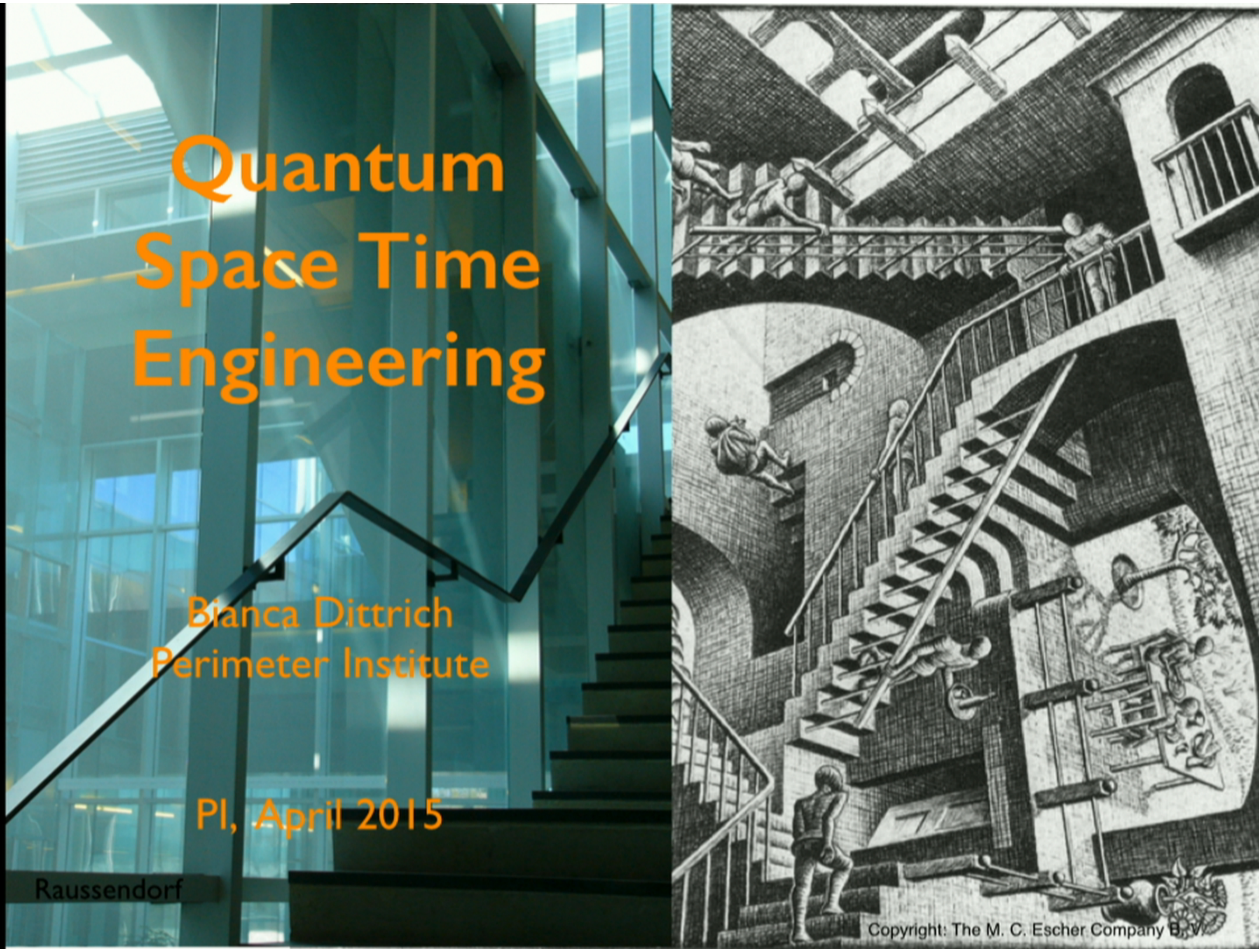
Title: Quantum Space Time Engineering

Date: Apr 08, 2015 02:00 PM

URL: http://pirsa.org/15040091

Abstract: <p>Modern physics rests on two basic frameworks, quantum theory and general relativity. Quantum gravity aims to unify these two frameworks into one consistent theory. One can expect that such a formulation delivers in particular a novel understanding of space and time as quantum objects.</p>

<p>I will give an introduction to some basic concepts in quantum gravity research and present possible models of quantum space time.</p>


Quantum
Space Time
Engineering
Bianca Dittrich
Perimeter Institute
PI, April 2015
Raussendorf
Copyright: The M. C. Escher Company B.V.

