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Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Uncertainty Relations, Classicalization and Superluminal propagation

arXiv:1208.3647
Europhysics Letters - 101 (2013) 34001

Alexander Vikman

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

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Plan

- Uncertainty Relations in Quantum Mechanics
- Uncertainty Relations in QFT
- Classicalization
- Quantum vacuum fluctuations (QF) for derivatively coupled Nambu-Goldstone bosons

Uncertainty Relations, QM

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- Werner Heisenberg (1927): $\Delta q \cdot \Delta p \gtrsim \hbar$
- Earle Kennard, Hermann Weyl (1928):

$$\delta_\Psi q \cdot \delta_\Psi p \geq \frac{\hbar}{2}$$

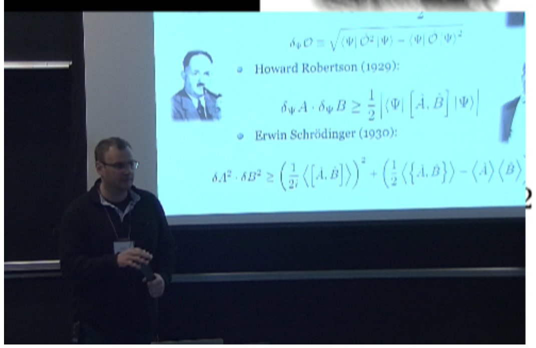
$$\delta_\Psi \mathcal{O} \equiv \sqrt{\langle\Psi|\,\hat{\mathcal{O}}^2\,|\Psi\rangle - \langle\Psi|\,\hat{\mathcal{O}}\,|\Psi\rangle^2}$$

- Howard Robertson (1929):

$$\delta_\Psi A \cdot \delta_\Psi B \geq \frac{1}{2}\left|\langle\Psi|\left[\hat{A},\hat{B}\right]|\Psi\rangle\right|$$

- Erwin Schrödinger (1930):

$$\cdot\, \delta B^2 \geq \left(\frac{1}{2i}\left\langle\left[\hat{A},\hat{B}\right]\right\rangle\right)^2 + \left(\frac{1}{2}\left\langle\left\{\hat{A},\hat{B}\right\}\right\rangle - \left\langle\hat{A}\right\rangle\left\langle\hat{B}\right\rangle\right)^2$$



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Averaging fields I

- Every device has a *finite* resolution
- One cannot measure a field at a point in *space*
- Every measurement of a field yields an eigenvalue of the corresponding *space-averaged* field operator

Averaging fields II

- A device with *finite adjustable* resolution ℓ measures eigenvalues of the field operator

$$\hat{\phi}_\ell(\mathbf{x},t) = \int \mathrm{d}^d\mathbf{x}'\, W^\phi_\ell(\mathbf{x}-\mathbf{x}')\,\hat{\phi}(\mathbf{x}',t)$$

smeared by a family of window functions

$$W^\phi_\ell(\mathbf{x}) = \ell^{-d}\cdot w^\phi\left(\frac{\mathbf{x}}{\ell}\right)$$

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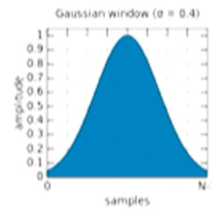

$$\hat{\phi}_\ell\left(\mathbf{x},t\right) = \int \mathrm{d}^d\mathbf{x}'\, W_\ell^\phi\left(\mathbf{x}-\mathbf{x}'\right)\hat{\phi}\left(\mathbf{x}',t\right)$$

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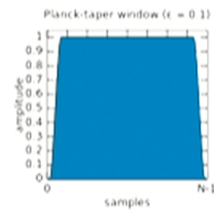

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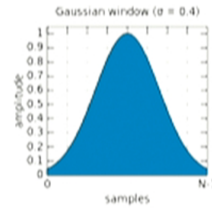

$$\hat{\phi}_\ell\left(\mathbf{x}, t\right) = \int \mathrm{d}^d\mathbf{x}'\, W_\ell^\phi\left(\mathbf{x} - \mathbf{x}'\right) \hat{\phi}\left(\mathbf{x}', t\right)$$

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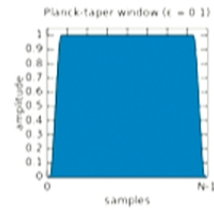

$$W_\ell^\phi\left(\mathbf{x}\right) = \ell^{-d} \cdot w^\phi\left(\frac{\mathbf{x}}{\ell}\right)$$

- Another device which we put *at the same place* measures canonical momentum $\hat{p}\left(t, \mathbf{x}\right)$ averaging it with some other family of window functions with the same scaling property above. Suppose that the resolution is the same.

