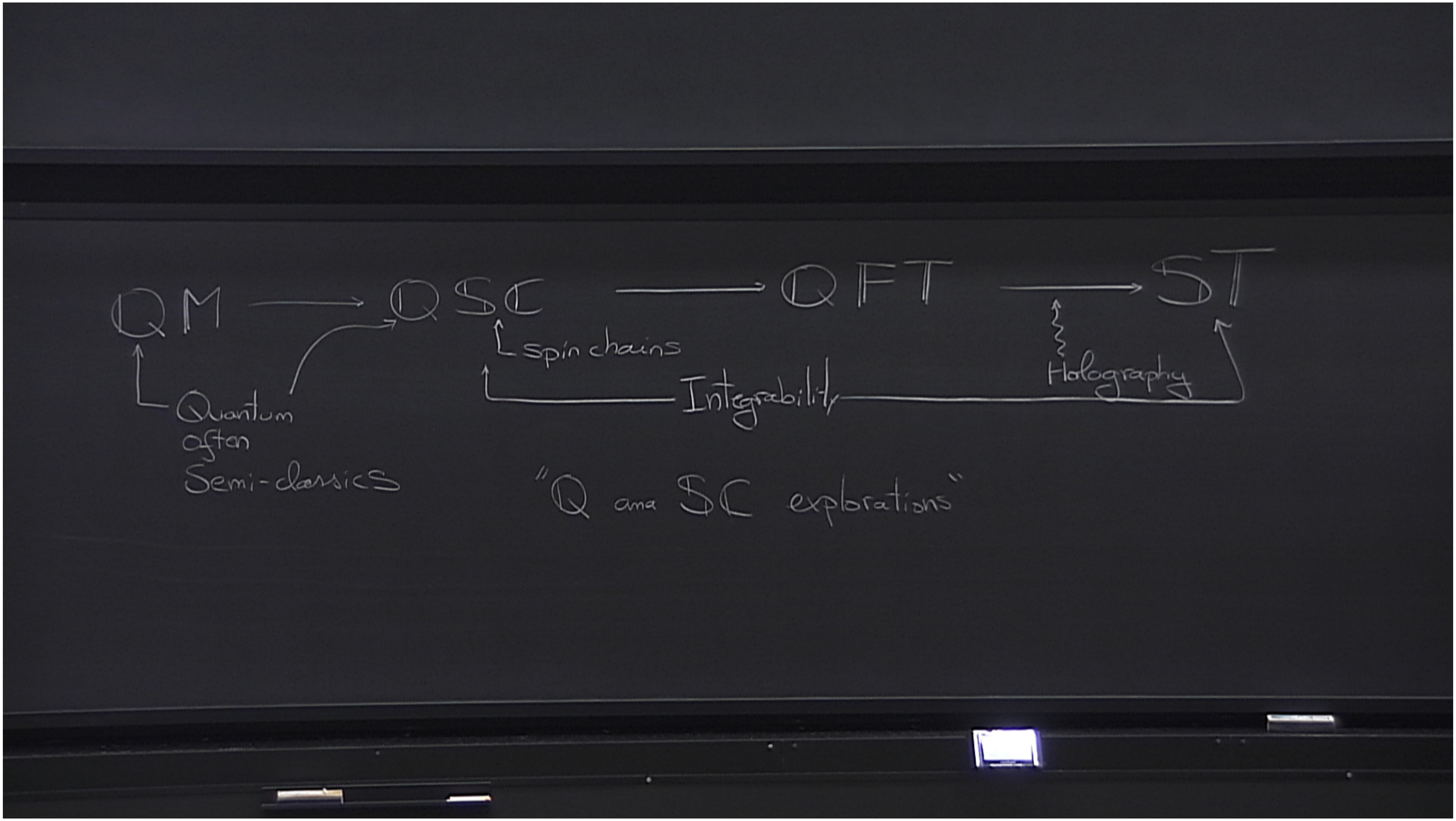
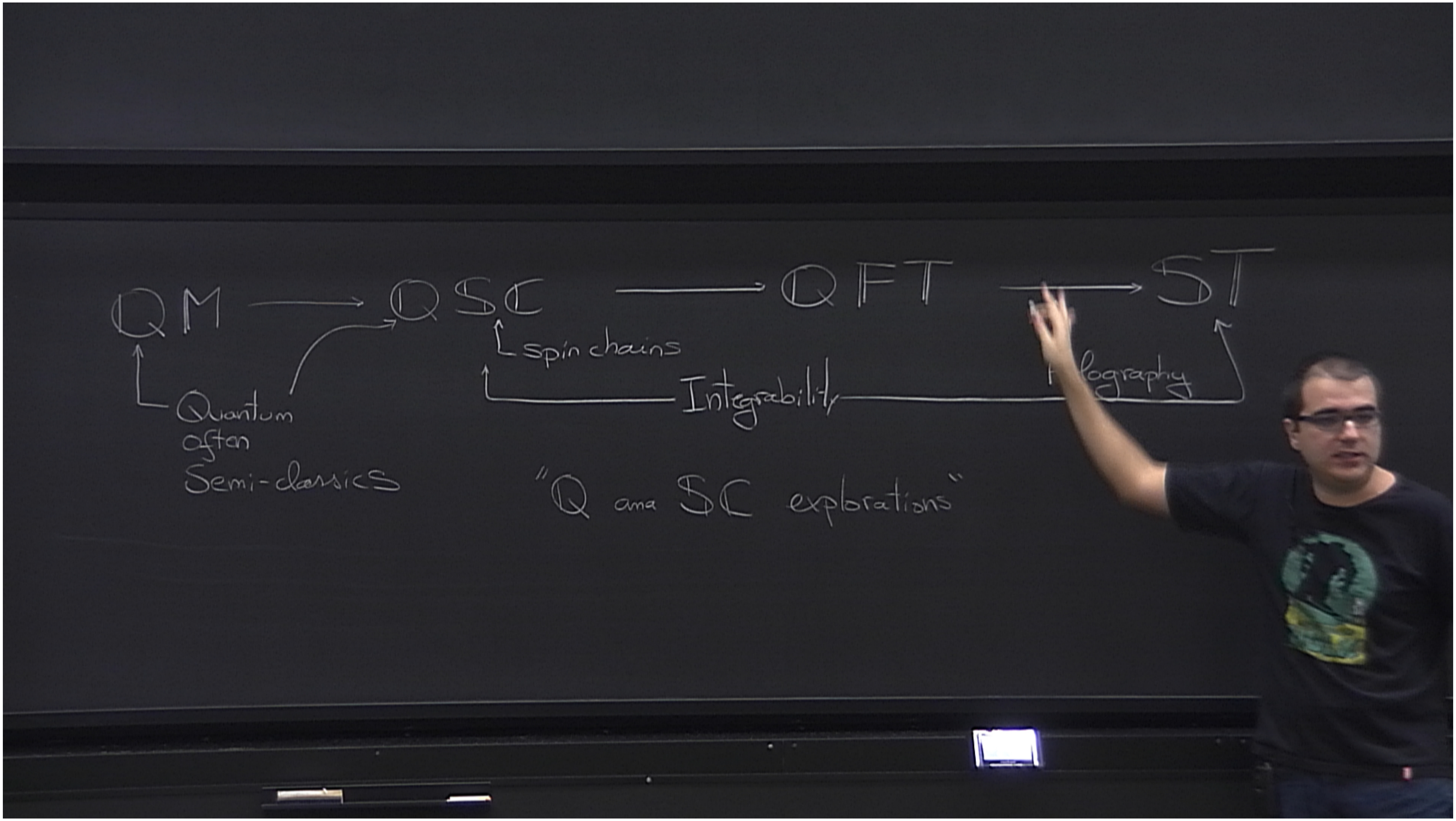
Title: Explorations in String Theory -1

Date: Apr 06, 2015 11:30 AM

URL: http://pirsa.org/15040117

Abstract:


QM
QSC
QFT
ST
spin chains
Integrability
Holography
Quantum
after
Semi-classics
"Q and SC explorations"

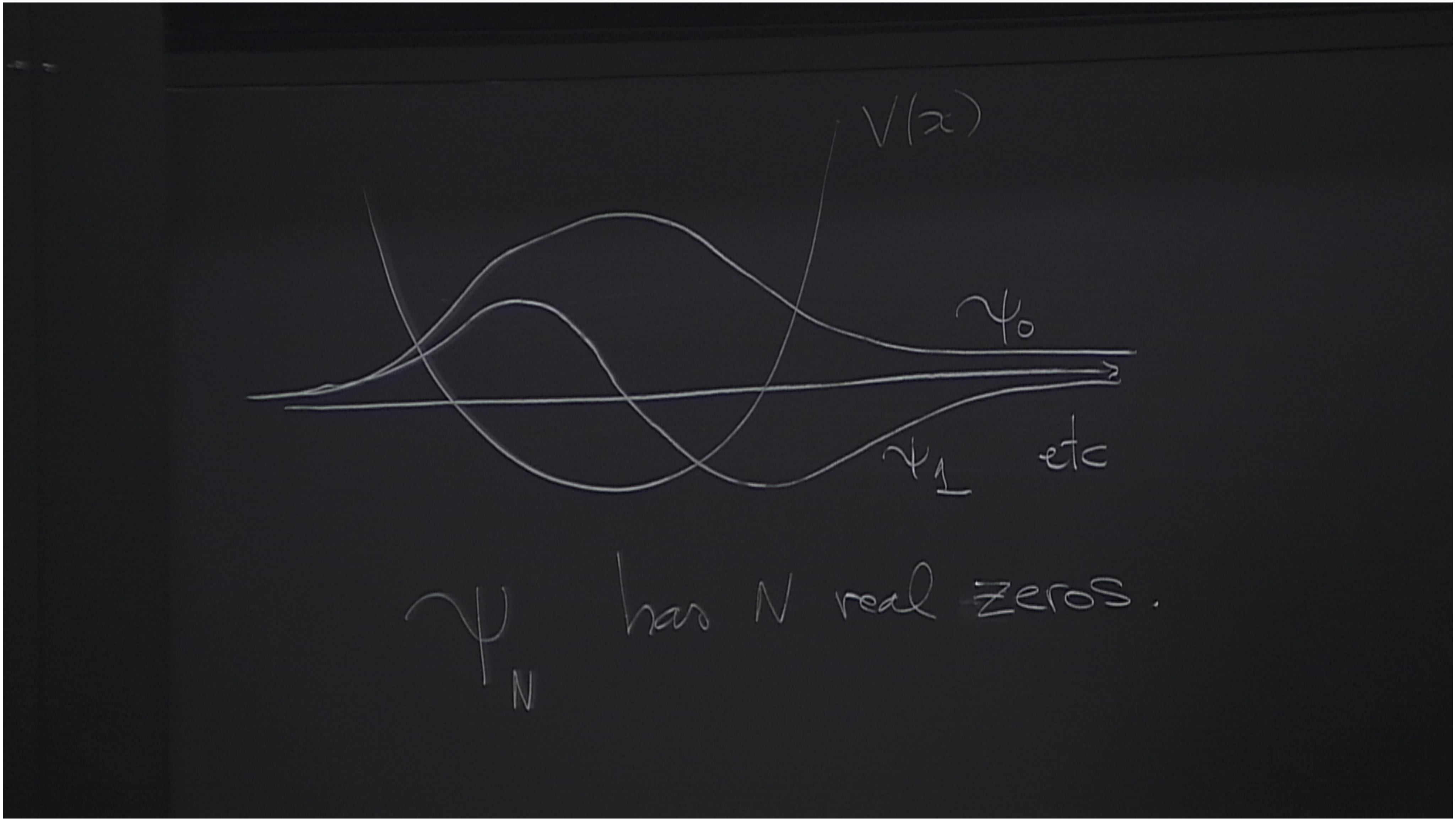

QM
QSC
QFT
ST
spin chains
Integrability
Quantum
often
Semi-classics
"Q and SC explorations"

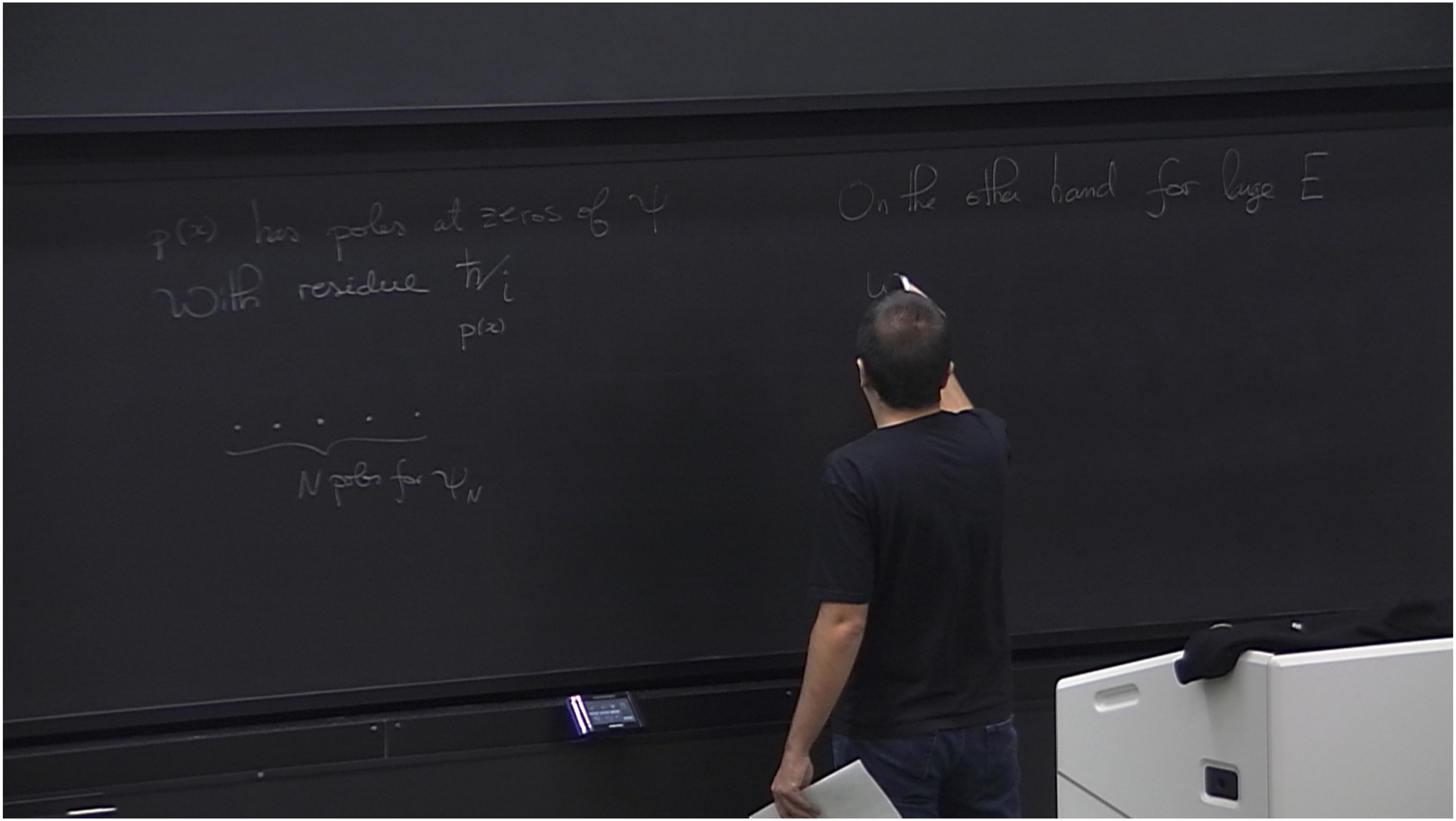
1.1) non-relativistic particle in 1D.

