Title: Superluminalities in Galileon theories

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Abstract:

Review on Superluminalities

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CASE WESTERN RESERVE UNIVERSITY EST. 1826
think beyond the possible™

Perimeter Institute April 9th 2015

Some Slides courtesy of
C. de Rham

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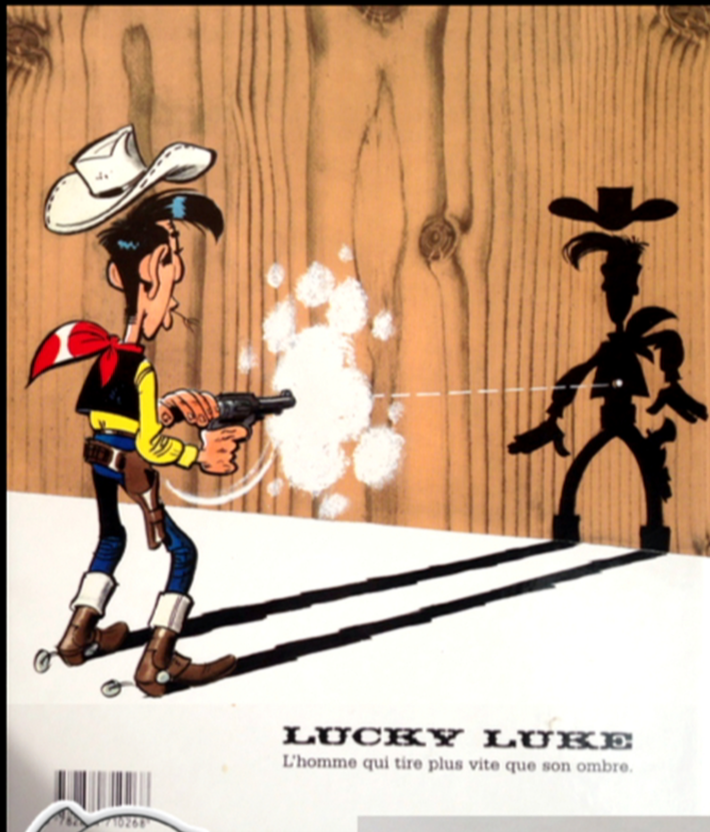
Sorry I couldn't be there
I am busy reading Lucky Luke



LUCKY LUKE

L'homme qui tire plus vite que son ombre.

The man that shoots faster than his shadow

What is the speed of information?

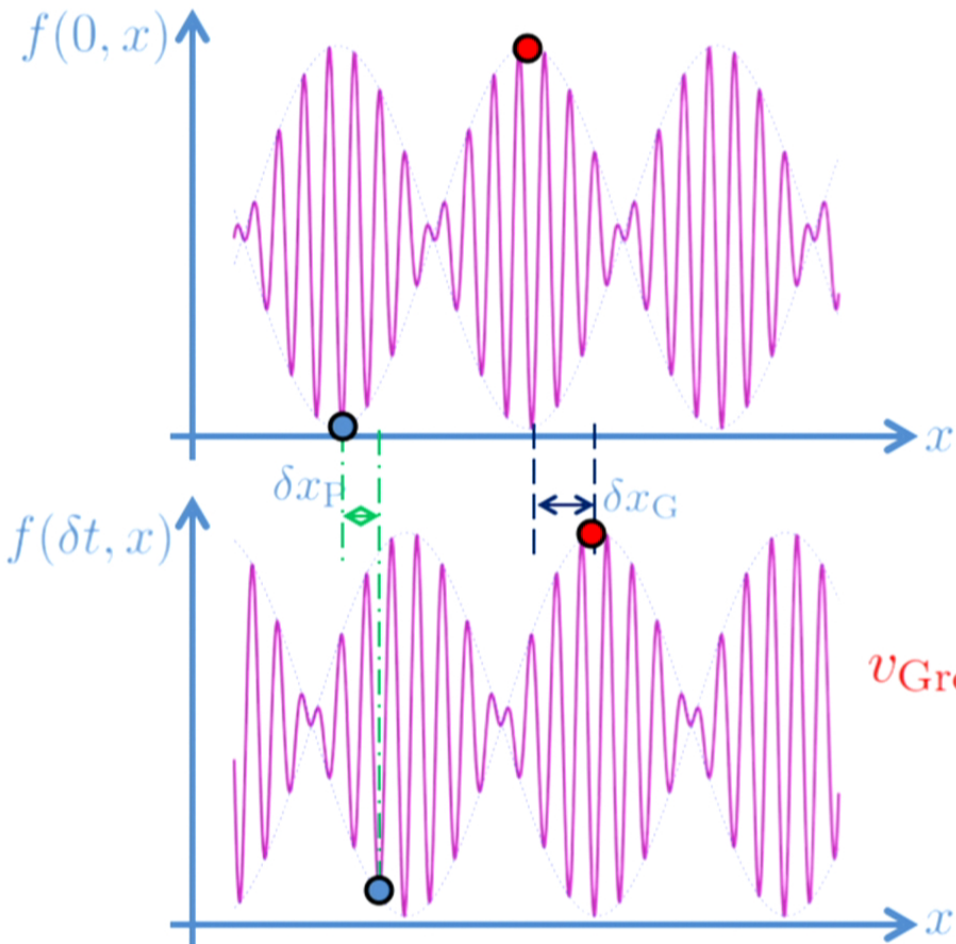
1. The velocity of a particle is unambiguous!
2. For a field/wave there are two natural notions of velocity: phase velocity and group velocity
3. Most undergraduates are told that the phase velocity can become superluminal without violating causality, but that the group velocity is the speed of information and must be (sub)luminal

What is the speed of information?

1. It was understood by Sommerfeld shortly after the invention of special relativity that the group velocity could become superluminal **without contradicting SR**
2. Only what Sommerfeld called the (wave)front velocity needed to be luminal

$$v_{\mathrm{front}} = \lim_{k \to \infty} \frac{\omega}{k}$$

Velocity



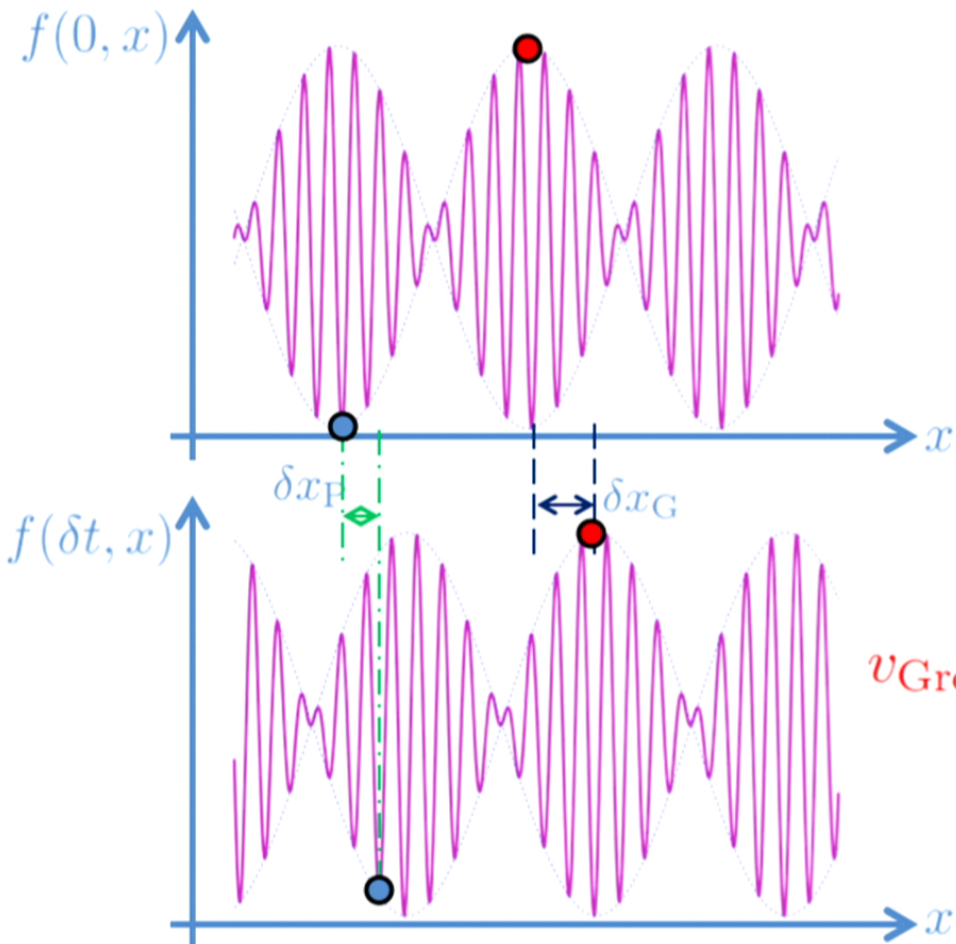
- Phase velocity

$$v_{\mathrm{Phase}} = \frac{\delta x_P}{\delta t} = \frac{\omega}{k}$$

- Group velocity

$$v_{\mathrm{Group}} = \frac{\delta x_G}{\delta t} = \frac{\partial \omega(k)}{\partial k}$$

$$f(t,x) = \int A(k) e^{i(k.x-\omega(k)t)} \mathrm{d}^3 k$$



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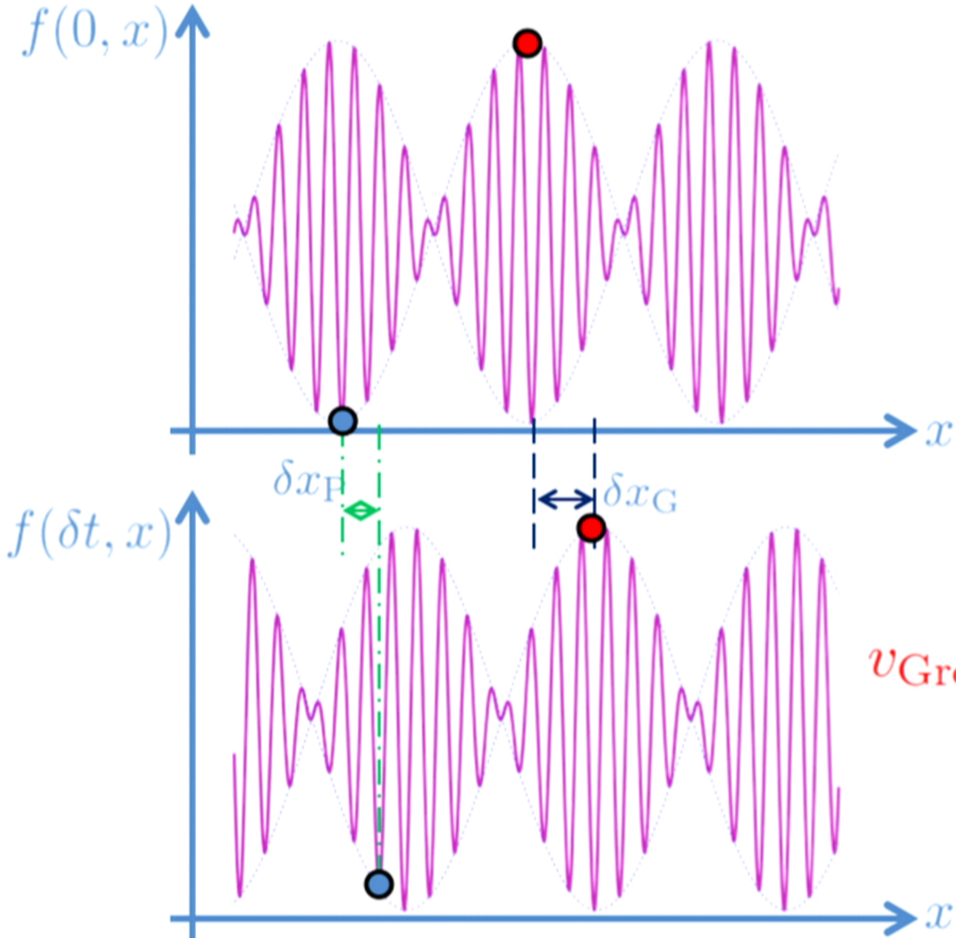

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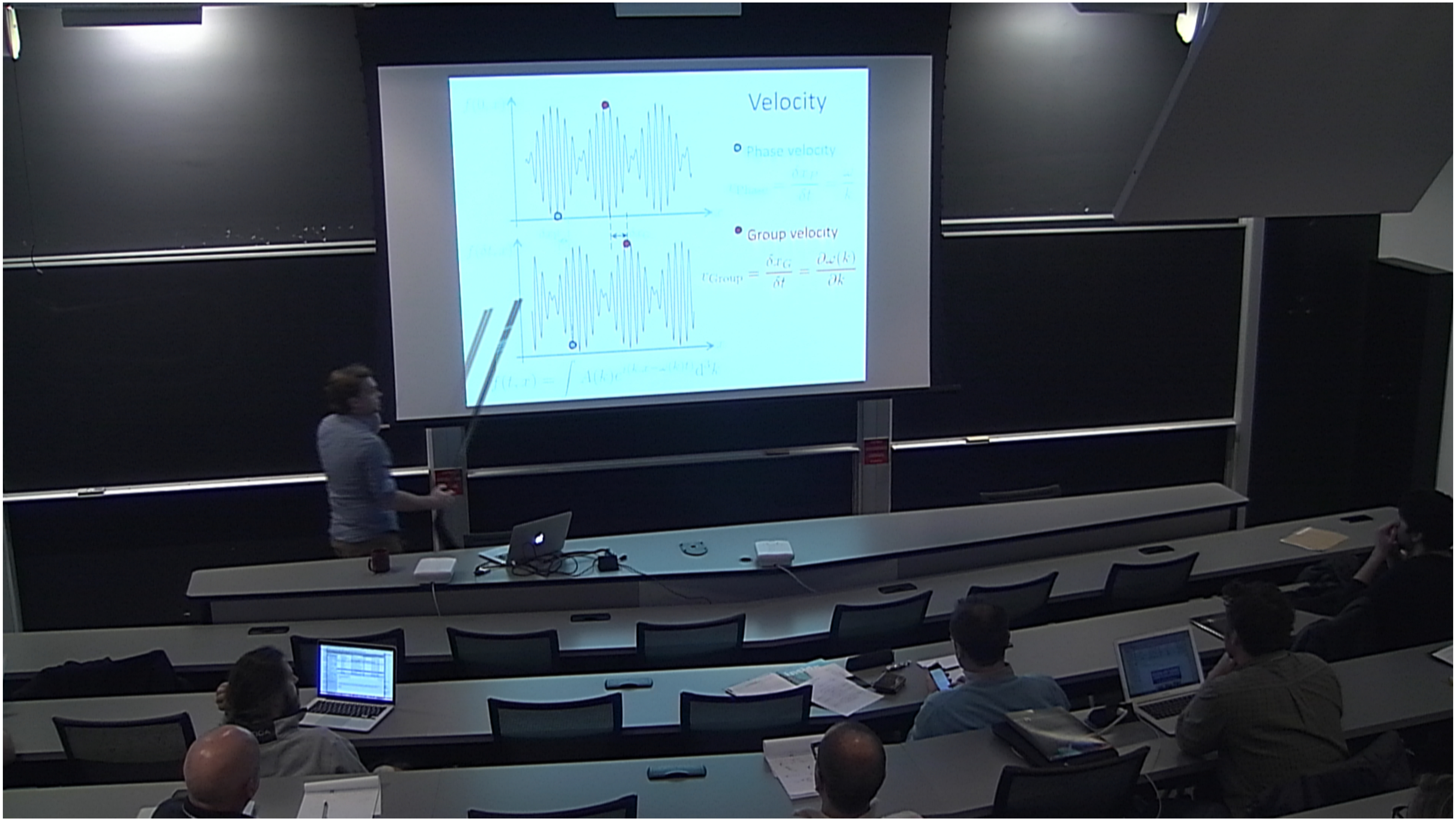
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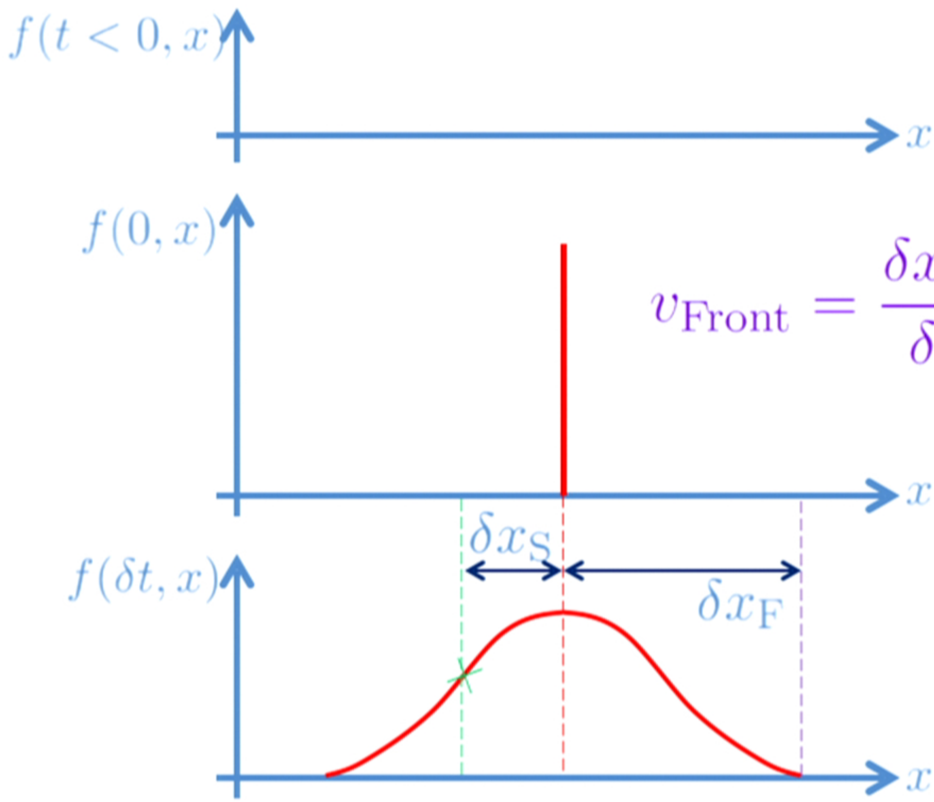
Phase velocity
Group velocity

Speed of Information



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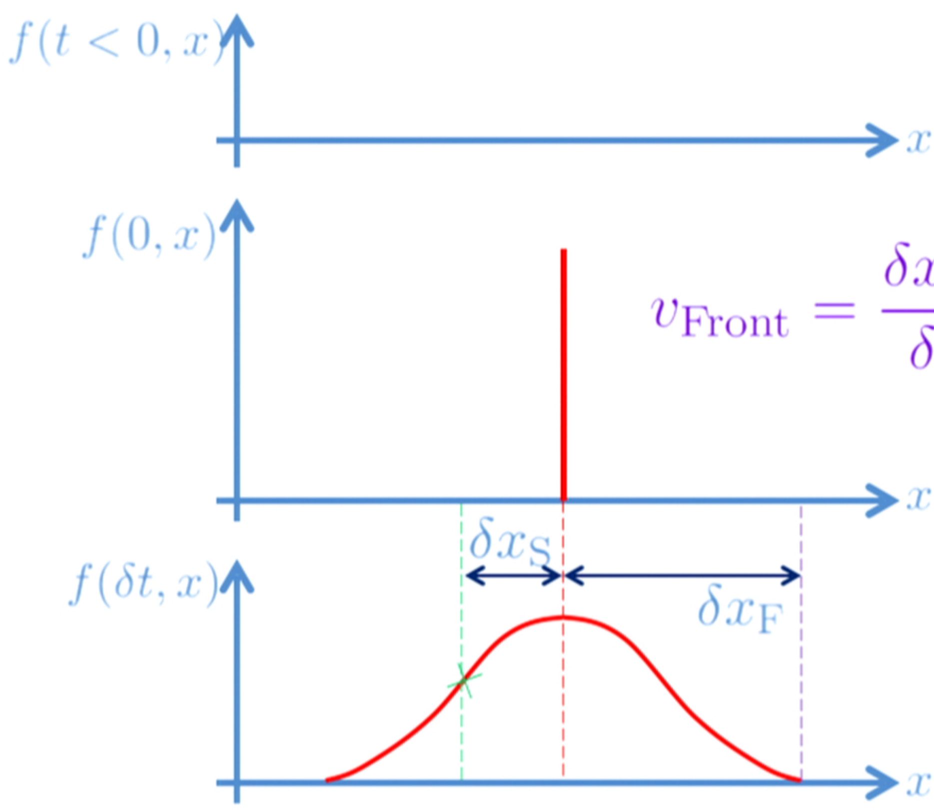
Speed of Information



$$v_{\rm Front} = \frac{\delta x_F}{\delta t} = v_{\rm Phase}(k \to \infty)$$

Why is it that only the front velocity is important?