Canonical Quantization

$$\left[\hat{\phi}\left(t,\mathbf{x}\right),\hat{p}\left(t,\mathbf{y}\right)\right]=i\hbar\,\delta\left(\mathbf{x}-\mathbf{y}\right)$$

$$\left[\hat{\phi}_{\ell}\left(\mathbf{x}\right),\hat{p}_{\ell}\left(\mathbf{y}\right)\right]=i\hbar\cdot\ell^{-d}\cdot\mathscr{D}\left(\frac{\mathbf{x}-\mathbf{y}}{\ell}\right)$$

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where

$$\mathscr{D}\left(\mathbf{r}\right)=\int\mathrm{d}^{d}\mathbf{r}'\,w^{\phi}\left(\mathbf{r}-\mathbf{r}'\right)w^{p}\left(\mathbf{r}'\right)$$

dimensionless

this convolution of shapes does not depend on scale ℓ but only on the way of averaging

Uncertainty Relations, QFT

fluctuations *on scale* ℓ:

$$\delta_\Psi \Phi_\ell \equiv \sqrt{\langle\Psi|\,\hat{\Phi}_\ell^2\,|\Psi\rangle - \langle\Psi|\,\hat{\Phi}_\ell\,|\Psi\rangle^2}$$

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$$\delta_\Psi \Phi_\ell \equiv \sqrt{\langle\Psi|\,\hat{\Phi}_\ell^2\,|\Psi\rangle - \langle\Psi|\,\hat{\Phi}_\ell\,|\Psi\rangle^2}$$

general uncertainty relation

$$\delta_\Psi A \cdot \delta_\Psi B \geq \frac{1}{2}\left|\langle\Psi|\left[\hat{A},\hat{B}\right]|\Psi\rangle\right|$$

$$\delta\phi_\ell(\mathbf{x}) \cdot \delta p_\ell(\mathbf{x}) \geq \frac{\hbar}{2} \cdot \mathscr{D}_0 \cdot \ell^{-d}$$

for two Gaussian window functions $\mathscr{D}_0 = \left(2\sqrt{\pi}\right)^{-d}$

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Two -Point Function / Correlator and the Power Spectrum

Two -Point Function / Correlator and the Power Spectrum

$$\langle 0|\, \hat{\varphi}\left(\mathbf{x}\right) \hat{\varphi}\left(\mathbf{y}\right) |0\rangle = \int \frac{dk}{k} \cdot \frac{\sin k\,|\mathbf{x}-\mathbf{y}|}{k\,|\mathbf{x}-\mathbf{y}|} \cdot \mathcal{P}\left(k\right)$$

power spectrum

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power spectrum

$$\langle 0|\,\hat{\varphi}_\ell^2\left(\mathbf{x}\right)|0\rangle = \int \frac{dk}{k}\cdot d^3\mathbf{y}'d^3\mathbf{x}'W_\ell\left(\mathbf{x}'\right)W_\ell\left(\mathbf{y}'\right)\frac{\sin k\left|\mathbf{x}'-\mathbf{y}'\right|}{k\left|\mathbf{x}'-\mathbf{y}'\right|}\cdot\mathcal{P}\left(k\right)$$

$$\delta\varphi_\ell^2 = \langle 0|\,\hat{\varphi}_\ell^2\left(\mathbf{x}\right)|0\rangle = \int dk\cdot D\left(k\ell\right)\cdot\mathcal{P}\left(k\right)$$

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$$\delta\varphi_\ell^2 \sim \mathcal{P}\left(\ell^{-1}\right)$$

Vacuum Fluctuations in 4d Minkowski spacetime for a canonical field in UV

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Classicalization & Gravity *motivation*

Dvali, Gomez;
Dvali, Giudice, Gomez, Kehagias;
Dvali, Pirtskhalava
(2010)

- How can non-renormalizable, strongly-coupled in UV, theories e.g. gravity maintain unitarity?
- In gravity any attempt to probe physics on scales $\ell \ll \ell_{\rm Pl}$ leads to a creation of a large *Black Hole* with the radius

$$R_{\rm g} = M_{\rm Pl}^{-1} \left(\ell \, M_{\rm Pl}\right)^{-1} = \ell_{\rm Pl} \left(\frac{\ell_{\rm Pl}}{\ell} \right)$$

