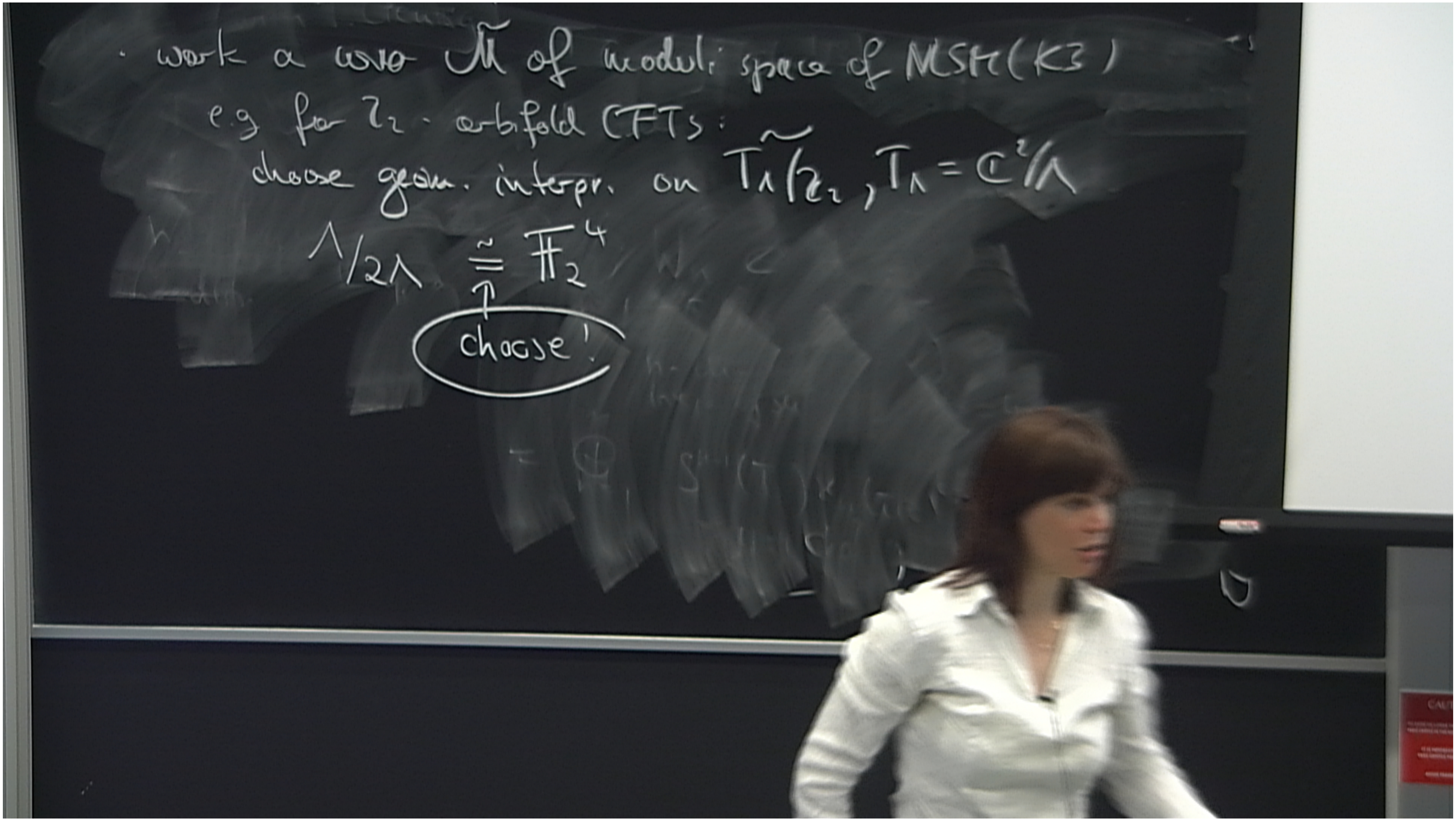
