Title: Causal structures in Massive gravity and Gauss-Bonnet gravity

Date: Apr 09, 2015 09:45 AM

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Abstract: In General Relativity, gravitons propagate to null directions, because of its well-organized structures. Modifying the gravity theory slightly, meanwhile, the beautiful structure is broken and gravitons can easily propagate superluminaly. Here, applying the characteristic method, which is the well-established powerful way to analyze causal structures, the causal structures in Massive gravity and Gauss-Bonnet gravity are analyzed. We discuss the superluminality, acausality and black holes.

Causal structures in Massive gravity

Class. Quant. Grav. 30 (2013) 184008, K. I., Y. C. Ong
Phys. Lett. B 726 (2013), 544, S. Deser, K. I., Y. C. Ong, A. Waldron
Mod. Phys. Lett. A Vol. 30, Nos. 3 & 4 (2015),
S. Deser, K. I., Y. C. Ong, A. Waldron
arXiv: 1504.????? (2015), K. I, N. Tanahashi

and

Gauss-Bonnet Gravity

K. I. Phys. Rev. D 90, 044037 (2014)

LeCosPA → July. 2015 → ICCUB

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  (National Taiwan University, LeCosPA)
  -> (University of Barcelona, ICCUB)

Modification of Gravity

- IR physics (cosmology)
  - Dark energy, Dark matter
- UV physics
  - Singularity
  - Quantization

⇒ **Modified Gravity**

- Modification of Lagrangian : f(R), Gauss–Bonnet
- Modification of vacuum state : ghost condensation
- Modification of concept of geometry
  - higher dimension : Braneworld
  - other manifold : Teleparallel gravity
- Introducing mass of graviton : massive gravity

Consistency Check of modified gravity

0-th order (of cosmology) : FLRW universe without perturbation

Consistency with standard cosmology
DM and DE??

1-th order : perturbation on FLRW background

Consistency with standard cosmology
Consistency with solar system physics
Stability

Nonlinear property

Causal structure
Nonlinear stability

Quantization

⋮

Consistency Check of modified gravity

0-th order (of cosmology) : FLRW universe without perturbation

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DM and DE??

1-th order : perturbation on FLRW background

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Consistency with solar system physics
Stability

