Title: Multi-Field Born-Infeld Lagrangians, Nilpotent Fields, N=2 Supersymmetry and Cubic Polynomials

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Abstract: We show how nonlinearly realized N=2 supersymmetry gives rise, in the low-energy limit, to an N=1 Born-Infeld U(1) Lagrangian. We then extend the construction to many N=2 vector multiplets. We show how the classification of inequivalent nilpotency constraints arising in the low-energy limit is connected to the theory of cubic polynomials and curves. We comment on causality of signal propagation in these systems.

MULTI-FIELD BORN-INFELD LAGRANGIANS, NILPOTENT N=2 FIELDS AND CUBIC POLYNOMIALS

(FERRARA, SAGNOTTI, M.P., YERANIAN)

- CONCRETE REALIZATION OF N=2 BROKEN TO N=1
- LOW ENERGY LIMIT: NILPOTENT FIELDS AND N=2 ==> N=1 VOLKOV-AKULOV ACTION (BORN-INFELD)
- MANY VECTOR MULTIPLETS
- FIELD REDEFINITIONS AND INEQUIVALENT ACTIONS. ROLE OF CUBIC PREPOTENTIALS

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A CONCRETE EXAMPLE OF N=2 BROKEN TO N=1

ANTONIADIS, PARTOUCHE & TAYLOR '95

ONE N=2 VECTOR MULTIPLET
=
ONE N=1 VECTOR MULTIPLET V +
ONE N=1 CHIRAL MULTIPLET X

$$L = i[X\bar{U}' - \bar{X}U']_D + i[U''W^2 + U'm + eX]_F + c.c.$$

$$W_\alpha = \bar{D}^2 D_\alpha V \qquad U = \tau X^3$$

THE APT ACTION CONTAINS BOTH MASSLESS AND MASSIVE DEGREES OF FREEDOM

IN THE IR IT REDUCES TO THE ACTION OF THE GOLDSTINO MULTIPLET FOR THE PARTIAL BREAKING
N=2 ⟶ N=1

EQUATIONS OF MOTION:

$$\bar{D}^2(\bar{U}' - \bar{X}U'') + U'''W^2 + U''m + e = 0$$

SETTING TO ZERO THE FERMIONS,
THE LAST COMPONENT OF THESE EQUATIONS IS

$$-U'''\bar{F}F + U'''G_+^2 + U'''Fm + \text{ derivative terms } = 0$$

THE RIGID SUPERSYMMETRY CURRENT ALGEBRA CAN HAVE A CENTRAL TERM (POLCHINSKI)

$$\int d^3y\{\bar{J}^{0\,a}_{\dot\alpha}(y)J^{b}_{\mu\,\alpha}(0)\} = 2\sigma^{\nu}_{\dot\alpha\alpha}T_{\nu\mu}(0)\delta^{ab} + \sigma_{\dot\alpha\alpha\,\mu}C^{ab}$$

HIS SUSY ALGEBRA RELATES FERMION SHIFTS UNDER SU TO THE SCALAR POTENTIAL

$$Z^{AB}\delta^{a}\bar{\chi}_A\delta^{b}\chi_B = V\delta^{ab} + C^{ab}$$

$$X = Y + x, \qquad U''(x)m + e = 0$$

THE IR LIMIT IS EQUIVALENT TO SENDING ALL MASS SCALES TO INFINITY, I.E.

$$|U'''| \gg |U''(x)|$$

IN THE LIMIT, A SELF-CONSISTENT SOLUTION FOR THE CHIRAL FIELD IS

$$-Y\bar{D}^2\bar{Y} + W^2 + Ym = 0$$

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$$-Y\bar{D}^2\bar{Y} + W^2 + Ym = 0$$

THE LOWEST COMPONENT IS

$$(m - \bar{F})y + \lambda\lambda = 0$$

SO THAT Y OBEYS THE NILPOTENCY CONSTRAINTS

$$Y^2 = 0, \qquad YW_\alpha = 0$$

THE FULL ACTION

$$L = \operatorname{Im} eF$$

IS THE SAME AS THE NONLINEAR ACTION
DESCRIBING A GOLDSTINO N=1 SUPERMULTIPLET
(N=1 VOLKOV-AKULOV ACTION)

