Title: Symmetries in large scale structure

Date: Apr 13, 2015 11:00 AM

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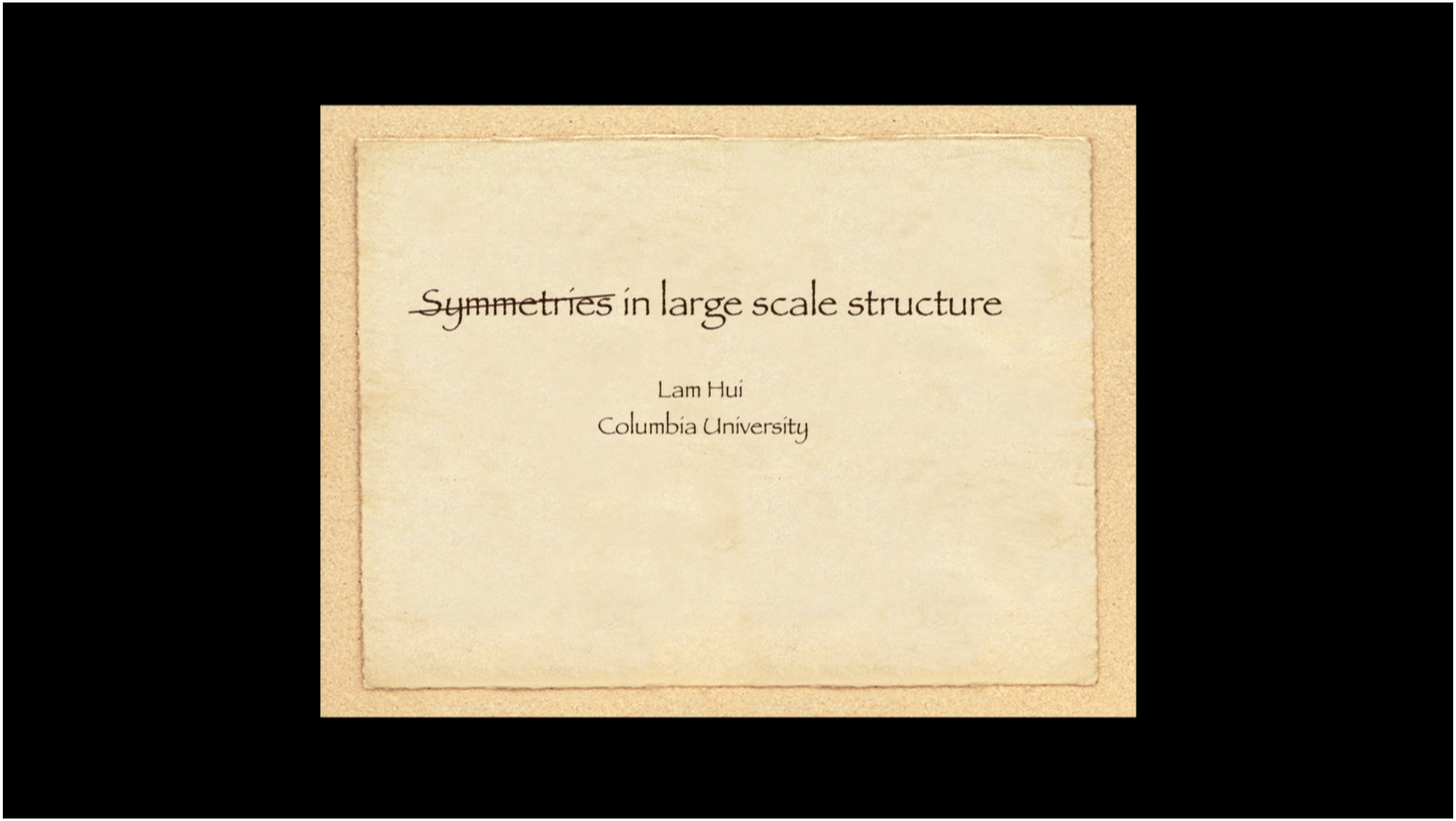
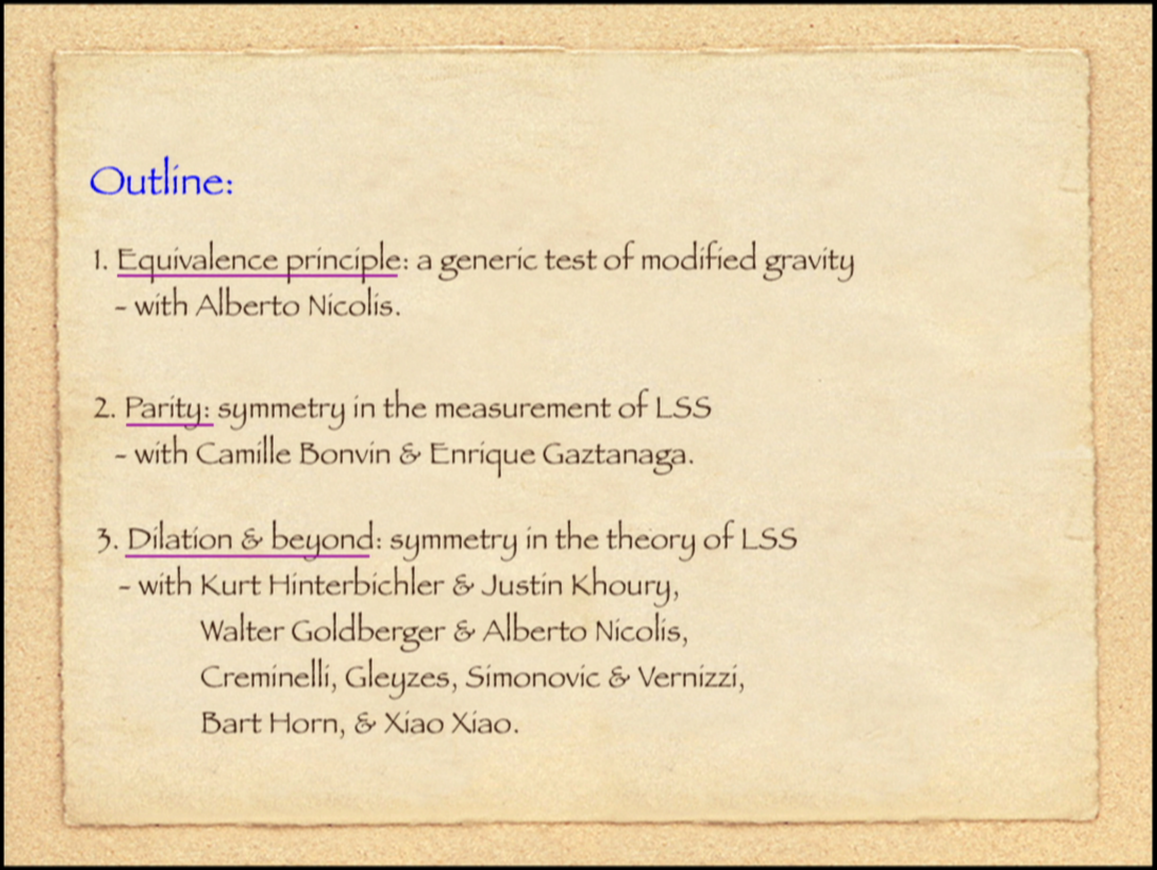
Abstract:



~~Symmetries~~ in large scale structure

Lam Hui
Columbia University

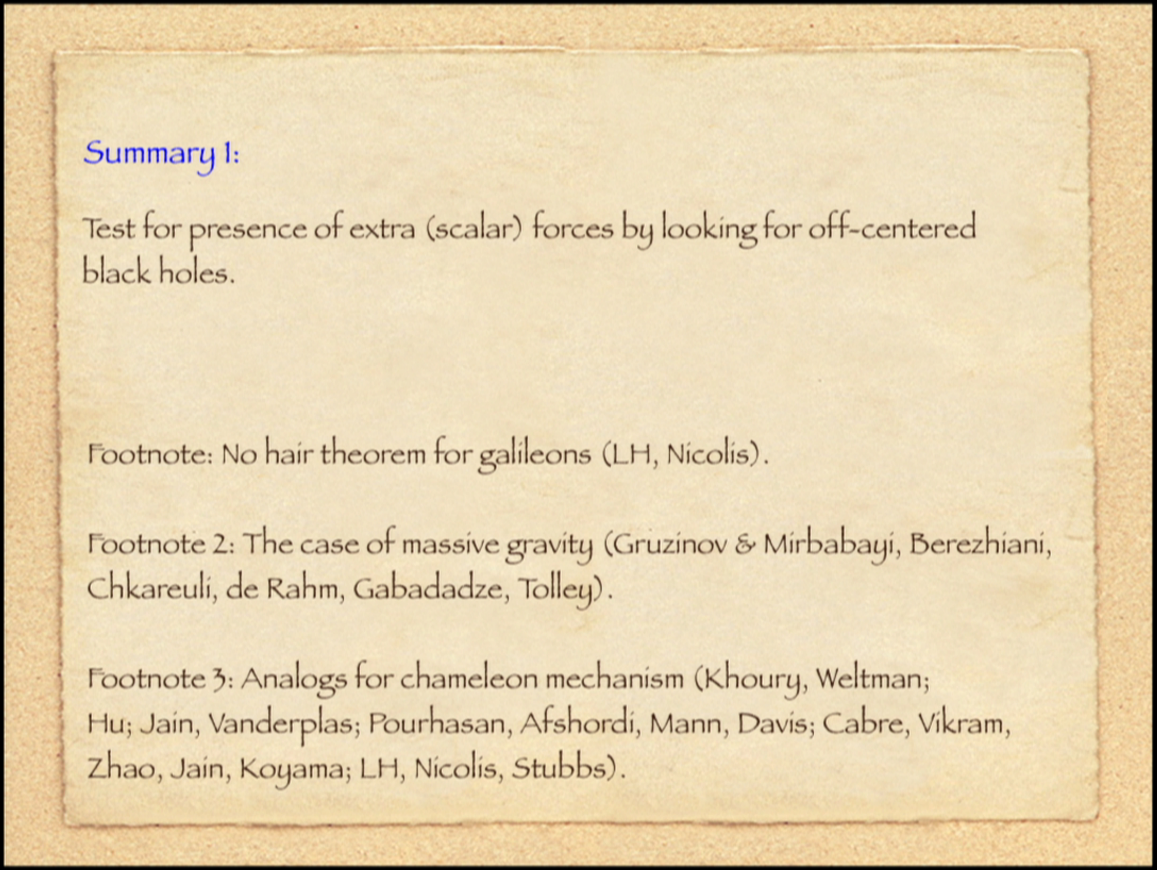


Outline:

1. Equivalence principle: a generic test of modified gravity
   - with Alberto Nicolis.

2. Parity: symmetry in the measurement of LSS
   - with Camille Bonvin & Enrique Gaztanaga.

3. Dilation & beyond: symmetry in the theory of LSS
   - with Kurt Hinterbichler & Justin Khoury,
     Walter Goldberger & Alberto Nicolis,
     Creminelli, Gleyzes, Simonovic & Vernizzi,
     Bart Horn, & Xiao Xiao.



Summary 1:

Test for presence of extra (scalar) forces by looking for off-centered black holes.

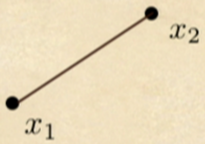
Footnote: No hair theorem for galileons (LH, Nicolis).

Footnote 2: The case of massive gravity (Gruzinov & Mirbabayi, Berezhiani, Chkareuli, de Rahm, Gabadadze, Tolley).

