Title: TBA

Date: Apr 10, 2015 04:45 PM

URL: http://pirsa.org/15040110

Abstract:

0709.1483

$\boxed{1+1}$

$$F(i\partial_t, -i\partial_x)\,\phi(t,x) = J(t,x)$$

$$\Longrightarrow \quad \phi(t,x) = \int dt'\,dx'\,G(t-t', x-x')\,J(t',x')$$

$$F \cdot G = \delta(t)\,\delta(x)$$

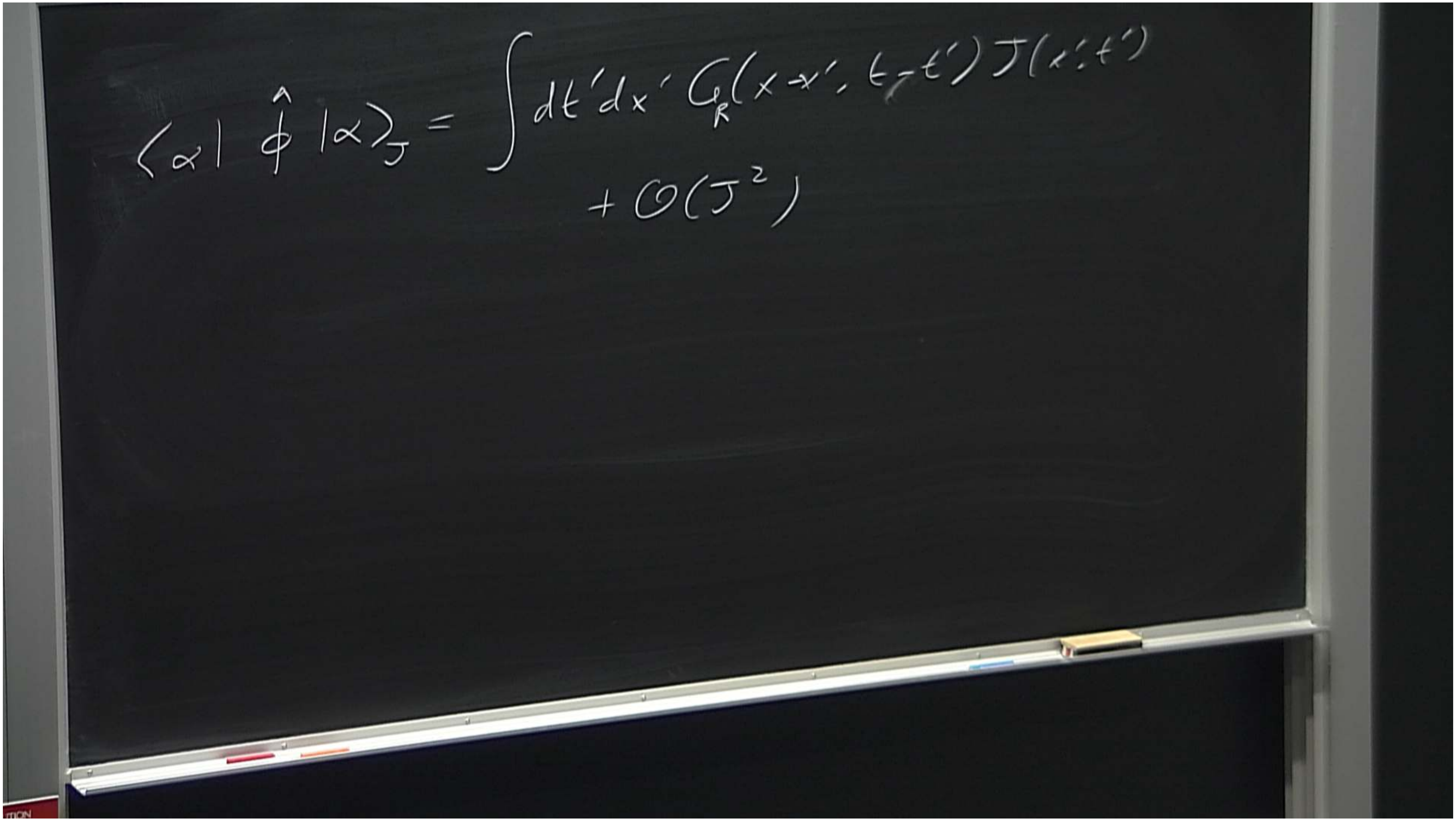
retarded (causality):

$$G_R(t<0, x) = 0$$

QFT: $$G_R(t,x) = \langle\alpha|[\hat\phi(t,x), \hat\phi(0)]|\alpha\rangle$$

$|\alpha\rangle$

$$S \to S + \int \hat\phi(x) J(x)\, d^2x$$


$\langle \alpha | \hat{\phi} | \alpha \rangle_J = \int dt' dx' \, G_R(x-x', t-t') J(x', t')$
$+ \mathcal{O}(J^2)$

retarded (<u>causality</u>):

$$G_R(t<0, x) = 0$$

QFT: $$\underline{G_R(t,x)} = \langle\alpha|[\hat{\phi}(t,x), \hat{\phi}(0)]|\alpha\rangle\,\theta(t)$$