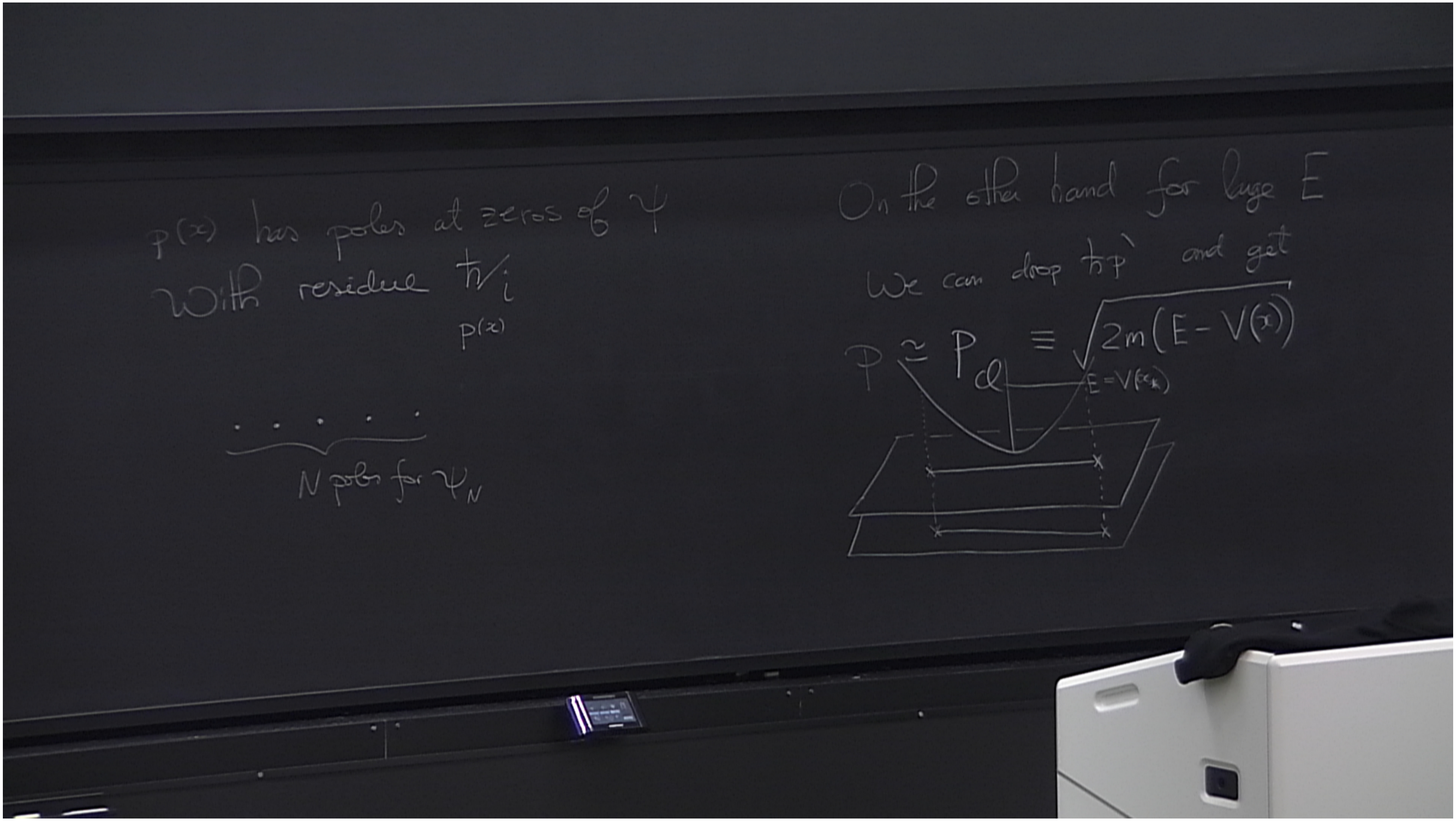
$$p(x) \equiv \frac{\hbar}{i} \frac{\psi'(x)}{\psi(x)}$$ quasi-momentum

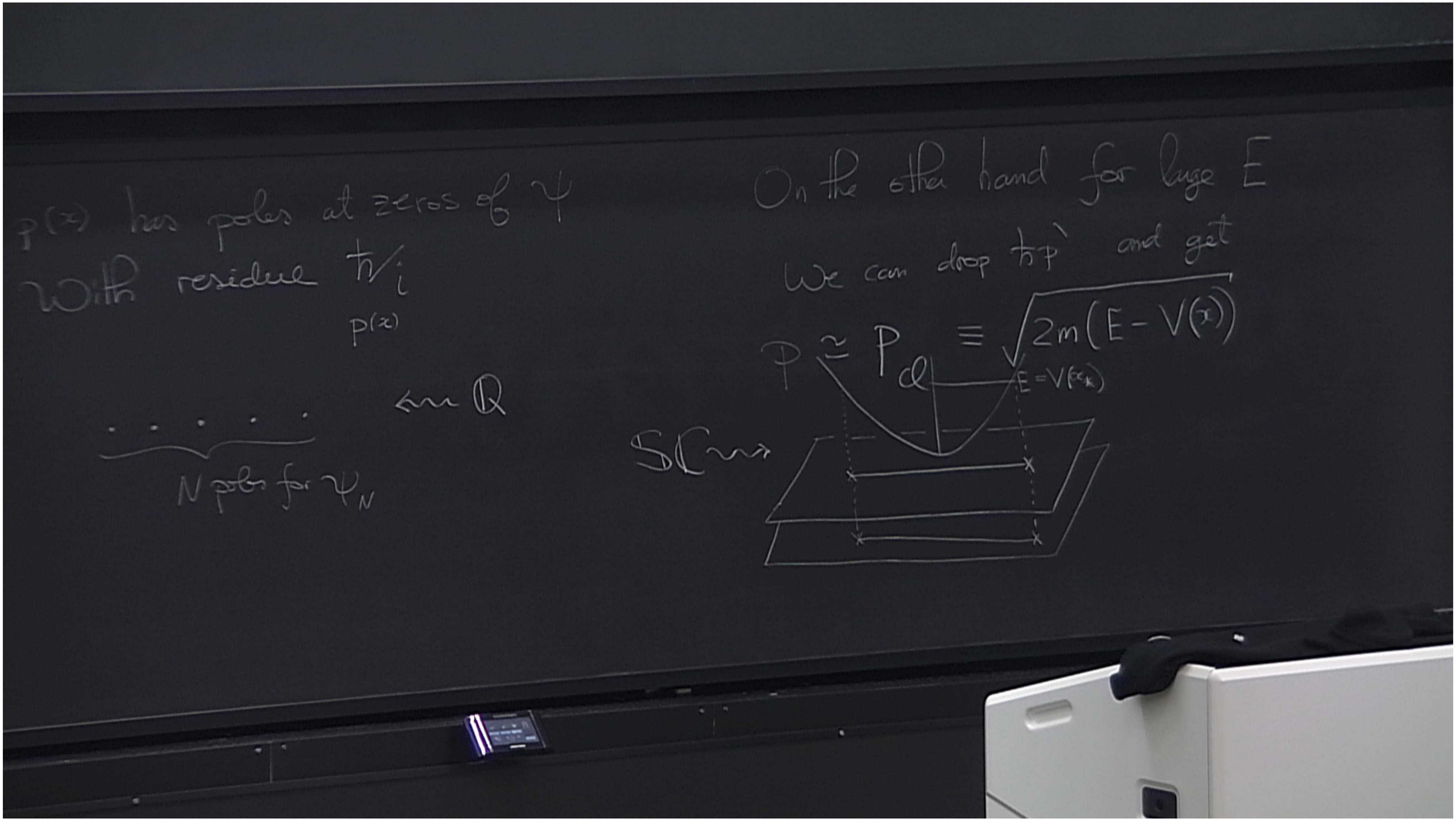
Schrodinger eq

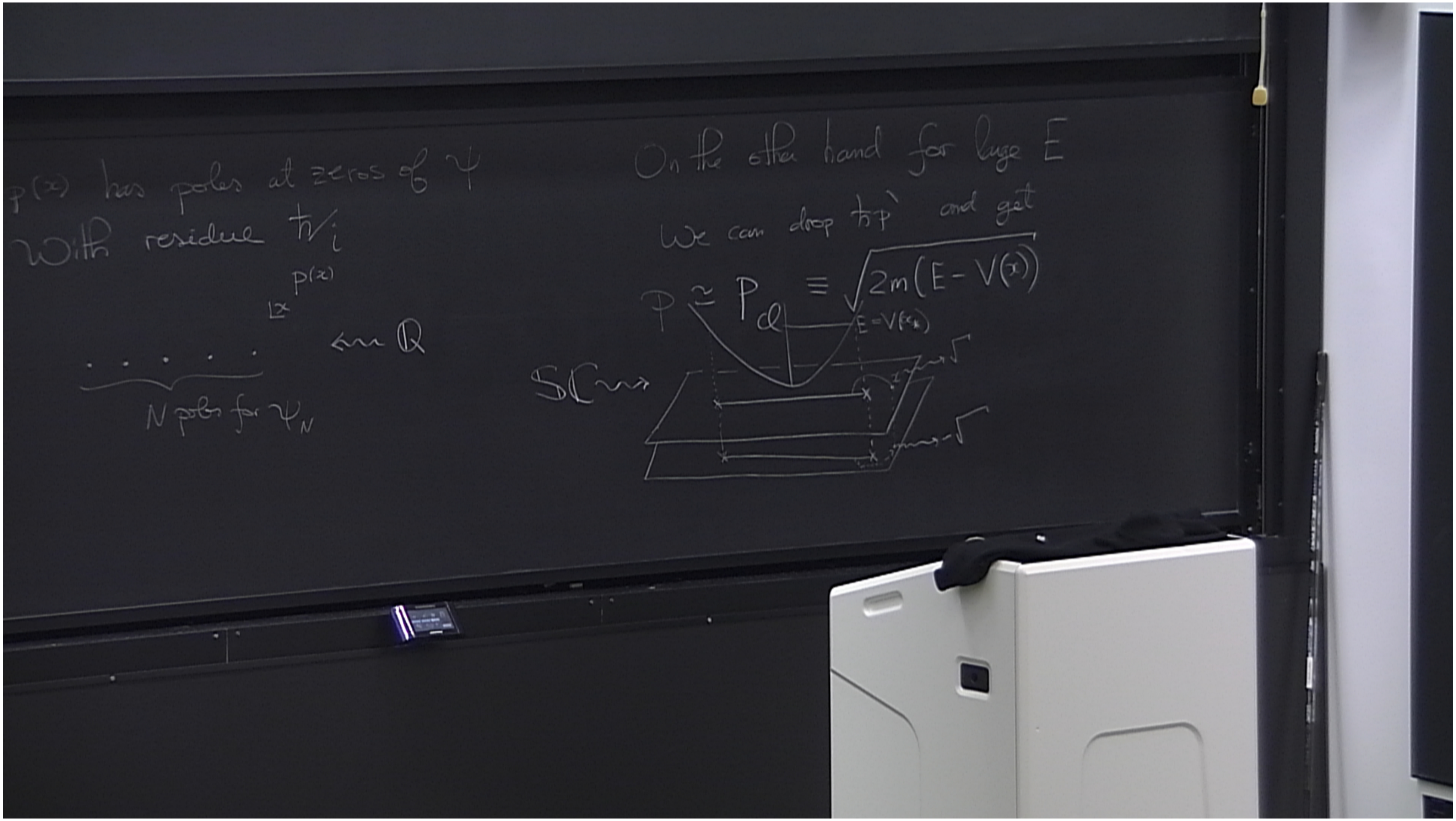
$\Rightarrow$ Riatti $p^2 - i\hbar p'(x) = 2m(E - V(x))$

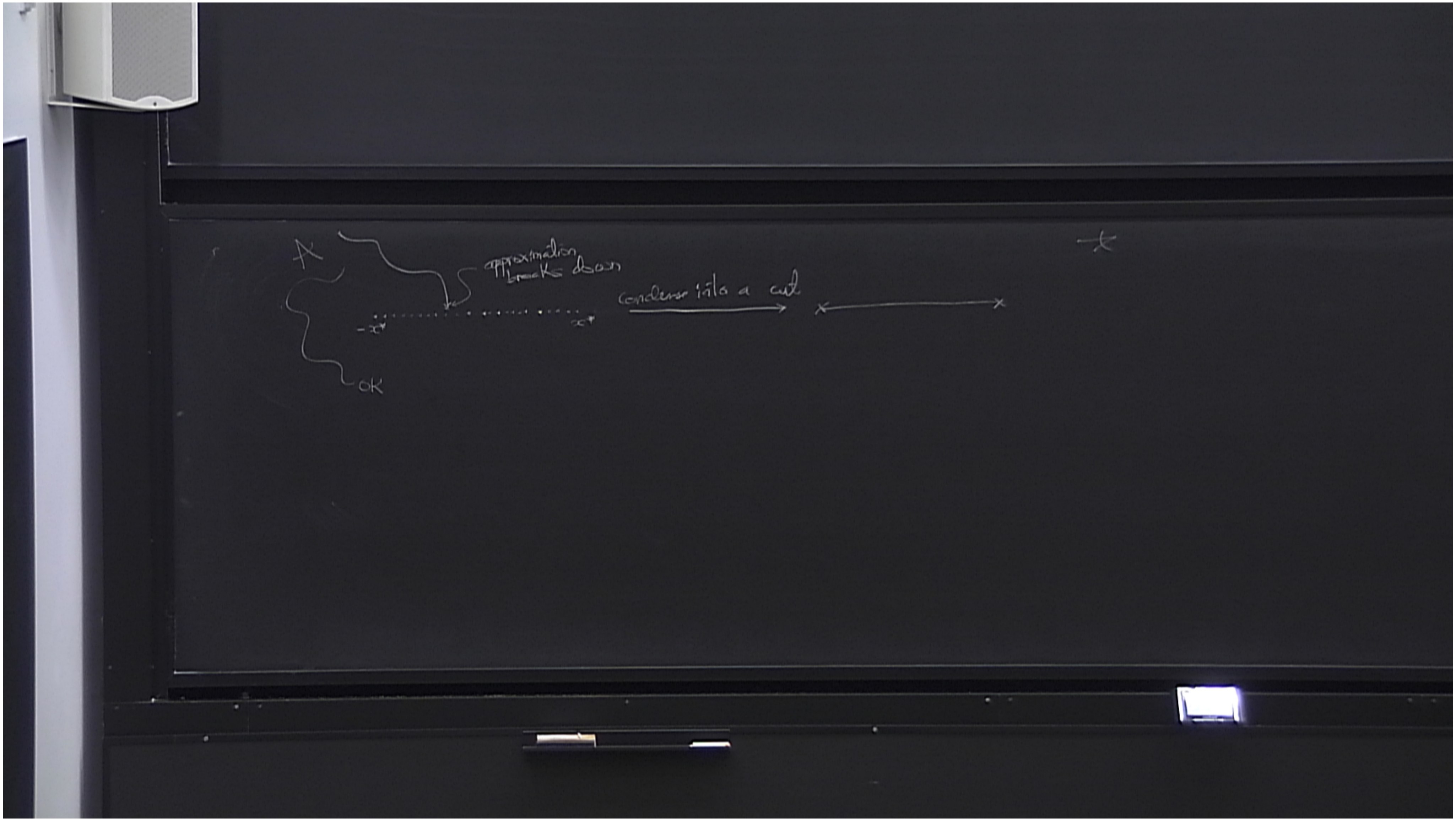

V(x)
ψ0
ψ1
etc
ψN has N real zeros.

