Title: Symmetries in large scale structure

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Abstract:

~~Symmetries~~ in large scale structure

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Columbia University

Outline:

1. Equivalence principle: a generic test of modified gravity
   - with Alberto Nicolis.

2. Parity: symmetry in the measurement of LSS
   - with Camille Bonvin & Enrique Gaztanaga.

3. Dilation & beyond: symmetry in the theory of LSS
   - with Kurt Hinterbichler & Justin Khoury,
     Walter Goldberger & Alberto Nicolis,
     Creminelli, Gleyzes, Simonovic & Vernizzi,
     Bart Horn, & Xiao Xiao.

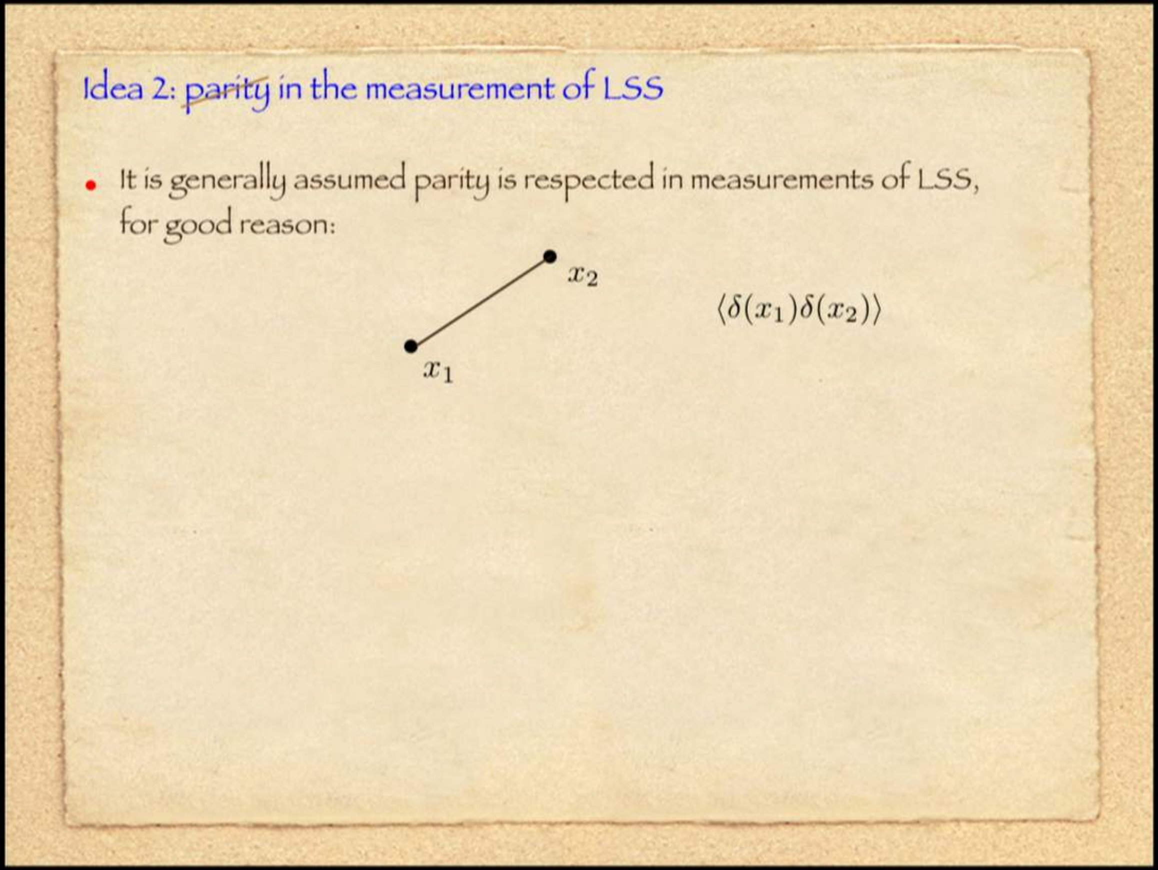
Summary 1:

Test for presence of extra (scalar) forces by looking for off-centered black holes.

Footnote: No hair theorem for galileons (LH, Nicolis).

Footnote 2: The case of massive gravity (Gruzinov & Mirbabayi, Berezhiani, Chkareuli, de Rahm, Gabadadze, Tolley).

Footnote 3: Analogs for chameleon mechanism (Khoury, Weltman; Hu; Jain, Vanderplas; Pourhasan, Afshordi, Mann, Davis; Cabre, Vikram, Zhao, Jain, Koyama; LH, Nicolis, Stubbs).



Idea 2: ~~parity~~ in the measurement of LSS

- It is generally assumed parity is respected in measurements of LSS, for good reason:

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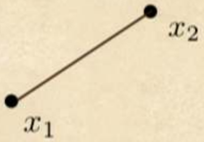

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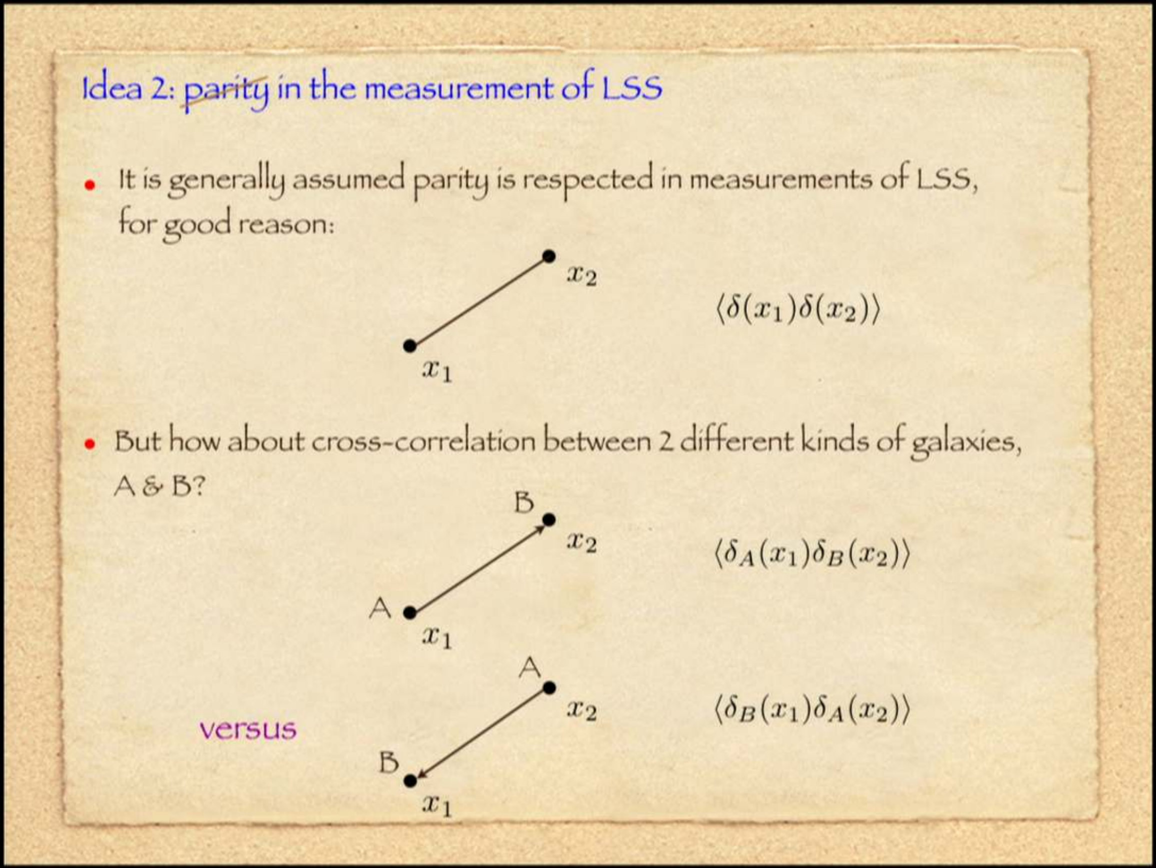

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$\langle\delta(x_1)\delta(x_2)\rangle$

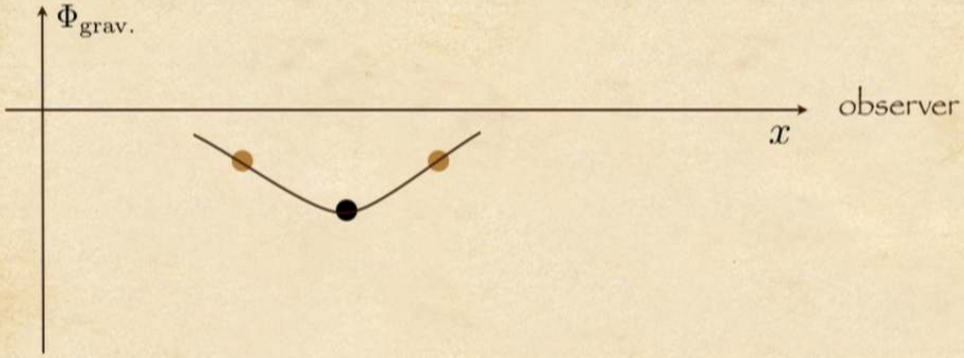
- But how about cross-correlation between 2 different kinds of galaxies, A & B?

$\langle\delta_A(x_1)\delta_B(x_2)\rangle$

versus

$\langle\delta_B(x_1)\delta_A(x_2)\rangle$

What could possibly violate parity?



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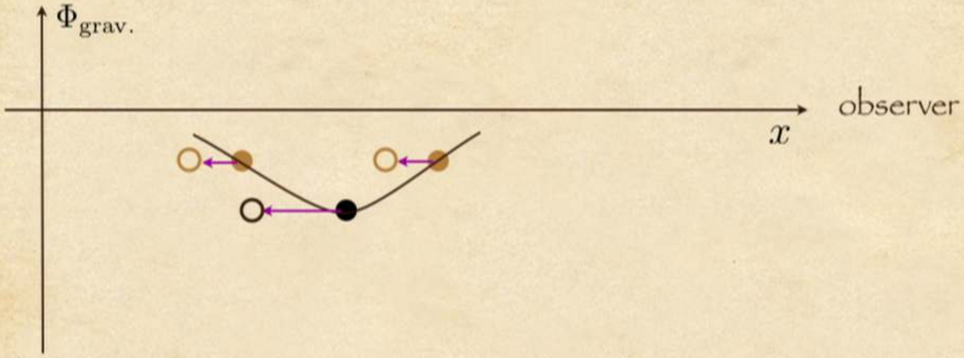

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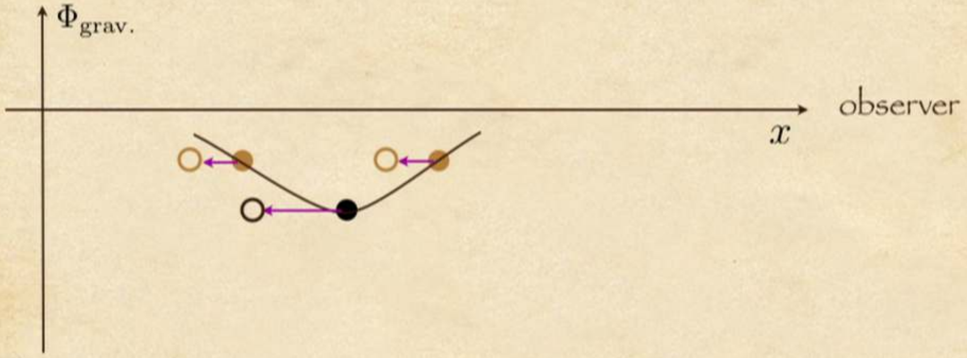

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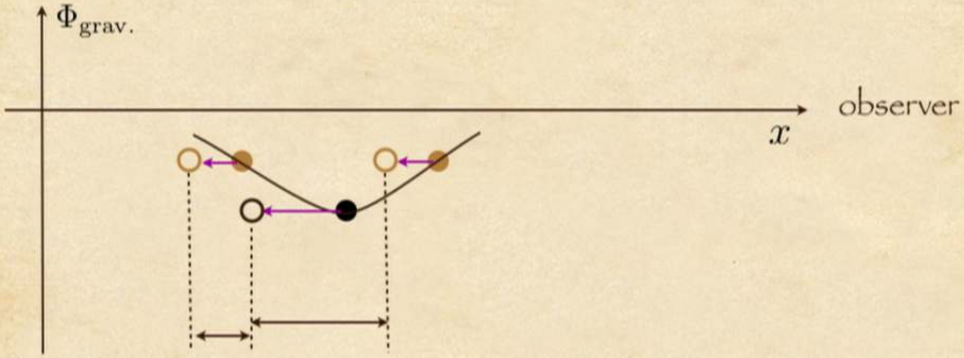

