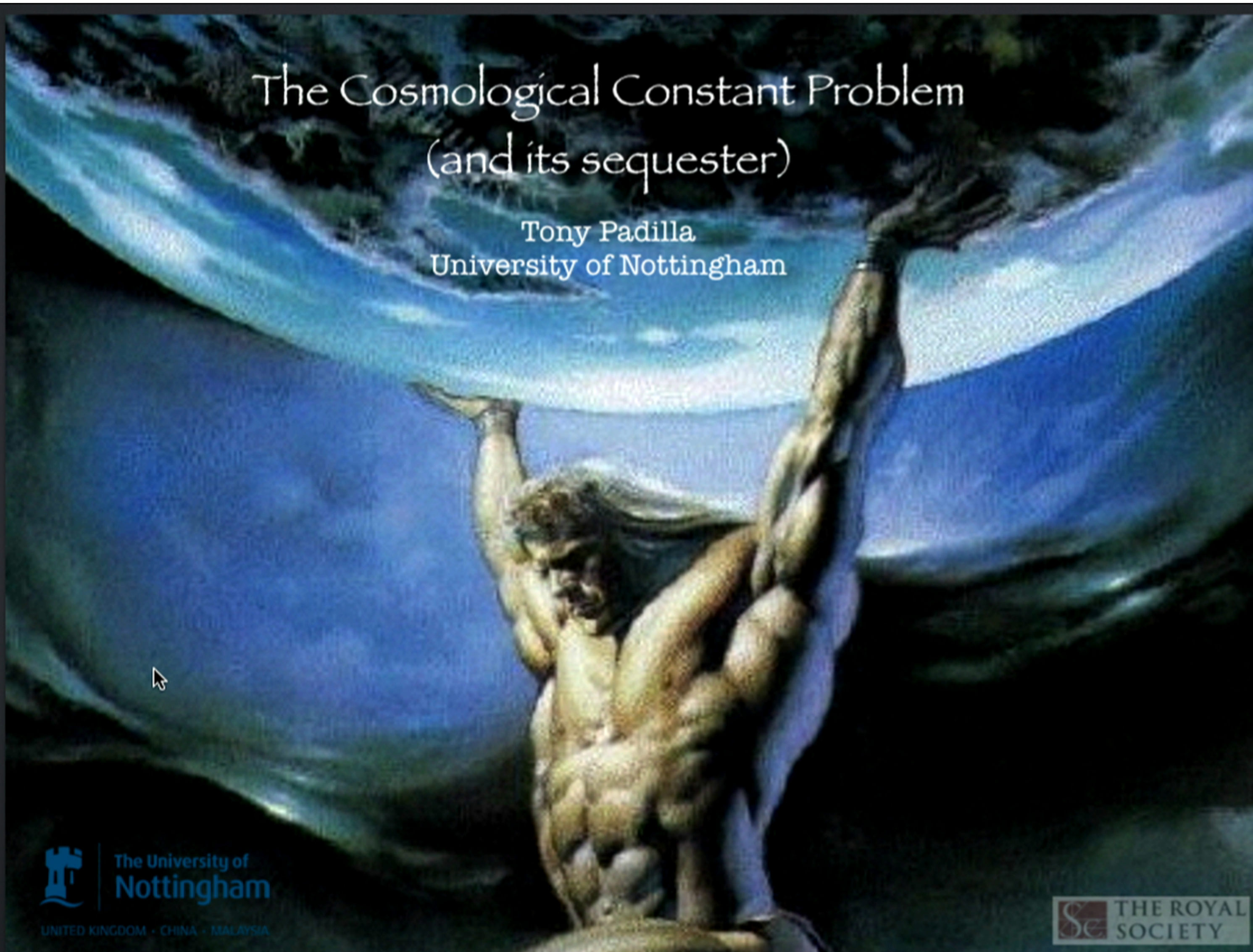
Title: The Cosmological Constant Problem (and its sequester)