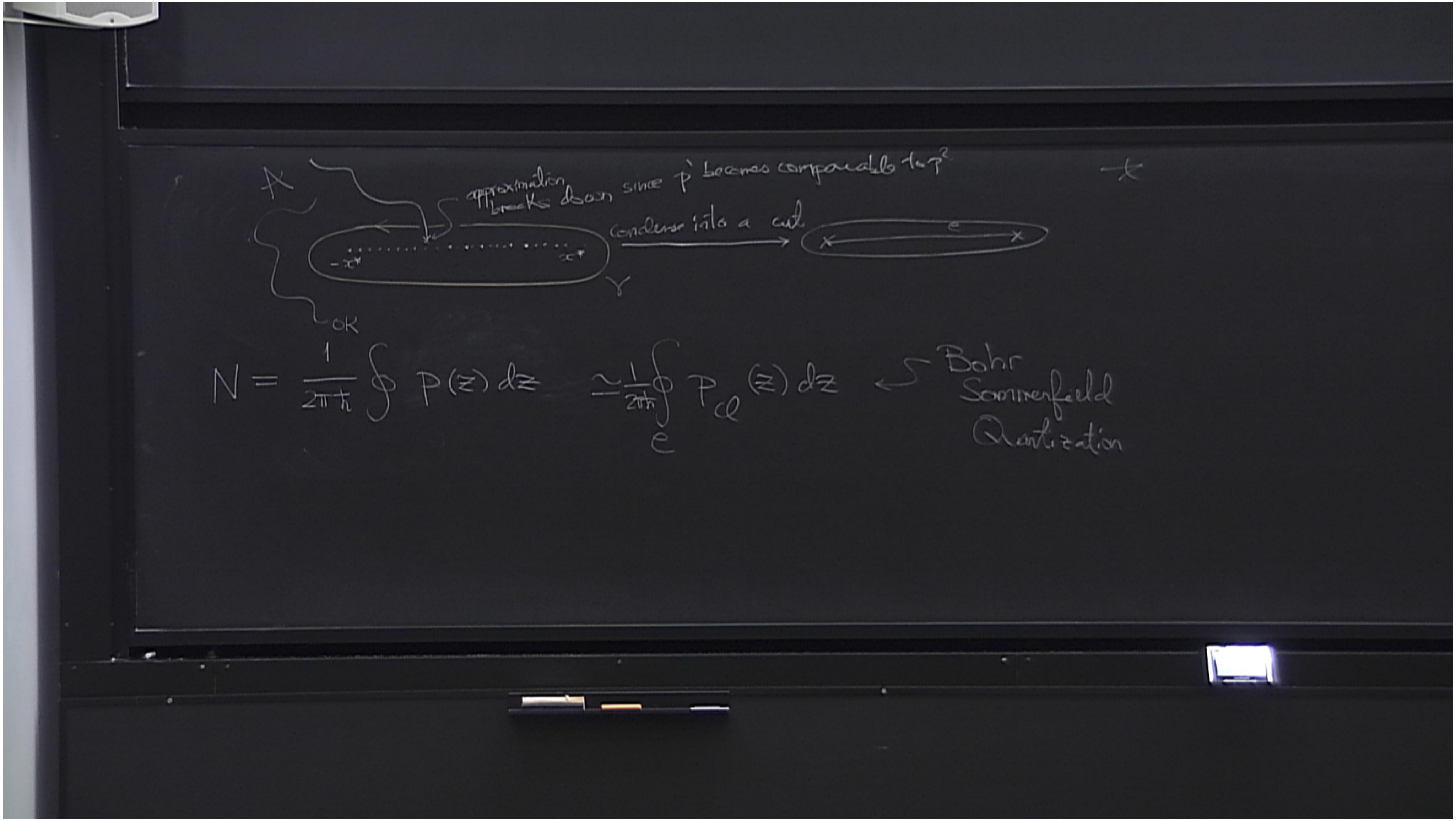

p(x) has poles at zeros of ψ
With residue ħ/i
p(x)
N poles for ψ_N
On the other hand for large E


p(x) has poles at zeros of ψ
with residue ħ/i p(x)
N poles for ψ_N
On the other hand for large E
We can drop ħp' and get
p ≃ p_cl ≡ √(2m(E − V(x)))
E = V(x_*)

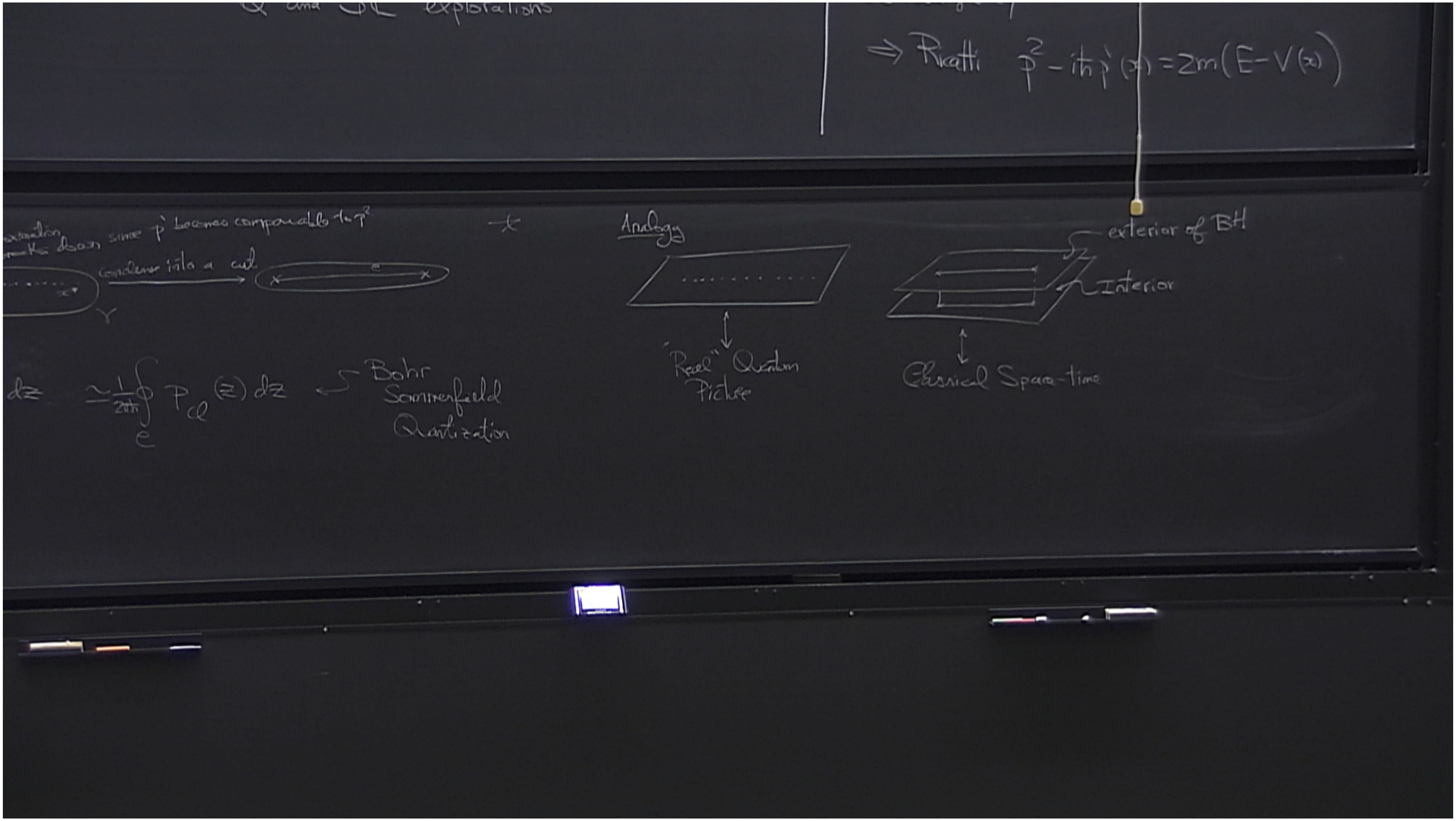

p(x) has poles at zeros of ψ
With residue ħ/i
p(x)
Q
N poles for ψ_N
On the other hand for large E
We can drop ħp' and get
p ≃ p_cl ≡ √(2m(E − V(x)))
E = V(x_*)

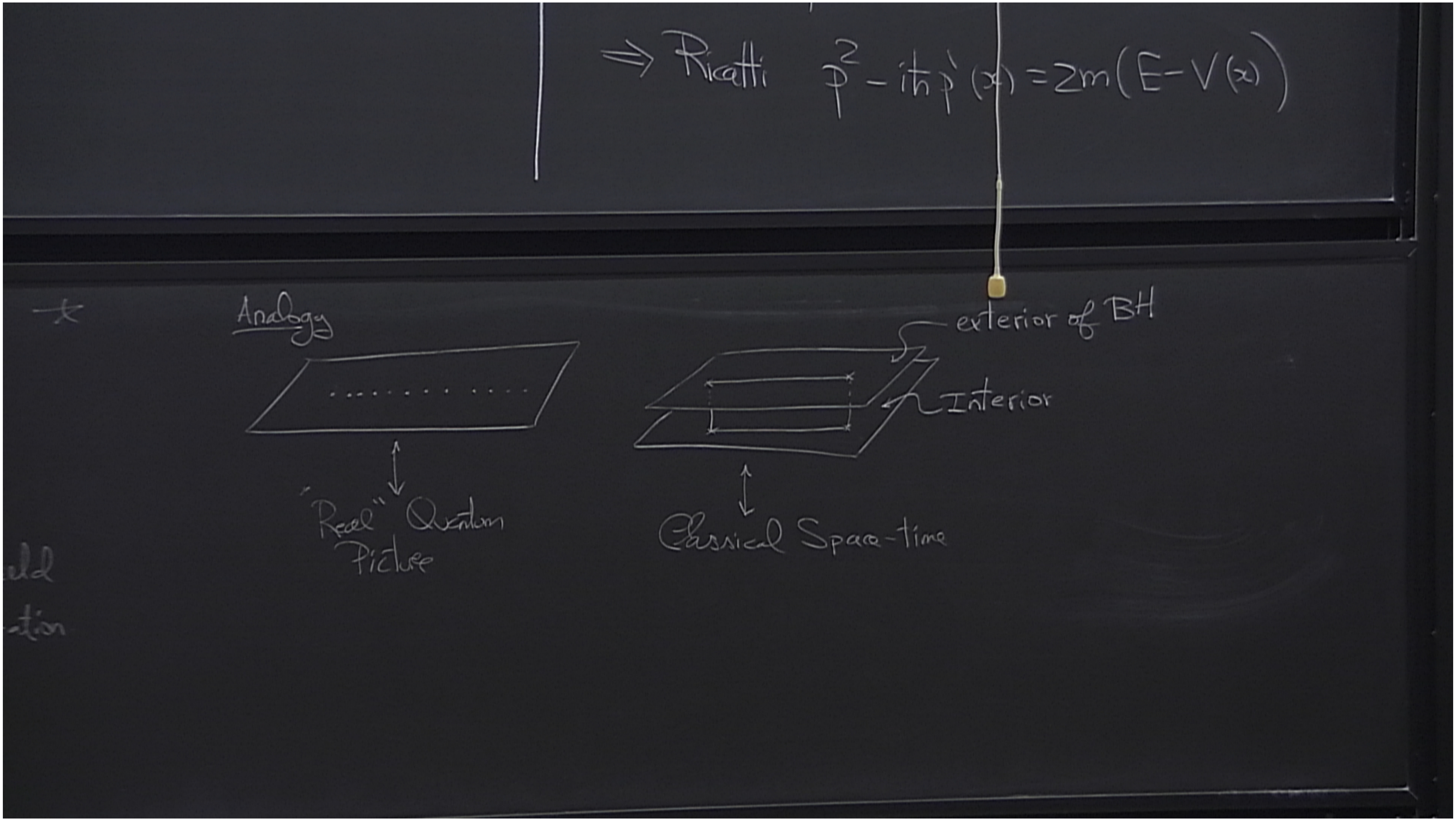

p(x) has poles at zeros of ψ
With residue ħ/i
p(x)
N poles for ψ_N
On the other hand for large E
We can drop ħp' and get
p ≃ p_cl ≡ √2m(E − V(x))
E = V(x*)
SC