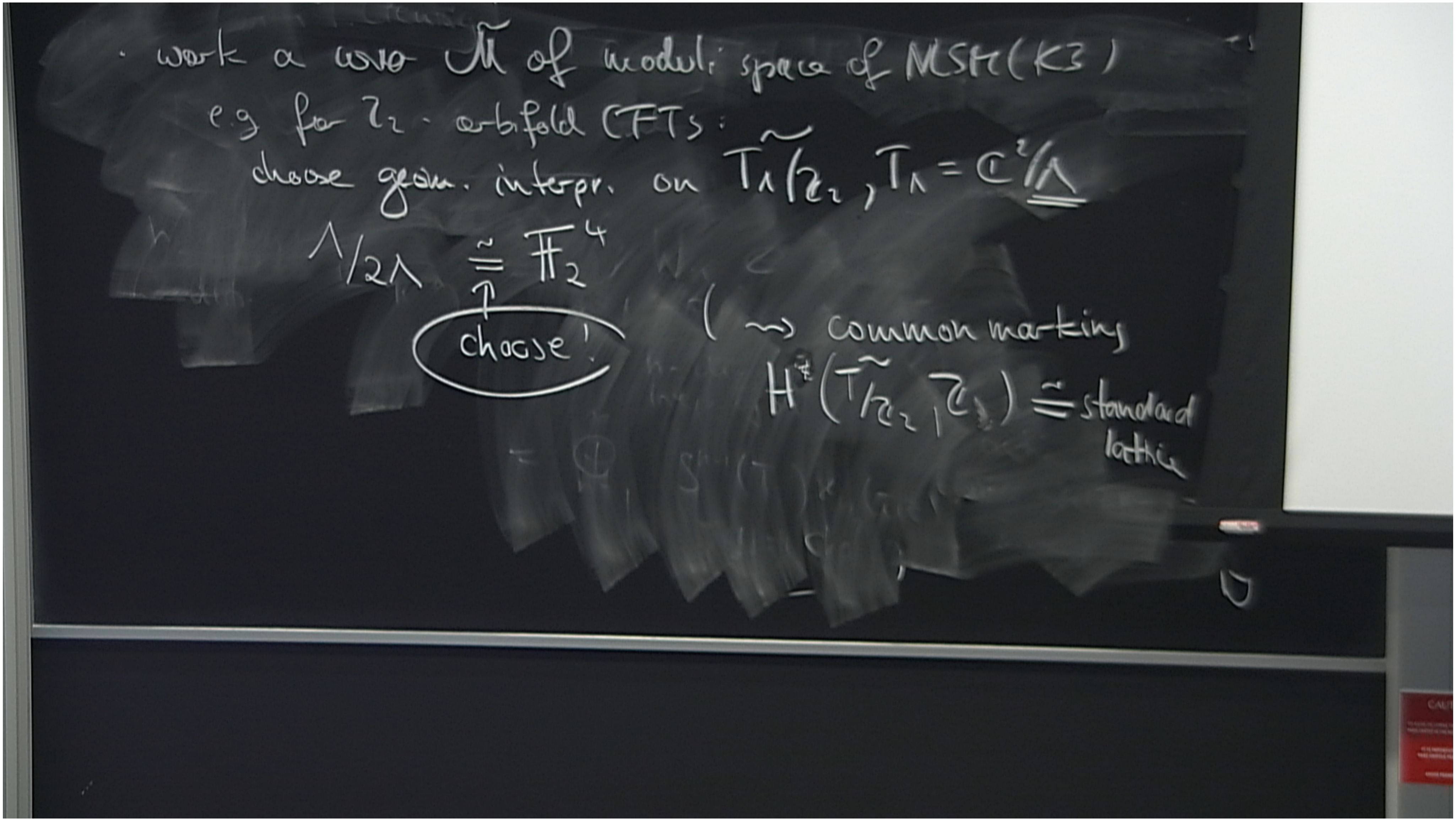