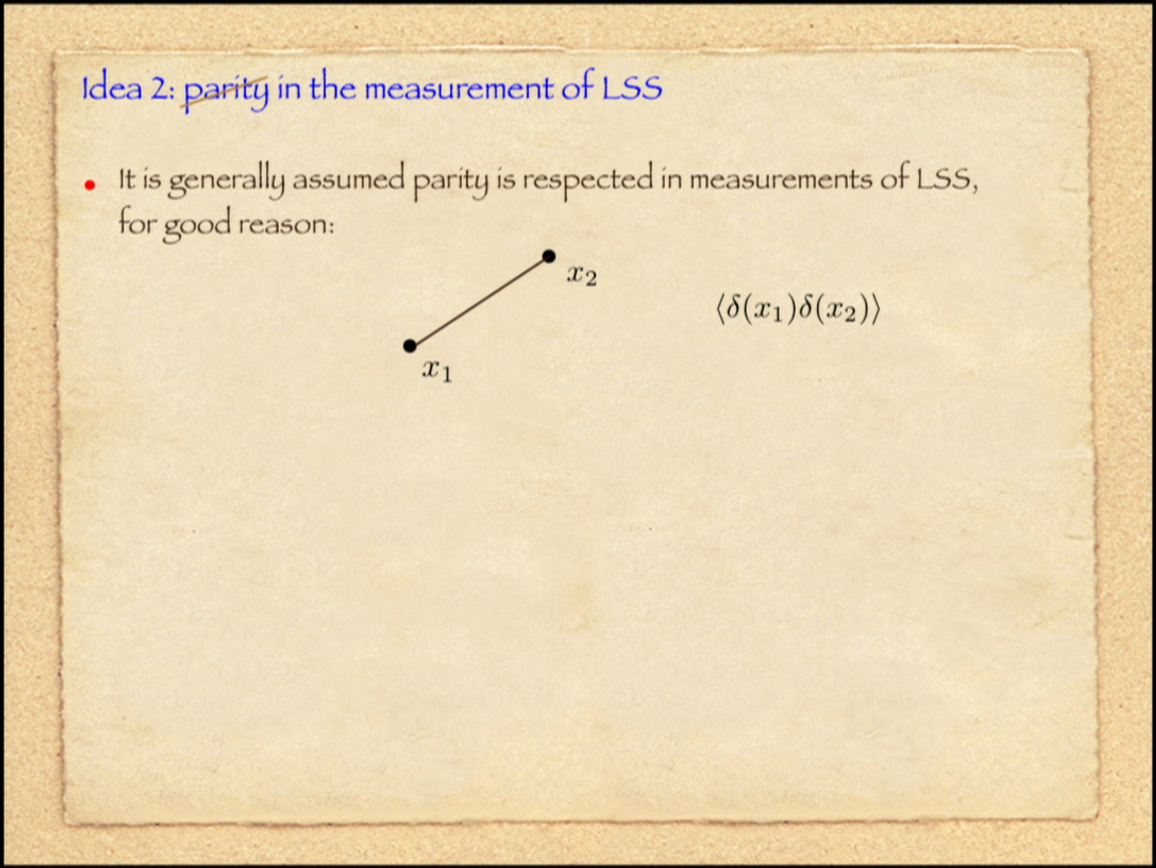
Footnote 3: Analogs for chameleon mechanism (Khoury, Weltman; Hu; Jain, Vanderplas; Pourhasan, Afshordi, Mann, Davis; Cabre, Vikram, Zhao, Jain, Koyama; LH, Nicolis, Stubbs).

Idea 2: ~~parity~~ in the measurement of LSS

- It is generally assumed parity is respected in measurements of LSS, for good reason:



$$\langle \delta(x_1)\delta(x_2) \rangle$$



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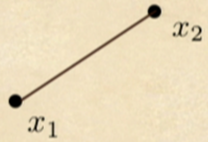
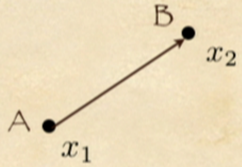


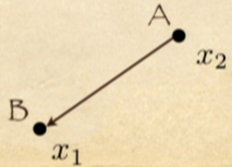
$$\langle \delta(x_1)\delta(x_2) \rangle$$

- But how about cross-correlation between 2 different kinds of galaxies, A & B?



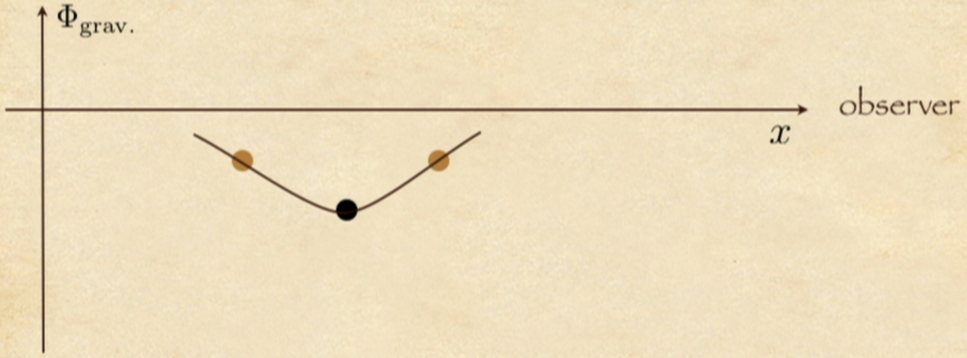
$$\langle \delta_A(x_1)\delta_B(x_2) \rangle$$

versus



$$\langle \delta_B(x_1)\delta_A(x_2) \rangle$$

What could possibly violate parity?



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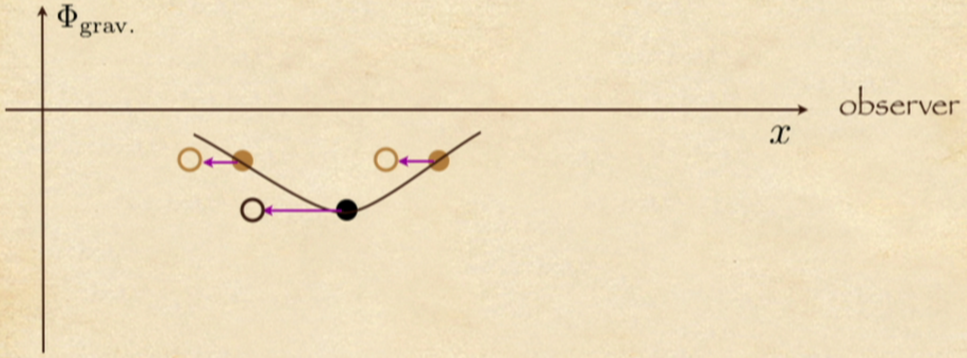

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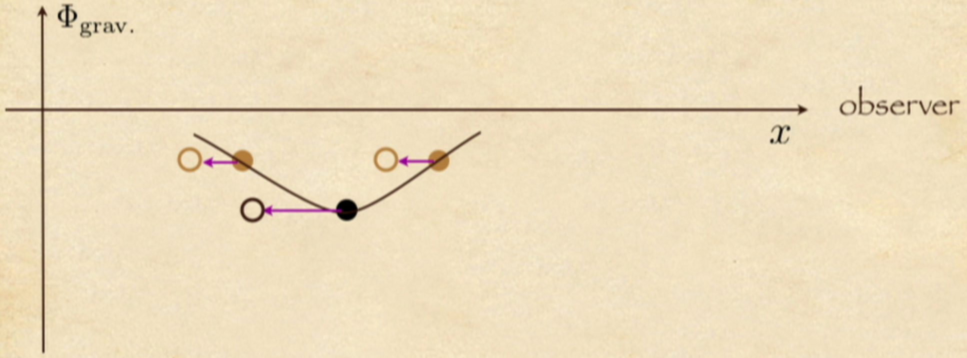

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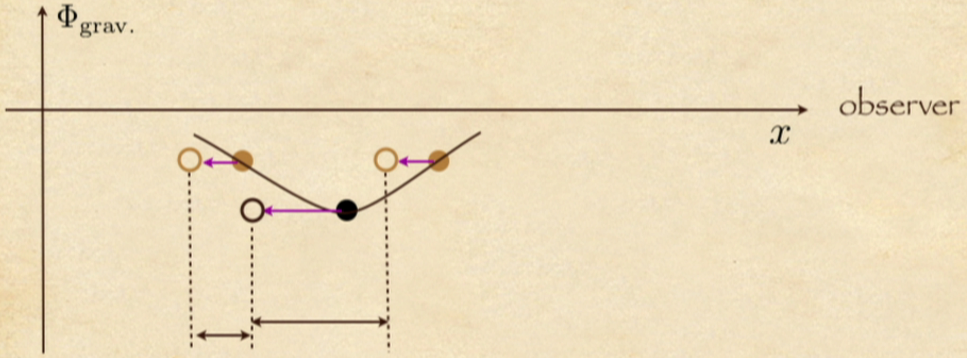

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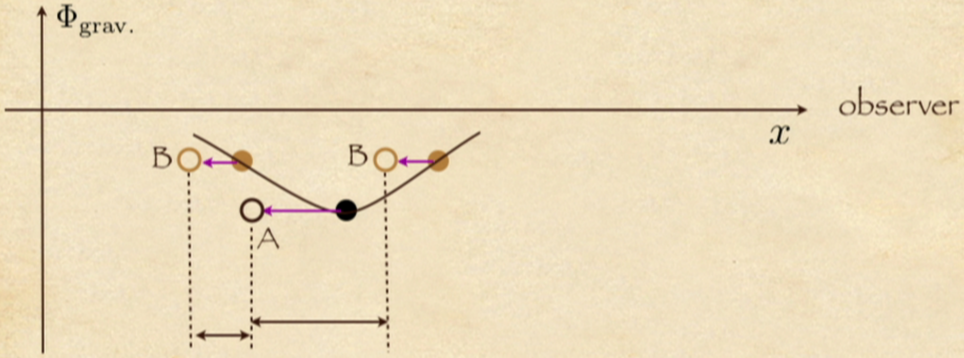
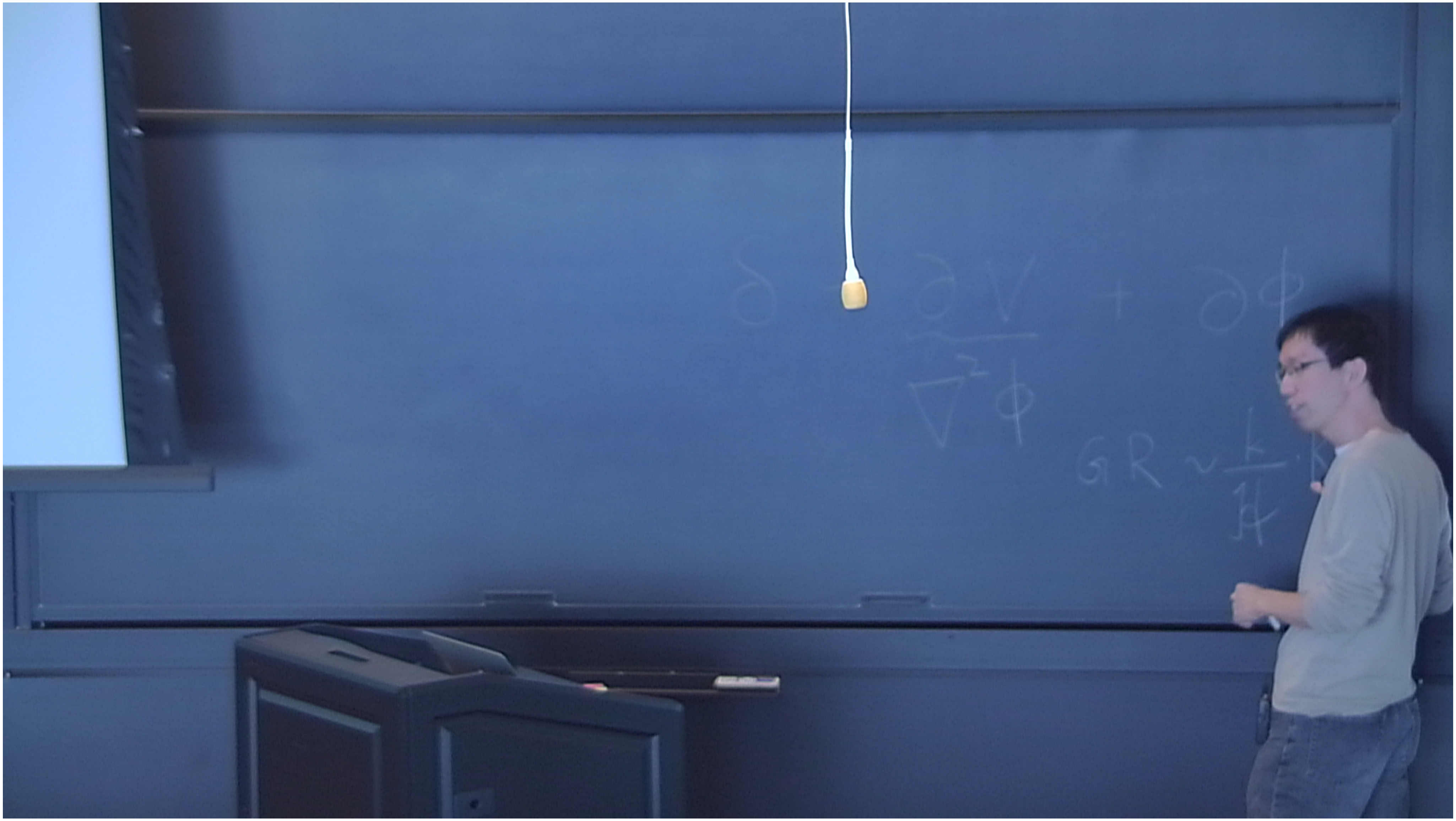
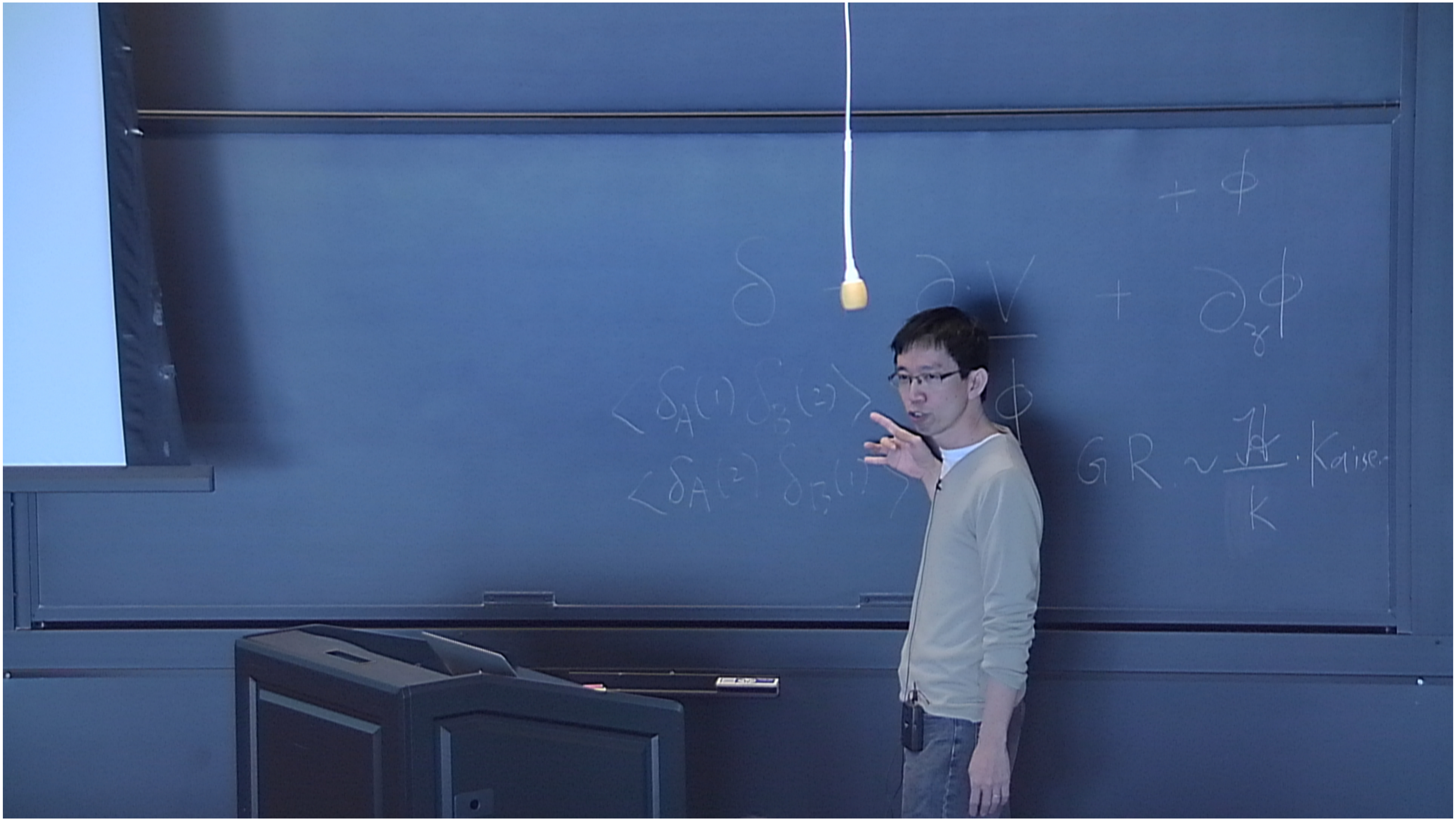



