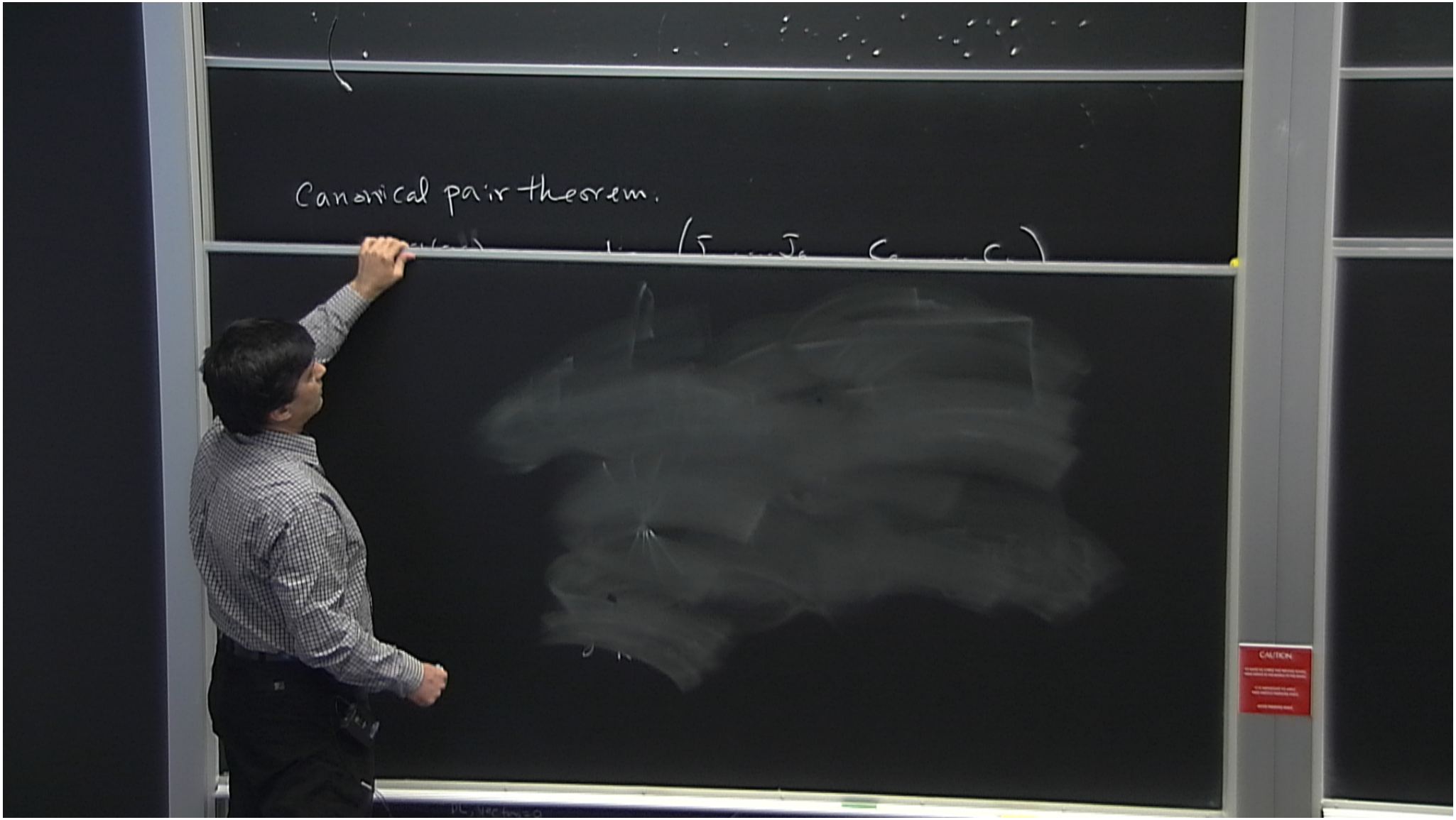
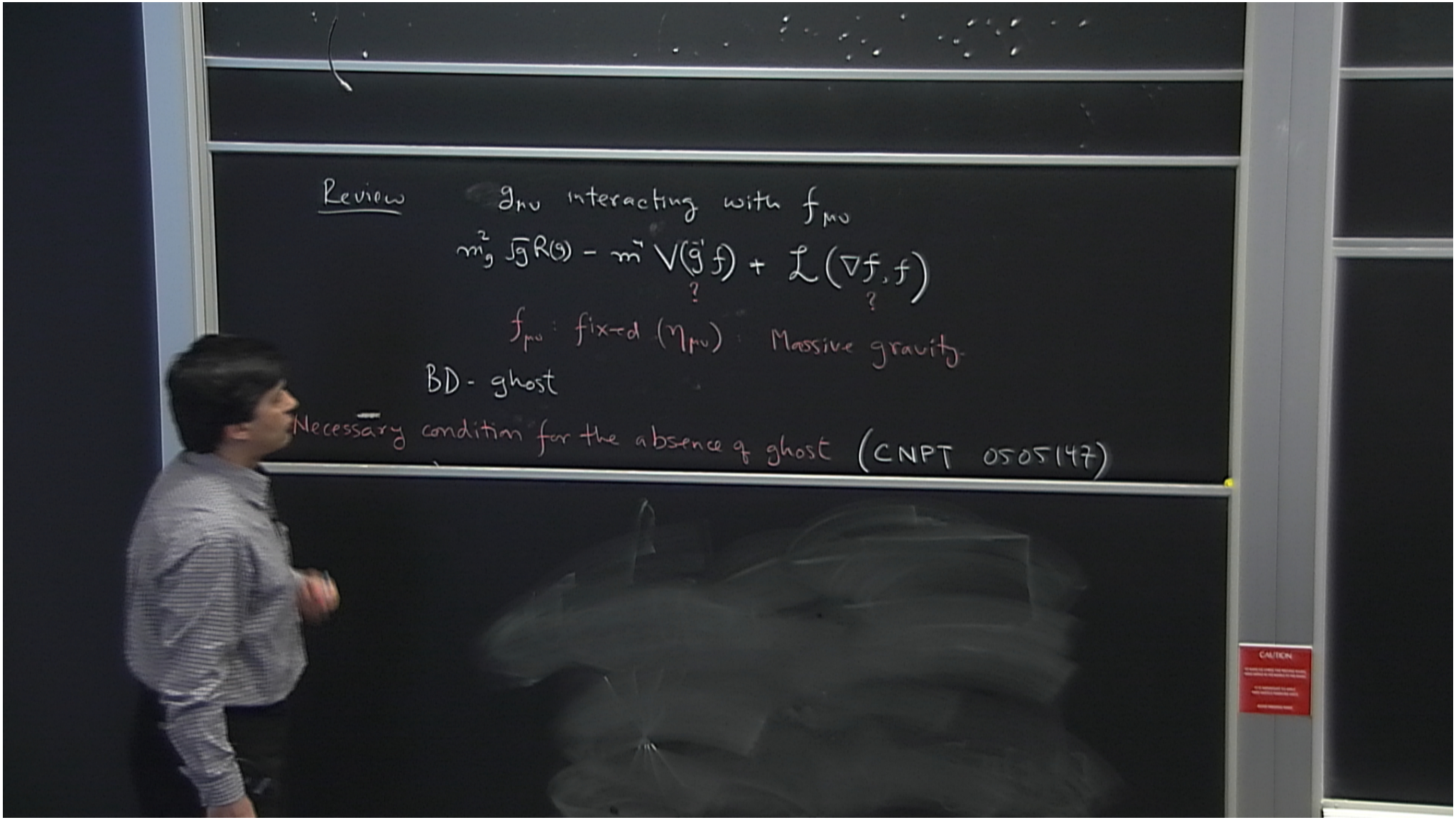
Title: Some recent results in Bimetric Theory

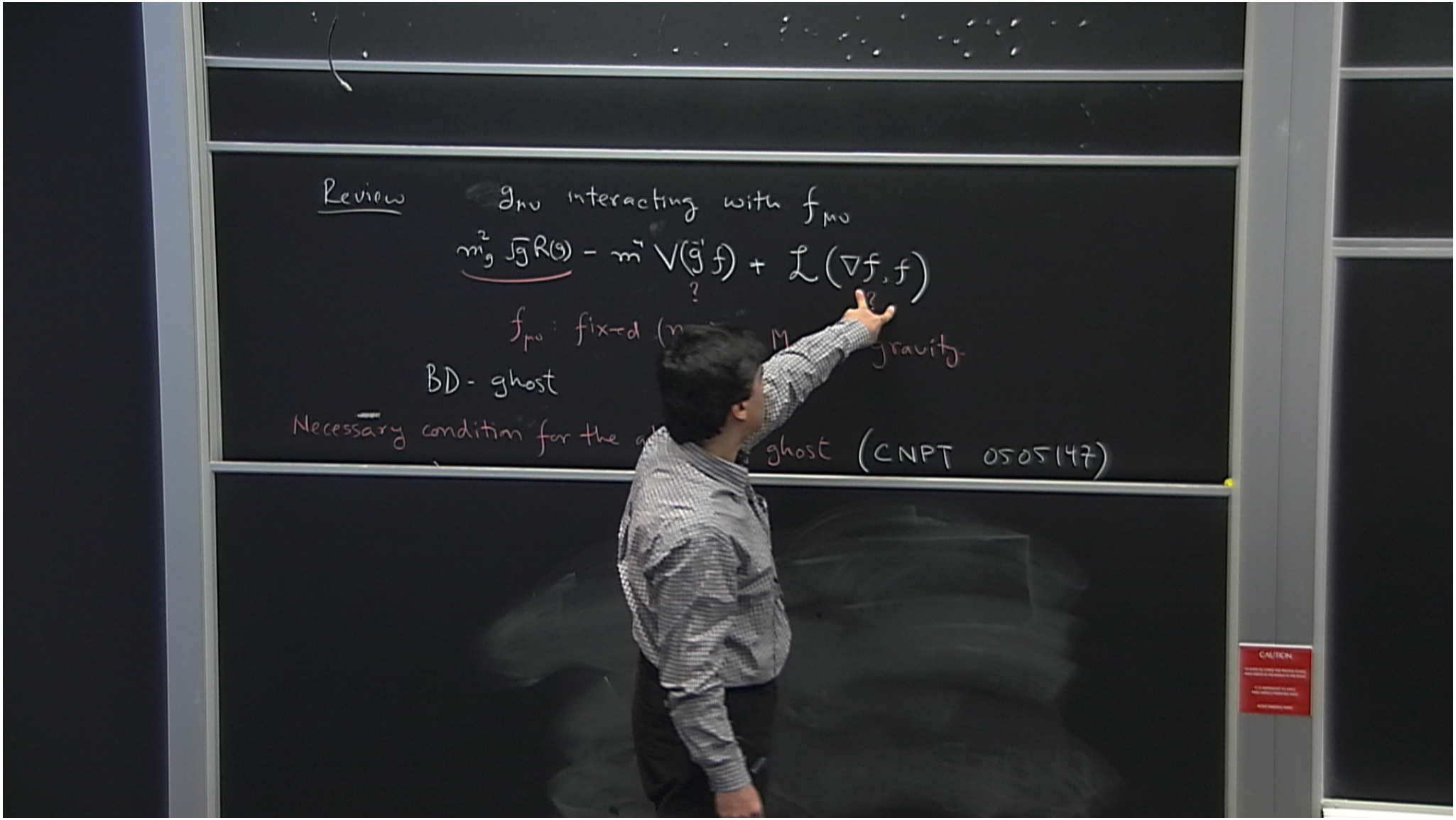
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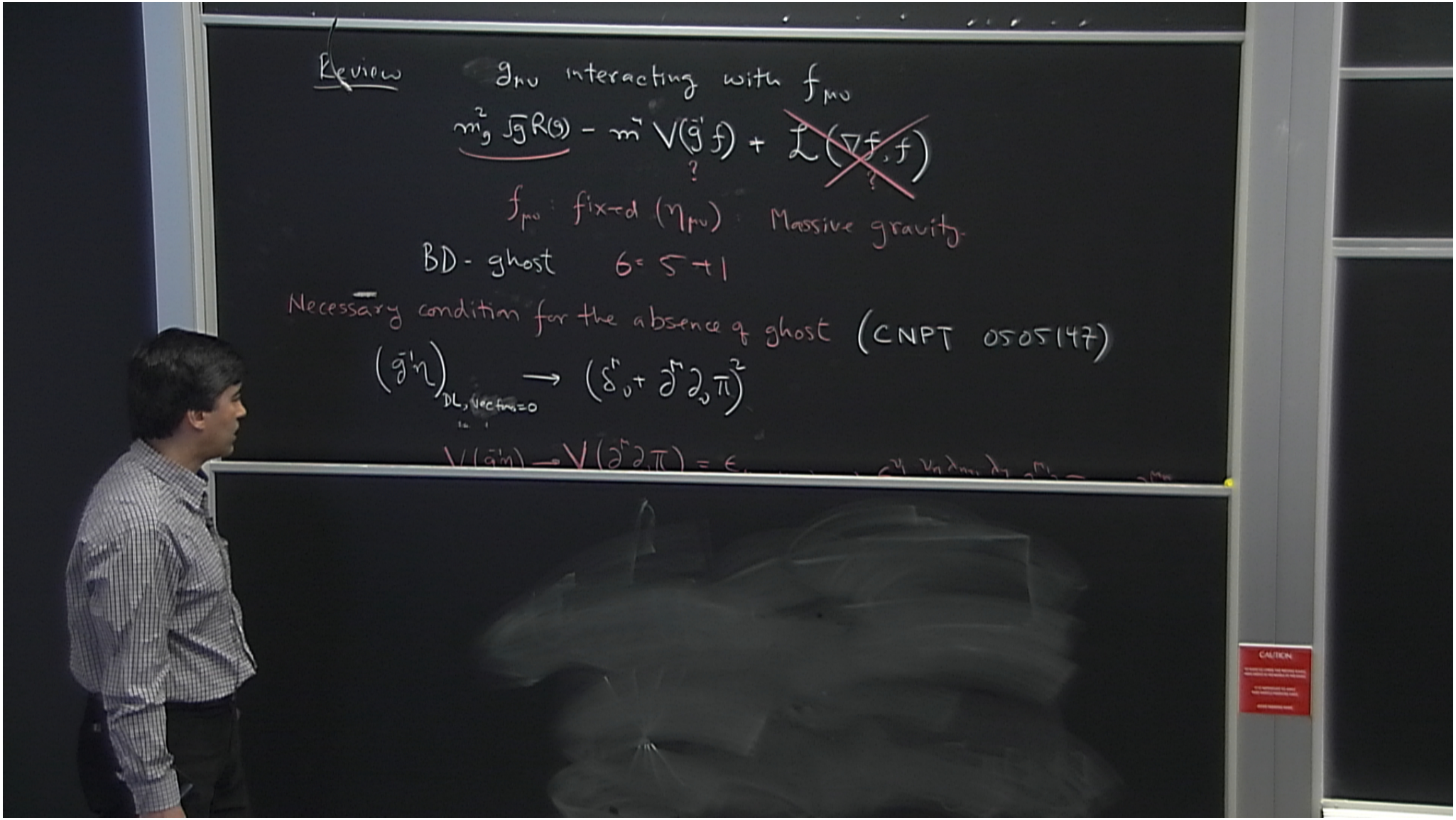
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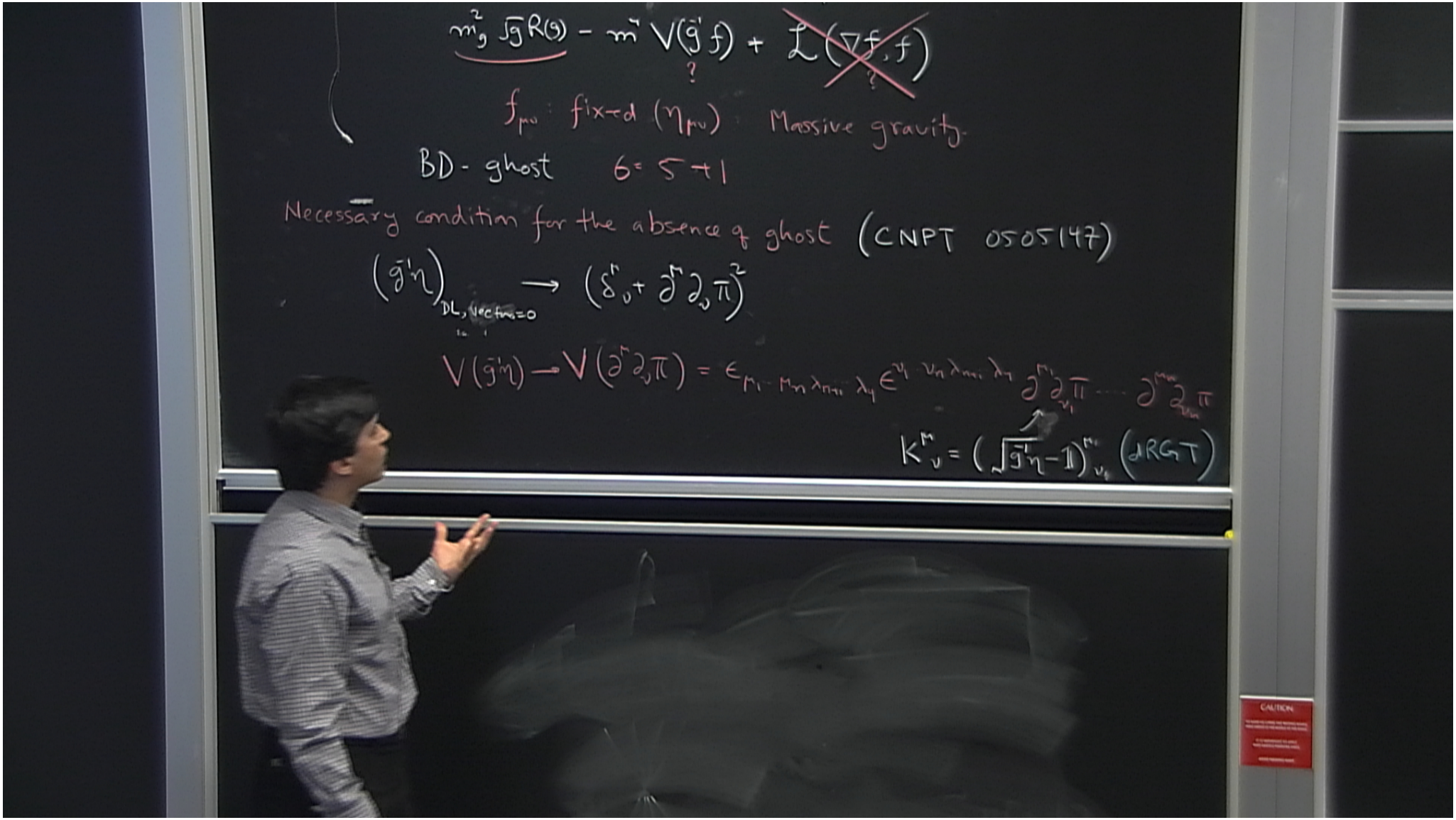
Abstract: This talk will summarize some recent results in bimetric theory, including the existence of the square-root matrix, possible connection to partial masslessness and conformal gravity, the structure of constraints and finally, the cosmological implications of the theory.

