Title: Fricke S-duality in CHL models

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Abstract: We consider dual pairs of four dimensional heterotic/type IIA CHL models with 16 space-time supersymmetries. We provide strong evidence for the existence of an S-duality acting on the heterotic axion-dilaton by a Fricke involution S --> -1/NS, where N is the order of the orbifold symmetry. While most models are self-dual, in some cases S-duality relates the CHL model to a compactification of type IIA on an orbifold of T^6. We provide a simple criterion to determine whether a model is self-dual or not. Finally, we argue that in self-dual CHL models the lattices of electric and magnetic charges must be N-modular and verify this prediction.

Fricke S-dualities in CHL models

Roberto Volpato
SLAC & SITP Stanford

(Mock) modularity, Moonshine and String Theory

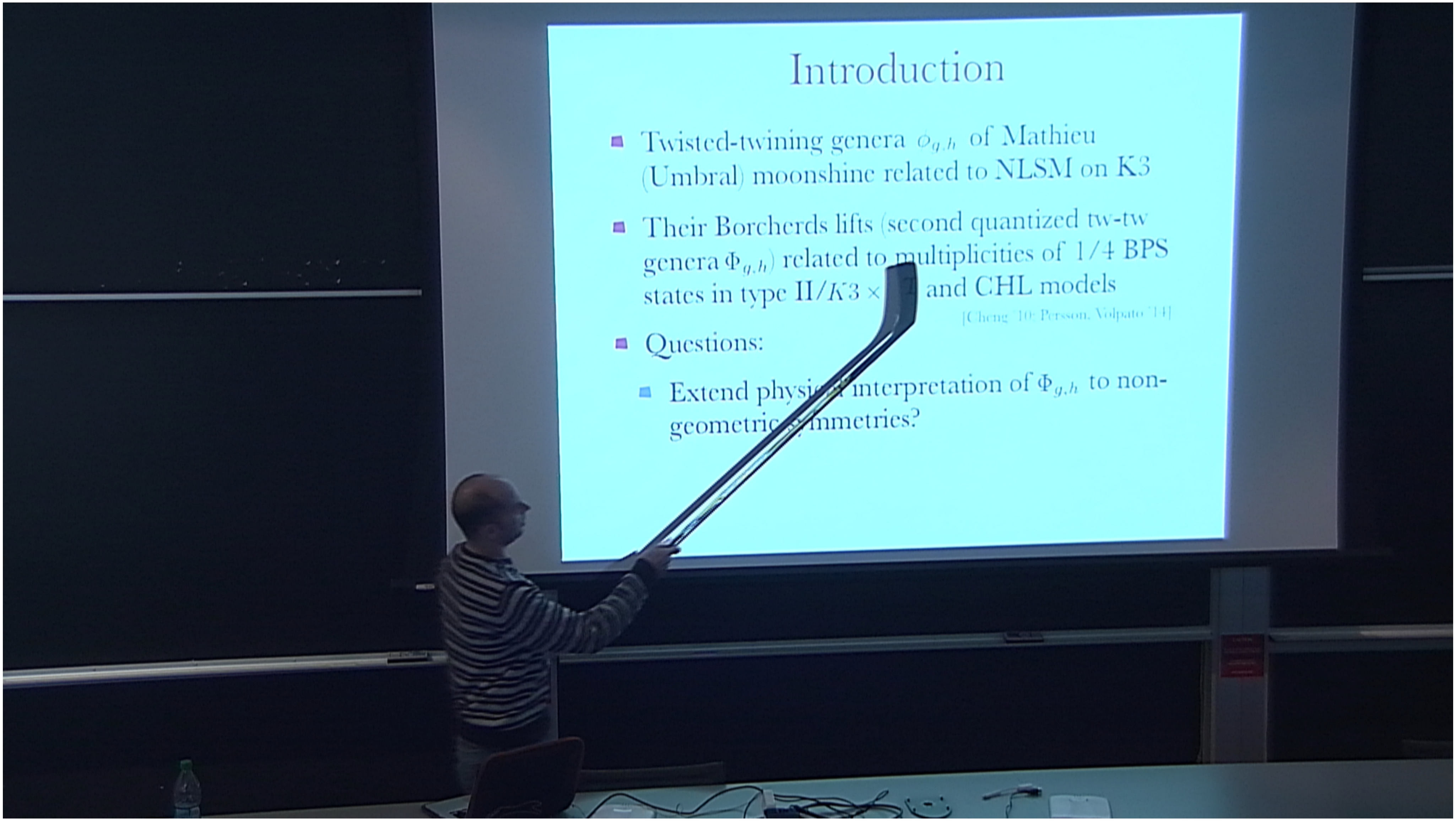
Perimeter Institute, April 14, 2015

work in collaboration with Daniel Persson

Introduction

- Twisted-twining genera $\phi_{g,h}$ of Mathieu (Umbral) moonshine related to NLSM on K3
- Their Borcherds lifts (second quantized tw-tw genera $\Phi_{g,h}$) related to multiplicities of $1/4$ BPS states in type II$/K3 \times T^2$ and CHL models

[Cheng '10; Persson, Volpato '14]


Introduction
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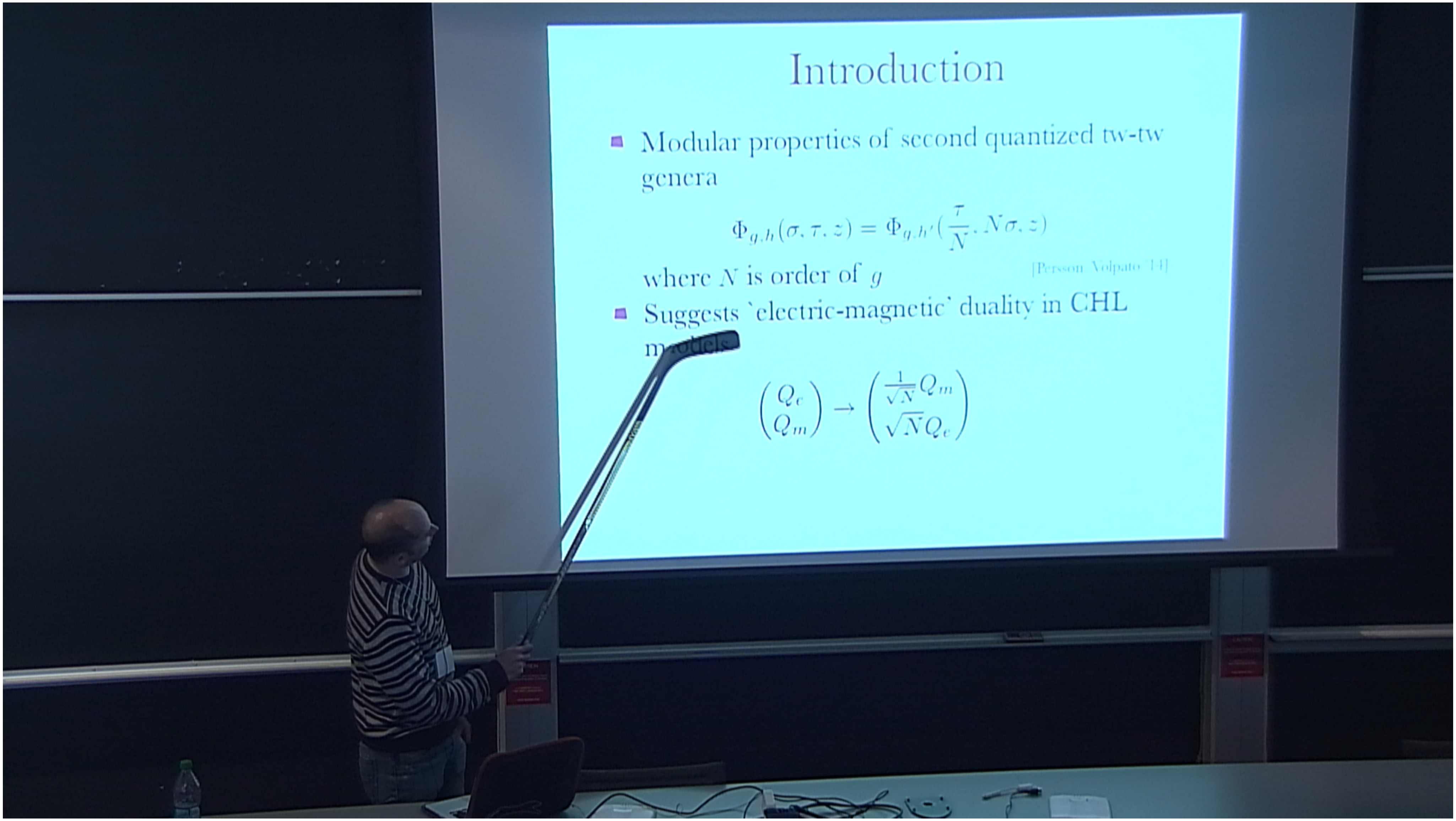
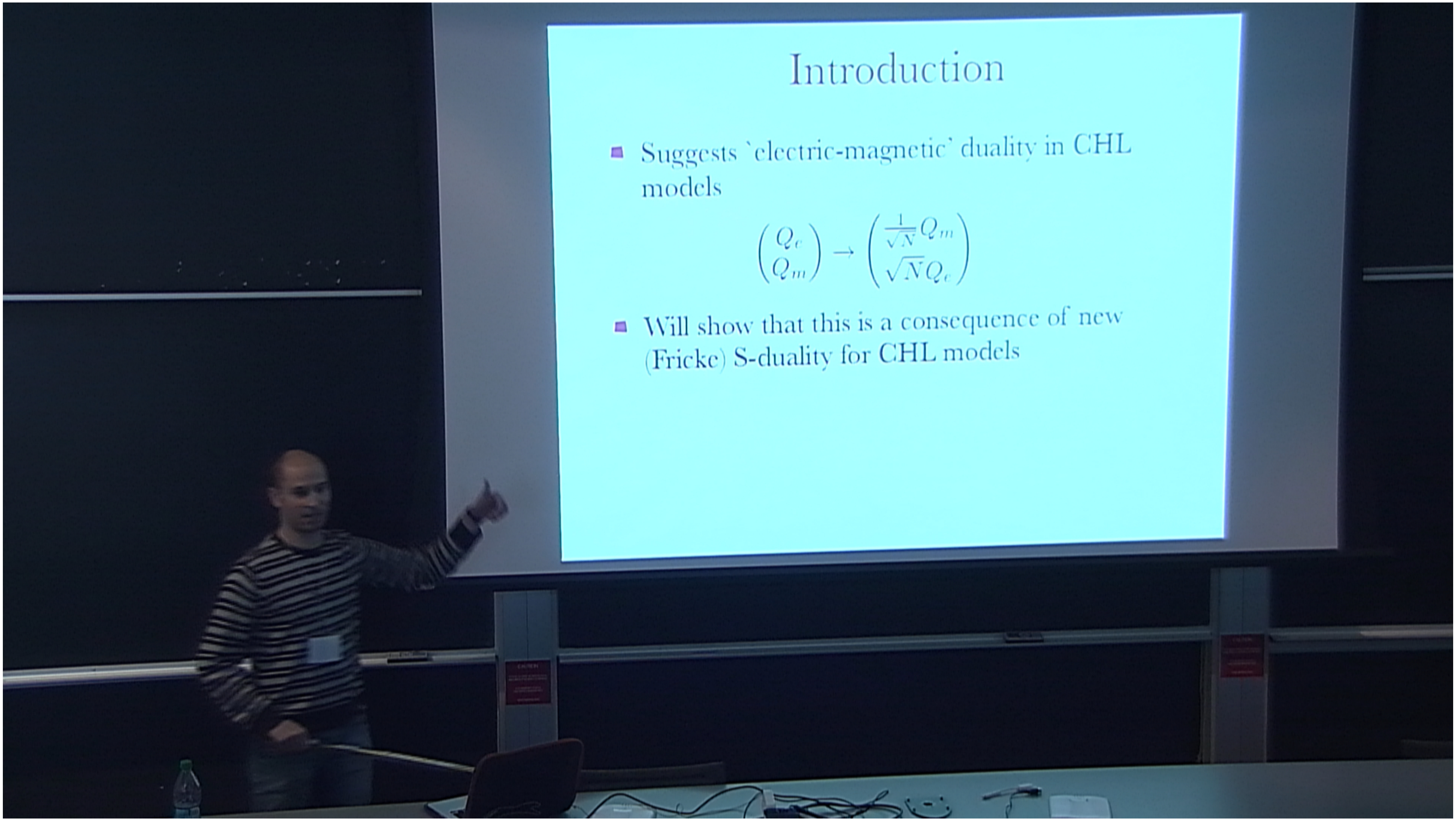
Their Borcherds lifts (second quantized tw-tw genera $\Phi_{g,h}$) related to multiplicities of 1/4 BPS states in type II/$K3 \times$ and CHL models
[Cheng '10; Persson, Volpato '14]
Questions:
Extend physi interpretation of $\Phi_{g,h}$ to non-
geometric mmetries?