$\delta \quad \frac{\partial V}{\nabla^2 \phi} + \partial \phi$
$GR \sim \frac{k}{H} \cdot k$


+ φ
δ + ∂·V / φ + ∂_γ φ
⟨δ_A(1) δ_B(2)⟩
⟨δ_A(2) δ_B(1)⟩
GR ~ H/k · Kaiser

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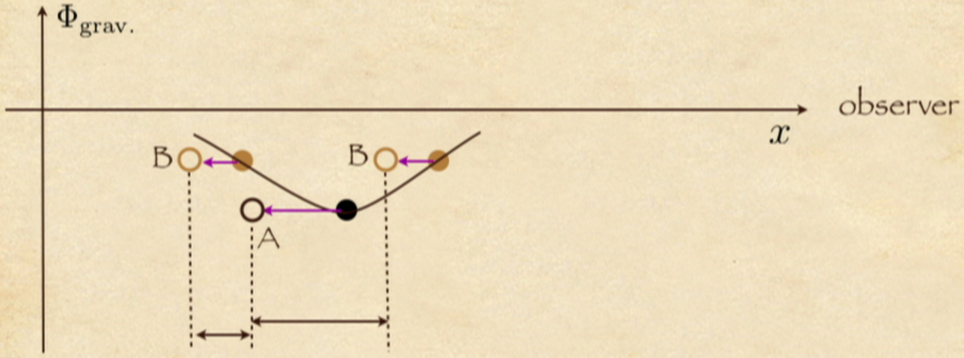
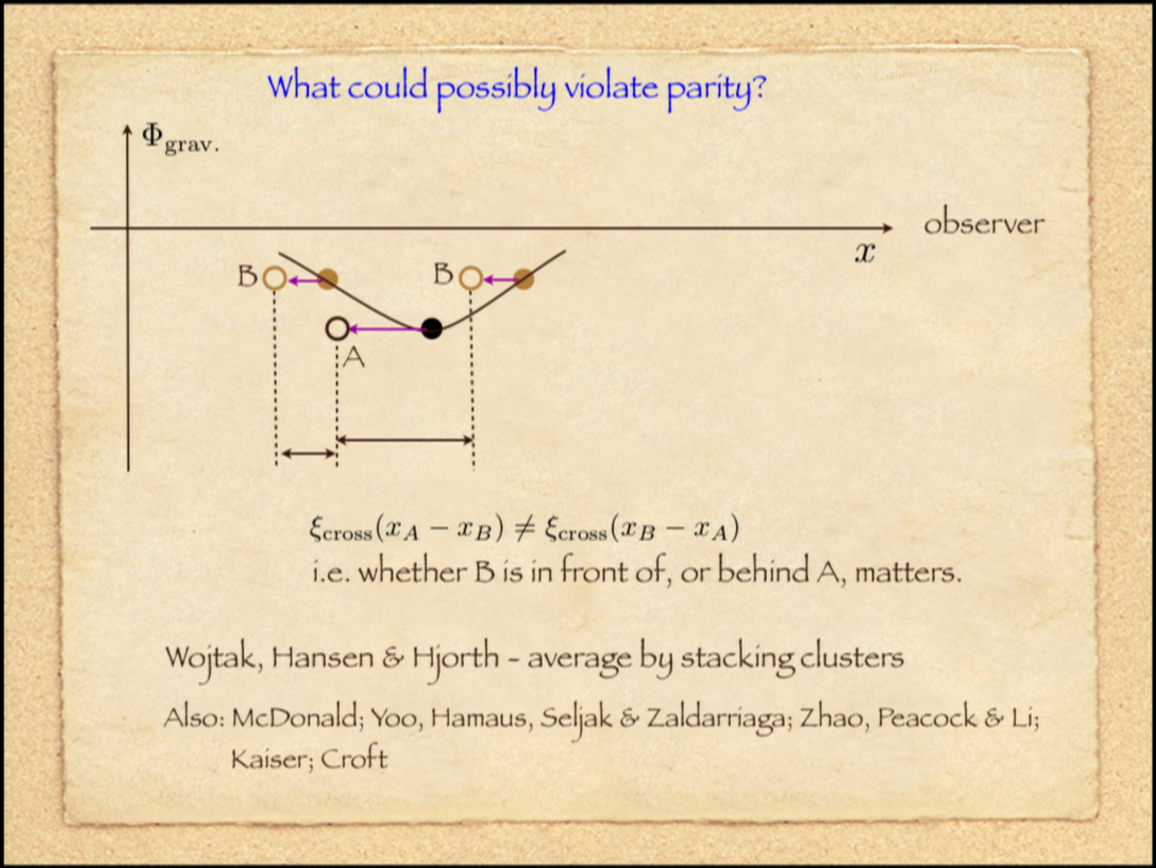


$$\xi_{\mathrm{cross}}(x_A - x_B) \neq \xi_{\mathrm{cross}}(x_B - x_A)$$

i.e. whether B is in front of, or behind A, matters.

Wojtak, Hansen & Hjorth - average by stacking clusters

Also: McDonald; Yoo, Hamaus, Seljak & Zaldarriaga; Zhao, Peacock & Li; Kaiser; Croft



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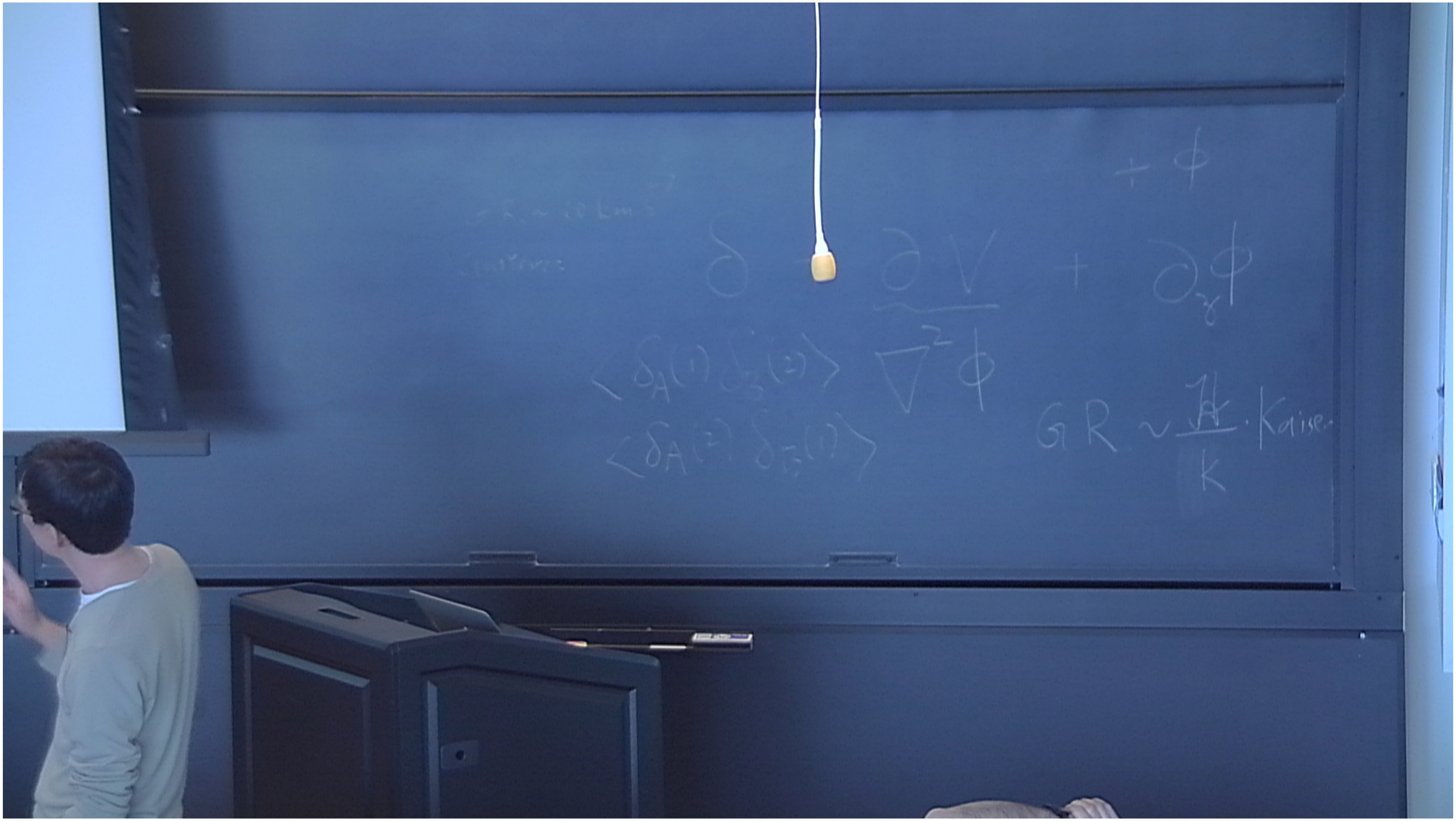

$+\phi$
$\delta + \underset{\sim}{\partial \cdot V} + \partial_\gamma \phi$
$\nabla^2 \phi$
$\langle \delta_A(1)\, \delta_B(2) \rangle$
$\langle \delta_A(2)\, \delta_B(1) \rangle$
GR $\sim \frac{\mathcal{H}}{k}$ · Kaise.

