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Abstract:

Propagation of massive gravity waves in a background field

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Motivation and goals

- Consistency requirements for higher spin theories
  (I) Degree of freedom count
  (II) Stability viz. no ghosts
  (III) Predictivity i.e. hyperbolicity
  (IV) (Sub)luminal propagation
- dRGT massive gravity: no Boulware-Deser ghost
  C. de Rham, G. Gabadadze, A. Tolley [2010,2011]
- Presence of superluminalities (cf Galileons in the decoupling limit)
- Interpretation of non-linear propagation analysis results
  S. Deser, M. Sandora, A. Waldron, GZ [arXiv:1408.0561]
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Methodology

- Linearize perturbations over an mGR solution
- Compute covariant constraints
- Compute characteristic matrix for perturbations
- Make analogy with charged spin 3/2 model
  W. Rarita, J. Schwinger [1941]
- Infer propagation properties
  G. Velo, D. Zwanziger [1969,1970]
  1. Hyperbolicity requirements
  2. Superluminality and causal structure(s)

Non-linear theory

Perturbative analysis

Spin 3/2 analogy

General setting

- dRGT massive gravity: first order Cartan formalism
  - 4 vierbein 1-forms $e^m := e_\mu{}^m dx^\mu$ (16 fields)
  - 6 connection 1-forms $\omega^{mn} := \omega_\mu{}^{mn} dx^\mu$ (24 fields)
- dRGT action (4 fiducial vierbein 1-forms $f^m := f_\mu{}^m dx^\mu$)

$$S = \frac{1}{4} \int \epsilon_{mnrs}\, e^m e^n \left[d\omega^{rs} + \omega^r{}_t \omega^{ts} \right]$$
$$+ m^2 \int \epsilon_{mnrs}\, e^m \left[\frac{\beta_0}{4} e^n e^r e^s + \frac{\beta_1}{3} e^n e^r f^s + \frac{\beta_2}{2} e^n f^r f^s + \beta_3 f^n f^r f^s \right]$$

Equations of motion

- Zero torsion condition

$$\mathcal{T}^m := \nabla e^m := de^m + \omega^m{}_n e^n \approx 0$$

- Einstein equations

$$\mathcal{G}_m := G_m - m^2 t_m \approx 0$$

  - Einstein 3-forms
    $G_m := \frac{1}{2}\epsilon_{mnrs} e^n \left[d\omega^{rs} + \omega^r{}_t \omega^{ts}\right]$
  - Mass term 3-forms
    $t_m := \epsilon_{mnrs}\left[\beta_0 e^n e^r e^s + \beta_1 e^n e^r f^s + \beta_2 e^n f^r f^s + \beta_3 f^n f^r f^s\right]$

- 40 eoms for 40 dynamical fields

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Primary constraints

- Space-time decomposition of a p-form ($p < 4$)

$$\theta := \mathring{\theta} + \boldsymbol{\theta}$$

  where $\mathring{\theta} \wedge dt = 0$

- Primary constraints: purely spatial part of the eoms

$$\boldsymbol{\mathcal{T}}^m = \boldsymbol{d}\boldsymbol{e}^m + \boldsymbol{\omega}^m{}_n \boldsymbol{e}^n \approx 0$$

$$\boldsymbol{G}_m \approx m^2 \boldsymbol{t}_m$$

- 12+4 =16 constraints

Secondary constraints

- Symmetry constraint

$$G_{[m}e_{n]} = \frac{1}{2}\epsilon_{mnrs}e^r \nabla \mathcal{T}^s \approx 0$$

So $t_{[m}e_{n]} \approx 0$ and generically

$$\mathcal{F} := e_m f^m \approx 0$$

- Vector constraint

$$\nabla G_m = \frac{1}{2}\epsilon_{mnrs}\mathcal{T}^n \left[d\omega^{rs} + \omega^r{}_t\omega^{ts}\right] \approx 0$$

So $\nabla t_m \approx 0$ which reduces to

$$\mathcal{V} := \epsilon_{mnrs}M^{mn}K^{rs} \approx 0$$

where $M^{mn}(e^r, f^s)$ and $K^{mn} := \omega^{mn} - \bar{\omega}^{mn}(f^r)$

- 6+4=10 constraints

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Tertiary constraints

- Curl of symmetry constraint

$$\nabla\mathcal{F} := K_{mn}e^{m}f^{n} \approx 0$$

where the purely spatial part is not new!

- Curl of vector constraint

$$\nabla\mathcal{V} := \epsilon_{mnrs}M^{mn}\nabla K^{rs} + \cdots \approx 0$$

  - ∇K^{rs}: no time derivatives on-shell
  - Scalar constraint:

$$\mathcal{S} := \nabla\mathcal{V} \approx 0$$

- 3+1=4 constraints
- 40-16-10-4=10 first order dofs OR 5 physical dofs

Secondary constraints

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Characteristic matrix

- $\chi := A^\mu \xi_\mu$ characteristic matrix
  - χ invertible on Σ: necessary condition for (locally) well-defined Cauchy problem
  - χ non-invertible at some point in Σ: Σ is a characteristic surface