Date: Apr 10, 2015 03:15 PM

URL: http://pirsa.org/15040109

Abstract: I will review the notorious cosmological constant problem, sometimes described as the worst fine tuning problem in Physics. I will explain the true nature of the problem, which is one of radiative instability against any change in the effective description. I will recall Weinbergâ€™s venerable no-go theorem that prohibits certain attempts to â€œsolveâ€• this problem before going on to explain a new mechanism that circumvents Weinberg. This is the vacuum energy sequester, a global modification of GR that results in the cancellation of large vacuum energy contributions from a protected matter sector (taken to include the Standard Model) at each and every order in the perturbative loop expansion. Cosmological consequences are a Universe which has finite space-time volume, will ultimately crunch, and for which dark energy can only be a transient. Furthermore, using a linear scalar potential within the sequestering set-up, I will show that dark energy today can be intimately related to the trigger that brings about cosmological collapse in the not too distant future, at the same time providing a possible solution to the â€œWhy Now?â€• problem.


The Cosmological Constant Problem
(and its sequester)
Tony Padilla
University of Nottingham
The University of
Nottingham
UNITED KINGDOM · CHINA · MALAYSIA
THE ROYAL SOCIETY

with Nemanja Kaloper

1309.6562

1406.0711

1409.7073

see also

1502.05296

The Cosmological Constant Problem

General Covariance & Equivalence Principle ⇒ Vacuum Energy Gravitates

$$-V_{vac}\int\sqrt{-g}d^4x \implies T_{\mu\nu} = -V_{vac}g_{\mu\nu}$$

The Cosmological Constant Problem

General Covariance & Equivalence Principle ⇒ Vacuum Energy Gravitates

$$-V_{vac}\int\sqrt{-g}\,d^4x \implies T_{\mu\nu} = -V_{vac}\,g_{\mu\nu}$$

…add a bare cosmological constant…

$$-(V_{vac}+\Lambda_{bare})\int\sqrt{-g}\,d^4x \implies T_{\mu\nu} = -\Lambda_{tot}\,g_{\mu\nu}$$

where $\Lambda_{tot} = V_{vac} + \Lambda_{bare} \lesssim (meV)^4$

Estimating the vacuum energy

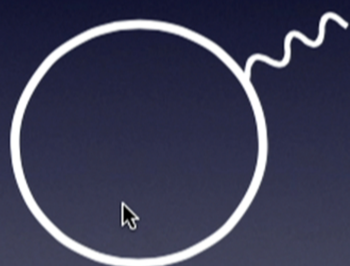
$$V_{vac} \supset \sum_m \int d^3k \frac{1}{2}\hbar\sqrt{k^2+m^2}$$

$$\sim c_\nu m_\nu^4 + c_e m_e^4 + c_\mu m_\mu^4 + \ldots + M_{\rm cut-off}^4$$

Estimating the vacuum energy

$$V_{vac} \supset \sum_m \int d^3k \frac{1}{2}\hbar\sqrt{k^2+m^2}$$

$$\sim c_\nu m_\nu^4 + c_e m_e^4 + c_\mu m_\mu^4 + \ldots + M_{\text{cut-off}}^4$$



$$\sim \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{k^\mu k^\nu - \frac{1}{2}\eta^{\mu\nu}\,(k^2+m^2)}{k^2+m^2}\right] = -\frac{i}{2}\eta^{\mu\nu}V_{vac}^{\phi,\text{1-loop}}$$

$$V_{vac}^{\phi,\text{1-loop}} = -\frac{m^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \text{finite} + \ln\left(\frac{M_{UV}^2}{m^2}\right)\right]$$

What the cosmological constant problem is not....

Why is the cosmological constant so small?