- Thus probing ℓ corresponds to $R_{\rm g}\,(\ell) \gg \ell_{\rm Pl} > \ell$
- *BH* slowly decay into weakly-coupled *soft* quanta
- This suggests that physics for UV $\ell \ll \ell_{\rm Pl}$ can be understood as (is equivalent to) weakly coupled quasiclassical gravity in IR

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- By scattering with a transferred momentum larger than the strong-coupling scale the system forms long lived *quasiclassical* configurations - *classicalons*
- The higher is the transferred momentum the larger is the size of the corresponding *classicalon*. *Classicalons* slowly decay into weakly-coupled *soft* quanta

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- In gravity: strong coupling-scale is the Planck scale and Black Holes are *classicalons*

Classicalization

Strongly coupled quantum physics in UV is equivalent to weakly coupled quasiclassical physics in IR!

Classicalization

Strongly coupled quantum physics in UV is equivalent to weakly coupled quasiclassical physics in IR!

classical scale of nonlinearity similar to the Schwarzschild radius

$$R_\star = M_s^{-1} \left(\ell M_s\right)^{-\alpha}$$

One cannot suppress the *product* of QF!

$$\delta\phi_\ell(\mathbf{x}) \cdot \delta p_\ell(\mathbf{x}) \geq \frac{\hbar}{2} \cdot \mathscr{D}_0 \cdot \ell^{-d}$$

The shorter is the scale of averaging the more quantum is any field theory!

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Suppressing Quantum Fluctuations (QF)?

- The higher is the transferred momentum the larger and the *more classical* is the corresponding ***field configuration*** - *classicalon*

- The shorter is the scale the softer are *quantum fluctuations of the field* $\delta\phi_\ell$.
  The power spectrum is *red* in UV

c.f. A. Kovner and M. Lublinsky (2012)

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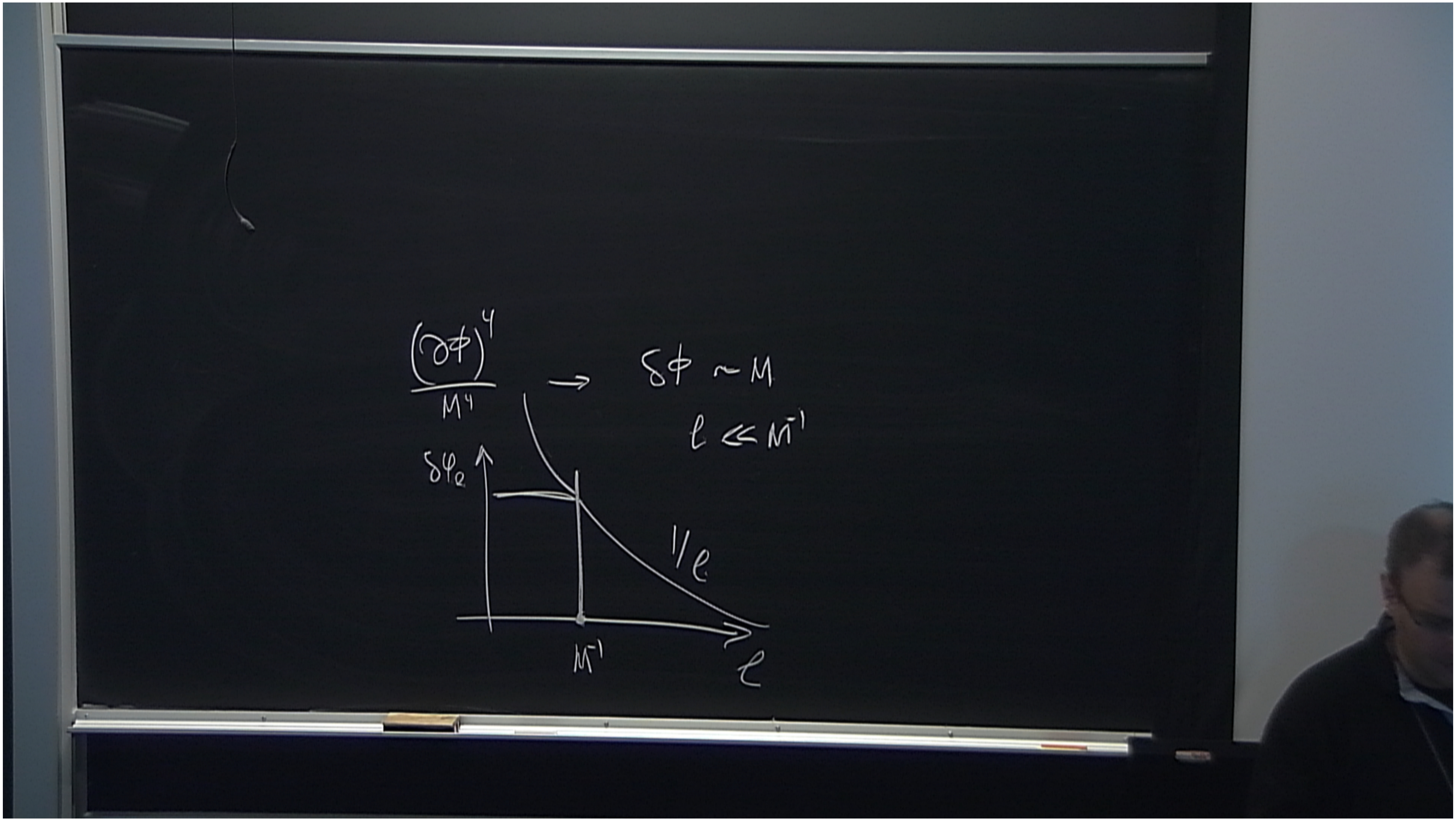