$|\alpha\rangle$

$$S \to S + \int \hat{\phi}(x) J(x)\, d^dx$$

$$\langle \alpha | \hat{\phi} | \alpha \rangle_J = \int dt' dx' \, G_R(x-x', t-t') J(x', t') + \mathcal{O}(J^2)$$

$$\tilde{G}_R(\omega, x) \equiv \int_0^\infty dt \, e^{i\omega t} G_R(t, x)$$

assume:

Stability: $G_R(t,x) \lesssim e^t \quad t \to \infty$

$\mathrm{Im}(\omega) > 0$

$$\frac{\partial}{\partial \omega} \tilde{G}(\omega, x) = \int_0^\infty dt\,(it)\, e^{i\omega t}\, G_R(x,t)$$

$$= \text{finite for } \mathrm{Im}(\omega) > 0$$

$$\Longrightarrow \tilde{G}(\omega, x) \ \underline{\text{analytic}} \quad '' \quad ''$$

$$\tilde{G}_R(\omega, p) = \int dx\, dt\; e^{+i\omega t - ipx}\, G_R(t, x)$$

$\Rightarrow$ analytic in ω for real p, $\mathrm{Im}(\omega) > 0$

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$\Rightarrow$ analytic in ω for real p, $\mathrm{Im}(\omega) > 0$

- Subluminality $\quad G_R(t, x) = 0$ for $|x| > t$

$$\tilde{\tilde{G}}_R(\omega, p) = \int_0^\infty dt \int_{-t}^{t} dx\; e^{+i\omega t - ipx}\, G_R(t, x)$$

$\Longrightarrow$ $\tilde{\tilde{G}}_R(\omega, p)$ analytic in ω & p for

$$\boxed{\mathrm{Im}(\omega) > |\mathrm{Im}(p)|}$$

Ex: (1) $F(\omega, p) = n^2(\omega)\,\omega^2 - p^2$

$$\int d\omega\, dp\; \phi^*(\omega, p)\left(n^2(\omega)\,\omega^2 - p^2\right)\phi(\omega, p)$$

Causality: $\Longrightarrow$ $n(\omega)$ analytic $\mathrm{Im}(\omega) > 0$

assume: 1) $n(\omega) \to 1$ for $\omega \to \infty$ (complex)

2) $\omega\, \mathrm{Im}(n(\omega)) > 0$ for real ω

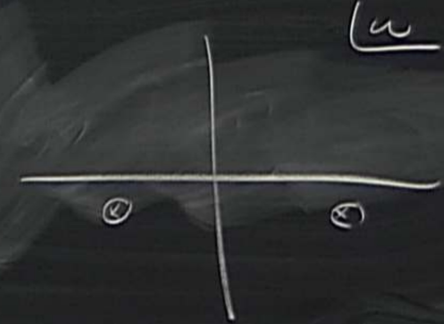
$\Longrightarrow$ $G_R(t, x)$ $|x| > t$ Subluminality!

② $F(\omega,p) = \omega^2 - f(p^2)$

$$S \sim \int d\omega\, dp\, \phi^*(\omega,p)\left(\omega^2 - f(p^2)\right)\phi(\omega,p)$$

- causality automatic

$$\tilde{\tilde{G}}_R(\omega,p) = \frac{1}{(\omega+i\varepsilon)^2 - f(p^2)}$$



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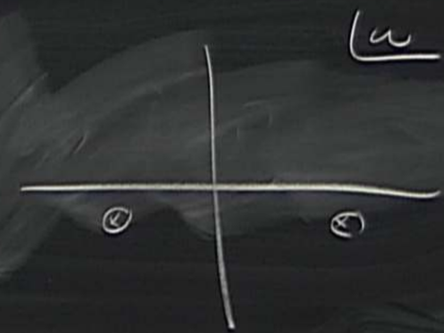


- causality automatic

$$\tilde{\tilde{G}}_R(\omega, p) = \frac{1}{(\omega + i\varepsilon)^2 - f(p^2)}$$

- Subluminality : $\dfrac{1}{\omega^2 - f(p^2)}$ analytic $\mathrm{Im}(\omega) > |\mathrm{Im}(p)|$