Nonlinear property

Causal structure ← Main topic in this talk
Nonlinear stability

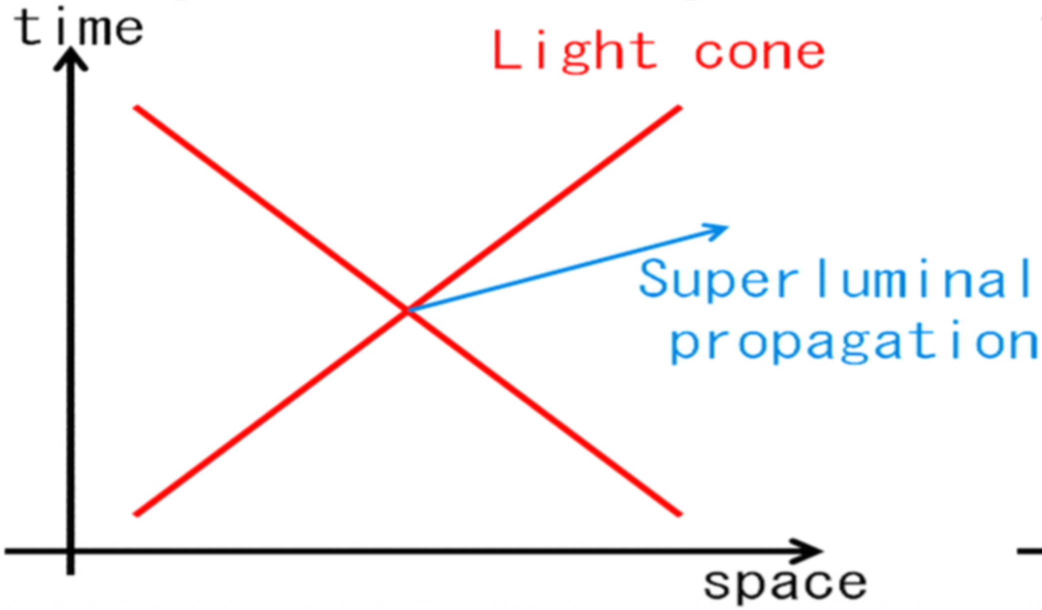
Quantization

⋮

Superluminal mode and Acausality

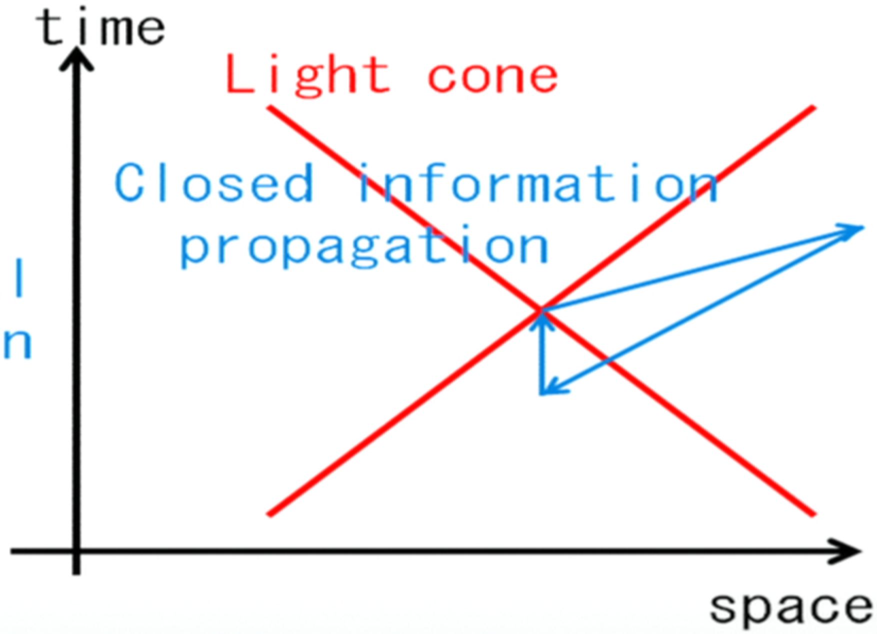
Superluminal mode

Propagation the speed of which is higher than that of light

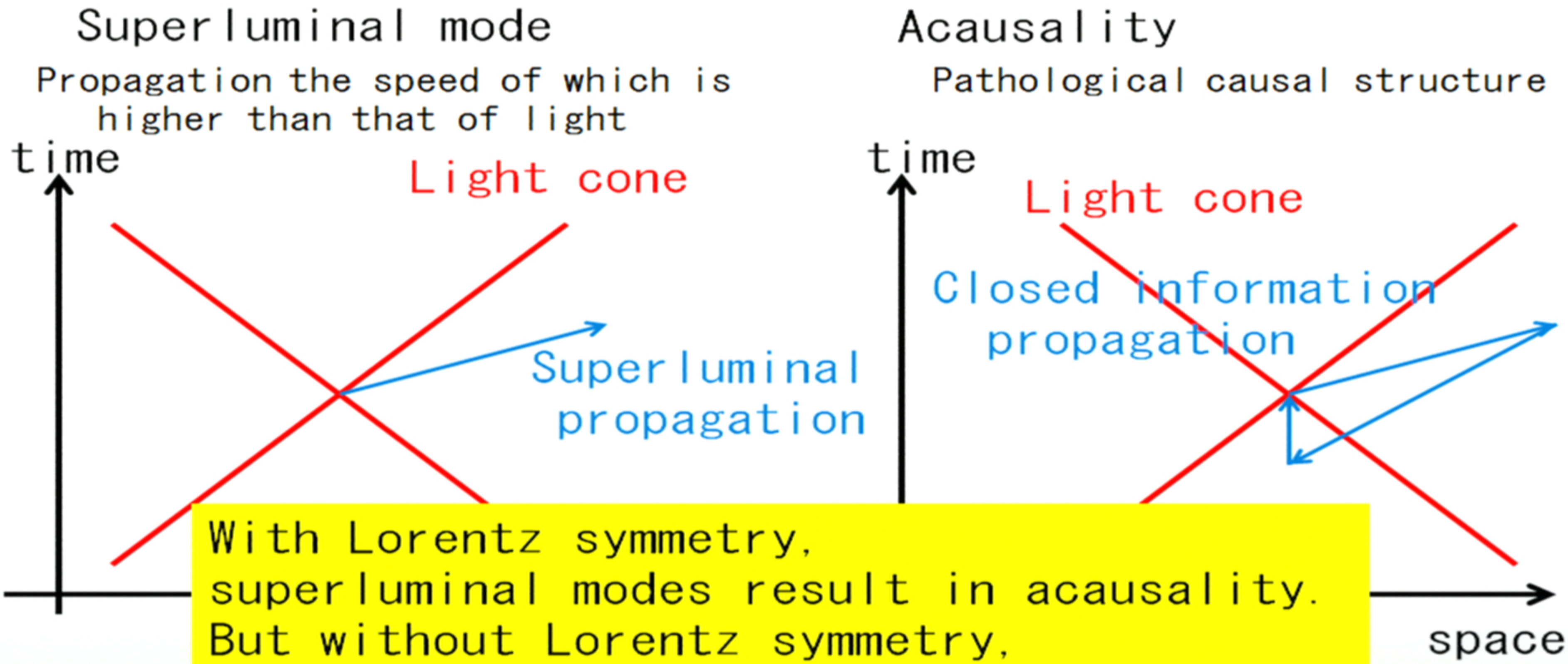


Acausality

Pathological causal structure



Superluminal mode and Acausality



With Lorentz symmetry,
superluminal modes result in acausality.
But without Lorentz symmetry,
it is not always true.

Causal structures

in

Massive gravity

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Three cases

FP vacuum

Time evolution from any spacelike hypersurface is unique.