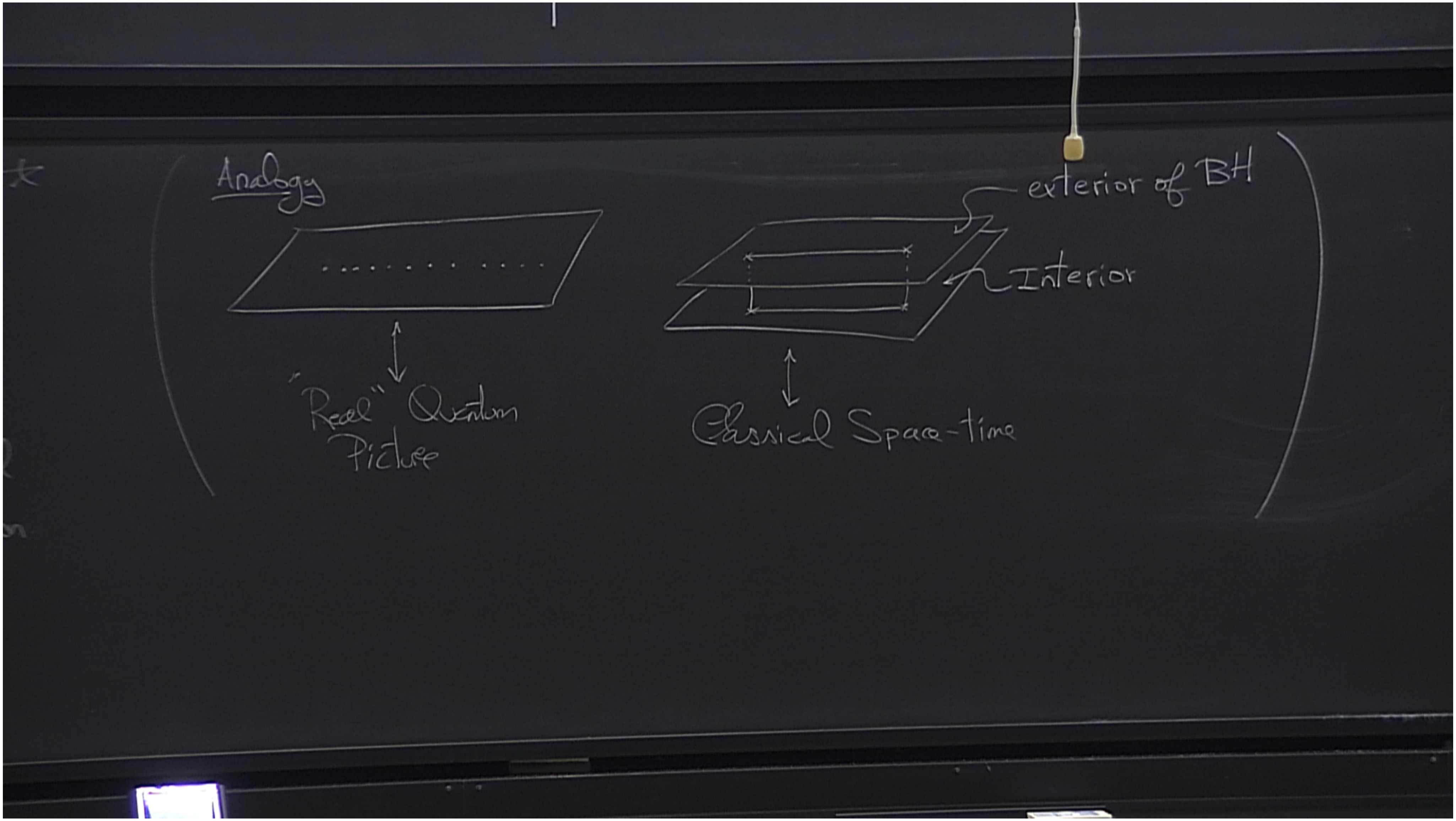

approximation
breaks down
condense into a cut
OK



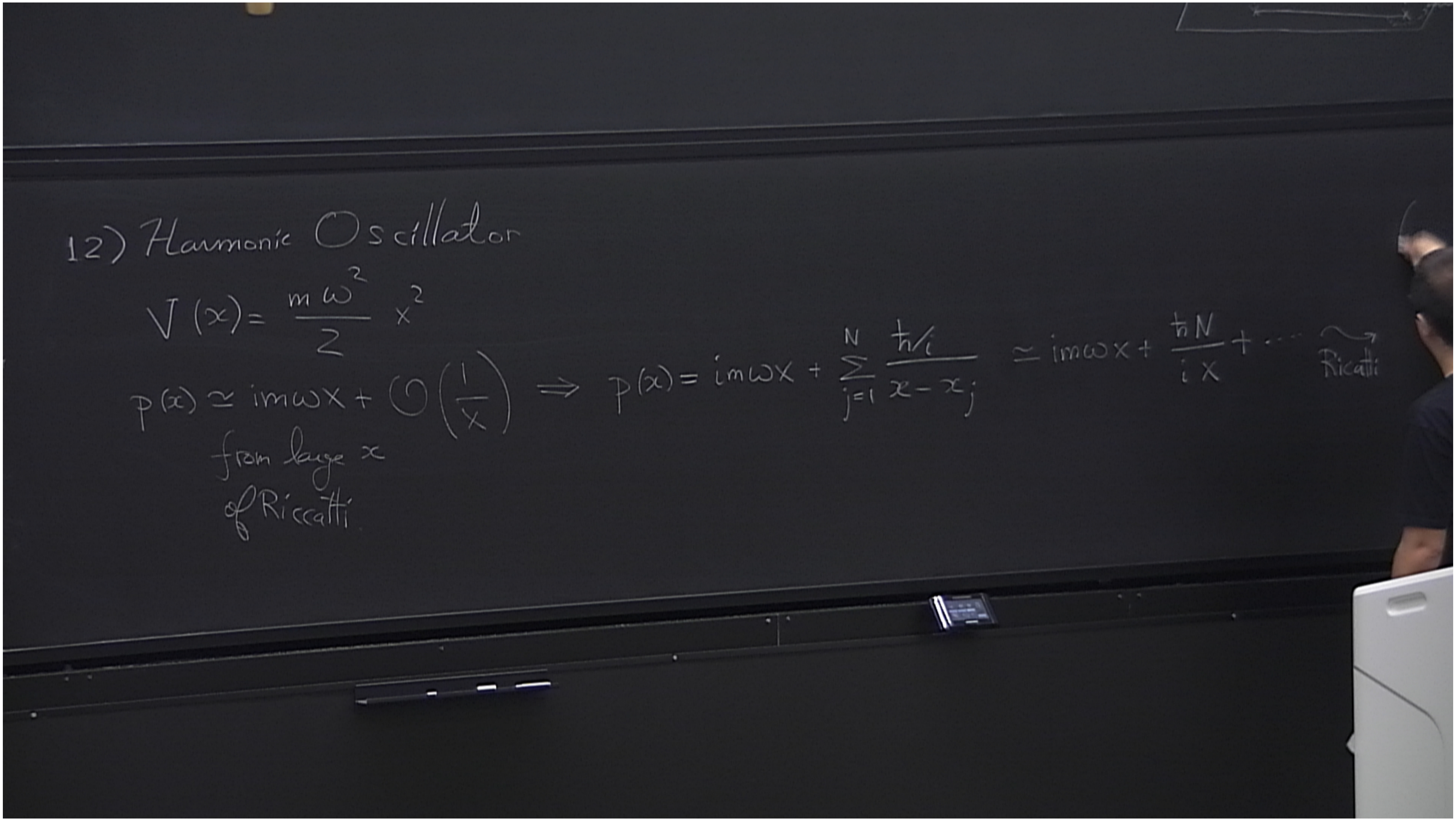
approximation breaks down since p' becomes comparable to p²
condense into a cut
OK
N = 1/2πħ ∮ p(z) dz ≃ 1/2πħ ∮_C p_cl(z) dz
Bohr Sommerfeld Quantization

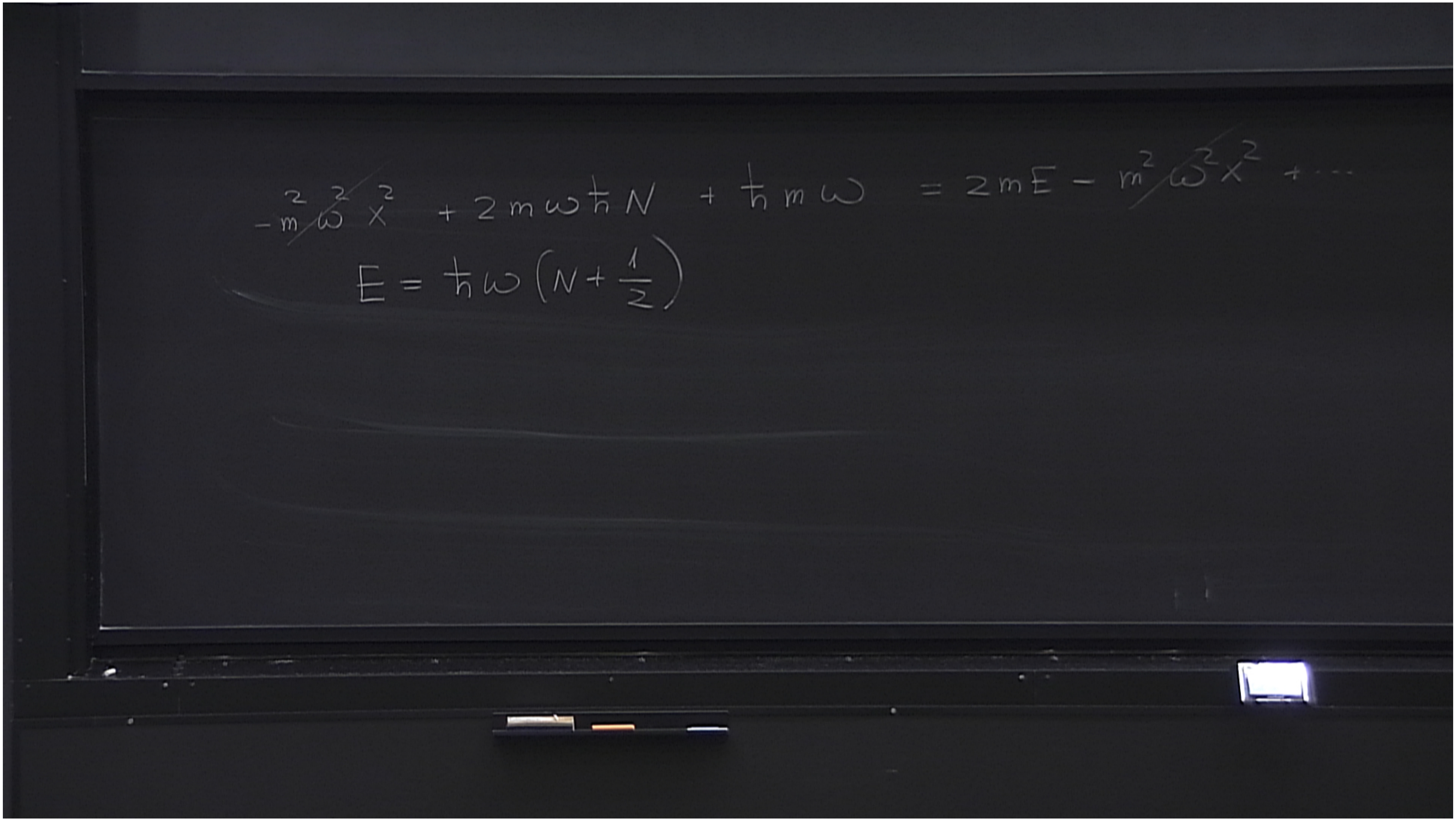

⇒ Ricatti $p^2 - i\hbar p'(x) = 2m(E - V(x))$
since p' becomes comparable to p^2
condense into a cut
Analogy
exterior of BH
Interior
"Real" Quantum Picture
Classical Space-time
$\simeq \frac{1}{2\pi\hbar} \oint_C p_{cl}(z)\, dz$
Bohr Sommerfeld Quantization

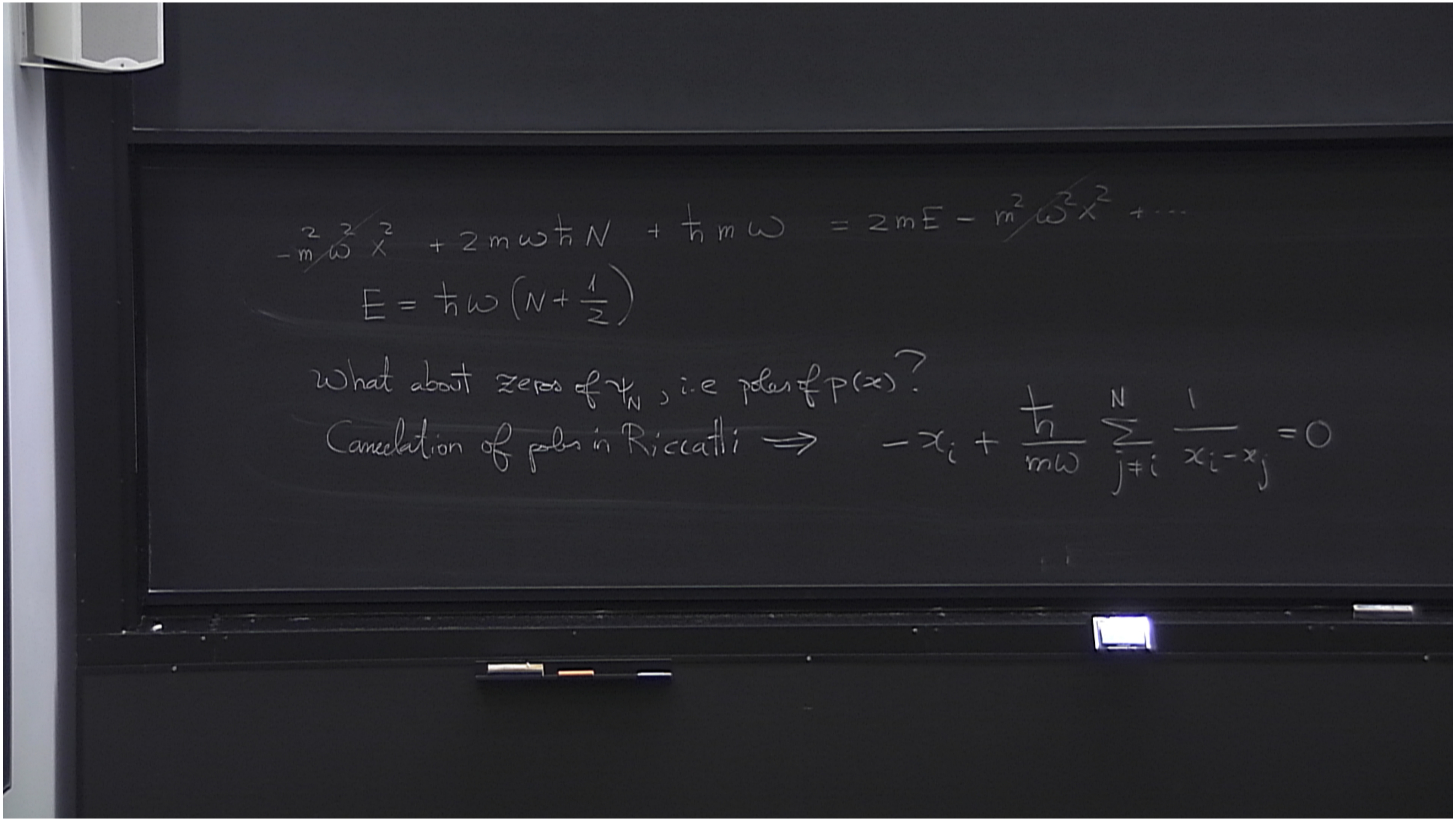

⇒ Ricatti $p^2 - i\hbar p'(x) = 2m(E - V(x))$
Analogy
exterior of BH
Interior
"Real" Quantum Picture
Classical Space-time