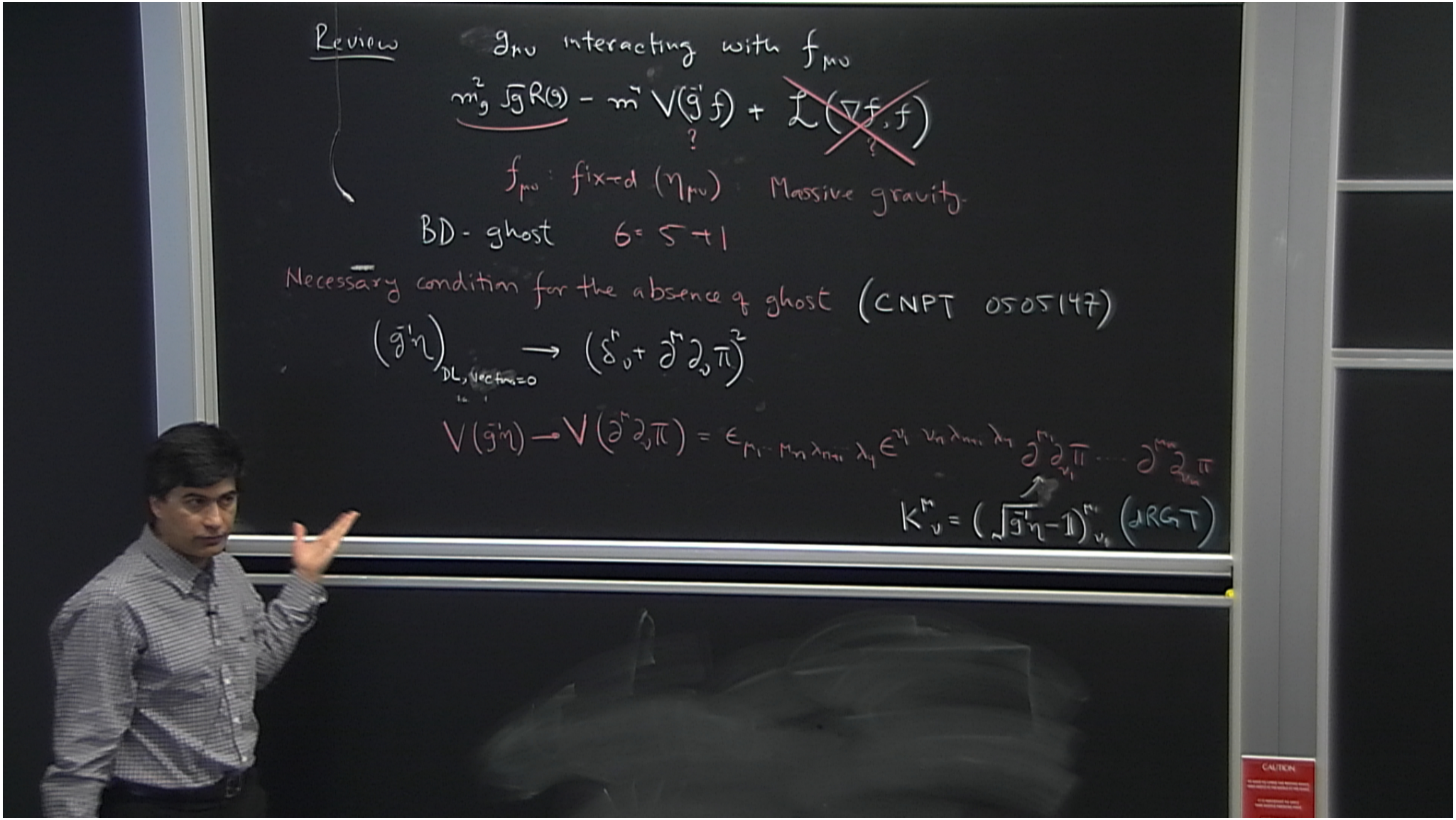

Canonical pair theorem.
CAUTION

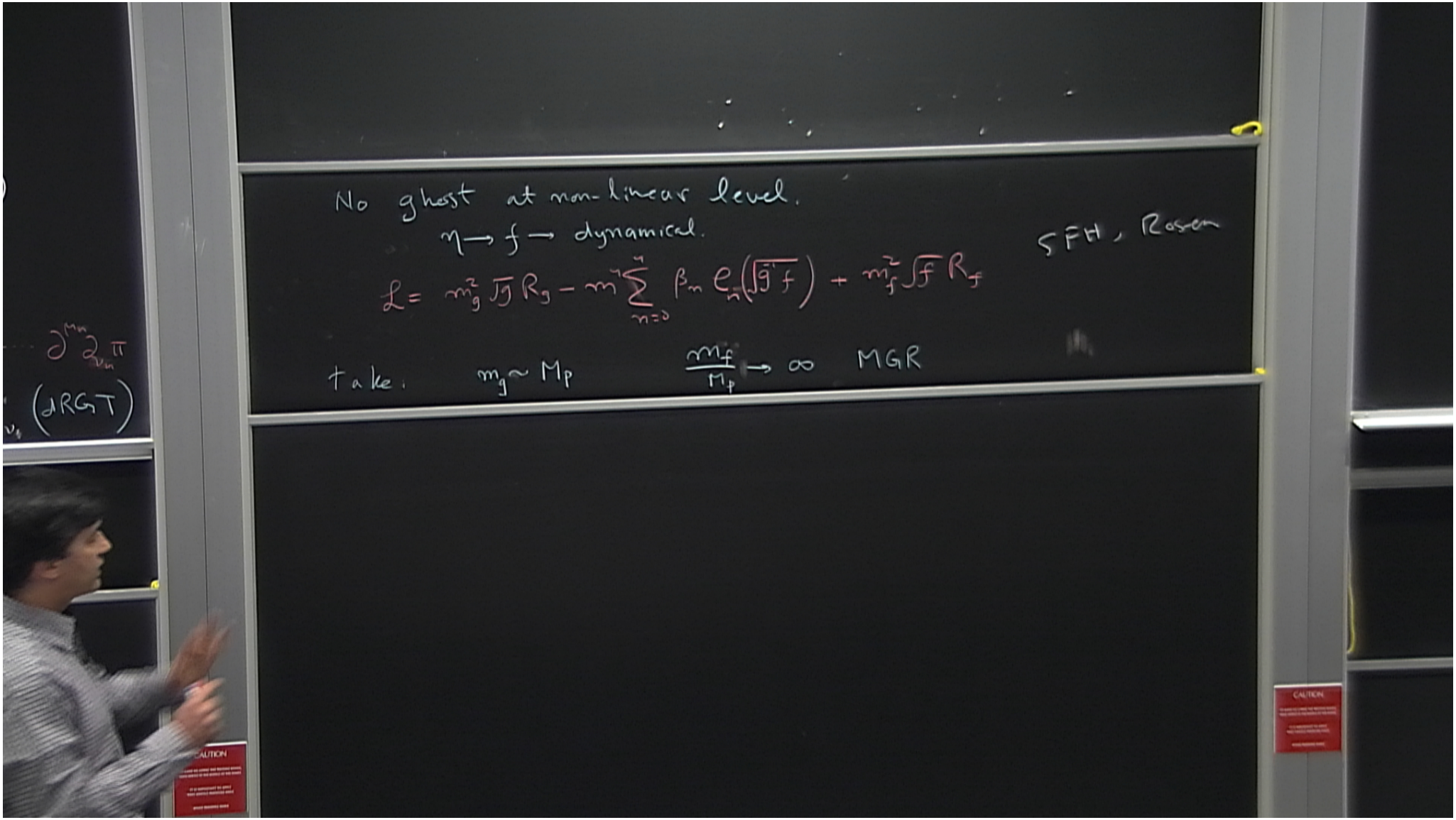

Review
$g_{\mu\nu}$ interacting with $f_{\mu\nu}$
$m_g^2 \sqrt{g} R(g) - m^4 V(\sqrt{g}^{-1} f) + \mathcal{L}(\nabla f, f)$
? ?
$f_{\mu\nu}$: fixed ($\eta_{\mu\nu}$) Massive gravity.
BD - ghost
Necessary condition for the absence of ghost (CNPT 0505147)
CAUTION

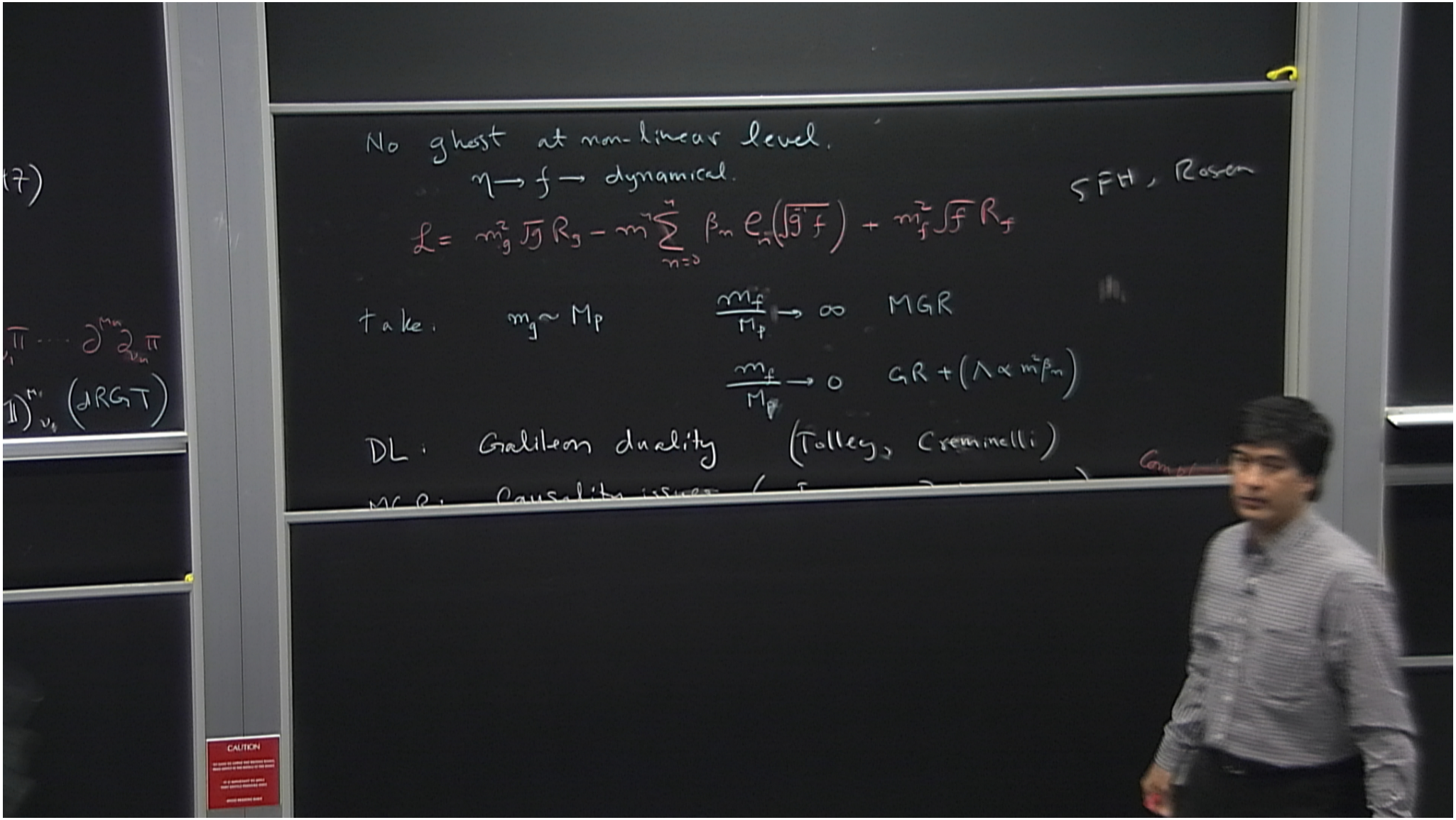

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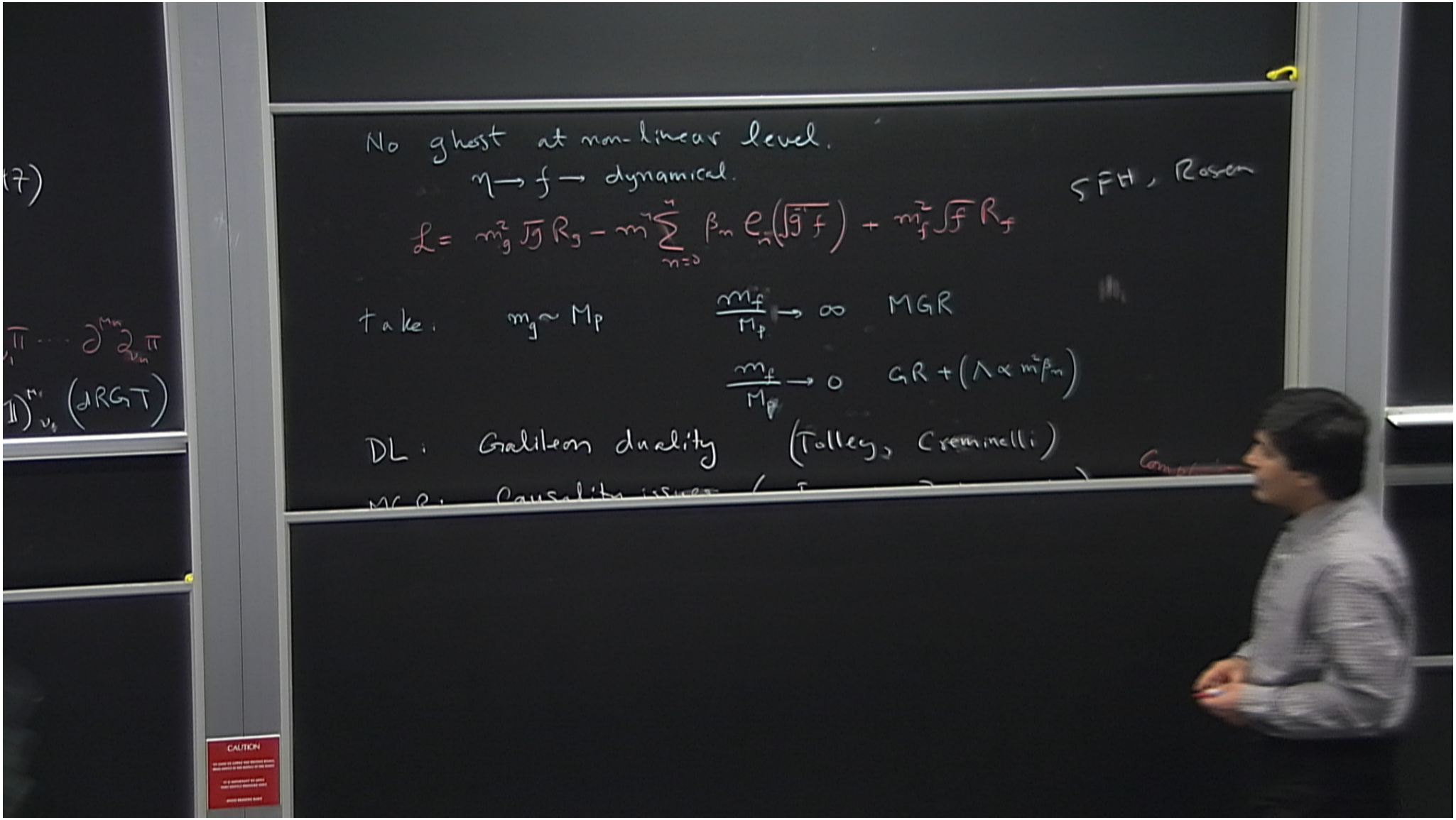

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BD - ghost $6 = 5 + 1$
Necessary condition for the absence of ghost (CNPT 0505147)
$(g^{-1}\eta)_{DL, \text{Vector}=0} \rightarrow (\delta^\mu_\nu + \partial^\mu \partial_\nu \pi)^2$
$V(g^{-1}\eta) \rightarrow V(\partial^\mu \partial_\nu \pi) = \epsilon$
CAUTION

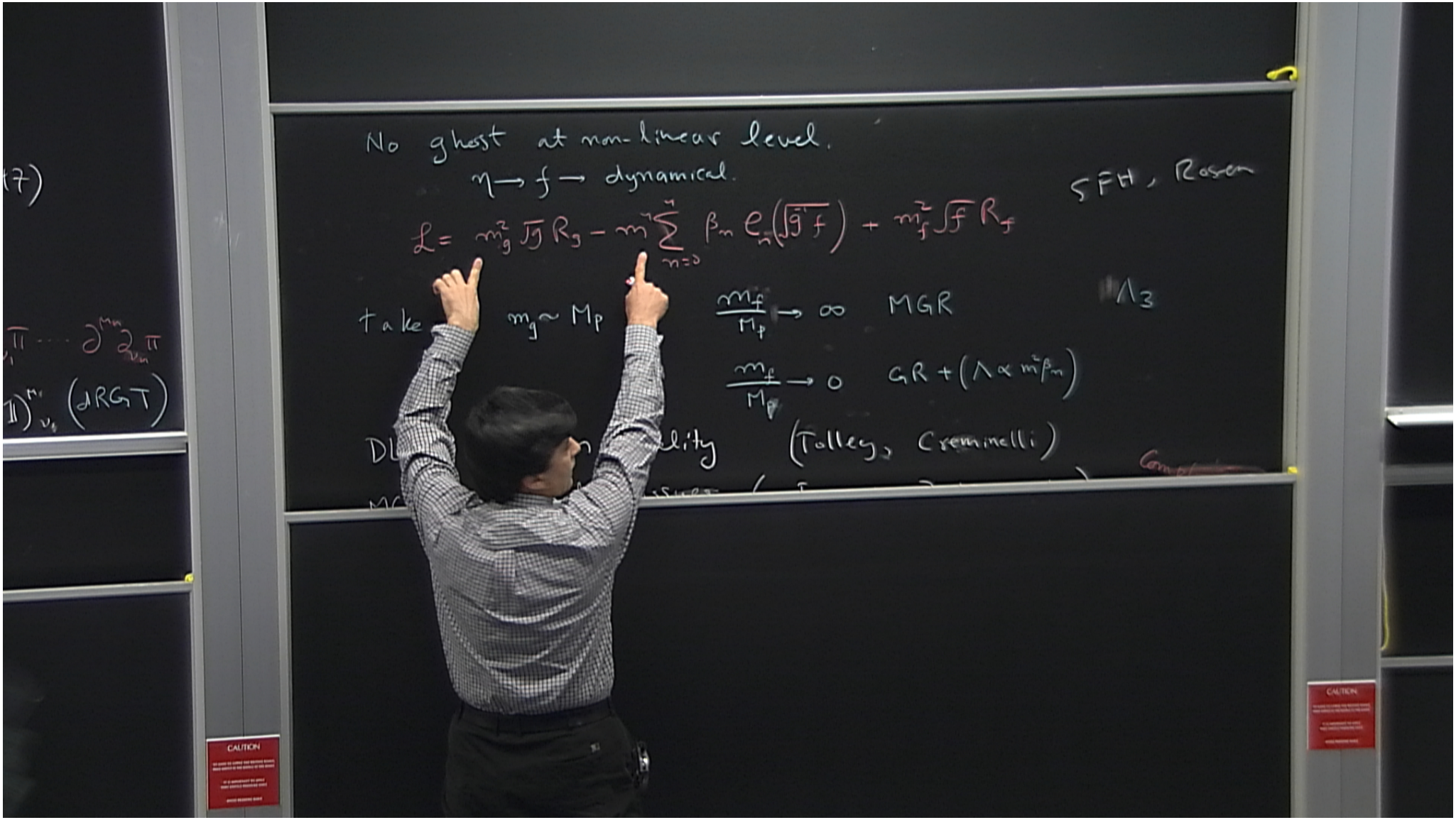

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$K^\mu_\nu = (\sqrt{g^{-1}\eta} - 1)^\mu_\nu$ (dRGT)
CAUTION