Instead of using the language of Sommerfeld - lets derived this from the perspective of quantum field theory

Relativistic causality is encoded in the statement that for every operator or pair of operators

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0\,, \text{ for } \ (x-y)^2 > 0$$

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Lets consider the retarded propagator in a non-vacuum state $|\alpha\rangle$

$$G_{\rm ret}(x,x') = -i\theta(t-t')\langle\alpha|[\mathcal{O}(x),\mathcal{O}(y)]|\alpha\rangle$$

Typically it looks something like

$$G_{\rm ret}(x,0) = -i\theta(t)\int \frac{\mathrm{d}^3k}{(2\pi)^3 2\omega(k)}\left(e^{i\vec{k}.\vec{x}-i\omega(k)t} - e^{i\vec{k}.\vec{x}+i\omega(k)t}\right)$$

where $\omega(k)$ depends on the state $|\alpha\rangle$

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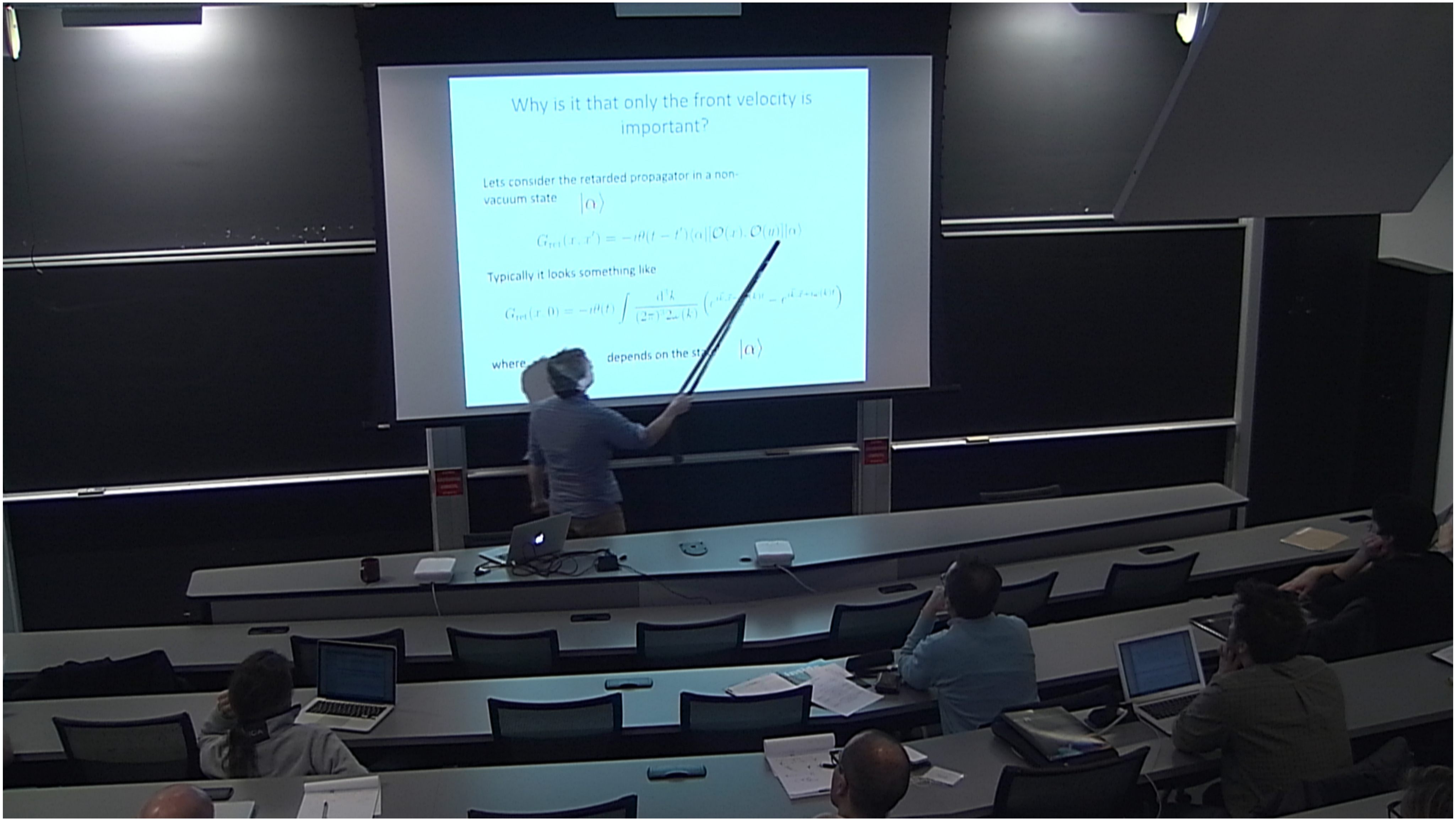
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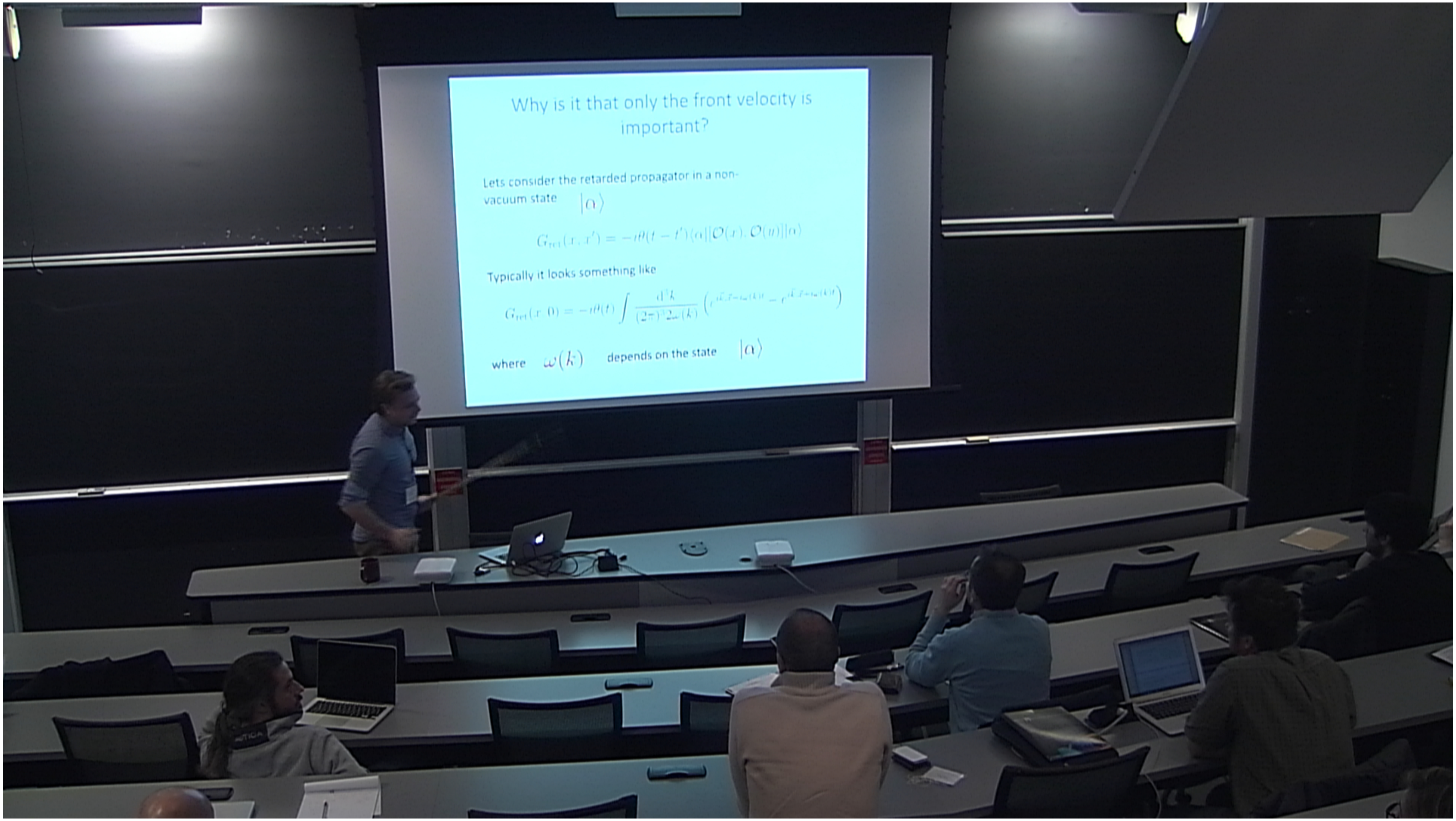
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Why is it that only the front velocity is important?

For simplicity lets assume the state $|\alpha\rangle$ is rotationally symmetric

$$G_{\text{ret}}(x, x') = -i\theta(t - t')\langle\alpha|[\mathcal{O}(x), \mathcal{O}(y)]|\alpha\rangle$$

Then the retarded propagator takes the form

$$G_{\text{ret}}(x, 0) = i\theta(t) \int_{-\infty}^{\infty} \mathrm{d}E \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{(2\pi)^3} \frac{k}{r} \frac{e^{-iEt}e^{ikr}}{\omega(k)^2 - (E + i\epsilon)^2}$$

Famous i epsilon prescription for retarded propagator

Introduce refractive index

Now we can define the refractive index $k = n(\omega)\omega$

$$n(-\omega) = n(\omega)$$

to rewrite this in the form

$$G_{\rm ret}(x,0) = i\theta(t)\int_{-\infty}^{\infty}\mathrm{d}E\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{(2\pi)^3}\frac{n(\omega)\omega}{r}\left(n(\omega)+\omega\frac{\mathrm{d}n(\omega)}{\mathrm{d}\omega}\right)\frac{e^{-iEt}e^{in(\omega)\omega r}}{\omega^2-(E+i\epsilon)^2}$$

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Refractive index analyticity

Now we can define the refractive index $k = n(\omega)\omega$

We now assume that the refractive index can be extended to an analytic function in the upper half complex plane!

In fact this must be the case since $G_{\rm ret}(x,0)$

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Refractive index analyticity

The pole contribution is

$$G_{\rm ret}(x,0) = -\theta(t)\int_{-\infty}^{\infty}\frac{{\rm d}E}{8\pi^2}\frac{n(E)}{r}\left(n(E)+E\frac{{\rm d}n(E)}{{\rm d}E}\right)e^{-iE(t-n(E)r)}$$

Physically this is the statement that the retarded propagator is made up out of radially outgoing waves!!!!!!!!!!!

This result was OBVIOUS classically, but its good to derive it from QFT

Now the punch line

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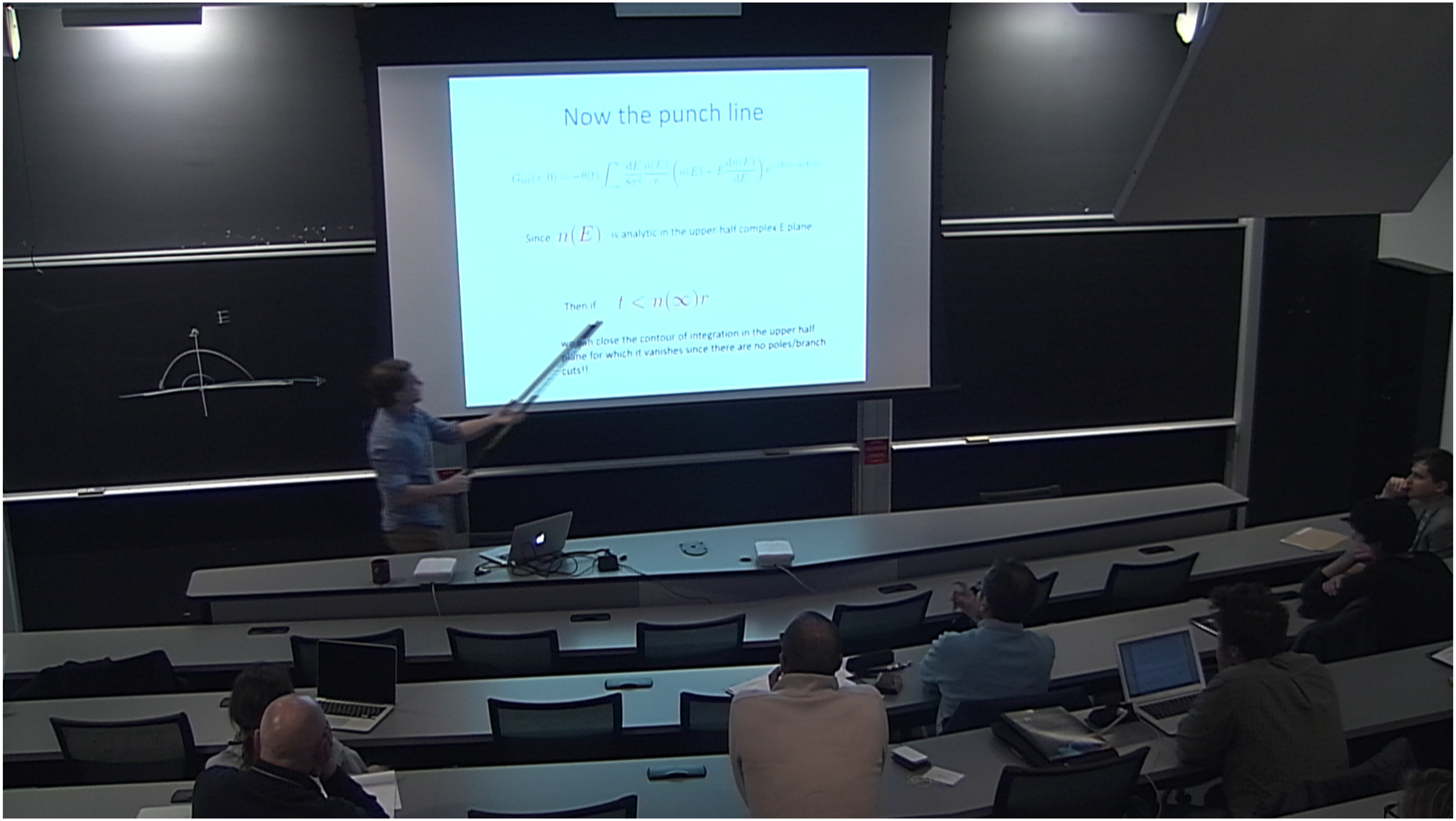
Since $n(E)$ is analytic in the upper-half complex E plane