$\frac{(\partial\phi)^4}{M^4}$ → $\delta\phi \sim M$
$\ell \ll M^{-1}$
$\delta\varphi_\ell$
$1/\ell$
M^{-1}
ℓ

Can a simple scalar field theory suppress $\delta\phi_\ell$?

Derivatively coupled Nambu-Goldstone bosons "k-essence"

- shift $\phi \to \phi + c$ symmetric action

$$S = \int \mathrm{d}^{1+d}x\, \mathscr{L}(X) \quad \text{where} \quad X = \frac{1}{2}(\partial\phi)^2$$

- canonical momentum $p = \frac{\partial \mathscr{L}}{\partial \dot{\phi}} = \mathscr{L}_X \dot{\phi}$
- one scale M_s :

$$\mathscr{L} = X\left(1 + \frac{X}{M_\mathrm{s}^{d+1}} + \ldots\right)$$

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Main Approximation $\langle f(\hat{\mathcal{O}})\rangle \simeq f(\langle\hat{\mathcal{O}}\rangle)$

$$\delta p\,(\ell) = \sqrt{\langle 0|\left(\mathscr{L}_X\dot{\phi}\right)^2_\ell|0\rangle} = \delta p_2\,(\delta\phi_\ell/\ell) + \text{higher order correlators}$$

where $\delta p_2\,(\delta\phi_\ell/\ell) \simeq \mathscr{L}_X\,(X_\ell)\,\delta\phi_\ell/\ell$, and $X_\ell = -\left(\frac{\delta\phi}{\ell}\right)^2$

- One can factorize and neglect higher order correlators, because the theory is supposed to be quasiclassical and weakly coupled
- the Lorentz invariance of the vacuum $|0\rangle$ allows to estimate space and time derivatives as

$$\delta\dot{\phi} \simeq \delta\partial_i\phi \simeq \delta\phi/\ell$$

- minus in front of X_ℓ corresponds to an Euclidean estimation

Vacuum minimizes product of QF out of all energy eigenstates

- $$\delta\phi(\ell)\cdot\delta p(\ell) = \frac{\hbar\mathscr{D}_0}{2}\cdot\Omega(\ell)\cdot\ell^{-d} \geq \frac{\hbar\mathscr{D}_0}{2}\cdot\ell^{-d},$$

characterizes vacuum

- $$\mathscr{L}_X(X_\ell)\,\delta\phi^2(\ell) \simeq \frac{\hbar\mathscr{D}_0}{2}\cdot\Omega(\ell)\cdot\ell^{1-d},$$

for a given $\Omega(\ell)$ this is an algebraic equation on $\delta\phi(\ell)$

differentiating with respect to ℓ we get rid of $\mathscr{D}_0$ and to find the slope $\mathrm{d}\delta\phi/\mathrm{d}\ell$

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Slope of QF in Vacuum

$$\frac{\mathrm{d}\delta\phi}{\mathrm{d}\ell} = \frac{\delta\phi}{\ell} \cdot \left[\frac{v^2 - d + \mathrm{d}\ln\Omega/\mathrm{d}\ln\ell}{v^2 + 1}\right]$$

Slope of QF in Vacuum

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where $v^2 = v^2\left(X_\ell\right) = 1 + \left.\frac{2X\mathscr{L}_{XX}}{\mathscr{L}_X}\right|_{X=X_\ell}$

is the speed of propagation for small perturbations around a *classical static* background $\phi\left(\mathbf{x}\right)$ such that $X_\ell = -\frac{1}{2}\partial_i\phi\partial_i\phi$

c.f. C. Armendariz-Picon and E. A. Lim (2005)