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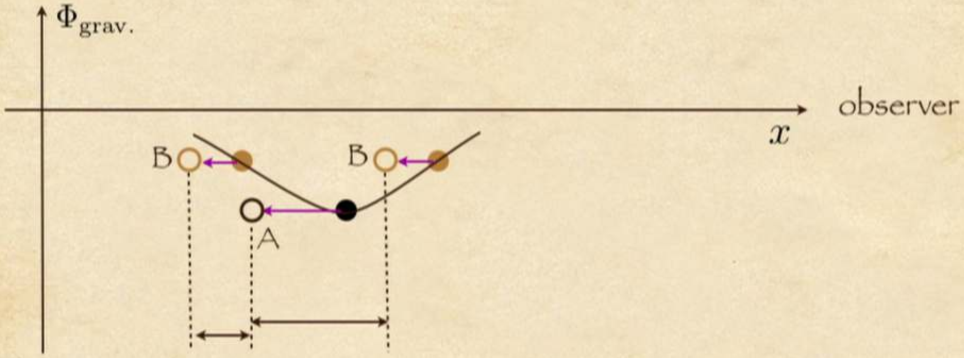
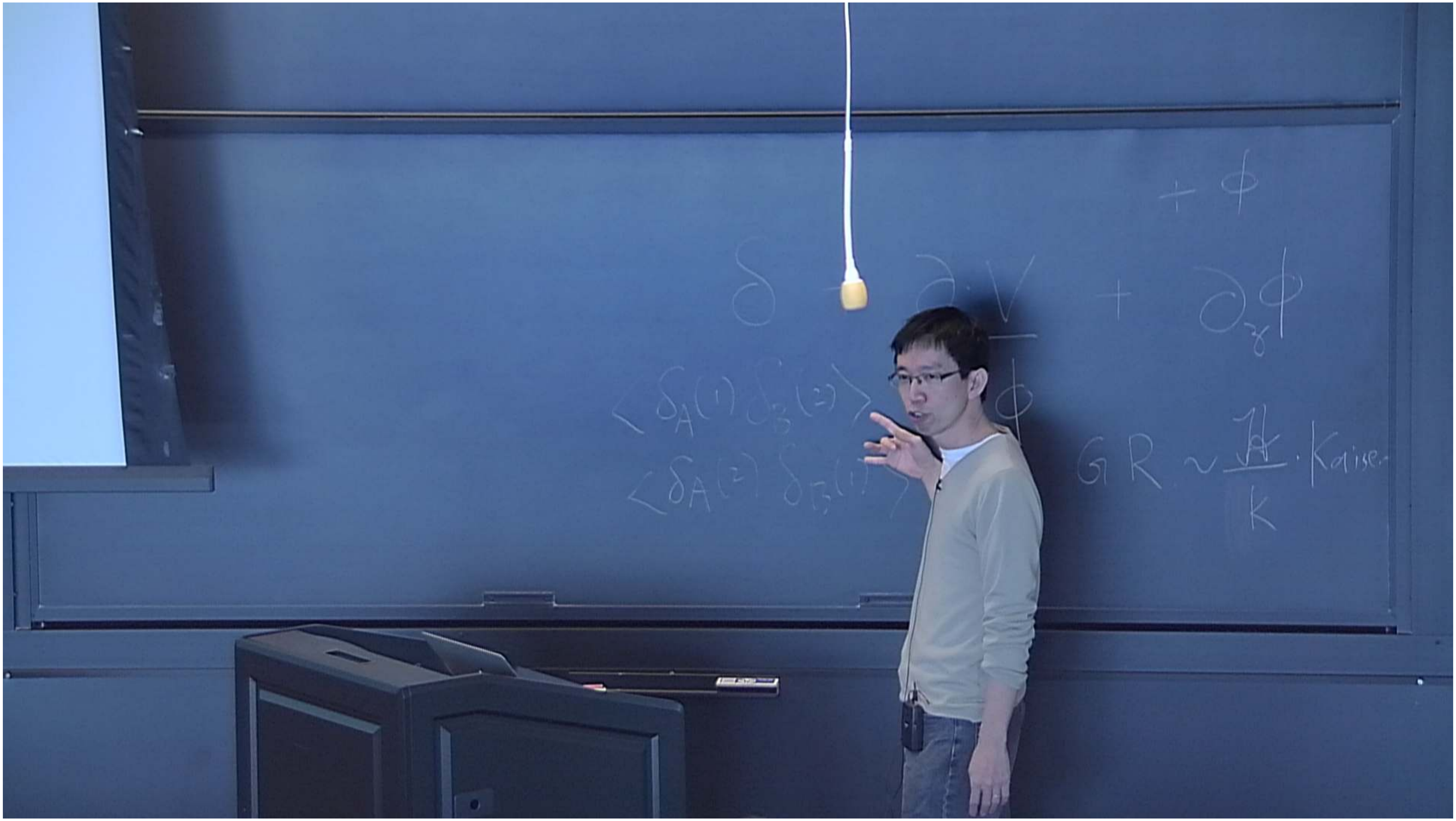



$\partial \quad \frac{\partial V}{\nabla^2 \phi} + \partial \phi$
$GR \sim \frac{k}{H} \cdot k$


+ φ
δ + ∂·V/φ + ∂_γ φ
< δ_A(1) δ_B(2) >
< δ_A(2) δ_B(1) >
GR ~ H/k · Kaiser

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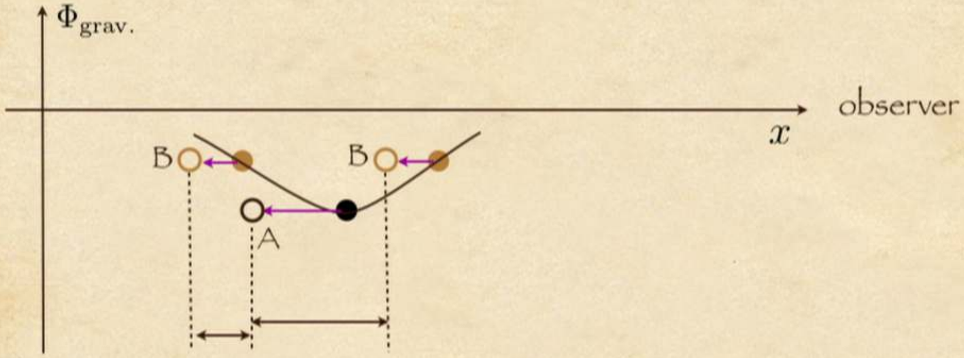

$\xi_{\rm cross}(x_A - x_B) \neq \xi_{\rm cross}(x_B - x_A)$
i.e. whether B is in front of, or behind A, matters.

Wojtak, Hansen & Hjorth - average by stacking clusters

Also: McDonald; Yoo, Hamaus, Seljak & Zaldarriaga; Zhao, Peacock & Li;
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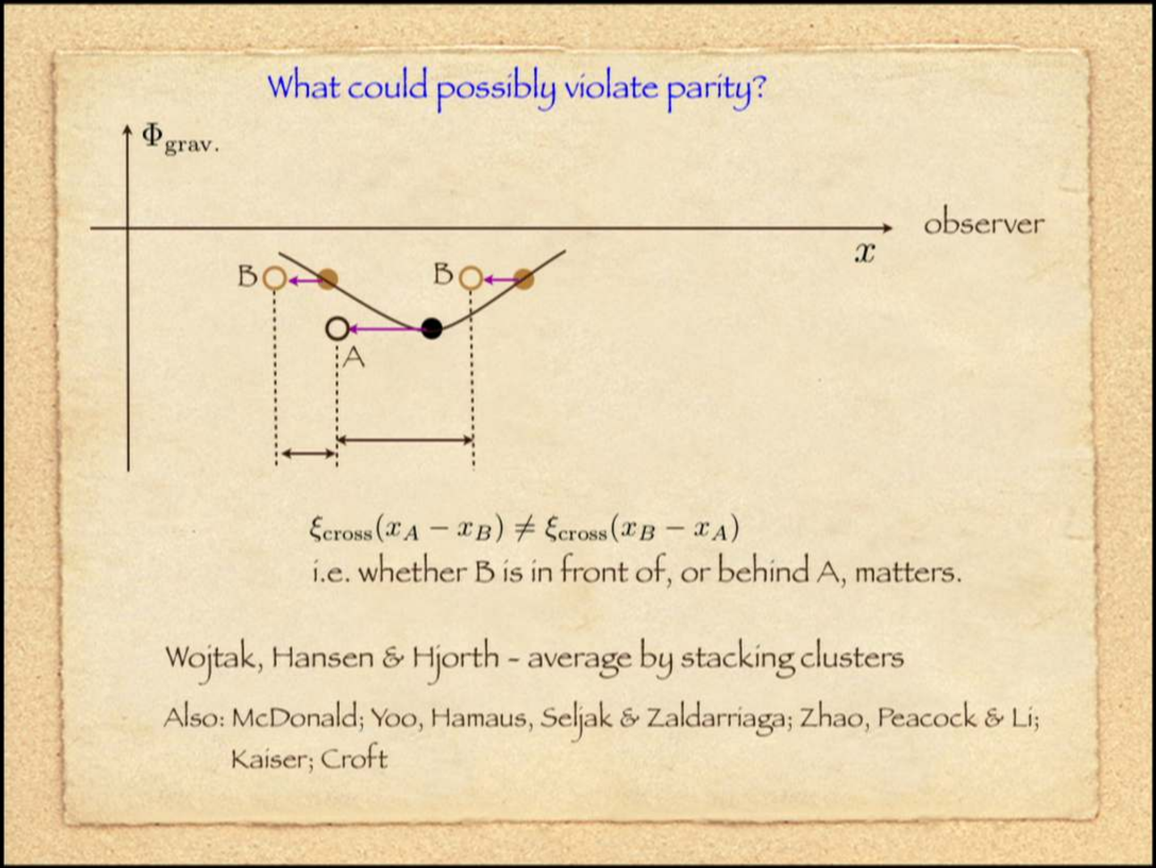

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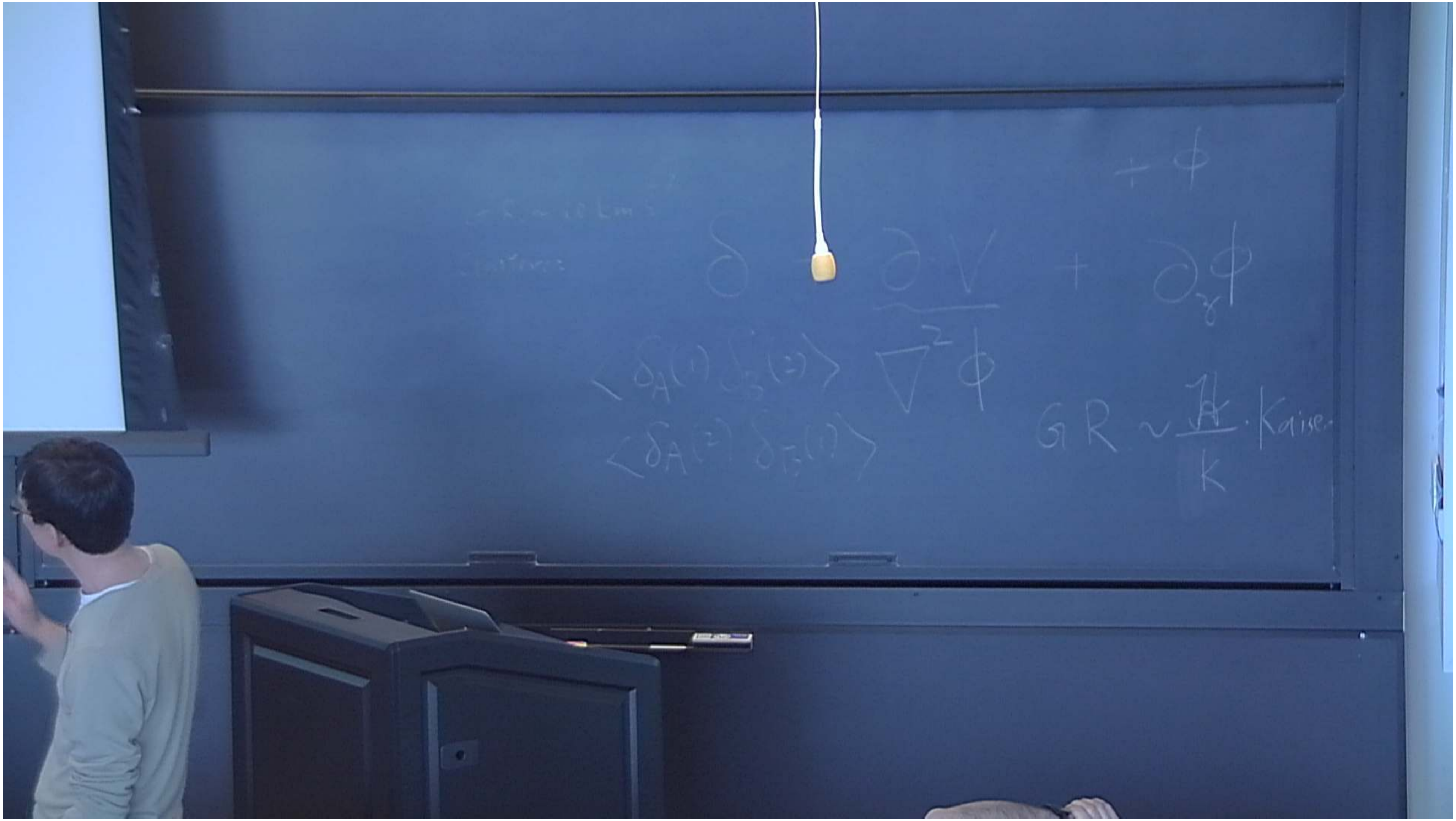