BUT IT IS ALSO AN N=1 BORN-INFELD
ACTION, WHOSE BOSONIC PART IS

$$L = \frac{m}{2}(\operatorname{Re} e)\left[1 - \sqrt{1 + \frac{4}{m^2}G^2 - \frac{4}{m^4}(G\tilde{G})^2}\right] - \frac{\operatorname{Im} e}{m}G\tilde{G}$$

$$-\bar{F}F + G_+^2 + Fm = 0$$

SOLVING FOR F AND PLUGGING BACK IN THE ACTION WE FIND A NONLINEAR (BORN-INFELD) ACTION FOR G

$$L = \mathrm{Im}\, eF$$

WE CAN DO BETTER AND SOLVE THE EQUATIONS OF MOTION SUPERSYMMETRICALLY TO EXPRESS X IN TERMS OF W

SHIFT THE CHIRAL SUPERFIELD X AS

$$X = Y + x, \qquad U''(x)m + e = 0$$

OTHER GENERALIZATIONS ARE POSSIBLE (ASCHIERI, BRACE, MORIARU, ZUMINO)

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OURS POSSESSES AN N=2 SUSY LINEARLY REALIZED IN THE UV

WE START FROM AN N=2 THEORY WRITTEN IN N=1 SUPERFIELD LANGUAGE:
n ABELIAN VECTOR MULTIPLETS AND n CHIRAL MULTIPLETS

N=2 MEANS THAT THE THEORY IS DEFINED BY A HOLOMORPHIC PREPOTENTIAL

$$U(X) = \frac{i}{2} C_{AB} X^A X^B + \frac{1}{3!} d_{ABC} X^A X^B X^C$$

NEEDED TO ENSURE POSITIVITY OF KINETIC TERMS

LAGRANGIAN

$$L = \mathrm{Im}\ \left\{[U_{AB}W^A W^B + U_A m^A - X^A e_A]_F + [X^A \bar{U}_A]_D\right\}$$

AS IN THE n=1 CASE IT IS CONVENIENT TO SHIFT THE SUPERFIELD X BY ITS VEV.

$$X^A = Y^A + x^A, \qquad U_{AB}(x) m^B = e_A$$

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E.O.M. FOR X IN LIMIT $\quad |U_{AB}(x)| \ll |d_{ABC}|$

$$d_{ABC} \left[W^A W^B + Y^B (m^C - \bar{D}^2 \bar{Y}^C) + \frac{1}{2} \bar{D}^2 (\bar{Y}^B \bar{Y}^C) \right] = 0$$

SELF-CONSISTENT ANSATZ IN IR LIMIT

$$d_{ABC}Y^BY^C = 0, \qquad d_{ABC}Y^BW^C = 0$$

GENERATES BI CONSTRAINTS

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SELF-CONSISTENT?
BECAUSE OF FIERZ IDENTITY

$$W_\alpha^{(A}W_\beta^BW_\gamma^{C)} = 0$$

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AFTER SOLVING THE CONSTRAINTS, IN THE IR LIMIT, THE BI LAGRANGIAN IS

$$L = \mathrm{Im}\left\{[U_{AB}(x)Y^A\bar{Y}^B]_D - [U_{AB}(x)W^A W^B]_F\right\}$$

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SECOND SUSY

$$\delta W^A_\alpha = m^A\eta_\alpha + \text{ terms vanishing on the vacuum}$$

OUR THEORIES ARE EQUIVALENT UNDER LINEAR FIELD REDEFINITIONS

INTERACTIONS AND CONSTRAINTS DEPEND ON CUBIC TERM ONLY

HOW MANY INEQUIVALENT INTERACTIONS ARE THERE, UP TO LINEAR FIELD REDEFINITIONS?