(Cauchy-Kovalevskaya theorem) Class.Quant.Grav. 30 (2013) 184008, K.I., Y.C. Ong

Any null hypersurface is characteristics. arXiv: 1504.????? (2015), K.I, N. Tanahashi

Around FP vacuum (Perturbation on FP vacuum)

Superluminal modes. arXiv: 1504.????? (2015), K.I, N. Tanahashi

Nonlinear regime

Acausality. Physics Letters B 726 (2013), 544, S. Deser, K.I., Y.C. Ong, A. Waldron
Modern Physics Letters A (2015), S. Deser, K.I., Y.C. Ong, A. Waldron

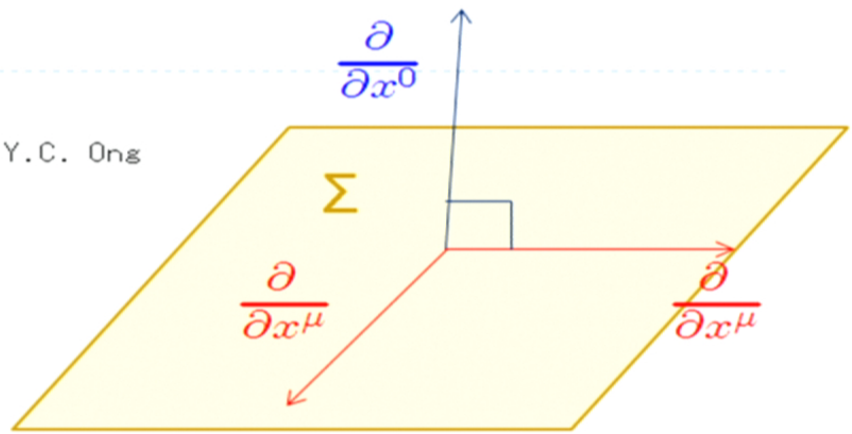
FP vacuum

Spacelike hypersurface

Class.Quant.Grav. 30 (2013) 184008, K.I., Y.C. Ong

For simplicity,
take the vecter normal to hypersuface
Any spacelike hypersurface never
be a characterisitcs.

Unique time evolution
(Cauchy-Kovalevskaya theorem).