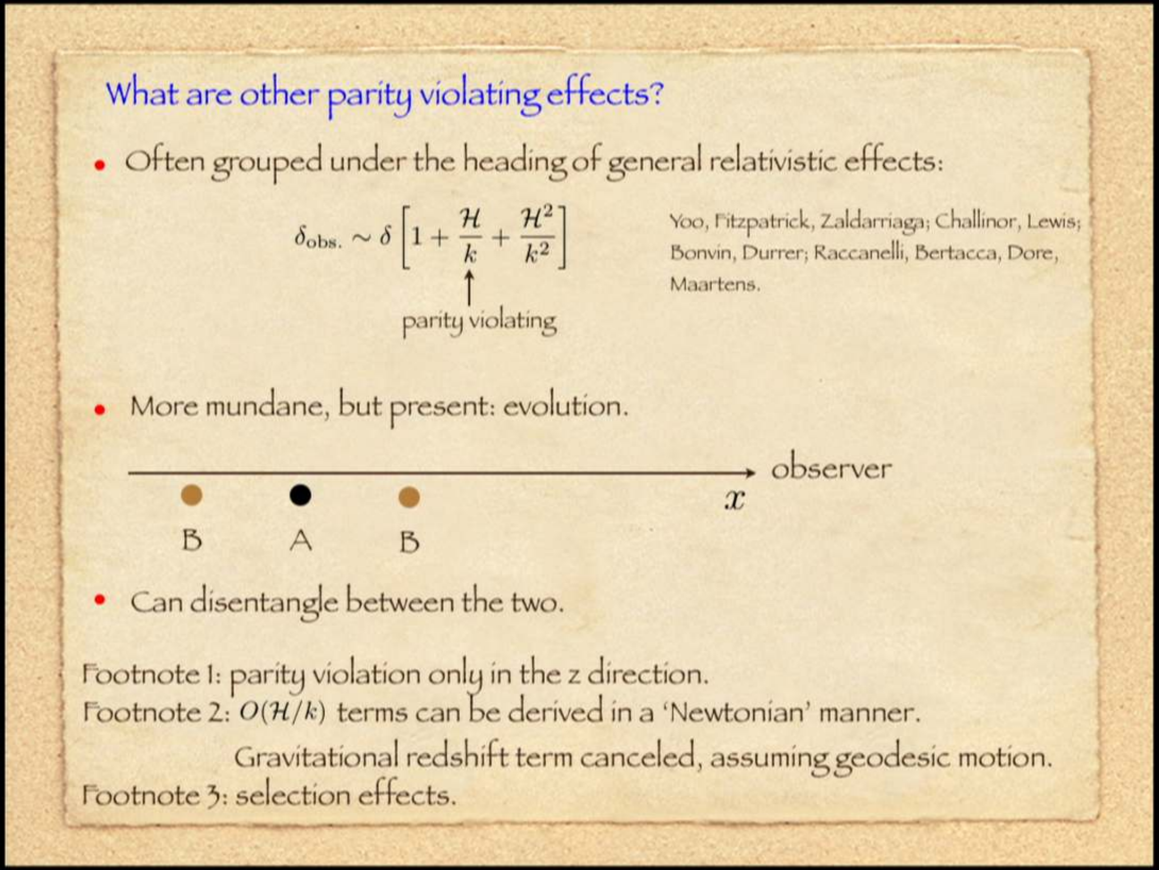
$+ \phi$

$\delta + \underset{\sim}{\partial \cdot V} + \partial_r \phi$

$\langle \delta_A(1)\, \delta_B(2) \rangle \quad \nabla^2 \phi$

$\langle \delta_A(2)\, \delta_B(1) \rangle$

$GR \sim \frac{\mathcal{H}}{k} \cdot$ Kaiser



What are other parity violating effects?

- Often grouped under the heading of general relativistic effects:

$$\delta_{\text{obs.}} \sim \delta\left[1+\underset{\substack{\uparrow\\ \text{parity violating}}}{\frac{\mathcal{H}}{k}}+\frac{\mathcal{H}^2}{k^2}\right]$$

Yoo, Fitzpatrick, Zaldarriaga; Challinor, Lewis; Bonvin, Durrer; Raccanelli, Bertacca, Dore, Maartens.

- More mundane, but present: evolution.

- Can disentangle between the two.

Footnote 1: parity violation only in the z direction.
Footnote 2: $O(\mathcal{H}/k)$ terms can be derived in a 'Newtonian' manner.
Gravitational redshift term canceled, assuming geodesic motion.
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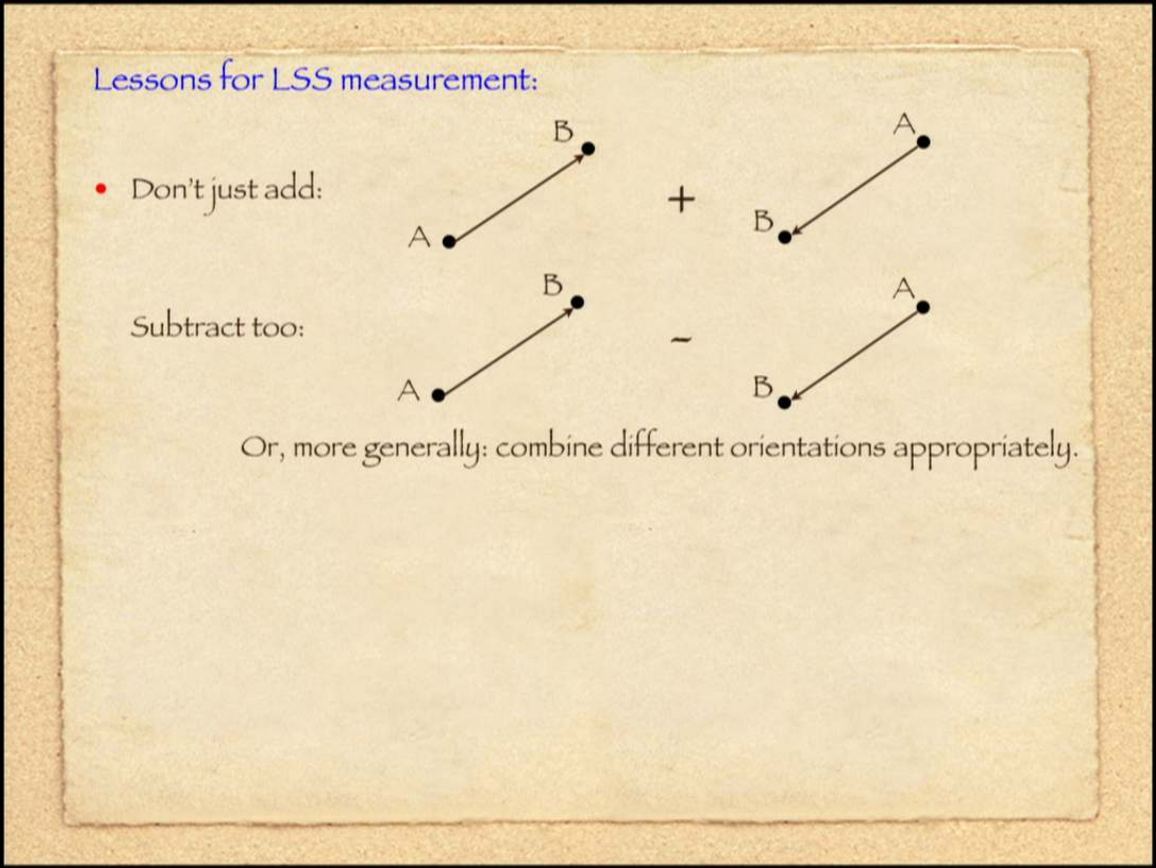

Lessons for LSS measurement:

- Don't just add:

  Subtract too:

Or, more generally: combine different orientations appropriately.

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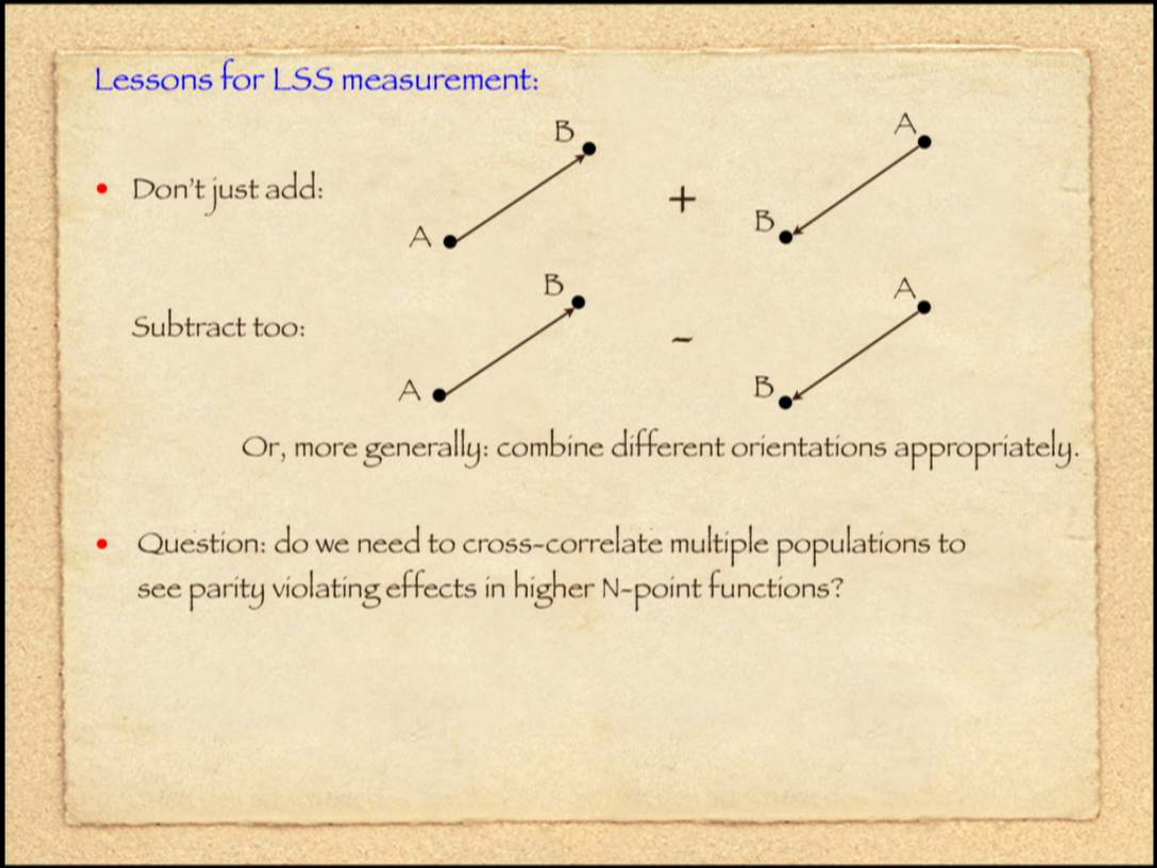

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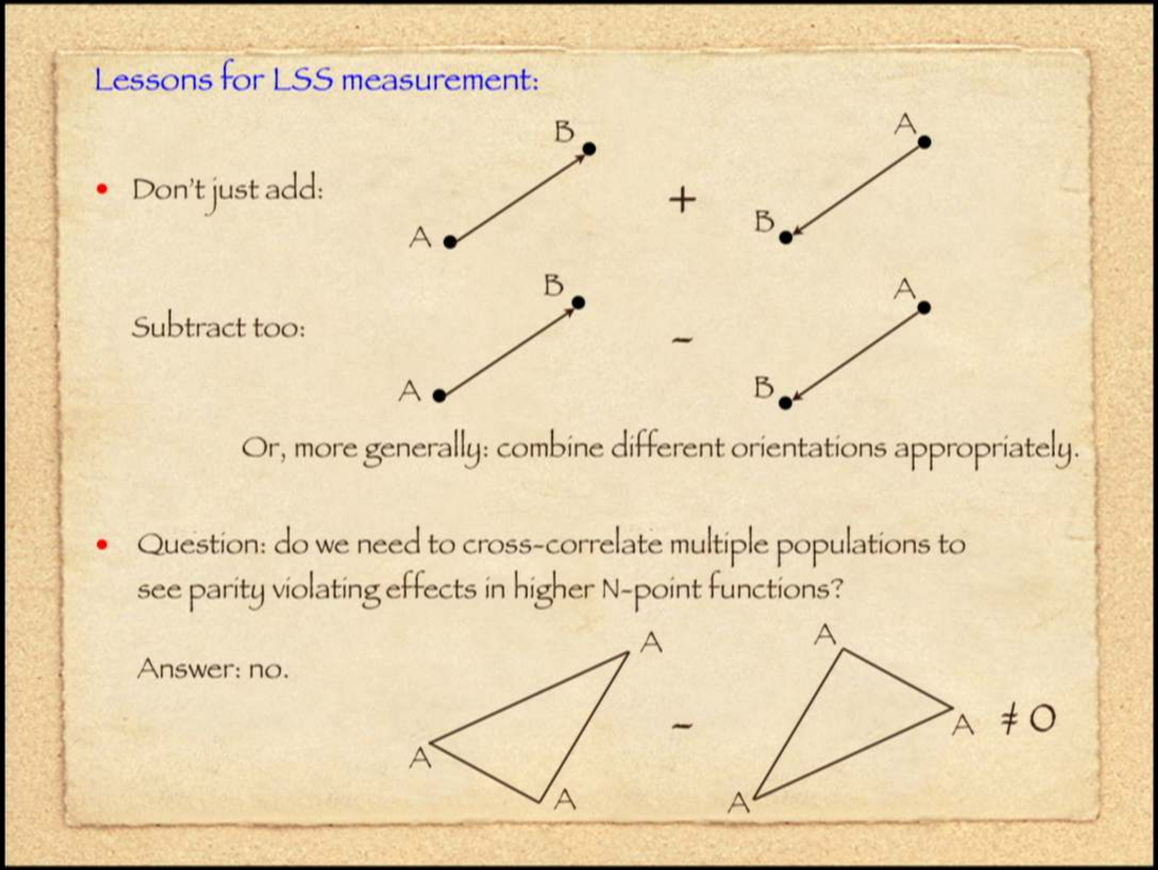

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- Don't just add:

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  Or, more generally: combine different orientations appropriately.