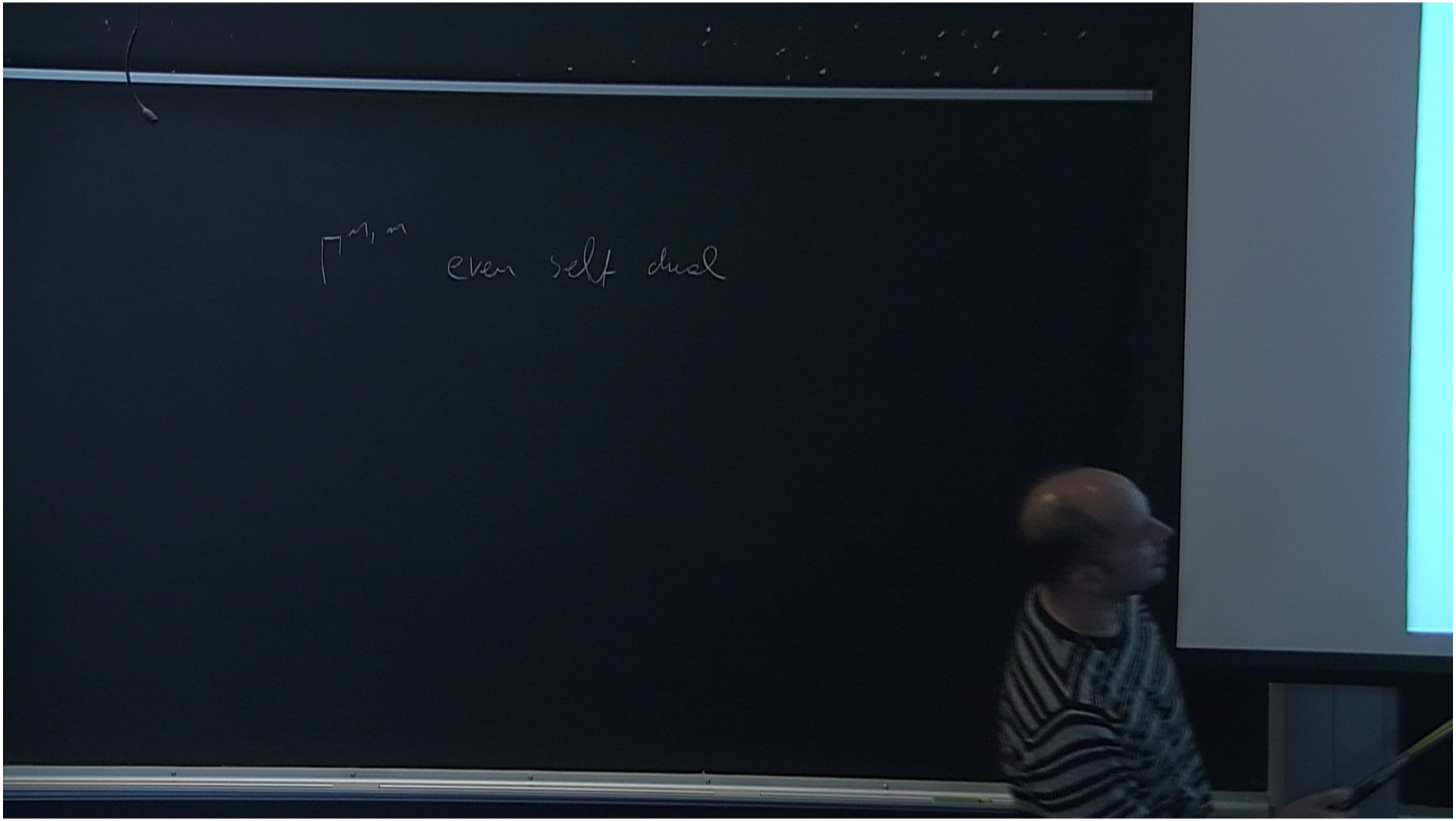

Introduction
Modular properties of second quantized tw-tw genera
$\Phi_{g,h}(\sigma,\tau,z) = \Phi_{g,h'}(\frac{\tau}{N}, N\sigma, z)$
where N is order of g
[Persson, Volpato '14]
Suggests `electric-magnetic' duality in CHL
$\begin{pmatrix} Q_e \\ Q_m \end{pmatrix} \to \begin{pmatrix} \frac{1}{\sqrt{N}} Q_m \\ \sqrt{N} Q_e \end{pmatrix}$

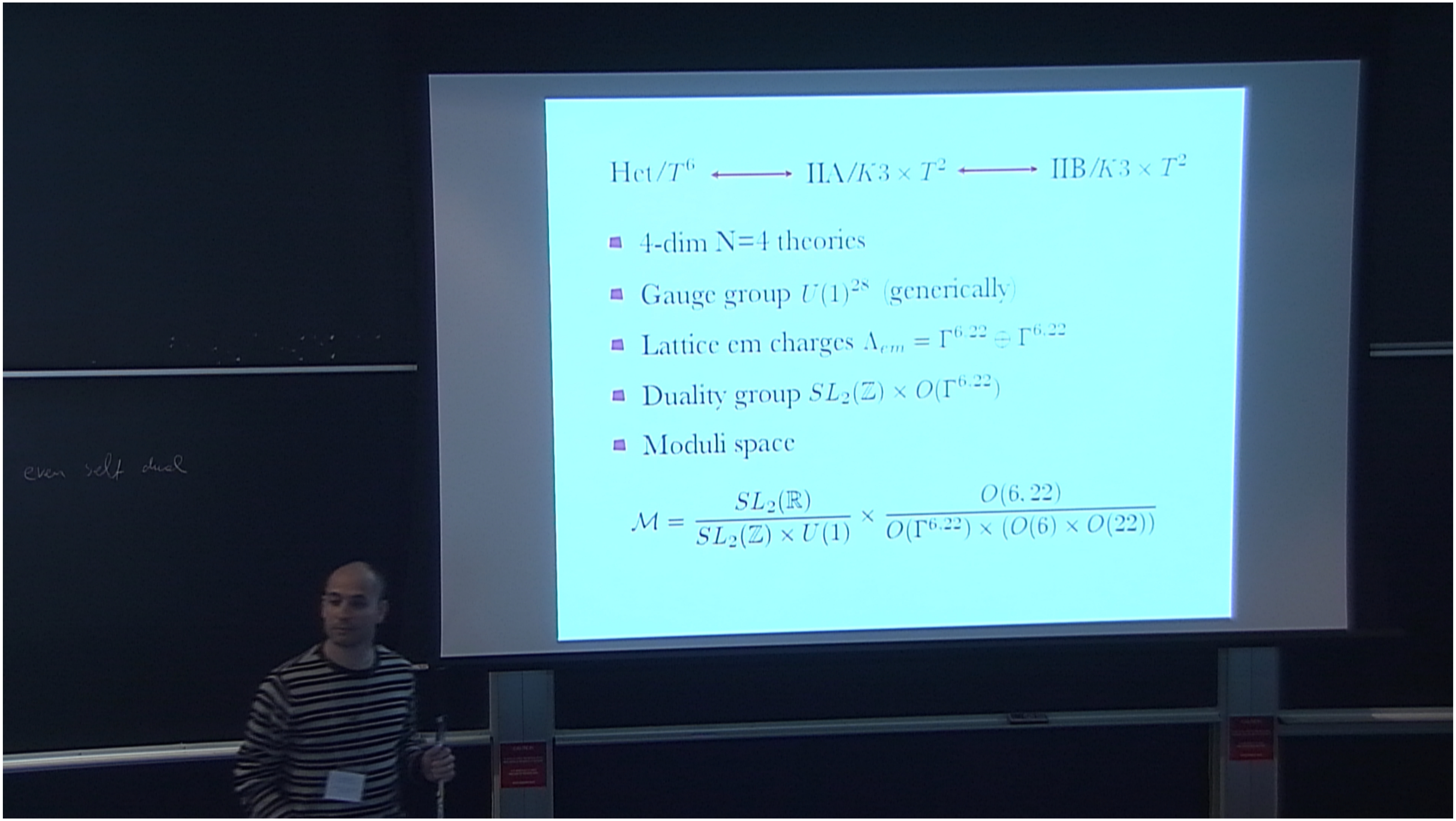

Introduction
Suggests `electric-magnetic' duality in CHL models
$\begin{pmatrix} Q_e \\ Q_m \end{pmatrix} \to \begin{pmatrix} \frac{1}{\sqrt{N}} Q_m \\ \sqrt{N} Q_e \end{pmatrix}$
Will show that this is a consequence of new (Fricke) S-duality for CHL models

$$\text{Het}/T^6 \longleftrightarrow \text{IIA}/K3 \times T^2 \longleftrightarrow \text{IIB}/K3 \times T^2$$

- 4-dim N=4 theories
- Gauge group $U(1)^{28}$ (generically)
- Lattice em charges $\Lambda_{em} = \Gamma^{6,22} \oplus \Gamma^{6,22}$
- Duality group $SL_2(\mathbb{Z}) \times O(\Gamma^{6,22})$
- Moduli space

$$\mathcal{M} = \frac{SL_2(\mathbb{R})}{SL_2(\mathbb{Z}) \times U(1)} \times \frac{O(6,22)}{O(\Gamma^{6,22}) \times (O(6) \times O(22))}$$