if E_H(τ,z) = str_HNS (. . . . .)
with extended SUSY : decompose !
χ(p_n(T)) → dim R_n = A_n ⟷ Rep(M24)
↓
H*(p_n(T))
[Eguchi / Ooguri / Tachikawa '10
the audience
Gannon '12]
? decompose E_{q,-y}
p_n T = S^2 T


VOA
pn(T) χn(T,z)
χ(H*(pn(T))) = An
M24
M24
DEA


work a cover of moduli space of NSM(K3)
e.g. for Z2-orbifold CFTs:
choose geom. interpr. on T/Z2, T = C²/Λ
Λ/2Λ ≅ F2⁴
choose!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choose geom. interpr. on $\widetilde{T_\Lambda/\mathbb{Z}_2}$, $T_\Lambda = \mathbb{C}^2/\Lambda$
$\Lambda/2\Lambda \cong \mathbb{F}_2^4$
choose!
(⇝ common marking
$H^*(\widetilde{T/\mathbb{Z}_2}, \mathbb{Z}) \cong$ standard lattice

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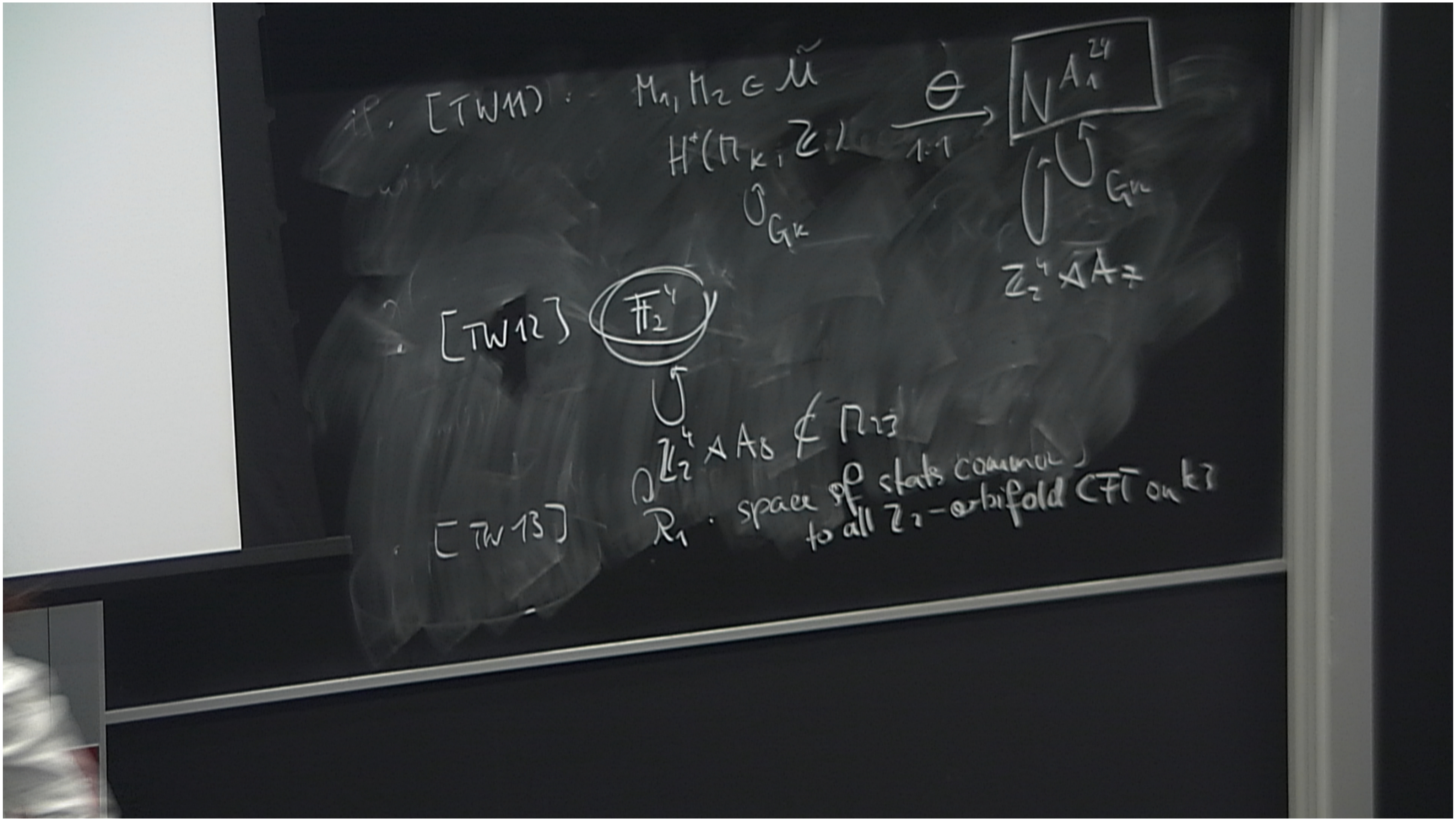
[TW11]
[TW12]
[TW13]
space of stats common
to all Z2-orbifold CFT on K3

Symmetry-surfing: Symmetries of $\mathbb{Z}_2$-orbifold CFTs on K3

G: geometric symmetry group of a $\mathbb{Z}_2$-orbifold CFT with geometric interpretation on $X = \widetilde{T_\Lambda/\mathbb{Z}_2}$, $T_\Lambda = \mathbb{C}^2/\Lambda$

$\Longrightarrow G = (\mathbb{Z}_2)^4 \rtimes G_\Lambda$,

using [Fujiki88] $\Longrightarrow$ $(\mathbb{Z}_2)^4$ acts by shifts on $\Lambda/2\Lambda$, $G_\Lambda \subset \mathrm{SO}(3)$