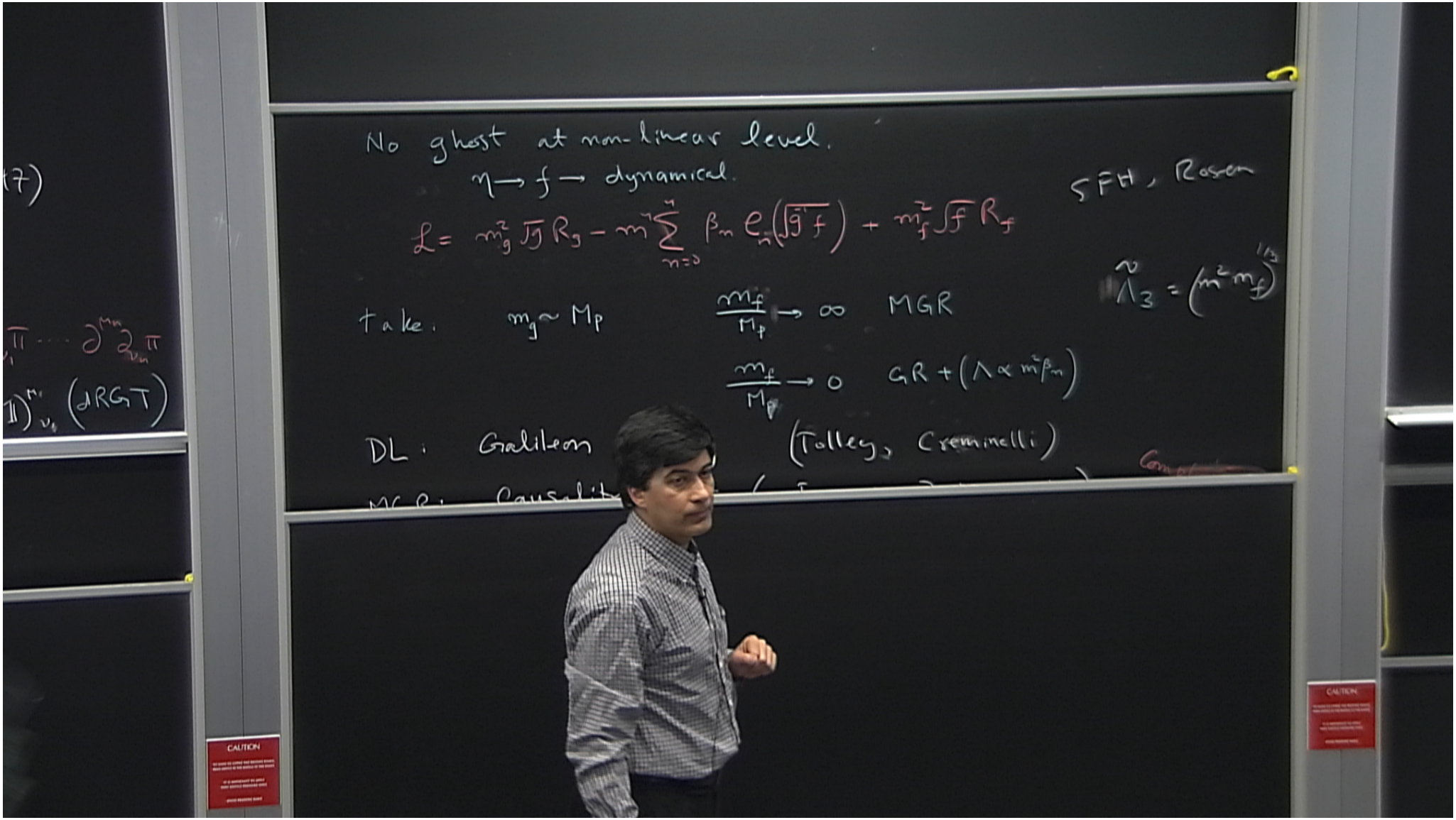

Review
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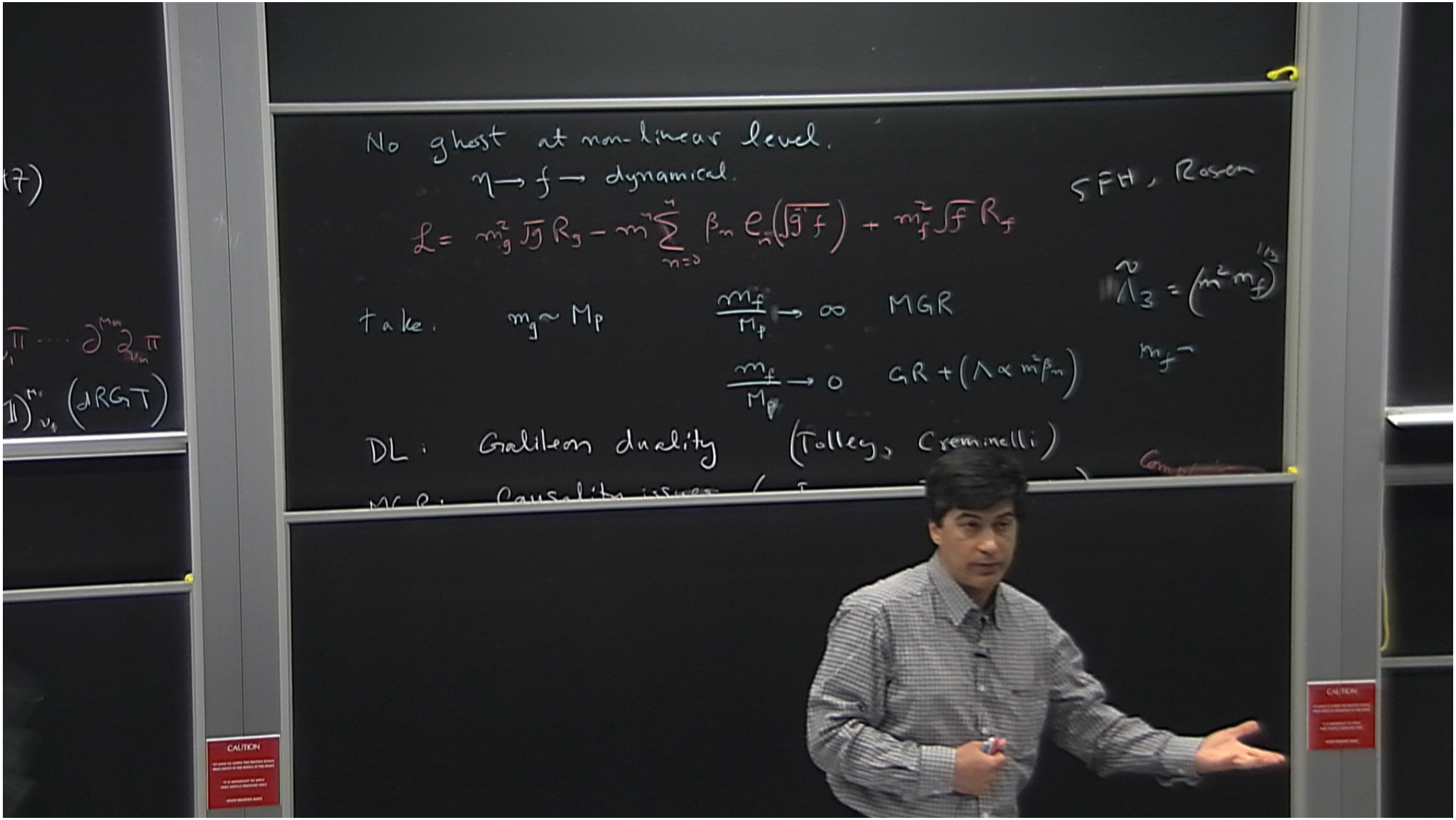

No ghost at non-linear level.
$\eta \to f \to$ dynamical.
$\mathcal{L} = m_g^2 \sqrt{g} R_g - m^2 \sum_{n=0}^{4} \beta_n e_n\left(\sqrt{g^{-1} f}\right) + m_f^2 \sqrt{f} R_f$
SFH, Rosen
take: $m_g \sim M_p$ $\frac{m_f}{M_p} \to \infty$ MGR
(dRGT)
CAUTION

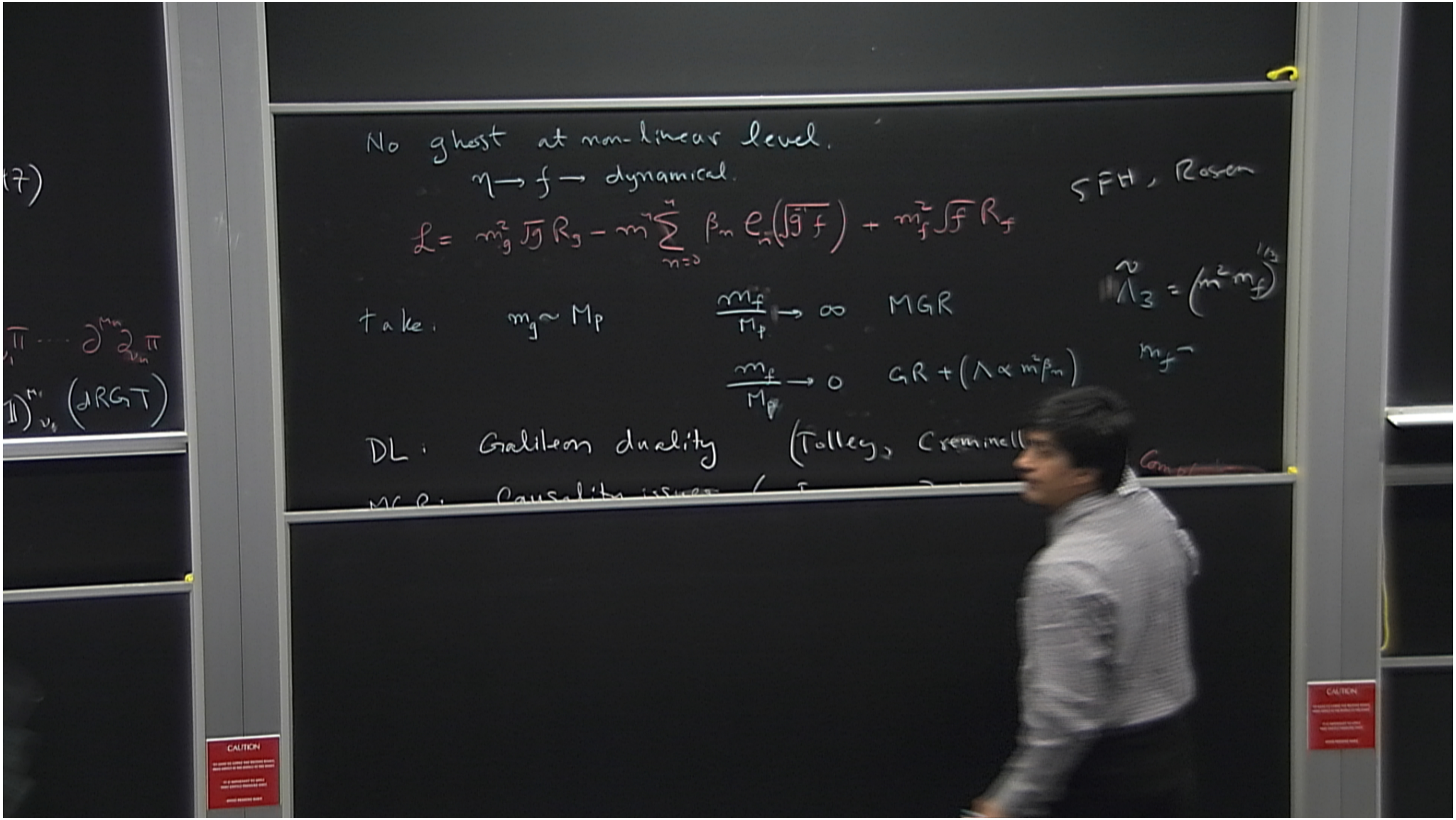

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SFH, Rosen
take: $m_g \sim M_P$
$\frac{m_f}{M_P} \to \infty$ MGR
$\frac{m_f}{M_P} \to 0$ GR + $(\Lambda \propto m^2 \beta_n)$
DL: Galileon duality (Talley, Creminelli)
CAUTION

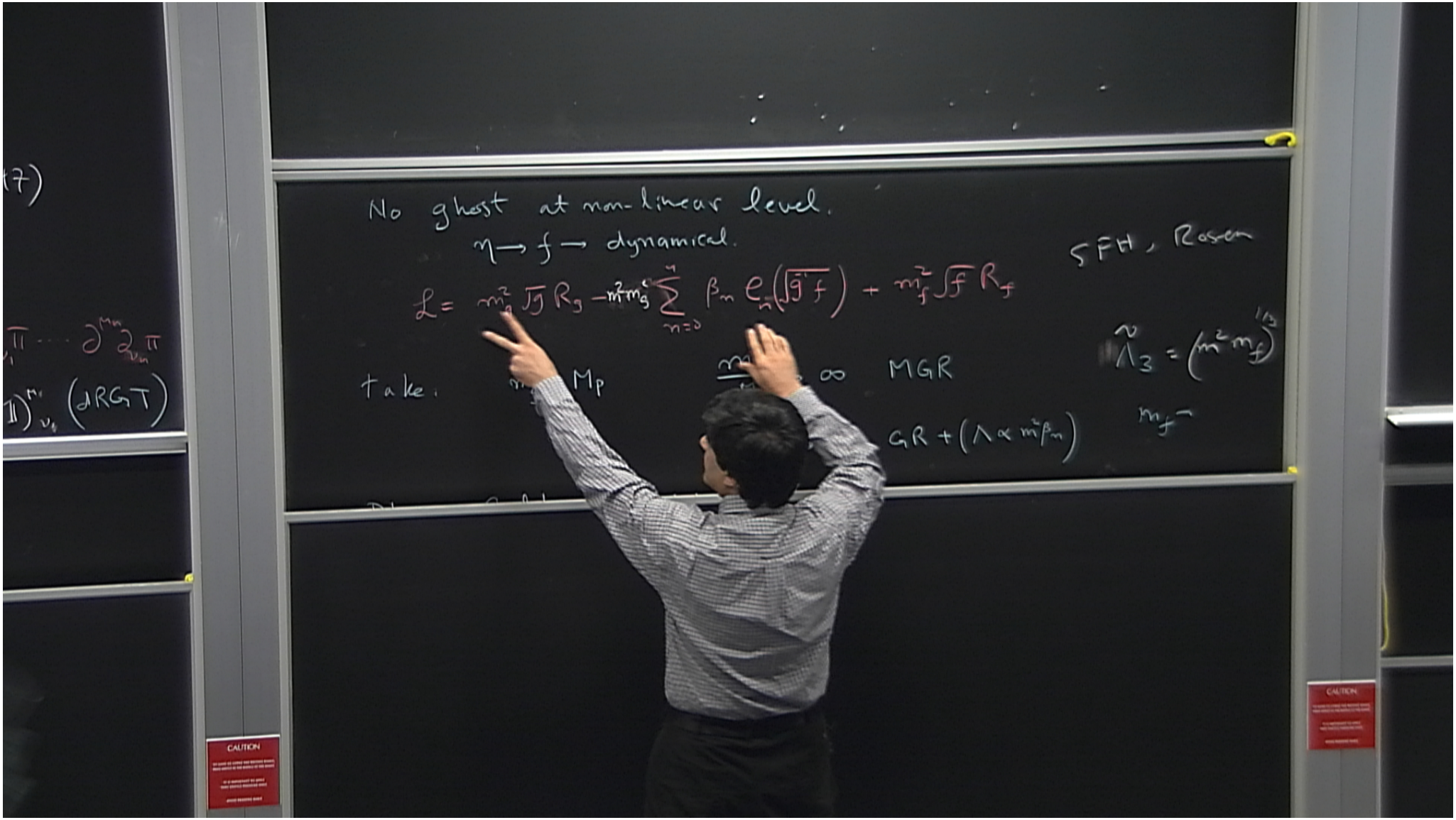

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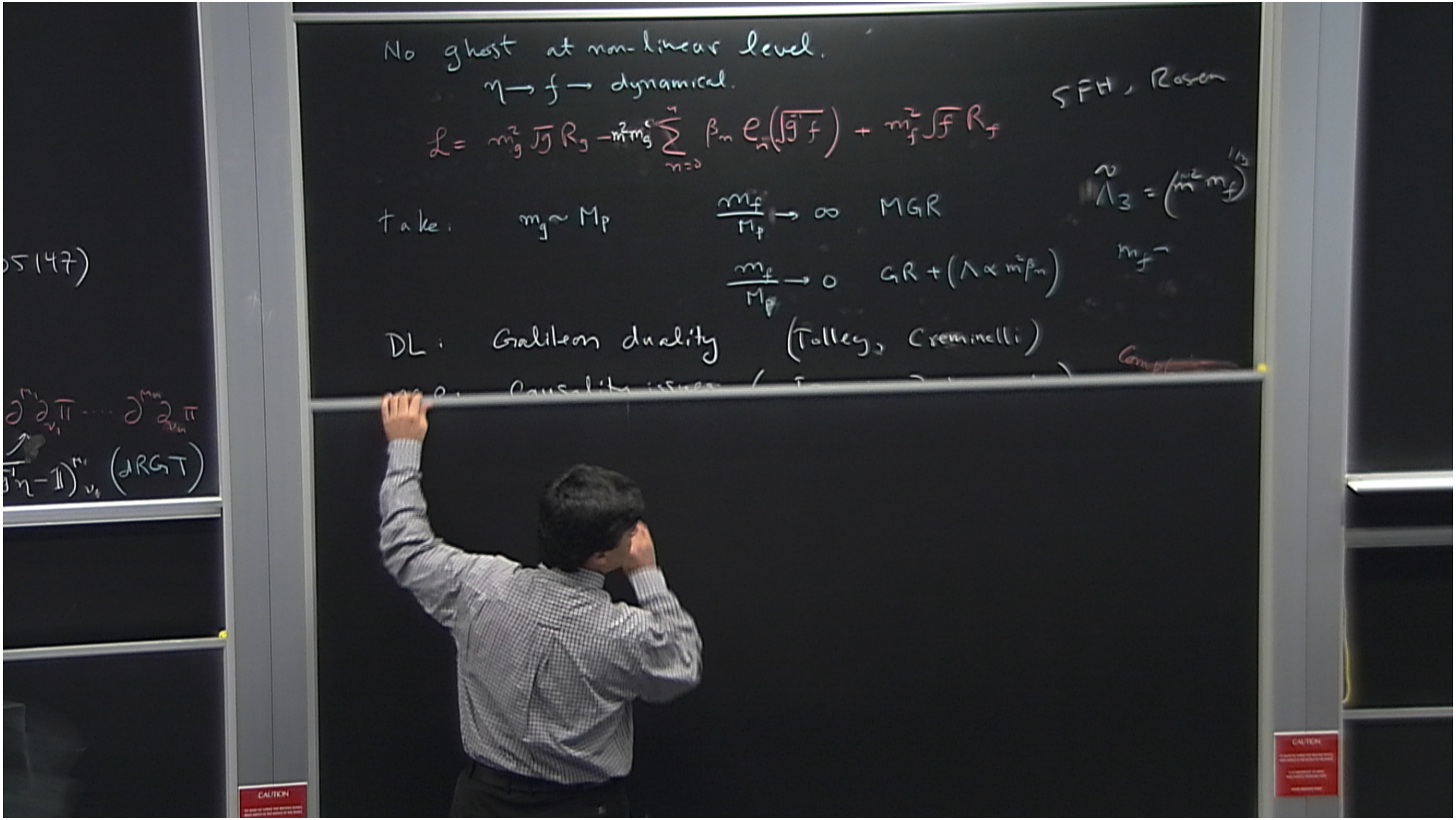

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(Talley, Creminelli)