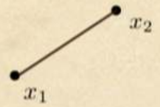
- Question: do we need to cross-correlate multiple populations to see parity violating effects in higher N-point functions?

  Answer: no.

Idea 3: non-perturbative consistency relations in LSS

- 1. Consider a familiar example of symmetry: spatial translation.

$$x \to x + \Delta x \quad , \quad \text{where } \Delta x = \text{const.}$$



Its consequence for correlation function is well known:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle = \langle \phi(x_1+\Delta x)\phi(x_2+\Delta x)\phi(x_3+\Delta x)\rangle$$

For small Δx , we have:

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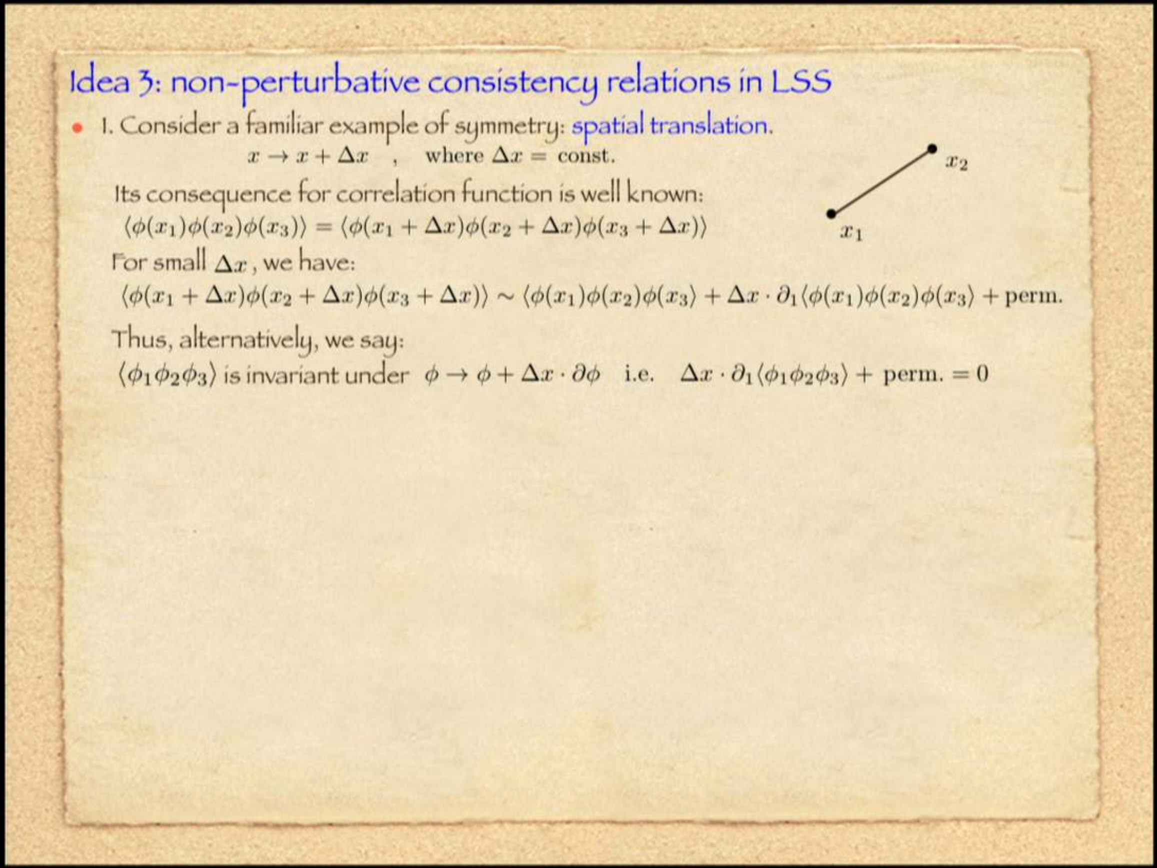
Thus, alternatively, we say:

$\langle \phi_1\phi_2\phi_3\rangle$ is invariant under $\phi \to \phi + \Delta x \cdot \partial\phi \quad \text{i.e.} \quad \Delta x \cdot \partial_1 \langle \phi_1\phi_2\phi_3\rangle + \text{perm.} = 0$



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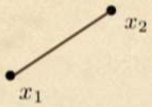

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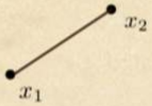

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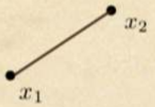

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$$\phi \rightarrow \phi + c \quad , \quad \text{where } c = \text{ const.}$$

For small c , we have:

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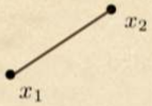

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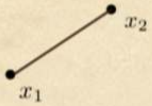

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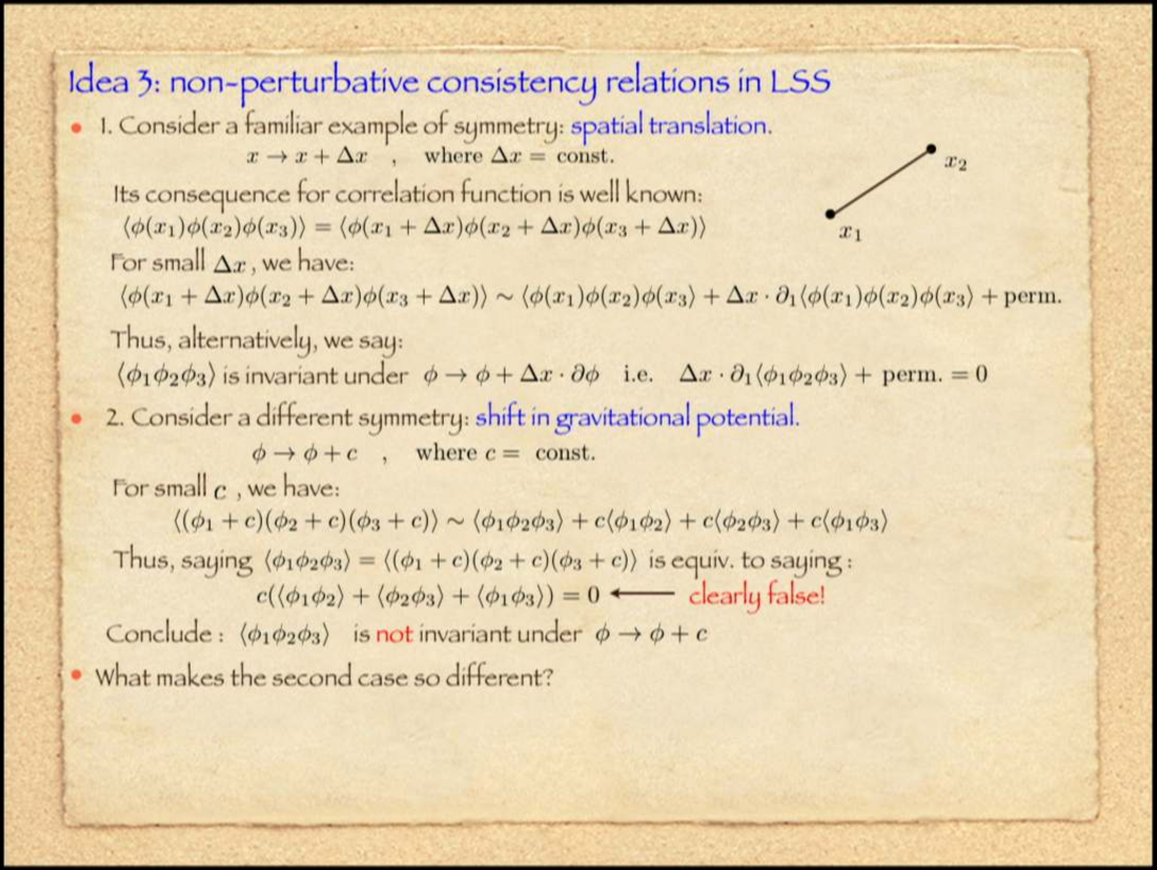

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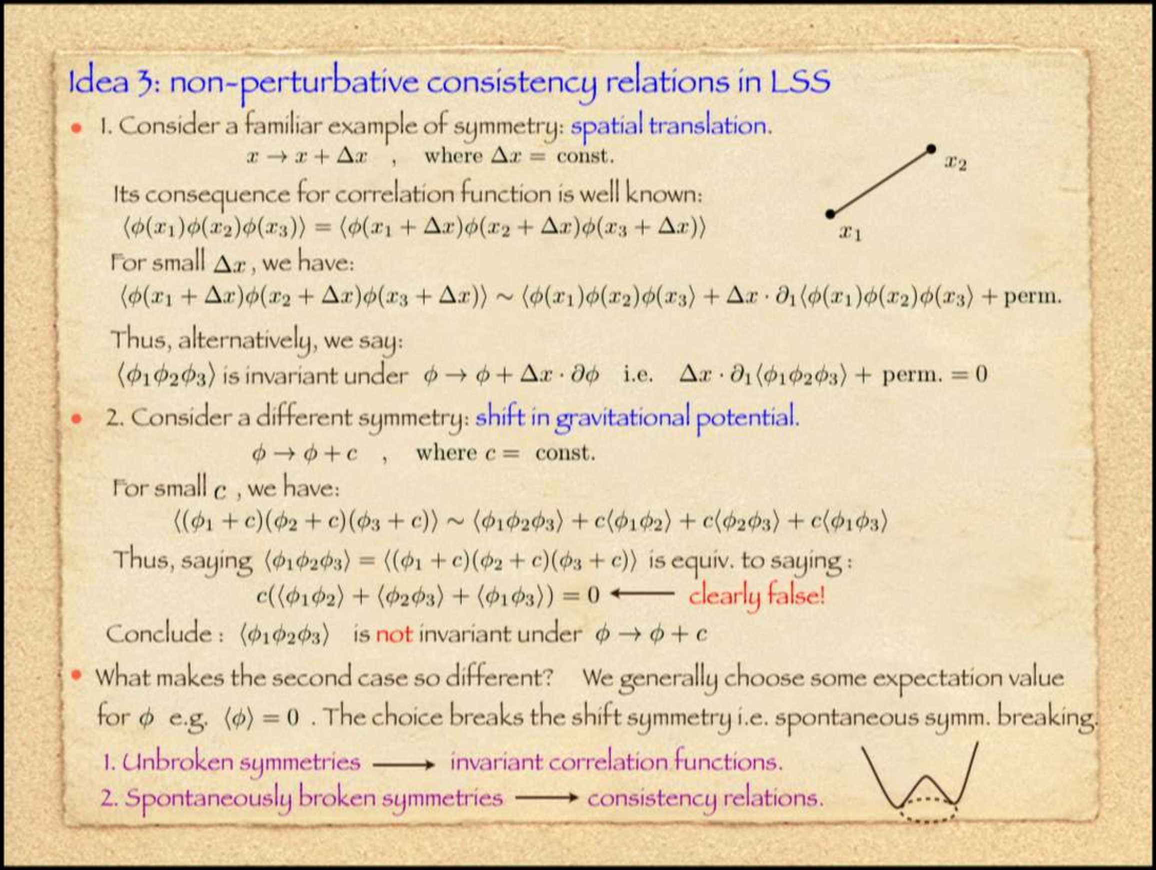

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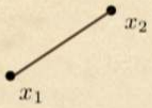

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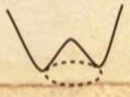

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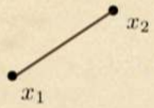

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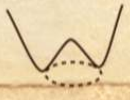