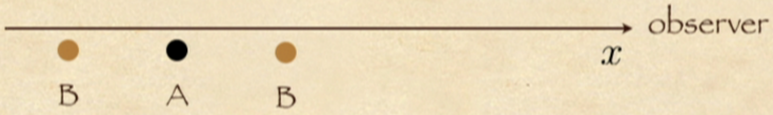
What are other parity violating effects?

- Often grouped under the heading of general relativistic effects:

$$\delta_{\rm obs.} \sim \delta\left[1+\underset{\substack{\uparrow\\ \text{parity violating}}}{\frac{\mathcal{H}}{k}}+\frac{\mathcal{H}^2}{k^2}\right]$$

Yoo, Fitzpatrick, Zaldarriaga; Challinor, Lewis; Bonvin, Durrer; Raccanelli, Bertacca, Dore, Maartens.

- More mundane, but present: evolution.



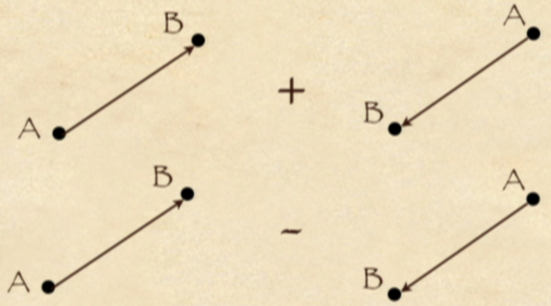
- Can disentangle between the two.

Footnote 1: parity violation only in the z direction.
Footnote 2: $O(\mathcal{H}/k)$ terms can be derived in a 'Newtonian' manner.
Gravitational redshift term canceled, assuming geodesic motion.
Footnote 3: selection effects.

Lessons for LSS measurement:

- Don't just add:

  Subtract too:



Or, more generally: combine different orientations appropriately.

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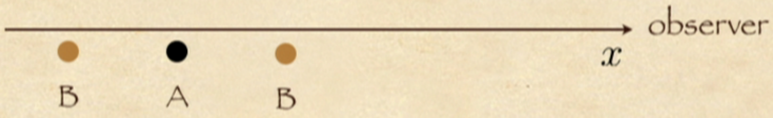

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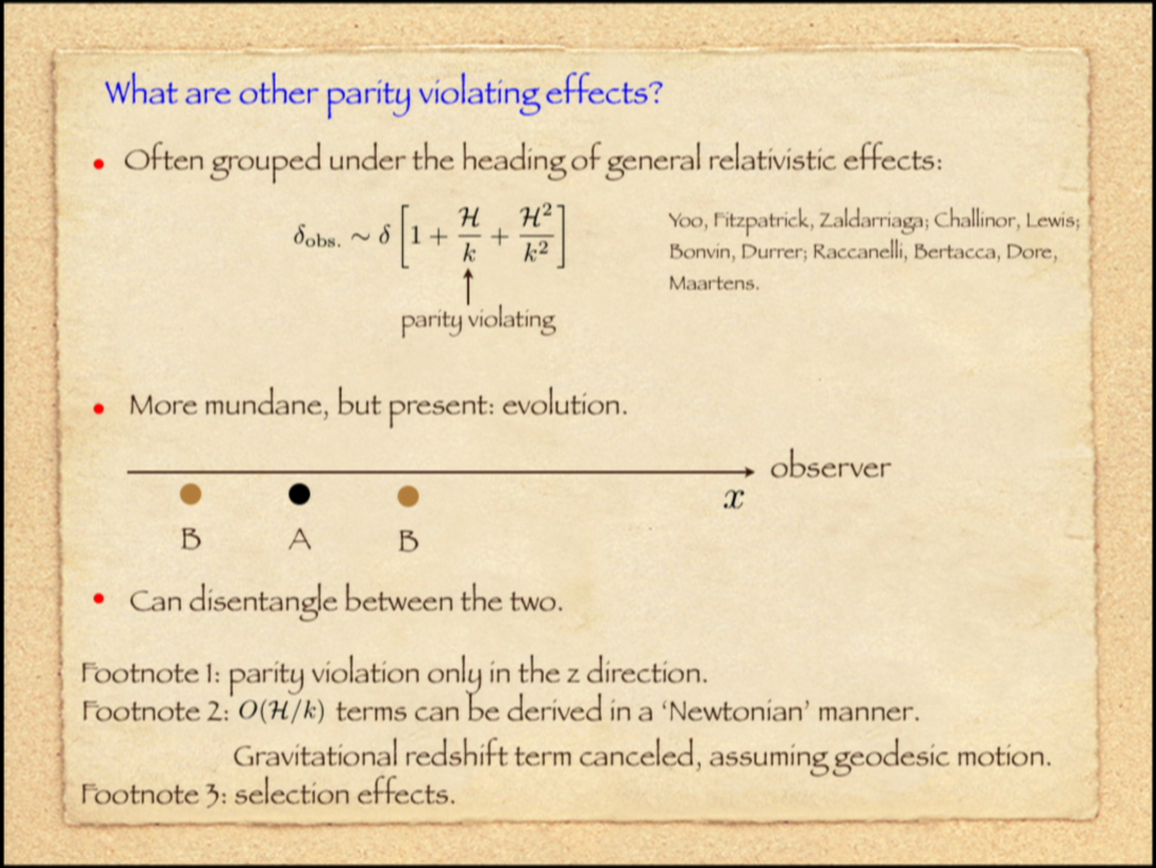

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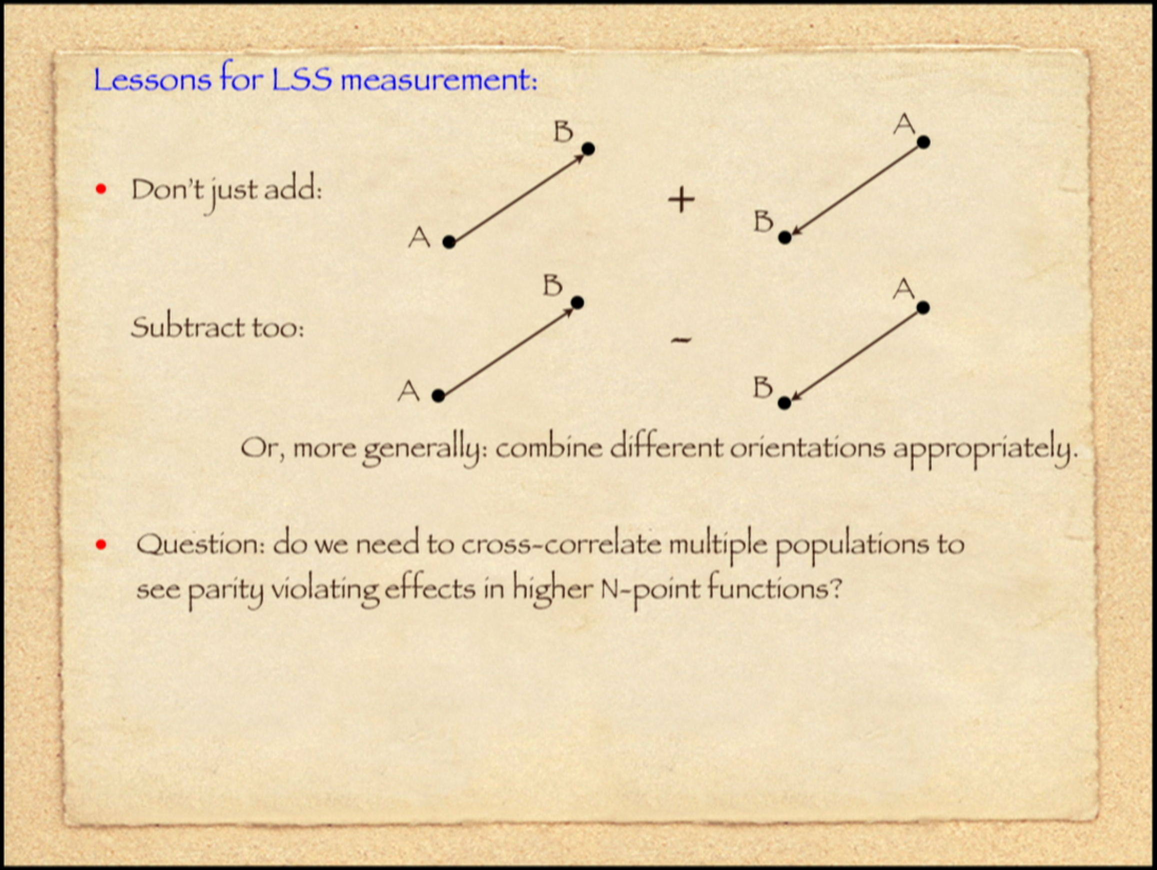

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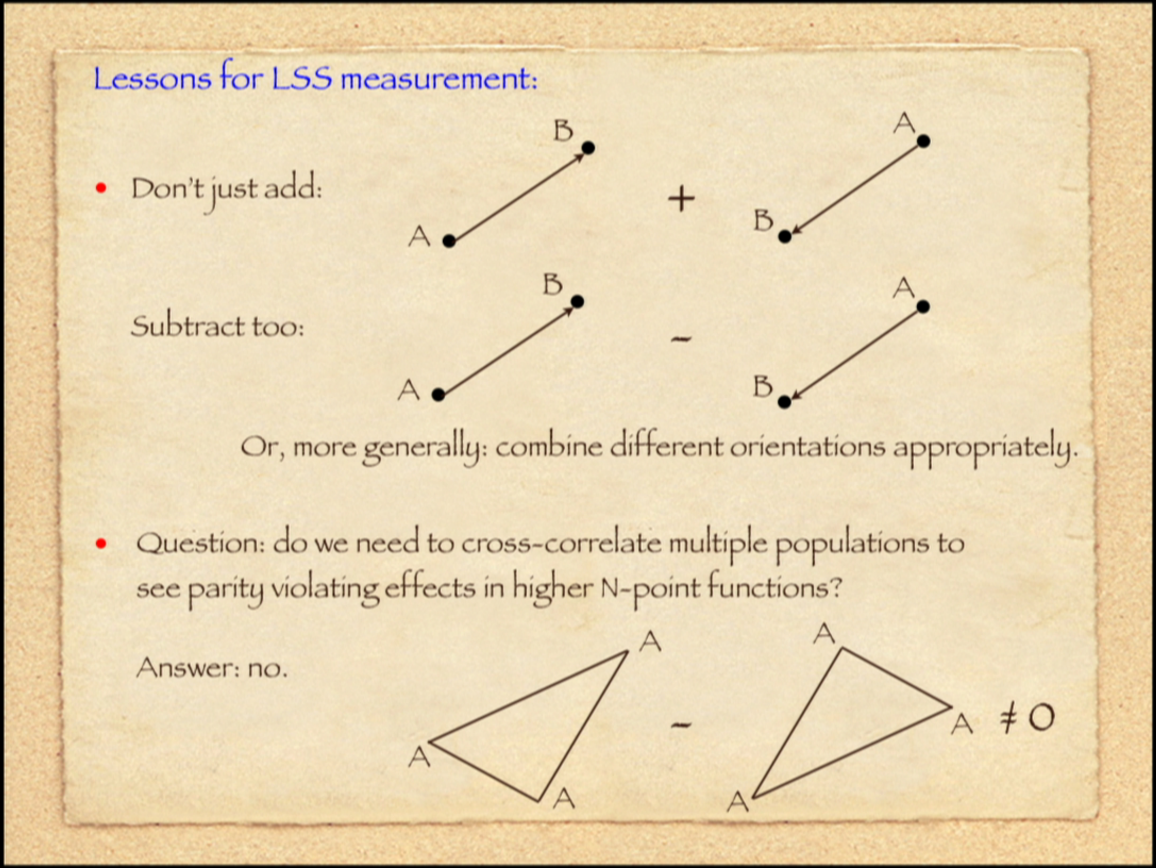
- Question: do we need to cross-correlate multiple populations to see parity violating effects in higher N-point functions?