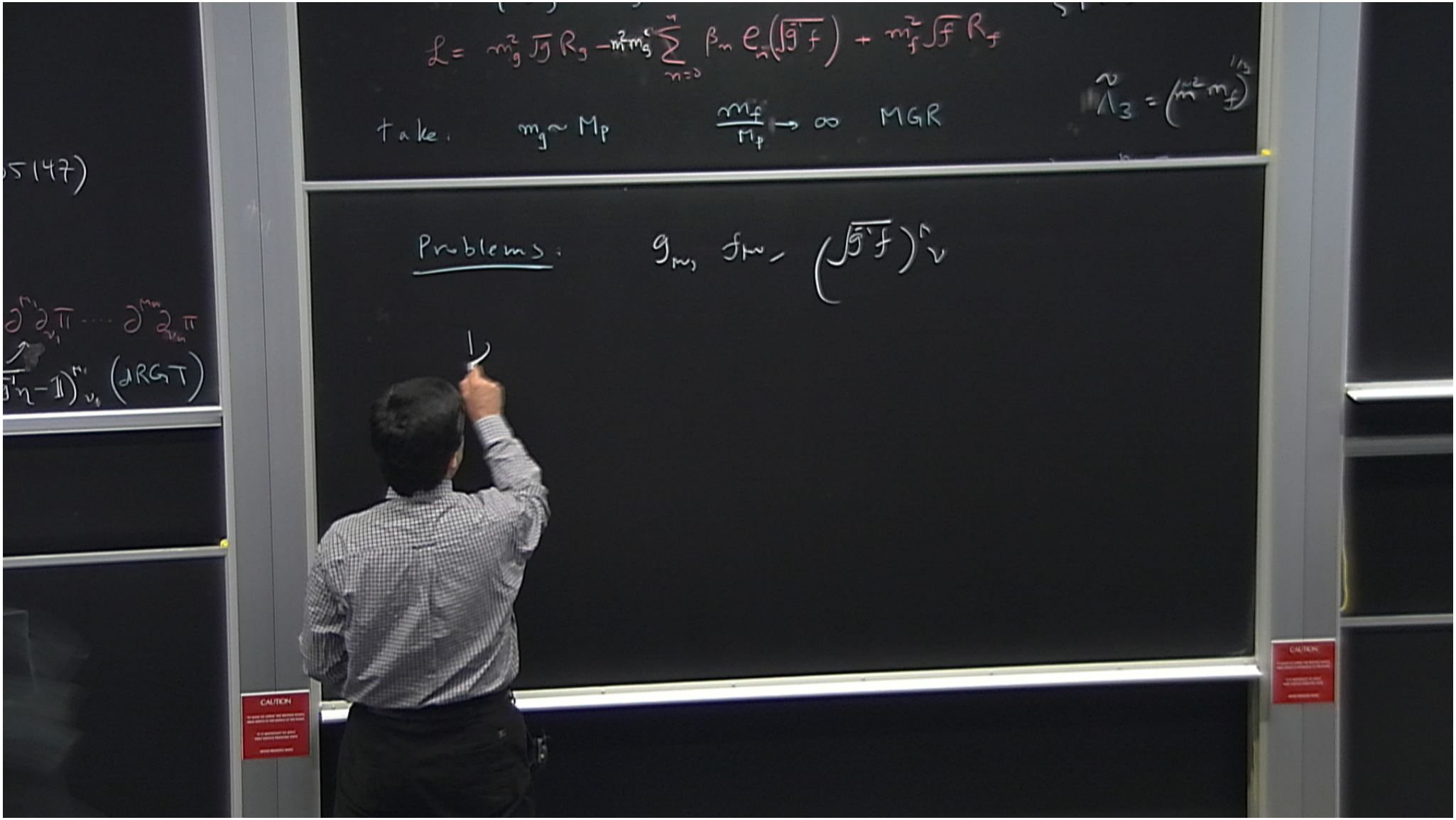

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CAUTION
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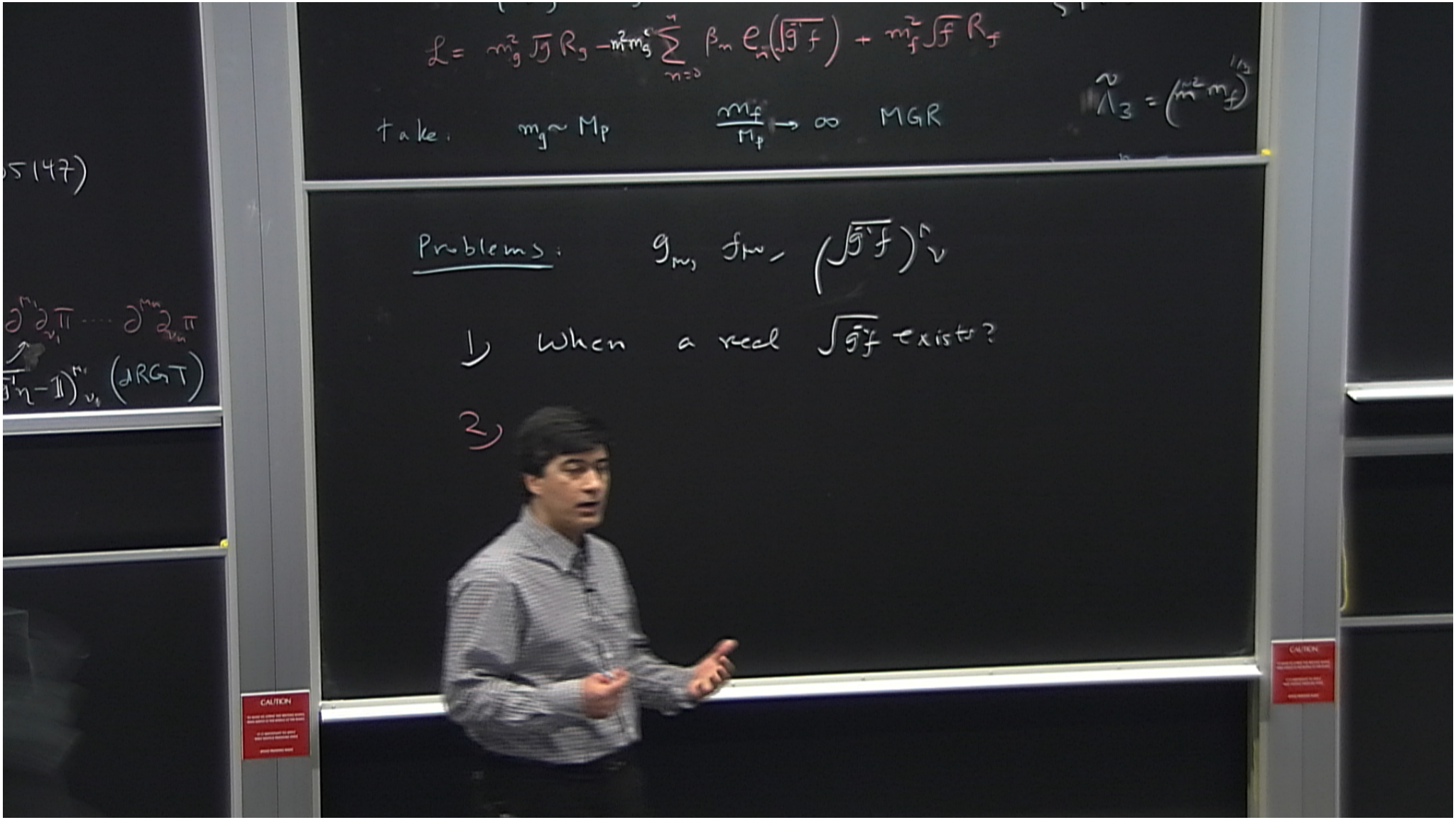

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$\tilde{\Lambda}_3 = (m^2 m_f)^{1/3}$
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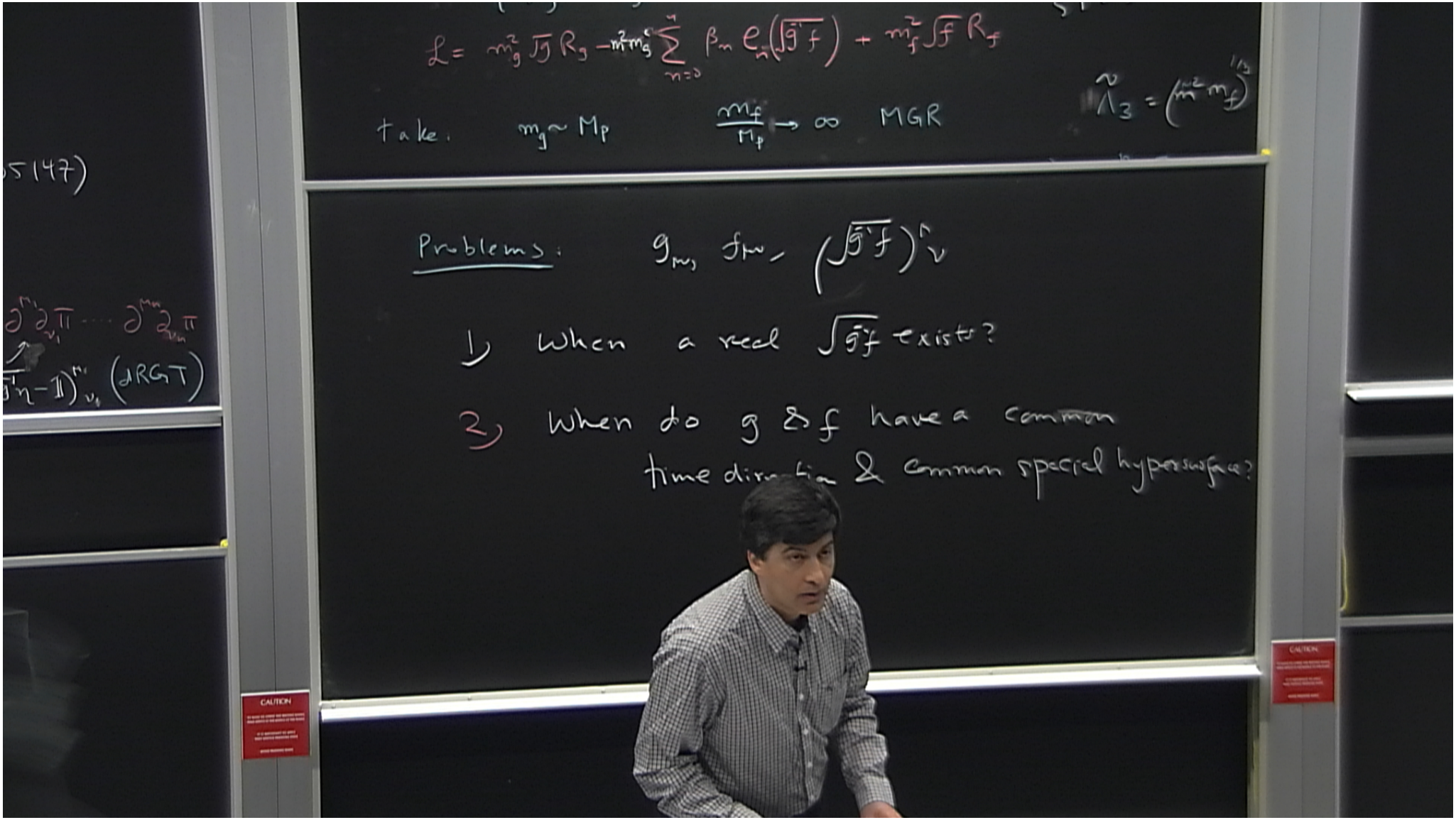

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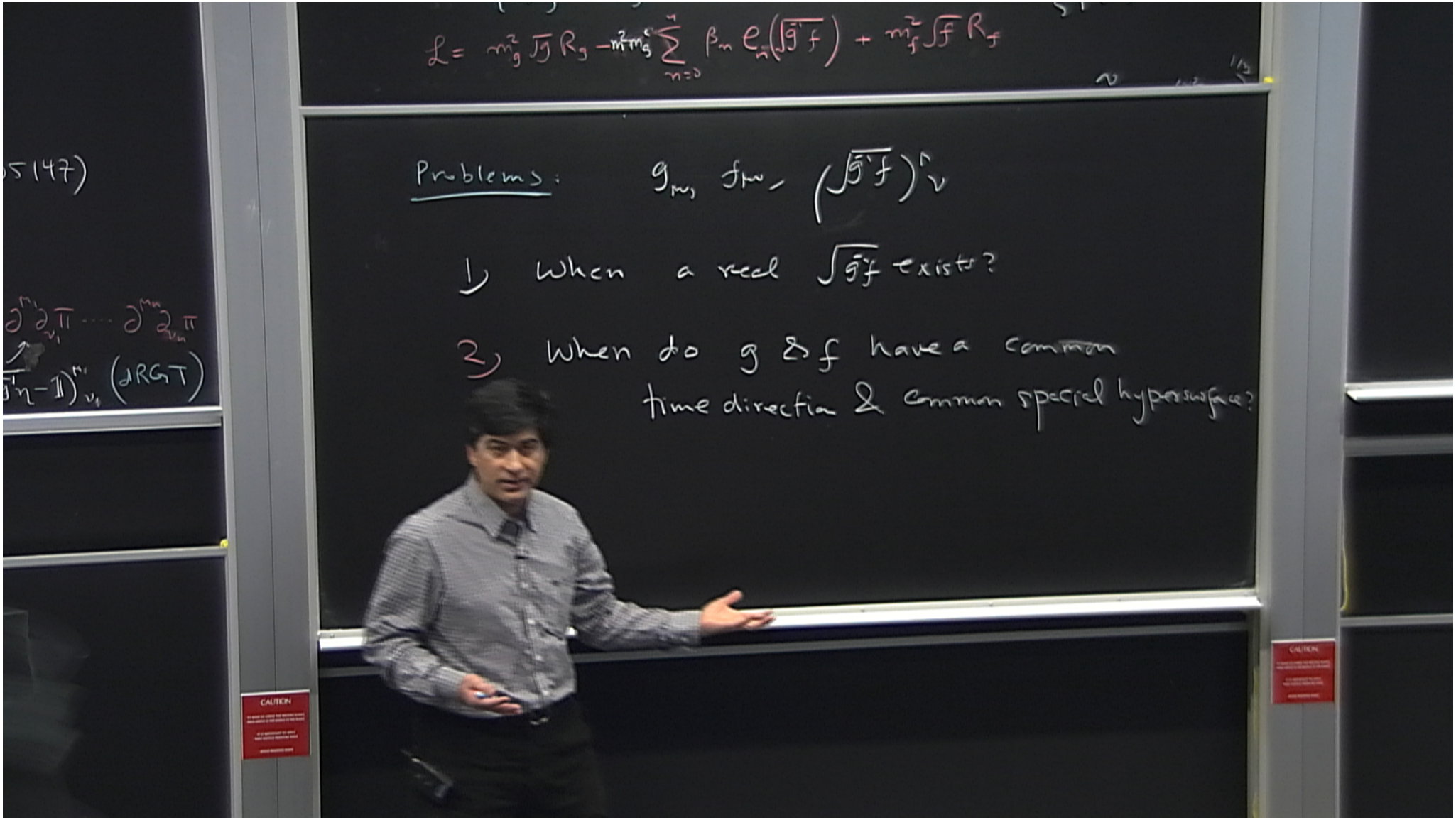

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SFH, Rosen
MGR
GR + (Λ ∝ m²β_n)
CAUTION

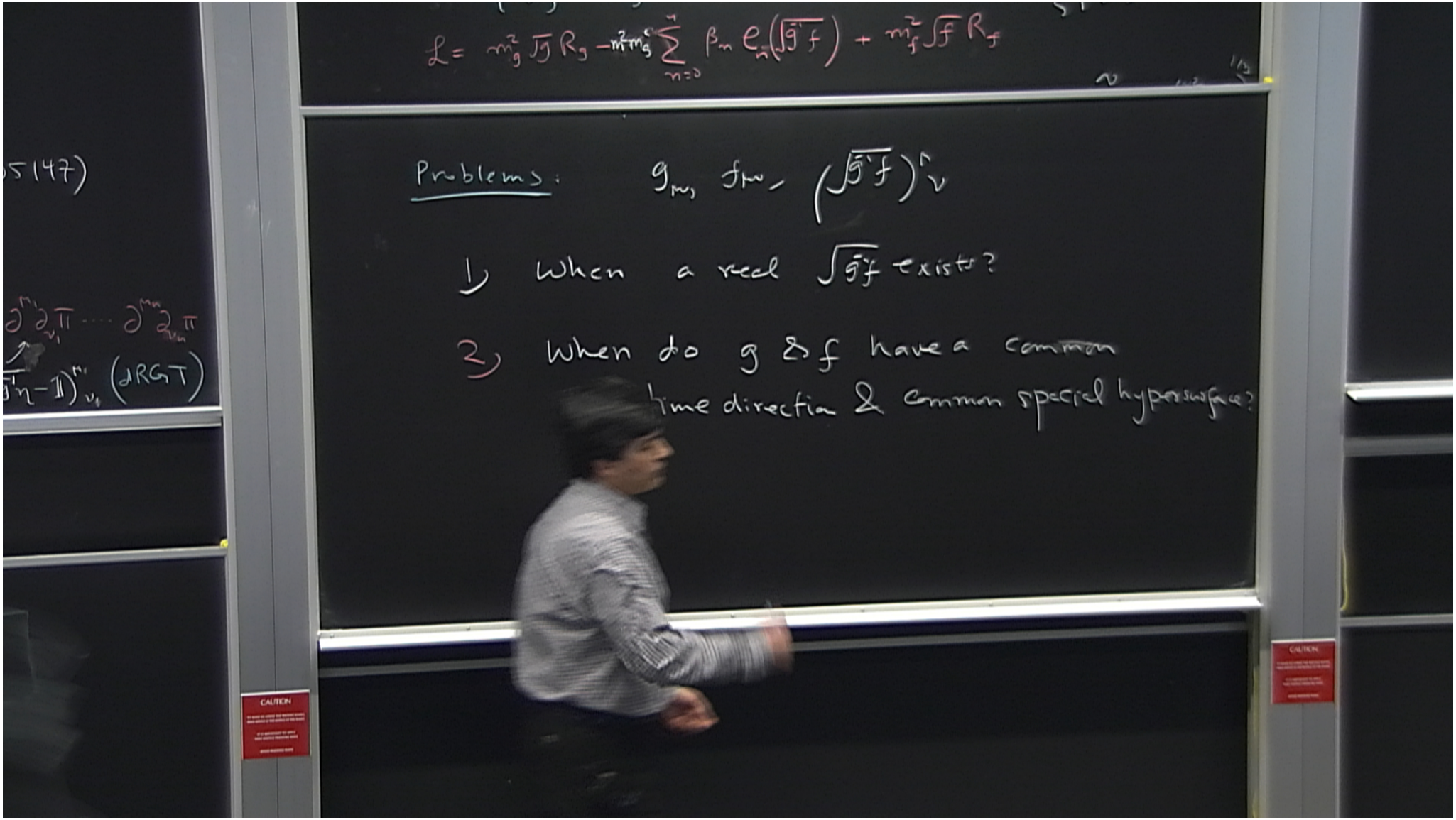

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$\frac{m_f}{M_P} \to 0$ GR + $(\Lambda \propto m^2 \beta_n)$
DL: Galileon duality (Tolley, Creminelli)
Comp
Causality issues
$\partial^{n_1} \partial_{\nu_1} \pi \cdots \partial^{n_m} \partial_{\nu_m} \pi$
$\left(\sqrt{g^{-1} f}\right)^{n}_{\nu_1}$ (dRGT)
5147)
CAUTION
CAUTION

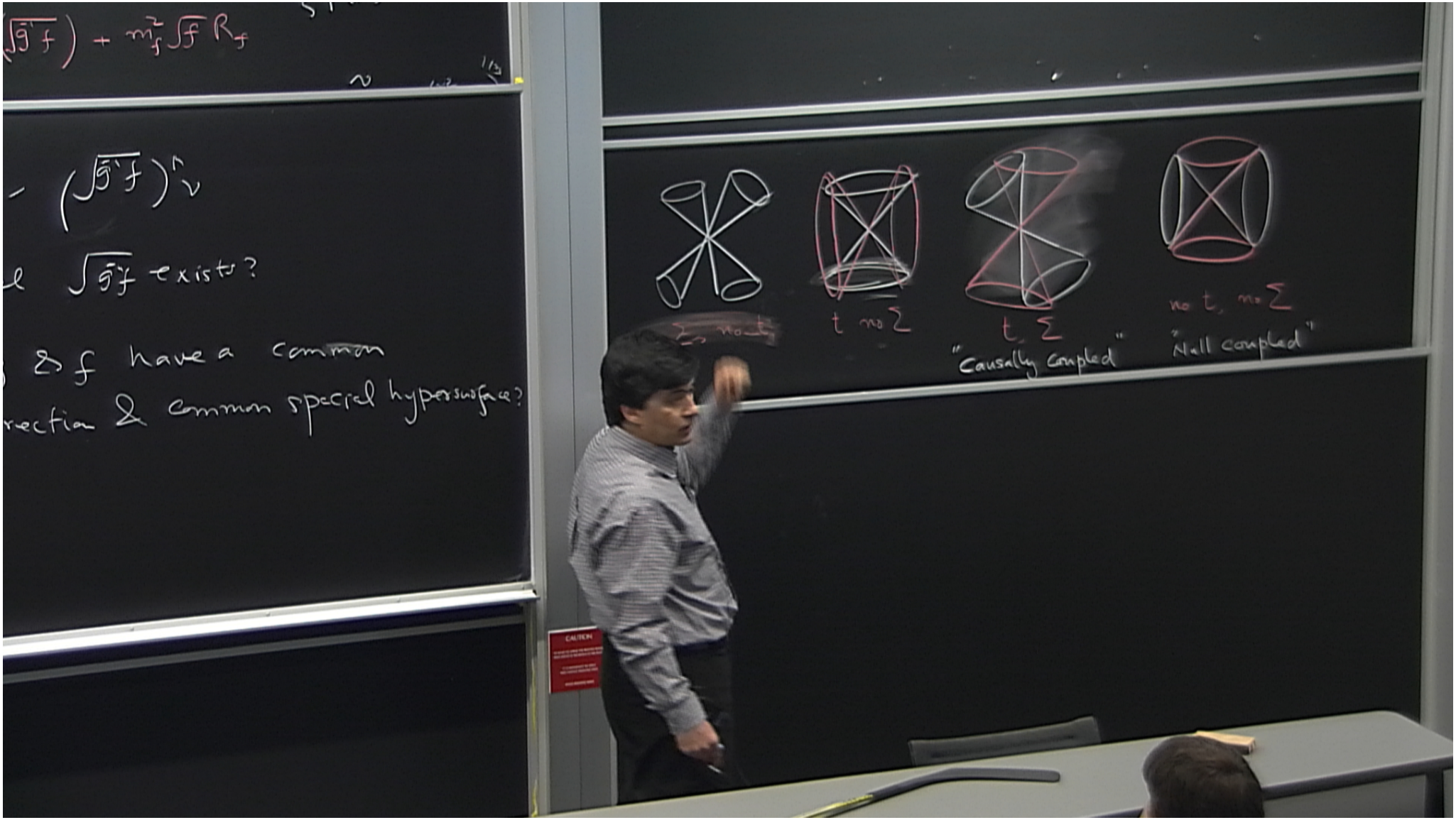

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Problems:
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CAUTION
CAUTION