Quantum Gravity cut-off	$(10^{18}\ GeV)^4$	fine tuning to 120 decimal places
SUSY cut-off	$(TeV)^4$	fine tuning to 60 decimal places
EW phase transition	$(200\ GeV)^4$	fine tuning to 56 decimal places
QCD phase transition	$(0.3\ GeV)^4$	
muon	$(100 MeV)^4$	fine tuning to 44 decimal places
electron	$(MeV)^4$	fine tuning to 36 decimal places
	$(meV)^4$	observed value

"Fine tuning" and renormalization

$$V_{vac}^{\phi,\text{1-loop}} = -\frac{m^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \text{finite} + \ln\left(\frac{M_{UV}^2}{m^2}\right)\right]$$ *Divergent vacuum energy*

"Fine tuning" and renormalization

$$V_{vac}^{\phi,\text{1-loop}} = -\frac{m^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \text{finite} + \ln\left(\frac{M_{UV}^2}{m^2}\right)\right]$$

Divergent vacuum energy

$$\Lambda_{bare} = \frac{m^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \ln\left(\frac{M_{UV}^2}{\mathcal{M}^2}\right)\right]$$

Bare counterterm

Renormalized vacuum energy:

$$\Lambda_{ren} = V_{vac}^{\phi,\text{1-loop}} + \Lambda_{bare} = \frac{m^4}{(8\pi)^2}\left[\ln\left(\frac{m^2}{\mathcal{M}^2}\right) - \text{finite}\right]$$

Depends on arbitrary subtraction scale so cannot be predicted...must be measured!


goglobal
Page 15 of 78

Radiative instability of Λ

At one loop, $V_{vac} = V_{vac}^{tree} + V_{vac}^{1loop} \sim M_{UV}^4 \gtrsim (TeV)^4$

tune finite part of Λ_{bare} to great precision

At two loops, $V_{vac} = V_{vac}^{tree} + V_{vac}^{1loop} + V_{vac}^{2loop}, \qquad V_{vac}^{2loop} \sim V_{vac}^{1loop}$

REtune finite part of Λ_{bare} to same precision

At three loops, ...

MISSION: IMPOSSIBLE™

"to make the cosmological constant radiatively stable"


Ask not why the radiative corrections are so small...


..but why they hardly gravitate.
Page 21 of 78


NO GO AREA!

The Sequester

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \Lambda - \mathcal{L}(g^{\mu\nu}, \Psi) \right]$$

Introduce global dynamical variables Λ

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \Lambda - \mathcal{L}(g^{\mu\nu}, \Psi) \right]$$

Introduce global dynamical variables Λ, λ

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \Lambda - \lambda^4 \mathcal{L}(\lambda^{-2} g^{\mu\nu}, \Psi) \right]$$

λ sets the hierarchy between matter scales and M_{pl}

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

Introduce global dynamical variables Λ, λ

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \Lambda - \lambda^4 \mathcal{L}(\lambda^{-2}g^{\mu\nu}, \Psi)\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right)$$

λ sets the hierarchy between matter scales and M_{pl}

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

Equations of motion

$$\Lambda \text{ equation} \quad : \quad \frac{\sigma'}{\lambda^4 \mu^4} = \int d^4x \sqrt{g}$$

$$\lambda \text{ equation} \quad : \quad 4\Lambda \frac{\sigma'}{\lambda^4 \mu^4} = \int d^4x \sqrt{g}\, \lambda^4\, \tilde{T}^{\mu}{}_{\mu}$$

$$g_{\mu\nu} \text{ equation} \quad : \quad M_{pl}^2 G^{\mu}_{\nu} = -\Lambda \delta^{\mu}_{\nu} + \lambda^4 \tilde{T}^{\mu}_{\nu}$$

$$T_{\mu\nu} = \lambda^4 \tilde{T}_{\mu\nu}$$

$$\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta \tilde{g}^{\mu\nu}} \int d^4x \sqrt{-\tilde{g}} \mathcal{L}(\tilde{g}^{\mu\nu}, \Psi)$$

Equations of motion

Λ equation :

λ equation :

$$\Lambda = \frac{1}{4}\langle T^{\alpha}{}_{\alpha}\rangle, \qquad \langle Q\rangle = \frac{\int d^4x Q\sqrt{g}}{\int d^4x\sqrt{g}}$$

$g_{\mu\nu}$ equation : $M_{pl}^2 G^{\mu}_{\nu} = -\Lambda\delta^{\mu}_{\nu} + T^{\mu}_{\nu}$