Then if $t < n(\infty)r$

we can close the contour of integration in the upper half plane for which it vanishes since there are no poles/branch cuts!!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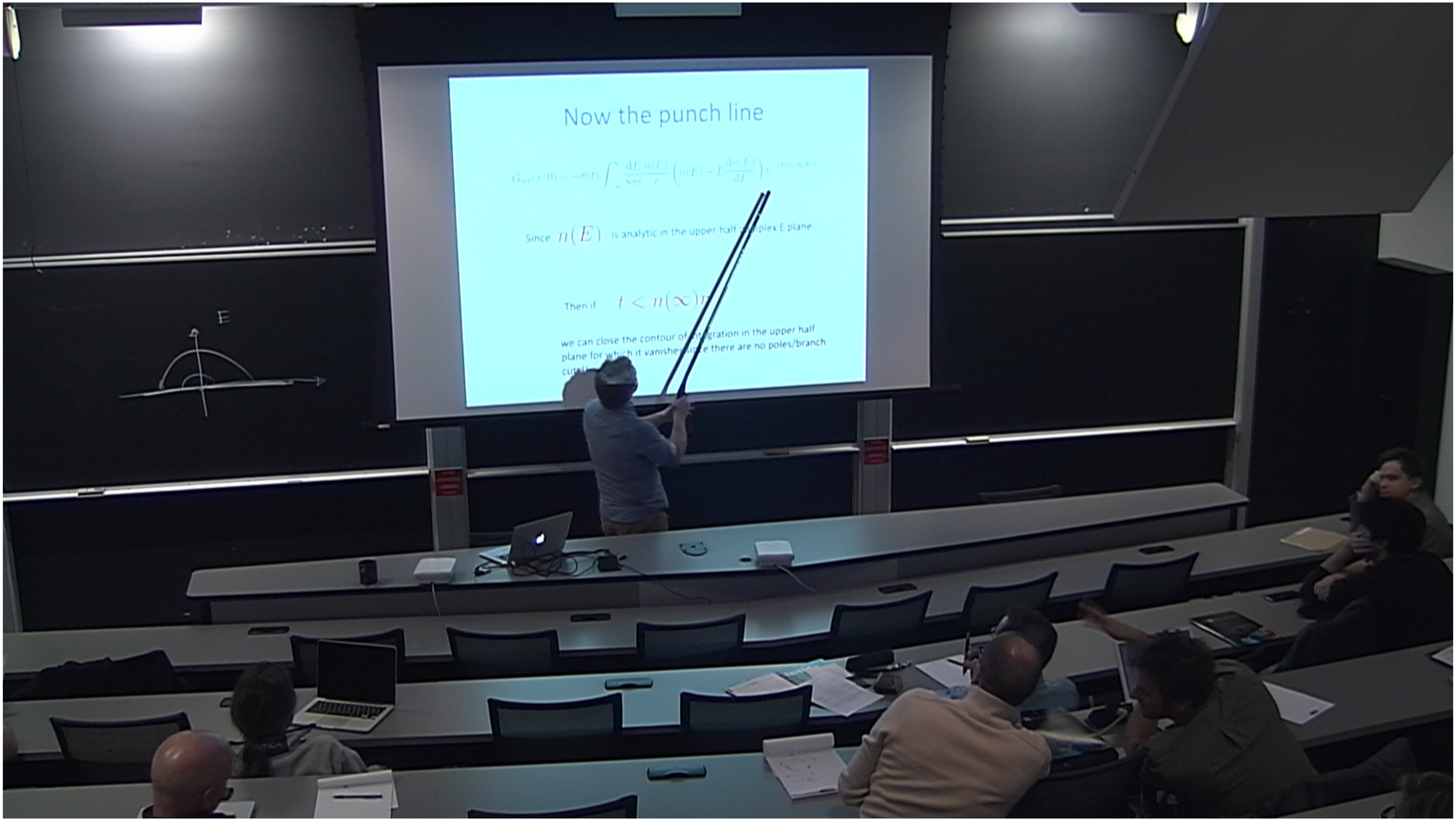

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vanishes outside the light cone defined by

$$t \geq n(\infty) r$$

for this lightcone to lie inside the Lorentz lightcone we require

$$v_{\rm front} = v_{\rm phase}(\infty) = \frac{1}{n(\infty)} \leq 1$$

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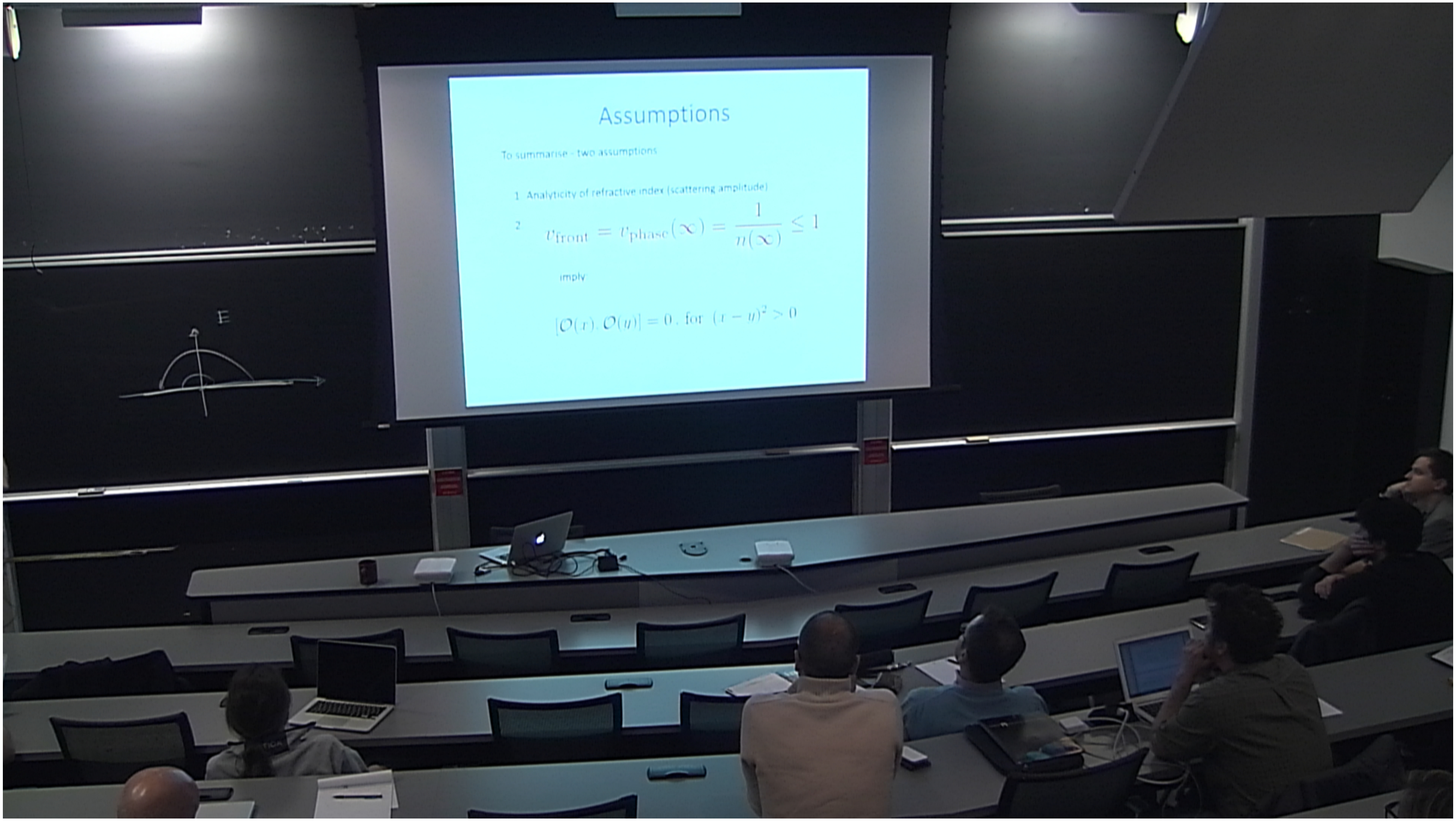
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To summarise - two assumptions
1. Analyticity of refractive index (scattering amplitude)
2. $v_{\rm front} = v_{\rm phase}(\infty) = \frac{1}{n(\infty)} \leq 1$
imply:
$[\mathcal{O}(x), \mathcal{O}(y)] = 0$, for $(x-y)^2 > 0$
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Implications

Both the low energy phase velocity and group velocity can become superluminal without contradicting causality as Sommerfeld noted in 1908!

$$v_{\text{front}} = v_{\text{phase}}(\infty) = \frac{1}{n(\infty)} \leq 1$$

Since the front velocity is

$$v_{\text{front}} = \lim_{k \to \infty} \frac{\omega(k)}{k}$$

it is sensitive to high k physics

But wait ……….. Does that mean i need to know my UV completion in order to determine how fast something like a pion propagates?

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LEEFT

Take two theories with identical interactions, one for which

$$\omega(k) = vk \qquad v > 1$$

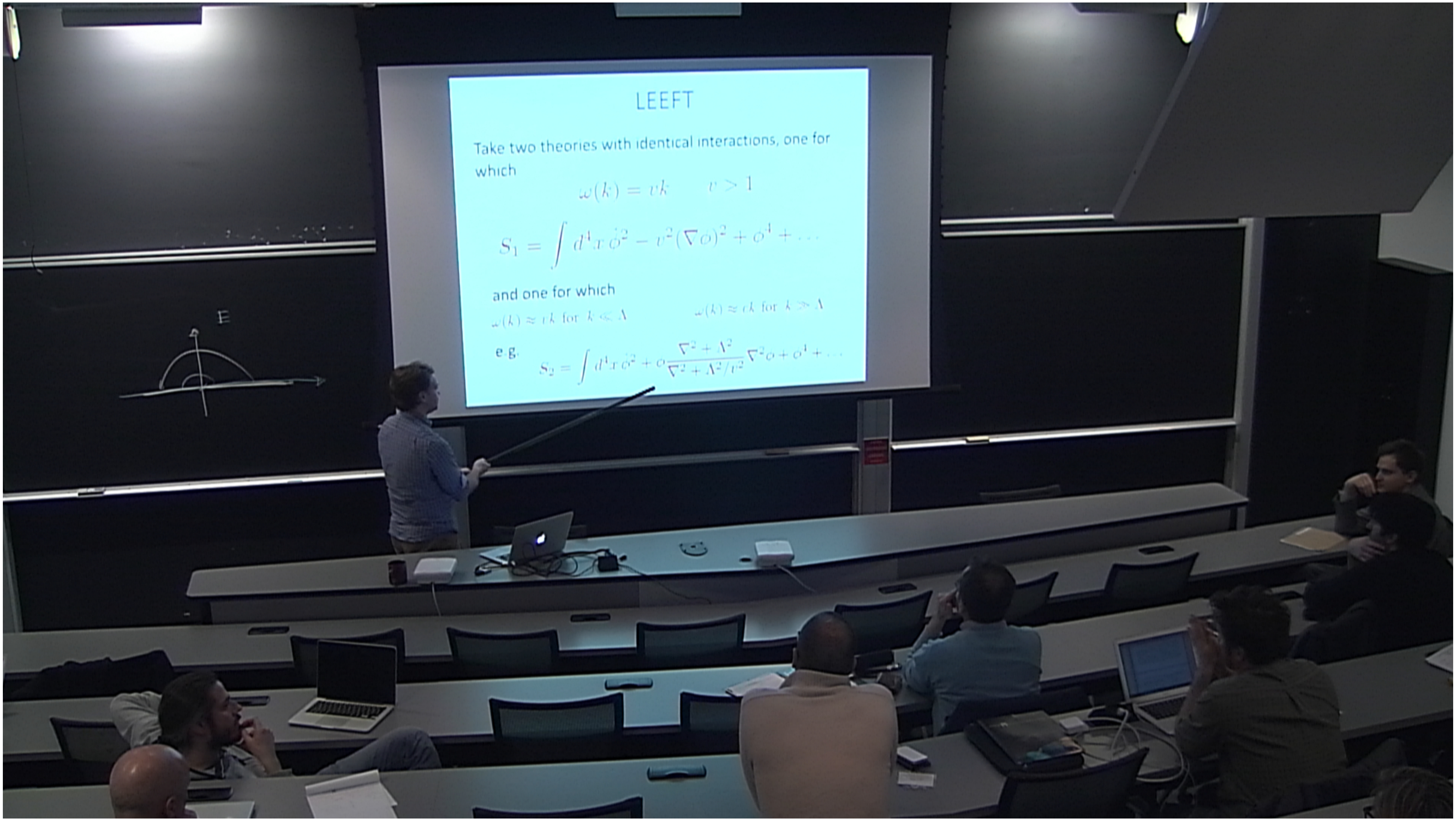
$$S_1 = \int d^4x\, \dot{\phi}^2 - v^2(\nabla\phi)^2 + \phi^4 + \ldots$$

and one for which

$\omega(k) \approx vk$ for $k \ll \Lambda$ $\qquad$ $\omega(k) \approx ck$ for $k \gg \Lambda$

e.g.

$$S_2 = \int d^4x\, \dot{\phi}^2 + \phi \frac{\nabla^2 + \Lambda^2}{\nabla^2 + \Lambda^2/v^2} \nabla^2\phi + \phi^4 + \ldots$$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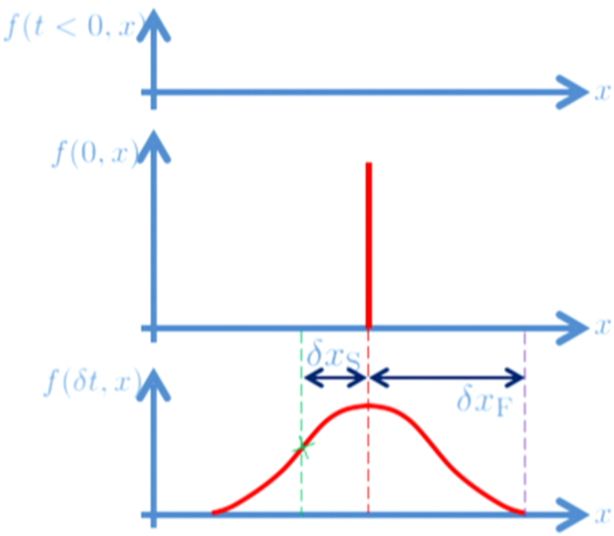
It is clear that these two theories lead to indistinguishable phenomenology for any physics e.g. scattering for which incoming momenta satisfy

$$k \ll \Lambda/v$$

However only theory 2 leads to a vanishing commutator outside the lightcone

Speed of Information in EFT

The point is that to answer Sommerfeld's question of what is the front velocity, we need to create an initial state with arbitrarily small time localization meaning



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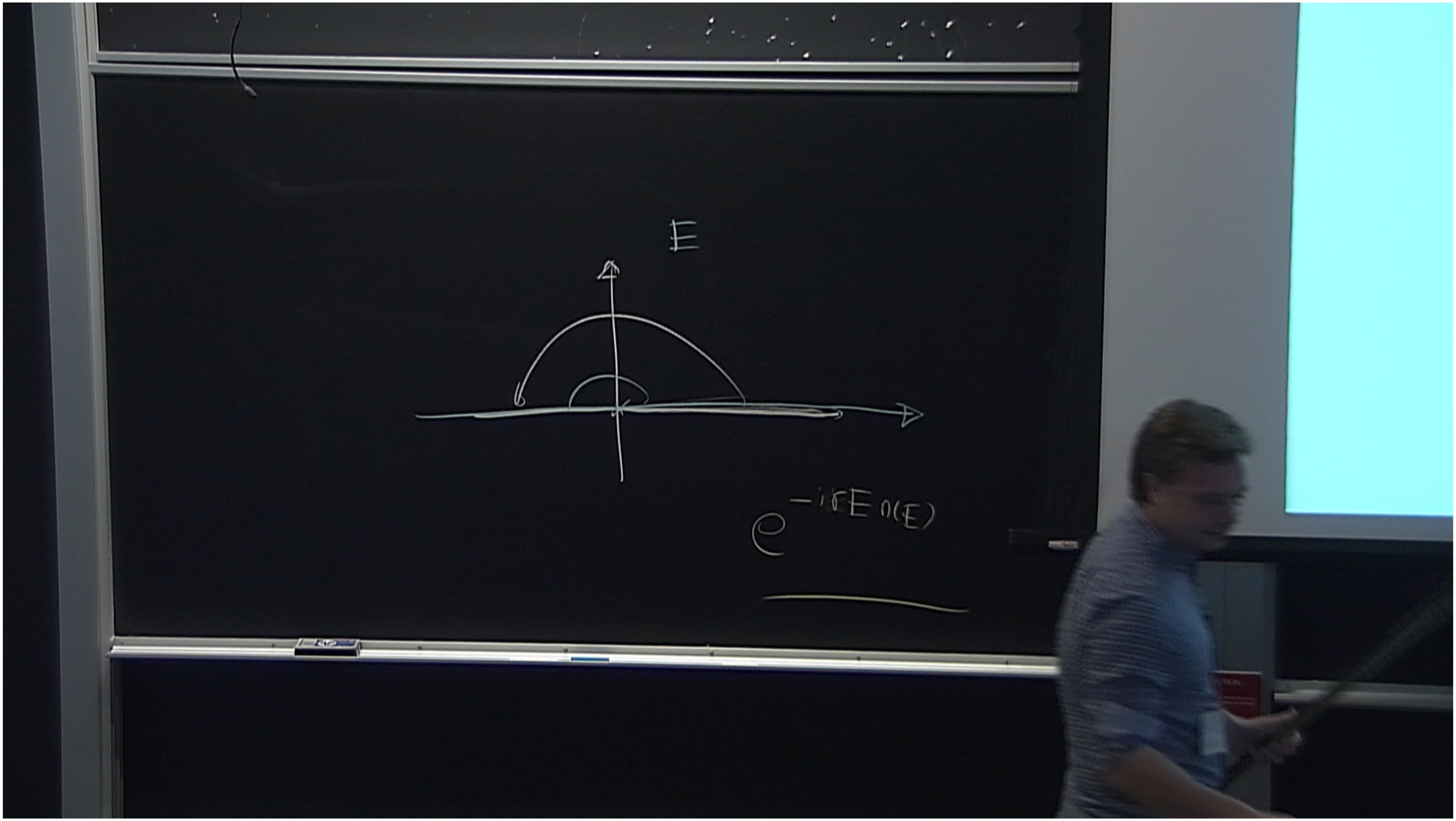
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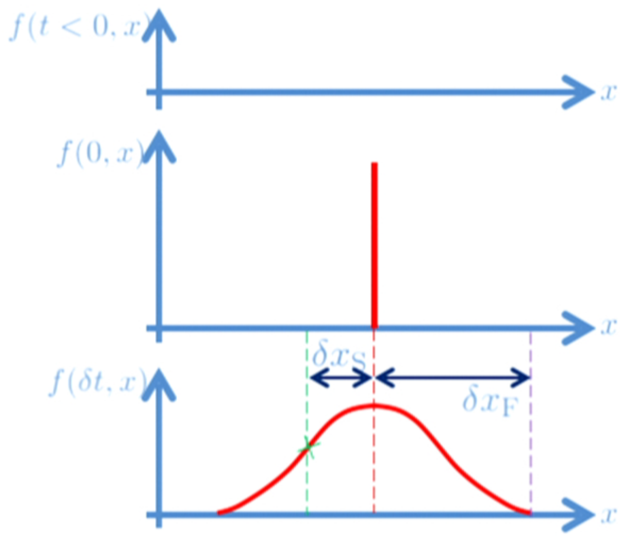
Speed of Information in EFT