$\Rightarrow f(p^2)$ <u>analytic</u> for all $p \in \mathbb{C}$

$$\tilde{G}(t,p) = -\theta(t)\, \frac{\sin(\sqrt{f}\, t)}{\sqrt{f}}$$

<u>assume</u> : $\tilde{G}(t,p)$ exponentially bounded in p

$$\Longrightarrow f(p^2) \sim p^2 \text{ at most}$$

$$\Longrightarrow \boxed{f(p^2) = ap^2 + bp + c}$$

$$m^2$$

$$F_1(\omega, p) = n^2(\omega)\,\omega^2 - p^2$$

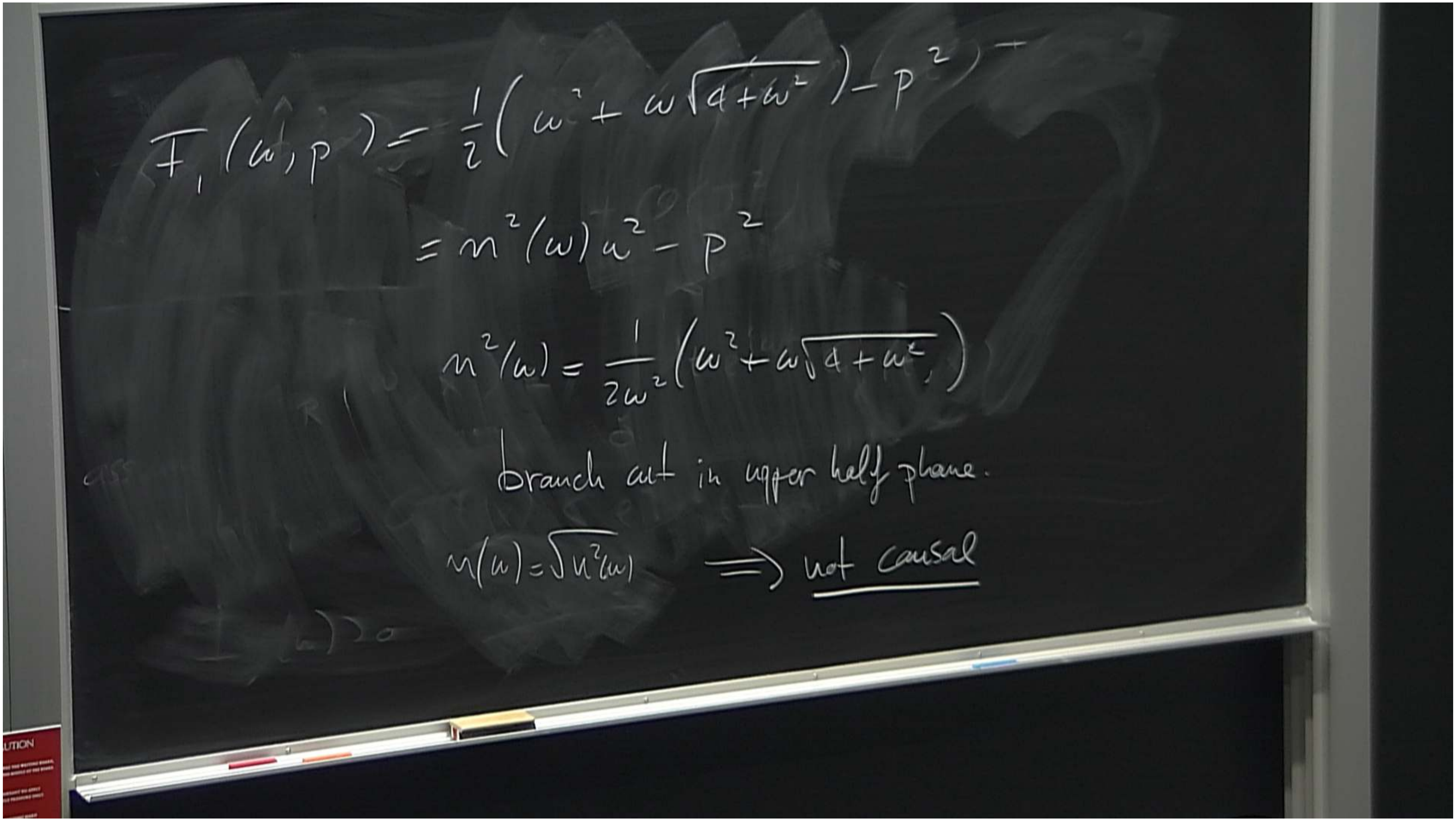
$$F_2(\omega, p) = \omega^2 - f(p^2)$$

$$\omega^2 - f(p^2) = 0$$

Concrete example:

$$F_2(\omega, p) = \omega^2 - \frac{p^4}{p^2+1}$$

Causal,
Stable,
Superluminal


$F_1(\omega, p) = \frac{1}{2}\left(\omega^2 + \omega\sqrt{4+\omega^2}\right) - p^2$
$= n^2(\omega)\,\omega^2 - p^2$
$n^2(\omega) = \frac{1}{2\omega^2}\left(\omega^2 + \omega\sqrt{4+\omega^2}\right)$
branch cut in upper half plane.
$n(\omega) = \sqrt{n^2(\omega)} \implies$ not causal

$$\left.\begin{aligned} F_1(\omega, p) &= n^2(\omega)\,\omega^2 - p^2 \\ F_2(\omega, p) &= \omega^2 - f(p^2) \end{aligned}\right\}\ \text{Same } \underline{\text{free solutions}}$$

$$\omega^2 - f(p^2) = 0$$

Concrete example:

$$F_2(\omega, p) = \omega^2 - \frac{p^4}{p^2+1}$$

Causal,
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