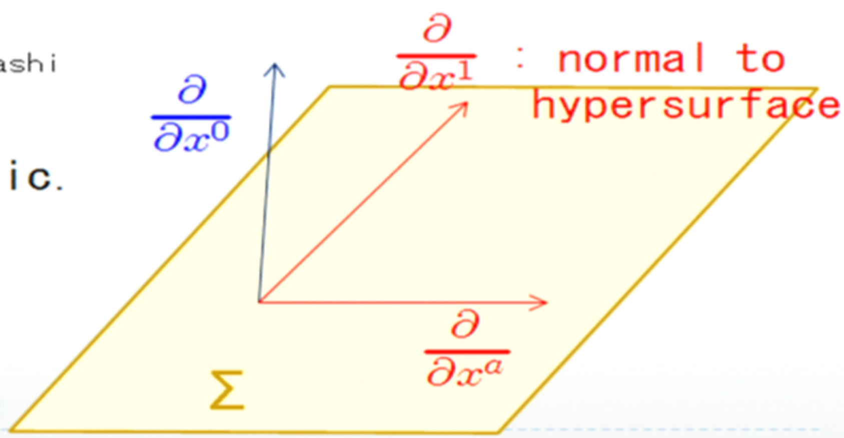
Null hypersurface

arXiv: 1504.????? (2015), K.I, N. Tanahashi

Check that any null hypersurface is
a characteristic.

Normal vector to hypersurface is
on the hypersurface.

Take a vector NOT tangent to hypersurface



Around FP vacuum

arXiv: 1504.????? (2015), K.I, N. Tanahashi

$$S = \int \epsilon_{abcd} \Big[-\frac{1}{4} e^a e^b \big(d\omega^{cd} + \omega^c_e \omega^{ed} \big) + m^2 e^a \big(\frac{\beta_0}{4} e^b e^c e^d + \frac{\beta_1}{3} e^b e^c f^d + \frac{\beta_2}{2} e^b f^c f^d + \beta_3 f^b f^c f^d \big) \Big]$$

Perturbation (GW) without matter

Flat background ➡ $\beta_0 + \beta_1 + \beta_2 + \beta_3 = 0$
$f^1_t = f^2_x = f^3_y = f^4_z = 1$

Graviton has mass. ➡ Rest frame.

Perturbative solution: $e^1_t = e^2_x = 1 \quad e^2_y = (1+\delta) \; e^3_z = (1-\delta)$

$$\delta = A \exp(i M_{FP}\ t) \quad M^2_{FP} = m^2(\beta_1 + 2\beta_2 + 3\beta_3)$$

⬇

Characteristic equations are perturbed.

Null characteristics on FP vacuum ➡ spacelike, timelike, or null?

Around FP vacuum

arXiv: 1504.????? (2015), K.I, N. Tanahashi

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$\beta_3 = 0$ ➡ Charecteristics for helicity-0 and 1 are generically not null, but spacelike or timelike.
Charecteristics of helicity-2 is null.

$\beta_3 \neq 0$ ➡ Charecteristics for helicity 1 are generically not null, but spacelike or timelike.
Charecteristics of helicity-2 are null.
Characteristics for helicity-0 are still under analysis

Around FP vacuum

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➡ This statement is true in nonlinear

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➡ This seems not generic in nonlinear.
One of helicity-2 modes is coupled with helicity-0 and 1.

Superluminal modes around FP vacuum

Superluminal modes may result in violation of (null) energy condition.

There may be negative energy solutions around FP vacuum.

↑

How to define energy??
In the sense of positive energy theorem,
we need to check ADM energy?

➡ Need diffeomorphism invariance??

Analyzing Bigravity is (perhaps) better.

Check PET in Bigravity. with N. Tanahashi

Around FP vacuum

arXiv: 1504.????? (2015), K.I, N. Tanahashi

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anashi

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Acausality

Phys. Lett. B 726 (2013), 544, S. Deser, K.I., Y.C. Ong, A. Waldron
Mod. Phys. Lett. A Vol. 30, Nos. 3 & 4 (2015), S. Deser, K.I., Y.C. Ong, A. Waldron

In nonlinear regime of massive gravity, acausality appears.

$$f^0 = dt, f^1 = dx, f^2 = dy, f^3 = dz$$
$$e^0 = A(x)dt, e^1 = B(x)dx, e^2 = C(x)dy, e^3 = B(x)dz$$

and if all connection are zero, local acausality appears

MG cannot be a fundamental, but should be effective theory.

What is the cutoff scale? $k_{cutoff} = (M_{pl}^a m^b)^{1/(a+b)}$??