The point is that to answer Sommerfeld's question of what is the front velocity, we need to create an initial state with arbitrarily small time localization meaning



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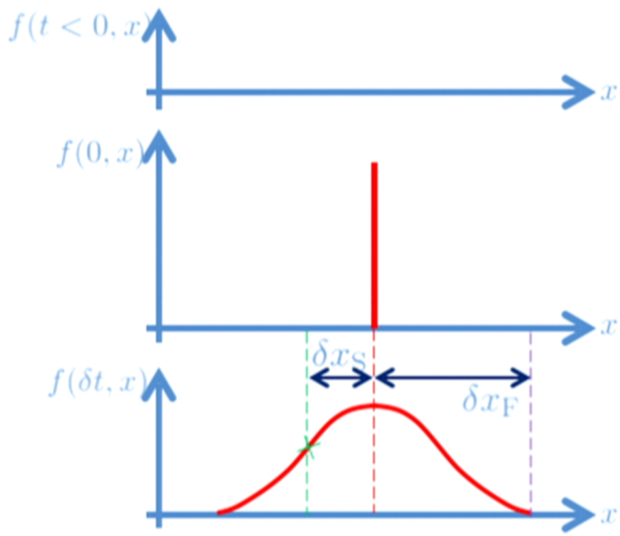

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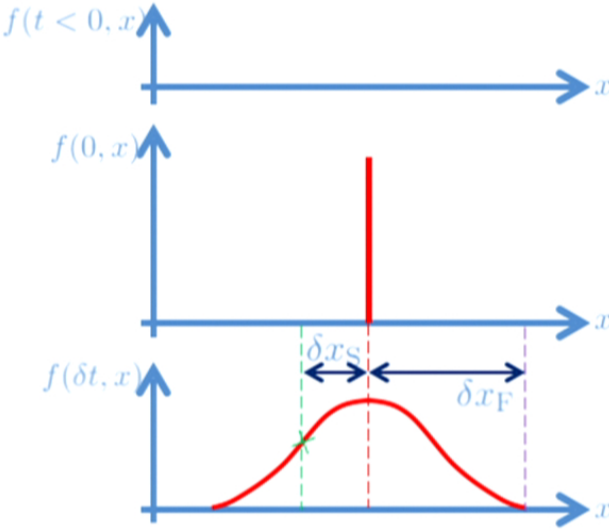

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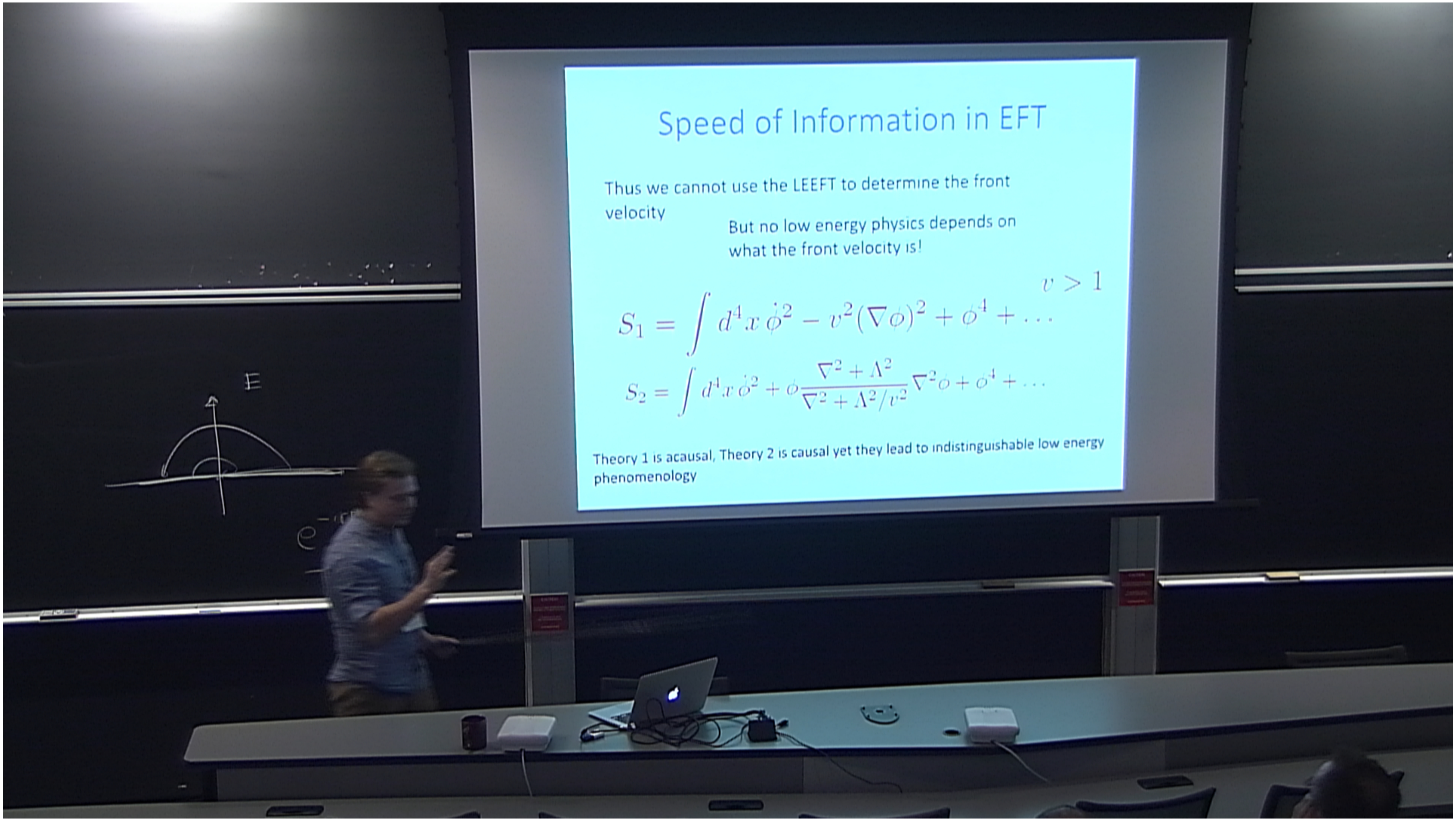
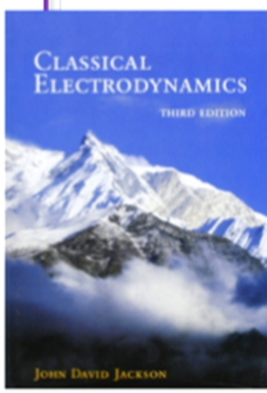


Speed of Information in EFT
Thus we cannot use the LEEFT to determine the front velocity
But no low energy physics depends on what the front velocity is!
$v > 1$
$S_1 = \int d^4x\, \dot\phi^2 - v^2(\nabla\phi)^2 + \phi^4 + \dots$
$S_2 = \int d^4x\, \dot\phi^2 + \phi \frac{\nabla^2 + \Lambda^2}{\nabla^2 + \Lambda^2/v^2} \nabla^2\phi + \phi^4 + \dots$
Theory 1 is acausal, Theory 2 is causal yet they lead to indistinguishable low energy phenomenology

Kramers-Kronig dispersion relations

where P means principal part. The delta function serves to pick up the contribution from the small semicircle going in a positive sense halfway around the pole at $\omega' = \omega$. Use of (7.117) and a simple rearrangement turns (7.116) into

$$\epsilon(\omega)/\epsilon_0 = 1 + \frac{1}{\pi i} P \int_{-\infty}^{\infty} \frac{[\epsilon(\omega')/\epsilon_0 - 1]}{\omega' - \omega} \, d\omega' \tag{7.118}$$



The real and imaginary parts of this equation are

$$\text{Re } \epsilon(\omega)/\epsilon_0 = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Im } \epsilon(\omega')/\epsilon_0}{\omega' - \omega} \, d\omega' \tag{7.119}$$

Jackson, Classical Electrodynamics

"simple" consequence of Cauchy integration formula

Assumes: Analyticity + Causality

Speed of Information in EFT

Thus we cannot use the LEEFT to determine the front velocity

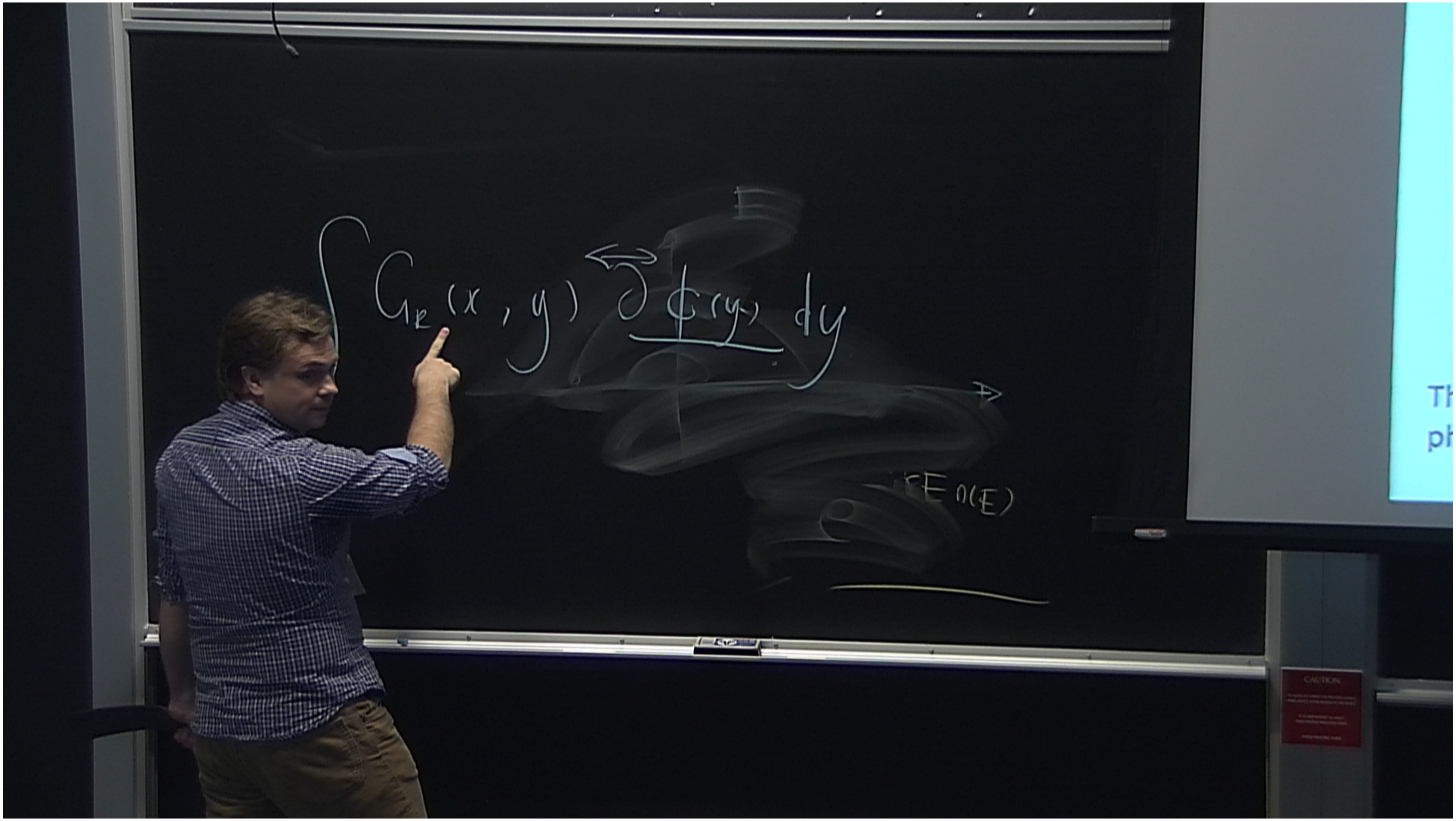
But no low energy physics depends on what the front velocity is!

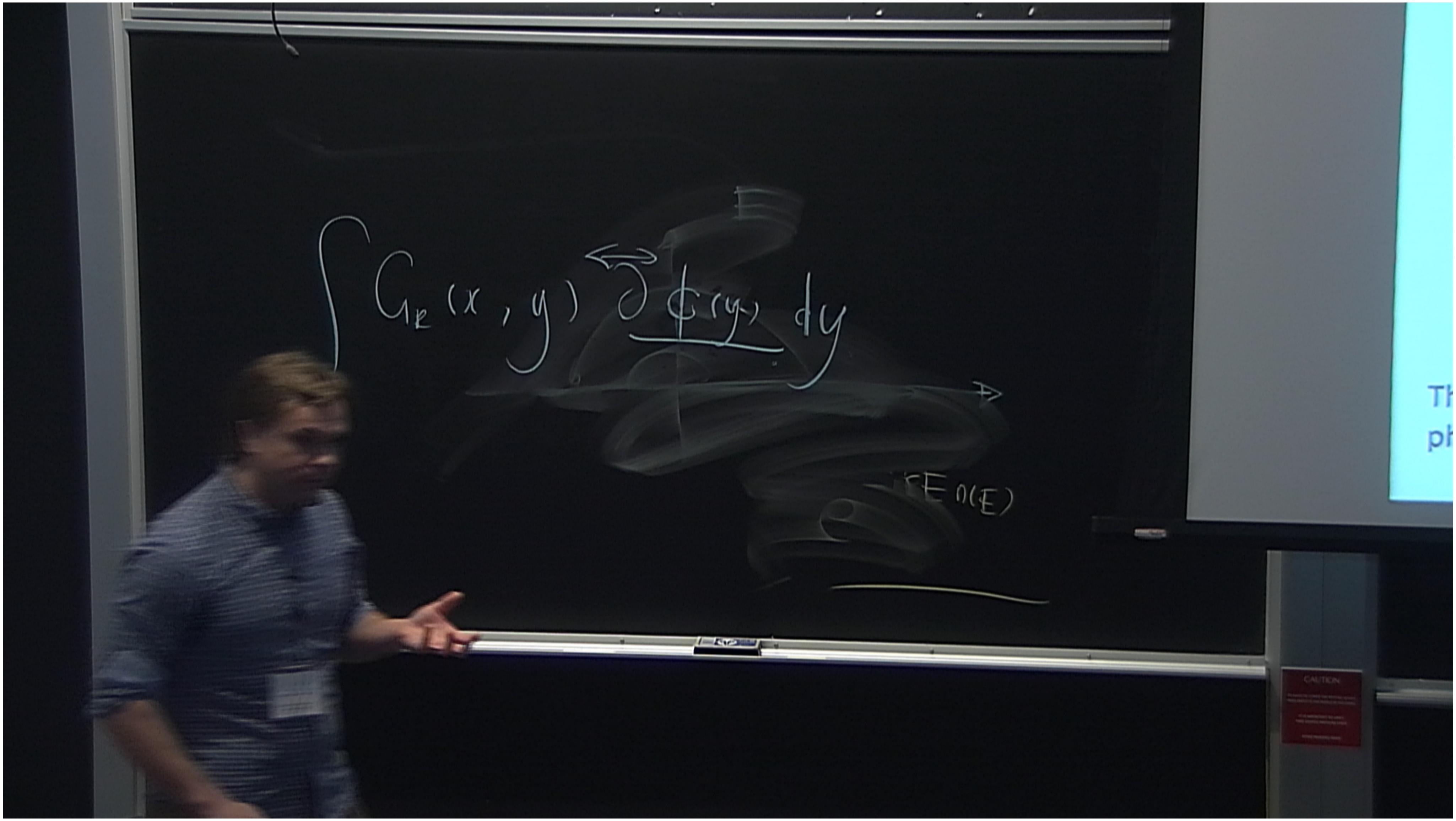
$$v > 1$$

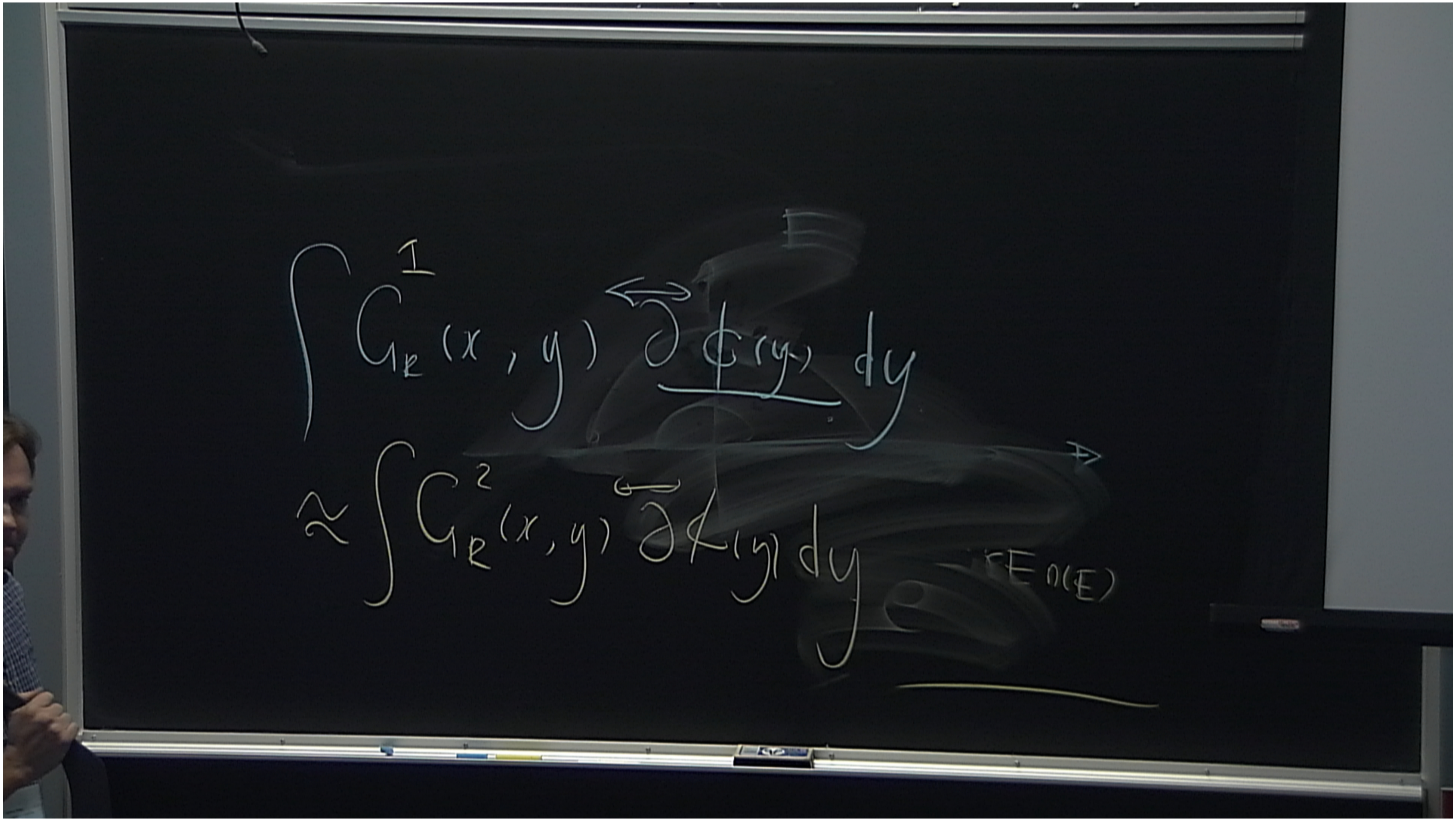
$$S_1 = \int d^4x\, \dot{\phi}^2 - v^2(\nabla\phi)^2 + \phi^4 + \ldots$$