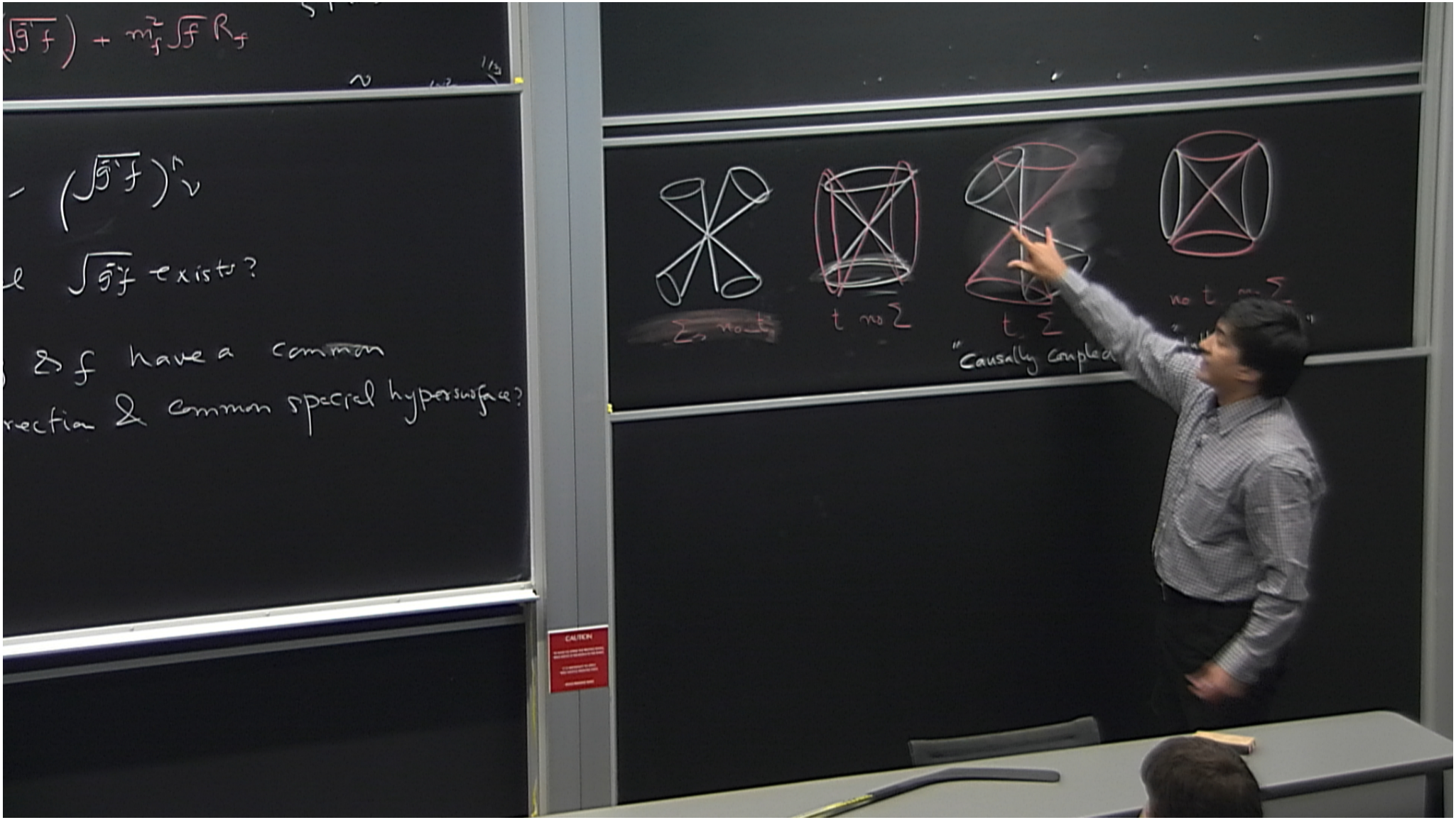

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Problems:
$g_{\mu\nu}, f_{\mu\nu} - (\sqrt{g^{-1} f})^{\mu}_{\nu}$
1) When a real $\sqrt{gf}$ exists?
2)
CAUTION
CAUTION


$\mathcal{L} = m_g^2 \sqrt{g} R_g - m^2 m_g^2 \sum_{n=0}^{4} \beta_n e_n\left(\sqrt{g^{-1} f}\right) + m_f^2 \sqrt{f} R_f$
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$\Lambda_3 = \left(m^2 m_f\right)^{1/3}$
Problems: $g_{\mu\nu}, f_{\mu\nu} \to \left(\sqrt{g^{-1} f}\right)^{\mu}_{\nu}$
1) When a real $\sqrt{g^{-1} f}$ exists?
2) When do g & f have a common time direction & common special hypersurface?

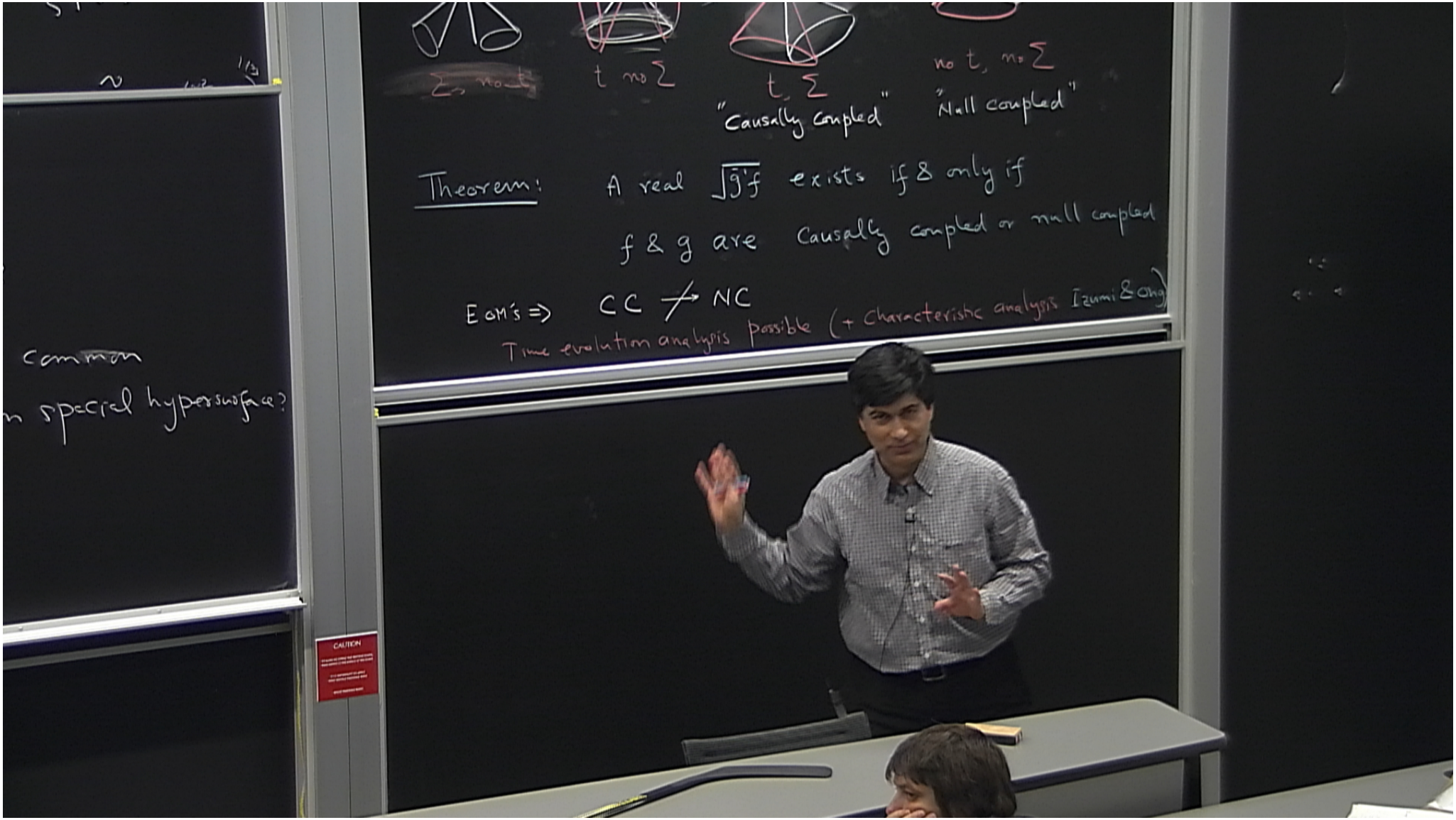

Problems:
1) When a real $\sqrt{g^{-1}f}$ exists?
3) When do g & f have a common time direction & common spacial hypersurface?

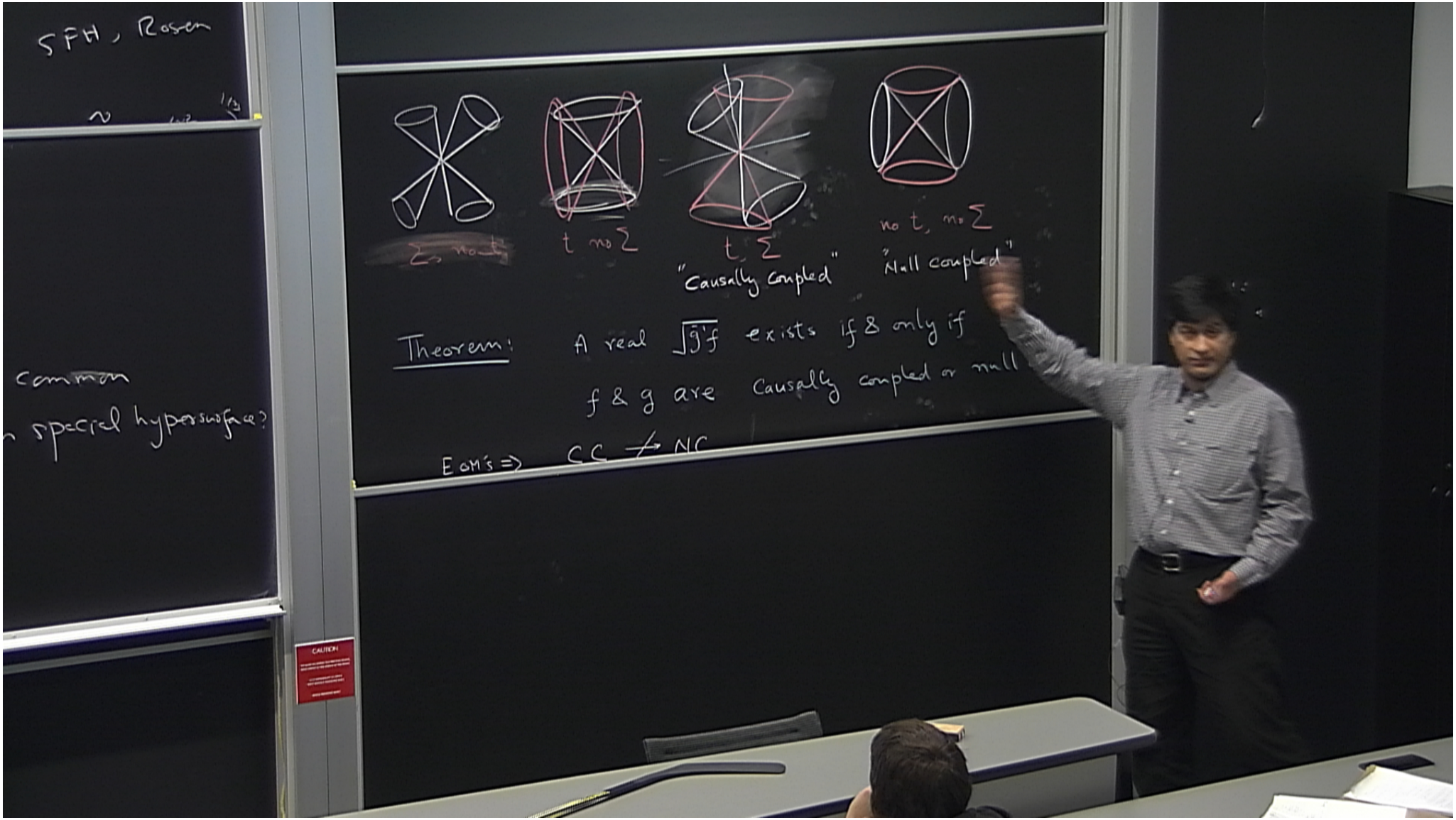

$\mathcal{L} = m_g^2 \sqrt{g} R_g - m^2 m_g^2 \sum_{n=0}^{4} \beta_n e_n\left(\sqrt{g^{-1} f}\right) + m_f^2 \sqrt{f} R_f$
Problems:
$g_{\mu\nu}$, $f_{\mu\nu}$ – $\left(\sqrt{g^{-1} f}\right)^{\mu}_{\nu}$
1) When a real $\sqrt{g^{-1} f}$ exists?
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CAUTION

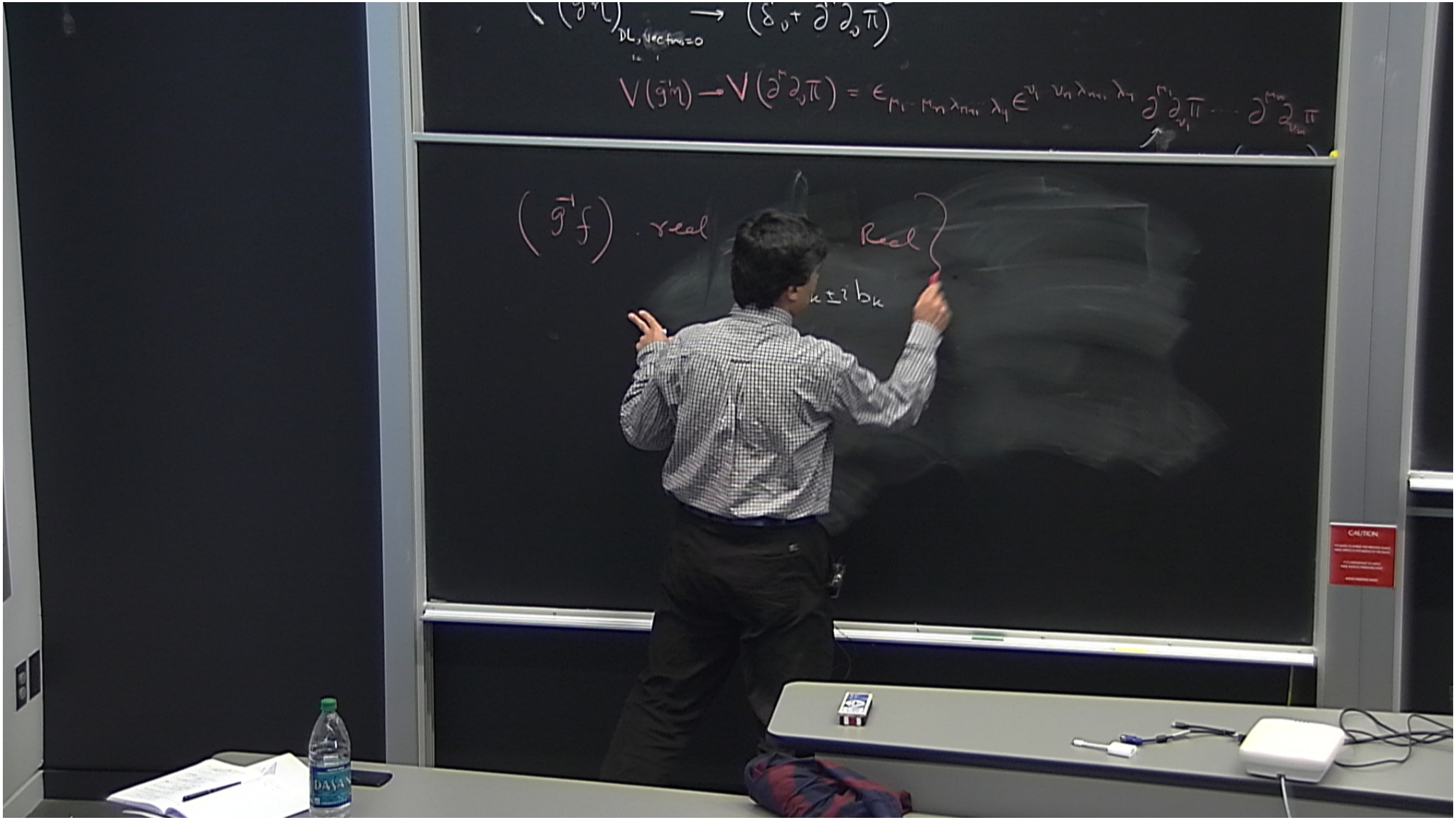

$\sqrt{\bar{g}}\,f\big) + m_f^2 \sqrt{f}\, R_f$
$(\sqrt{\bar{g}}\,f)^{\wedge}_{\vee}$
$\sqrt{\bar{g}}f$ exists?
& f have a common
& common special hypersurface?
t no Σ
t, Σ
"Causally coupled"
no t, no Σ
"Null coupled"

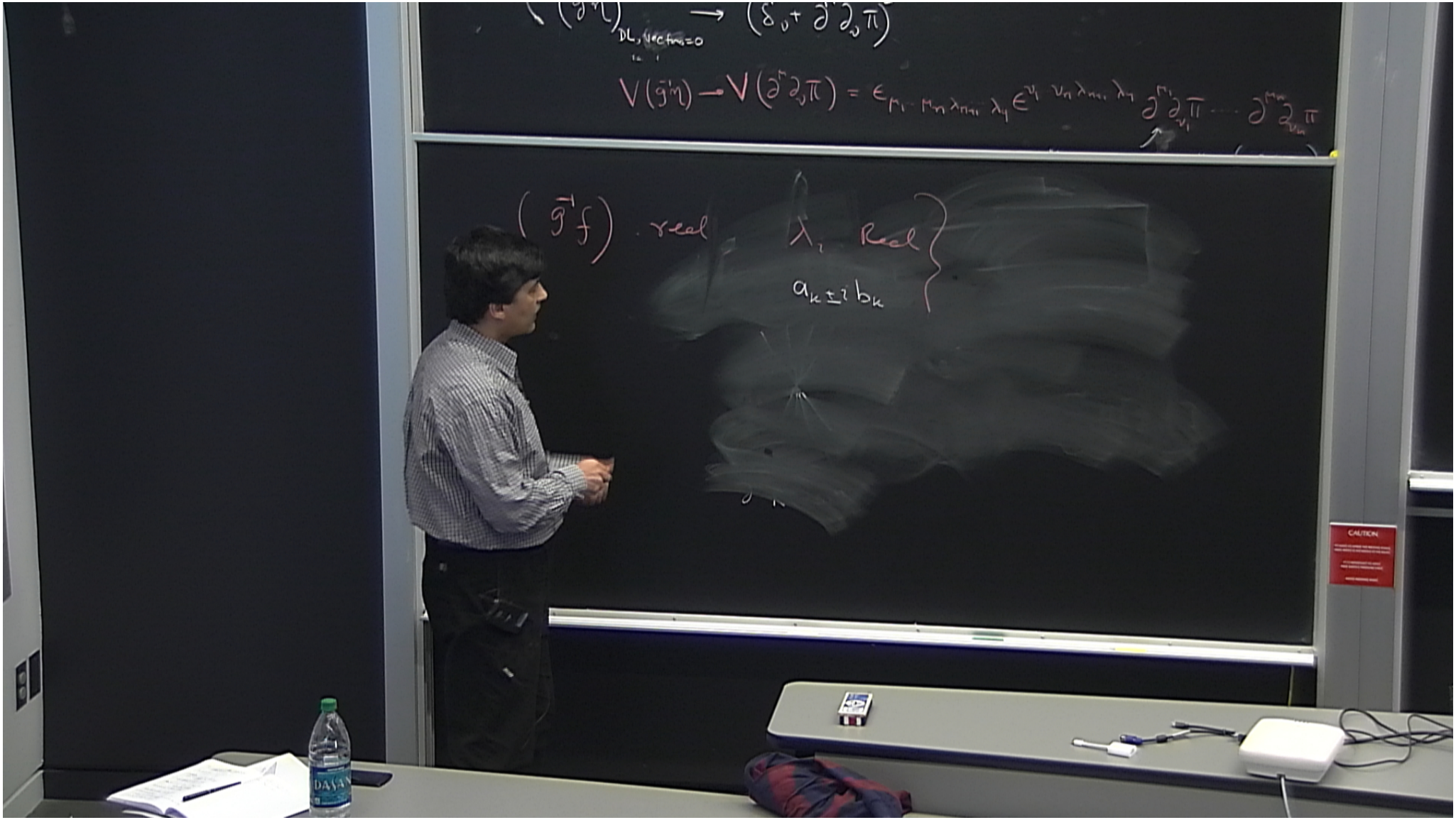

$\sqrt{g}f) + m_f^2 \sqrt{f} R_f$
$(\sqrt{g}f)^{\wedge}_{\vee}$
$\sqrt{g}f$ exists?
& f have a common
rection & common special hypersurface?
t no Σ
t, Σ
"Causally Complete"
CAUTION