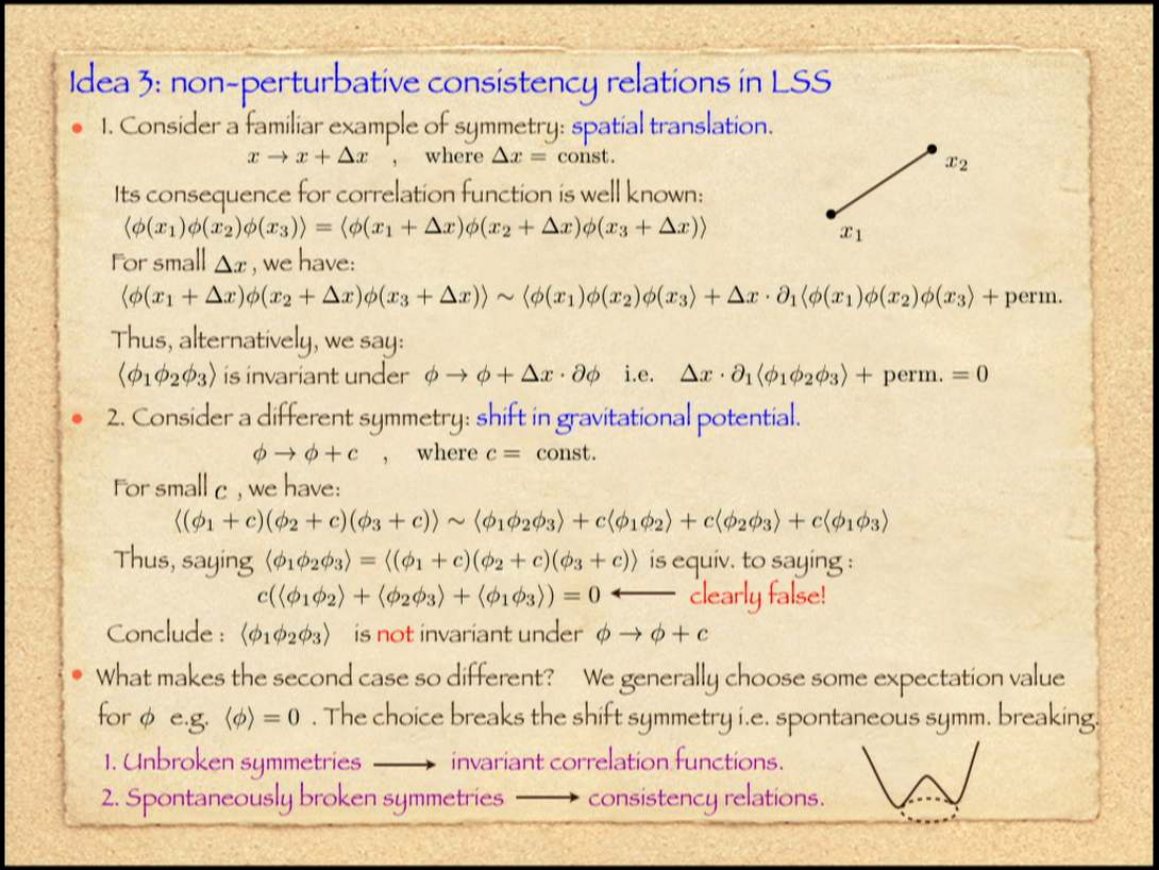

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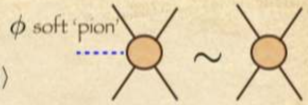
- Schematic form: $\lim_{q\to 0} \frac{1}{P_\phi(q)} \langle \phi(q)\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle \sim \langle \mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle$



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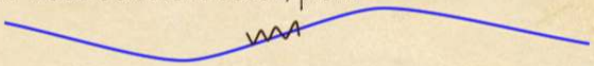

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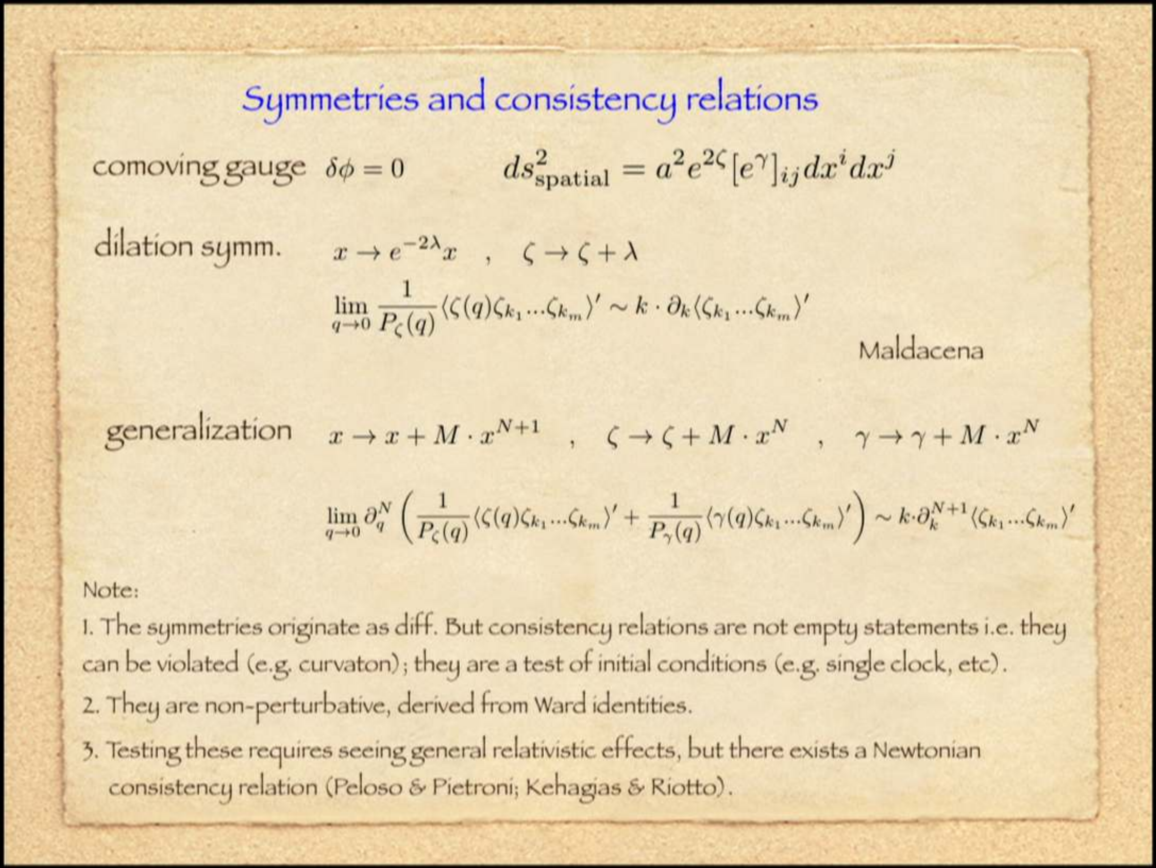

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Maldacena

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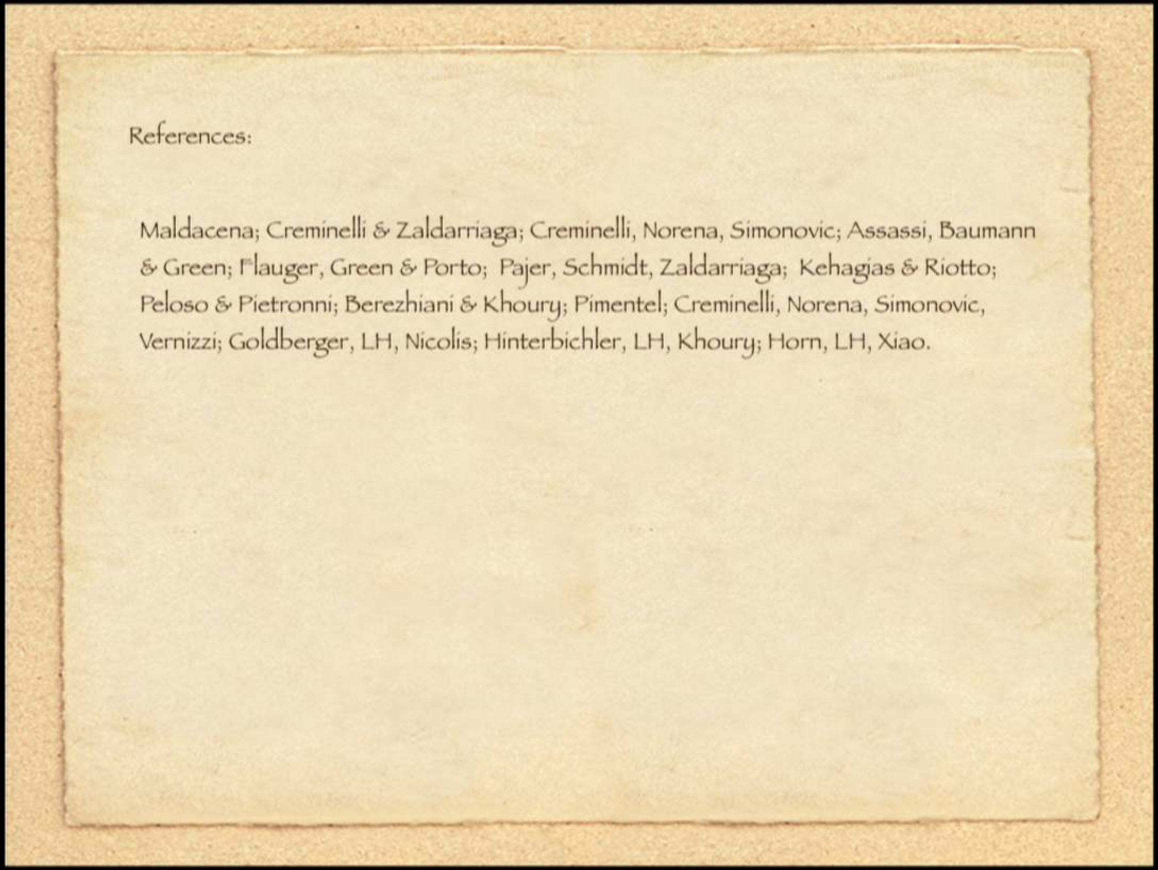
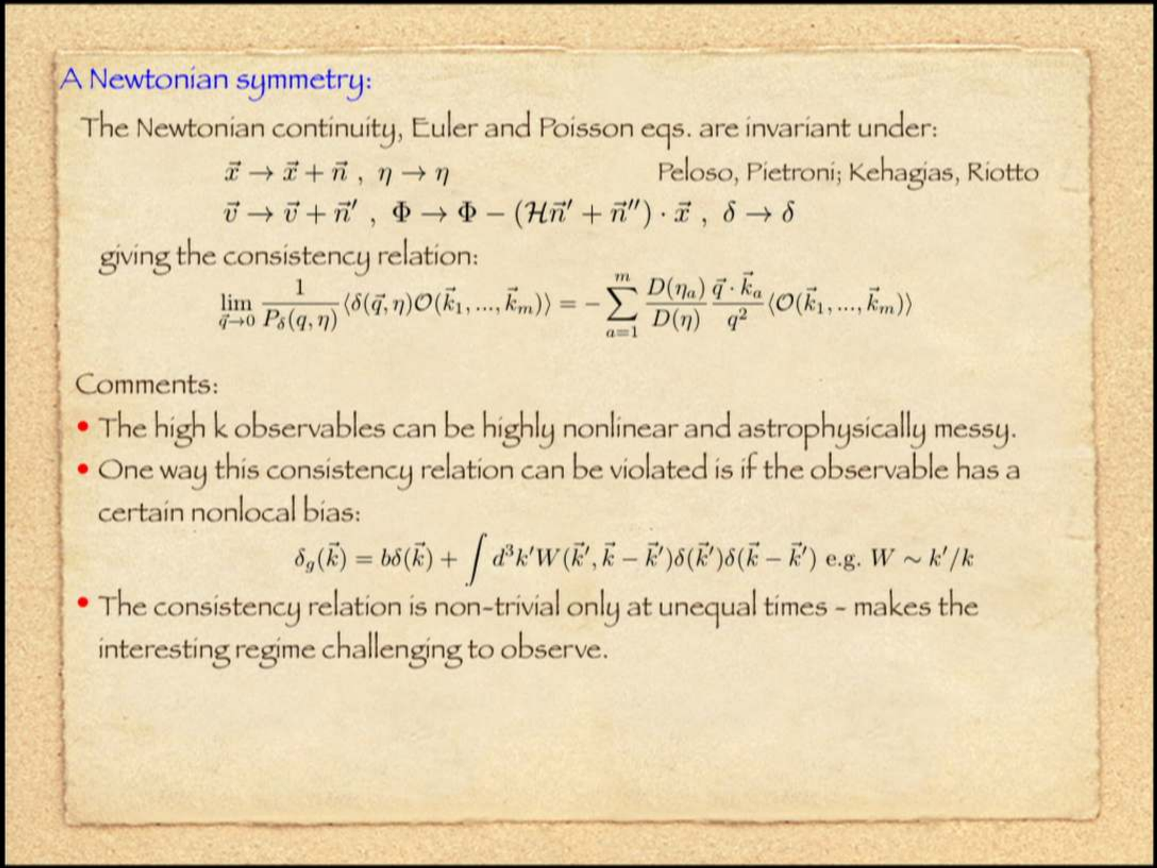


References:

Maldacena; Creminelli & Zaldarriaga; Creminelli, Norena, Simonovic; Assassi, Baumann & Green; Flauger, Green & Porto; Pajer, Schmidt, Zaldarriaga; Kehagias & Riotto; Peloso & Pietronni; Berezhiani & Khoury; Pimentel; Creminelli, Norena, Simonovic, Vernizzi; Goldberger, LH, Nicolis; Hinterbichler, LH, Khoury; Horn, LH, Xiao.