SOLUTION KNOWN FOR $n=2,3$ AND, PARTIALLY FOR $n=4$; UNKNOWN FOR GENERAL n

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TWO RELATED PROBLEMS

1) FIND CANONICAL FORMS FOR THE (CUBIC) PREPOTENTIAL

2) FIND INVARIANTS CHARACTERIZATIONS OF INEQUIVALENT THEORIES

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1) FIND CANONICAL FORMS FOR THE (CUBIC) PREPOTENTIAL

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WE WILL START WITH THE SIMPLEST NONTRIVIAL CASE: n=2

CUBIC PART OF PREPOTENTIAL
=
REAL HOMOGENEOUS CUBIC IN X,Y

$$U = (X - aY)(X - bY)(X - cY)$$

$$a, b, c = \text{real or } a = \text{real }, b = c*$$

LINEAR, HOMOGENEOUS COORDINATE CHANGES = GL(2,R)

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A) THREE REAL, DISTINCT a,b,c

WITH GL(2,R) SET a=1,b=0,c=-1

$$U = X(X^2 - Y^2)$$

INVARIANT POLYNOMIALS: THEY CLASSIFY THE ORBITS OF GL(2,R) [SAME AS SL(2,R)]

$$I_4 = -27d_{222}^2 d_{111}^2 + d_{221}^2 d_{112}^2 + 18 d_{222} d_{111} d_{112} d_{221} - 4 d_{111} d_{122}^3 - 4 d_{222} d_{211}^3$$

CASE A): $I_4 > 0$

CASE B): $I_4 < 0$

CASE C): $I_4 = 0$

CASE D): $I_4 = 0, \qquad \partial I_4 = 0$

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INEQUIVALENT QUADRATIC TERMS (UP TO SHIFTS IN X,Y)

$A)\ a(X^2+Y^2) \qquad B)\ bXY \qquad C)\ cY^2 \qquad D)\ aY^2 + bXY$

CASE n=3: CUBIC POLYNOMIALS UP TO GL(3,R): REPRESENTATIVE POLYNOMIALS

A BIT OF (PROJECTIVE) GEOMETRY:
A CUBIC HOMOGENEOUS POLYNOMIAL
IN 3 REAL VARIABLES DEFINES A
CUBIC CURVE ON THE PLANE

(ON THE COMPLEX, IT DEFINES A GENUS ONE CURVE)

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THE EASIEST CASE IS THE NON-DEGENERATE ONE:

$$d_{ABC}X^BX^C \text{ not all } = 0 \text{ on curve } d_{ABC}X^AX^BX^C = 0$$

IRREDUCIBLE AND SINGULAR: EITHER NODE OR CUSP

$$U = Y^2 Z - X^3 + \epsilon X^2 Z, \qquad \epsilon = 0, \pm 1$$

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REDUCIBLE: QUADRIC TIMES LINE: QL

$$Q = X^2 + Y^2 + Z^2, \qquad L = X + Y + Z$$

$$Q = X^2 + Y^2 - Z^2, \qquad L = X \text{ or } L = Z \text{ or } L = X + Z$$

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REDUCIBLE: THREE LINES

$$U = XYZ, \qquad U = X(Y^2 + Z^2) \qquad \text{non concurrent}$$

$$U = XY(X + Y), \qquad U = X(X^2 + Y^2) \qquad \text{concurrent}$$

$$XY^2, \qquad X^3 \qquad \text{2 or 3 coincident}$$

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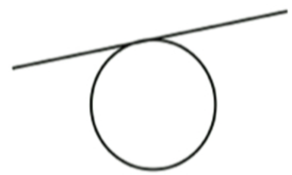
A GRAPHIC SUMMARY (OVER THE COMPLEX)