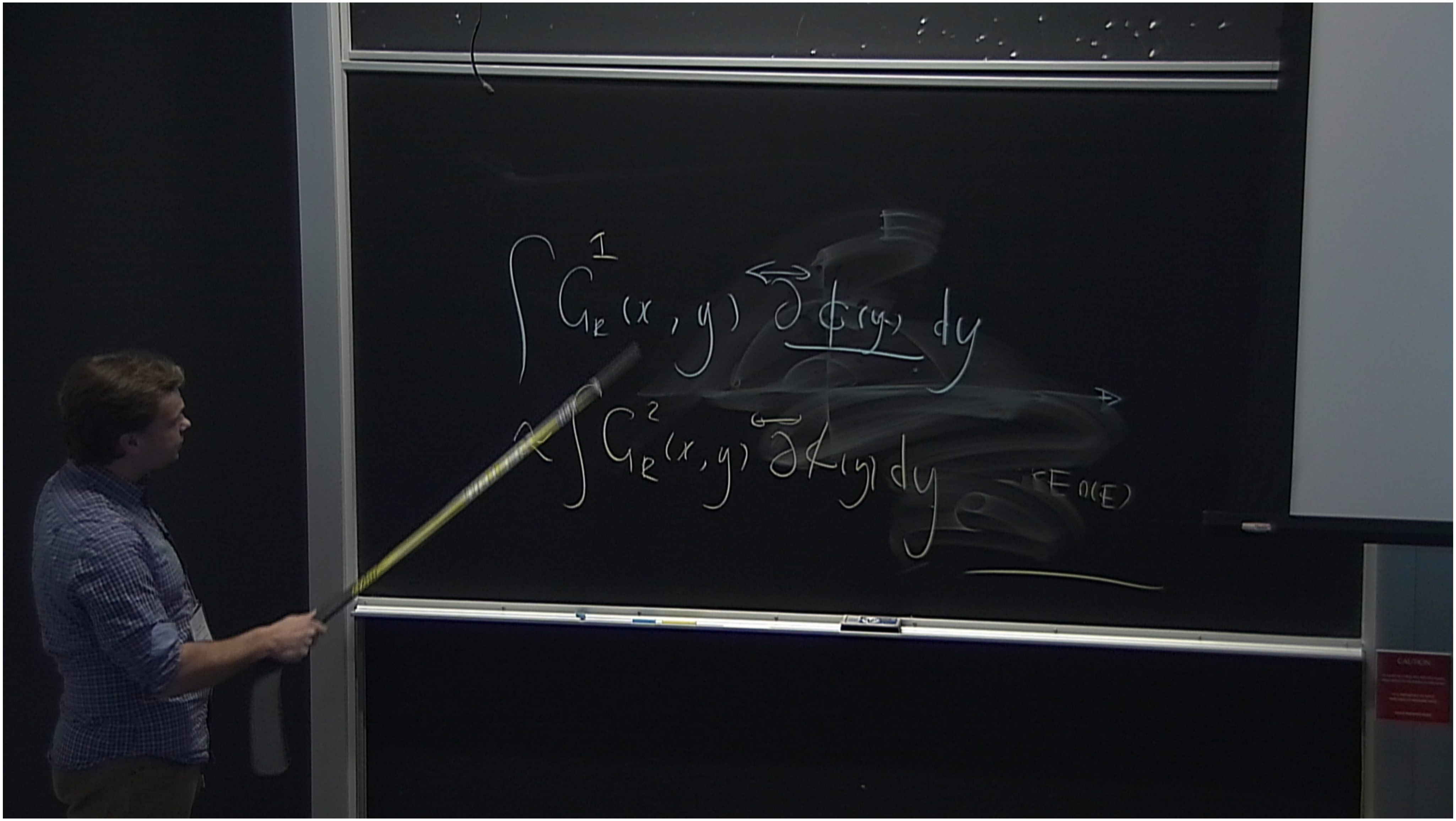
$$S_2 = \int d^4x\, \dot{\phi}^2 + \phi \frac{\nabla^2 + \Lambda^2}{\nabla^2 + \Lambda^2/v^2} \nabla^2\phi + \phi^4 + \ldots$$

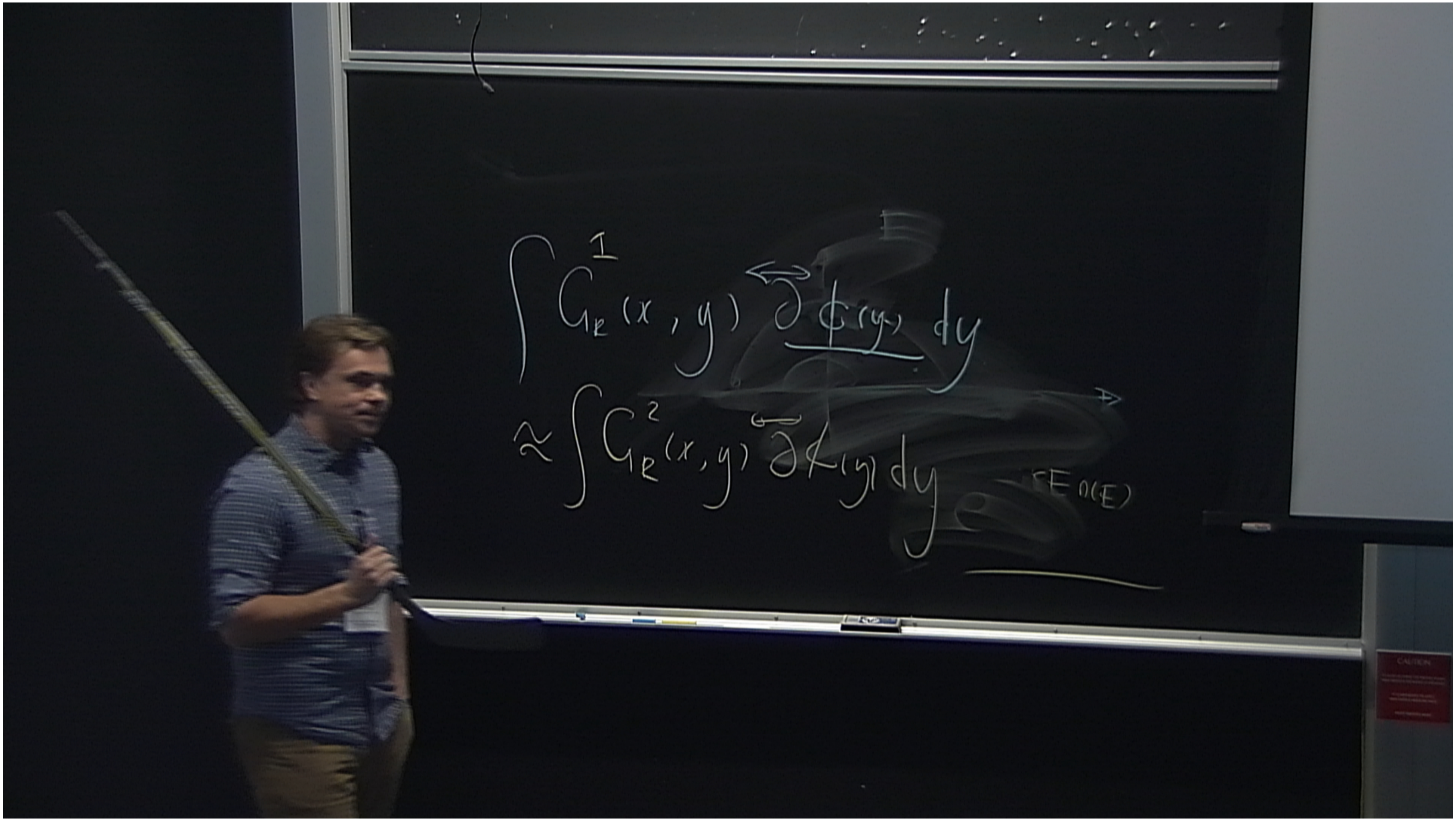
Theory 1 is acausal, Theory 2 is causal yet they lead to indistinguishable low energy phenomenology






$\int G_R^1(x,y)\overleftrightarrow{\partial}\phi(y)\,dy$
$\approx \int G_R^2(x,y)\overleftrightarrow{\partial}\phi(y)\,dy$





Kramers-Kronig dispersion relations

Refractive index $n(w)$ for a dilute gas of atoms

$$v_{\text{Phase}}(\omega) = \frac{c}{\text{Re}[n(\omega)]}$$

$\text{Im}[n(\omega)]$ is related to the Scattering amplitude

Causality + Analyticity $\Longrightarrow$ $\text{Re}[n(\infty)] - \text{Re}[n(0)] = -\frac{2}{\pi}\int_0^\infty \text{Im}[n(w)]\frac{\text{d}\omega}{\omega}$

Unitarity $\Longrightarrow$ $\text{Im}[n(\omega)] > 0 \quad \forall\, \omega$

Causality + Analyticity + Unitarity $\Longrightarrow$ $v_{\text{Phase}}(\infty) > v_{\text{Phase}}(0)$

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Yet… Superluminal phase&group velocities are observed…

Direct measurement of superluminal group velocity and of signal velocity in an optical fiber

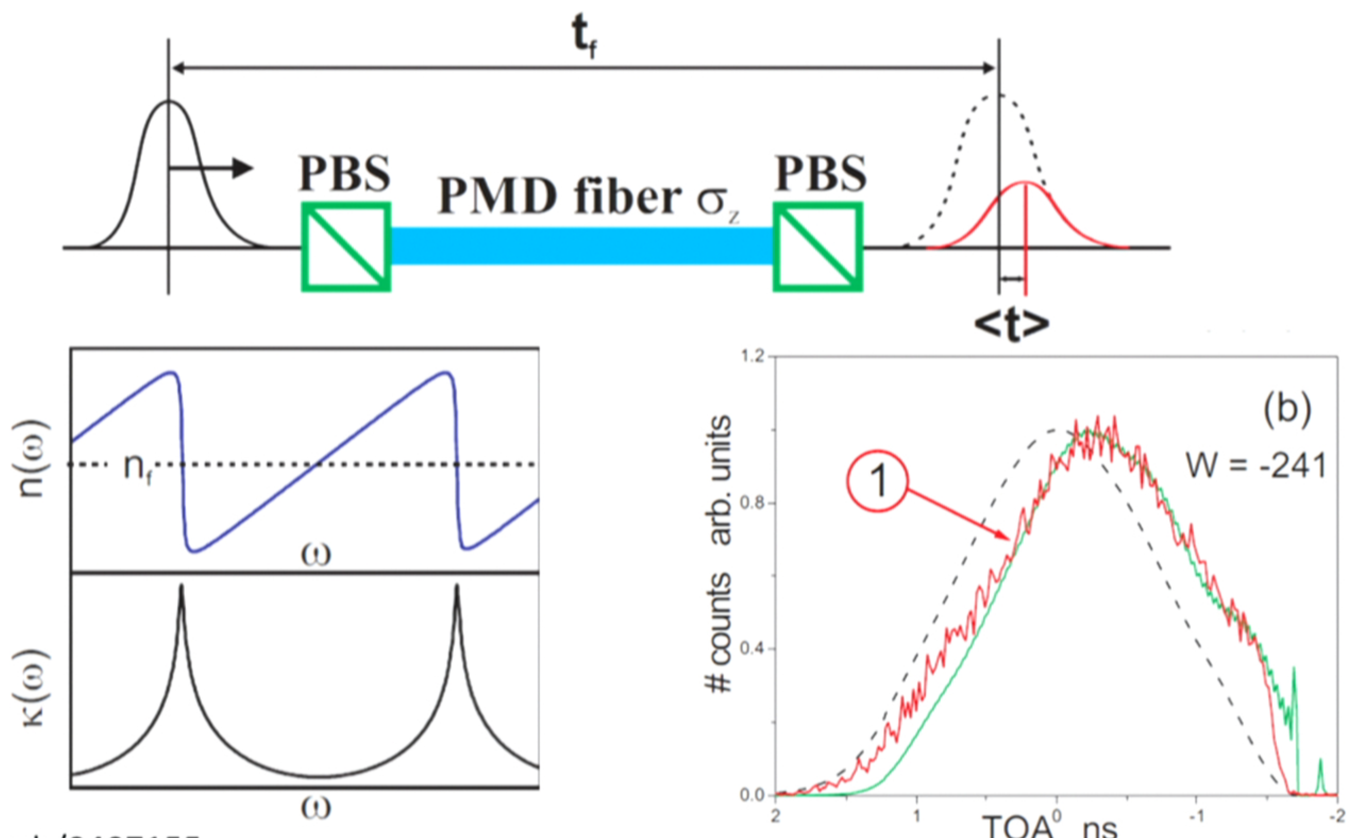
Nicolas Brunner, Valerio Scarani, Mark Wegmüller, Matthieu Legré and Nicolas Gisin
Group of Applied Physics, University of Geneva,
20 rue de l'Ecole-de-Médecine, CH-1211 Geneva 4, Switzerland
(February 1, 2008)

quant-ph/0407155

We present an easy way of observing superluminal group velocities using a birefringent optical fiber and other standard devices. In the theoretical analysis, we show that the optical properties of the setup can be described using the notion of "weak value". The experiment shows that the group velocity can indeed exceed c in the fiber; and we report the first direct observation of the so-called "signal velocity", the speed at which information propagates and that cannot exceed c.

Same group that patterned entanglement technologies that transfers information between major swiss banks…

Yet… Superluminal phase&group velocities are observed…



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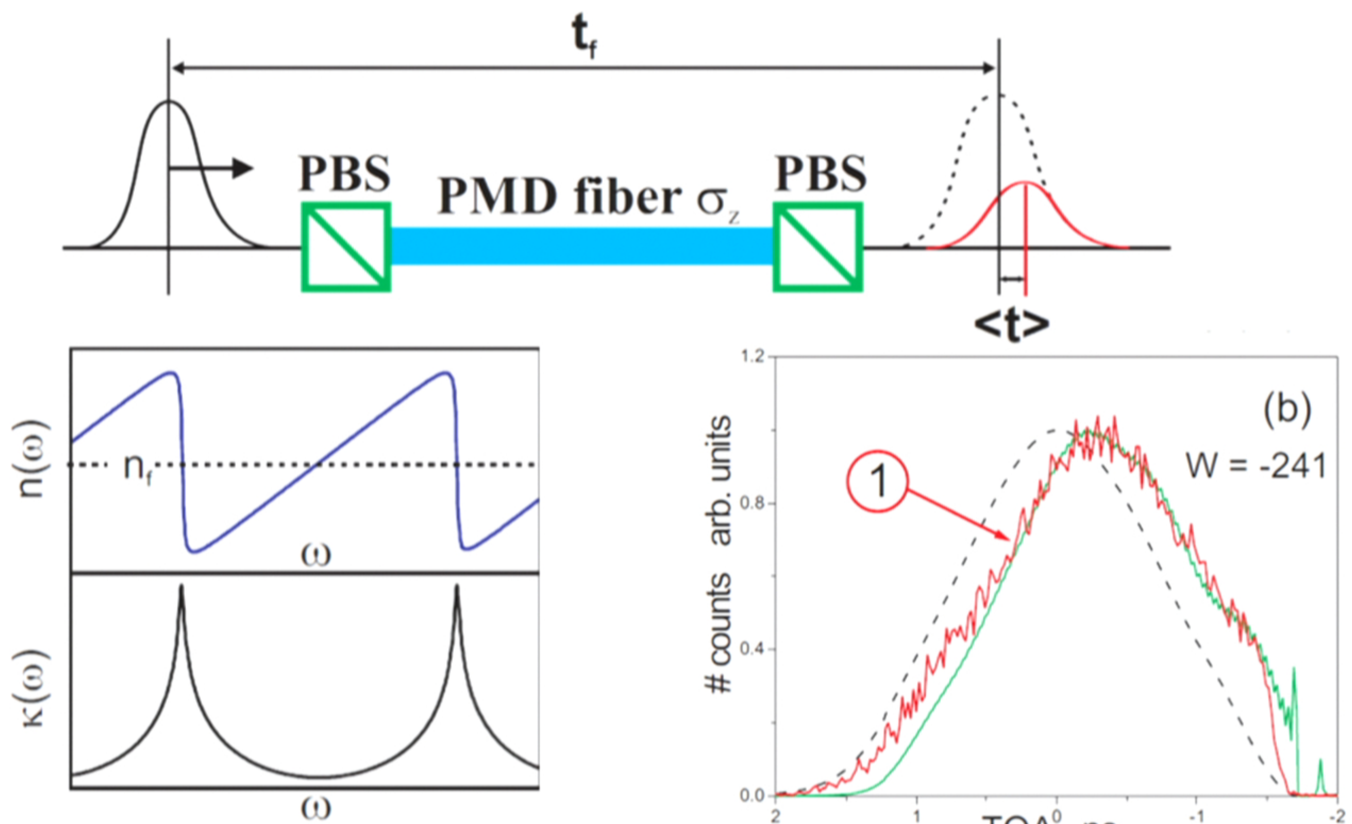