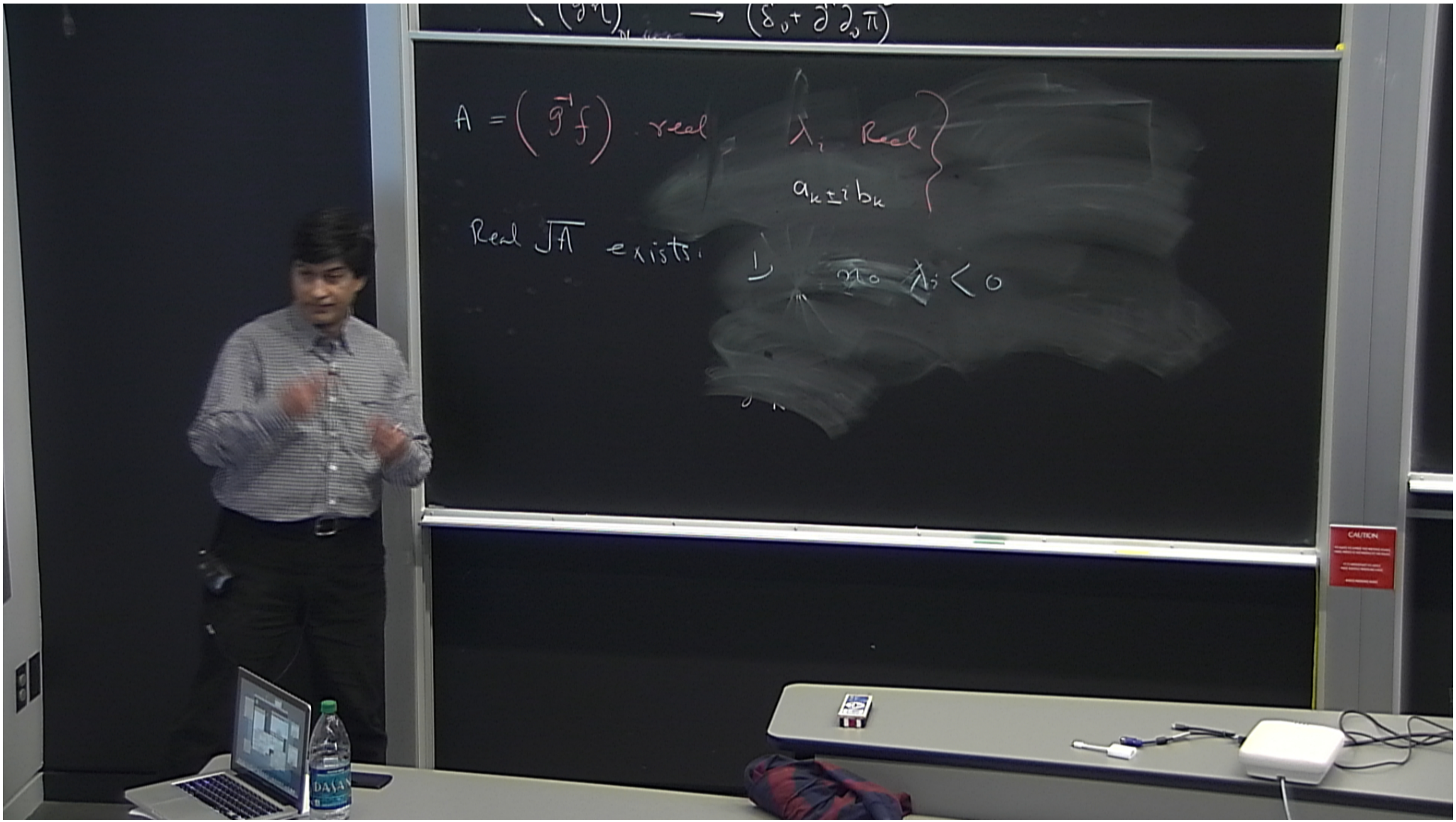

$+ m_f^2 \sqrt{f} R_f$
$\sqrt{f})^{\wedge}_{\nu}$
$\sqrt{f}$ exists?
have a common
& common special hypersurface?
Σ, no t
t, no Σ
t, Σ
"Causally coupled"
no t, no Σ
"Null coupled"
Theorem:
CAUTION


"Causally coupled"
"Null coupled"
Theorem:
A real $\sqrt{g'f}$ exists if & only if
f & g are Causally coupled or null coupled
EoM's ⇒ CC ⇏ NC
Time evolution analysis possible (+ Characteristic analysis Izumi & Ong)
common
special hypersurface?
CAUTION

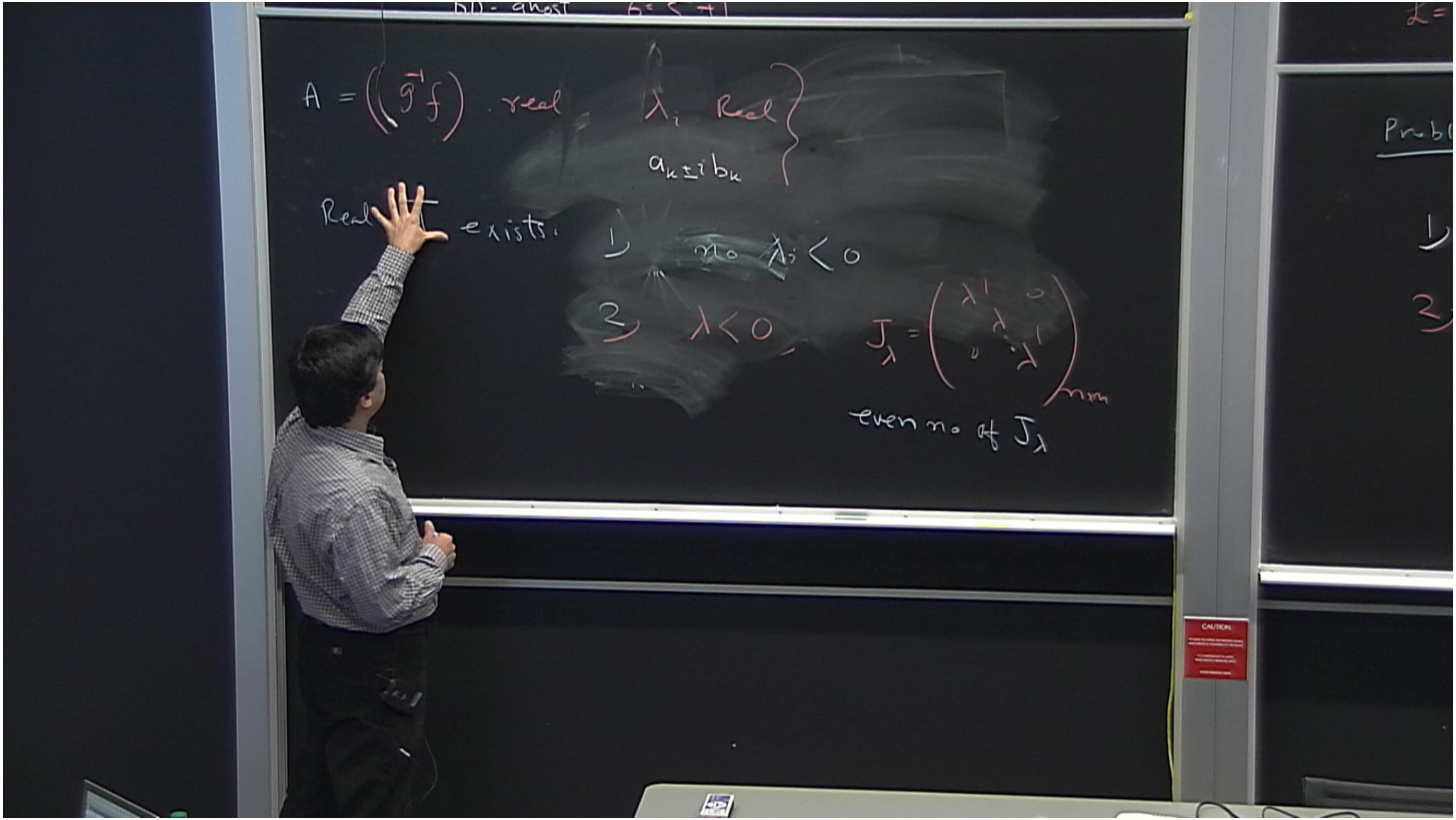

SFH, Rosen
common
special hypersurface?
t no Σ
t, Σ
"Causally Coupled"
no t, no Σ
"Null Coupled"
Theorem:
A real √g f exists if & only if
f & g are Causally coupled or null
EoM's ⇒ CC ≠ NC

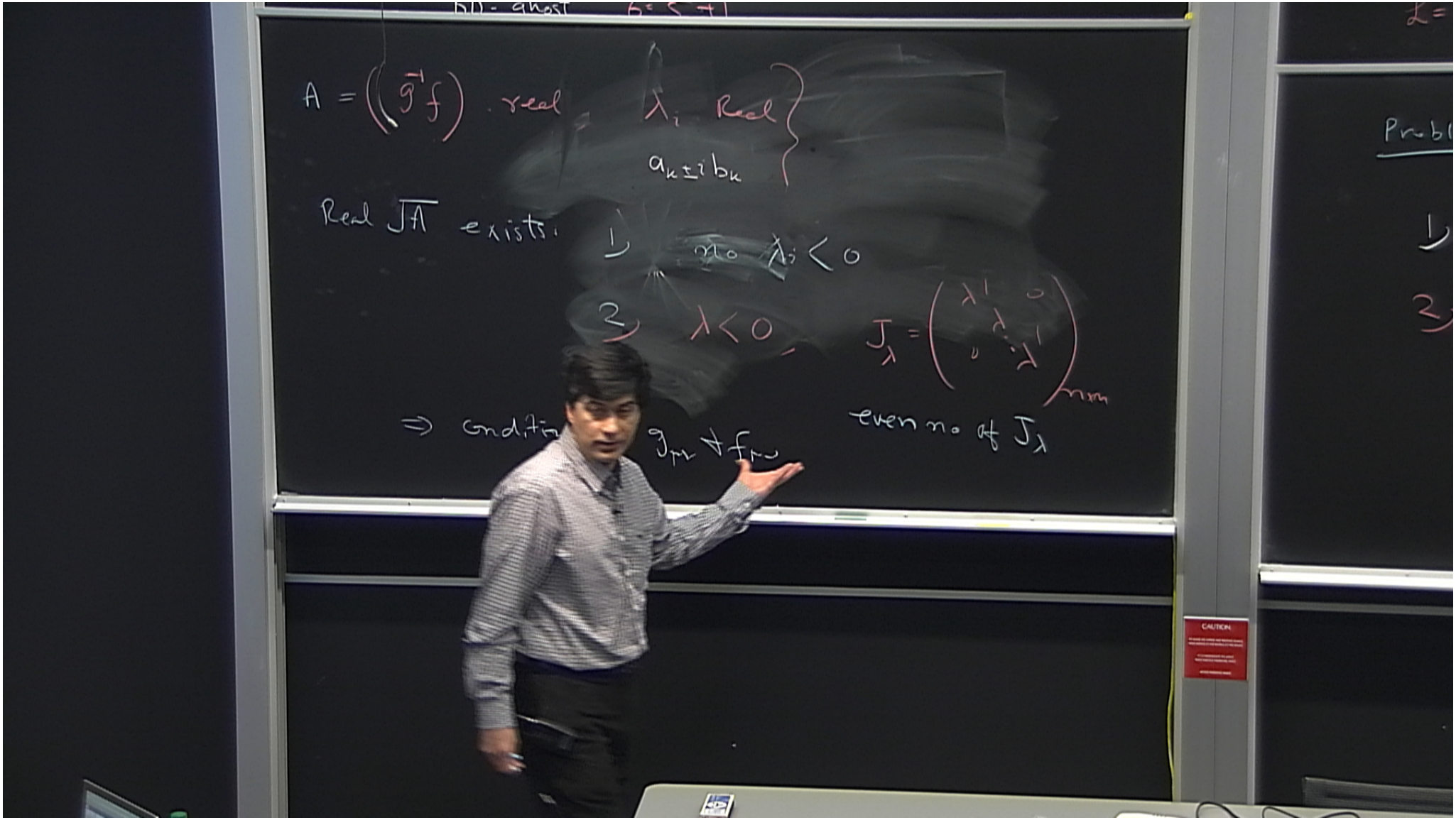

$\to (\delta_{\mu\nu} + \partial_\mu \partial_\nu \pi)$
$V(\vec{g}^{\,i}) \to V(\partial^{\mu} \partial_\nu \pi) = \epsilon_{\mu_1 \cdots \mu_n \lambda_{n+1} \cdots \lambda_4} \epsilon^{\nu_1 \cdots \nu_n \lambda_{n+1} \cdots \lambda_4} \partial^{\mu_1} \partial_{\nu_1} \pi \cdots \partial^{\mu_n} \partial_{\nu_n} \pi$
$(\vec{g}^{\,i} f)$ real
Real
$a_k \pm i\, b_k$
CAUTION

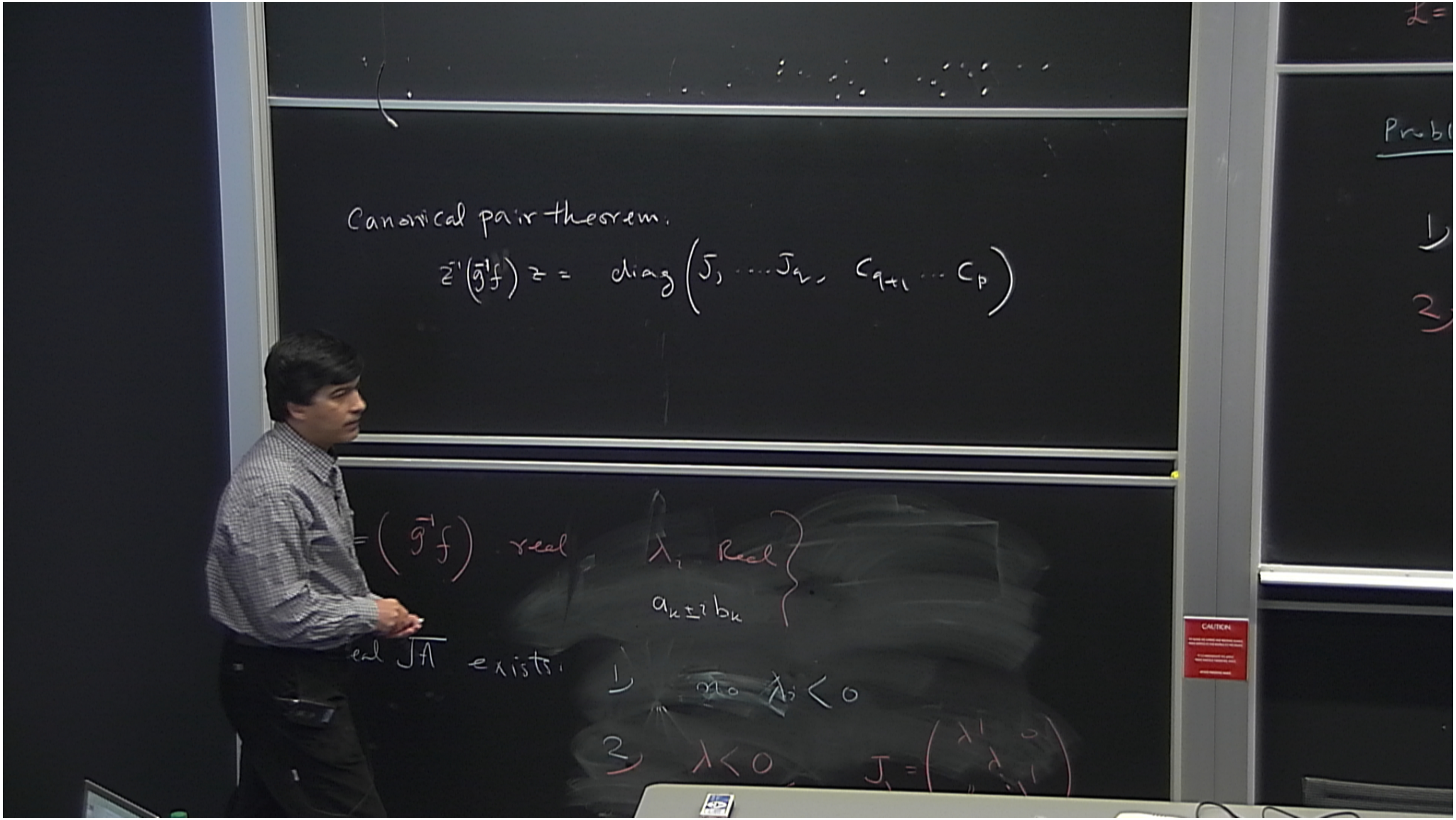

DL, Vector=0
$\rightarrow (\delta_{ij} + \partial_i \partial_j \pi)$
$V(\vec{g}^{\,i}m) \rightarrow V(\partial^{\mu}\partial_{\nu}\pi) = \epsilon_{\mu_1 \ldots \mu_m \lambda_{m+1} \ldots \lambda_4} \epsilon^{\nu_1 \ldots \nu_m \lambda_{m+1} \ldots \lambda_4} \partial^{\mu_1}\partial_{\nu_1}\pi \cdots \partial^{\mu_m}\partial_{\nu_m}\pi$
$(\vec{g}^{\,i}f)$ real
λ_i Real
$a_k \pm i\, b_k$
CAUTION

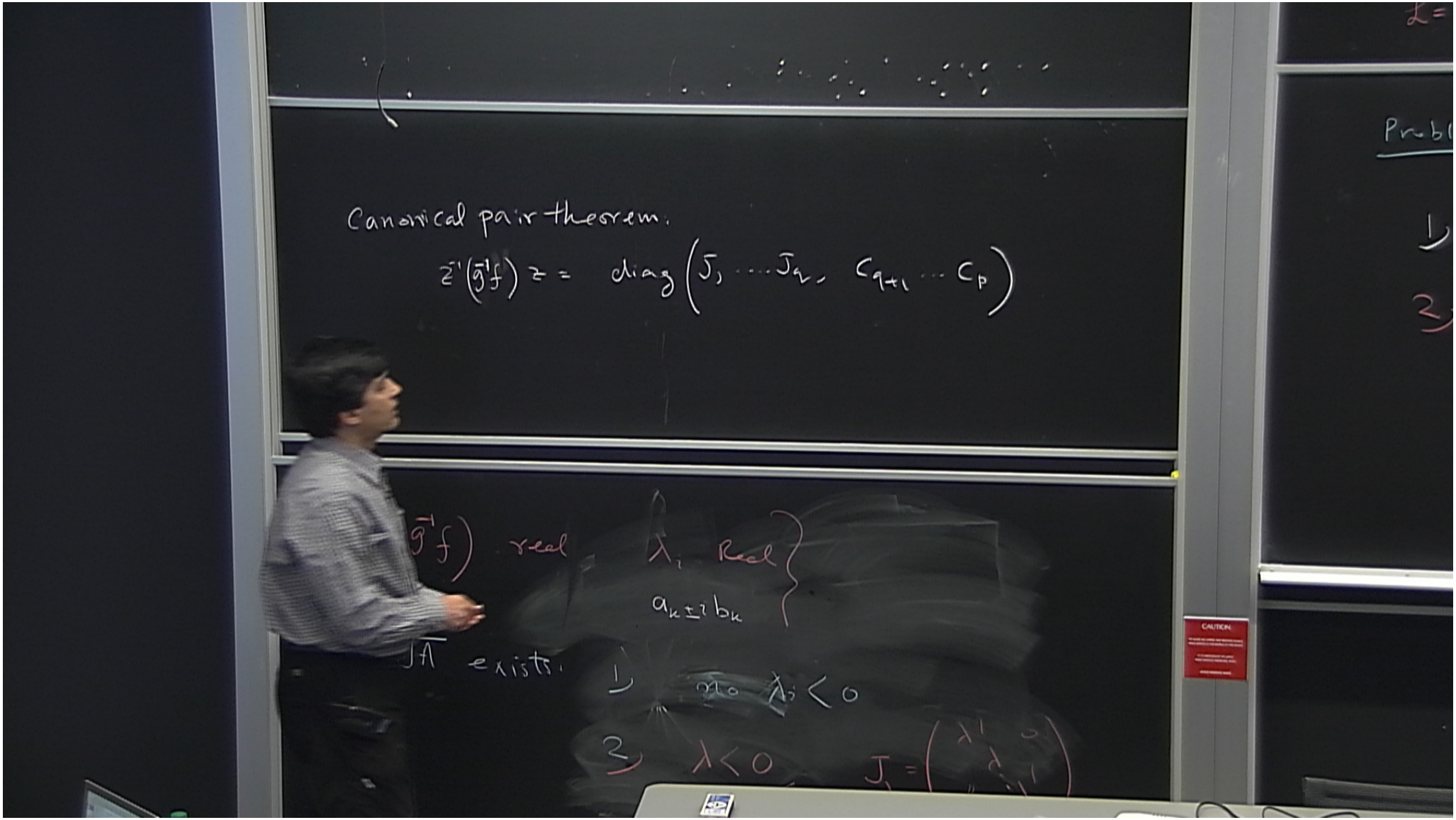

$A = (\vec{\nabla} f)$ real
λ_i Real
$a_k \pm i b_k$
Real $\sqrt{A}$ exists:
1) no $\lambda_i < 0$
CAUTION