Lessons for LSS measurement:

- Don't just add:

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  Or, more generally: combine different orientations appropriately.

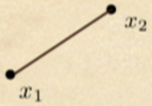
- Question: do we need to cross-correlate multiple populations to see parity violating effects in higher N-point functions?

  Answer: no.

Idea 3: non-perturbative consistency relations in LSS

- 1. Consider a familiar example of symmetry: spatial translation.

$$x \to x + \Delta x \quad , \quad \text{where } \Delta x = \text{const.}$$



Its consequence for correlation function is well known:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle = \langle \phi(x_1+\Delta x)\phi(x_2+\Delta x)\phi(x_3+\Delta x) \rangle$$

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Thus, alternatively, we say:

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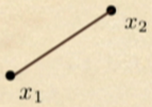

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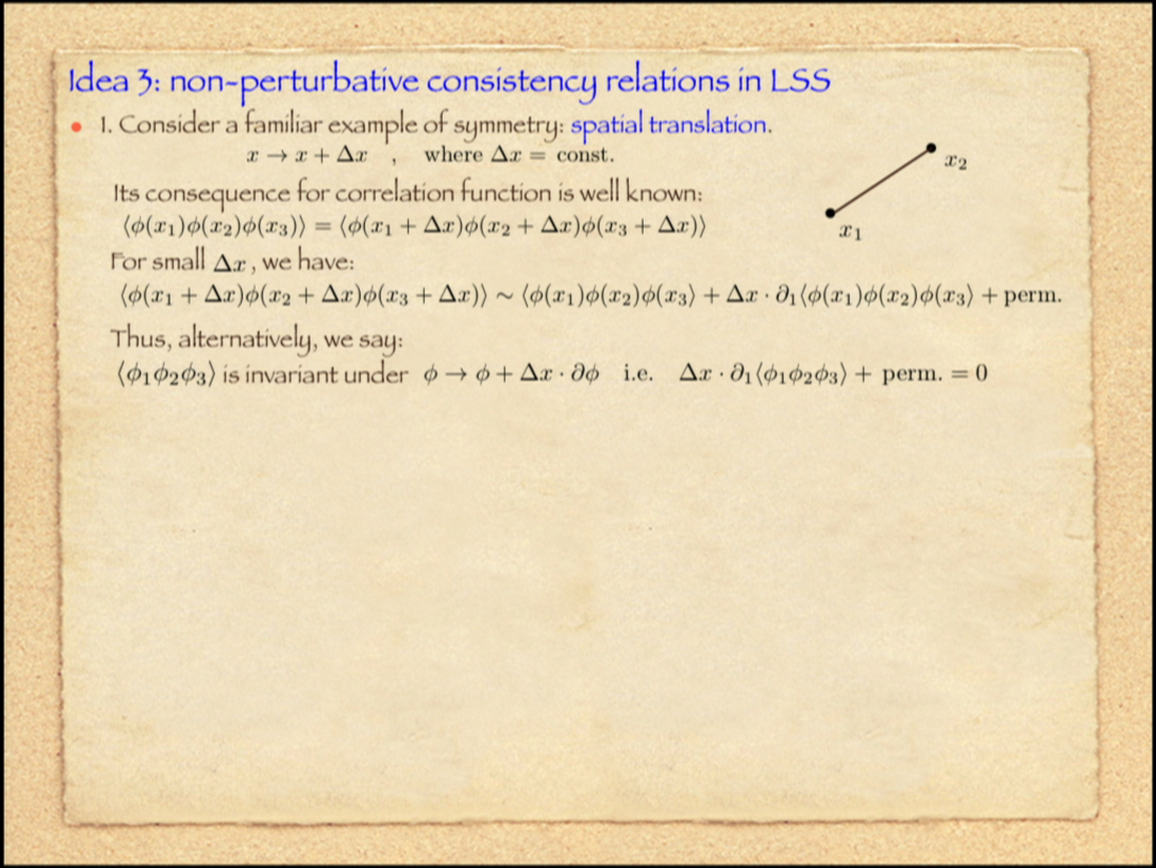

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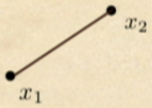

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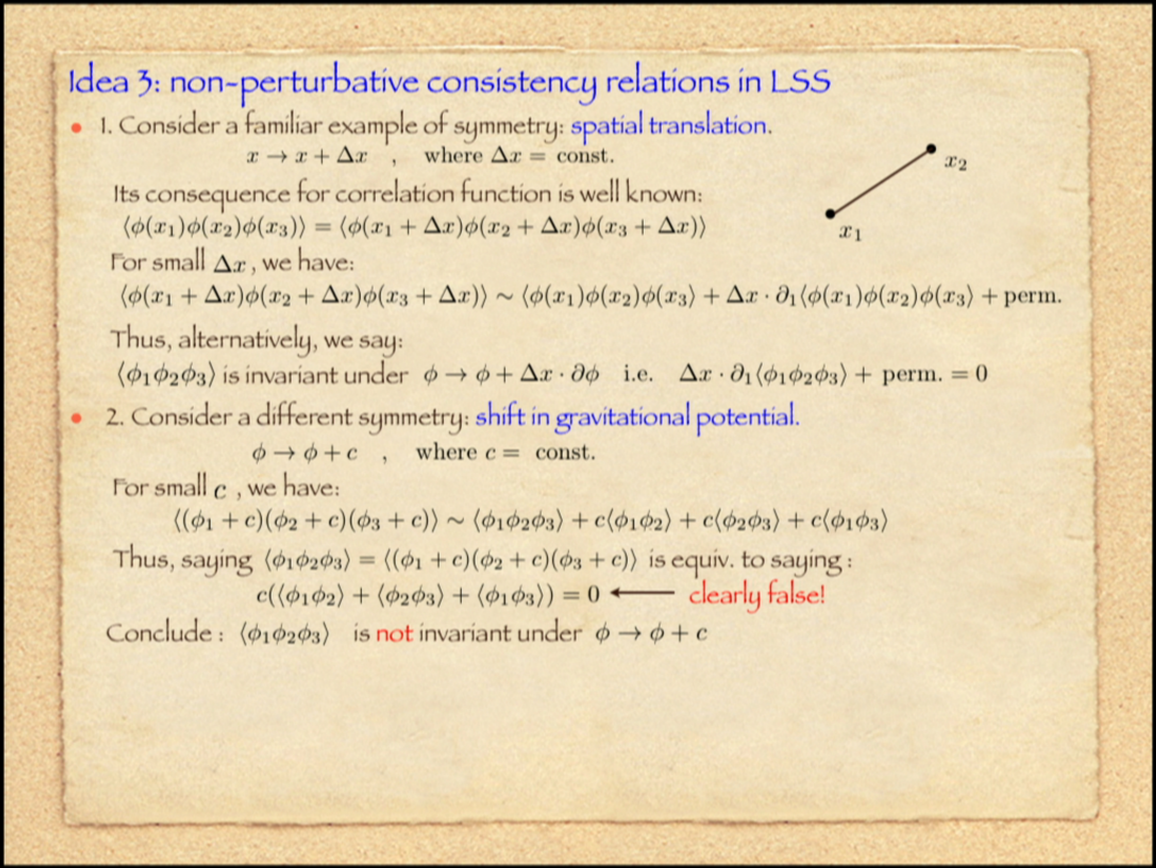

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- 2. Consider a different symmetry: shift in gravitational potential.

$$\phi \to \phi + c \quad , \quad \text{where } c = \text{ const.}$$

For small c, we have:

$$\langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle \sim \langle \phi_1\phi_2\phi_3\rangle + c\langle \phi_1\phi_2\rangle + c\langle \phi_2\phi_3\rangle + c\langle \phi_1\phi_3\rangle$$

Thus, saying $\langle \phi_1\phi_2\phi_3\rangle = \langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle$ is equiv. to saying :

$$c(\langle \phi_1\phi_2\rangle + \langle \phi_2\phi_3\rangle + \langle \phi_1\phi_3\rangle) = 0 \longleftarrow \text{clearly false!}$$

Conclude : $\langle \phi_1\phi_2\phi_3\rangle$ is not invariant under $\phi \to \phi + c$

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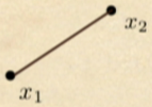

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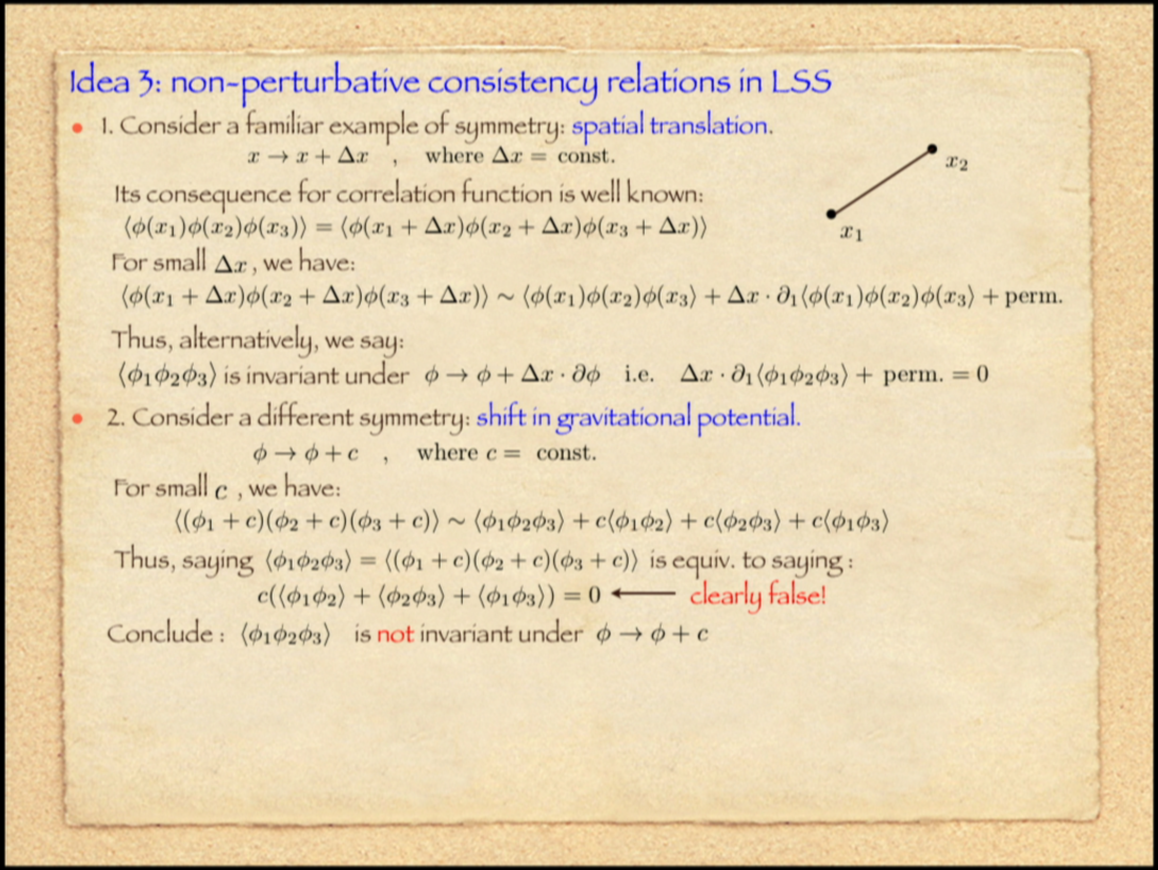

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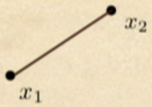

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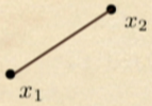

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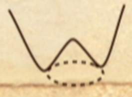