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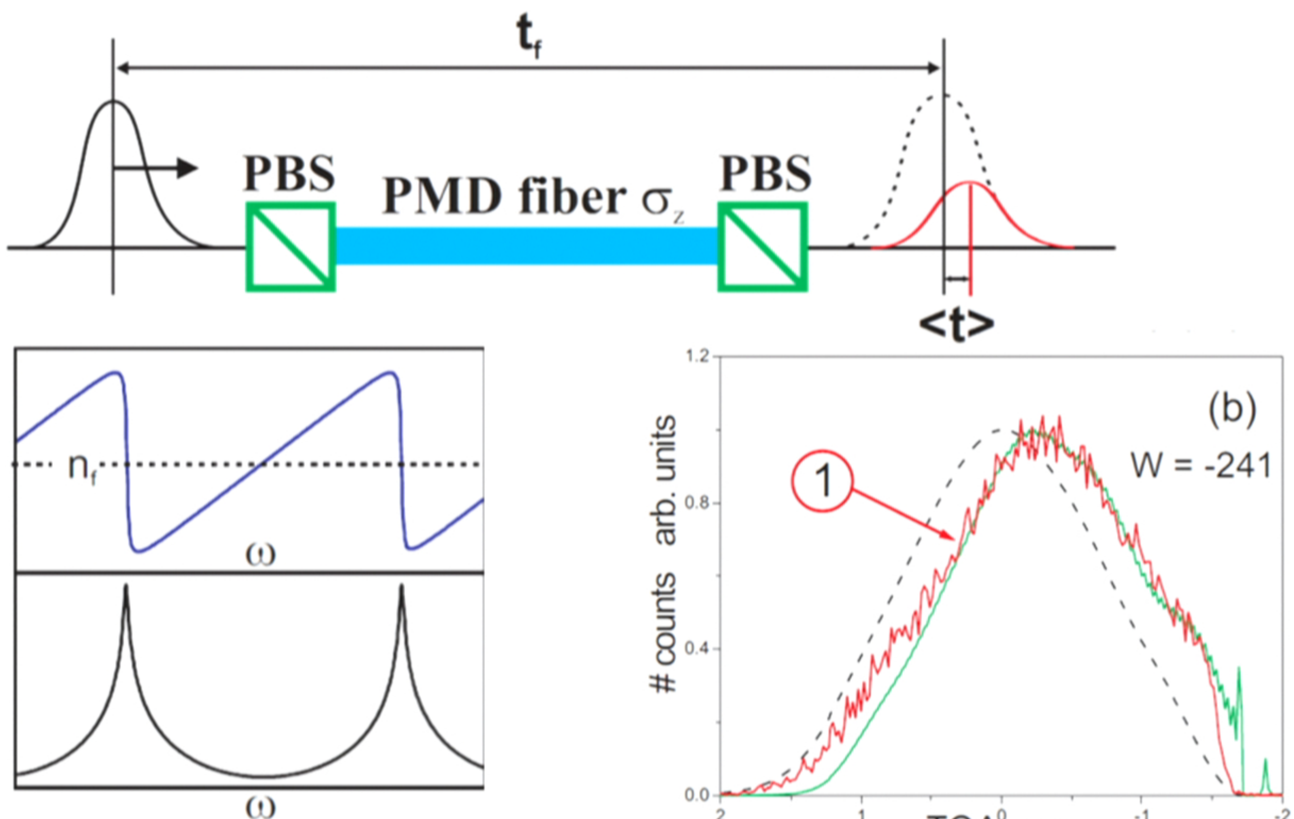

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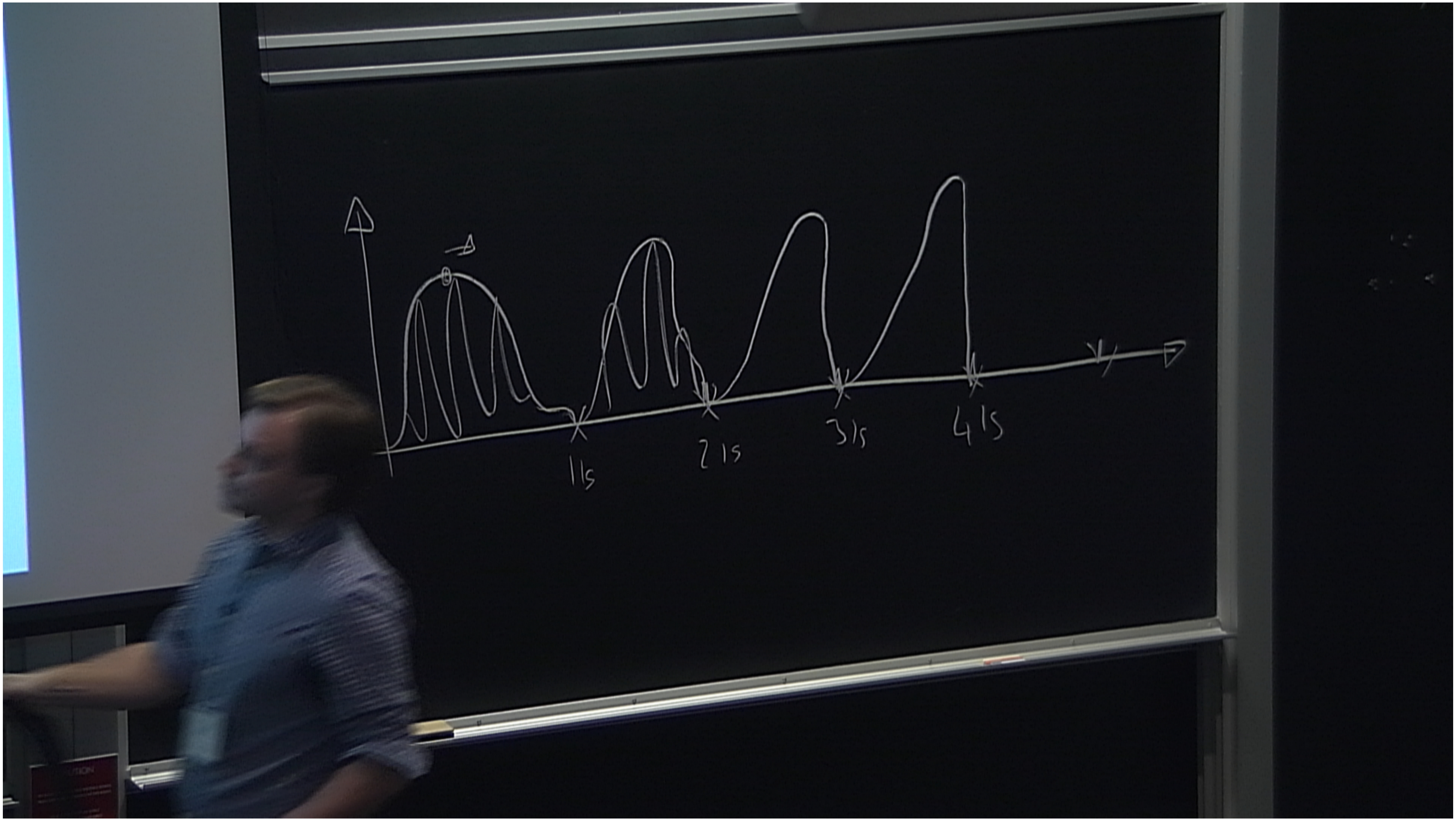
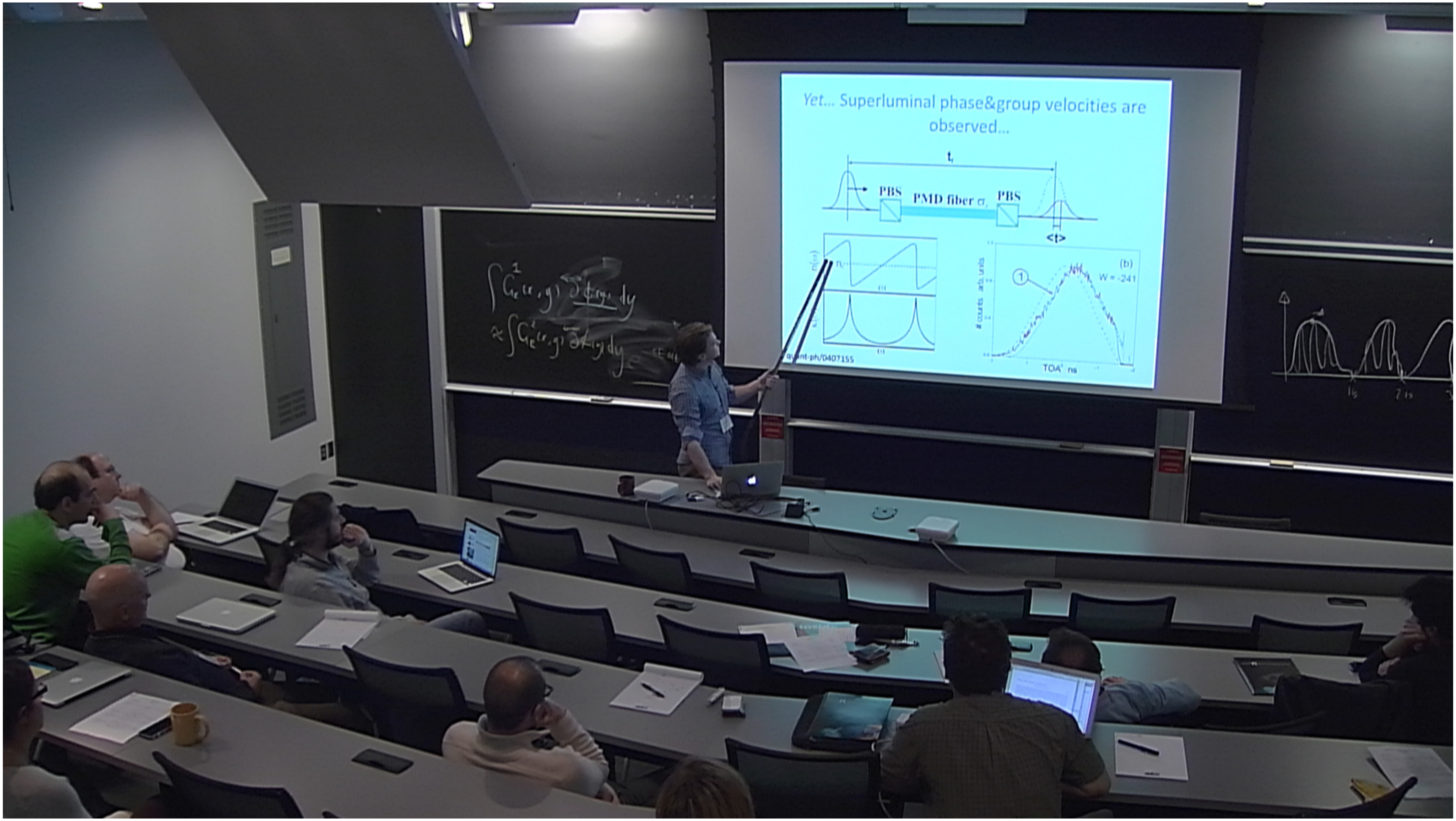

1s
2s
3s
4s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PBS
PMD fiber
PBS
quant-ph/0407155
W = -241
TOA

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Breaking our assumptions

Our current physics paradigms strongly rely on the assumptions of

Causality + Analyticity + Unitarity

The real world MUST break one of these assumptions

1. Front velocity is not luminal, and causality is violated…

2. Some medium can allow for $Im(n(w)) < 0$ meaning "exhibiting gain" (ie. Laser takes energy from the system)

3. The conditions of analyticity are violated

Eg. QED on curved spacetime. See Hollowood's talk and Hollowood & Shore 0707.2302, 0707.2303, 0806.1019, 1006.1238

Does the same occur for Galileons/massive gravity?

A few Misconceptions

- SL in Galileons/MG are in the front/signal velocity

  SL are computed in classical theory which is only valid till strong coupling scale Λ and by definition does not determine the front velocity

- Characteristic analysis can diagnose something beyond a classical SL analysis

  SL in Galileon/MG have been pointed out more than a decade ago. Characteristic analysis is a classical analysis which is not valid beyond Λ

- Characteristic analysis has diagnosed acausality

  All a Characteristic analysis can diagnose is a dof becoming infinitively strongly coupled on a background and the breakdown of the classical theory on that bckgd.

- Constraint that removes BD ghost in MG is responsible for acausality

  The 2 are completely independent. SL are equally present (if not more) for theories with a ghost. No acausality has been shown in neither cases.

For instance: Take EFT point of view

In Massive gravity/Bi-gravity Perturbation theory breaks down at scale

$$\Lambda = (m^2 M_{\rm pl})^{1/3}$$

If this is the cutoff then a traditional EFT would say, massive gravity really takes form

$$\mathcal{L} = \frac{M_{\rm Planck}}{2} R + m^2([K]^2 - [K^2] + \alpha_1 (K^3 + \dots) + \alpha_2 (K^4 + \dots))$$

$$+ \sum_n c_n \Lambda^4 K \frac{\Box^{2n}}{\Lambda^{2n}} K + \Lambda^4 \sum_{p,q} \frac{\partial^{2p}}{\Lambda^{2p}} K^q + \dots$$


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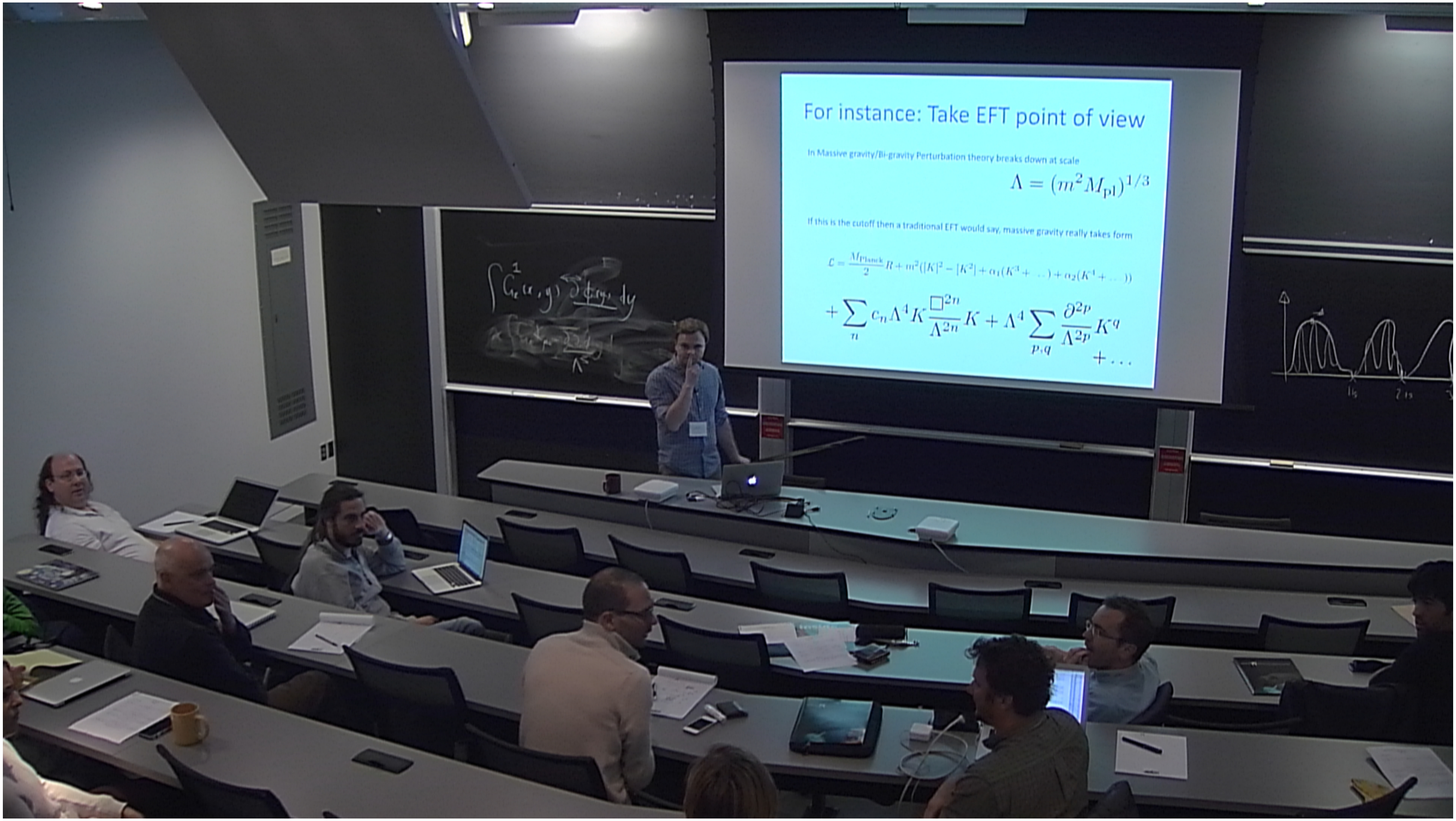