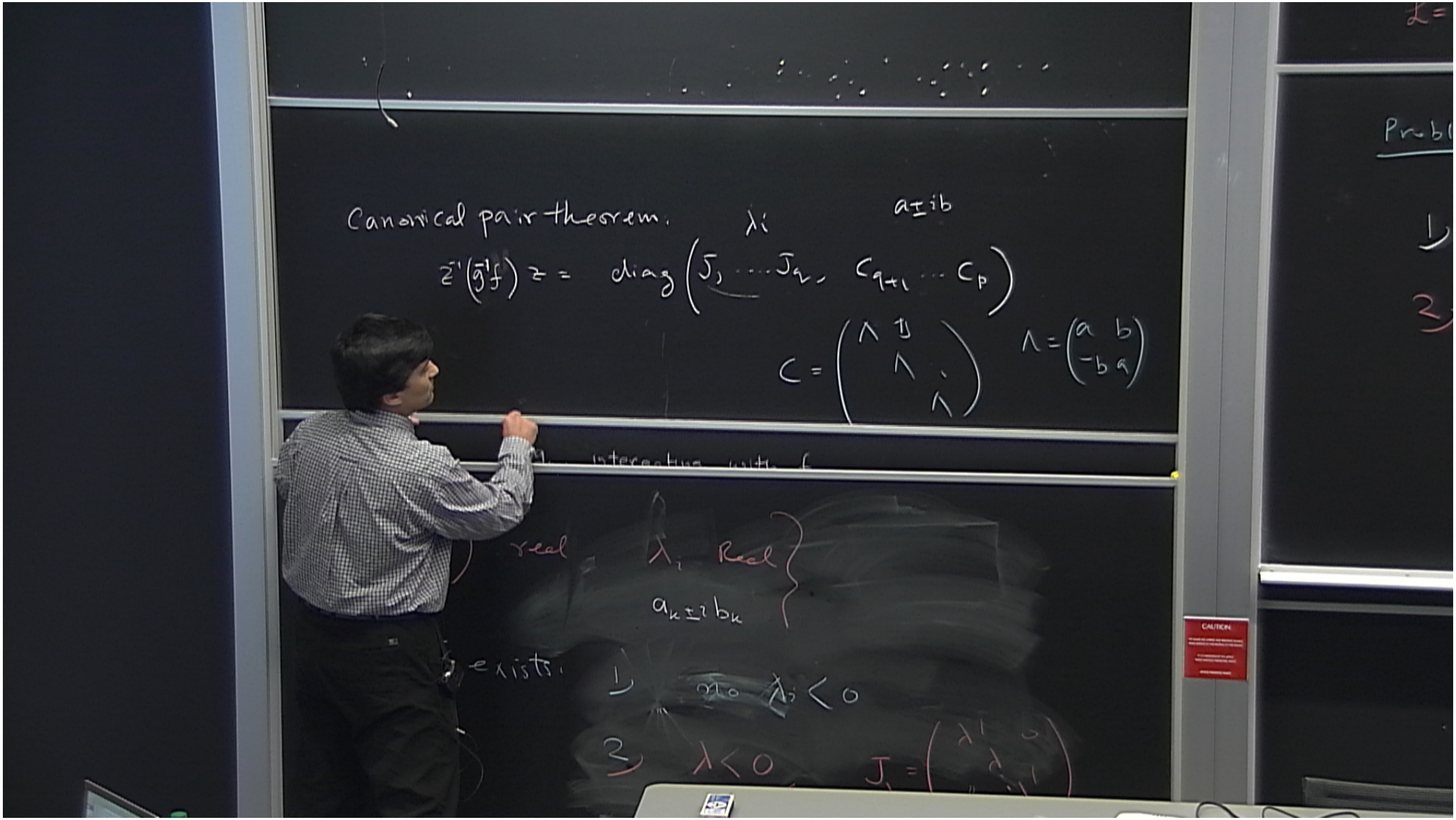

BD - ghost
A = (g⁻¹f) real
λ_i Real
a_k ± i b_k
Real √A exists:
1)
no λ_i < 0
λ < 0
J
CAUTION

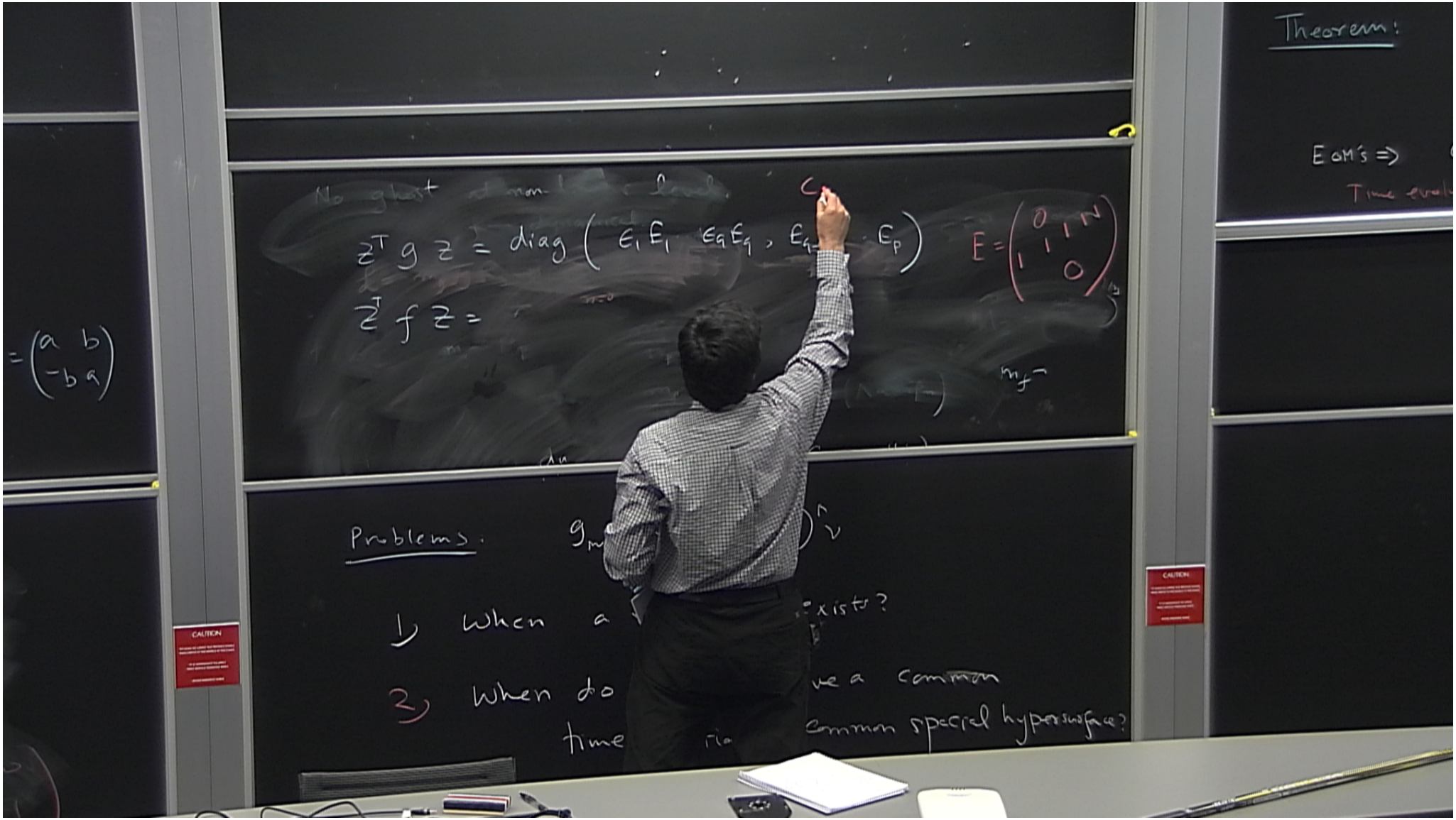

A = (($\vec{g}$ f) real
λ_i Real
$a_k \pm i b_k$
Real H exists:
1) no $\lambda_i < 0$
2) $\lambda < 0$
J_λ = (λ 1 0 ; 0 λ 1 ; 0 0 λ) nxn
even no of J_λ

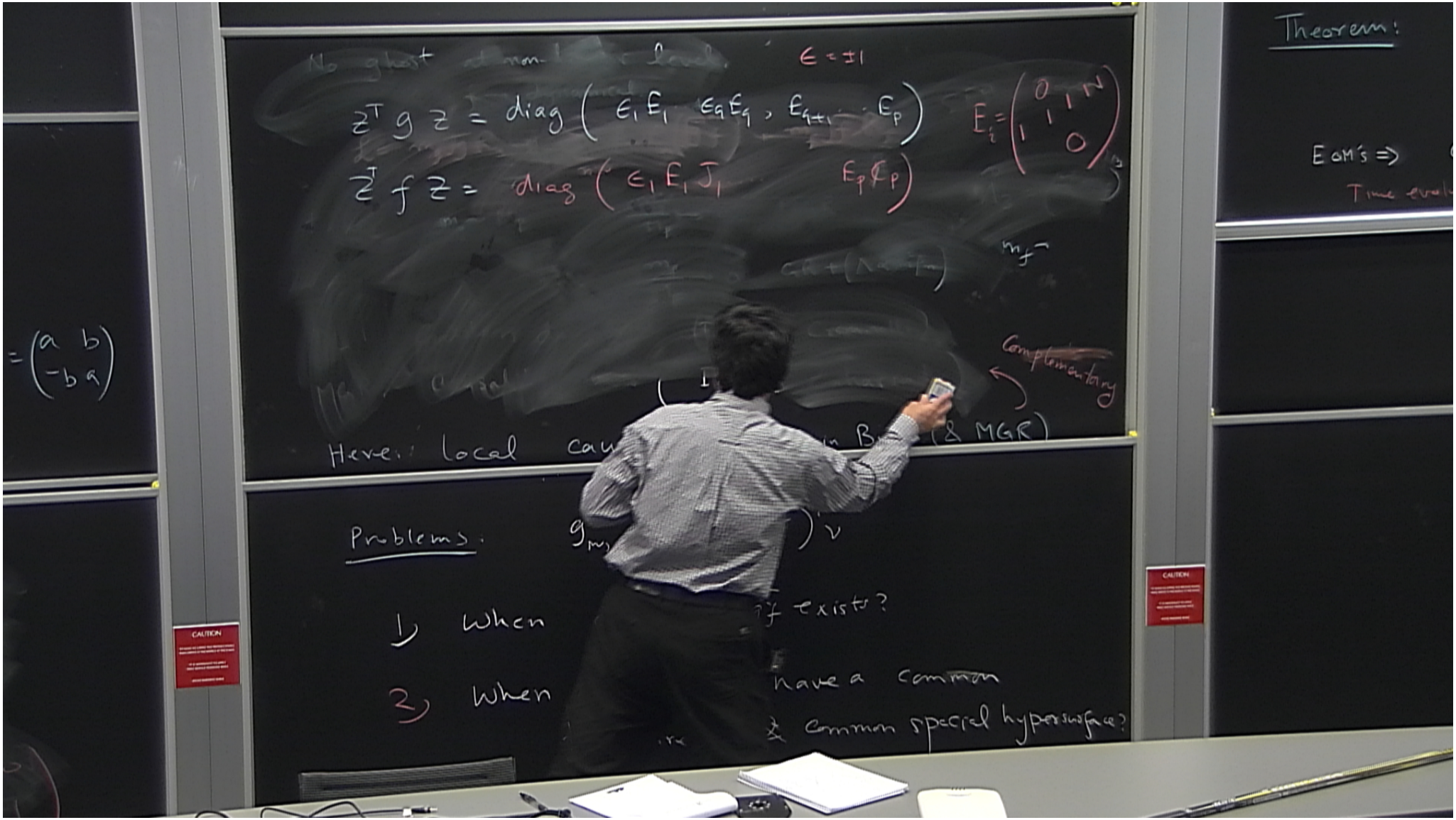

$A = (\vec{g}^{-1} f)$ real
λ_i Real
$a_k \pm i b_k$
Real $\sqrt{A}$ exists:
1) no $\lambda_i < 0$
2) $\lambda < 0$
$J_\lambda = \begin{pmatrix} \lambda & 1 & 0 \\ & \ddots & 1 \\ 0 & & \lambda \end{pmatrix}_{n \times n}$
even no. of J_λ
$\Rightarrow$ Condition
$g_{\mu\nu}$ & $f_{\mu\nu}$
CAUTION

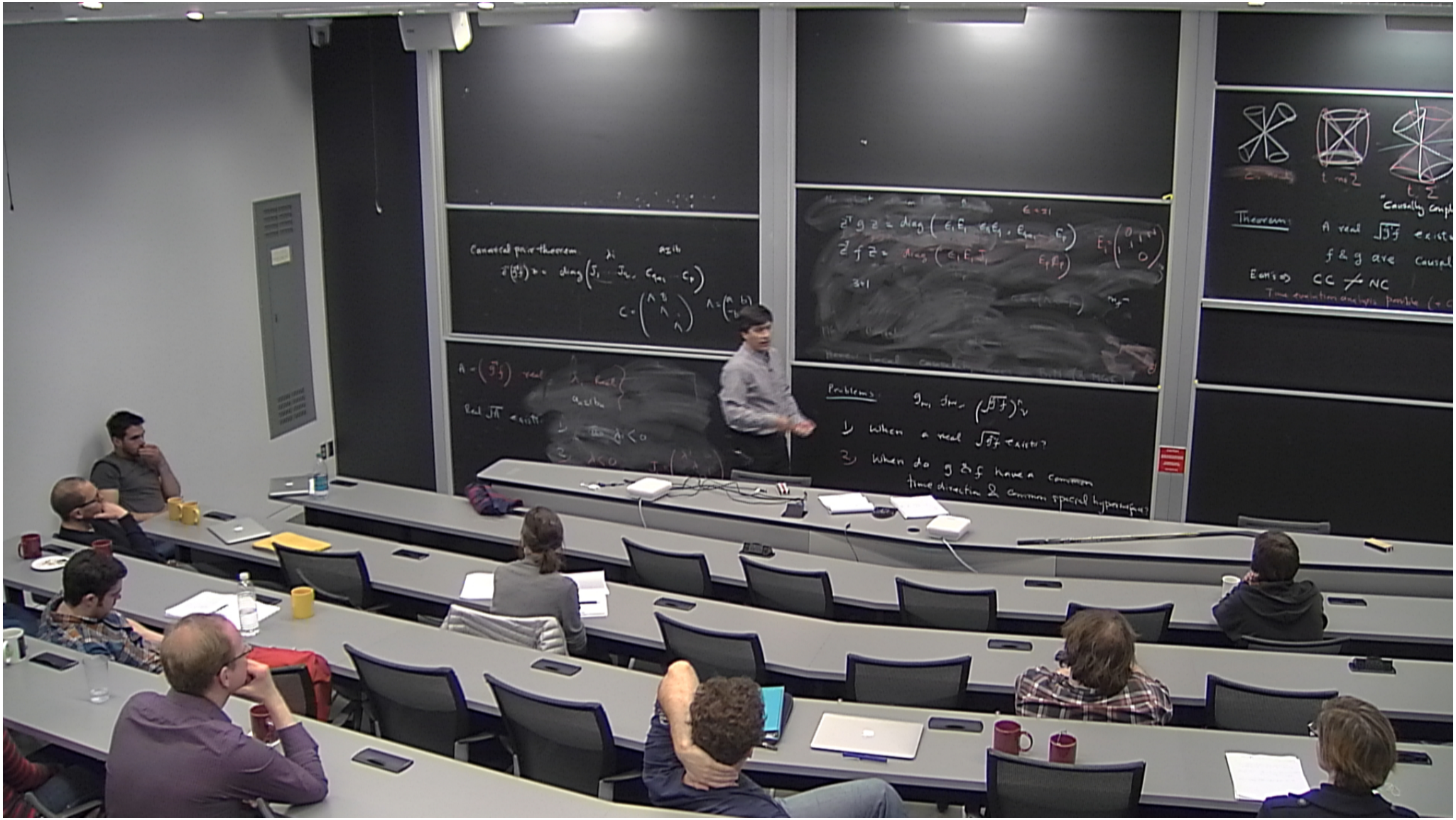

Canonical pair theorem.
$z^{-1}(\bar{g}^{-1}f)z = \mathrm{diag}(J_1 \cdots J_q, c_{q+1} \cdots c_p)$
real
λ_i Real
$a_k \pm i b_k$
exists.
1)
2)
$\lambda < 0$
CAUTION

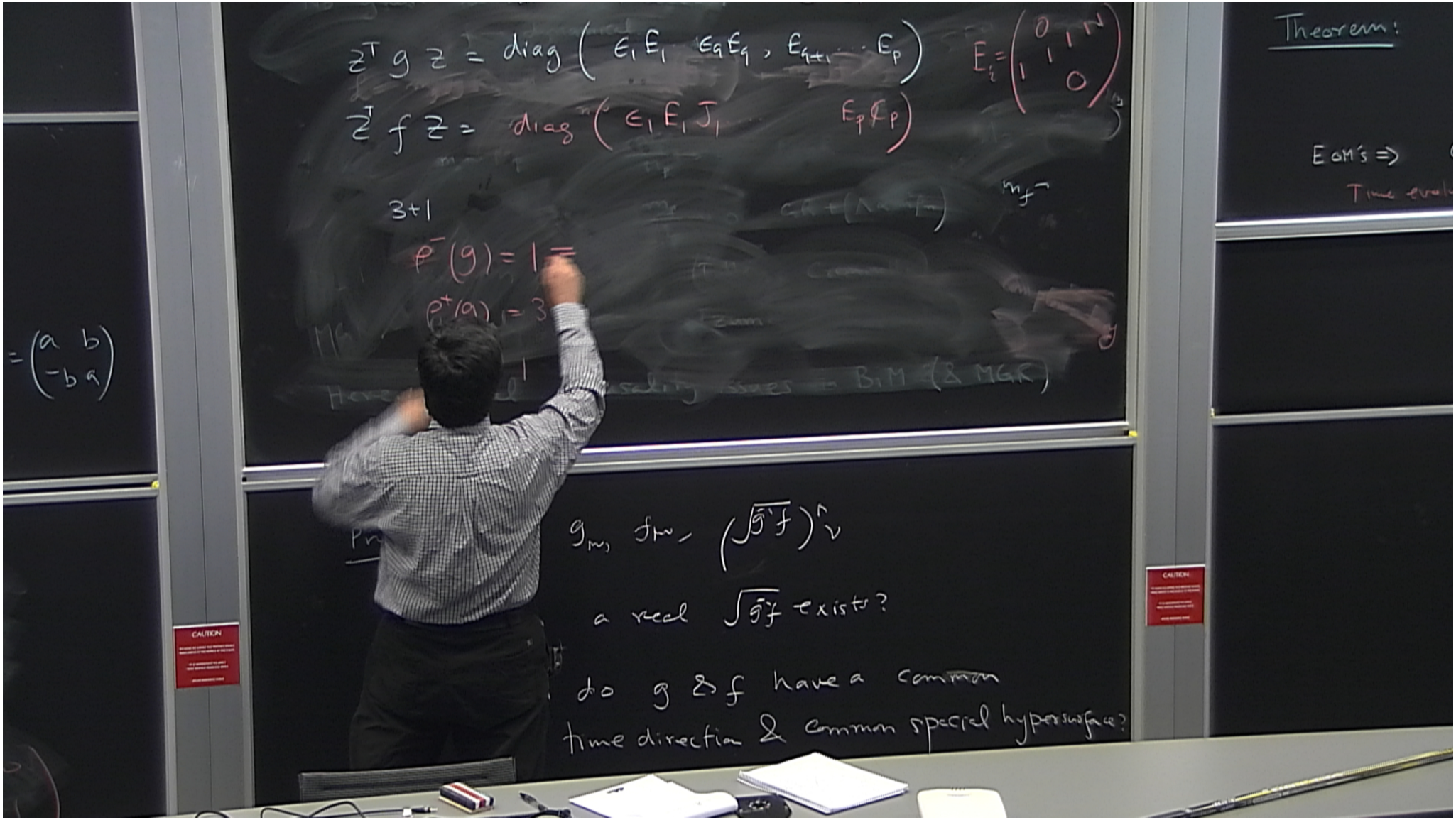

Canonical pair theorem.
$z^{-1}(\bar{g}^{-1}f)z = \mathrm{diag}(J_1 \cdots J_q, c_{q+1} \cdots c_p)$
real
λ_i Real
$a_k \pm i b_k$
exists.
1) $\alpha_i \lambda_i < 0$
2) $\lambda < 0$
$J_i =$
CAUTION