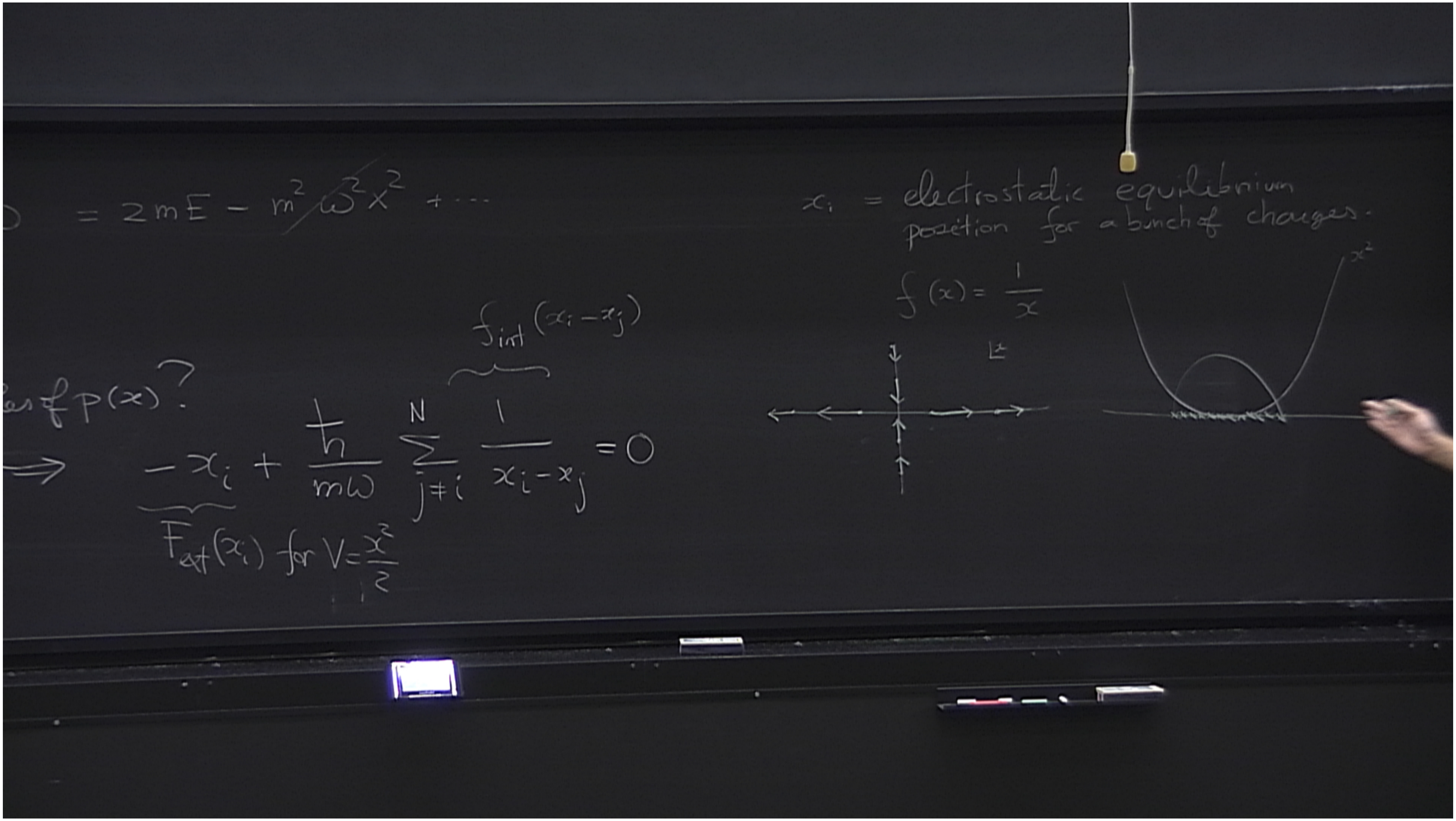


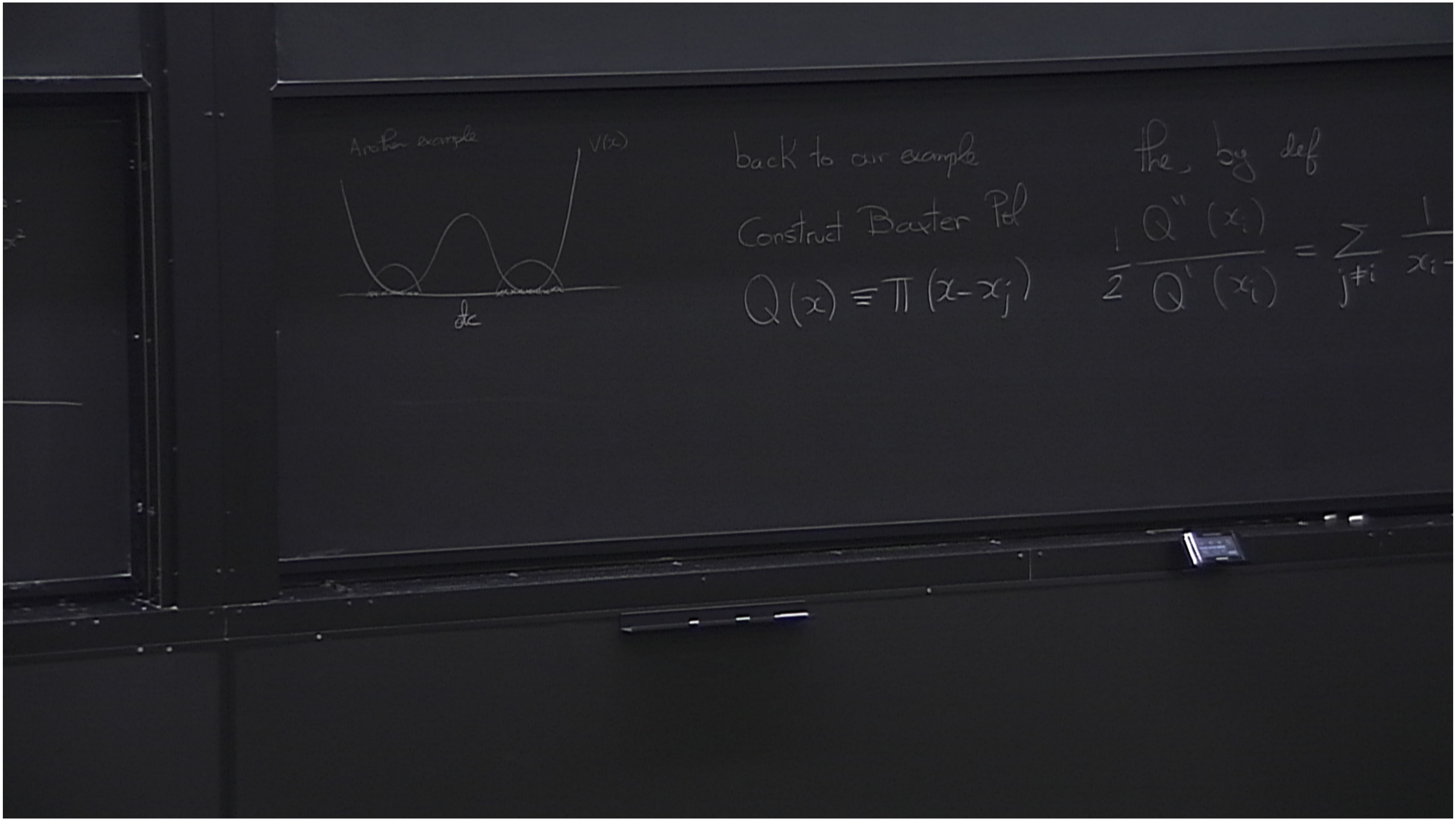
Analogy
exterior of BH
Interior
"Real" Quantum
Picture
Classical Space-time

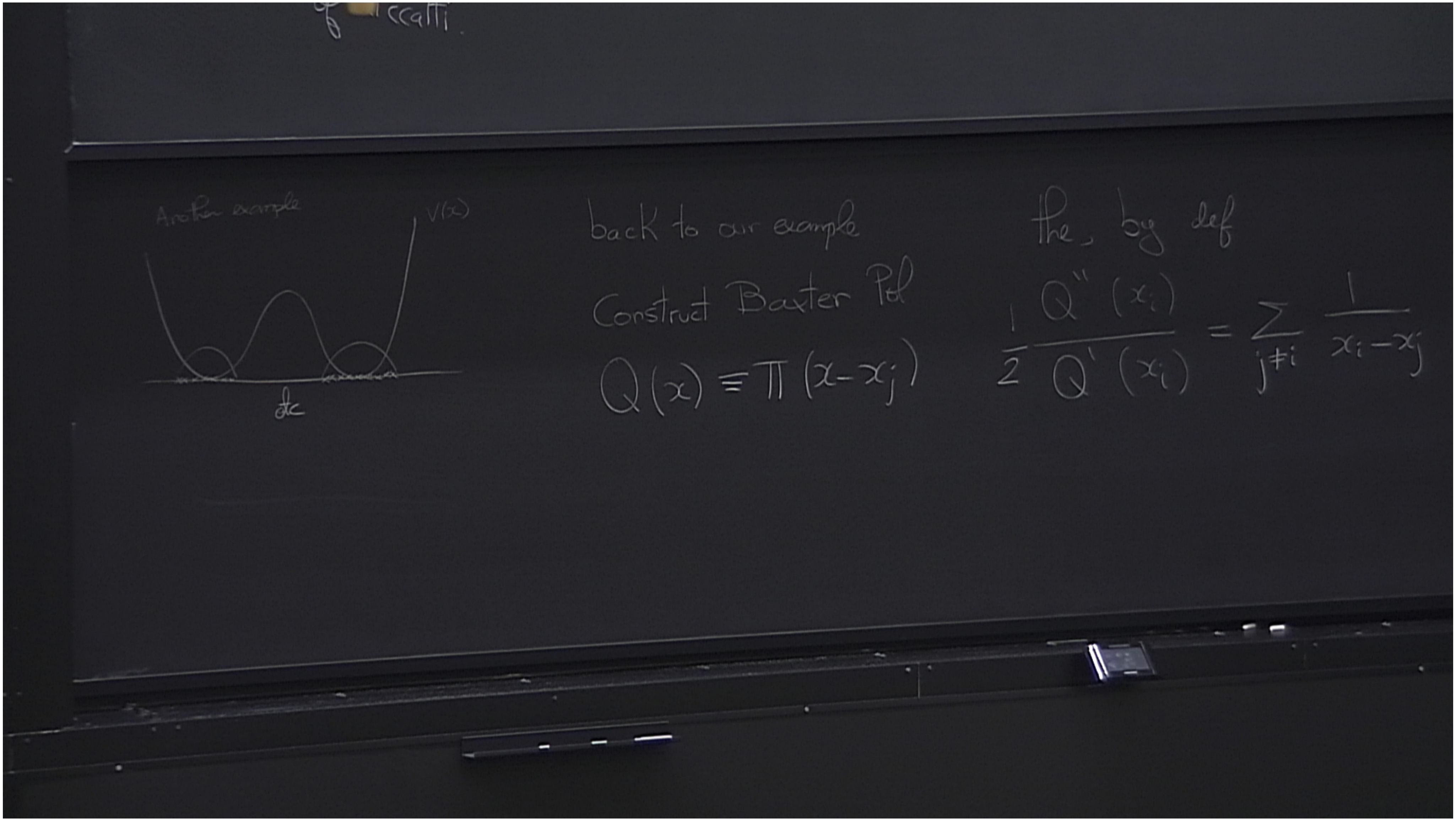

12) Harmonic Oscillator
$V(x) = \frac{m\omega^2}{2} x^2$
$p(x) \simeq im\omega x + O\left(\frac{1}{x}\right) \Rightarrow p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar/i}{x - x_j} \simeq im\omega x + \frac{\hbar N}{i x} + \cdots$
Ricatti
from large x
of Riccatti


$-m^2\omega^2x^2 + 2m\omega\hbar N + \hbar m\omega = 2mE - m^2\omega^2x^2 + \cdots$
$E = \hbar\omega\left(N + \frac{1}{2}\right)$


$-m^2\omega^2x^2 + 2m\omega\hbar N + \hbar m\omega = 2mE - m^2\omega^2x^2 + \cdots$
$E = \hbar\omega\left(N + \frac{1}{2}\right)$
What about zeros of ψ_N, i.e poles of $p(x)$?
Cancellation of poles in Riccati $\Rightarrow$ $-x_i + \frac{\hbar}{m\omega}\sum_{j\neq i}^{N}\frac{1}{x_i - x_j} = 0$


= 2mE − m²ω²x² + · · ·
x_i = electrostatic equilibrium position for a bunch of charges.
f(x) = 1/x
x²
f_int(x_i − x_j)
p(x)?
⟹ −x_i + ℏ/mω Σ_{j≠i}^N 1/(x_i − x_j) = 0
F_ext(x_i) for V = x²/2


Another example
V(x)
back to our example
Construct Baxter Pol
$Q(x) \equiv \prod (x - x_j)$
then, by def
$\frac{1}{2} \frac{Q''(x_i)}{Q'(x_i)} = \sum_{j \neq i} \frac{1}{x_i -}$