Γ^{n,m} even self dual


$Het/T^6 \longleftrightarrow IIA/K3 \times T^2 \longleftrightarrow IIB/K3 \times T^2$
4-dim N=4 theories
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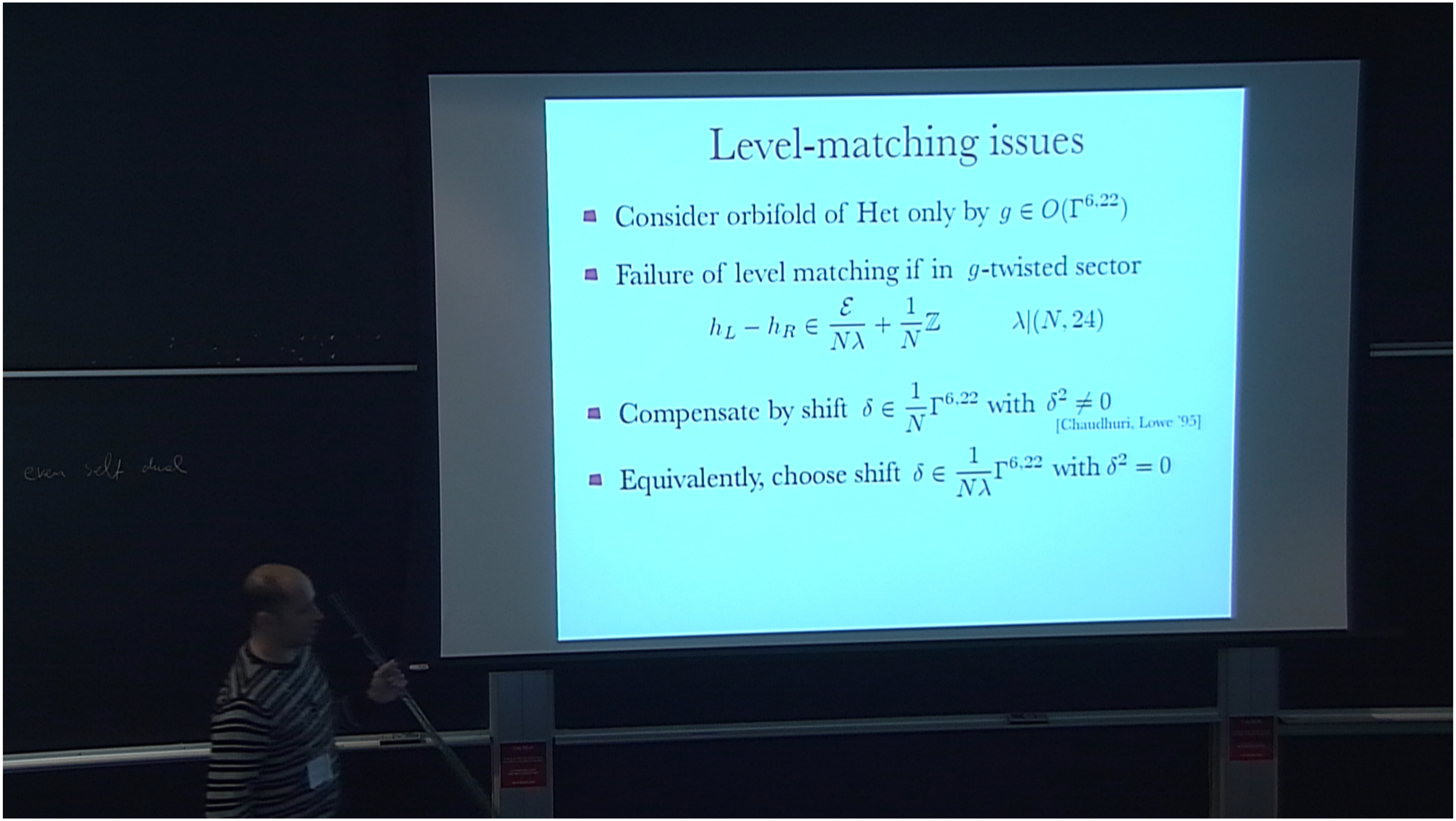
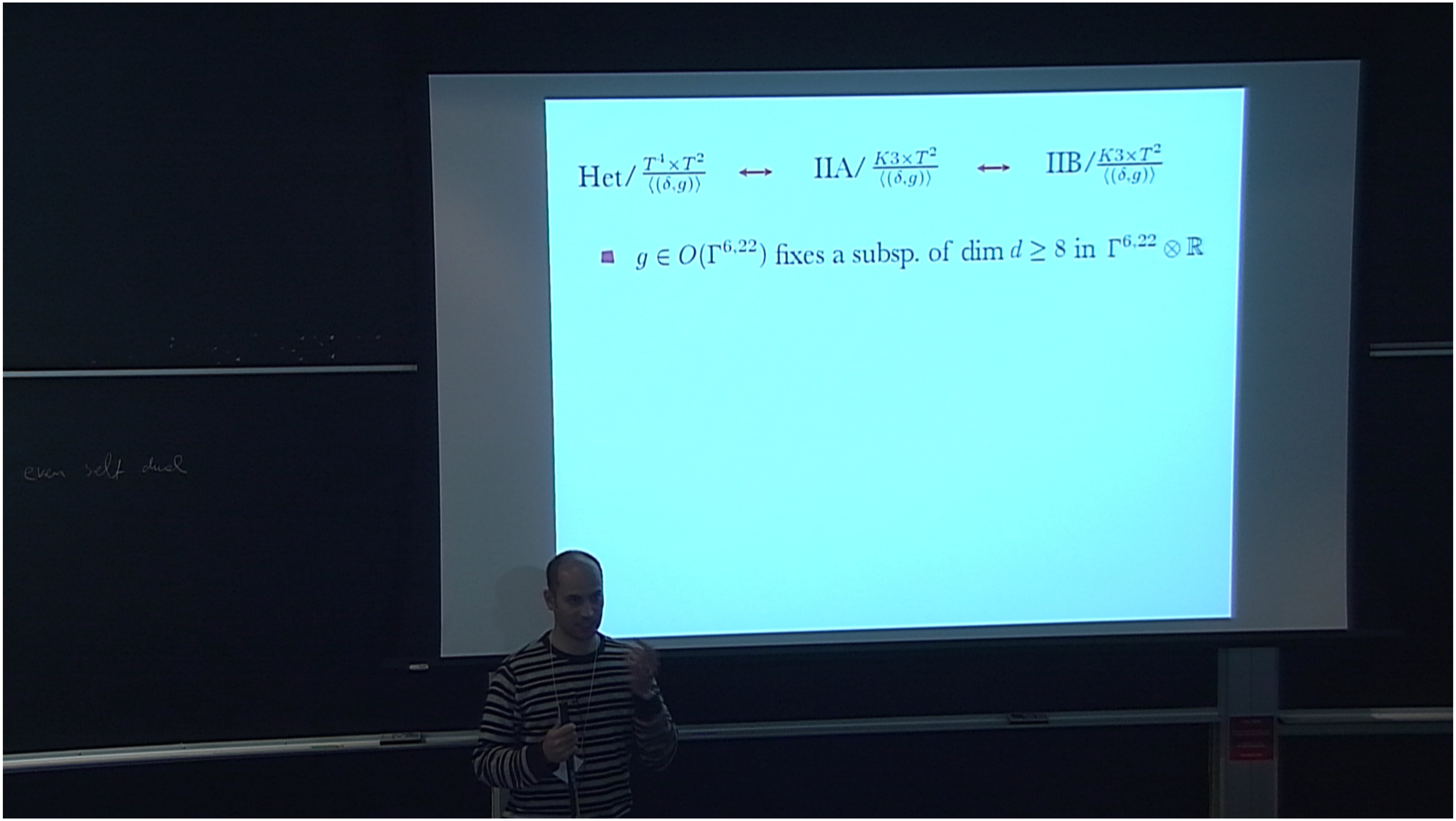

Level-matching issues
Consider orbifold of Het only by $g \in O(\Gamma^{6,22})$
Failure of level matching if in g-twisted sector
$h_L - h_R \in \frac{\mathcal{E}}{N\lambda} + \frac{1}{N}\mathbb{Z}$ $\lambda|(N,24)$
Compensate by shift $\delta \in \frac{1}{N}\Gamma^{6,22}$ with $\delta^2 \neq 0$
[Chaudhuri, Lowe '95]
Equivalently, choose shift $\delta \in \frac{1}{N\lambda}\Gamma^{6,22}$ with $\delta^2 = 0$
even self dual


Het/ $\frac{T^4 \times T^2}{\langle(\delta,g)\rangle}$ ↔ IIA/ $\frac{K3 \times T^2}{\langle(\delta,g)\rangle}$ ↔ IIB/ $\frac{K3 \times T^2}{\langle(\delta,g)\rangle}$
$g \in O(\Gamma^{6,22})$ fixes a subsp. of dim $d \geq 8$ in $\Gamma^{6,22} \otimes \mathbb{R}$
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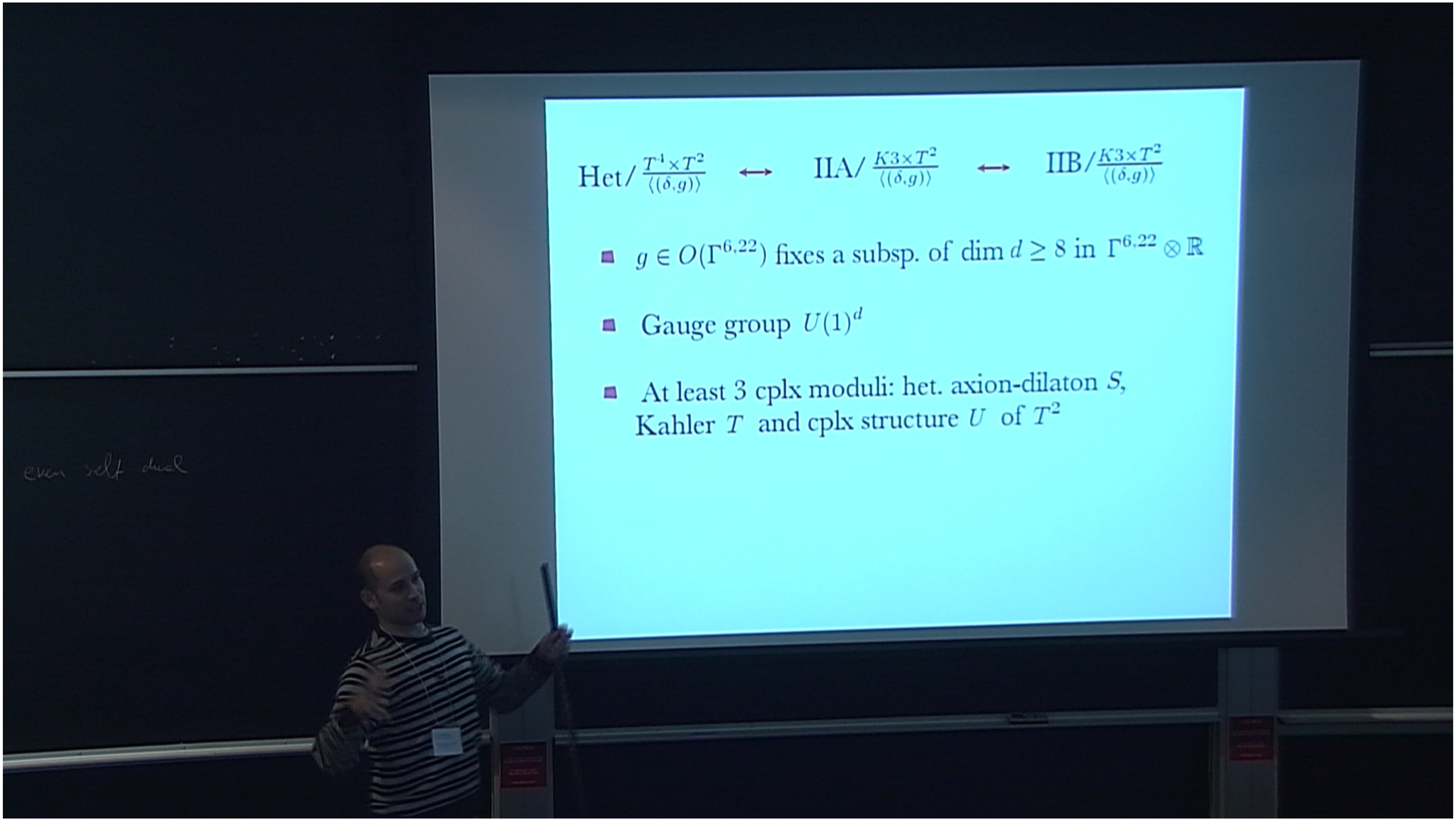
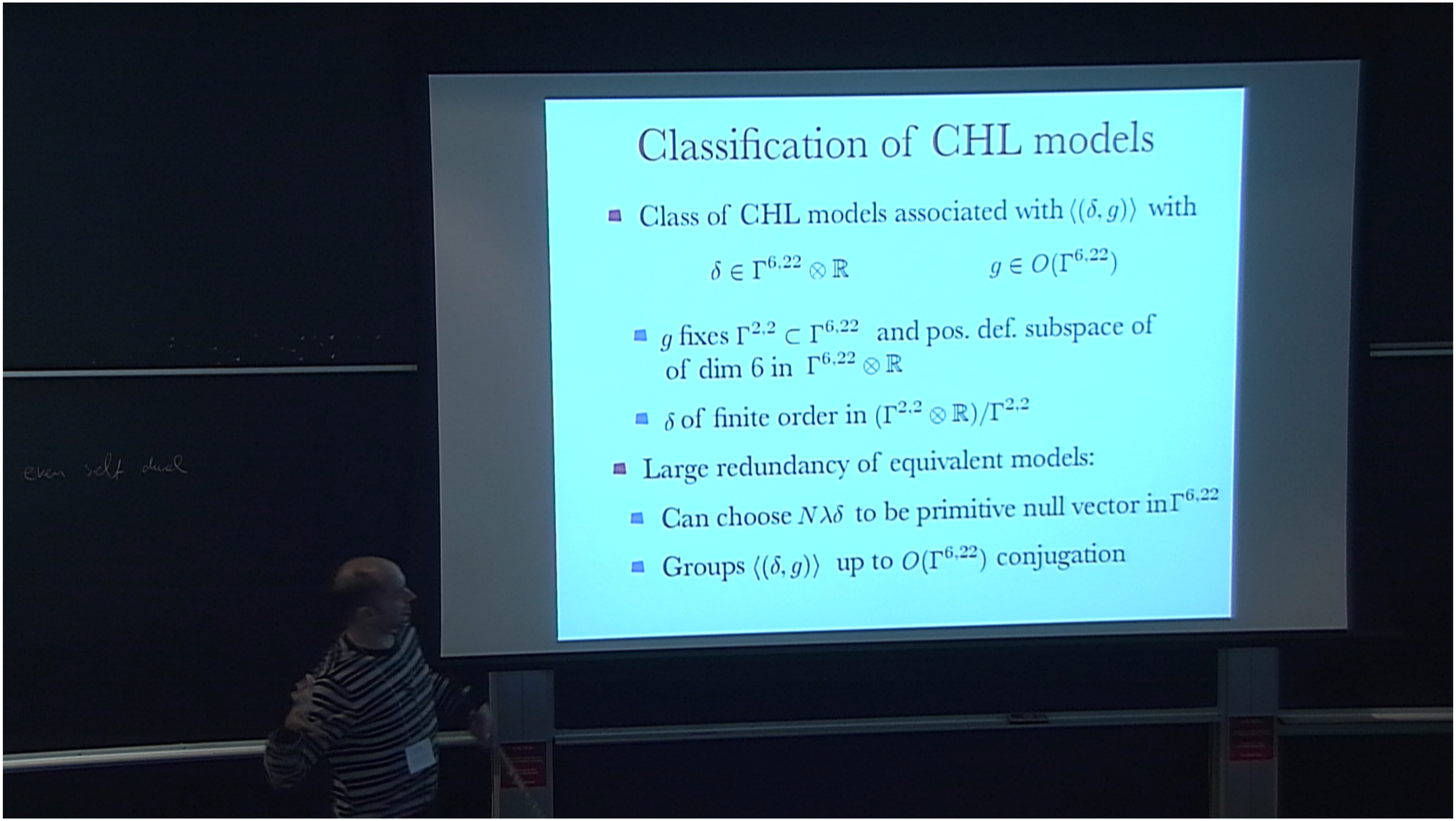
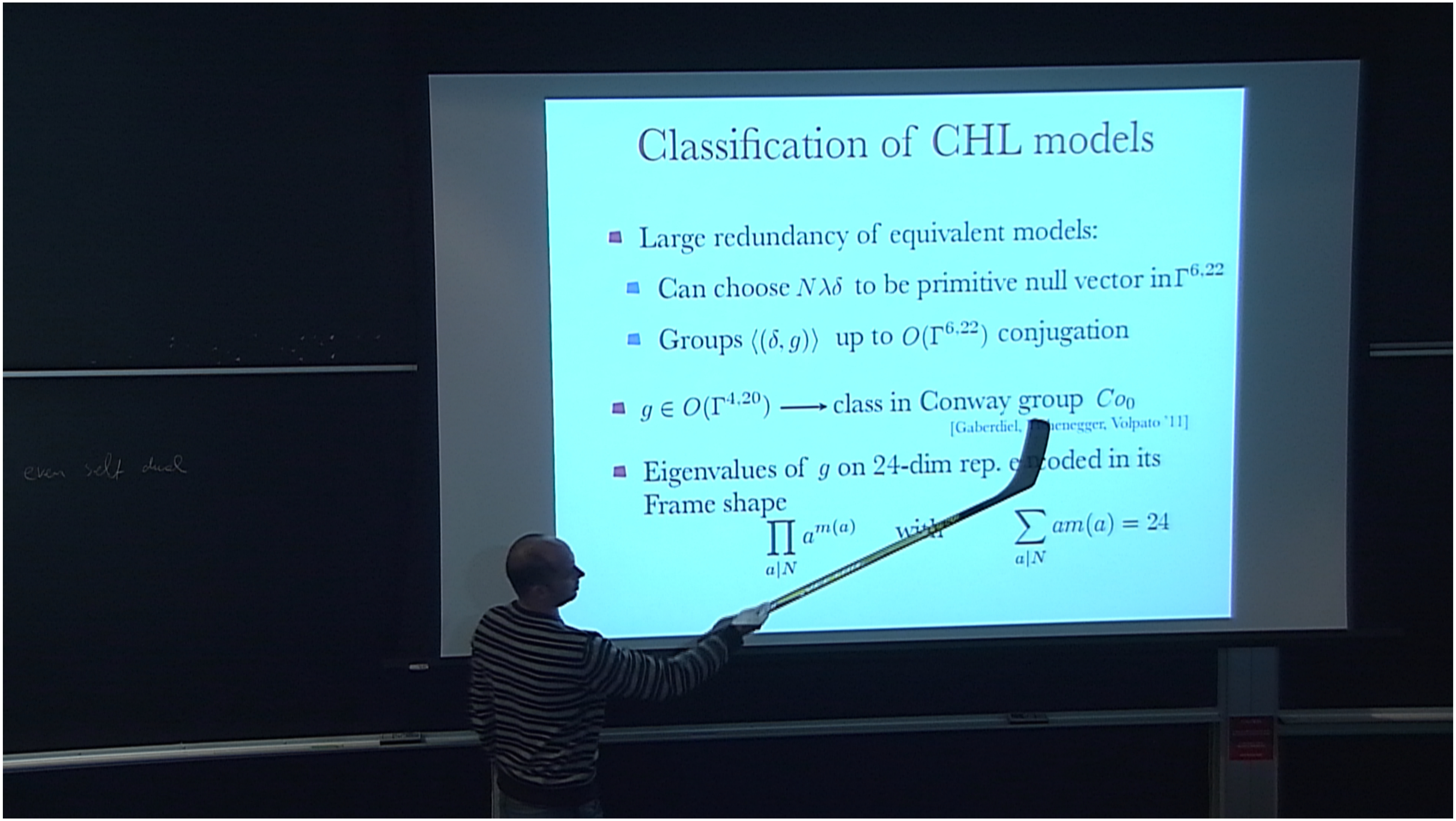