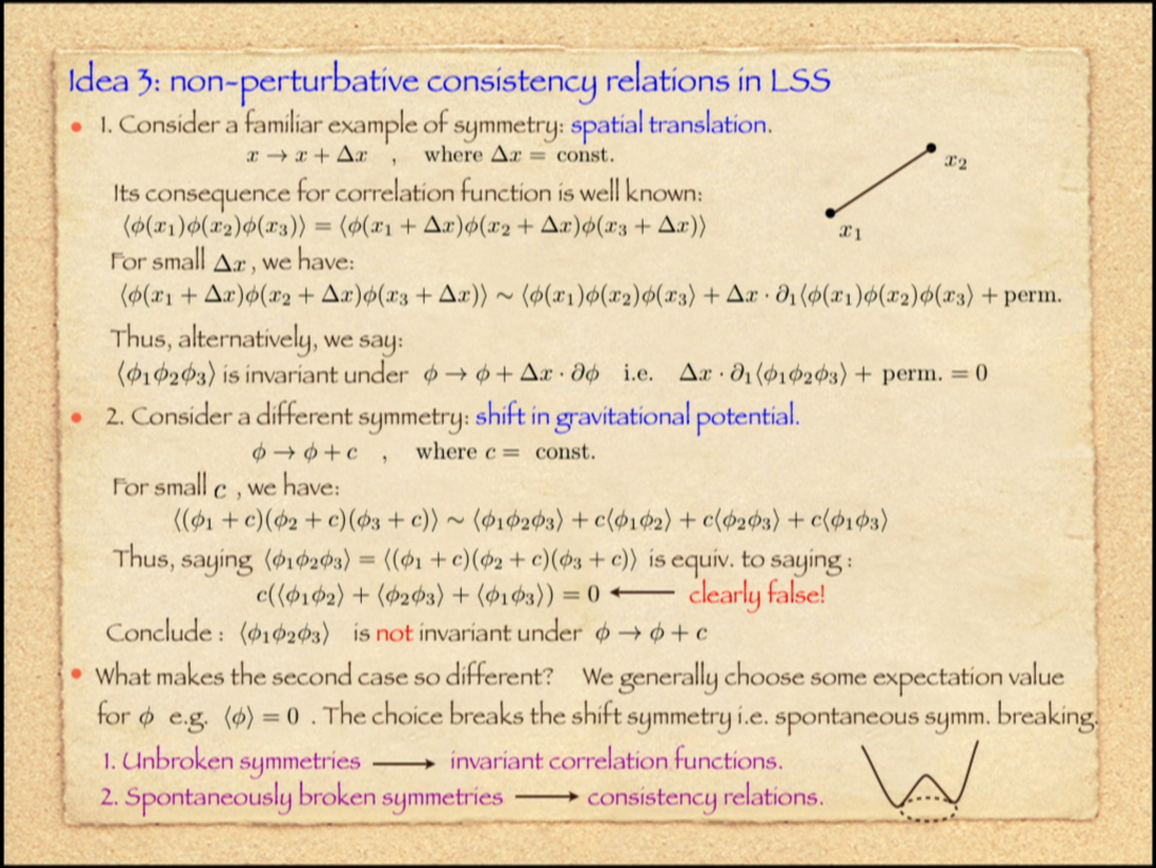

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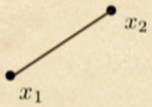

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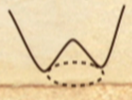

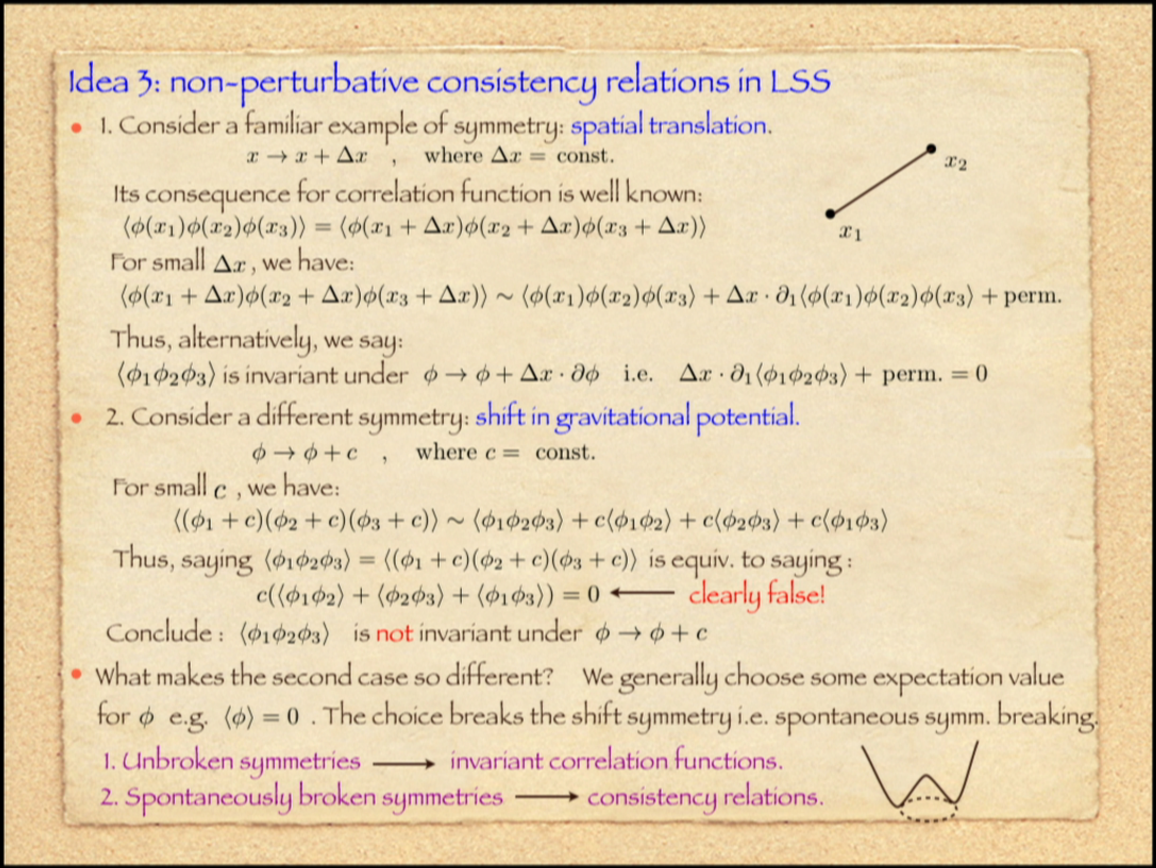

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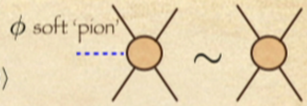
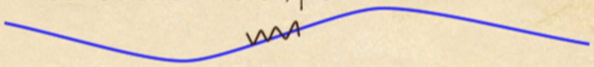


- Schematic form: $\lim_{q\to 0}\frac{1}{P_\phi(q)}\langle\phi(q)\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle \sim \langle\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle$

They are (momentum space) statements about how correlations of observables $\mathcal{O}$ behave in the presence of a long wave-mode Goldstone boson/pion.



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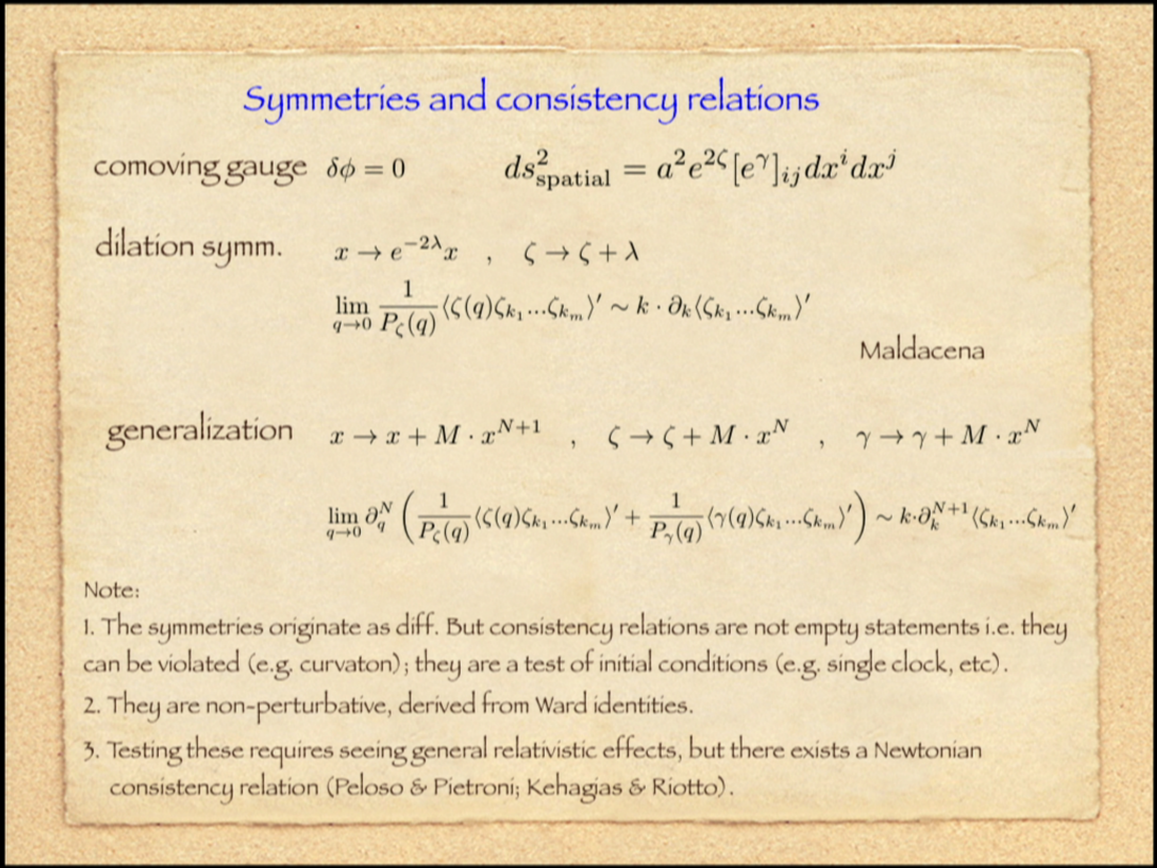

Symmetries and consistency relations

comoving gauge $\delta\phi = 0$ $\qquad ds^2_{\text{spatial}} = a^2 e^{2\zeta} [e^{\gamma}]_{ij} dx^i dx^j$

dilation symm. $x \to e^{-2\lambda} x \quad , \quad \zeta \to \zeta + \lambda$

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Maldacena

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References:

Maldacena; Creminelli & Zaldarriaga; Creminelli, Norena, Simonovic; Assassi, Baumann & Green; Flauger, Green & Porto; Pajer, Schmidt, Zaldarriaga; Kehagias & Riotto; Peloso & Pietronni; Berezhiani & Khoury; Pimentel; Creminelli, Norena, Simonovic, Vernizzi; Goldberger, LH, Nicolis; Hinterbichler, LH, Khoury; Horn, LH, Xiao.