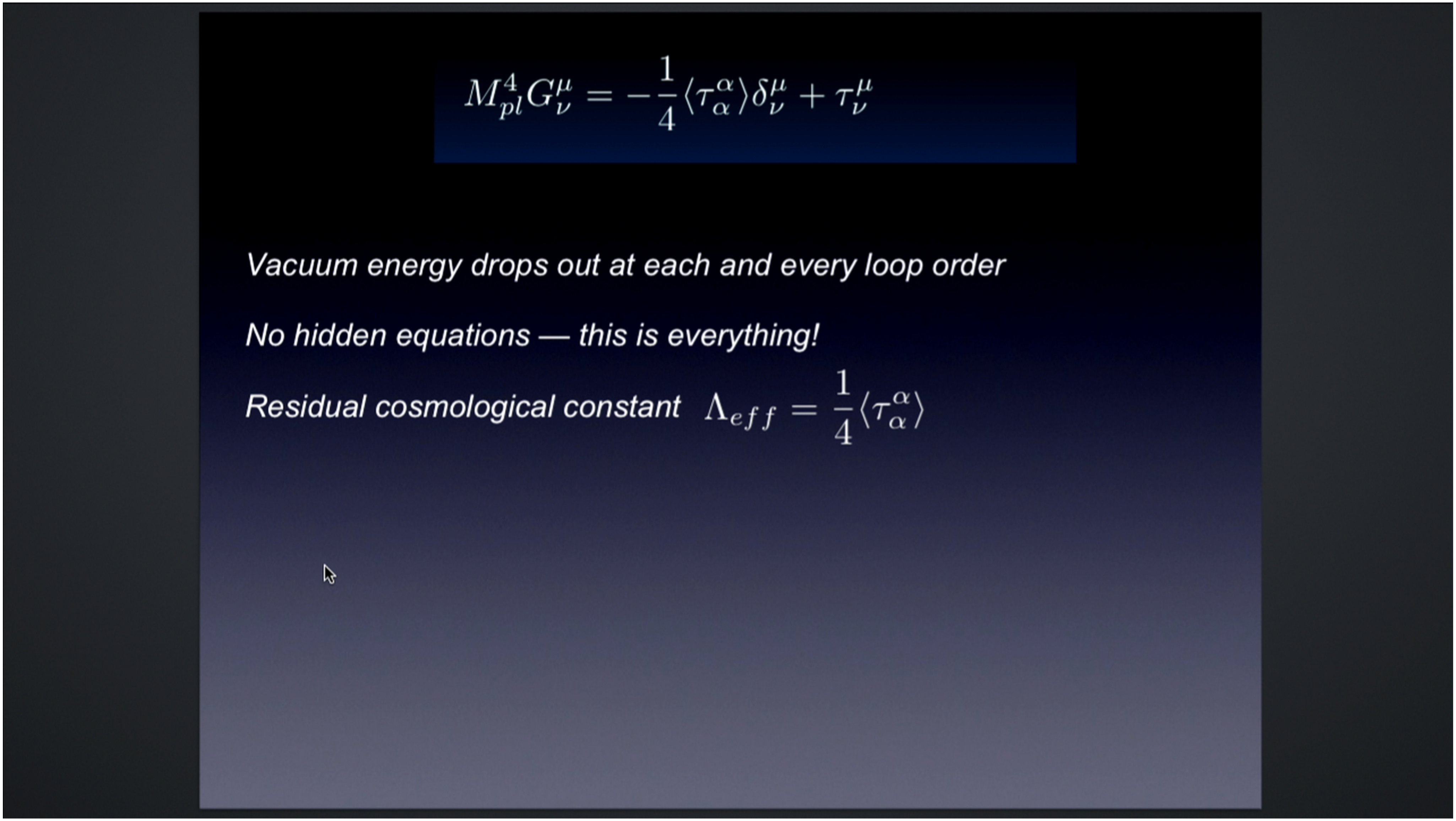
Equations of motion

$$M_{pl}^2 G^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle T^{\alpha}{}_{\alpha}\rangle$$