Het/ $\frac{T^4 \times T^2}{\langle(\delta, g)\rangle}$ $\longleftrightarrow$ IIA/ $\frac{K3 \times T^2}{\langle(\delta, g)\rangle}$ $\longleftrightarrow$ IIB/ $\frac{K3 \times T^2}{\langle(\delta, g)\rangle}$
$g \in O(\Gamma^{6,22})$ fixes a subsp. of dim $d \geq 8$ in $\Gamma^{6,22} \otimes \mathbb{R}$
Gauge group $U(1)^d$
At least 3 cplx moduli: het. axion-dilaton S, Kahler T and cplx structure U of T^2
even self dual


Classification of CHL models
Class of CHL models associated with $\langle(\delta, g)\rangle$ with
$\delta \in \Gamma^{6,22} \otimes \mathbb{R}$
$g \in O(\Gamma^{6,22})$
g fixes $\Gamma^{2,2} \subset \Gamma^{6,22}$ and pos. def. subspace of of dim 6 in $\Gamma^{6,22} \otimes \mathbb{R}$
δ of finite order in $(\Gamma^{2,2} \otimes \mathbb{R})/\Gamma^{2,2}$
Large redundancy of equivalent models:
Can choose $N\lambda\delta$ to be primitive null vector in $\Gamma^{6,22}$
Groups $\langle(\delta, g)\rangle$ up to $O(\Gamma^{6,22})$ conjugation
even self dual


Classification of CHL models
Large redundancy of equivalent models:
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Groups $\langle(\delta, g)\rangle$ up to $O(\Gamma^{6,22})$ conjugation
$g \in O(\Gamma^{4,20}) \longrightarrow$ class in Conway group Co_0
[Gaberdiel, henegger, Volpato '11]
Eigenvalues of g on 24-dim rep. e coded in its Frame shape
$\prod_{a|N} a^{m(a)}$
$\sum_{a|N} am(a) = 24$
even self dual

Classification of CHL models

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  - Can choose $N\lambda\delta$ to be primitive null vector in $\Gamma^{6,22}$
  - Groups $\langle(\delta, g)\rangle$ up to $O(\Gamma^{6,22})$ conjugation
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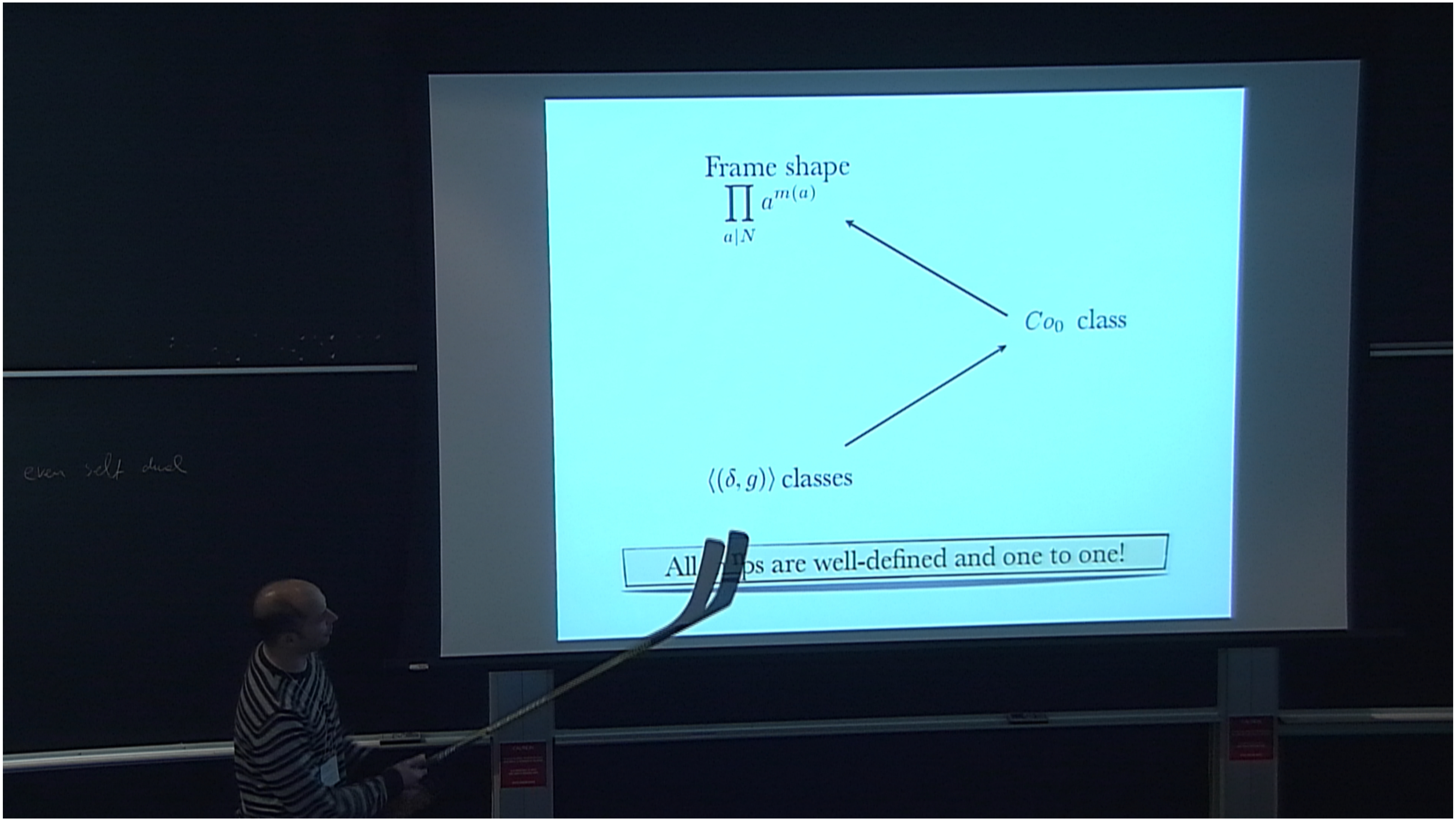
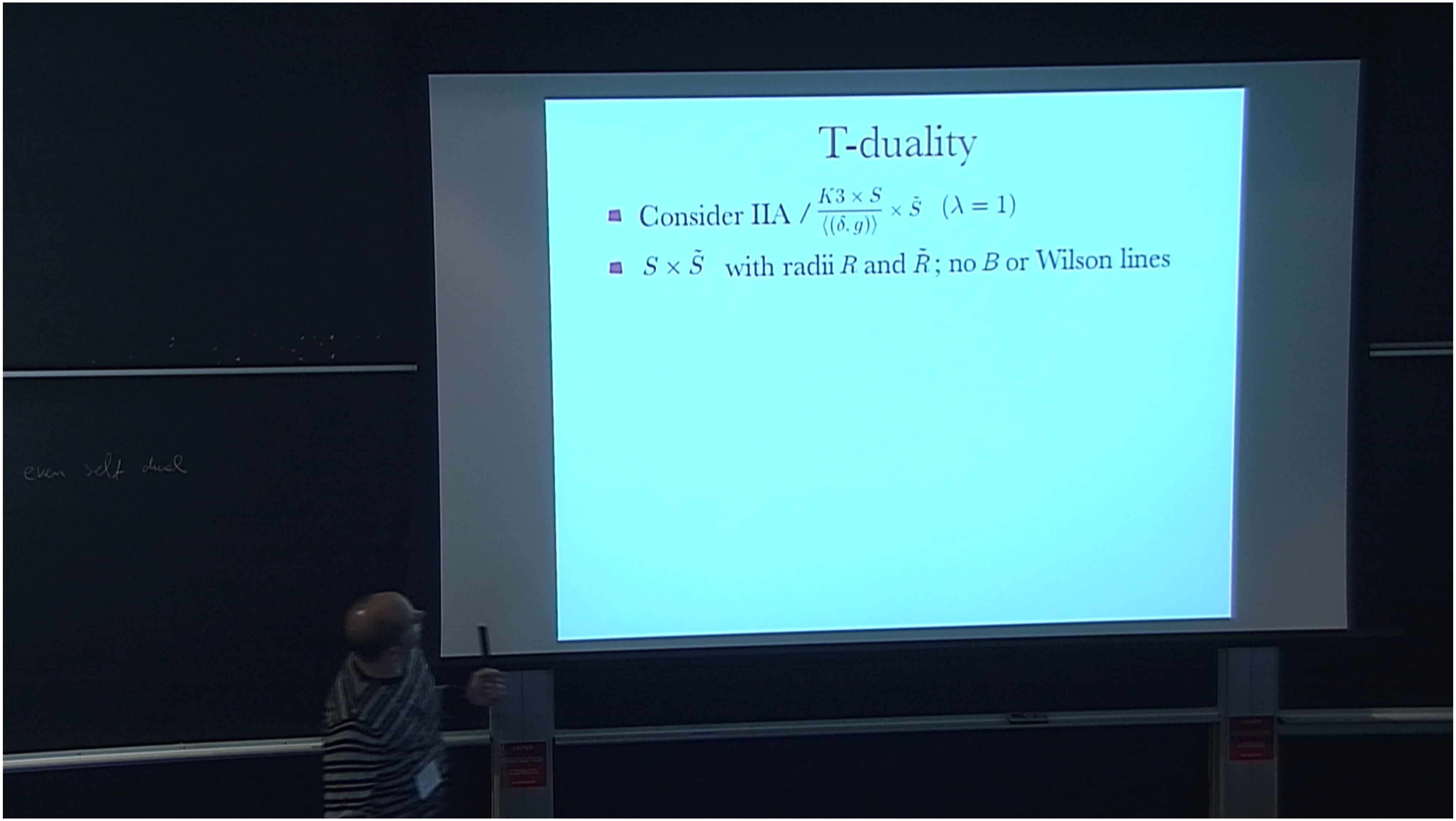
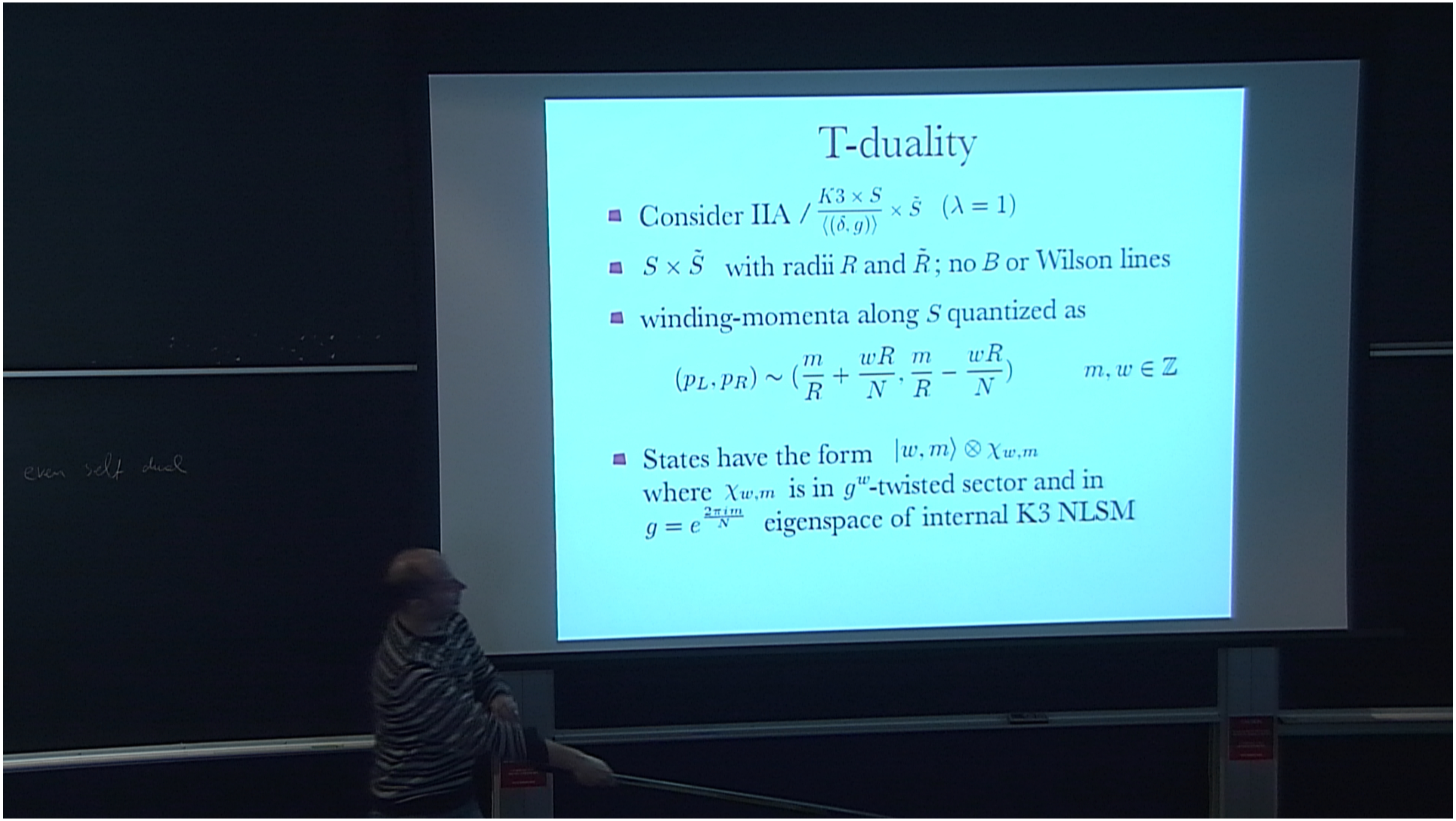
  [Gaberdiel, Hohenegger, Volpato '11]
- Eigenvalues of g on 24-dim rep. encoded in its Frame shape