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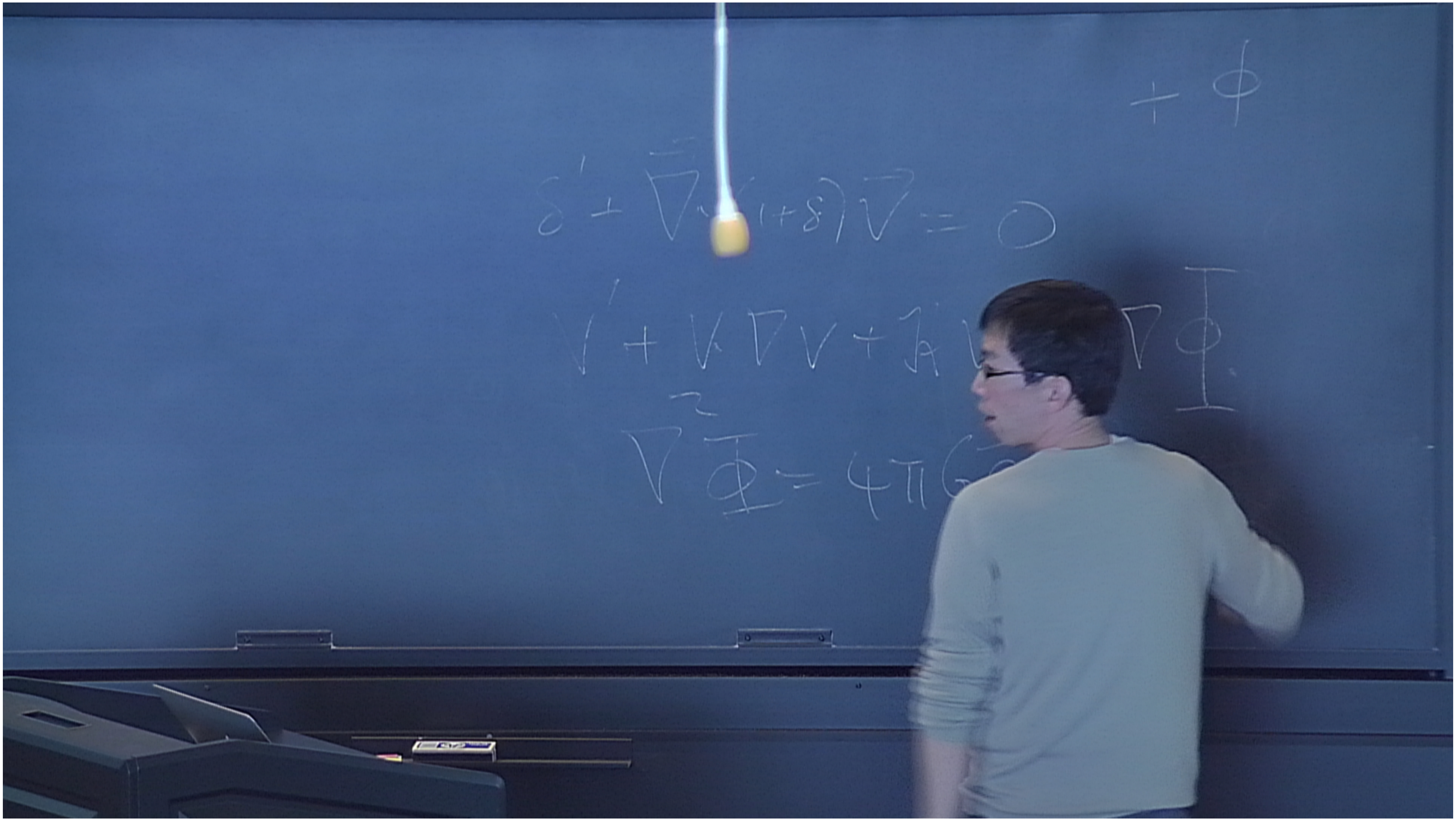

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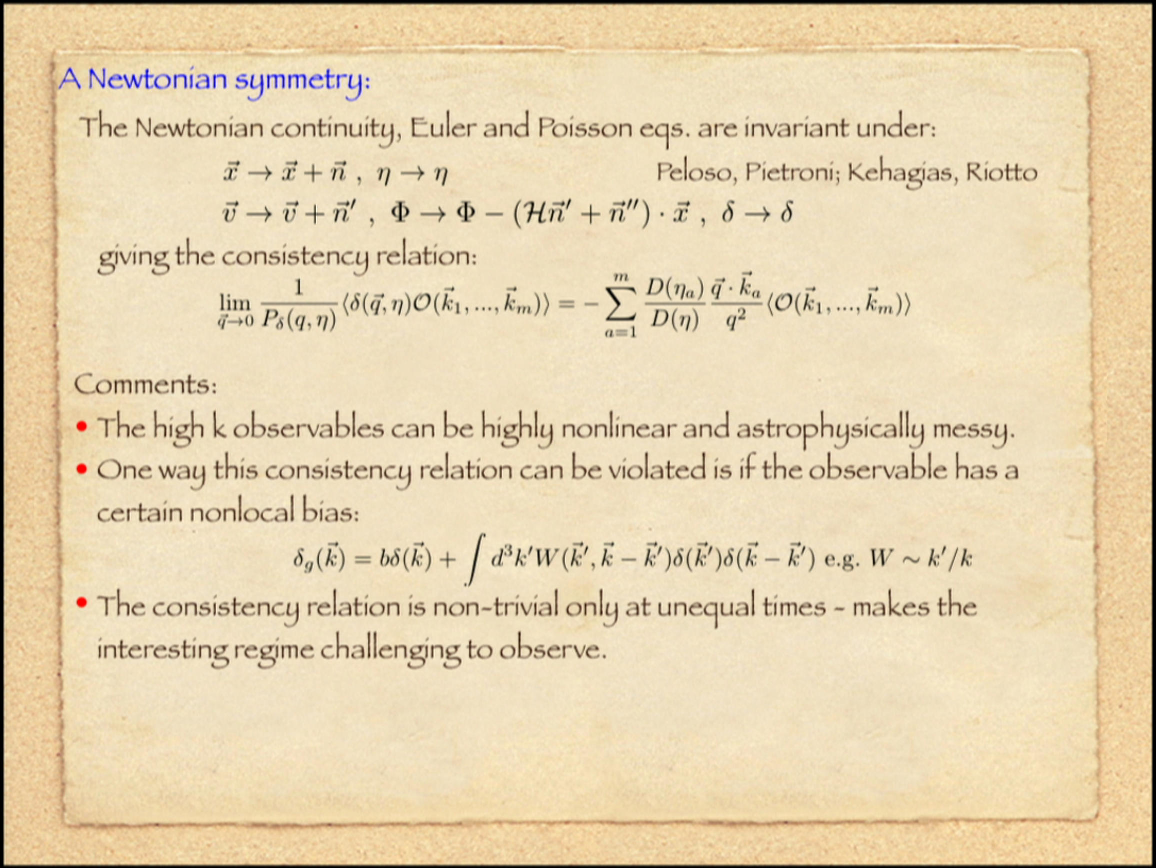

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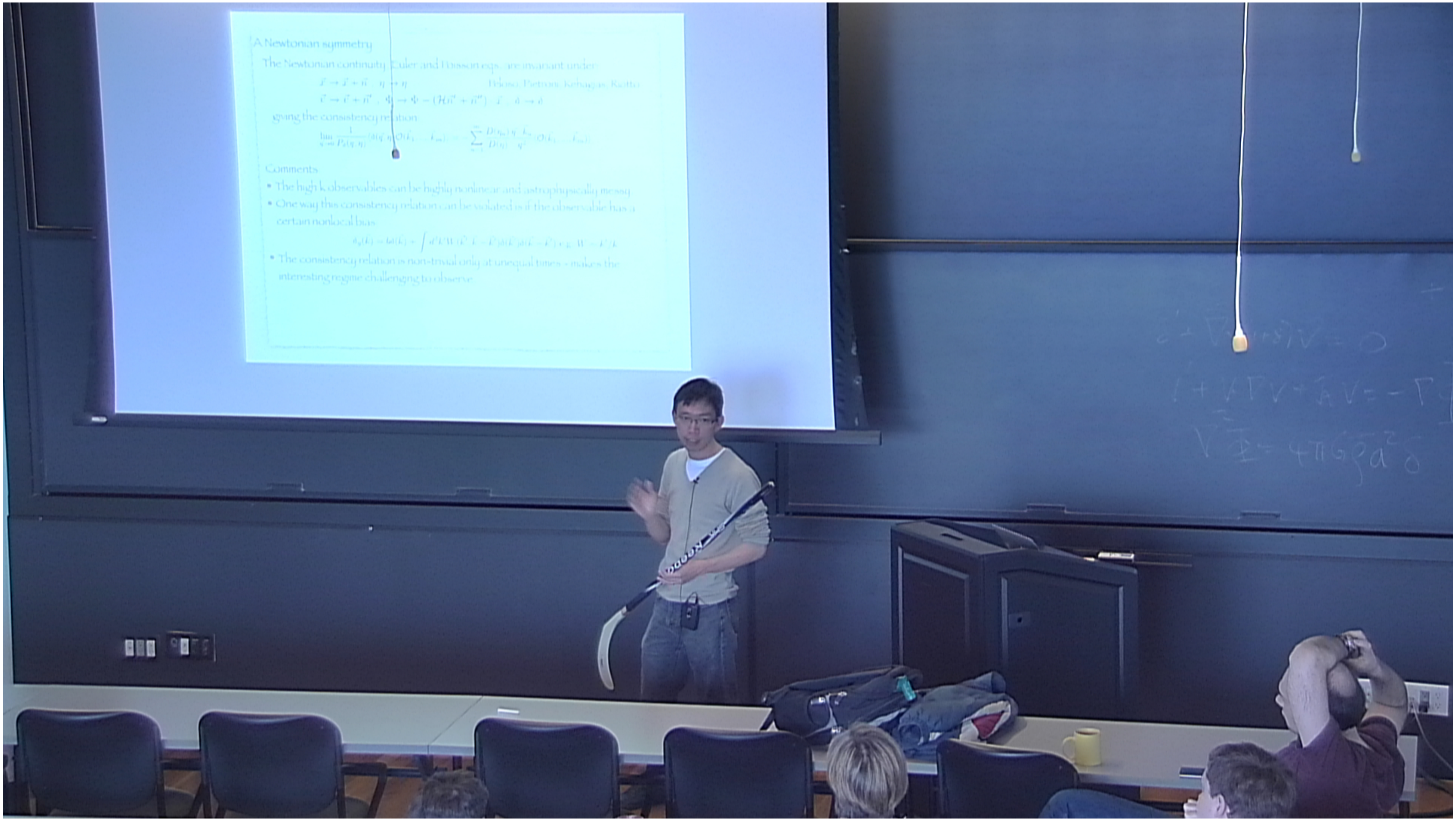