Acausal solution is nonlinear zero-mode.
It seems irrelevant to the cutoff of momentum.

We need not only the cutoff scale of momentum
but also the cutoff scale of field value ϕ_{cutoff} .

We should derive ϕ_{cutoff}
and estimate the energy scale associated with ϕ_{cutoff} .

Then we can discuss
whether the acausal solution is outside of effective theory.

Around FP vacuum

arXiv: 1504.????? (2015), K.I, N. Tanahashi

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Causal structures

in

Gauss–Bonnet gravity

Gauss-Bonnet gravity

action

$$S = \int d^n x \left[\frac{1}{2\kappa^{D-2}}\{R - 2\Lambda + \alpha(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD})\} + L_m\right]$$

GB terms

Matter contribution

EoM $G_{AB} + \Lambda g_{AB} - \frac{\alpha}{2}H_{AB} = 2\kappa^{D-2}T_{AB}$

$$H_{AB} := (R^2 - 4R_{CD}R^{CD} + R_{CDEF}R^{CDEF})g_{AB}$$
$$- 4(RR_{AB} - 2R_{AC}R_B{}^C - 2R_{ACBD}R^{CD} + R_{ACDE}R_B{}^{CDE})$$

Check the characteristic for EoM

Assumption : T_{AB} does not involve the highest order derivatives of metric

Decomposition

A hypersurface Σ

$\frac{\partial}{\partial x^\mu}$ are vectors on Σ

$\frac{\partial}{\partial t} = \frac{\partial}{\partial x^0}$ is independent vector from $\frac{\partial}{\partial x^\mu}$

$$\xi^A\left(\frac{\partial}{\partial x^A}\right) := \frac{\partial}{\partial t}$$

$$\zeta_A dx^A = dt$$

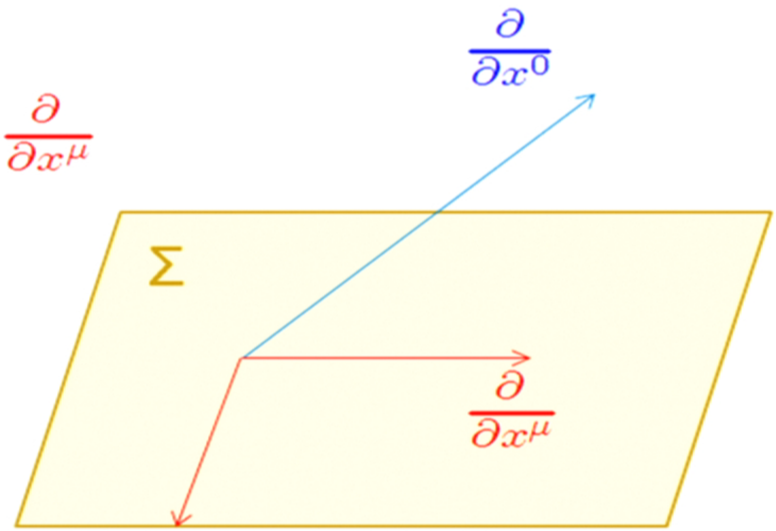
Projection operator

$$P^A_B = \delta^A_B - \xi^A \zeta_B$$

Decomposition of vector

$$V_A : V_0 := \xi^A V_A \quad , \quad V_\mu := P^A_\mu V_A$$

$$V^A : V^0 := \zeta_A V^A \quad , \quad V^\mu := P^\mu_A V^A$$



Gauge modes

We have fixed time evolution of g_{AB} , $\Gamma_{00\alpha}$, $\Gamma_{\alpha\beta\gamma}$, $\Gamma_{\alpha\beta 0}$

Remaining variables are Γ_{000} , $\Gamma_{\alpha 00}$, $\Gamma_{0\alpha\beta}$

DoF: 1 , $(D-1)$, $(D-1)D/2$

Diff. inv. : # of gauge DoF is D

$\partial_0\Gamma_{000}$, $\partial_0\Gamma_{\alpha 00}$ never appear in EoM. ⇒ Gauge modes

$\Gamma_{0\alpha\beta}$ are physical.

Check the characteristics for $\Gamma_{0\alpha\beta}$!