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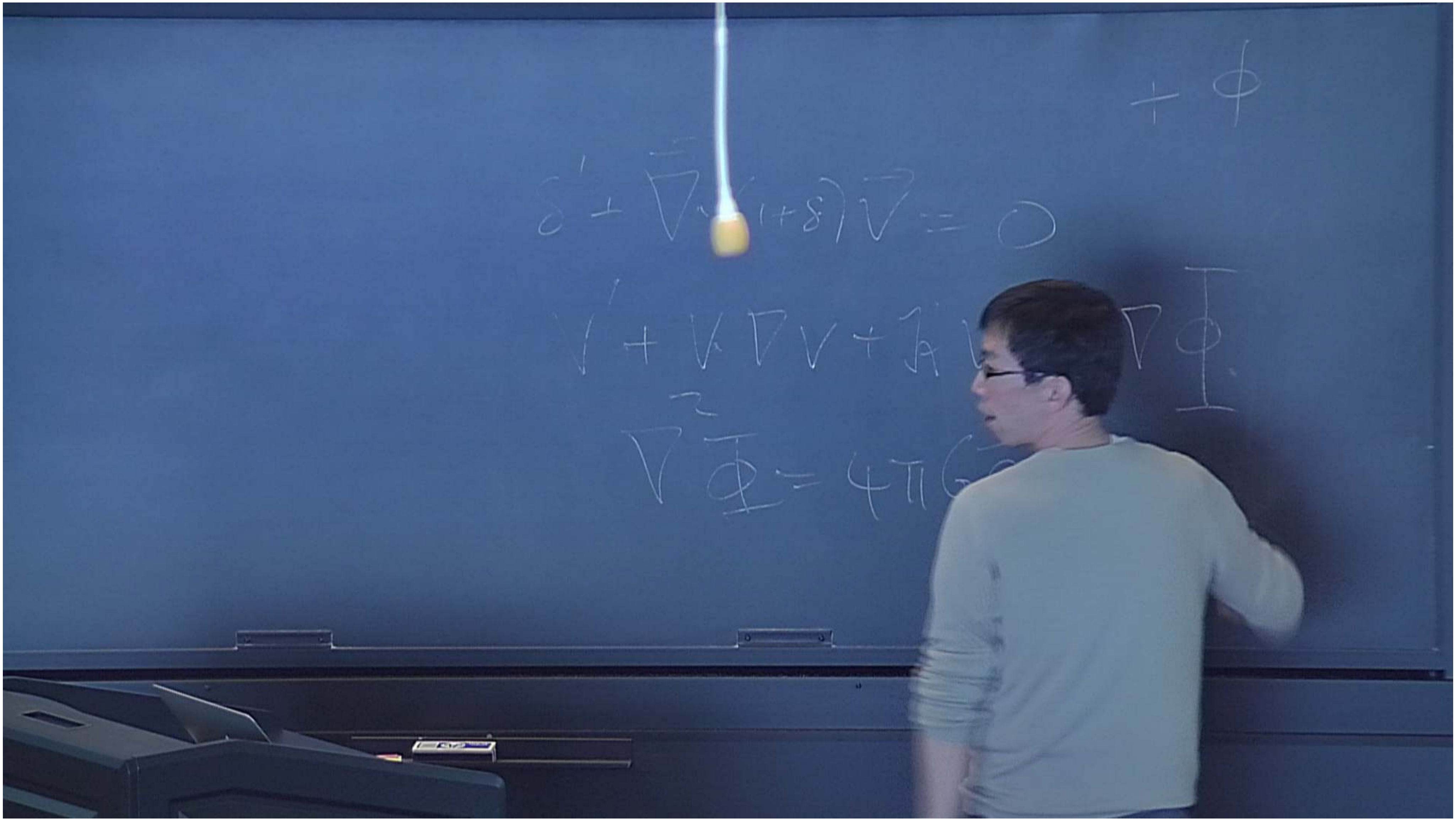
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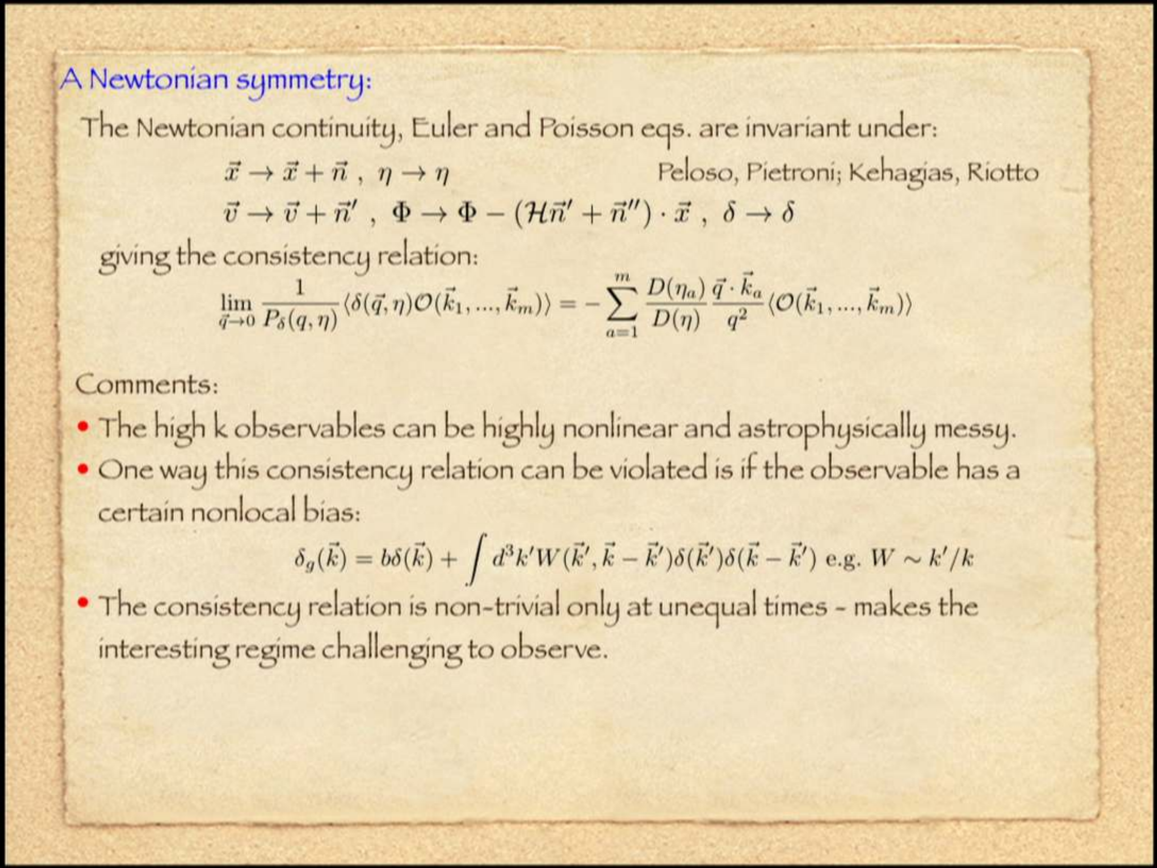

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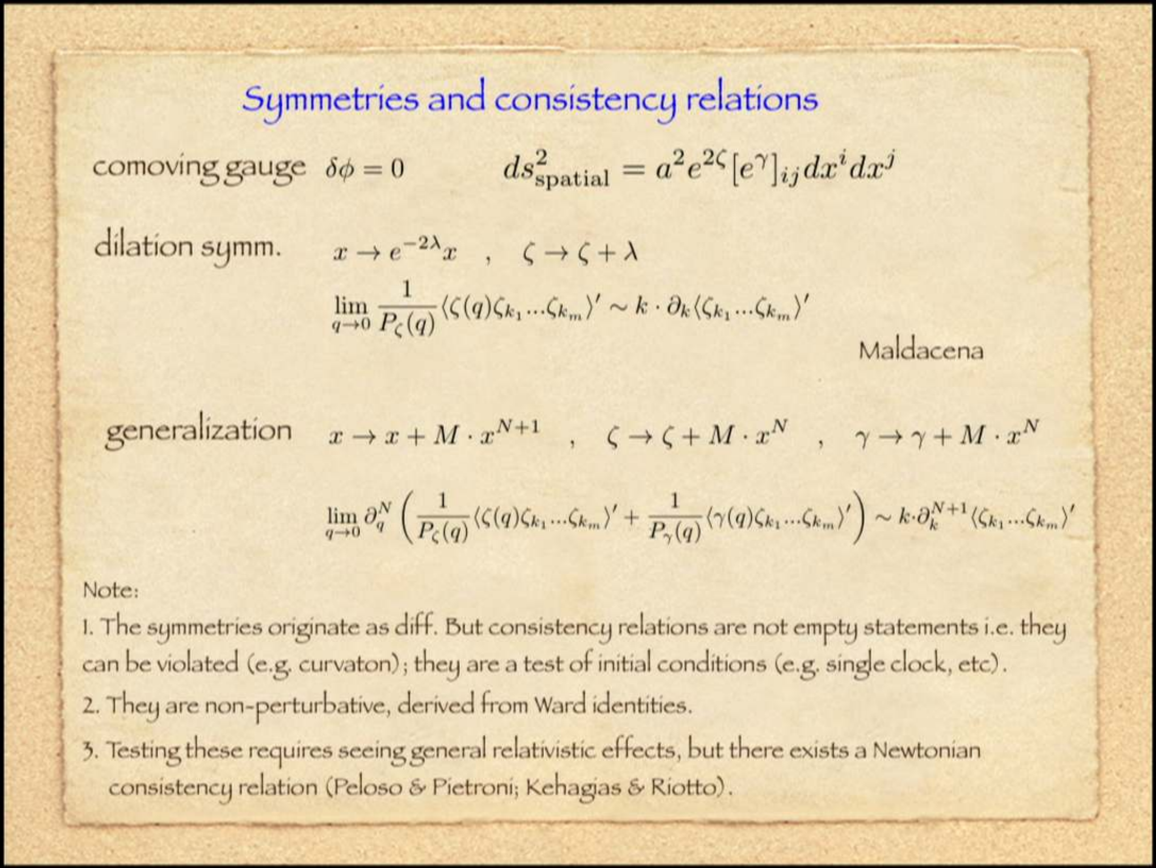

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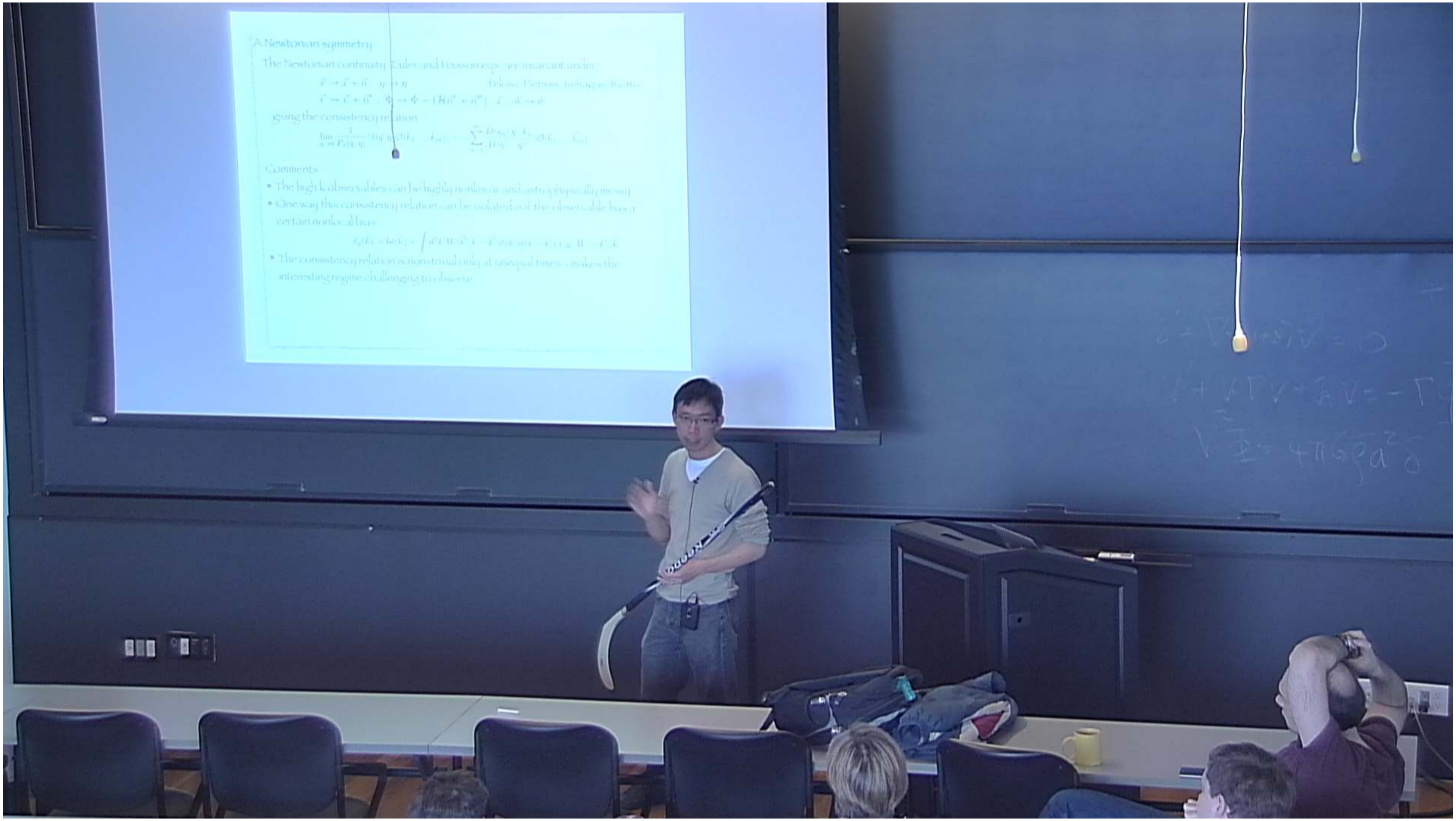