! χ closely related to kinetic matrix for perturbations

→ May become non-invertible during time-evolution: sign of strong coupling

- Solving for ξ_μ such that $|\chi| = 0$ determines the causal structure of the theory: a cone sheet for each field component

! Hyperbolic system: real cone sheets

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Non-linear theory

Perturbative analysis

Spin 3/2 analogy

Non-linear theory

Perturbative analysis

Spin $3/2$ analogy

Rarita-Schwinger model

- Action

$$S_{3/2} = \int d^4x \overline{\psi}_\mu \gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\mathcal{D}_\nu\psi_\rho$$

where $\mathcal{D}_\nu := \partial_\nu + ieA_\nu + \frac{m}{2}\gamma_\nu$

- Equations of motion

$$\mathcal{R}^\mu := \gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\mathcal{D}_\nu\psi_\rho \approx 0$$

- Constraints
  - 4 primaries: $\mu = 0$ part of the eom
  - 4 secondaries: $\mathcal{D}_\mu\mathcal{R}^\mu \approx 0$ since $[\mathcal{D}_\mu, \mathcal{D}_\nu] = ieF_{\mu\nu} + \frac{m^2}{2}\gamma_{[\mu}\gamma_{\nu]}$
- Degree of freedom count: 16-4-4=8

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Velo-Zwanziger propagation analysis

- Characteristic determinant

$$|\chi| = (\xi^2)^4 \left[\xi^2 + \left(\frac{2e}{3m^2}\right)(\tilde{F}\cdot\xi)^2\right]^4$$

where $\tilde{F}_{\mu\nu} := \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

- Two different cone sheets $\xi^2 = 0$ and $\xi^2 + \left(\frac{2e}{3m^2}\right)(\tilde{F}\cdot\xi)^2 = 0$
- Is the extraordinary sheet spacelike?
    - If $\xi_\mu = (\xi, 0, 0, 0)$ then

$$|\chi| = \xi^{16}\left[1 - \left(\frac{2e}{3m^2}\right)^2 \boldsymbol{B}^2\right]^4$$

    - Zero after appropriate boosting!
- Different modes propagate on different cone sheets (some of them wider than Minkowski): crystal-like behavior

Analogy and expectations

- Analogy with perturbative mGR model

$$\psi_\mu \to (\varepsilon_\mu^m, \lambda_\mu^{mn})$$

$$A_\mu \to (e_\mu^m, \omega_\mu^{mn})$$

$$\eta_{\mu\nu} \to f_\mu^m$$

- Same propagation peculiarities as spin 3/2?
- Hyperbolic regime with different propagation speeds for different modes...

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VZ-like result

- Special parameter choice: $\beta_2 = \beta_3 = 0$
- Fiducial vierbein flat: $f^m = \delta_\mu{}^m dx^\mu$
- Characteristic matrix written for $\xi_\mu = (1, 0, 0, 0)$

$$\begin{pmatrix} \mathbf{1}_{30\times 30} & 0 & 0 \\ 0 & \boldsymbol{f}_m & 0 \\ 0 & 2\epsilon_{mnrs}\boldsymbol{e}^n \times \boldsymbol{K}^{rs} & \epsilon_{mnrs}\boldsymbol{e}^r \times \boldsymbol{e}^s \\ 0 & \boldsymbol{f}^n \times \boldsymbol{K}_{nm} & \boldsymbol{f}_m \times \boldsymbol{e}_n \\ 0 & \mathcal{R}_m(\boldsymbol{e}, \boldsymbol{K}) & \mathcal{K}_{mn}(\boldsymbol{e}, \boldsymbol{f}, \boldsymbol{K}) \end{pmatrix}$$

$\rightarrow$ Already different cone sheets seem to exists: spacelike?

VZ-like result

- Electric and magnetic part of $\boldsymbol{K}^{mn}$

$$\boldsymbol{E}^a := \boldsymbol{K}^{0a} \quad \text{and} \quad \boldsymbol{B}_a := \epsilon_{abc} \boldsymbol{K}^{bc}$$

- Background choice $\boldsymbol{e}^0 = \boldsymbol{E}^a = 0$
- Characteristic determinant ("roughly")

$$(\boldsymbol{B}^a \cdot \tilde{\mathrm{e}}^b)(\boldsymbol{B}_{[a} \cdot \tilde{\mathrm{e}}_{b]}) + \frac{1}{2}(\boldsymbol{B}^a \cdot \tilde{\mathrm{e}}_{[a})(\boldsymbol{B}^b \cdot \tilde{\mathrm{e}}_{b]}) - 6 m_{\mathrm{FP}}^2 \left((\boldsymbol{f}^a \cdot \tilde{\mathrm{e}}_a) - 4\right)$$

where $\tilde{\boldsymbol{e}}_a$ is the 3-inverse of $\boldsymbol{e}^a$

\+ VZ boosting argument

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Conclusion

- dRGT covariant constraint analysis
- Characteristic matrix computation in non-linear theory = Characteristic matrix computation for linearized theory about some solution
- Analogy with Rarita-Schwinger model
- Some backgrounds: non-hyperbolicity, acausality

→ Non-hyperbolic case can be reached dynamically: strong coupling

- When hyperbolic: multiple causal cone sheets...

→ Some modes are superluminal!