$$\prod_{a|N} a^{m(a)} \qquad \text{with} \qquad \sum_{a|N} am(a) = 24$$


Frame shape
$\prod_{a|N} a^{m(a)}$
Co_0 class
$\langle(\delta, g)\rangle$ classes
All
ps are well-defined and one to one!
even self dual


T-duality
Consider IIA / $\frac{K3 \times S}{\langle (\delta, g) \rangle} \times \tilde{S} \quad (\lambda = 1)$
$S \times \tilde{S}$ with radii R and $\tilde{R}$; no B or Wilson lines
even self dual


T-duality
Consider IIA / $\frac{K3 \times S}{\langle(\delta, g)\rangle} \times \tilde{S}$ $(\lambda = 1)$
$S \times \tilde{S}$ with radii R and $\tilde{R}$; no B or Wilson lines
winding-momenta along S quantized as
$(p_L, p_R) \sim (\frac{m}{R} + \frac{wR}{N}, \frac{m}{R} - \frac{wR}{N})$ $m, w \in \mathbb{Z}$
States have the form $|w, m\rangle \otimes \chi_{w,m}$
where $\chi_{w,m}$ is in g^w-twisted sector and in
$g = e^{\frac{2\pi i m}{N}}$ eigenspace of internal K3 NLSM
even self dual

T-duality

- Consider IIA / $\frac{K3 \times S}{\langle(\delta, g)\rangle} \times \tilde{S} \quad (\lambda = 1)$
- $S \times \tilde{S}$ with radii R and $\tilde{R}$; no B or Wilson lines
- winding-momenta along S quantized as

$$(p_L, p_R) \sim (\frac{m}{R} + \frac{wR}{N}, \frac{m}{R} - \frac{wR}{N}) \qquad m, w \in \mathbb{Z}$$