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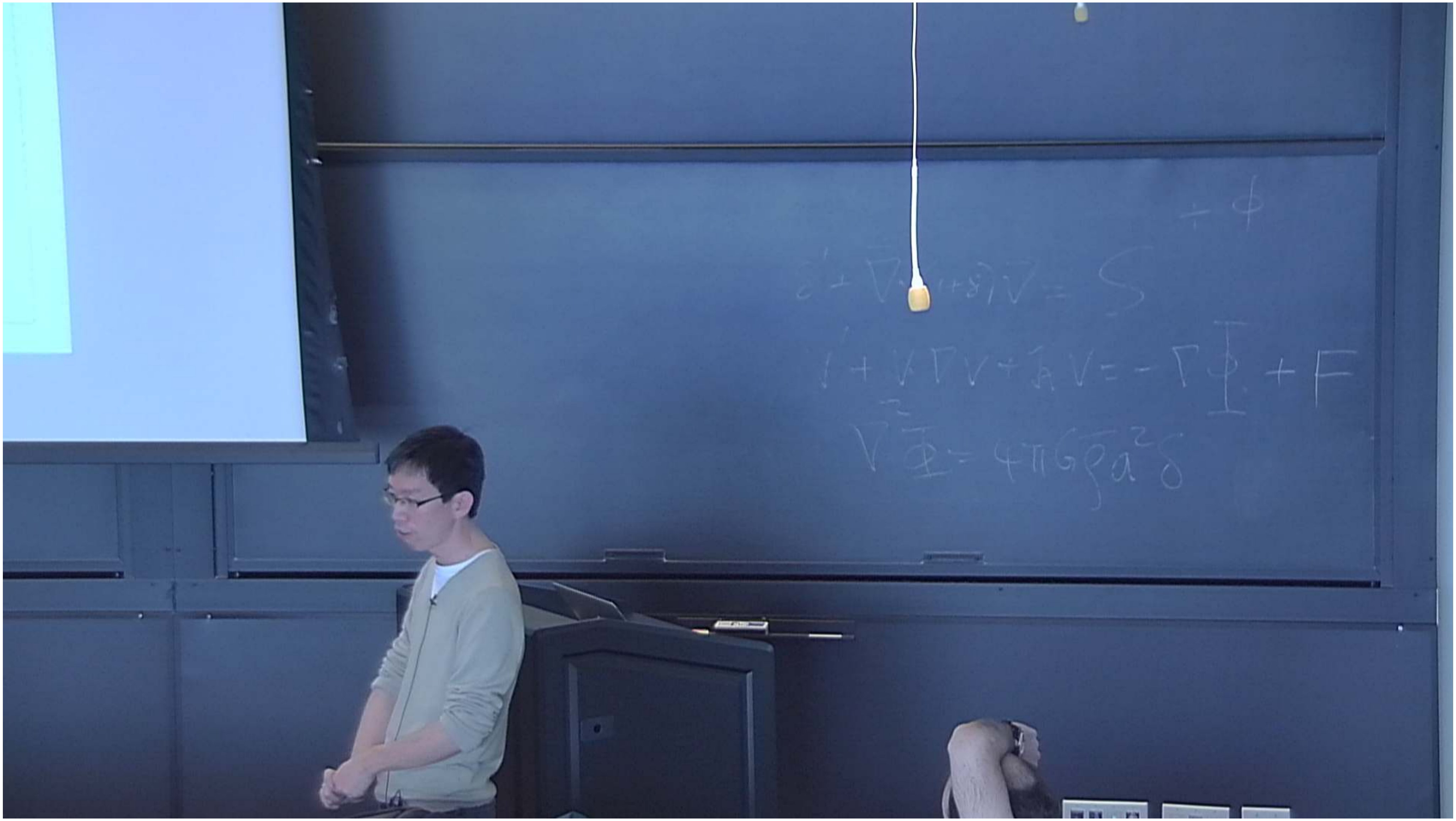

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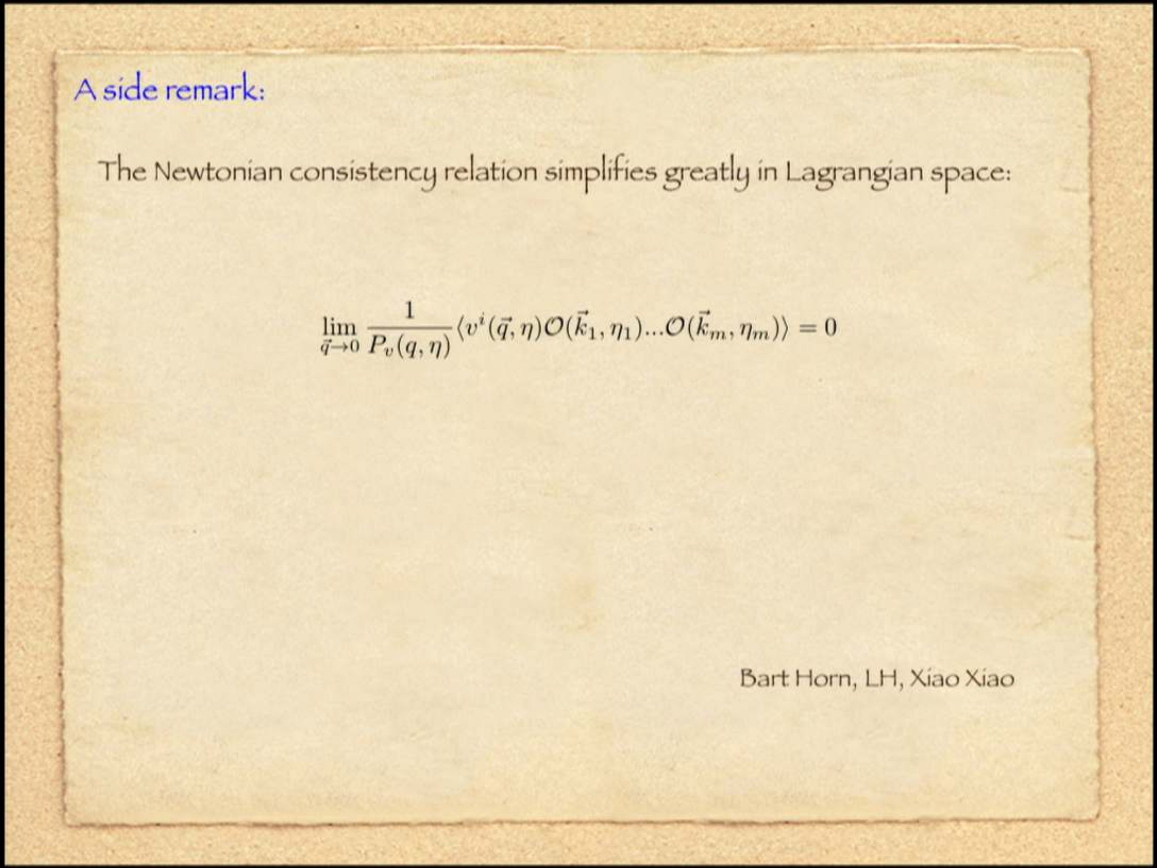

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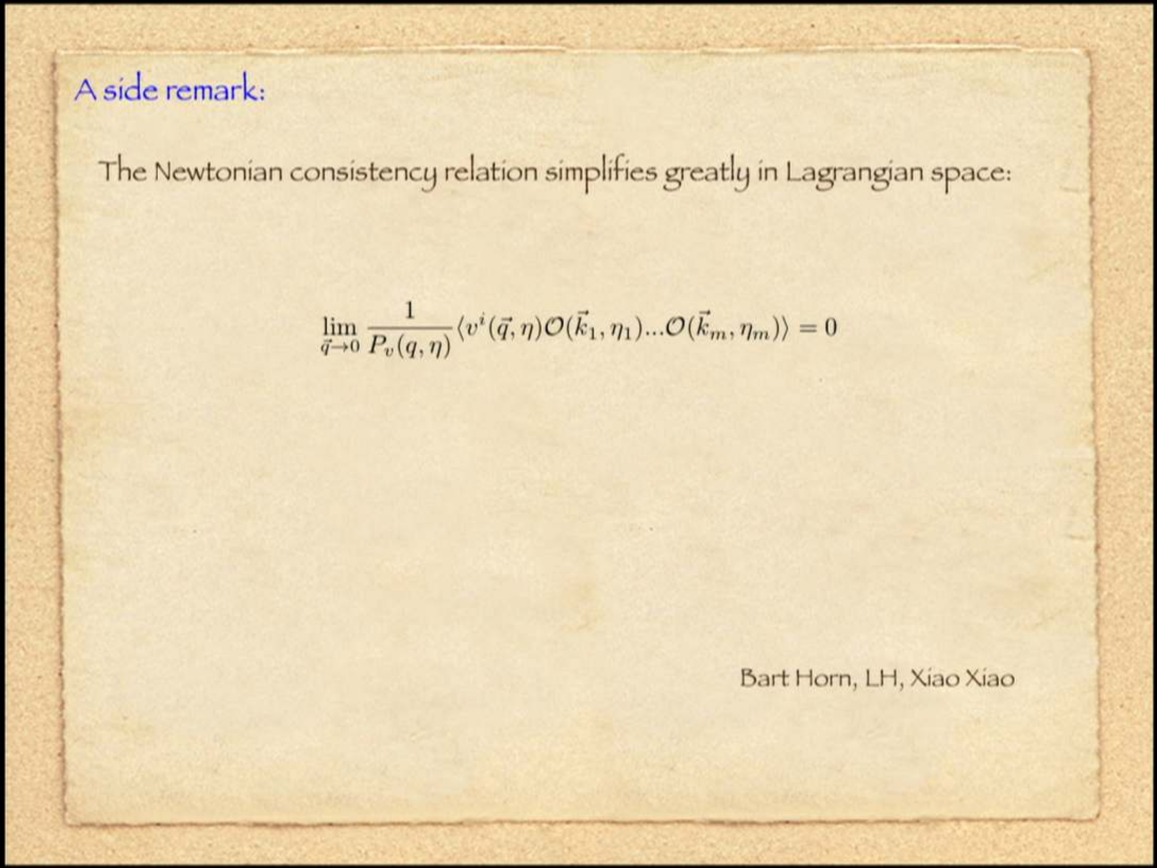