General Relativity

$(\partial_0 \Gamma_{\alpha\beta 0} = -\partial_0 \Gamma_{0\alpha\beta} + \cdots)$

$\partial_0 \Gamma_{0\alpha\beta}$ appears only in $R_{0\alpha 0\beta}$ $(= -R_{\alpha 00\beta} = \cdots)$

$G^{AB} = R_{0\alpha 0\beta} A^{AB,\alpha\beta} + \cdots$ ⟶ $\boxed{A^{AB,\alpha\beta} \partial_0 \Gamma_{0\alpha\beta} + \cdots = 0}$

Characteristic equation

$A^{00,\alpha\beta} = A^{0\mu,\alpha\beta} = A^{\mu 0,\alpha\beta} = 0$

$A^{\mu\nu,\alpha\beta} = g^{00}(h^{\alpha\mu} h^{\beta\nu} - h^{\alpha\beta} h^{\mu\nu})$

$h^{\mu\nu} := g^{\mu\nu} - g^{0\mu} g^{0\nu} / g^{00}$

($h_{\mu\nu}$ is induced metric of Σ)

Null hypersurface $g^{00} = 0$ $h_{\mu\nu} =$ diag $(0, 1, 1, \cdots)$ $h^{11} = O[(g^{00})^{-1}]$

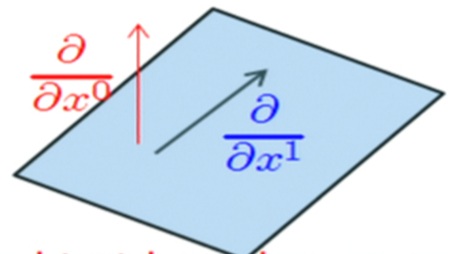
$-g^{00} h^{11} \sum_i \partial_0 \Gamma_{0ii} + \cdots = 0$ $(1,1)$ -component

$g^{00} h^{11} \partial_0 \Gamma_{01i} + \cdots = 0$ $(1,i)$ -component

$-g^{00} h^{11} \partial_0 \Gamma_{011} \delta_{ij} + \cdots = 0$ (i,j) -component

1 : null direction

i : normal to 1



All component have the same kinetic term

$D(D-3)/2$ degeneracies ⟵ Gravitational wave

Gauss-Bonnet correction

$$G_{AB} + \Lambda g_{AB} - \frac{\alpha}{2} H_{AB} = 2\kappa^{D-2} T_{AB}$$

$$H^{AB} = R_{0\alpha 0\beta} B^{AB,\alpha\beta} + \cdots$$

$$B^{00,\alpha\beta} = B^{0\mu,\alpha\beta} = B^{\mu 0,\alpha\beta} = 0$$

$$B^{\mu\nu,\alpha\beta} = 4g^{00} R_{\lambda\omega\gamma\delta}(h^{\lambda\gamma}h^{\omega\delta}h^{\mu\nu}h^{\alpha\beta} - h^{\lambda\gamma}h^{\omega\delta}h^{\mu\alpha}h^{\nu\beta} + 2h^{\lambda\mu}h^{\gamma\alpha}h^{\omega\delta}h^{\nu\beta} + 2h^{\lambda\nu}h^{\gamma\alpha}h^{\omega\delta}h^{\mu\beta}$$
$$-2h^{\lambda\alpha}h^{\gamma\beta}h^{\omega\delta}h^{\mu\nu} - 2h^{\lambda\mu}h^{\gamma\nu}h^{\omega\delta}h^{\alpha\beta} + 2h^{\lambda\mu}h^{\omega\alpha}h^{\gamma\nu}h^{\delta\beta}).$$

GR: $A^{AB,\alpha\beta}\partial_0\Gamma_{0\alpha\beta} + \cdots = 0$

Null hypersurface

$-g^{00}h^{11}\sum_i \partial_0\Gamma_{0ii} + \cdots = 0$	$(1,1)$ -component
$g^{00}h^{11}\sum_i \partial_0\Gamma_{01i} + \cdots = 0$	$(1,i)$ -component
$-g^{00}h^{11}\sum_i \partial_0\Gamma_{011}\delta_{ij} + \cdots = 0$	(i,j) -component

GB: $\left(A^{AB,\alpha\beta} - \frac{\alpha}{2}B^{AB,\alpha\beta}\right)\partial_0\Gamma_{0\alpha\beta} + \cdots = 0$