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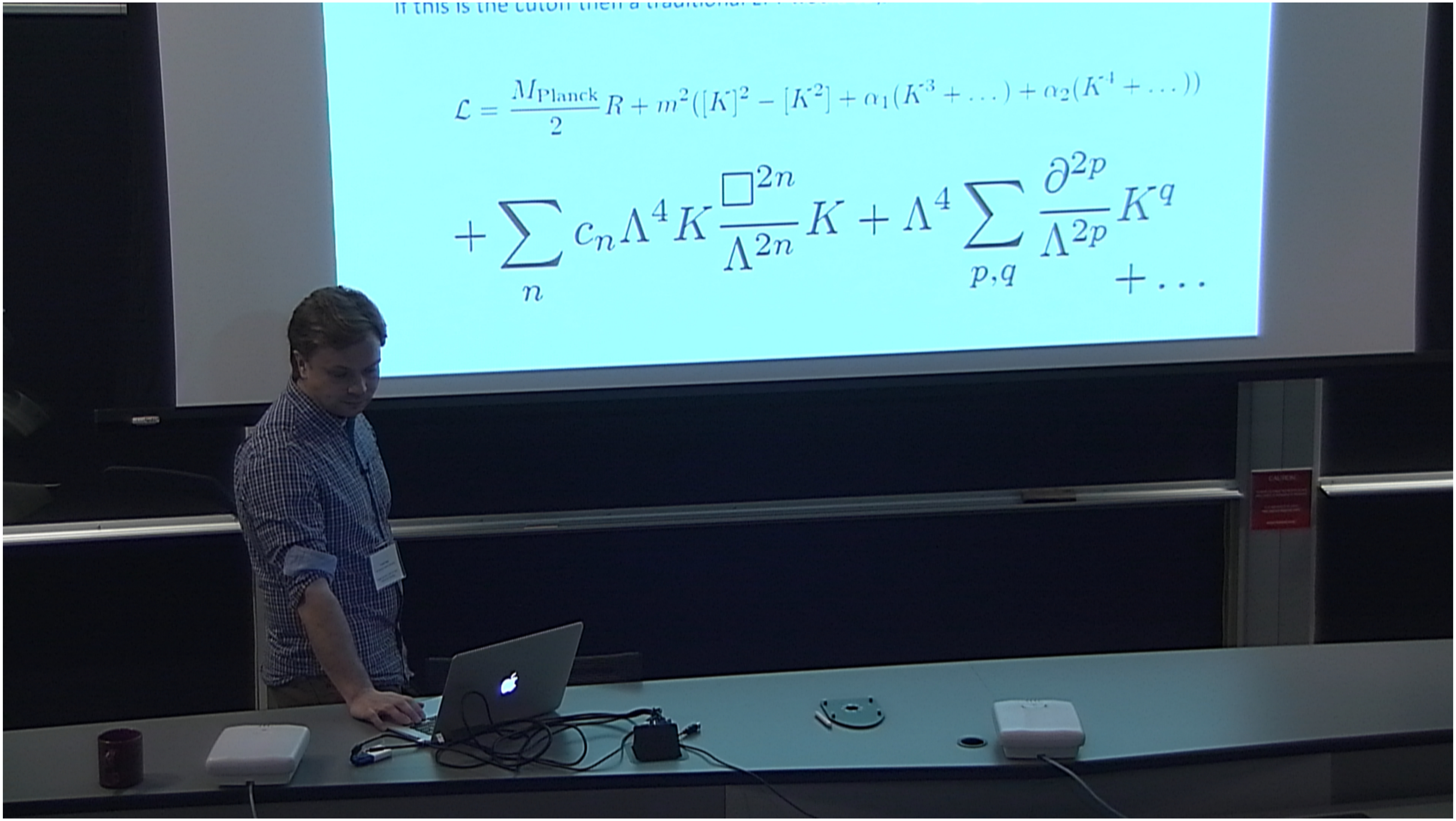
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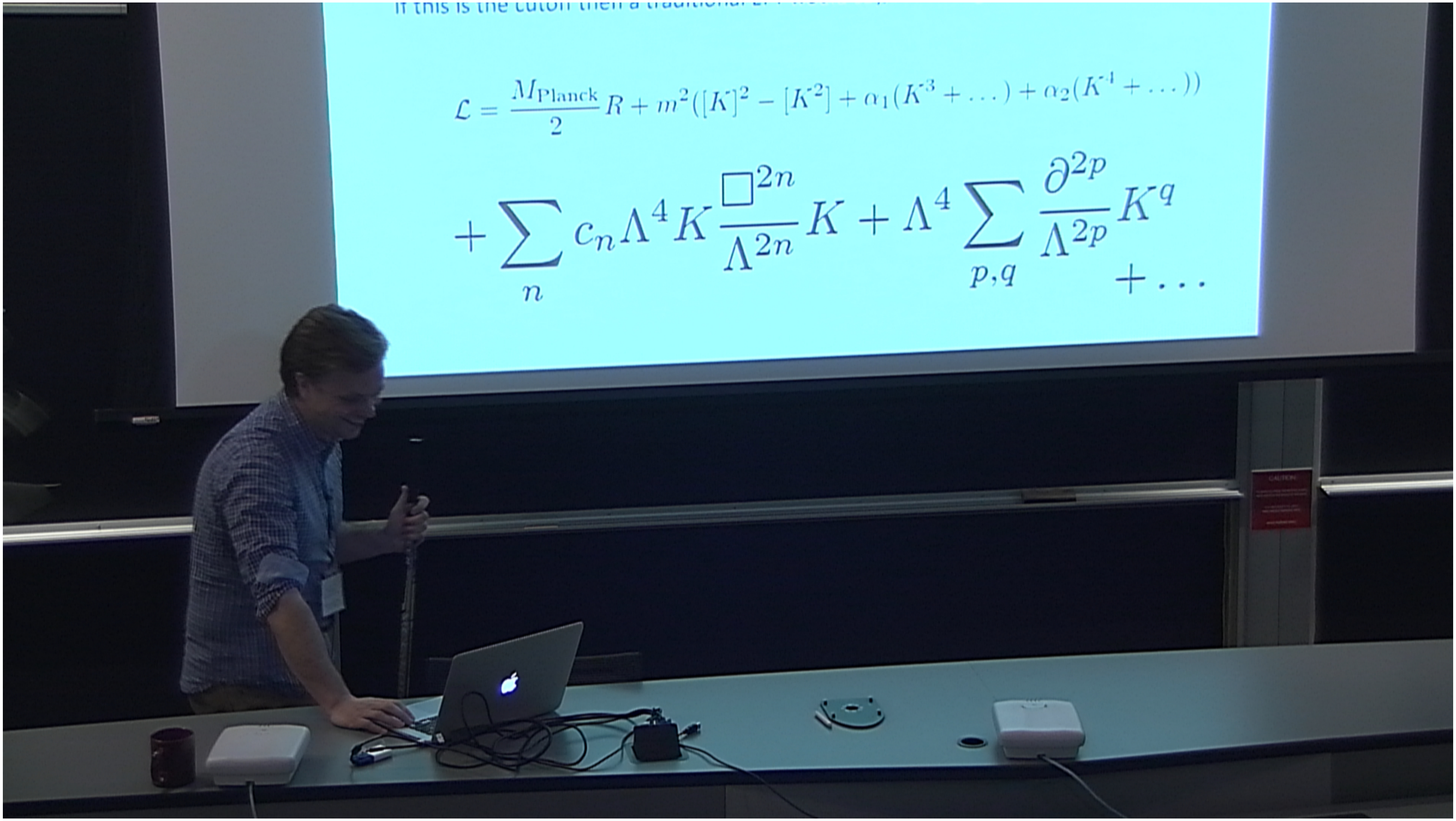
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Eg. Interacting Proca

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2A^2 - \frac{\lambda}{4}A^4$$

Characteristic analysis replaces the highest derivative terms $\partial^N A \to k^N \tilde{A}$

$$\left[(m^2 + \lambda A^2)k^2 + 2\lambda(A.k)^2\right]k^\mu \tilde{A}_\mu = 0$$

If $\lambda \neq 0$, there are solutions with $k^\mu \tilde{A}_\mu \neq 0$

There are field configurations for which the normal to the surface is timelike

→ modes with $k^\mu \tilde{A}_\mu \neq 0$ can propagate SL

Velo & Zwanziger, "Noncausality and other defects of interaction Lagrangians for particles with spin one and higher", Phys.Rev., 188, 2218 2222 (1969).
Ong, Izumi, Nester & Chen, PRD88, 024019 (2013).

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$\mathcal{Z} = (m^2 + \lambda A^2)k^2 + 2\lambda(A.k)^2$ is precisely the kinetic term of the helicity-0 mode π

$$A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \pi$$

→ The helicity-0 mode is infinitely strongly coupled on these solutions and these classical considerations are obsolete.

CdR Living Rev.Rel. 17 (2014) 7

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$$Z^{\mu\nu} = (1 + \frac{\lambda}{m^2}(\partial\pi)^2)\eta^{\mu\nu} + 2\frac{\lambda}{m^2}\partial^\mu\pi\partial^\nu\pi$$

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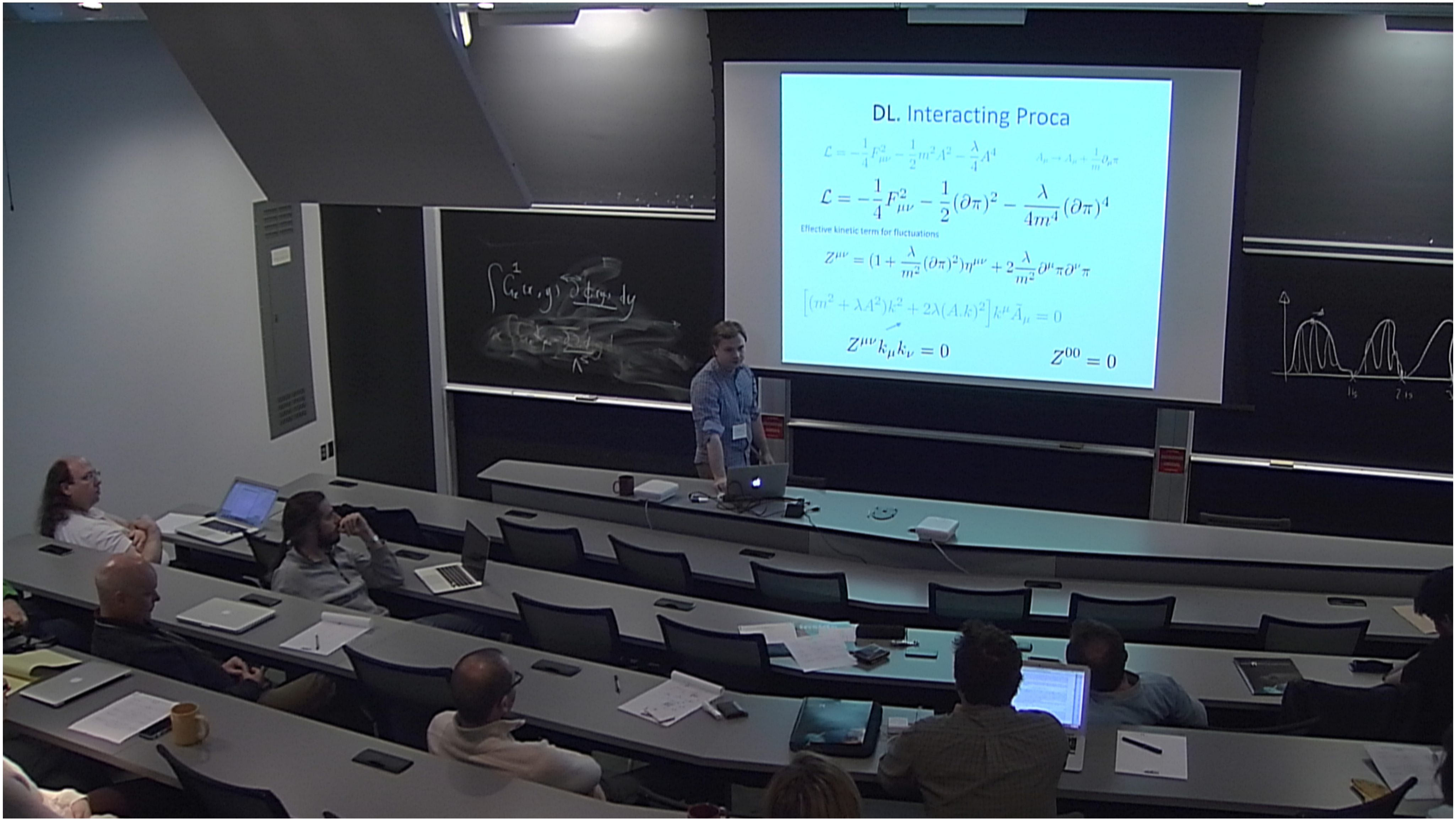
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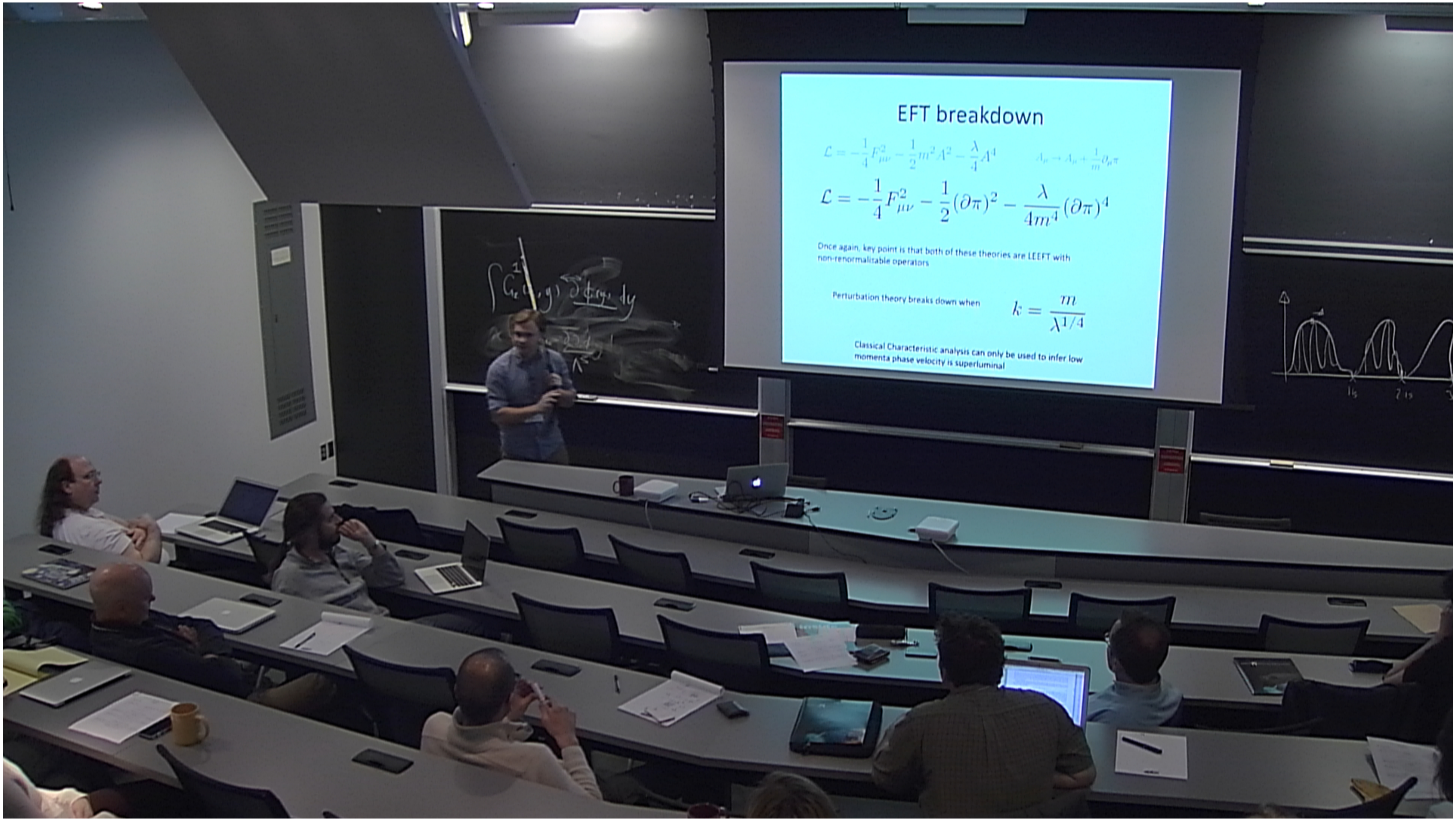
EFT breakdown
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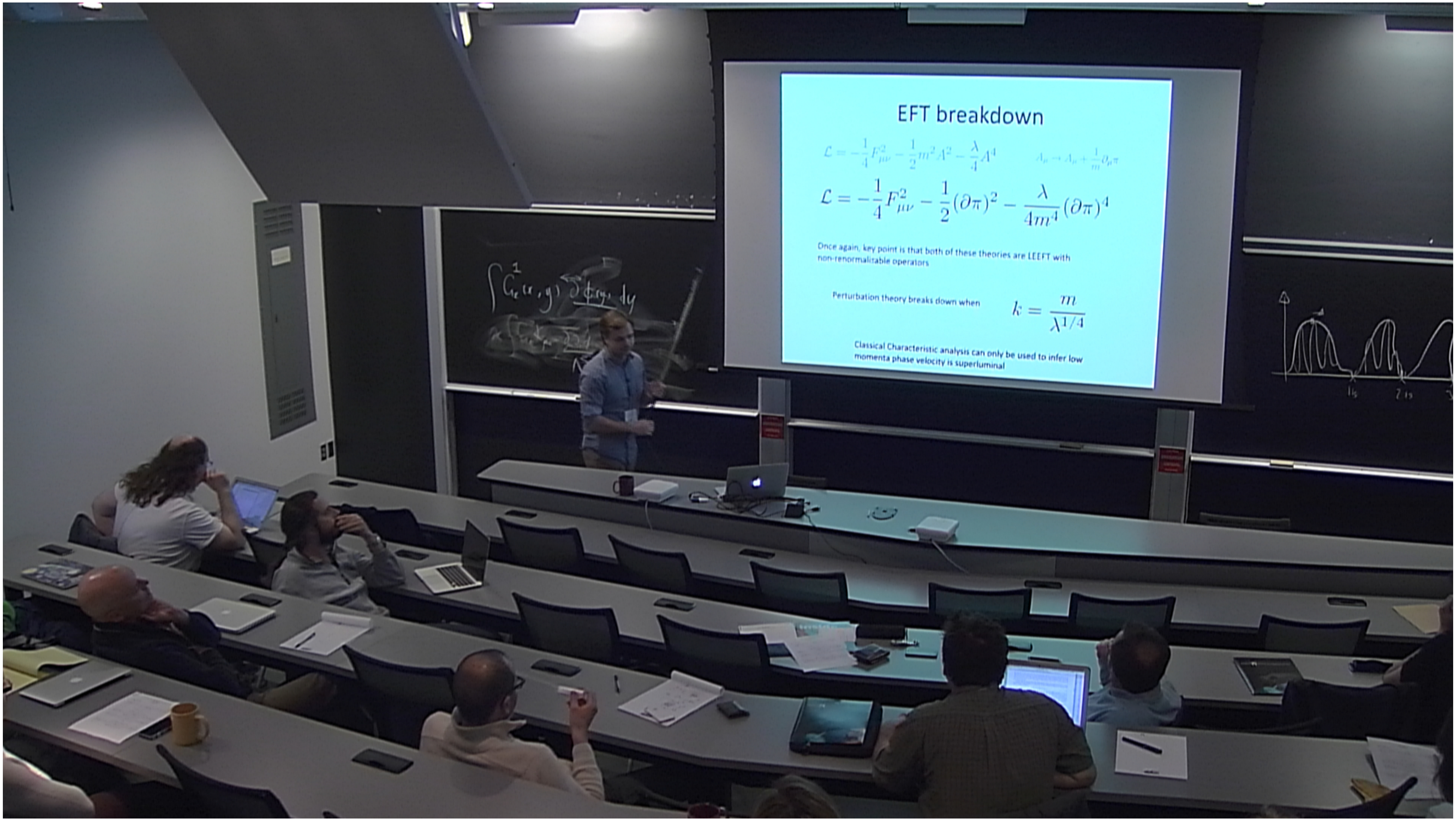
Once again, key point is that both of these theories are LEEFT with non-renormalisable operators
Perturbation theory breaks down when
$k = \frac{m}{\lambda^{1/4}}$
Classical Characteristic analysis can only be used to infer low momenta phase velocity is superluminal



EFT breakdown
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial\pi)^2 - \frac{\lambda}{4m^4}(\partial\pi)^4$
Once again, key point is that both of these theories are LEEFT with non-renormalizable operators
Perturbation theory breaks down when
$k = \frac{m}{\lambda^{1/4}}$
Classical Characteristic analysis can only be used to infer low momenta phase velocity is superluminal

The real issue:

Can a Lorentz invariant LEEFT which admits superluminal phase velocities admit a Lorentz invariant UV completion?