A_c^1

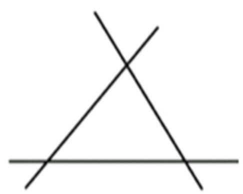
A_c^2

B_c^1

B_c^2



C_c^1



C_c^2

C_c^3

C_c^4

SUMMARY TABLE OF SINGULAR CASES

R	C	Polynomial	P_4	Q_6	∂I_{12}	∂P_4	∂Q_6	$\partial^2 I_{12}$	$\partial^2 P_4$	$\partial^2 Q_6$
A^1	A_c^1	$-x^3 - x^2 z + y^2 z$	$\frac{8}{27}$	$\frac{16}{243}$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
A^2	A_c^1	$-x^3 + x^2 z + y^2 z$	$\frac{8}{27}$	$-\frac{16}{243}$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
A^3	A_c^2	$-x^3 + y^2 z$	0	0	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
B^1	B_c^1	$(x+y+z)(x^2+y^2+z^2)$	$\frac{8}{3}$	$-\frac{16}{9}$	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
B^2	B_c^1	$x(x^2+y^2-z^2)$	$\frac{8}{27}$	$\frac{16}{243}$	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
B^3	B_c^1	$z(x^2+y^2-z^2)$	$\frac{8}{27}$	$-\frac{16}{243}$	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
B^4	B_c^2	$(x+z)(x^2+y^2-z^2)$	0	0	0	$\neq 0$	0	0	$\neq 0$	$\neq 0$
C^1	C_c^1	$6xyz$	24	48	0	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$
C^2	C_c^1	$x(y^2+z^2)$	$\frac{8}{27}$	$-\frac{16}{243}$	0	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$
C^3	C_c^2	$xy(x+y)$	0	0	0	0	0	0	$\neq 0$	$\neq 0$
C^4	C_c^2	$x(x^2+y^2)$	0	0	0	0	0	0	$\neq 0$	$\neq 0$
C^5	C_c^3	xy^2	0	0	0	0	0	0	$\neq 0$	0
C^6	C_c^4	x^3	0	0	0	0	0	0	0	0

FURTHER DEVELOPMENTS

- EQUATIONS FOR AUXILIARY FIELDS CAN BE SOLVED ALGEBRAICALLY FOR ALL CASE FOR $n=2,3$ .... BUT THE SOLUTION MAY NOT BE ILLUMINATING....

(1) $$\mathrm{Im}\, F^i \;=\; \frac{G^i\cdot\tilde{G}^i\, m^j\, m^k \;+\; A^i\,\sigma \;+\; B^i\,\sigma^2 \;+\; C^i\,\sigma^3}{\left[(m^i)^3 \;+\; (m^j)^3 \;+\; (m^k)^3\right]\,\sigma^2 \;-\; m^i\, m^j\, m^k\;(1+2\sigma^3)}\;,$$

where $(i,j,k)=1,2,3,\ (i\neq j\neq k)$

$$\begin{aligned} A^i &= 2\,G^j\cdot\tilde{G}^k\, m^j\, m^k \;-\; G^j\cdot\tilde{G}^j\,(m^k)^2 \;-\; G^k\cdot\tilde{G}^k\,(m^j)^2\;, \\ B^i &= G^j\cdot\tilde{G}^j\, m^i\, m^j \;+\; G^k\cdot\tilde{G}^k\, m^i\, m^k \;-\; G^i\cdot\tilde{G}^i\,(m^i)^2 \\ &\qquad -\,2\,G^i\cdot\tilde{G}^j\,(m^j)^2 \;-\; 2\,G^i\cdot\tilde{G}^k\,(m^k)^2\;, \end{aligned}$$

(2) $$C^i \;=\; 2\,m^i\left(-G^j\cdot\tilde{G}^k\, m^i \;+\; G^i\cdot\tilde{G}^k\, m^j \;+\; G^i\cdot\tilde{G}^j\, m^k\right)\;.$$

(3) $$H^1 \;=\; \sqrt{\frac{A_{11}\,A_{22}}{A_{22}+2\,U\,\sigma\left(A_{33}\,\sigma+\sqrt{A_{22}\,A_{33}\,U^2-2A_{22}^2\,U\,\sigma+A_{33}^2\,\sigma^2}\right)}}\;,$$

where U is a solution of the fourth–order equation

$$U^4\left(A_{11}^2 \;-\; 4\,A_{22}\,A_{33}\,\sigma^2\right) \;+\; 4\,U^3\,\sigma^2\left(A_{11}\,A_{33} \;+\; 2\,A_{22}^2\,\sigma\right) \;-\; 2\,A_{11}\,A_{22}\,U^2\left(1 \;+\; 8\,\sigma^3\right)$$

(4) $$+\;4\,U\,\sigma^2\left(A_{22}\,A_{33} \;+\; 2\,A_{11}^2\,\sigma\right) \;+\; A_{22}^2 \;-\; 4\,A_{11}\,A_{33}\,\sigma^2 \;=\; 0$$

that is consistent with the weak–field limit.

Definitions:

(5) $$A_{i\,i} \;=\; R^{i\,i} \;+\; 2\,\sigma R^{j\,k}\;,$$

with $(i,j,k)=1,2,3,\ (i\neq j\neq k)$.