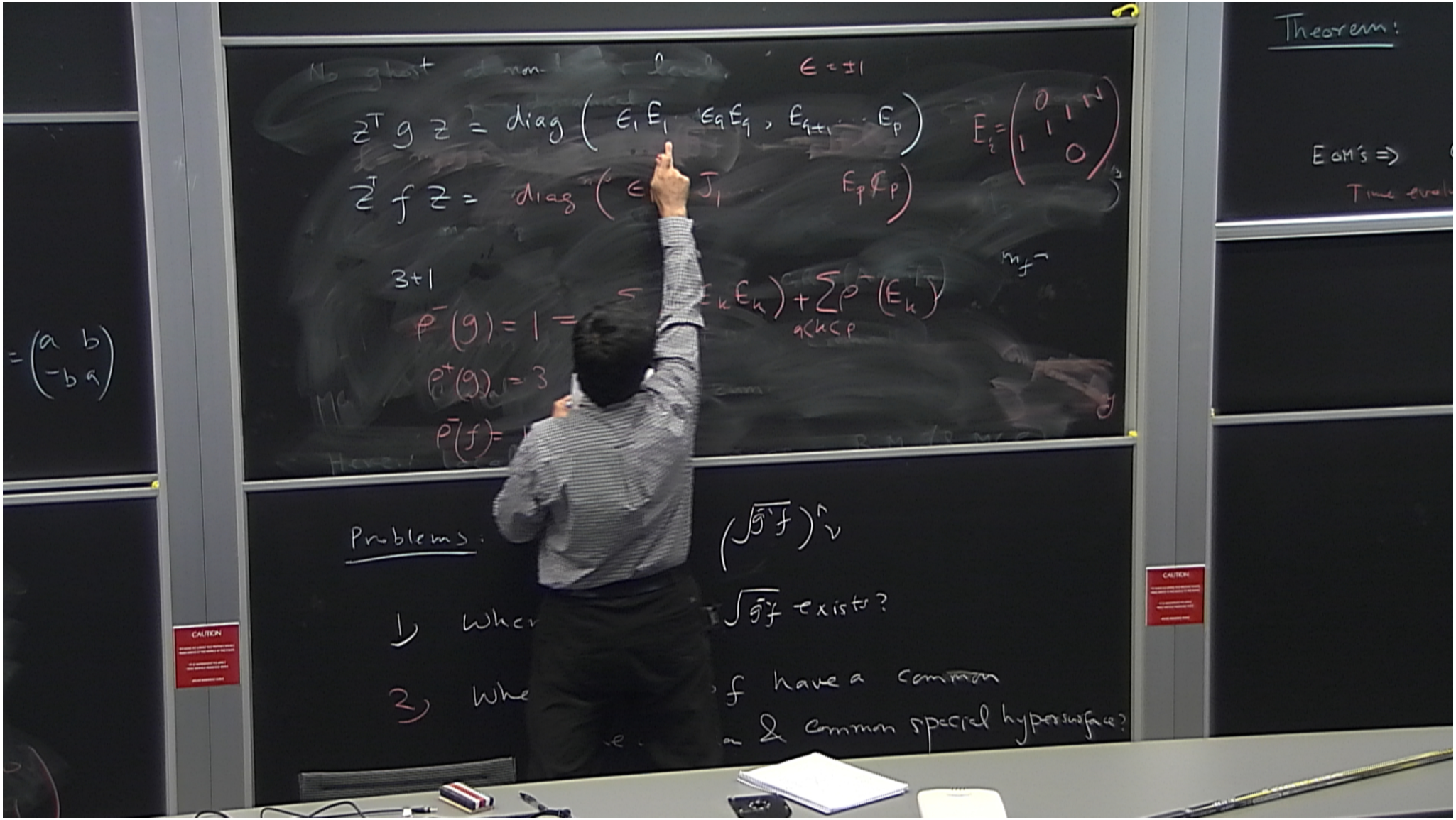

Canonical pair theorem.
λ_i
$a \pm ib$
$z^T(\bar{g}^{-1}f)z = \mathrm{diag}(J_1 \cdots J_k, c_{q+1} \cdots c_p)$
$C = \begin{pmatrix} \Lambda & & \mathbb{1} \\ & \Lambda & \\ & & \Lambda \end{pmatrix}$
$\Lambda = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$
real
λ_i Real
$a_k \pm i b_k$
exists.
1) x_0 $\lambda_i < 0$
2) $\lambda < 0$
$J_i = \begin{pmatrix} \lambda' & 0 \\ \cdot & \cdot \\ 0 & \lambda' \end{pmatrix}$

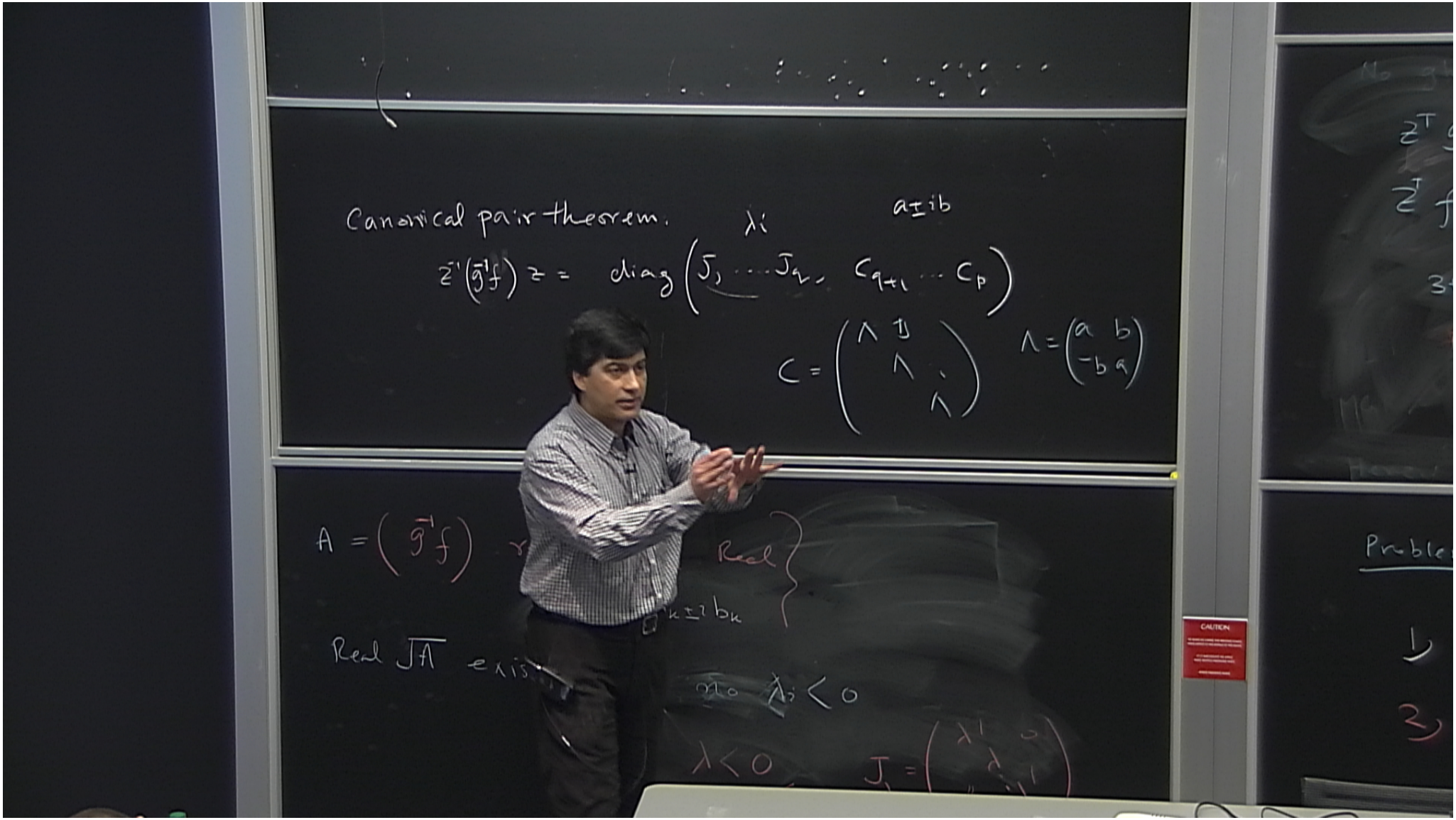

Theorem:
E.o.M's ⇒
Problems.
1) When a
2) When do
time
exists?
ve a common
common special hypersurface?
CAUTION

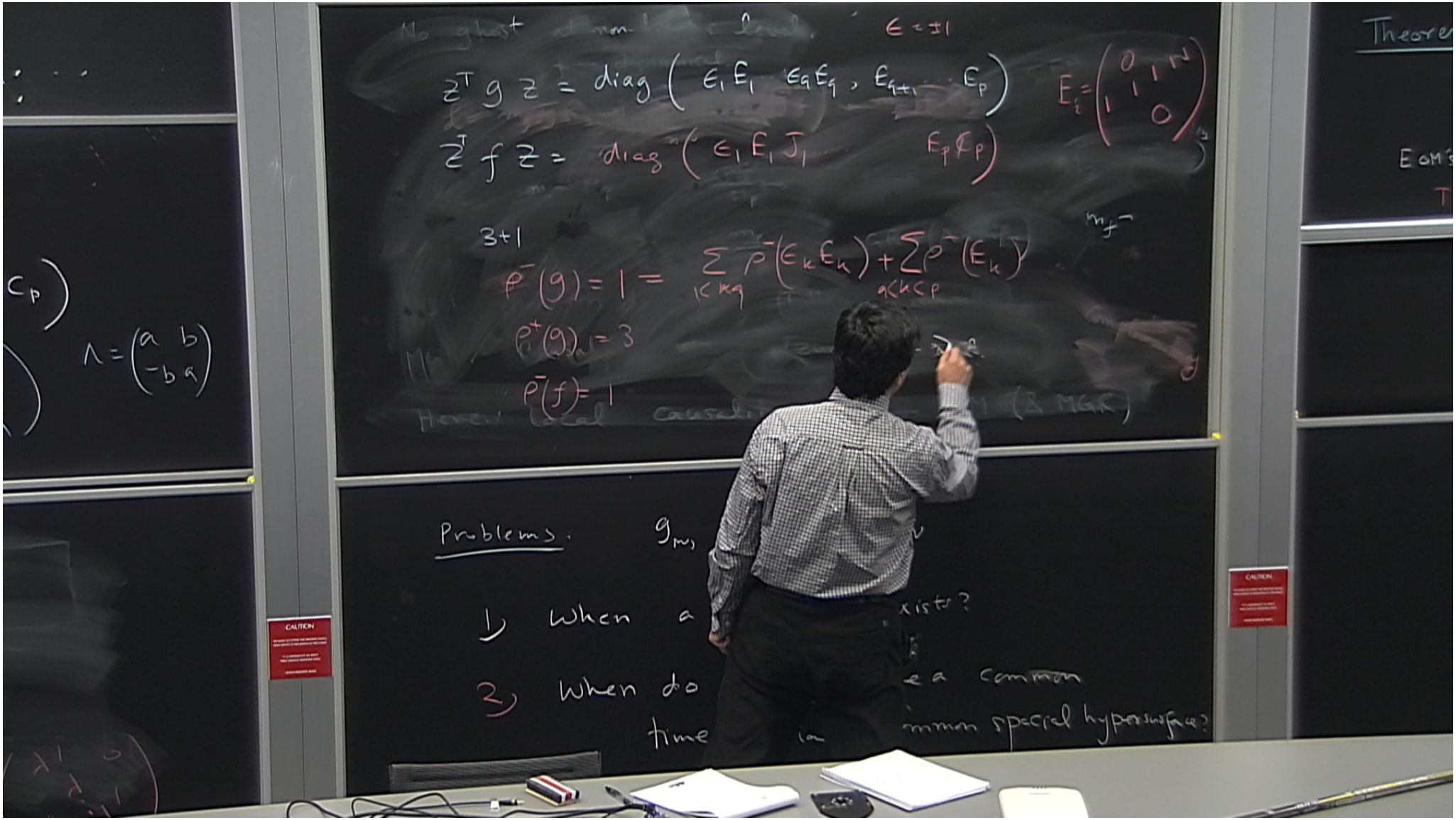

Theorem:
$\epsilon = \pm 1$
$z^T g z = \mathrm{diag}(\epsilon_1 E_1, \epsilon_q E_q, E_{q+1}, \ldots E_p)$
$z^T f z = \mathrm{diag}(\epsilon_1 E_1 J_1, \ldots E_p J_p)$
$E_i = \begin{pmatrix} 0 & 1 & & \\ 1 & 1 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix}$
EoM's ⇒
Time evol
Complementary
Here: local
(& MGR)
Problems.
1) When
3) When
exists?
have a common
& common special hypersurface?
CAUTION

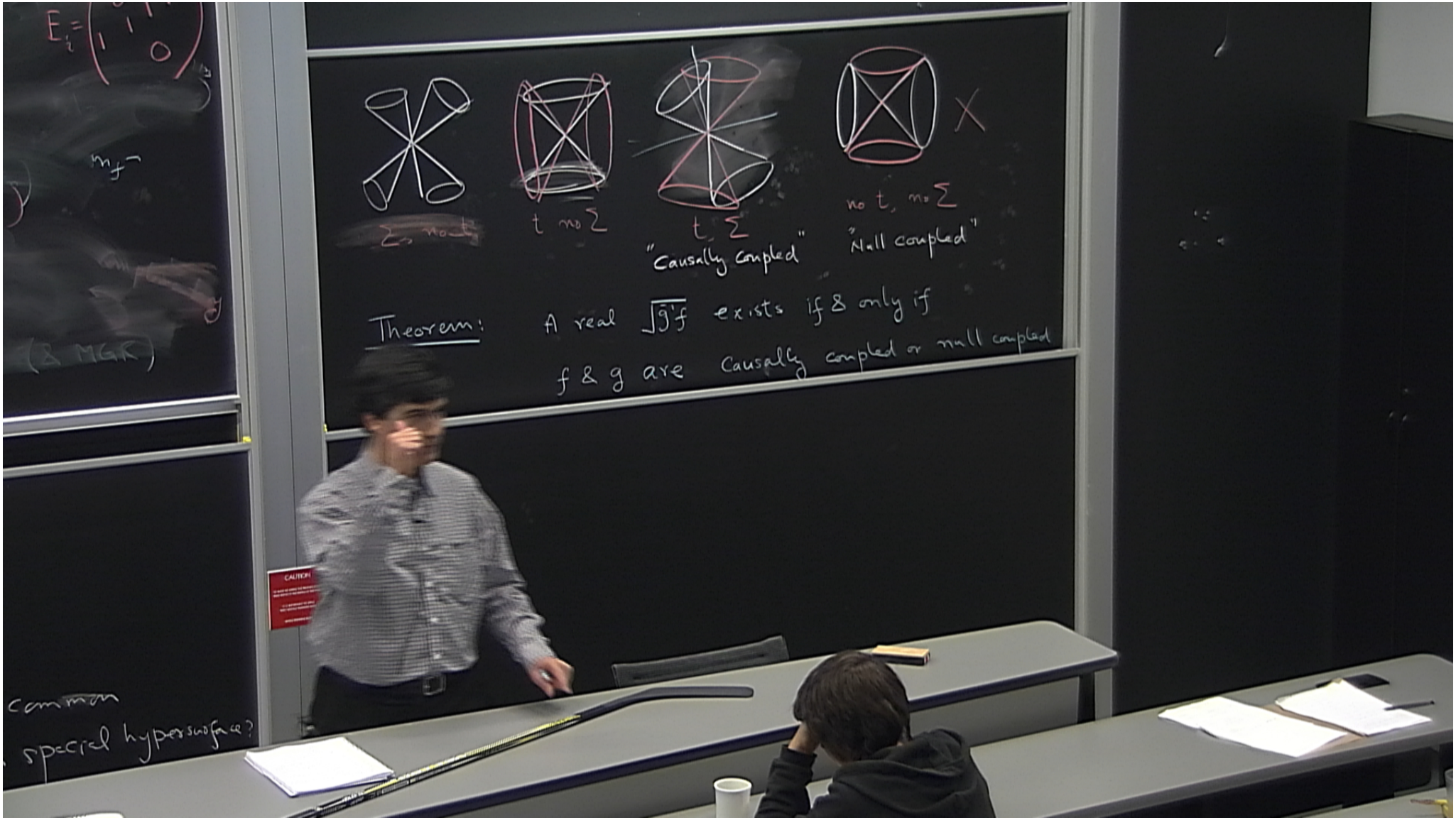

Canonical pair theorem
Theorem:
Problems:
When a real $\sqrt{gf}$ exists?
When do g & f have a common time direction & common spacial hypersurface?

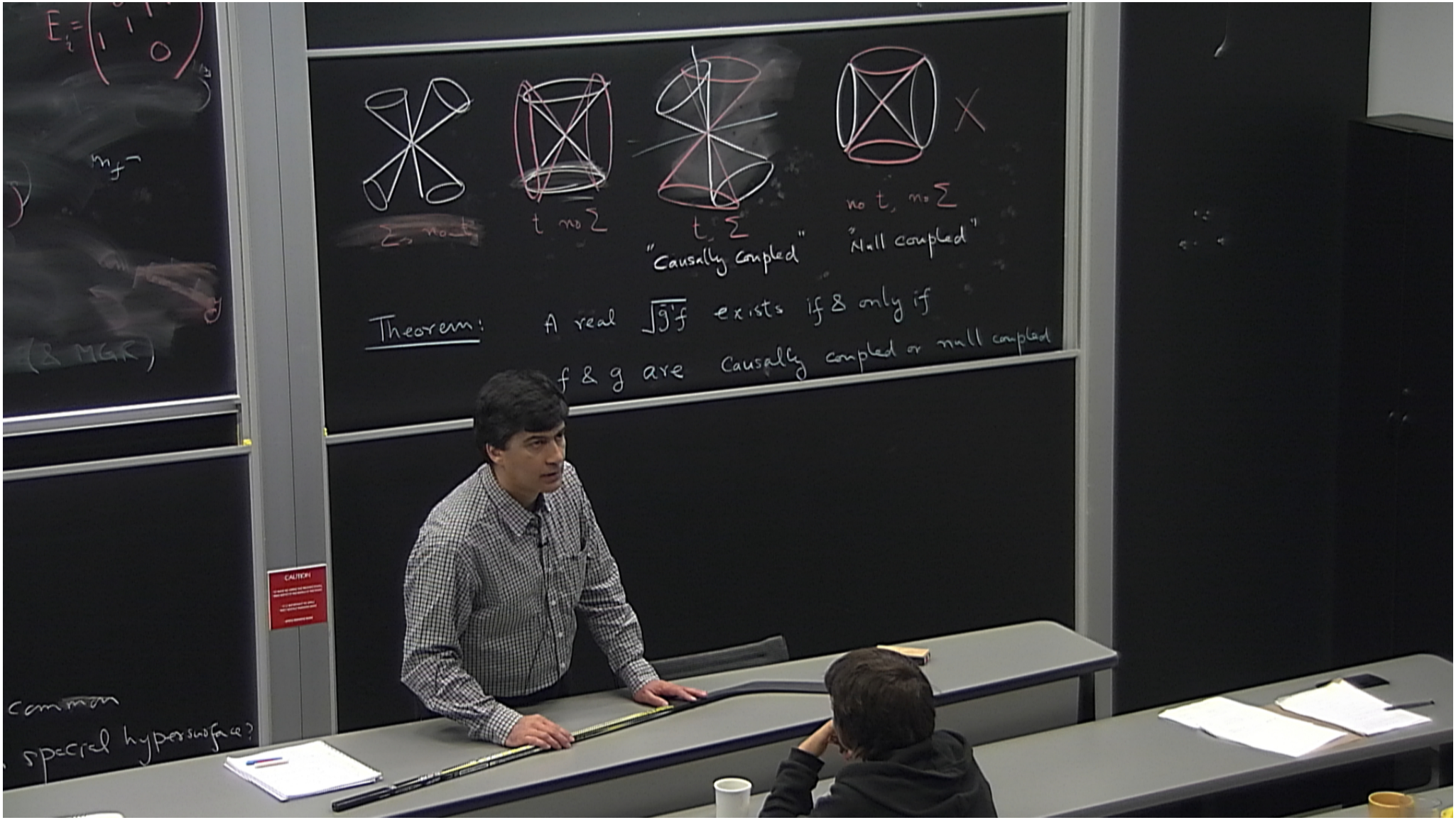

Theorem:
3+1
a real $\sqrt{gf}$ exists?
do g & f have a common
time direction & common spacial hypersurface?


Theorem:
Problems.
CAUTION


Canonical pair theorem.
λ_i
$a \pm ib$
$z^T(\vec{g}^{-1}f)z = \mathrm{diag}(J_1 \cdots J_q, c_{q+1} \cdots c_p)$
$C = \begin{pmatrix} \Lambda & \mathbb{1} & & \\ & \Lambda & \ddots & \\ & & & \Lambda \end{pmatrix}$
$\Lambda = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$
$A = (\vec{g}^{-1}f)$
Real
$a_k \pm ib_k$
Real $\sqrt{A}$
$\lambda < 0$
$J_i =$
CAUTION


Problems.
1) When a
2) When do
time
Theore
E om's


t, no Σ
t, Σ
"Causally Coupled"
no t, no Σ
"Null coupled"
Theorem:
A real √g f exists if & only if
f & g are Causally coupled or null coupled
common
special hypersurface?


"Causally Coupled"
"Null coupled"
Theorem:
A real $\sqrt{g}f$ exists if & only if
f & g are Causally coupled or null coupled
common
special hypersurface?