Another example
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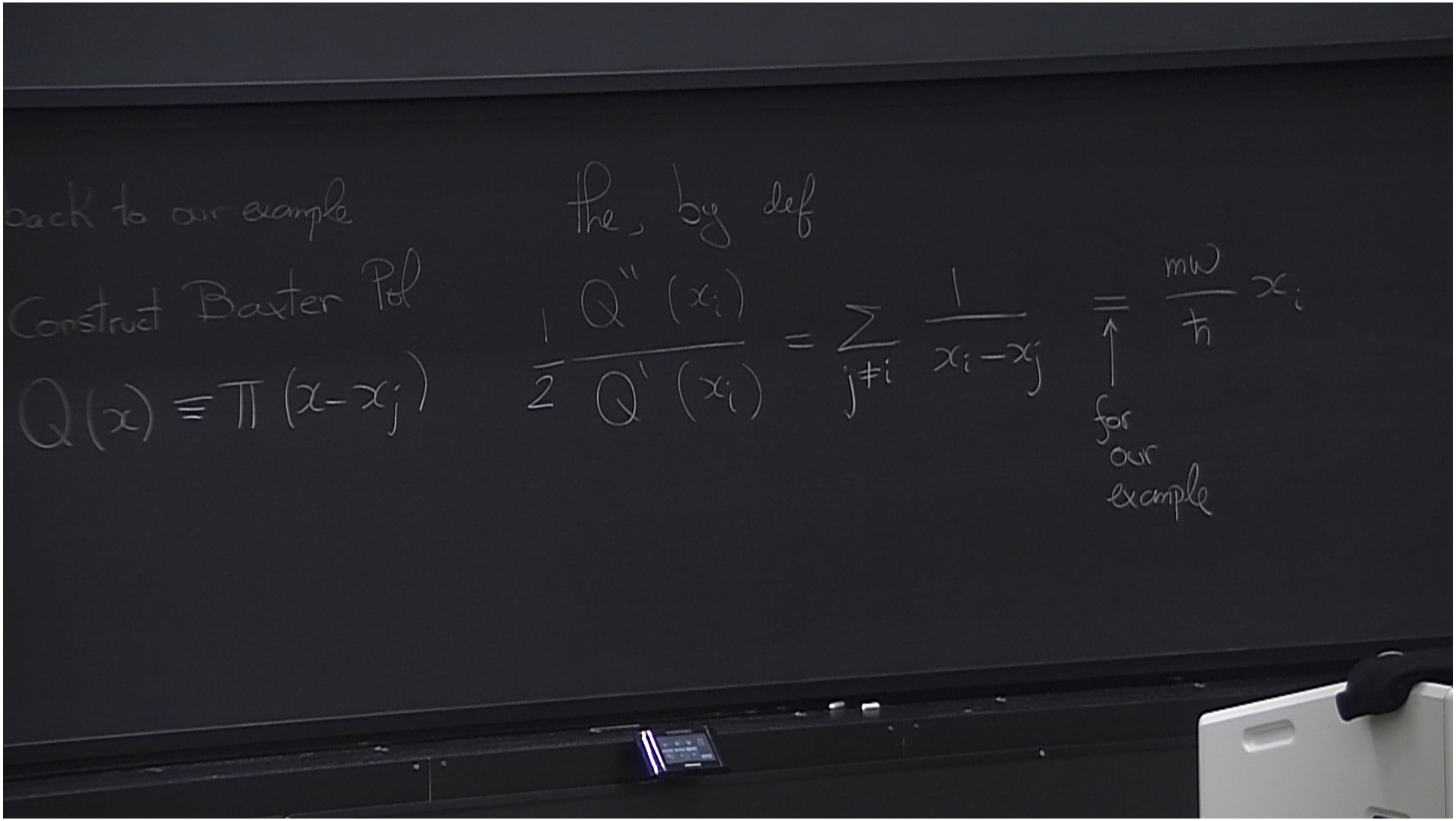
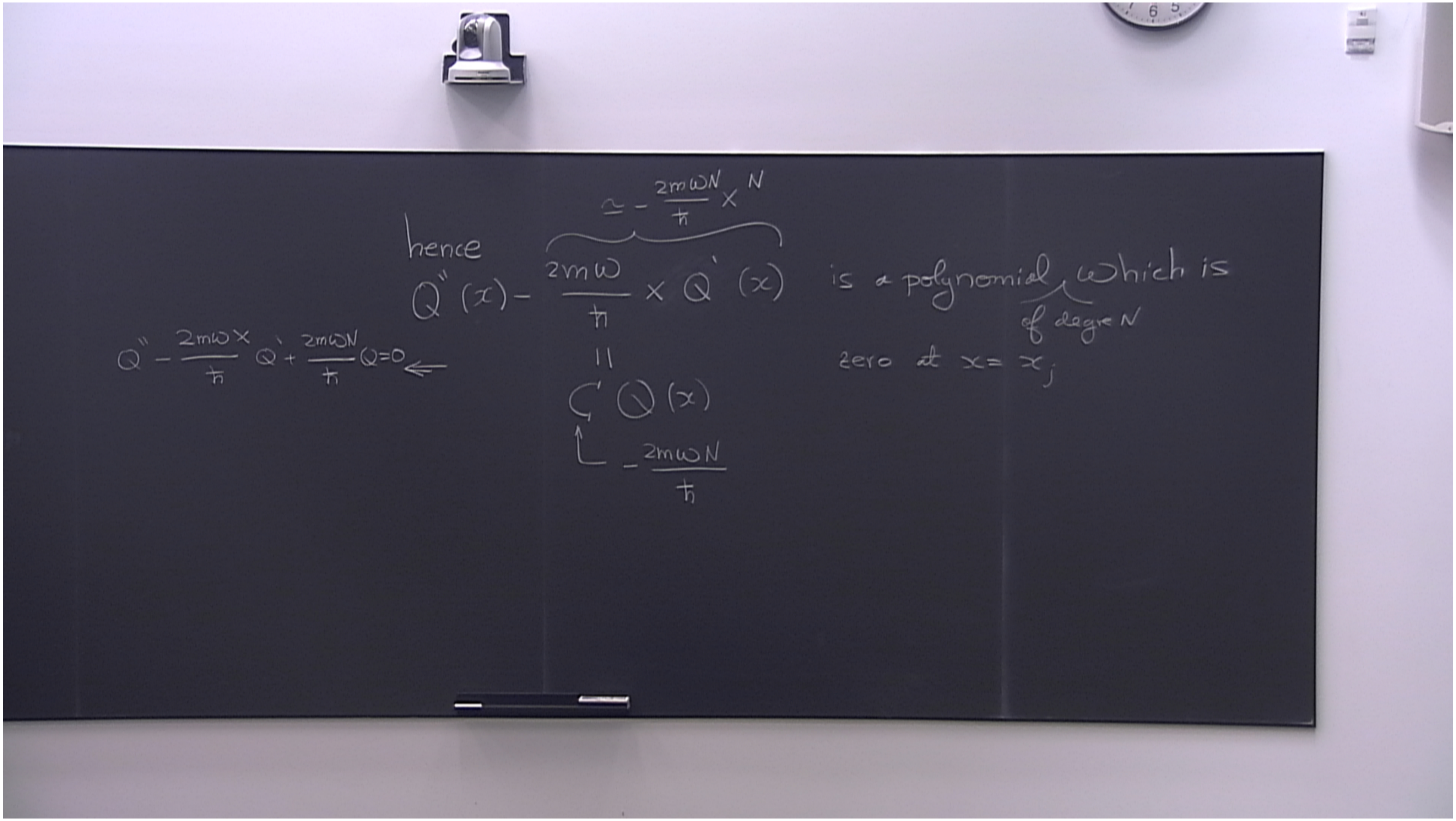
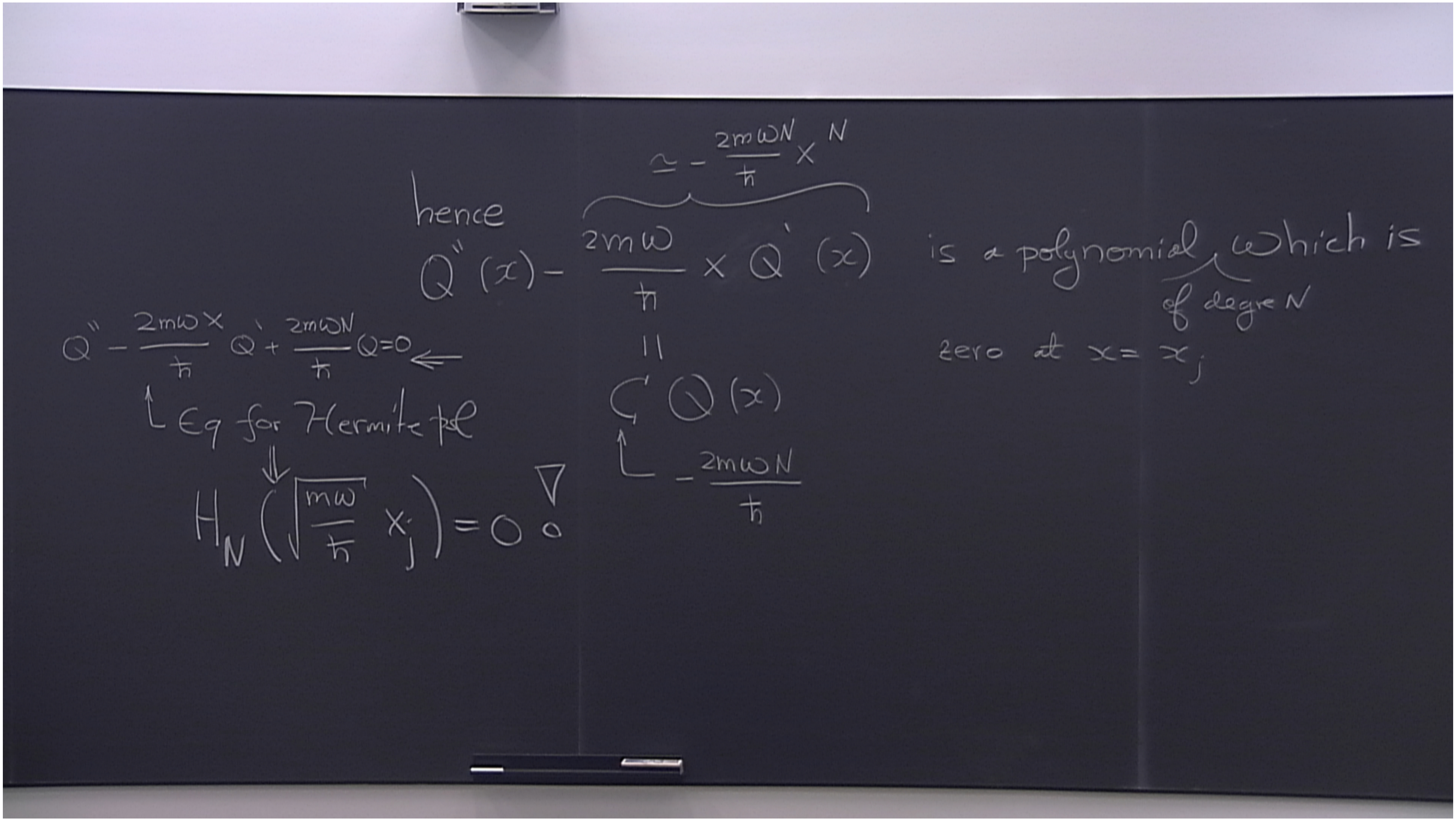
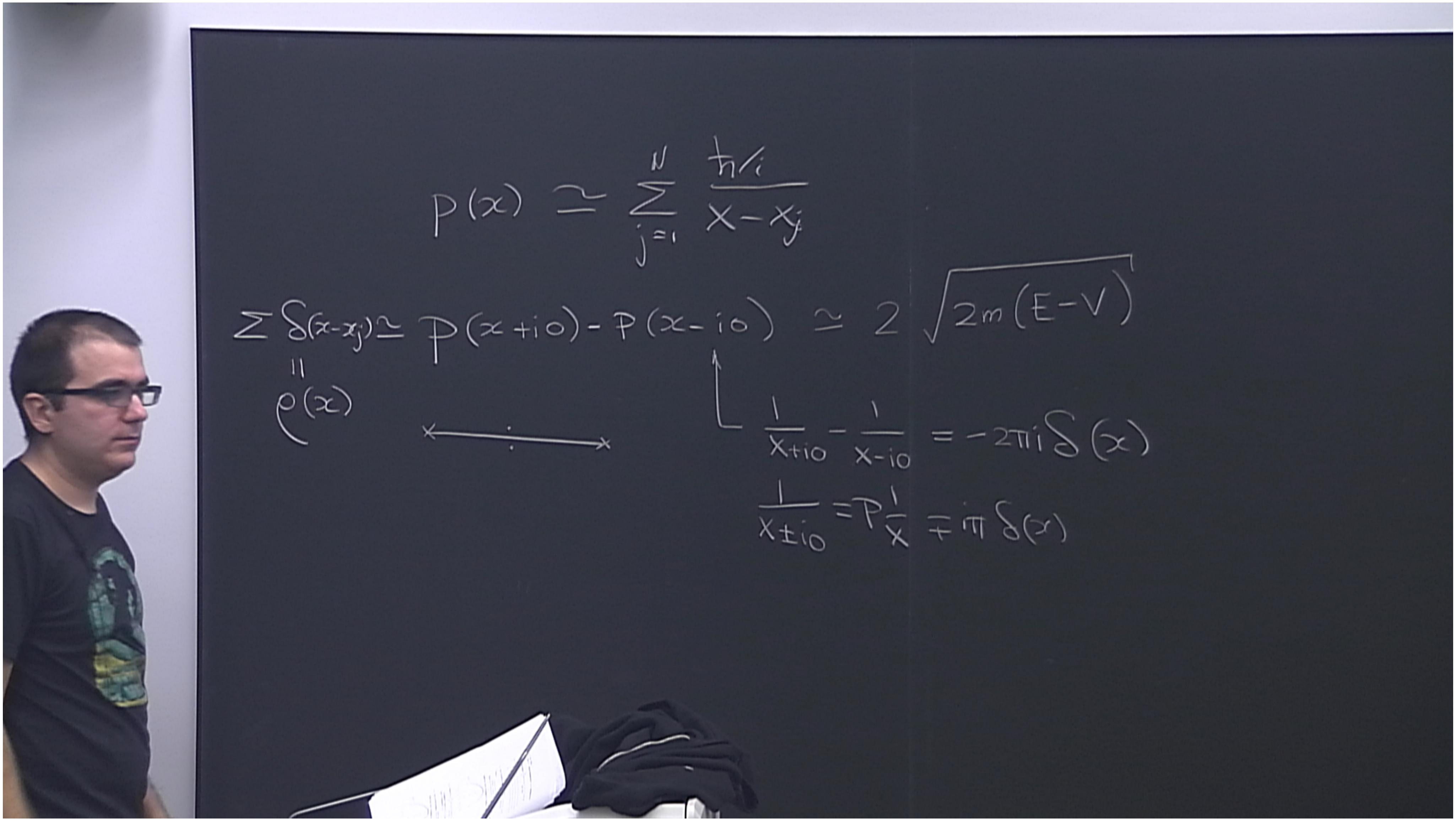
back to our example
Construct Baxter Pol
$Q(x) \equiv \prod (x - x_j)$
the, by def
$\frac{1}{2}\frac{Q''(x_i)}{Q'(x_i)} = \sum_{j \neq i} \frac{1}{x_i - x_j}$

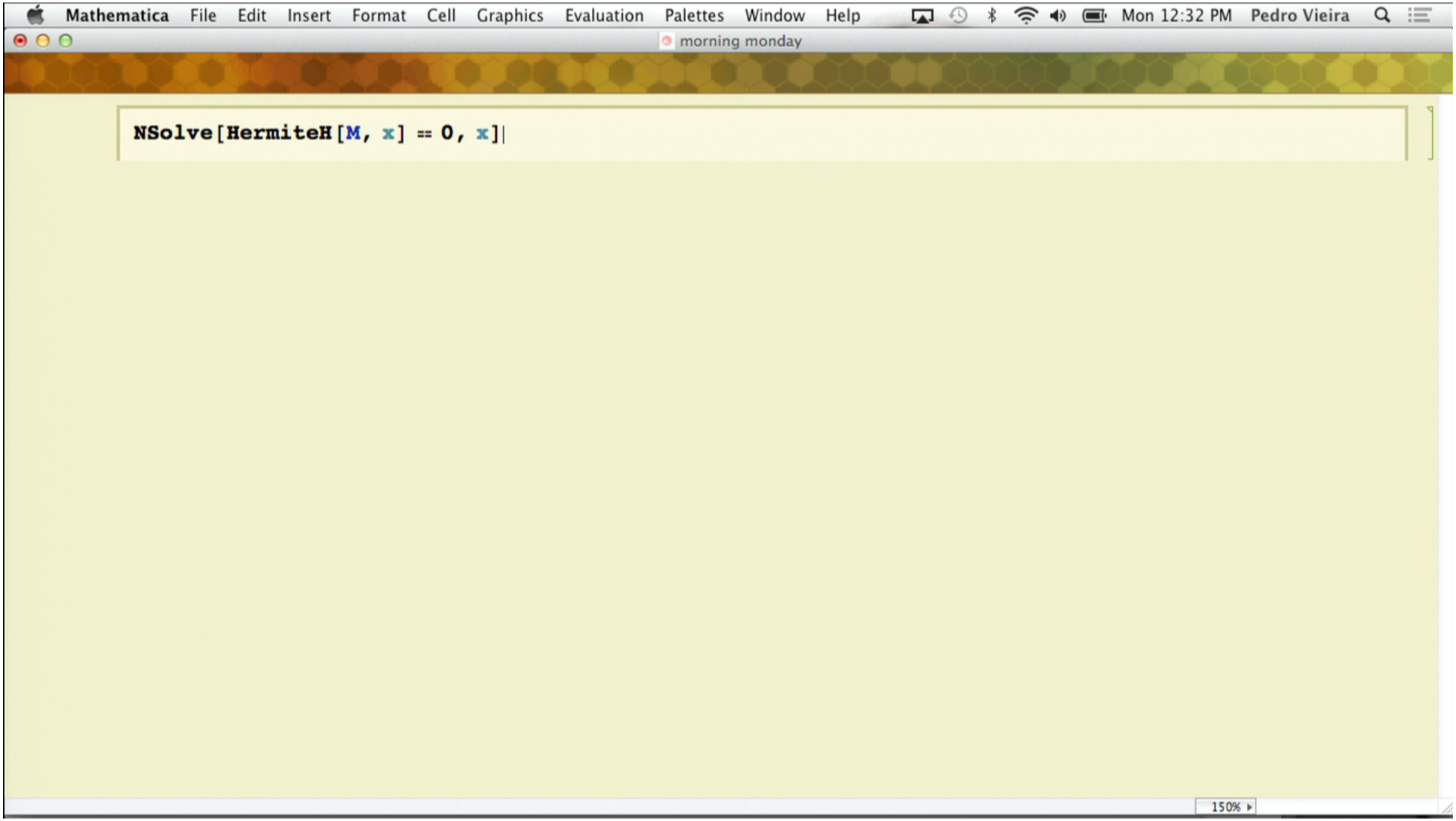

back to our example
Construct Baxter Pol
$Q(x) \equiv \prod (x - x_j)$
the, by def
$\frac{1}{2}\frac{Q''(x_i)}{Q'(x_i)} = \sum_{j \neq i} \frac{1}{x_i - x_j} = \frac{m\omega}{\hbar} x_i$
for our example


hence
$\simeq -\frac{2m\omega N}{\hbar} x^N$
$Q''(x) - \frac{2m\omega}{\hbar} x \, Q'(x)$ is a polynomial of degree N, which is zero at $x = x_j$
$= C \, Q(x)$
$C = -\frac{2m\omega N}{\hbar}$
$Q'' - \frac{2m\omega x}{\hbar} Q' + \frac{2m\omega N}{\hbar} Q = 0$



hence
$Q''(x) - \frac{2m\omega}{\hbar} x\, Q'(x)$ is a polynomial, which is of degree N
$\simeq -\frac{2m\omega N}{\hbar} x^N$
zero at $x = x_j$
$= c\, Q(x)$
$c = -\frac{2m\omega N}{\hbar}$
$Q'' - \frac{2m\omega x}{\hbar} Q' + \frac{2m\omega N}{\hbar} Q = 0$
Eq for Hermite pol
$H_N\left(\sqrt{\frac{m\omega}{\hbar}}\, x_j\right) = 0$!