- States have the form $|w, m\rangle \otimes \chi_{w,m}$
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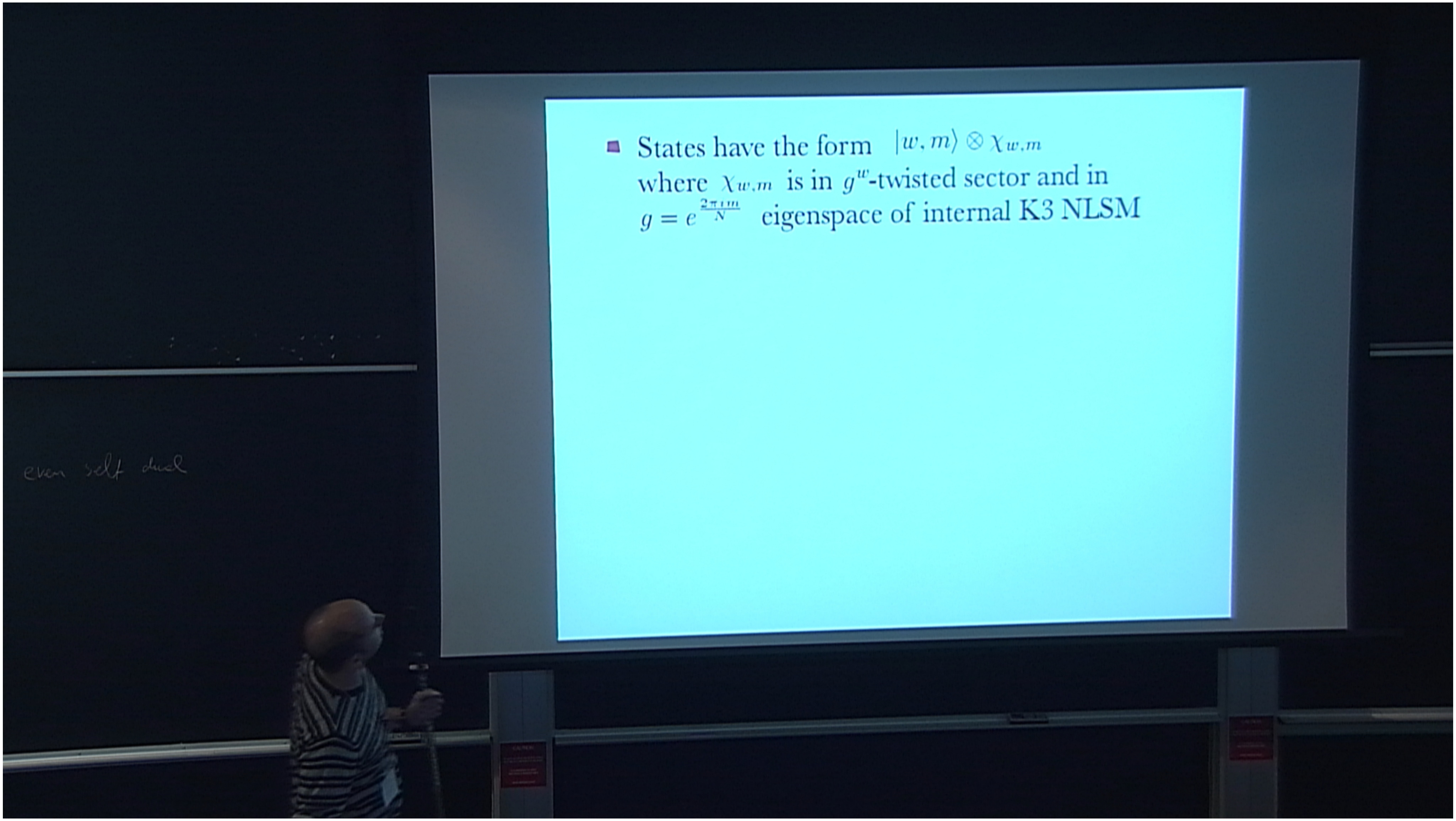
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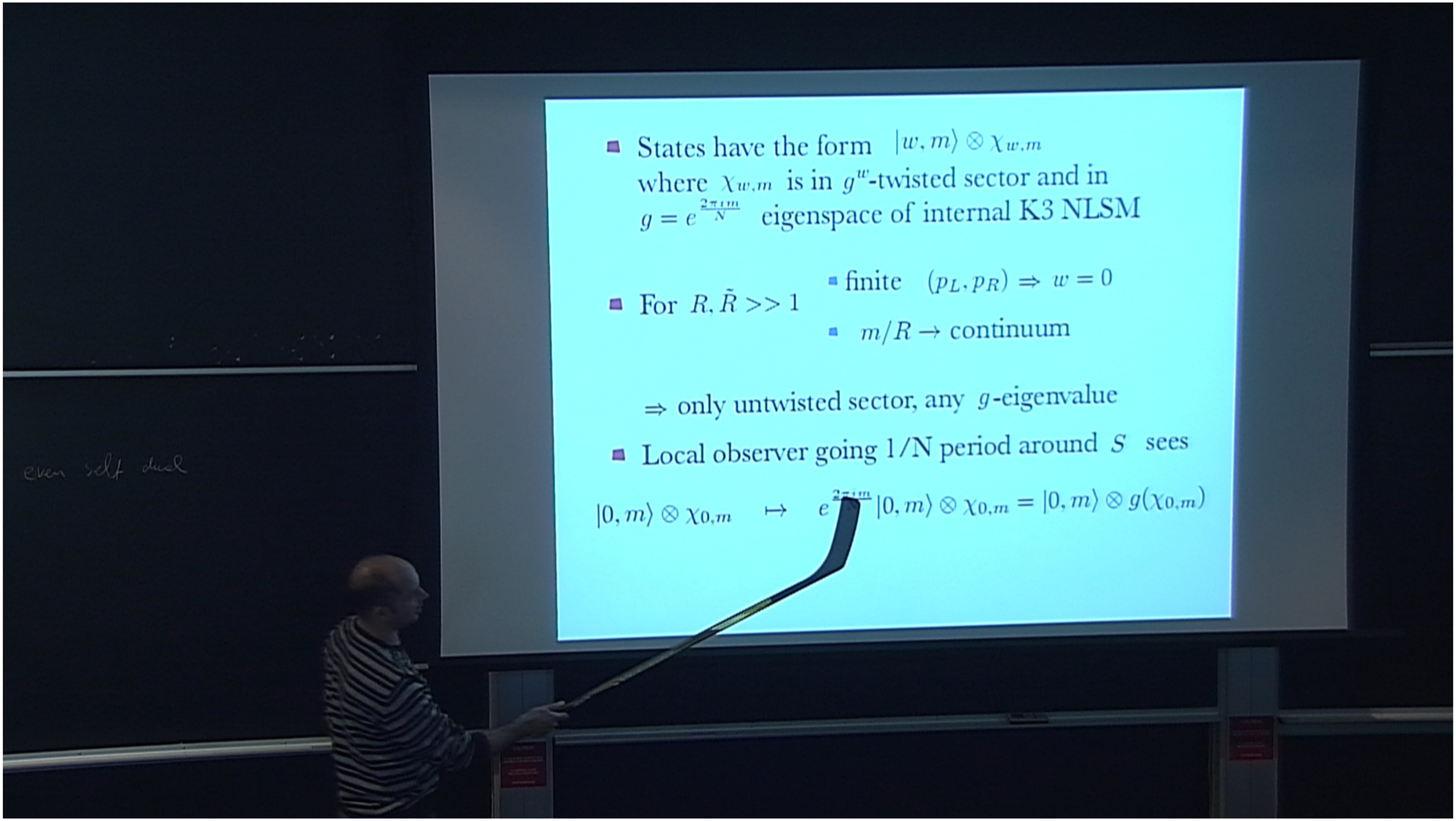
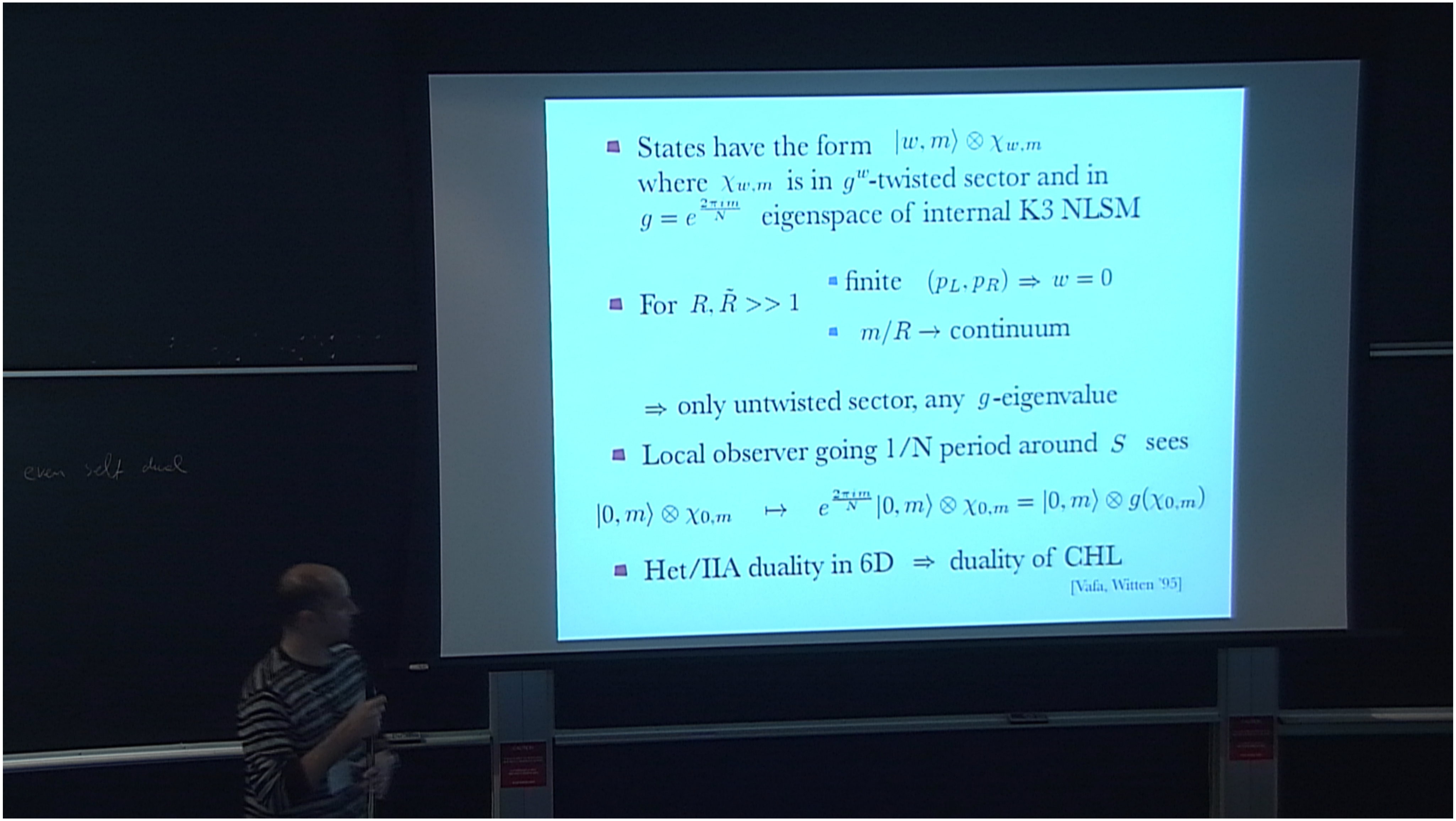
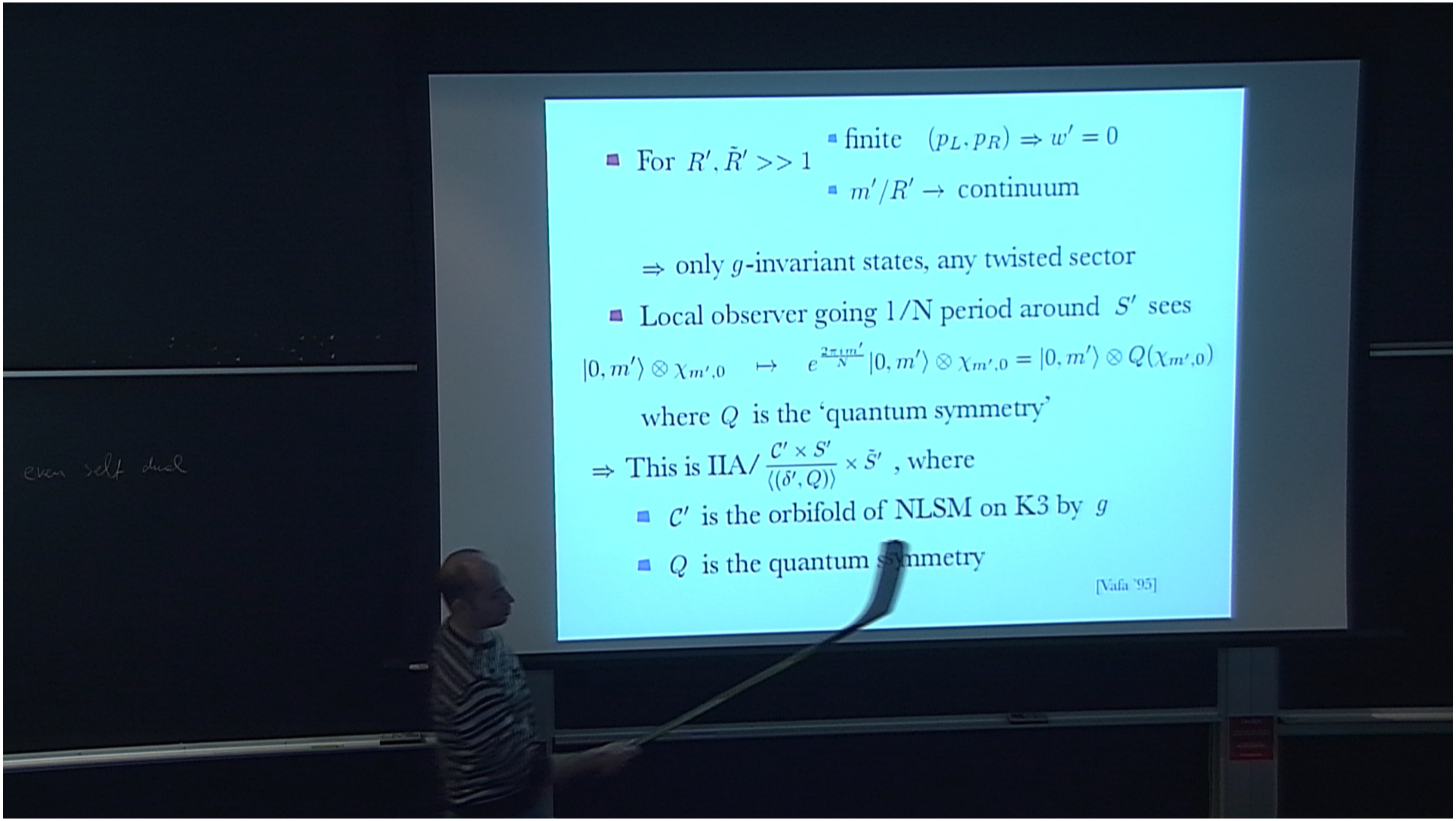
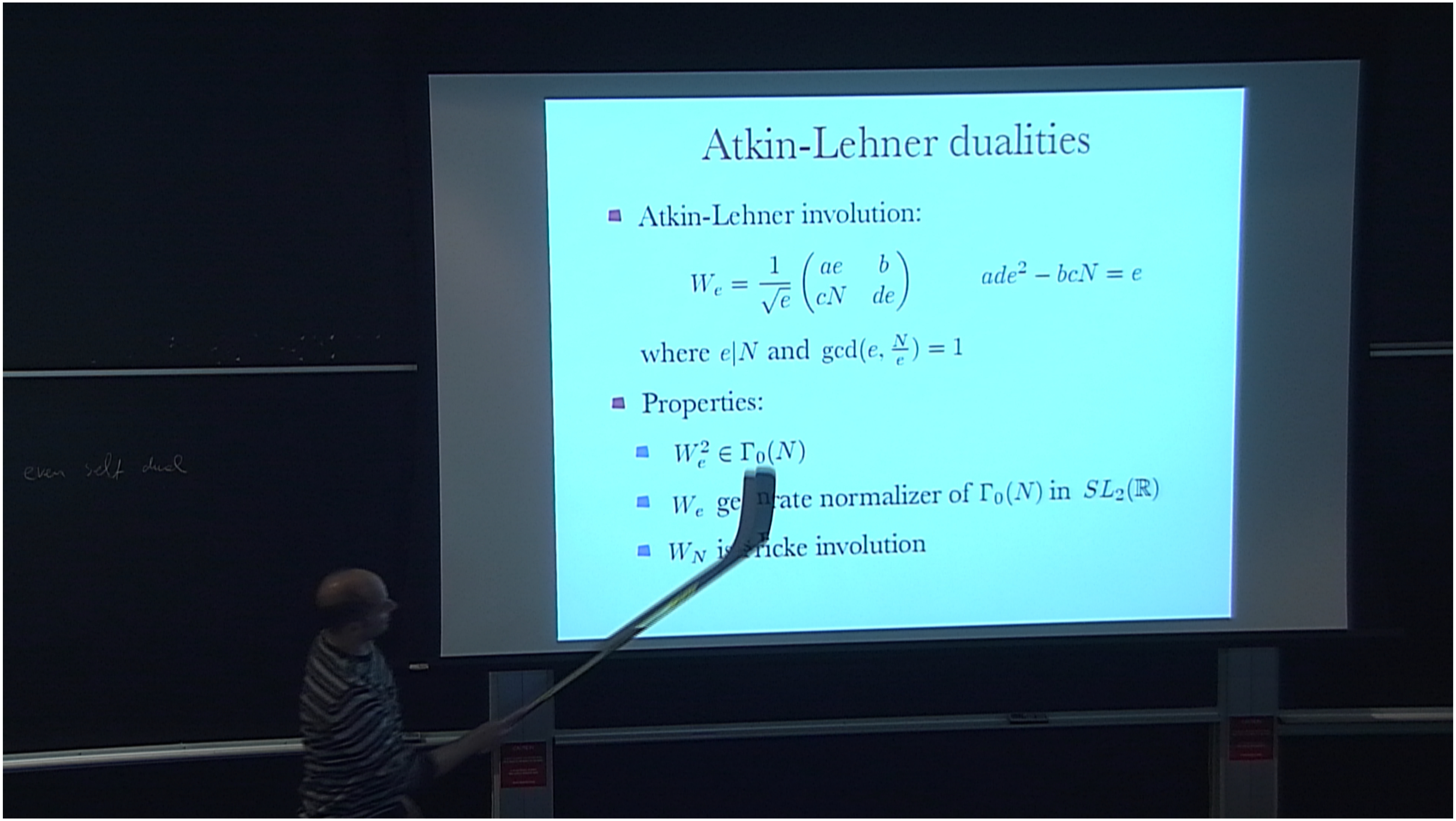
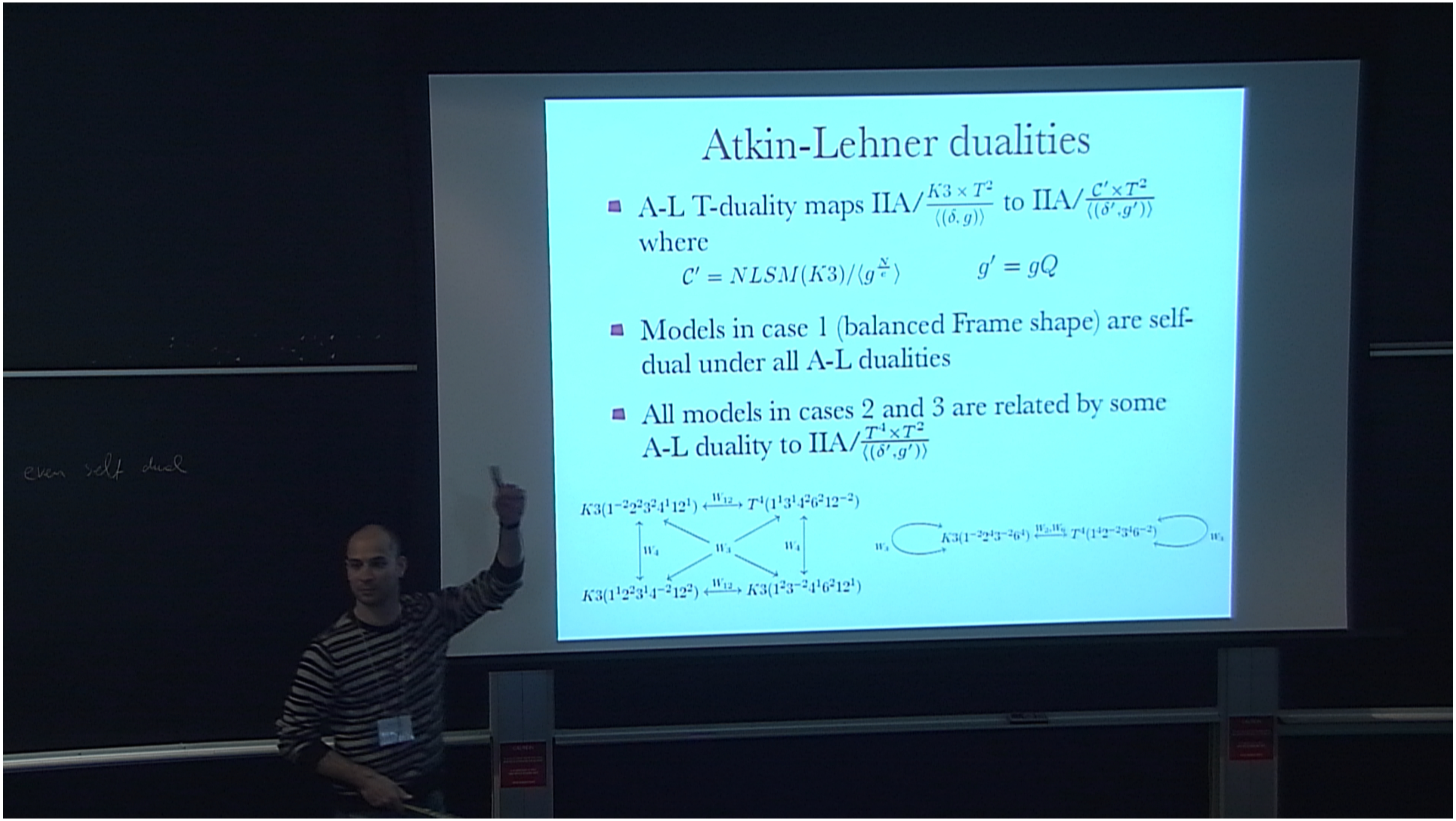