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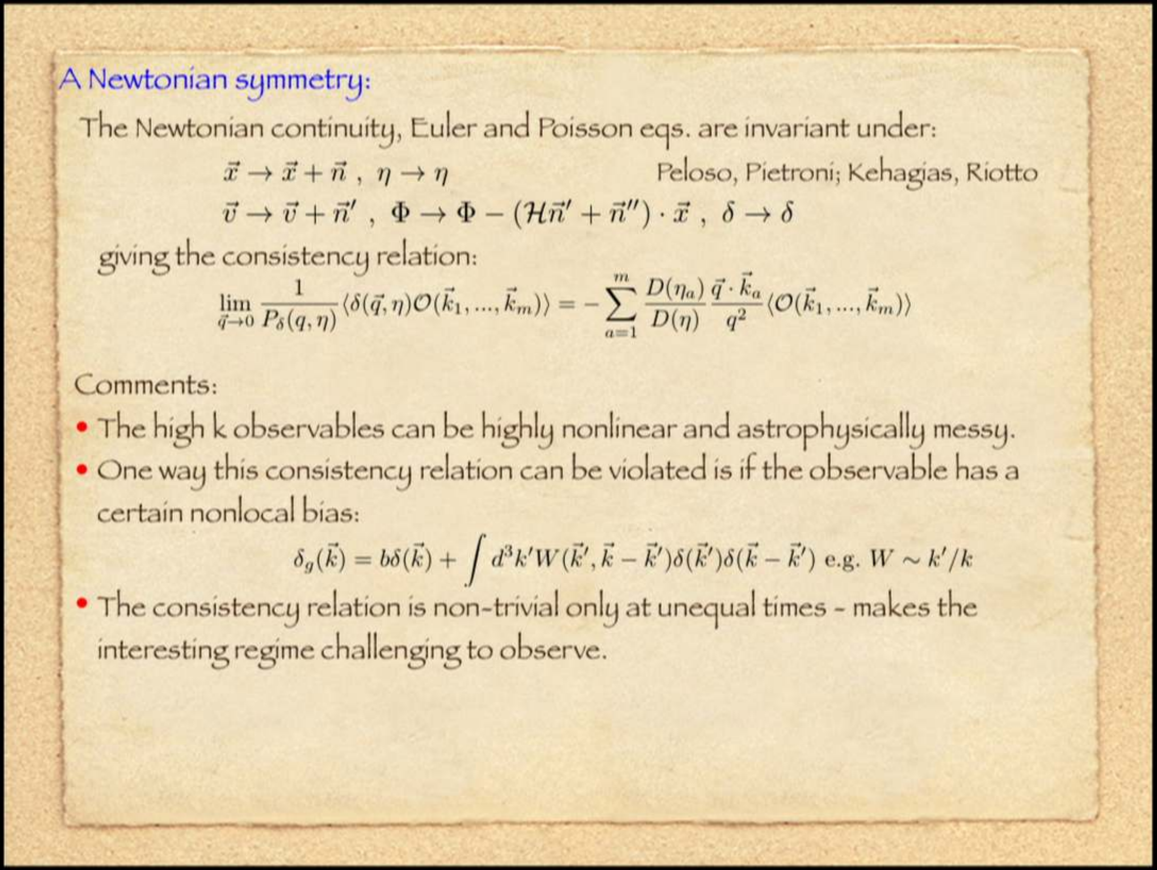

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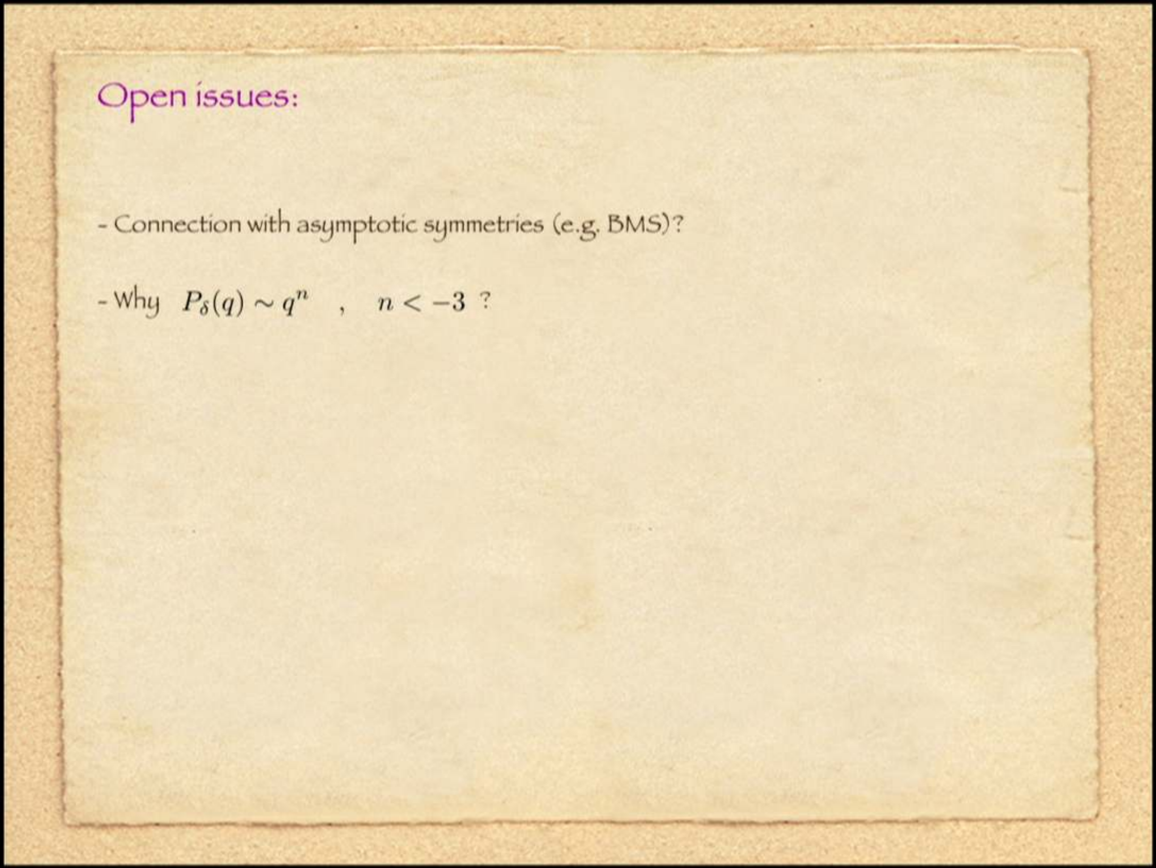
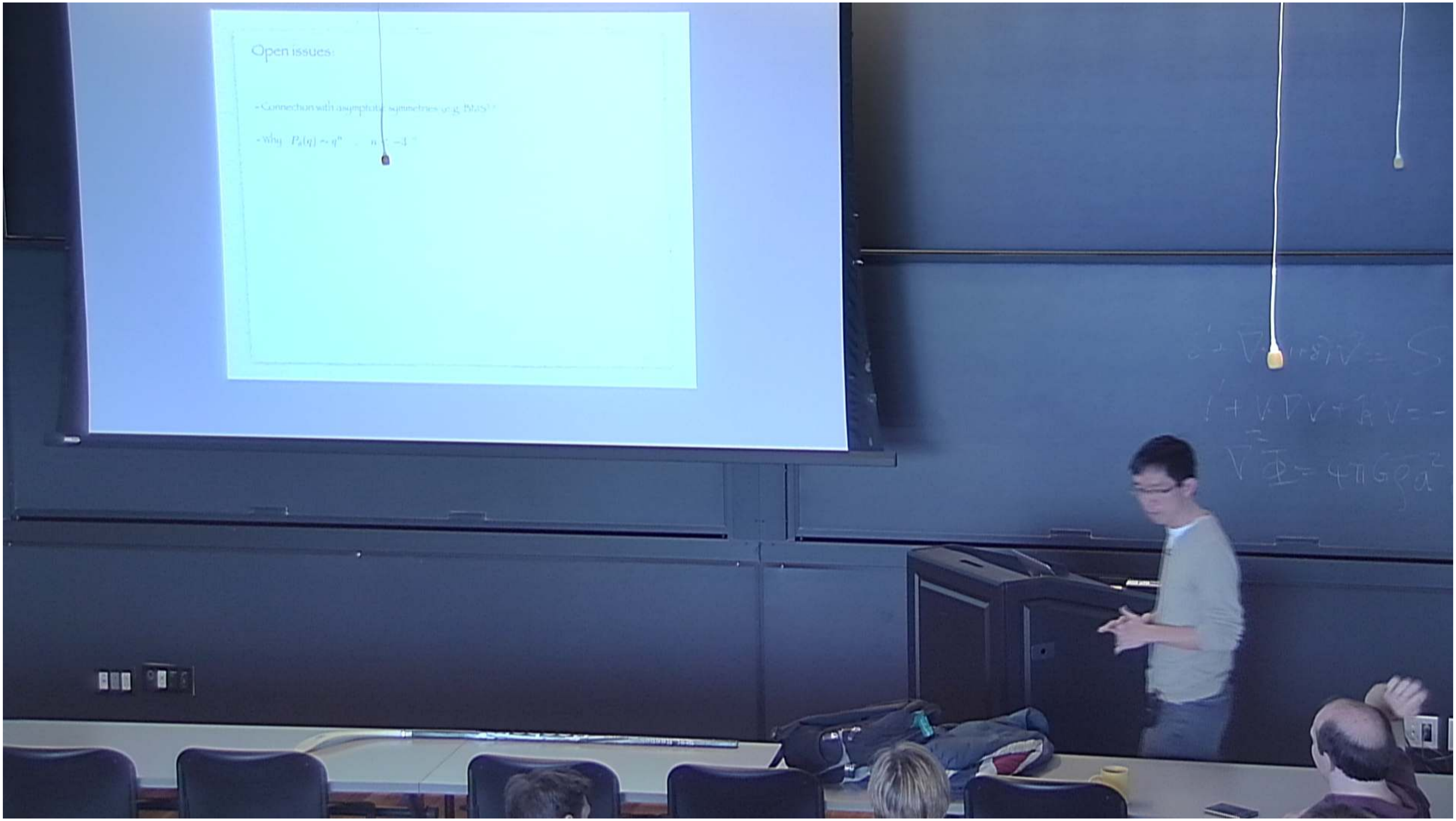


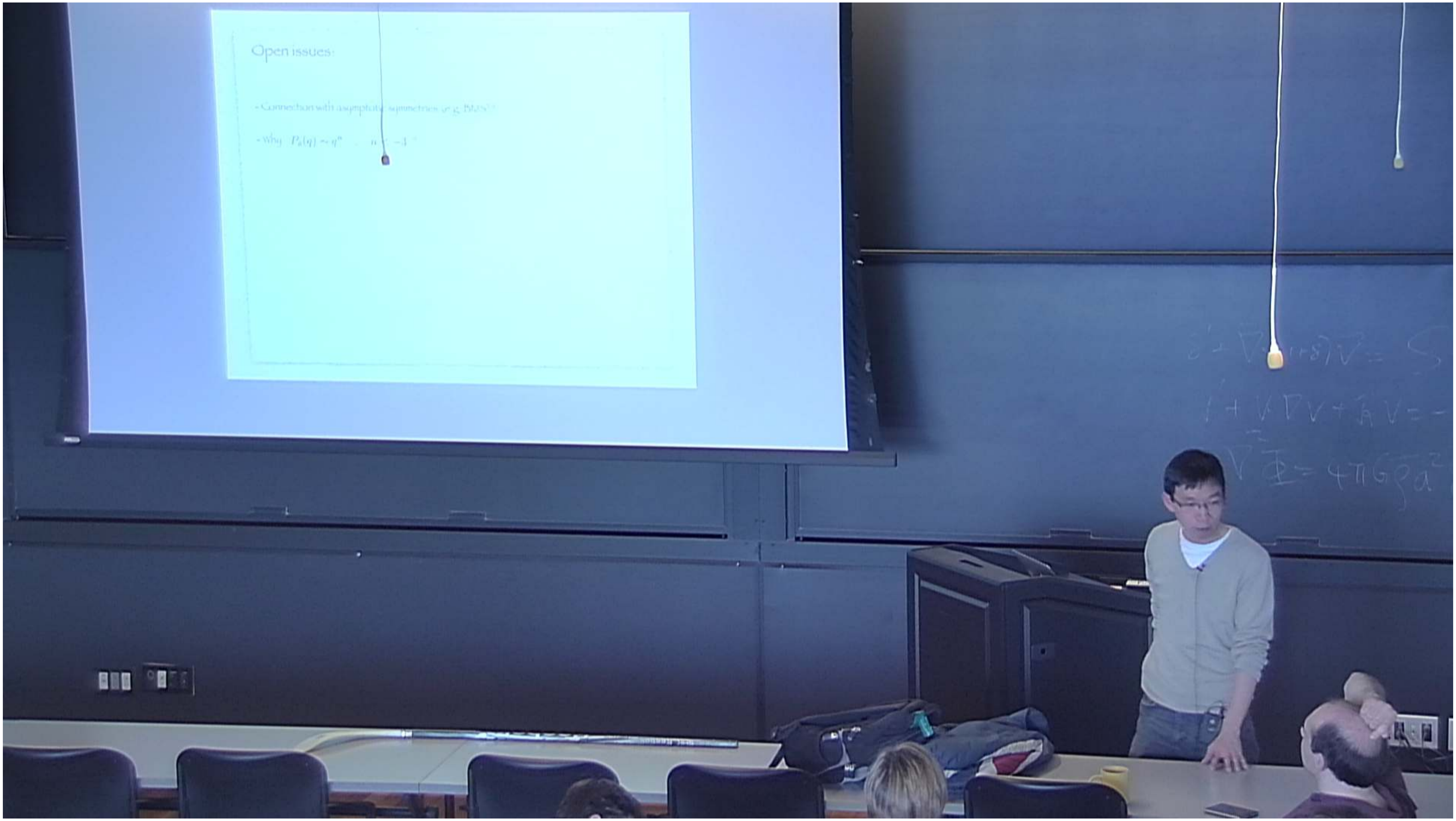
Open issues:

- Connection with asymptotic symmetries (e.g. BMS)?
- Why $P_\delta(q) \sim q^n \quad , \quad n < -3$?



Open issues:

- Connection with asymptotic symmetries (e.g. BMS)?
- Why $P_\delta(q) \sim q^n$, $n \neq -4$?



Open issues:

- Connection with asymptotic symmetries (e.g. BMS)?

- Why $P_\phi(q) \sim q^n$, $n \leq -4$?