Equations of motion

$$M_{pl}^2 G^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle T^{\alpha}{}_{\alpha}\rangle$$

$$T^{\mu}_{\nu} = -V_{vac}\delta^{\mu}_{\nu} + \tau^{\mu}_{\nu}$$


$M_{pl}^4 G^\mu_\nu = -\frac{1}{4}\langle \tau^\alpha_\alpha \rangle \delta^\mu_\nu + \tau^\mu_\nu$
Vacuum energy drops out at each and every loop order
No hidden equations — this is everything!
Residual cosmological constant $\Lambda_{eff} = \frac{1}{4}\langle \tau^\alpha_\alpha \rangle$

$$M_{pl}^4 G^\mu_\nu = -\frac{1}{4}\langle\tau^\alpha_\alpha\rangle\delta^\mu_\nu + \tau^\mu_\nu$$

Vacuum energy drops out at each and every loop order

No hidden equations — this is everything!

Residual cosmological constant $\Lambda_{eff} = \frac{1}{4}\langle\tau^\alpha_\alpha\rangle$

Λ_{eff} has nothing to do with vacuum energy

It is radiatively stable

OK to fix it empirically

How big is Λ_{eff}?

For standard matter, space-time integrals dominated by time when universe is largest

$$\int d^4x\sqrt{-g} \sim \frac{1}{H_{age}^4}$$

where lifetime $t_{age} \sim \frac{1}{H_{age}} \gtrsim 13.7$ Gyrs

$$\langle \tau^\alpha_\alpha \rangle \sim \rho_{age} \sim \text{energy density at largest size} < \rho_c$$

$\Rightarrow \Lambda_{eff}$ *is not dark energy … too small!*

Universe has finite spacetime volume

Universe has finite spacetime volume

$$\frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 \sqrt{g}$$

space-time volume must be finite or else $\lambda \to 0$

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

if $\lambda \to 0$ particle masses go to zero

Page 41 of 78

Universe has finite spacetime volume

$$\frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 \sqrt{g}$$

space-time volume must be finite or else $\lambda \to 0$

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

if $\lambda \to 0$ particle masses go to zero

Universe has finite spacetime volume

$$\frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 \sqrt{g}$$

space-time volume must be finite or else $\lambda \to 0$

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

if $\lambda \to 0$ particle masses go to zero

Ends in a crunch
w=-1 is transient
Ω_k>0

Universe has finite spacetime volume

$$\frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 \sqrt{g}$$

space-time volume must be finite or else $\lambda \to 0$

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

if $\lambda \to 0$ particle masses go to zero

Ends in a crunch
w=-1 is transient
Ω_k>0

circles in the sky?
possible correlation between 1+w and Ω_k

COLLAPSE TRIGGER = DARK ENERGY

Linear potential $V=m^3\varphi$

form protected by shift symmetry,
size of m^3 technically natural

Radiatively stable choice of collapse time?

Radiatively stable choice of φ_{in}?

Radiatively stable choice of collapse time?

Yes, thanks to m^3

Radiatively stable choice of φ_{in}?

Yes, thanks to shift symmetry

But its not even a "choice".... $\langle R \rangle = 0$ picks out precisely those solutions with $\varphi_{in} > M_{pl}$!!!!!!!

Radiatively stable choice of collapse time?

Yes, thanks to m^3

Radiatively stable choice of φ_{in}?

Yes, thanks to shift symmetry

But its not even a "choice".... <R>=0 picks out precisely those solutions with $\varphi_{in} > M_{pl}$!!!!!!!

(Non) Issues

Fine tuning Λeff

Silly — loops are gone!

Acausality

Some acausality essential for solving CCP
...need to scan all of spactime to measure true vacuum energy

We fix one number, there are no local pathologies

Summary

Found a way to cancel SM vacuum energy at any order in loops

Locally theory looks just like GR with a small Λ

Small Λ is radiatively stable & determined by a global boundary condition

Universe is doomed