Historical: Velo and Zwanziger (1969)/Johnson and Sudarshan (1961) were the first to argue that a naively Lorentz invariant field theory may not admit a Lorentz invariant UV completion - they say NO

VZ - Causality restricts the possible interaction Lagrangians

The real issue:

Historical: Velo and Zwanziger (1969)/Johnson and Sudarshan (1961) say NO

VZ/JS - Causality restricts the possible interaction Lagrangians

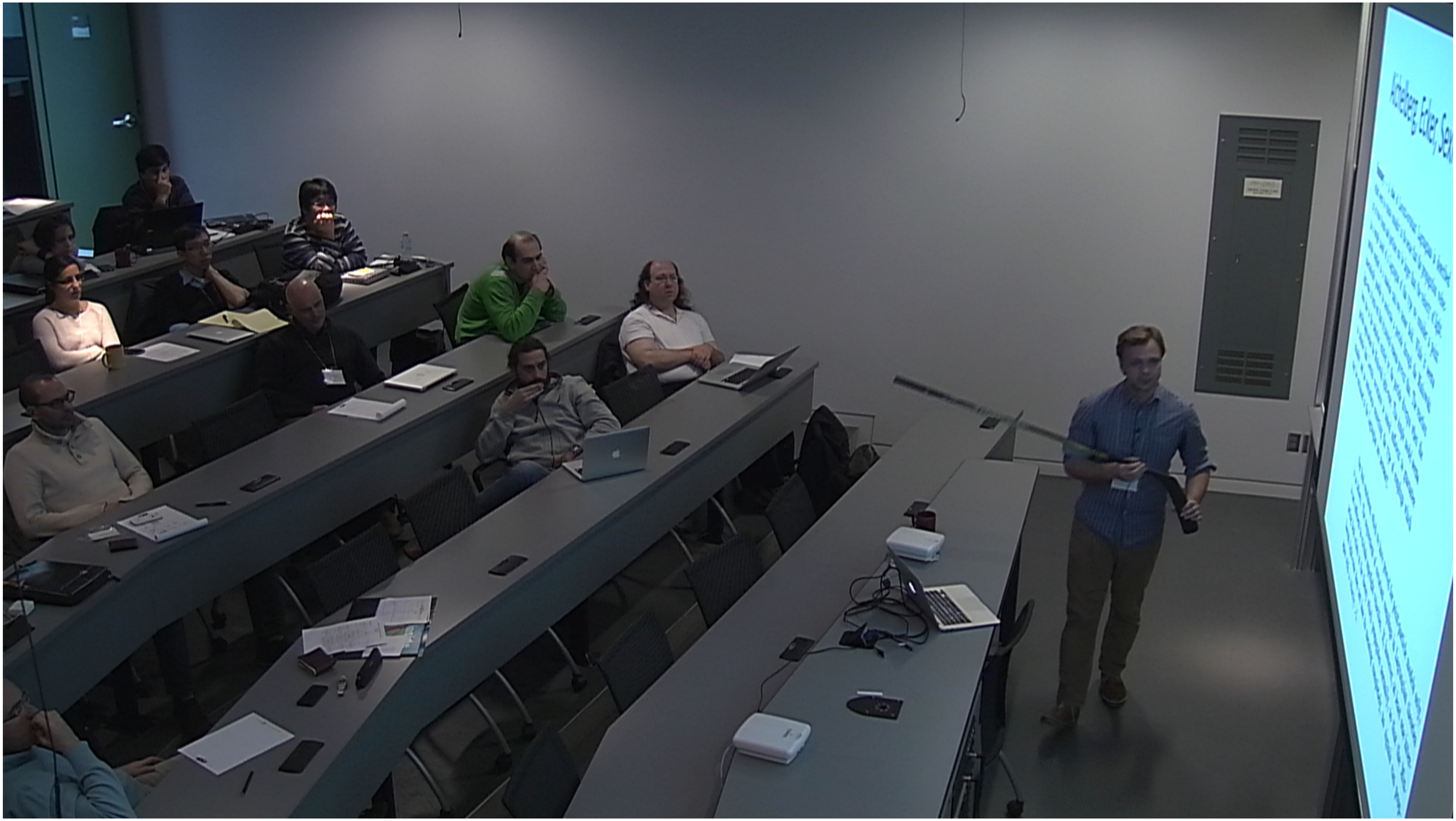
Historical: Aichelberg, Ecker, Sexl (1971) immediately criticise the VZ result and say YES

AES - **Lorentz invariance ensures causality**

Aichelberg, Ecker, Sexl (1971)

Summary. — A class of Lorentz-covariant Lagrangians is described which seem to violate causality in the sense that the propagation velocity of wave fronts and particles can be larger than the velocity of light. As a simple model of a Lagrangian of this type we consider a point particle coupled to a massless rank-two tensor field. While it seems kinematically possible to accelerate a particle through the Minkowski light-cone, it turns out that dynamical reasons prevent this. The reaction force due to the radiation emitted by the particle diverges when the particle approaches the Minkowski light-cone. This simple model seems to indicate that Lorentz covariance is indeed sufficient to guarantee causality and no restrictions concerning the type of couplings which may be contained in the Lagrangian are necessary.

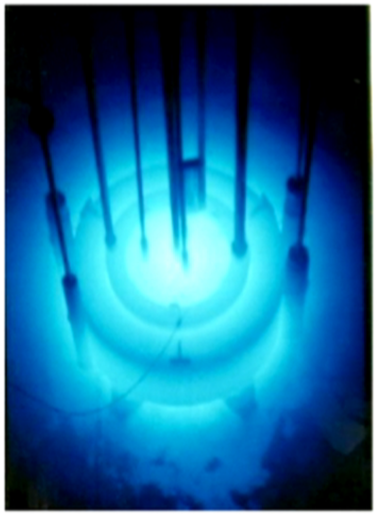
Our final conclusion, which we have proved for point-particle models only, is as follows. The requirement of covariant couplings in Lagrangian theories is not sufficient to exclude superlight velocities. If, however, at least one interacting field exists in the theory which propagates normally, so that the Minkowski light-cone does not lose its meaning completely, no signals can propagate with a velocity >1.



Aichelberg, Ecker, Sexl (1971)

The Punch line of this argument, take a theory with two coupled fields Alice, Bob in which one field Alice can naively be superluminal. Propagate an Alice particle (wave-packet) in a background for which Alice is superluminal and Bob is luminal

Generically Alice will Cherenkov radiate into Bob



AES example was a bad one but now we could do this easily with Galileon coupled to some other field

Cherenkov radiation

The amount of Cerenkov radiation will be given by some analogue (dependent on the precise field theory coupings) of the Frank Tamm formula

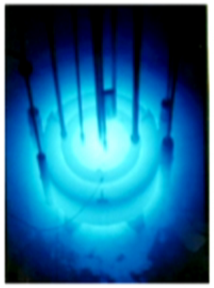
$$\frac{dE}{dL} = \frac{q^2}{4\pi} \int_{v > \frac{c}{n(\omega)}} \mu(\omega)\omega \left(1 - \frac{c^2}{v^2 n^2(\omega)}\right) d\omega$$

$$v > \frac{c}{n(\omega)}$$

The integral is performed over all frequencies for which

It is dominated by the highest frequencies for which this condition is satisfied -

which is why its BLUE!!!!!!!!!



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Front velocity again

The field theory analogue of this statement is the following:

If

$$v_{\text{front}}(\text{Alice}) = \frac{1}{n_{\text{Alice}}(\infty)} > v_{\text{front}}(\text{BOB}) = \frac{1}{n_{\text{BOB}}(\infty)}$$

then there will be an infintite backreaction

the only conclusion is that

$$v_{\text{front}}(\text{Alice}) = v_{\text{front}}(\text{BOB}) = 1$$

Analyticity again

In a local field theory, operators commute outside the lightcone

$$[\pi(x), \pi(y)] = 0 \qquad (x-y)^2 > 0$$

From this, and the assumption of stability (all states have positive energy and mass) we derive the Jost-Lehmann-Dyson representation

$$\langle P_f | [\pi(x/2), \pi(-x/2)] | P_i \rangle = \int_0^\infty d\mu D(\mu, P_i, P_f, x) \Delta_\mu(x)$$

$$\Delta_\mu(x) = 0, \quad x^2 > 0$$

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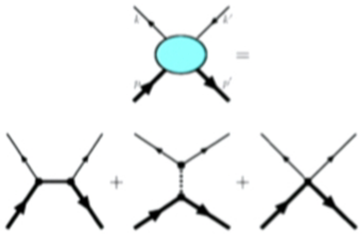
Dispersion relations

$$\langle P_f | [\pi(x/2), \pi(-x/2)] | P_i \rangle = \int_0^\infty d\mu D(\mu, P_i, P_f, x) \Delta_\mu(x)$$

In a generic field theory, D grows as a polynomial in μ



From this one can prove that the
forward scattering amplitude A(s,0)
is analytic in the complex s plane
(modulo branch cuts and poles on real axis)
with a **finite** number of subtractions

Hepp 1964

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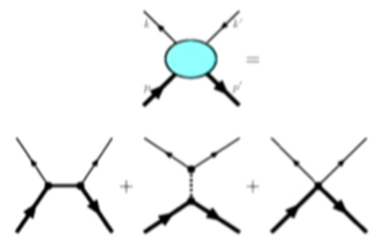

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POLYNOMIAL BOUNDEDNESS

In a local field theory:
Wightman functions are tempered distributions

$$W(\{k_i\}) = \prod_i \int d^d k_i \langle 0|\pi(x_1)\pi(x_2)\ldots\pi(x_N)e^{-\sum_i ik_i.x_i}|\rangle$$

Momentum space growth is bounded by a polynomial in k

$$W(\{k_i\}) < C|\sum_i |k_i||^N$$

Scattering amplitudes are bounded by a polynomial in k for complex k

$$A(\{k_i\}) < C|\sum_i |k_i||^N$$

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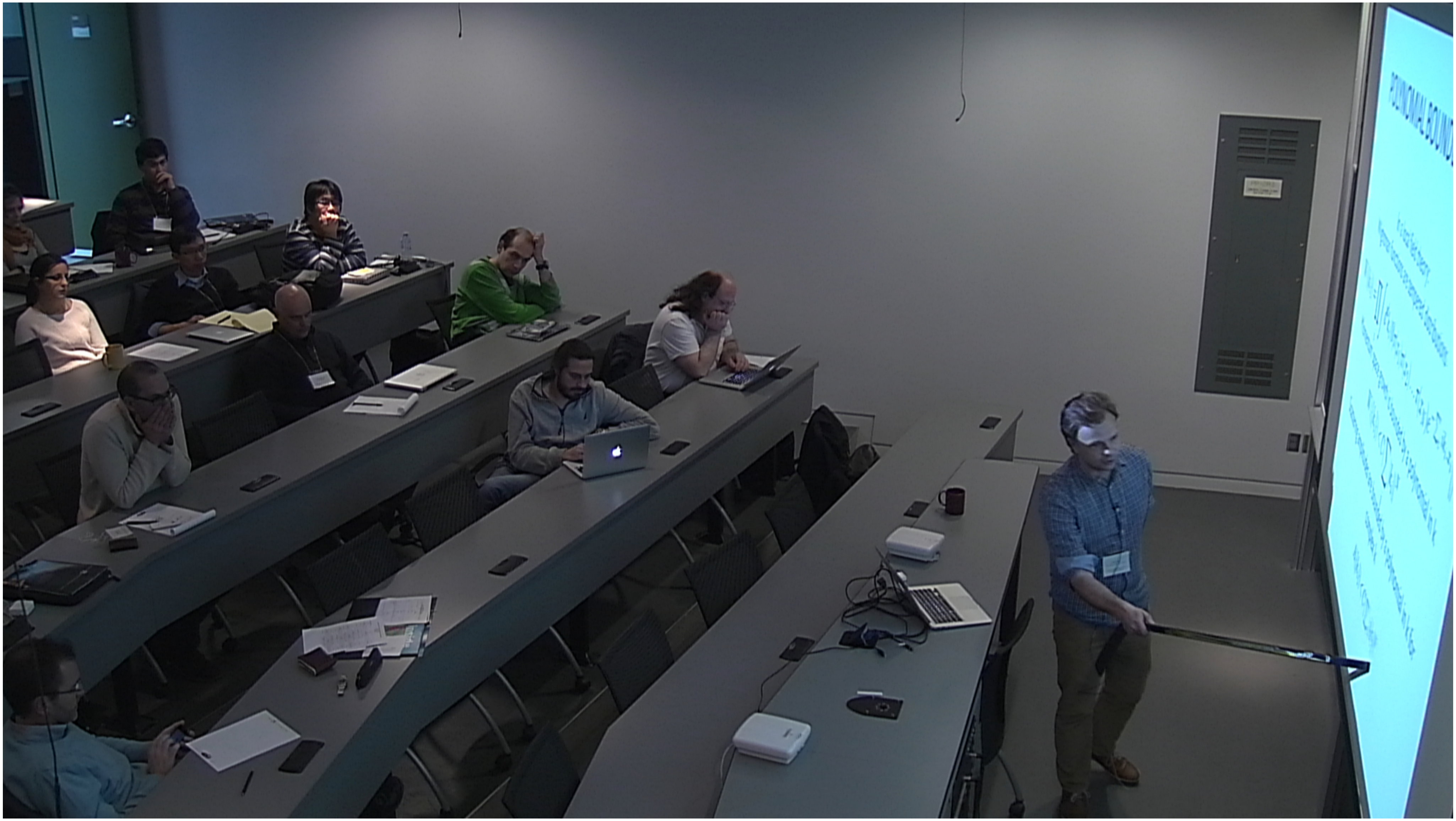

Froissart-Martin bound

Assuming only Polynomial Boundedness, and (proven) analyticity in the Martin-Lehmann ellipse

If the theory admits a mass gap!!!

Number of subtractions in dispersion relation is never more than 2!

$$A(s) \leq \frac{c}{m^2} s (\ln s)^2$$

$$\sigma(s) \sim \frac{Im(A(s))}{s} < \frac{c}{m^2} (\ln s)^2$$

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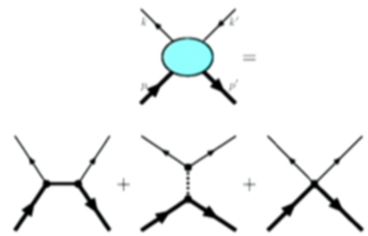
Forward scattering dispersion



Unsubtracted Dispersion Relation

$$A(s) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}(A(s)}{s'-s} ds' + \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}(A(s)}{s'+4m^2+s} ds' + \text{pole terms}$$

after subtractions

$$A(s) = A_0 + sA_1 + \frac{1}{\pi}s^2 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^2(s'-s)} + \frac{1}{\pi}(4m^2-s)^2 \int_{4m^2}^{\infty} \frac{\mathrm{Im}[A(s')]}{s'^2(s'-4m^2+s)} + \text{pole terms}$$

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crossing symmetry

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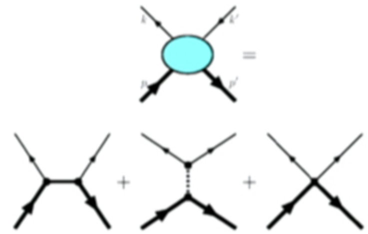

$$A(s) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}(A(s)}{s' - s} ds' + \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}(A(s)}{s' + 4m^2 + s} ds' + \text{pole terms}$$

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Differentiating and taking limit

$$m \to 0$$

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Adams et al 2006

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But for ALL Galileon models!

at best lowest order contribution

$$A(s,t,u) \sim \frac{1}{\Lambda^6}(s^3+t^3+u^3)+\ldots$$

$$A''(0) = 0 \qquad \text{in limit} \quad m \to 0$$

which violates

$$A''(s=s_0) = \frac{4}{\pi}\int \mathrm{d}s \frac{s\sigma(s)}{s^3} + \cdots > 0$$

Galileons do not admit a local, Lorentz invariant, UV completion

Adams et al 2006

ASSUMPTIONS

Locality/causality implies analyticity through JLD representation

Locality implies polynomial boundedness (temperedness assumption)

Mass Gap

Polynomial boundedness plus analyticity plus mass gap implies Froissart-Martin bound

Together with unitarity imply $A''(s) > 0$

All of these statements (assuming mass gap) are
Rigorously proven in Axiomatic Field Theory

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DGP

These arguments were originally used to argue the DGP model is inconsistent

However in the DGP model the Galileon is the helicity-zero mode which arises as a resonance state

There is no mass gap, the branch cut on the right hand plane extends to s=0.

This means we cannot prove analyticity in Martins extension of the Lehmann ellipse:
Froissart bound does not have to apply

Conclusions

My only real conclusion is that assessing superluminality is far more complicated that you might expect.

LEEFT cannot do it.

Superluminalities in LEEFT are not inconsistent with relativity.

Real question is that of UV completion - which for Galileons/massive gravity is still up in the air.

A UV completion of Galileons/massive gravity may resolve apparent SL in low energy theory.

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