Gauss-Bonnet correction

$$\bar{\Gamma}_{0\alpha\beta} \sim \partial_0 \Gamma_{0\alpha\beta}$$

$$-g^{00}h^{11}\left[\sum_i \bar{\Gamma}_{0ii} + 2\alpha\left(\sum_{i,k,l} R_{klkl}\bar{\Gamma}_{0ii} - 2\sum_{i,j,k} R_{ikjk}\bar{\Gamma}_{0ij}\right)\right] + \cdots = 0$$

(1, 1) -component

$$g^{00}h^{11}\left[\bar{\Gamma}_{01i} + 2\alpha\left(\sum_{k,l} R_{klkl}\bar{\Gamma}_{01i} - 2\sum_{j,k} R_{ikjk}\bar{\Gamma}_{01j}\right) + 8\alpha\sum_{j,k}\left(R_{1kik}\bar{\Gamma}_{0jj} - R_{1kjk}\bar{\Gamma}_{0ij} - R_{1jik}\bar{\Gamma}_{0jk}\right)\right] + \cdots = 0$$

(1, i) -component

$$-g^{00}h^{11}\left[\delta_{ij}\bar{\Gamma}_{011} + 2\alpha\left(\sum_{k,l} R_{klkl}\delta_{ij} - 2R_{ikjk}\right)\bar{\Gamma}_{011} + \alpha\sum_k\left(R_{1ijk} + R_{1jik}\right)\bar{\Gamma}_{01k} + 4\alpha\left\{\delta_{ij}\sum_{k,l}\left(R_{1k1k}\bar{\Gamma}_{0ll} - R_{1k1l}\bar{\Gamma}_{0kl}\right) + \sum_k\left(R_{1i1k}\bar{\Gamma}_{0kj} + R_{1j1k}\bar{\Gamma}_{0ki} + R_{1k1k}\bar{\Gamma}_{0ij} + R_{1i1j}\bar{\Gamma}_{0kk}\right)\right\}\right] + \cdots = 0$$

(i, j) -component

Example 1: All degeneracies are resolved

$$R_{ijkl} = R_{1ijk} = 0$$

$$R_{1i1j} = C\delta_{ij}$$

$$-g^{00}h^{11}\sum_i \bar{\Gamma}_{0ii} + \cdots = 0 \qquad (1,1) \text{ -component}$$

$$g^{00}h^{11}\sum_i \bar{\Gamma}_{01i} + \cdots = 0 \qquad (1,i) \text{ -component}$$

$$-g^{00}h^{11}\left[\delta_{ij}\bar{\Gamma}_{011} + 4\alpha(D-4)C\left(\delta_{ij}\sum_k \bar{\Gamma}_{0kk} - \bar{\Gamma}_{0ij}\right)\right] + \cdots = 0$$

(i,j) -component

Characteristic hypersurface is not null.

The speed of graviton is not that of light.

Dynamical case (spherically symmetric case)

$$R_{AB}U^AU^B > 0$$ Subluminal

$$R_{AB}U^AU^B < 0$$ Superluminal

H. Maeda, M. Nozawa (2008)

Einstein Branch

$$T_{AB}U^AU^B \geq 0 \Rightarrow R_{AB}U^AU^B \geq 0$$

GB Branch

$$T_{AB}U^AU^B \geq 0 \Rightarrow R_{AB}U^AU^B \leq 0$$

Evaporating BH

Contraposition of Area-increasing low

$$R_{AB}U^AU^B < 0 \Rightarrow \text{Superluminal}$$

Graviton can escape from Evaporating "BH" defined by null curves

Summary of Gauss-Bonnet gravity

In GB gravity, gravitational propagation
potentially becomes superluminal.

Causal structure based on null curve is meaningless.
We need to analyze it by using the fastest propagation.

Causal structure is analyzed with method of characteristics,
which is powerful technique.

Killing horizon is the event horizon in the sense of causality.

On spherically symmetric background,

$R_{AB}U^AU^B = 0$ luminal.

$R_{AB}U^AU^B > 0$ subluminal. ➡ gravitational Cherenkov

$R_{AB}U^AU^B < 0$ superluminal. ➡ Acausality? Need to check it.