even self dual
States have the form $|w,m\rangle \otimes \chi_{w,m}$
where $\chi_{w,m}$ is in g^w-twisted sector and in
$g = e^{\frac{2\pi i m}{N}}$ eigenspace of internal K3 NLSM
For $R, \tilde{R} >> 1$
finite $(p_L, p_R) \Rightarrow w = 0$
$m/R \rightarrow$ continuum
$\Rightarrow$ only untwisted sector, any g-eigenvalue
Local observer going 1/N period around S sees
$|0,m\rangle \otimes \chi_{0,m} \mapsto e^{\frac{2\pi i m}{N}} |0,m\rangle \otimes \chi_{0,m} = |0,m\rangle \otimes g(\chi_{0,m})$

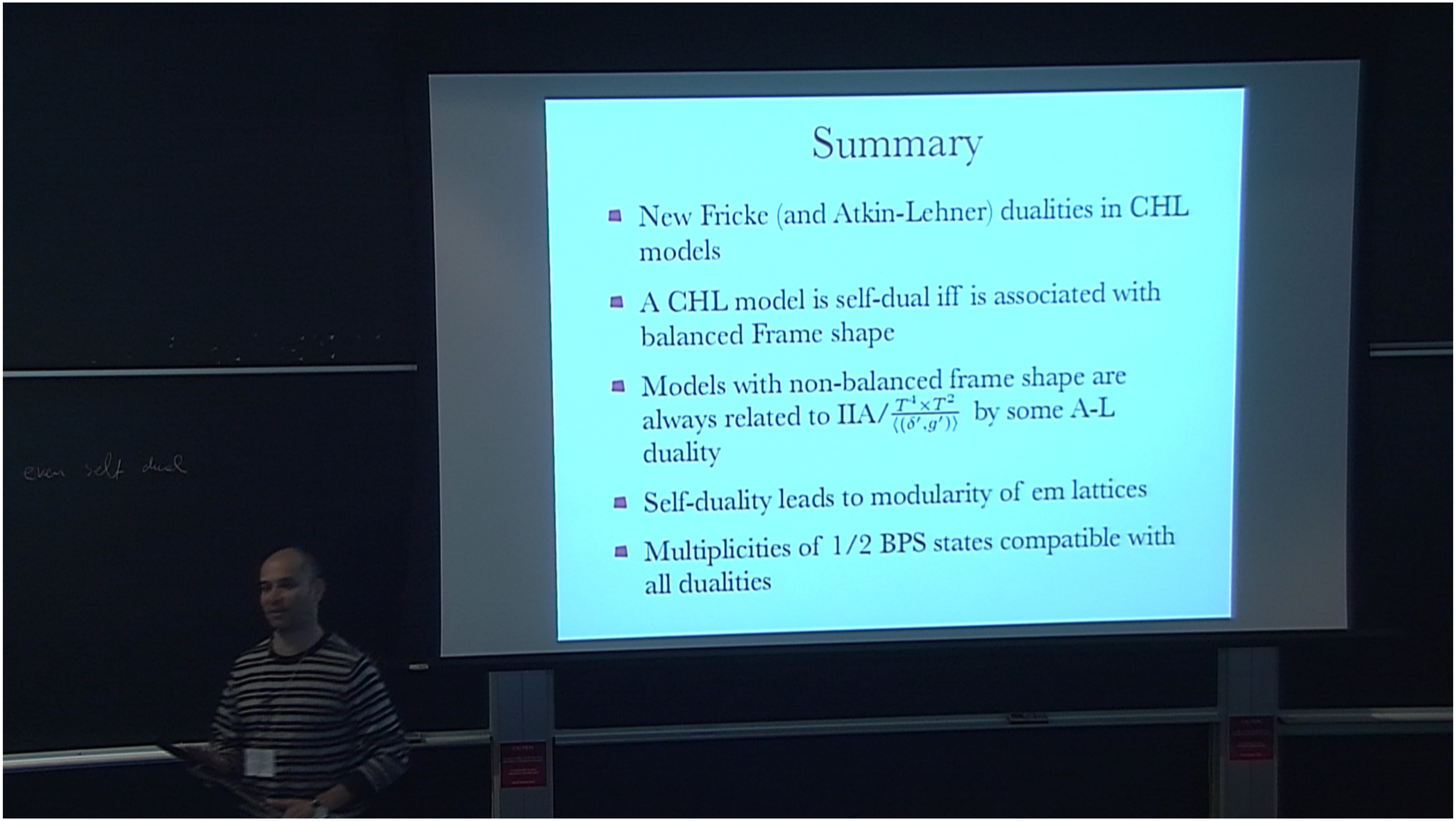

States have the form $|w, m\rangle \otimes \chi_{w,m}$
where $\chi_{w,m}$ is in g^w-twisted sector and in
$g = e^{\frac{2\pi i m}{N}}$ eigenspace of internal K3 NLSM
For $R, \tilde{R} >> 1$
finite $(p_L, p_R) \Rightarrow w = 0$
$m/R \to$ continuum
$\Rightarrow$ only untwisted sector, any g-eigenvalue
Local observer going $1/N$ period around S sees
$|0, m\rangle \otimes \chi_{0,m} \mapsto e^{\frac{2\pi i m}{N}} |0, m\rangle \otimes \chi_{0,m} = |0, m\rangle \otimes g(\chi_{0,m})$
Het/IIA duality in 6D $\Rightarrow$ duality of CHL
[Vafa, Witten '95]
even self dual