- $G_\Lambda \subset G_{\Lambda_k}$, one of three maximal finite groups $G_{\Lambda_1} = A_4,\ G_{\Lambda_2} = S_3,\ G_{\Lambda_0} = \mathbb{Z}_2^2$
- our choice of lattice generators $\Lambda = \mathrm{span}_\mathbb{C}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ for the corresponding maximally symmetric lattices Λ_k is

$$\Lambda_1 = \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), \tfrac{1}{2}(1,1,1,1)\right\},$$
$$\Lambda_2 = \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), \tfrac{1}{2}(-1,\sqrt{3},0,0), (0,0,1,0), \tfrac{1}{2}(0,0,-1,\sqrt{3})\right\},$$
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with geometric interpretation on $X = T_\Lambda/\mathbb{Z}_2$, $T_\Lambda = \mathbb{C}^2/\Lambda$

$$\Longrightarrow G = (\mathbb{Z}_2)^4 \rtimes G_\Lambda \subset (\mathbb{Z}_2)^4 \rtimes \mathrm{GL}_4(\mathbb{F}_2) \overset{\text{[Jordan1870]}}{\cong} (\mathbb{Z}_2)^4 \rtimes A_8 \subset M_{24},$$

$$\mathbb{F}_2^4 \cong \Lambda/2\Lambda, \; G_\Lambda \subset \mathrm{SO}(3)$$

using [Fujiki88] $\Longrightarrow$

- $G_\Lambda \subset G_{\Lambda_k}$, one of three maximal finite groups
$$G_{\Lambda_1} = A_4,\; G_{\Lambda_2} = S_3,\; G_{\Lambda_0} = \mathbb{Z}_2^2$$
- our choice of lattice generators $\Lambda = \mathrm{span}_\mathbb{C}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ for the corresponding maximally symmetric lattices Λ_k is

$$\Lambda_1 = \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), \tfrac{1}{2}(1,1,1,1)\right\},$$
$$\Lambda_2 = \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), \tfrac{1}{2}(-1,\sqrt{3},0,0), (0,0,1,0), \tfrac{1}{2}(0,0,-1,\sqrt{3})\right\},$$
$$\Lambda_0 = \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\right\}.$$

Symmetry-surfing: Symmetries of $\mathbb{Z}_2$-orbifold CFTs on K3

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$$\begin{aligned}
\Lambda_1 &= \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), \tfrac{1}{2}(1,1,1,1)\right\},\\
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\Lambda_0 &= \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\right\}.
\end{aligned}$$

Symmetry surfing the moduli space of Kummer K3s

Results [Taormina/W11&12&13]

- For the $\mathbb{Z}_2$-orbifold CFTs on K3 with geometric interpretation on some $X = T_\Lambda/\mathbb{Z}_2$ ($T_\Lambda = \mathbb{C}^2/\Lambda$), the joint action of all symmetry groups yields the maximal subgroup $(\mathbb{Z}_2)^4 \rtimes A_8 \subset M_{24}$.

  Note: $(\mathbb{Z}_2)^4 \rtimes A_8$ is not a subgroup of M_{23}.

- There is a 90-dim. space $\mathcal{R}_1$ of states common to all K3-theories that are $\mathbb{Z}_2$-orbifolds of toroidal theories. As common representation space of all geometric symmetry groups of Kummer K3s, $\mathcal{R}_1$ carries an action of $(\mathbb{Z}_2)^4 \rtimes A_8$ induced from $\mathcal{R}_1 \cong \mathbf{45} \oplus \overline{\mathbf{45}}$ with irreps $\mathbf{45}$, $\overline{\mathbf{45}}$ of M_{24}.

  Note: There is a twist in this action of the groups $G_{\Lambda_1}, G_{\Lambda_2}, G_{\Lambda_0}$ on $\mathcal{R}_1 \cong \mathbf{15} \otimes (\mathbf{3} \oplus \overline{\mathbf{3}})$.

Symmetry surfing the moduli space of Kummer K3s

Results [Taormina/W11&12&13]

- For the $\mathbb{Z}_2$-orbifold CFTs on K3 with geometric interpretation on some $X = \widetilde{T_\Lambda/\mathbb{Z}_2}$ ($T_\Lambda = \mathbb{C}^2/\Lambda$), the joint action of all symmetry groups yields the maximal subgroup $(\mathbb{Z}_2)^4 \rtimes A_8 \subset M_{24}$.

  Note: $(\mathbb{Z}_2)^4 \rtimes A_8$ is not a subgroup of M_{23}.

- There is a 90-dim. space $\mathcal{R}_1$ of states common to all K3-theories that are $\mathbb{Z}_2$-orbifolds of toroidal theories. As common representation space of all geometric symmetry groups of Kummer K3s, $\mathcal{R}_1$ carries an action of $(\mathbb{Z}_2)^4 \rtimes A_8$ induced from $\mathcal{R}_1 \cong \mathbf{45} \oplus \overline{\mathbf{45}}$ with irreps $\mathbf{45}$, $\overline{\mathbf{45}}$ of M_{24}.

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