$p(x) \simeq \sum_{j=1}^{N} \frac{\hbar/i}{x - x_j}$
$\sum \delta(x-x_j) \simeq p(x+i0) - p(x-i0) \simeq 2\sqrt{2m(E-V)}$
$\rho(x)$
$\frac{1}{x+i0} - \frac{1}{x-i0} = -2\pi i \delta(x)$
$\frac{1}{x \pm i0} = P\frac{1}{x} \mp i\pi \delta(x)$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Mon 12:32 PM Pedro Vieira
morning monday
NSolve[HermiteH[M, x] == 0, x]
150%

```
M = 100;
NSolve[HermiteH[M, x] == 0, x]
```

In[101]:=

```
M = 100;
x /. NSolve[HermiteH[M, x] == 0, x]
```

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In[103]:=

```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x];
```

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In[103]:=

```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x];
```

```
Table[xs[[i]] - Sum[If[i == j, 0,
```

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In[103]:=

```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x];
```

$$\texttt{Table}\left[\texttt{xs[[i]]} - \texttt{Sum}\left[\texttt{If}\left[\texttt{i == j, 0, }\frac{1}{\texttt{xs[[i]] - xs[[j]]}}\right], \right]\right]$$

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In[103]:=

```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x];
```

$$\text{Table}\left[xs[[i]] - \sum_{j}^{M} \text{If}\left[i == j, 0, \frac{1}{xs[[i]] - xs[[j]]}\right], \{i, M\}\right]$$

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In[103]:=

```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x];
```

In[105]:=

$$\text{Table}\left[xs[[i]] - \sum_{j=1}^{M} \text{If}\left[i == j, 0, \frac{1}{xs[[i]] - xs[[j]]}\right], \{i, M\}\right]$$

Out[105]=

{3.42782, 3.39147, -1.85326, -4.15298, -3.33581, -7.12456, -4.1448,
-4.0329, -1.37246, -2.52987, -0.821888, -0.275577, 0.805505,
2.55809, 59.5767, -60.9795, 224.385, -220.66, 12.2793, -6.6583,
11.3536, 37.2559, -27.0392, 10.3027, 12.2443, -18.2003, 4.77018,
2.93676, 16.1712, -17.7238, 13.3476, -10.1736, 4.26087, 0.118306,
-6.8731, -2.42054, -2.15149, -1.38605, -1.04379, -0.807337,
-0.643104, -0.519636, -0.422441, -0.343079, -0.276248, -0.21838,
-0.166947, -0.120053, -0.0762064, -0.0341577, 0.00720696,
0.0489546, 0.0921843, 0.138113, 0.188181, 0.244197, 0.308556,

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In[106]:=

```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x, WorkingPrecision → 50];
```

In[108]:=

$$\text{Table}\left[\text{xs}[[i]] - \sum_{j=1}^{M} \text{If}\left[i == j,\ 0,\ \frac{1}{\text{xs}[[i]] - \text{xs}[[j]]}\right],\ \{i, M\}\right]$$

Out[108]=

$\{0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48}, 0. \times 10^{-48},$
$0. \times 10^{-48}, 0. \times 10^{-49}, 0. \times 10^{-49}, 0. \times 10^{-49}, 0. \times 10^{-49}, 0. \times 10^{-49}$

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```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x, WorkingPrecision → 50];
```

In[109]:=

$$\texttt{Table}\left[\texttt{xs}[\![i]\!] - \sum_{j=1}^{M} \texttt{If}\left[i == j,\ 0,\ \frac{1}{\texttt{xs}[\![i]\!] - \texttt{xs}[\![j]\!]}\right],\ \{i, M\}\right]\ \texttt{// Chop}$$

Out[109]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

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```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x, WorkingPrecision → 50];
```

In[109]:=

$$\text{Table}\left[xs[[i]] - \sum_{j=1}^{M} \text{If}\left[i == j, 0, \frac{1}{xs[[i]] - xs[[j]]}\right], \{i, M\}\right] \text{ // Chop}$$

Out[109]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

$$\frac{1}{xs[[i]] - xs[[j]]}$$

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```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x, WorkingPrecision → 50];
```

In[109]:=

$$\text{Table}\left[xs[[i]] - \sum_{j=1}^{M} \text{If}\left[i == j, 0, \frac{1}{xs[[i]] - xs[[j]]}\right], \{i, M\}\right] \text{ // Chop}$$

Out[109]=

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

$$\frac{1}{xs[[i+1]] - xs[[i]]}$$

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```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x, WorkingPrecision → 50];
```

In[109]:=

$$\texttt{Table}\left[\texttt{xs[[i]]} - \sum_{j=1}^{M} \texttt{If}\left[i == j,\ 0,\ \frac{1}{\texttt{xs[[i]]} - \texttt{xs[[j]]}}\right],\ \{i, M\}\right] \texttt{ // Chop}$$

Out[109]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

$$\frac{(\texttt{i}|+|\texttt{1} - \texttt{i}}{\texttt{xs[[i + 1]]} - \texttt{xs[[i]]}}$$

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```
M = 100;
xs = x /. NSolve[HermiteH[M, x] == 0, x, WorkingPrecision → 50];
```

In[109]:=

$$\text{Table}\left[\text{xs}[[i]] - \sum_{j=1}^{M} \text{If}\left[i == j,\ 0,\ \frac{1}{\text{xs}[[i]] - \text{xs}[[j]]}\right],\ \{i, M\}\right] \text{ // Chop}$$

Out[109]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

$$\text{ListPlot@Table}\left[\left\{\frac{\text{xs}[[i+1]] + \text{xs}[[i]]}{2},\ \frac{(i+1) - (i)}{\text{xs}[[i+1]] - \text{xs}[[i]]}\right\},\ \{i, M-1\}\right];$$

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```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

In[111]:=

$$\text{lp} = \text{ListPlot@Table}\left[\left\{\frac{\text{xs}[[i+1]] + \text{xs}[[i]]}{2},\ \frac{(i+1)-(i)}{\text{xs}[[i+1]] - \text{xs}[[i]]}\right\},\ \{i,\ M-1\}\right];$$

$$\text{plot} = \text{Plot}\left[\frac{\sqrt{2\left(M - 1/2 - x^2/2\right)}}{\pi}\right.$$

200%

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morning monday

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[111]:=

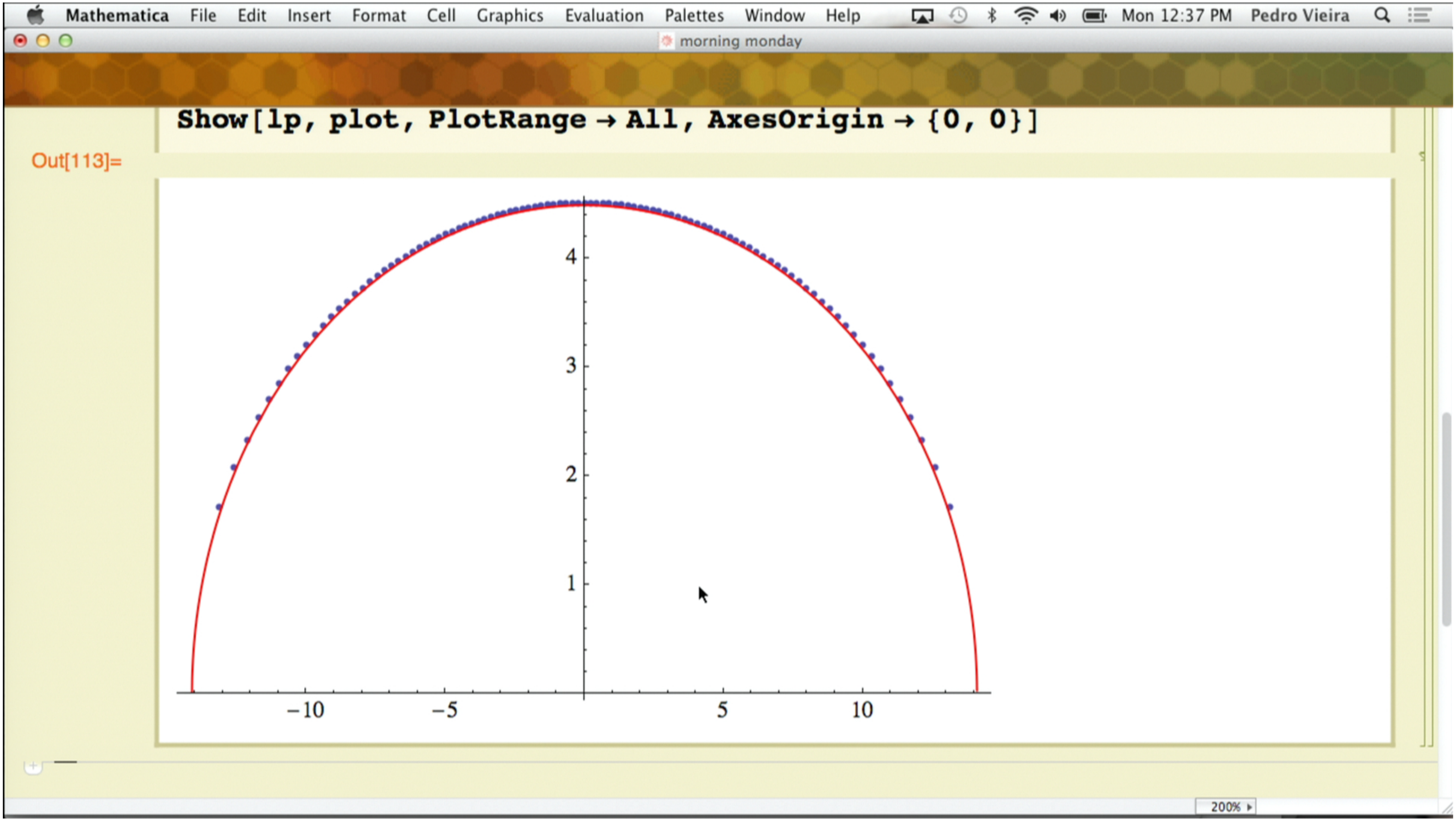
$$\text{lp} = \text{ListPlot@Table}\left[\left\{\frac{\text{xs}[\![i+1]\!] + \text{xs}[\![i]\!]}{2}, \frac{(i+1)-(i)}{\text{xs}[\![i+1]\!] - \text{xs}[\![i]\!]}\right\}, \{i, M-1\}\right];$$

In[112]:=

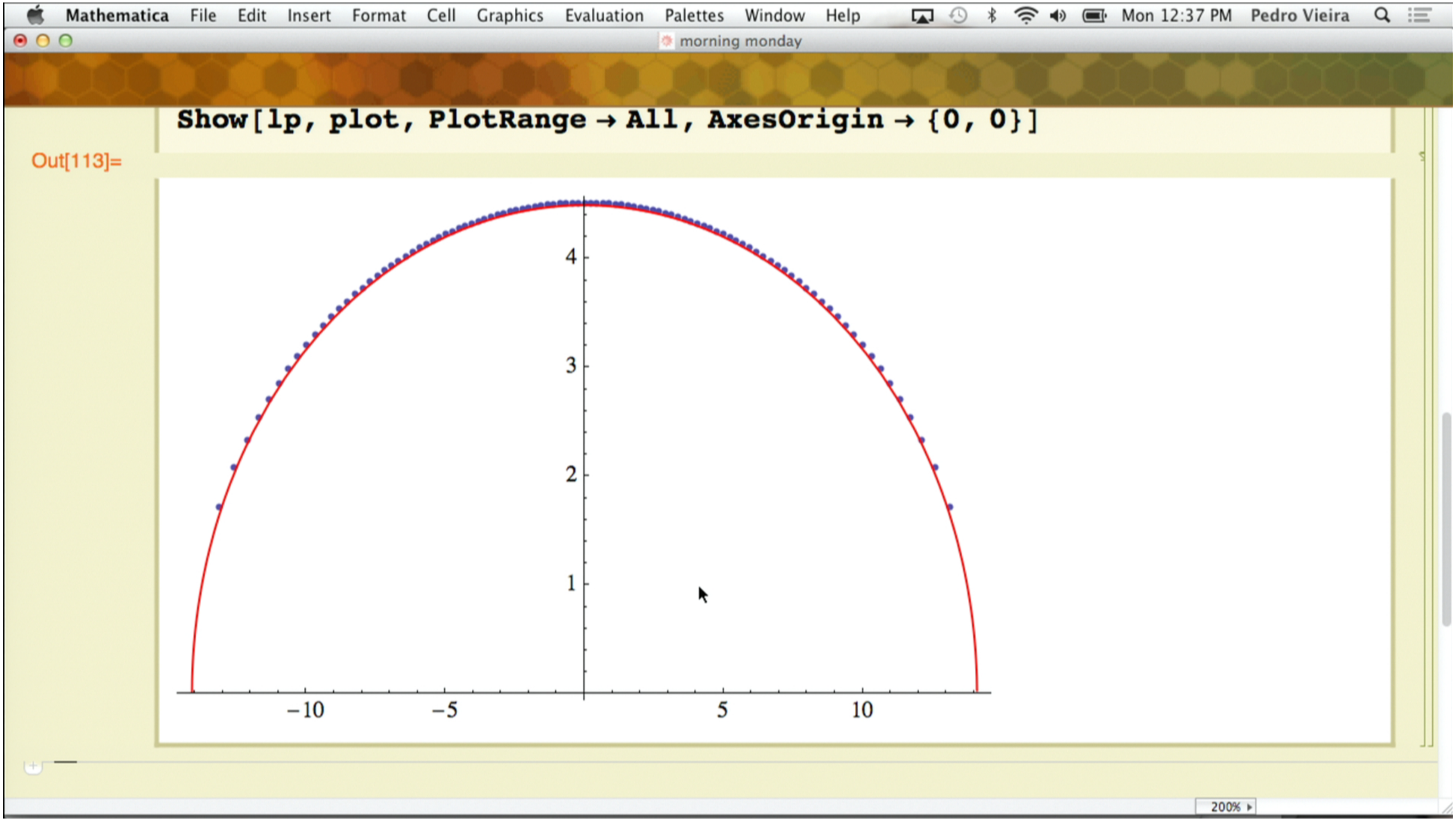
$$\text{plot} = \text{Plot}\left[\frac{\sqrt{2\,(M-1/2-x^2/2)}}{\pi}, \left\{x, -\sqrt{2\,(M+1/2)}, \sqrt{2\,(M+1/2)}\right\}, \text{PlotStyle} \to \text{Red}\right];$$

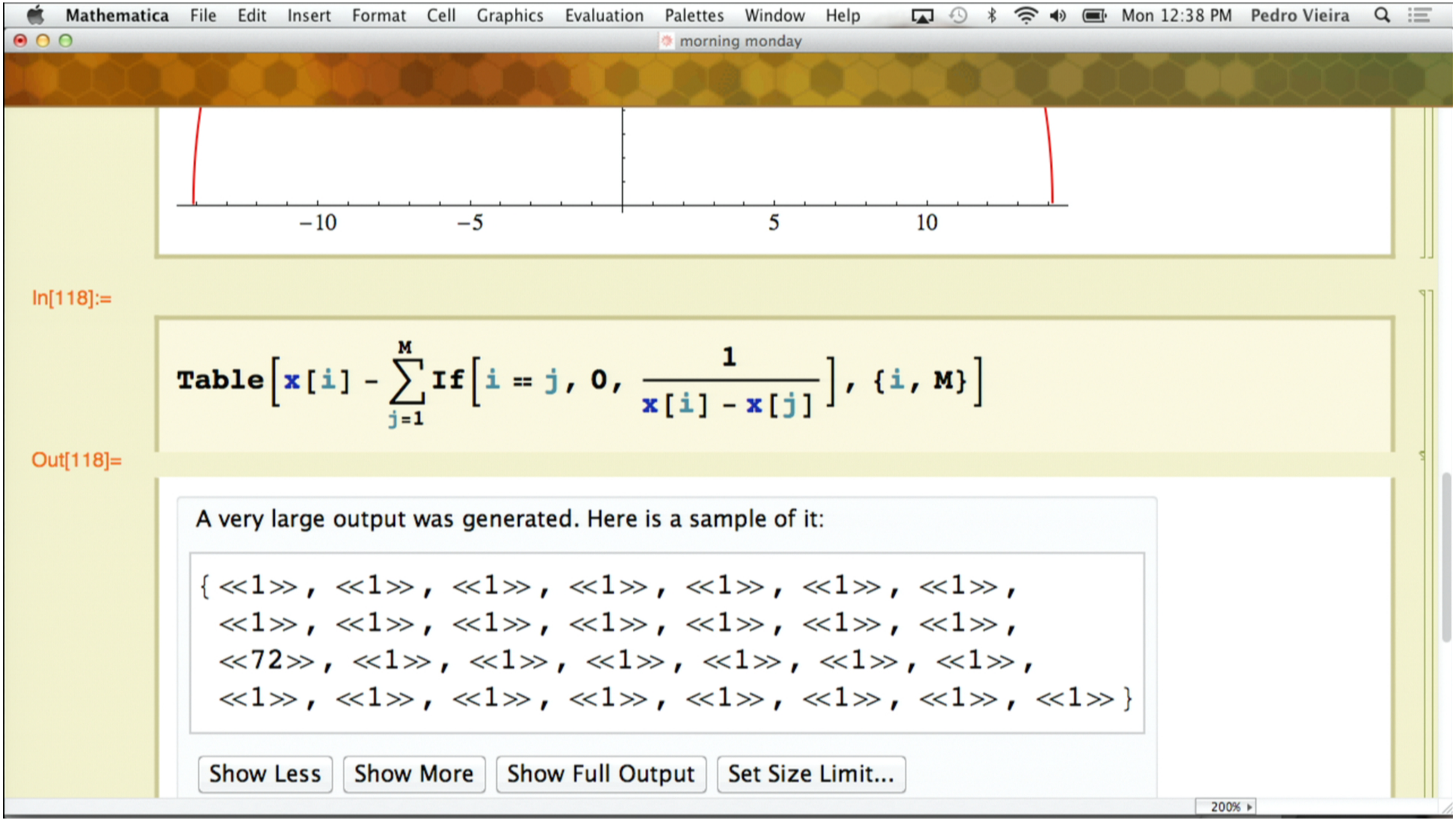
```
Show[lp, plot,
```

200%



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:37 PM Pedro Vieira
morning monday
Show[lp, plot, PlotRange → All, AxesOrigin → {0, 0}]
Out[113]=
4
3
2
1
−10
−5
5
10
200%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:37 PM Pedro Vieira
morning monday
Show[lp, plot, PlotRange → All, AxesOrigin → {0, 0}]
Out[113]=
4
3
2
1
−10
−5
5
10
200%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:38 PM Pedro Vieira
morning monday
−10 −5 5 10
In[118]:=
Table[x[i] - Sum[If[i == j, 0, 1/(x[i] - x[j])], {j, 1, M}], {i, M}]
Out[118]=
A very large output was generated. Here is a sample of it:
{≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫,
≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫,
≪72≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫,
≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫, ≪1≫}
Show Less
Show More
Show Full Output
Set Size Limit...
200%

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morning monday

$$\text{Table}\left[x[i]-\sum_{j=1}^{M}\text{If}\left[i==j,\,0,\,\frac{1}{x[i]-x[j]}\right],\,\{i,M\}\right]\ //\ \text{FindRoot}[\#,\,\{x$$

200%

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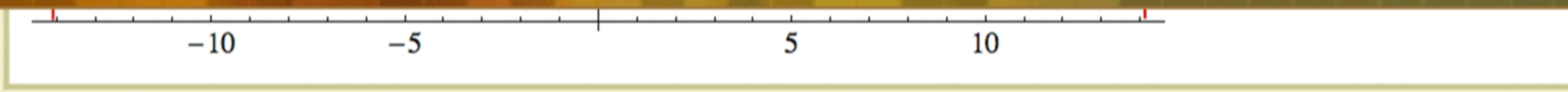

In[119]:=