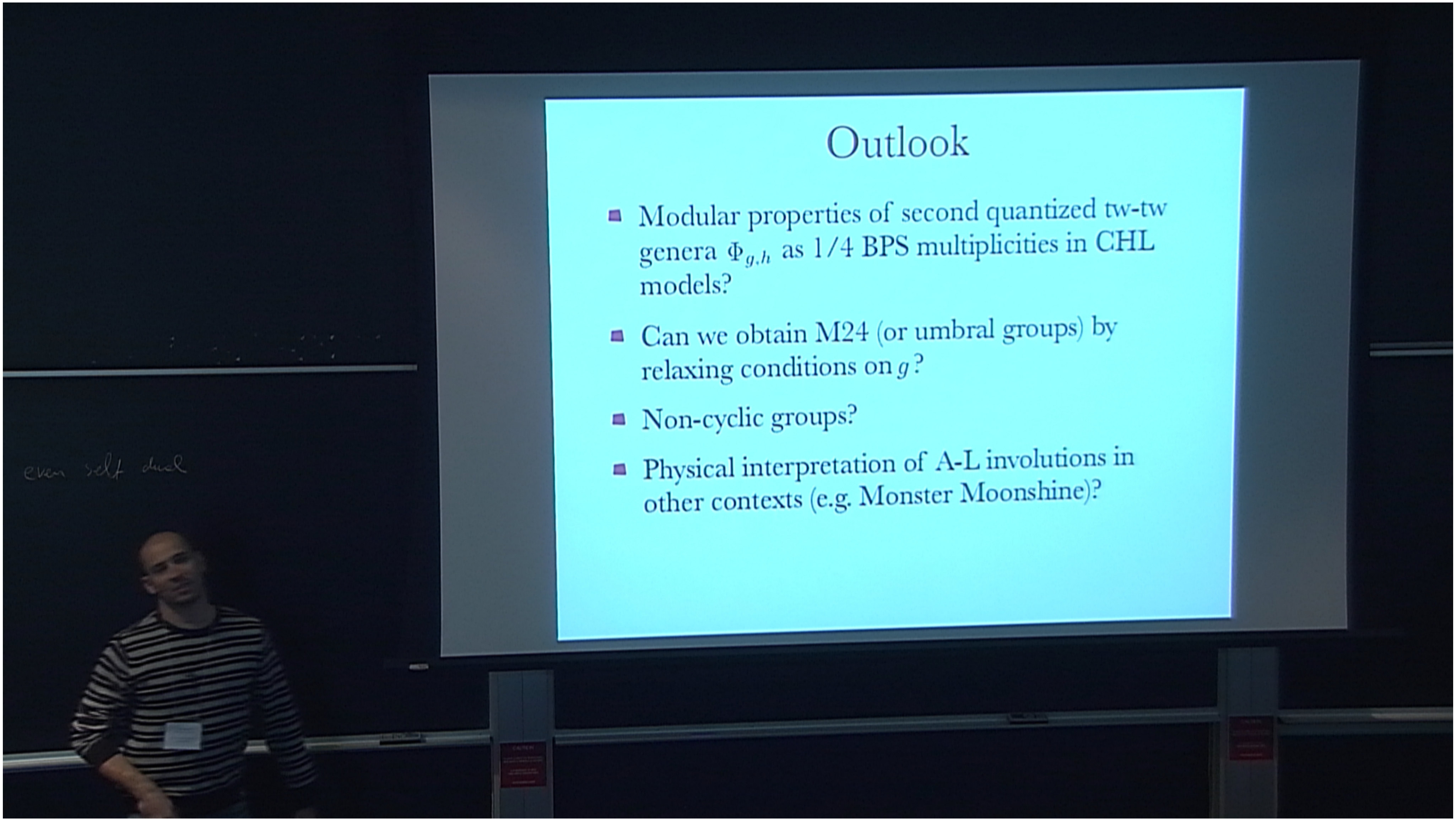

For $R', \tilde{R}' >> 1$
finite $(p_L, p_R) \Rightarrow w' = 0$
$m'/R' \rightarrow$ continuum
$\Rightarrow$ only g-invariant states, any twisted sector
Local observer going 1/N period around S' sees
$|0, m'\rangle \otimes \chi_{m',0} \mapsto e^{\frac{2\pi i m'}{N}} |0, m'\rangle \otimes \chi_{m',0} = |0, m'\rangle \otimes Q(\chi_{m',0})$
where Q is the 'quantum symmetry'
$\Rightarrow$ This is IIA/ $\frac{\mathcal{C}' \times S'}{\langle(\delta', Q)\rangle} \times \tilde{S}'$, where
$\mathcal{C}'$ is the orbifold of NLSM on K3 by g
Q is the quantum symmetry
[Vafa '95]
even self dual

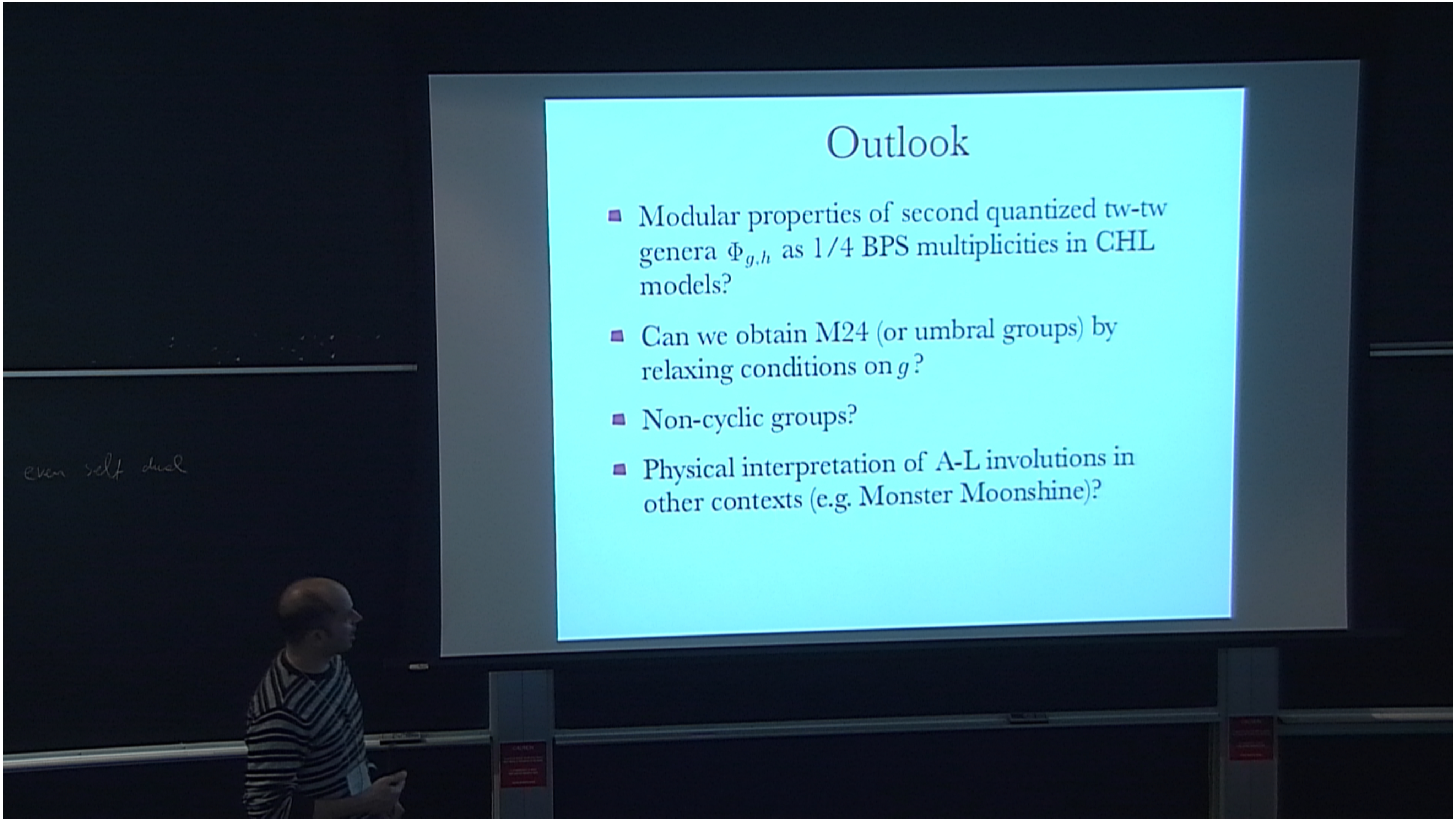


Atkin-Lehner dualities
Atkin-Lehner involution:
$W_e = \frac{1}{\sqrt{e}} \begin{pmatrix} ae & b \\ cN & de \end{pmatrix}$
$ade^2 - bcN = e$
where $e|N$ and $\gcd(e, \frac{N}{e}) = 1$
Properties:
$W_e^2 \in \Gamma_0(N)$
W_e ge rate normalizer of $\Gamma_0(N)$ in $SL_2(\mathbb{R})$
W_N i ricke involution
even self dual


Atkin-Lehner dualities
A-L T-duality maps IIA/$\frac{K3 \times T^2}{\langle(\delta, g)\rangle}$ to IIA/$\frac{\mathcal{C}' \times T^2}{\langle(\delta', g')\rangle}$ where
$\mathcal{C}' = NLSM(K3)/\langle g^{\frac{N}{e}} \rangle$ $g' = gQ$
Models in case 1 (balanced Frame shape) are self-dual under all A-L dualities
All models in cases 2 and 3 are related by some A-L duality to IIA/$\frac{T^4 \times T^2}{\langle(\delta', g')\rangle}$
$K3(1^{-2}2^2 3^2 4^1 12^1)$
W_{12}
$T^4(1^1 3^1 4^2 6^2 12^{-2})$
W_4
W_3
W_4
$K3(1^1 2^2 3^1 4^{-2} 12^2)$
W_{12}
$K3(1^2 3^{-2} 4^1 6^2 12^1)$
$K3(1^{-2}2^4 3^{-2}6^4)$
$T^4(1^4 2^{-2} 3^4 6^{-2})$
even self dual


Summary
New Fricke (and Atkin-Lehner) dualities in CHL models
A CHL model is self-dual iff is associated with balanced Frame shape
Models with non-balanced frame shape are always related to IIA/$\frac{T^4\times T^2}{\langle(\delta',g')\rangle}$ by some A-L duality
Self-duality leads to modularity of em lattices
Multiplicities of 1/2 BPS states compatible with all dualities
even self dual


Outlook
Modular properties of second quantized tw-tw genera $\Phi_{g,h}$ as 1/4 BPS multiplicities in CHL models?
Can we obtain M24 (or umbral groups) by relaxing conditions on g?
Non-cyclic groups?
Physical interpretation of A-L involutions in other contexts (e.g. Monster Moonshine)?
even self dual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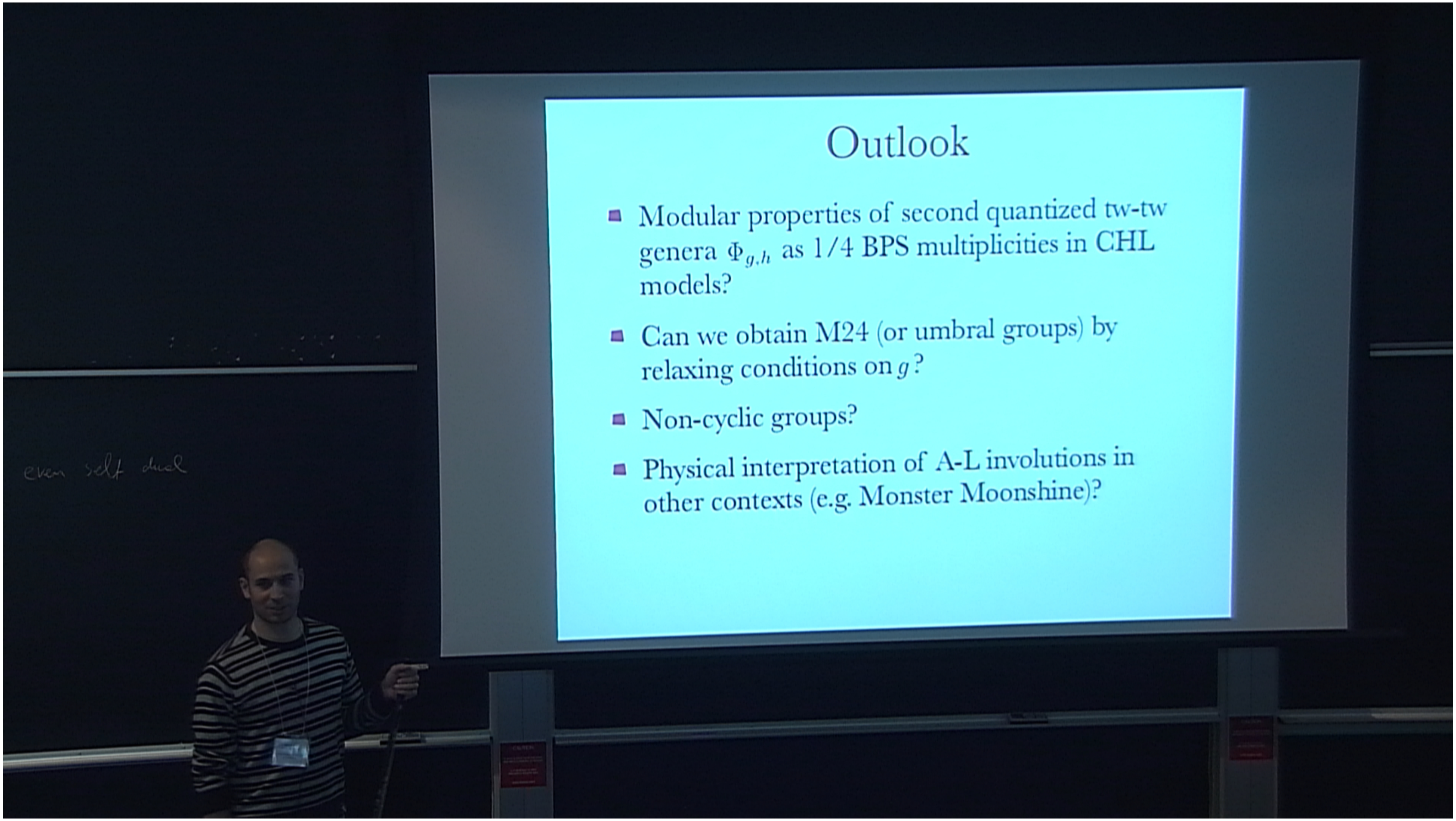
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