(6) $$R^{AB} \;=\; G^A\cdot G^B \;+\; \frac{m^A\, m^B}{4} \;-\;\mathrm{Im}\,F^B\,\mathrm{Im}\,F^C$$

POSITIVITY OF THE BI LAGRANGIAN IS MUCH EASIER TO STUDY IN THE UV N=2 THEORY. IN SOME CASES POSITIVITY DOES NOT REQUIRE A QUADRATIC TERM IN THE PREPOTENTIAL (STUDY USING SYLVESTER'S CRITERION)

Polynomial	Determinant	Minor2	Minor1
$x^3+y^3+z^3+6\,\sigma\,x\,y\,z$	$x\,y\,z\,(1+2\,\sigma^3)$ $-\left(x^3+y^3+z^3\right)\,\sigma^2$	$xy-z^2\,\sigma^2$	x
$-x^3-x^2z+y^2z$	$3xy^2-x^2z+y^2z$	$-\frac{z}{9}\,(3x+z)$	$-x-\frac{z}{3}$
$-x^3+x^2z+y^2z$	$\frac{xy^2}{9}-\frac{1}{27}\left(x^2+y^2\right)z$	$\frac{z}{9}(-3x+z)$	$-x+\frac{z}{3}$
$-x^3+y^2z$	$\frac{xy^2}{9}$	$-\frac{xz}{3}$	$-x$
$(x+y+z)\left(x^2+y^2+z^2\right)$	$\frac{1}{27}(x+y+z)\left[x^2+y^2\right.$ $\left.+8yz+z^2+8x(y+z)\right]$	$\frac{2}{9}\left(x^2+4xy+y^2\right)$ $+\frac{1}{9}\left[4(x+y)z+z^2\right]$	$x+\frac{y+z}{3}$
$x\left(x^2+y^2-z^2\right)$	$-\frac{x}{27}\left(3x^2-y^2+z^2\right)$	$\frac{1}{9}\left(3x^2-y^2\right)$	x
$z\left(x^2+y^2-z^2\right)$	$-\frac{z}{27}\left(x^2+y^2+3z^2\right)$	$\frac{z^2}{9}$	$\frac{z}{3}$
$(x+z)\left(x^2+y^2-z^2\right)$	$-\frac{4}{27}(x+z)^3$	$\frac{1}{9}\left[(x+z)(3x+z)-y^2\right]$	$x+\frac{z}{3}$
$6\,x\,y\,z$	$2\,x\,y\,z$	$-z^2$	0
$x\left(y^2+z^2\right)$	$-\frac{x}{27}\left(y^2+z^2\right)$	$-\frac{y^2}{9}$	0
$x\,y\,(x+y)$	0	$\frac{1}{9}\left(-x^2-xy-y^2\right)$	$\frac{y}{3}$
$x\left(x^2+y^2\right)$	0	$\frac{1}{9}\left(3x^2-y^2\right)$	x
$x\,y^2$	0	$-\frac{y^2}{9}$	0
x^3	0	0	x

FURTHER DEVELOPMENTS

- EQUATIONS FOR AUXILIARY FIELDS CAN BE SOLVED ALGEBRAICALLY FOR ALL CASE FOR n=2,3
- IN SOME CASES THE ELECTRIC FIELD OF POINT SOURCES IS BOUNDED, AS IN THE BOSONIC BORN-INFELD LAGRANGIAN. THIS SEEMS TO HAPPEN WHEN A PURELY CUBIC PREPOTENTIAL GIVES A POSITIVE KINETIC TERM

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- THE CLASSIFICATION OF THE n=4 CASE IS DOABLE BUT NOT FULLY DONE IN THE MATHEMATICAL LITERATURE

SUMMARY:

WE PROPOSED A MULTI-FIELD EXTENSION OF BORN-INFELD THAT POSSESSES A HIDDEN, BROKEN N=2 SUPERSYMMETRY.

IT ARISES AS AN INFRARED LIMIT OF AN N=2 THEORY BROKEN SPONTANEOUSLY BY (RELEVANT) FLUXES

CLASSIFICATION OF INEQUIVALENT BI THEORIES BECOMES A NONTRIVIAL ALGEBRAIC GEOMETRY PROBLEM, SOLVABLE FOR LOW n

CAUSALITY AND (NON) SUPERLUMINALITY RELATIVELY EASY TO CHECK IN THE UV, DIFFICULT IN THE IR EFFECTIVE THEORY