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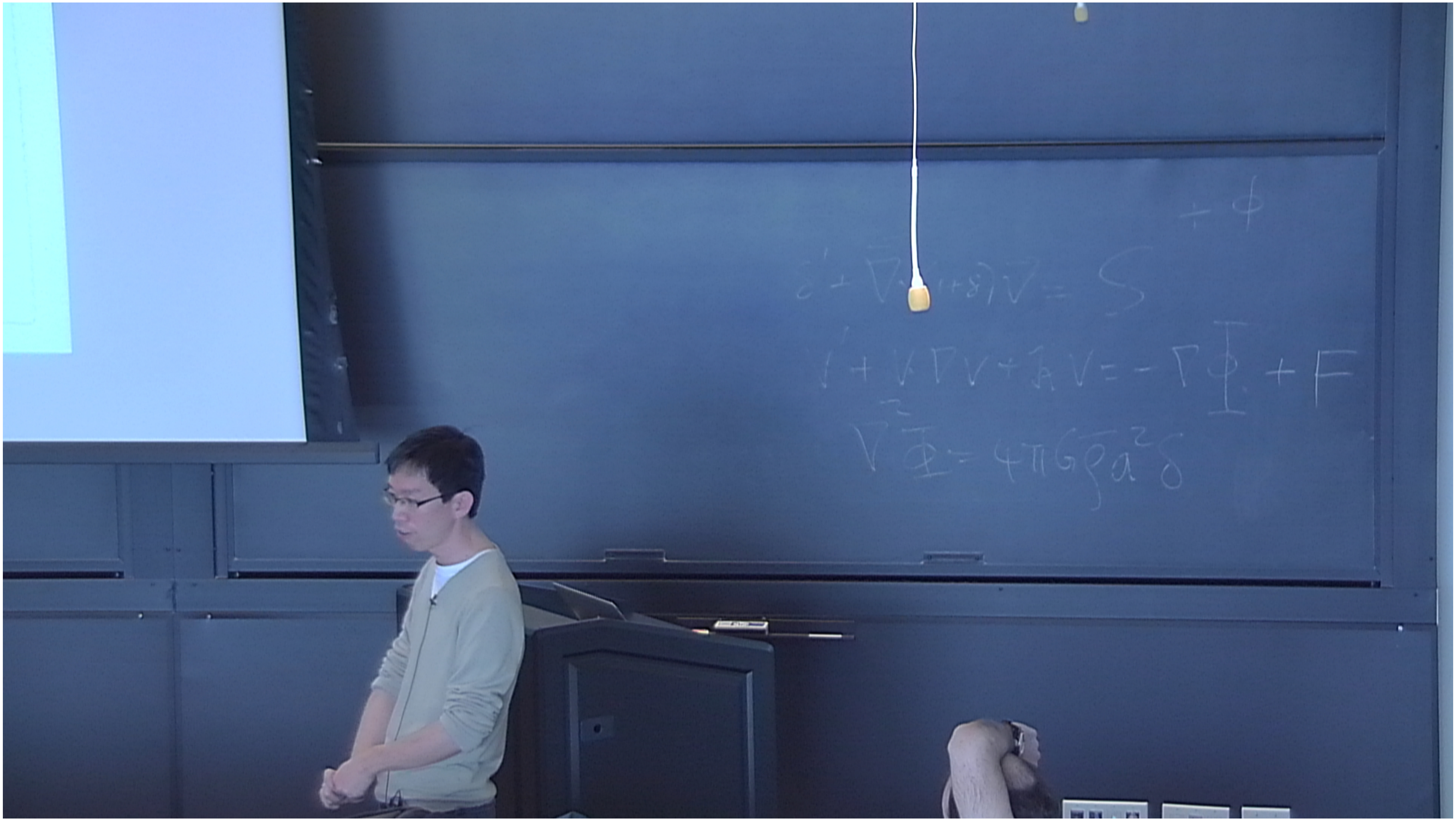
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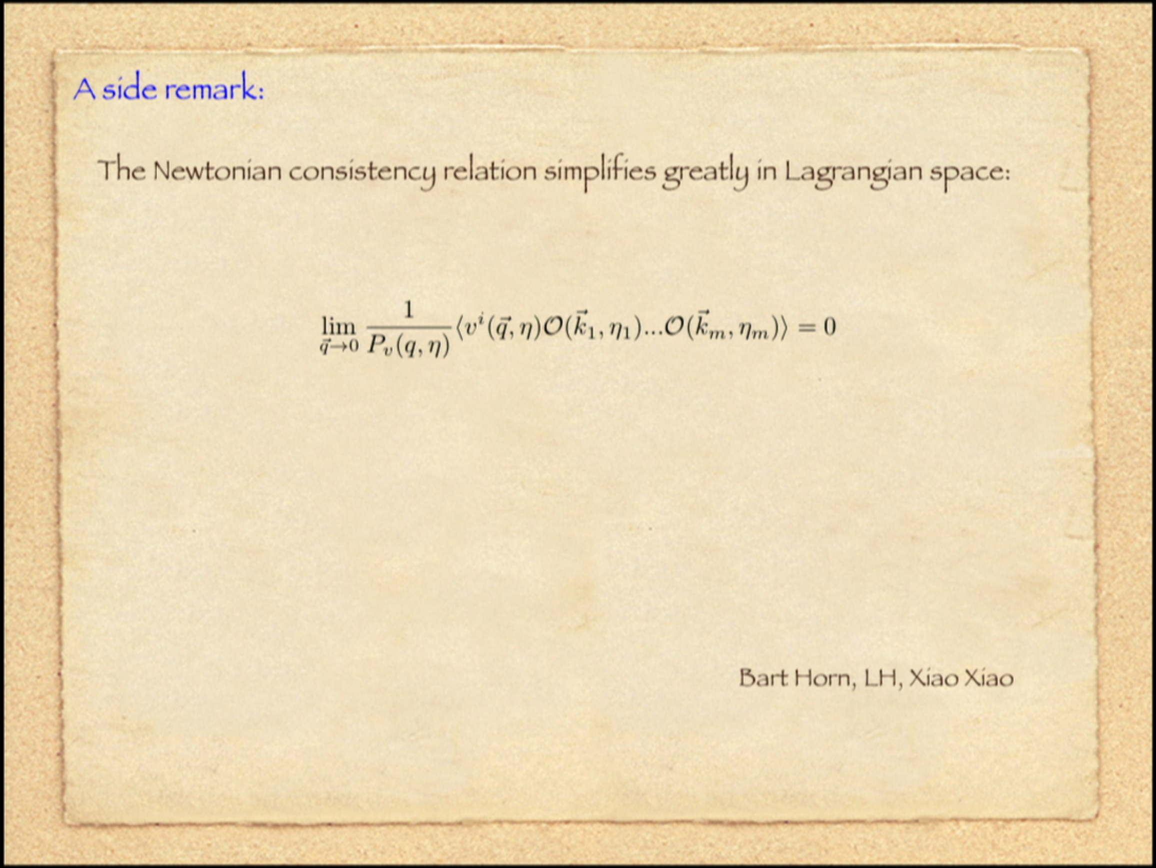
$\nabla^2\Phi = 4\pi G\bar{\rho}a^2\delta$

A side remark:

The Newtonian consistency relation simplifies greatly in Lagrangian space:

$$\lim_{\vec{q}\to 0} \frac{1}{P_v(q,\eta)} \langle v^i(\vec{q},\eta)\mathcal{O}(\vec{k}_1,\eta_1)...\mathcal{O}(\vec{k}_m,\eta_m)\rangle = 0$$

Bart Horn, LH, Xiao Xiao



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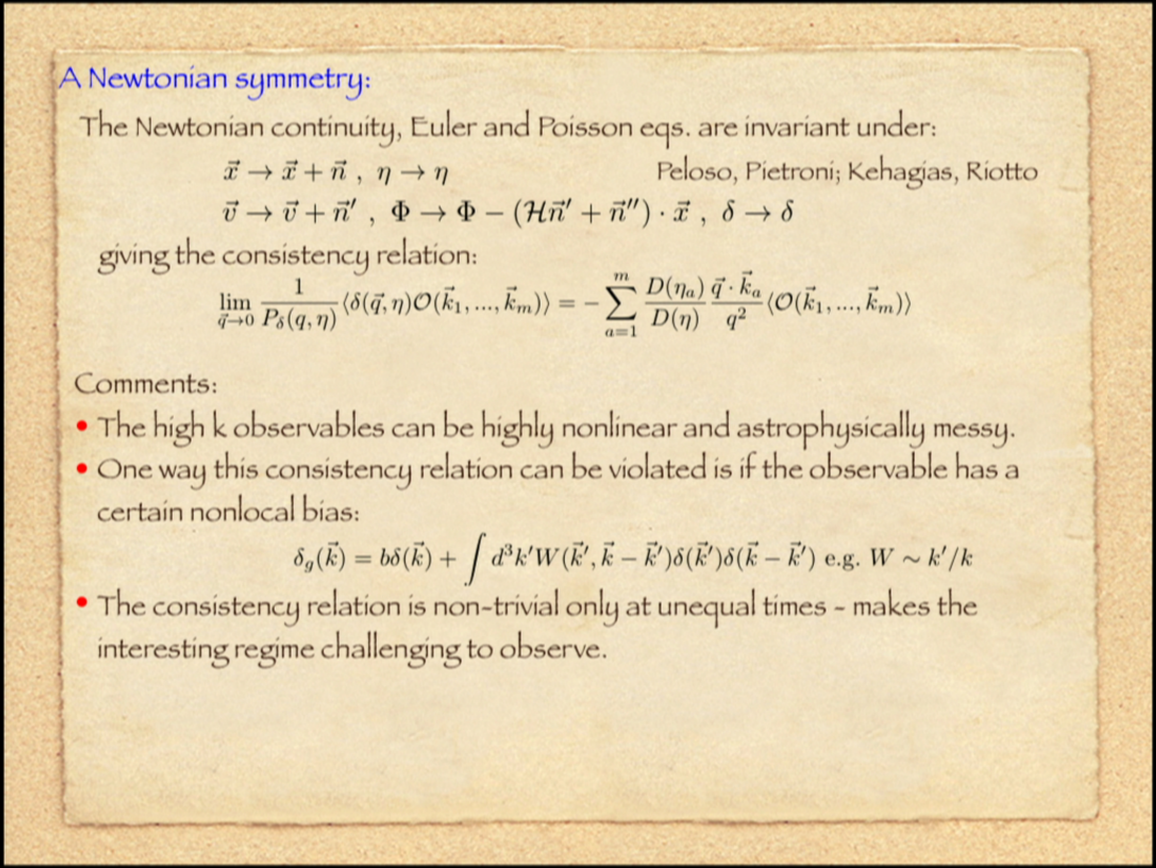


A Newtonian symmetry:

The Newtonian continuity, Euler and Poisson eqs. are invariant under:

$$\vec{x} \to \vec{x} + \vec{n} \ , \ \eta \to \eta$$

Peloso, Pietroni; Kehagias, Riotto

$$\vec{v} \to \vec{v} + \vec{n}' \ , \ \Phi \to \Phi - (\mathcal{H}\vec{n}' + \vec{n}'') \cdot \vec{x} \ , \ \delta \to \delta$$

giving the consistency relation:

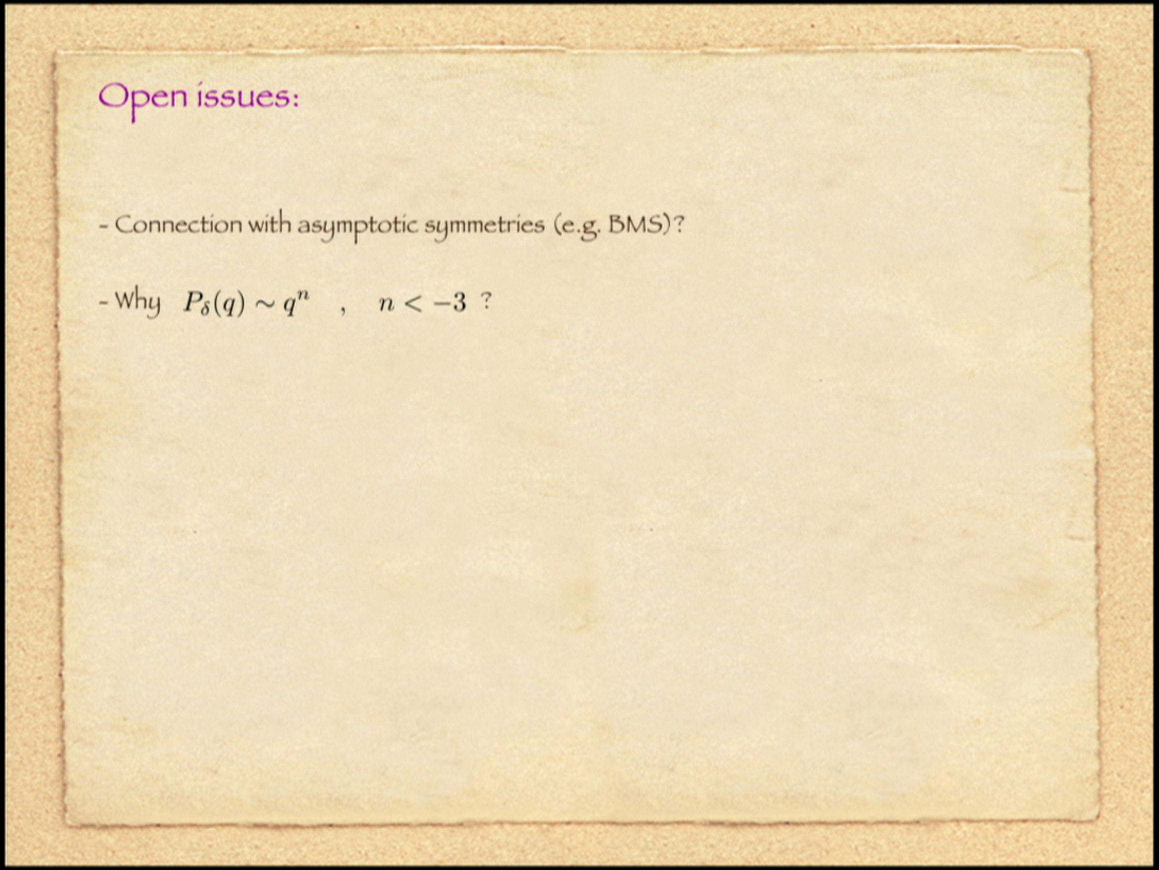
$$\lim_{\vec{q}\to 0} \frac{1}{P_\delta(q,\eta)} \langle \delta(\vec{q},\eta) \mathcal{O}(\vec{k}_1, ..., \vec{k}_m) \rangle = - \sum_{a=1}^{m} \frac{D(\eta_a)}{D(\eta)} \frac{\vec{q}\cdot\vec{k}_a}{q^2} \langle \mathcal{O}(\vec{k}_1, ..., \vec{k}_m) \rangle$$

Comments:

- The high k observables can be highly nonlinear and astrophysically messy.
- One way this consistency relation can be violated is if the observable has a certain nonlocal bias:

$$\delta_g(\vec{k}) = b\delta(\vec{k}) + \int d^3k' W(\vec{k}', \vec{k} - \vec{k}') \delta(\vec{k}') \delta(\vec{k} - \vec{k}') \text{ e.g. } W \sim k'/k$$

- The consistency relation is non-trivial only at unequal times - makes the interesting regime challenging to observe.



Open issues:

- Connection with asymptotic symmetries (e.g. BMS)?

- Why $P_\delta(q) \sim q^n \quad , \quad n < -3$?

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- Why $P_\delta(q) \sim q^n$, $n \le -3$?