Conditions for suppression of QF $\delta\phi(\ell)$ in vacuum

$$\frac{\mathrm{d}\delta\phi}{\mathrm{d}\ell} = \frac{\delta\phi}{\ell} \cdot \left[\frac{1-d}{2} + \frac{(1+d)\left(v^2-1\right) + 2\mathrm{d}\ln\Omega/\mathrm{d}\ln\ell}{2\left(v^2+1\right)} \right]$$

- QF suppressed with respect to the free field:
$$v^2 > 1 - \frac{2}{1+d} \cdot \frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln\ell} \geq 1$$
- Red spectrum of QF: $v^2 > d - \frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln\ell} \geq d$

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$$\frac{d\delta\phi}{d\ell} = \frac{\delta\phi}{\ell} \cdot \left[\frac{1-d}{2} + \frac{(1+d)\left(v^2-1\right) + 2d\ln\Omega/d\ln\ell}{2\left(v^2+1\right)}\right]$$

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quasiclassical suppression of

$$\delta\phi\left(\ell\right)$$

in Lorentz invariant vacuum

The theory should possess such a *classical static* background $\phi\left(\mathbf{x}\right)$ where the speed of propagation v for small classical perturbations is superluminal!

cf. Dvali (2011); Dvali, Franca, Gomez (2012)

- superluminal propagation of perturbations on nontrivial backgrounds $\phi(x^\mu)$ does not automatically imply any causal paradoxes and inconsistencies in effective field theory

Babichev, Mukhanov, AV (2007); Geroch (2010)

- this superluminal propagation implies the absence of the Lorentz invariant local and weakly coupled Wilsonian UV completion

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

⇓

- classicalization and Wilsonian UV-completions are mutually exclusive

Dvali (2011); Dvali, Franca, Gomez (2012)

Lagrange multiplier and “renormalization” of $\hbar$

- One can rewrite the theory as

$$S = \int \mathrm{d}^{1+d}x \, \left(\lambda \cdot X - V(\lambda)\right)$$

- Lagrange multiplier $\lambda = \mathscr{L}_X$ rescales the kinetic term

$$\delta\phi(\ell) \cdot \delta\dot{\phi}(\ell) \geq \frac{\mathscr{D}_0}{2} \left(\frac{\hbar}{\lambda(\ell)}\right) \cdot \ell^{-d} \, .$$

DBI vs anti-DBI

$$\mathscr{L} = \sigma M_{\rm s}^{d+1}\left[\sqrt{1+\frac{2X}{\sigma M_{\rm s}^{d+1}}}-1\right], \quad X = \frac{1}{2}(\partial\phi)^2$$

- Never ghosty
- "always" hyperbolic - free of gradient instabilities
- $\sigma = -1$ standard DBI, never superluminal
- $\sigma = +1$ anti-DBI, never subluminal Mukhanov & Vikman (2005)

Extensively studied as a toy model for classicalization:

Dvali, Giudice, Gomez, Kehagias; (2010)
Dvali, Franca, Gomez (2012)
Rizos, Tetradis (2011); Rizos, Tetradis, Tsolias; (2012)
Alberte, Bezrukov (2012)

(anti)-DBI QF in **UV**, $\ell \ll M_s^{-1}$

- DBI- hard / blue spectrum

$$\delta\phi_{\mathrm{DBI}}(\ell) \sim M_s^{-(d+1)/2} \cdot \ell^{-d}$$

- anti-DBI- soft / red spectrum

$$\delta\phi_{\mathrm{aDBI}}(\ell) \sim M_s^{(d+1)/2} \cdot \ell$$

vacuum state is squeezed in orthogonal directions
for these theories

$$\frac{\delta p_{\mathrm{aDBI}}}{\delta\phi_{\mathrm{aDBI}}/\ell} \sim (\ell M_s)^{-(d+1)} \gg 1\,,$$

$$\frac{\delta p_{\mathrm{DBI}}}{\delta\phi_{\mathrm{DBI}}/\ell} \sim (\ell M_s)^{+(d+1)} \ll 1\,.$$

Conclusions and open questions

- The superluminality seems to be necessary for a *quasiclassical* softening of quantum fluctuations i.e. for the suggested *classicalization* of the Nambu-Goldstone bosons.
- Can one prove this result and estimations by some other calculation?
- Is it the case for other systems?

Thanks a lot for attention!