```
Table[x[i] - Sum[If[i == j, 0, 1/(x[i] - x[j])], {j, 1, M}], {i, M}] //
 FindRoot[#, Table[{x[i], (i - M/2)/M 10}, {i, M}]] &
```

$$\text{Table}\left[x[i]-\sum_{j=1}^{M}\text{If}\left[i==j,0,\frac{1}{x[i]-x[j]}\right],\{i,M\}\right]\ //\ \text{FindRoot}\left[\#,\text{Table}\left[\left\{x[i],\frac{i-M/2}{M}10\right\},\{i,M\}\right]\right]\ \&$$

Out[119]=

```
{x[1] → -13.4065, x[2] → -12.8238, x[3] → -12.343, x[4] → -11.9151,
 x[5] → -11.5214, x[6] → -11.1524, x[7] → -10.8023, x[8] → -10.4672,
 x[9] → -10.1445, x[10] → -9.83227, x[11] → -9.52897,
 x[12] → -9.23342, x[13] → -8.94469, x[14] → -8.662,
 x[15] → -8.3847, x[16] → -8.11225, x[17] → -7.84418,
 x[18] → -7.5801, x[19] → -7.31965, x[20] → -7.06253,
 x[21] → -6.80846, x[22] → -6.55721, x[23] → -6.30854,
 x[24] → -6.06228, x[25] → -5.81823, x[26] → -5.57624,
```

200%

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In[122]:=

```
Table[x[i] - Sum[If[i == j, 0, 1/(x[i] - x[j])], {j, 1, M}], {i, M}] //
  FindRoot[#, Table[{x[i], (i - M/2)/M 10}, {i, M}]] &;
xs2 = x /@ Range[M] /. %;
```

$$\text{Table}\left[x[i]-\sum_{j=1}^{M}\text{If}\left[i==j,0,\frac{1}{x[i]-x[j]}\right],\{i,M\}\right]\ //\ \text{FindRoot}\left[\#,\text{Table}\left[\left\{x[i],\frac{i-M/2}{M}10\right\},\{i,M\}\right]\right]\&;$$

In[125]:=

```
xs2 - xs // Chop
```

Out[125]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

200%

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morning monday

```
xs2 - xs // Chop
```

Out[125]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

μ = 8;

$$\text{Table}\left[x[i] - \sum_{j=1}^{M} \text{If}\left[i == j, 0, \frac{1}{x[i] - x[j]}\right], \{i, M\}\right] //$$

$$\text{FindRoot}\left[\#, \text{Table}\left[\left\{x[i], \frac{i - M/2}{M} 10\right\}, \{i, M\}\right]\right] \&;$$

```
xs2 = x /@ Range[M] /. %;
```

200%

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```
j=1                x[i] - x[j]
```

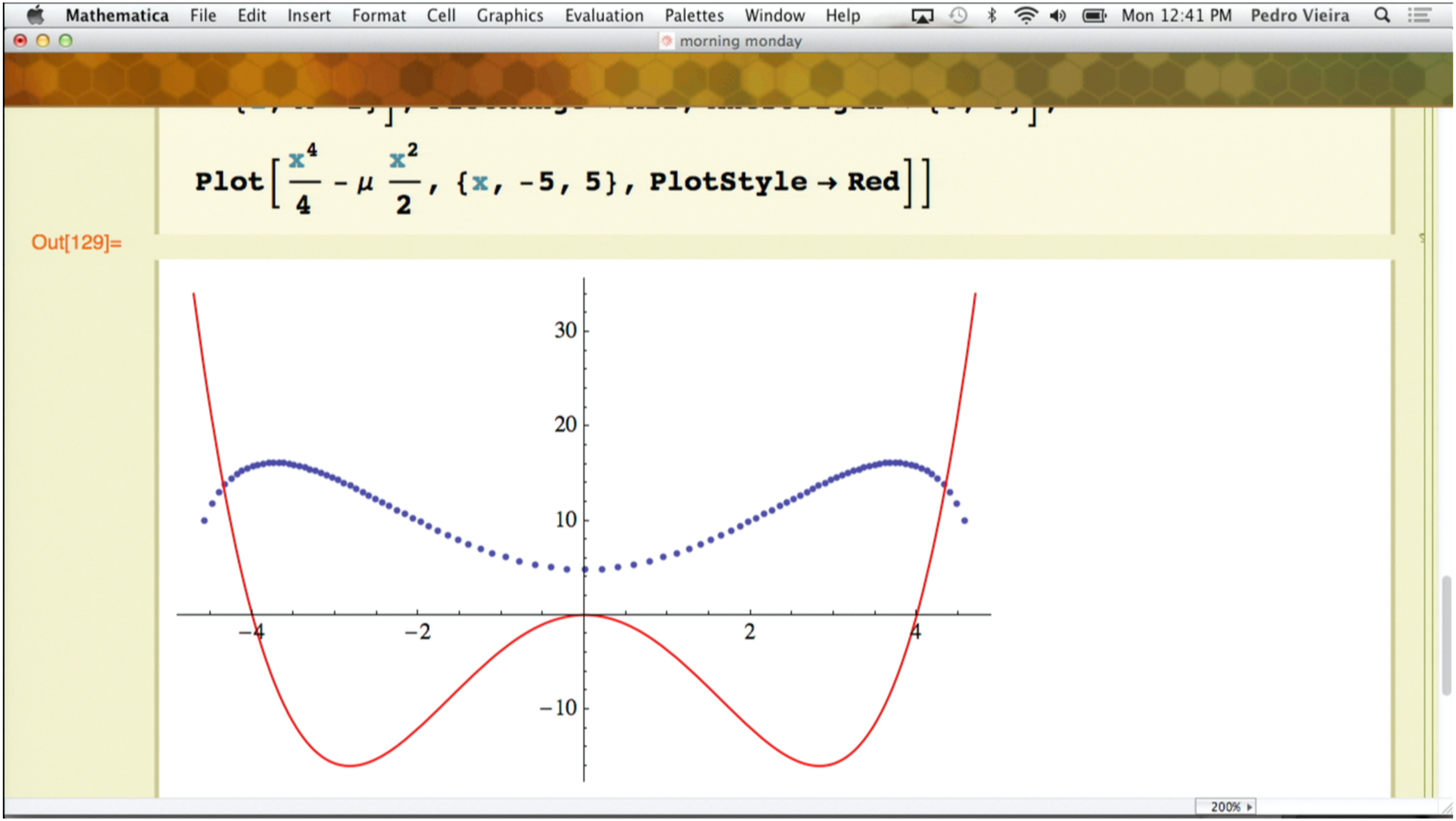
$$\texttt{FindRoot}\left[\#,\ \texttt{Table}\left[\left\{x[i],\ \frac{i - M/2}{M}\ 10\right\},\ \{i,\ M\}\right]\right]\ \&;$$

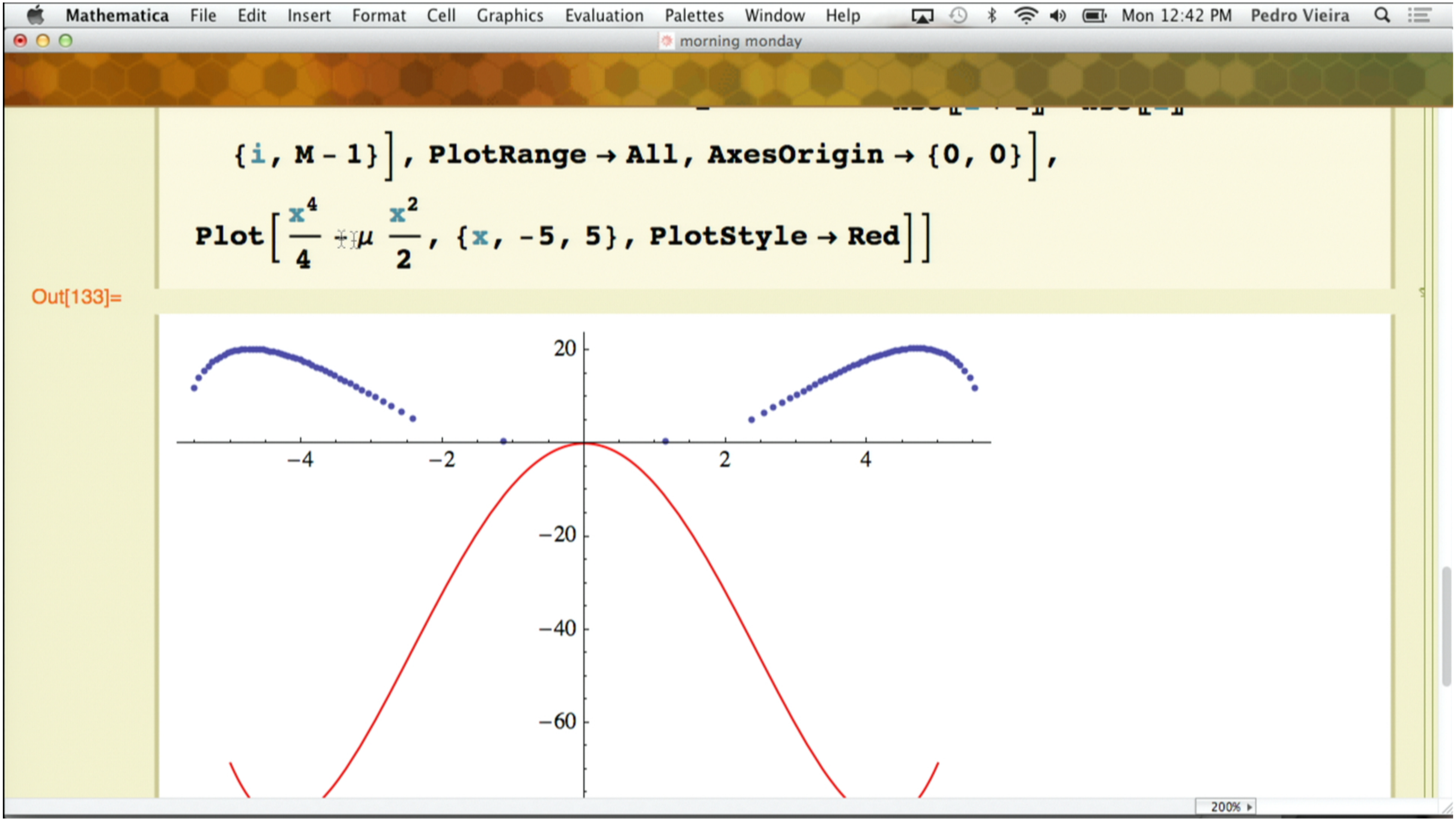
```
xs3 = x /@ Range[M] /. %;
```

$$\texttt{Show}\left[\texttt{ListPlot@Table}\left[\left\{\frac{xs3[[i+1]] + xs3[[i]]}{2},\ \frac{1}{xs3[[i+1]] - xs3[[i]]}\right\},\right.\right.$$

$$\left.\{i,\ M-1\}\right],\ \texttt{Plot}\left[\frac{x^4}{4} - \mu\ \frac{x^2}{2}\right.$$

200%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:41 PM Pedro Vieira
morning monday
Plot[x^4/4 - μ x^2/2, {x, -5, 5}, PlotStyle → Red]]
Out[129]=
30
20
10
−10
−4
−2
2
4
200%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Mon 12:42 PM Pedro Vieira
morning monday
{i, M - 1}], PlotRange → All, AxesOrigin → {0, 0}],
Plot[x^4/4 + μ x^2/2, {x, -5, 5}, PlotStyle → Red]]
Out[133]=
20
-4
-2
2
4
-20
-40
-60
200%

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morning monday

```
xs2 = x /@ Range[M] /. %;
```

In[125]:=

```
xs2 - xs // Chop
```

Out[125]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
μ = 18; (* try μ=8 also *)
```

$$\text{Table}\left[x[i]^3 - \mu\, x[i] - \sum_{j=1}^{M} \text{If}\left[i == j,\ 0,\ \frac{1}{x[i] - x[j]}\right],\ \{i, M\}\right]\ //$$

$$\text{FindRoot}\left[\#,\ \text{Table}\left[\left\{x[i],\ \frac{i - M/2}{M}\, 10\right\},\ \{i, M\}\right]\right]\ \&;$$

```
xs3 = x /@ Range[M] /. %;
```

200%