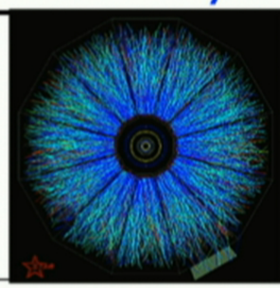

PERIMETER
INSTITUTE

20th century accomplishments

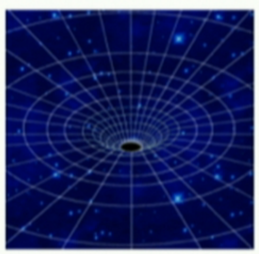
matter
(+gravitons)

gravity
(+classical matter)

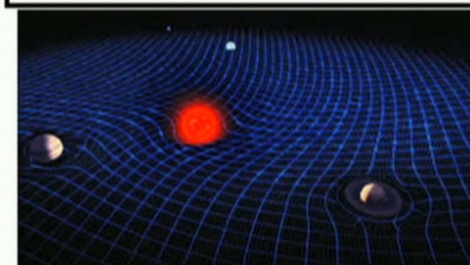
Quantum (field) theory



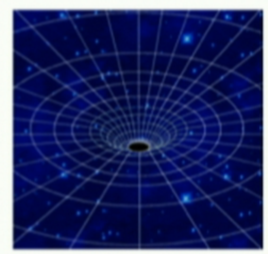
lives on a classical space time



General Relativity



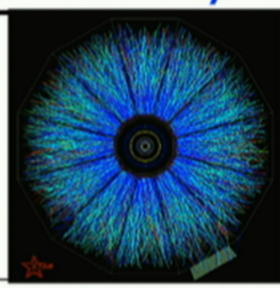
describes shape of classical space time



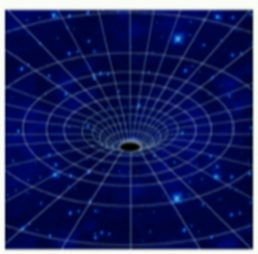
20th century accomplishments

matter
(+gravitons)

Quantum (field) theory

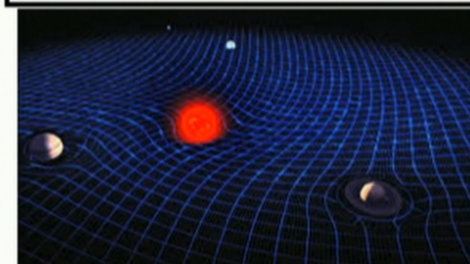


lives on a classical space time

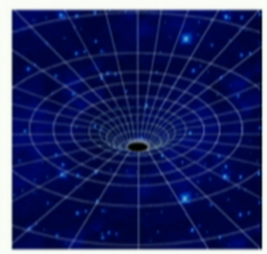


gravity
(+classical matter)

General Relativity



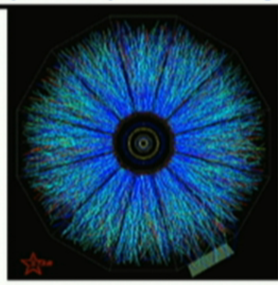
describes shape of classical space time



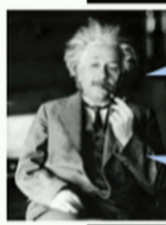

20th century accomplishments
matter
(+gravitons)
gravity
(+classical matter)
Quantum (field)
theory
General
Relativity
lives on a classical
space time
describes shape of classical
space time
Break down in
extreme (but interesting)
regimes
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS

One theory to rule them all

Quantum (field) theory

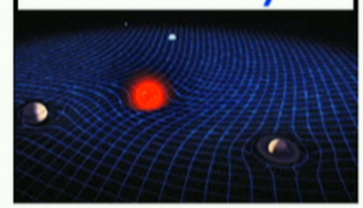


Quantum Gravity

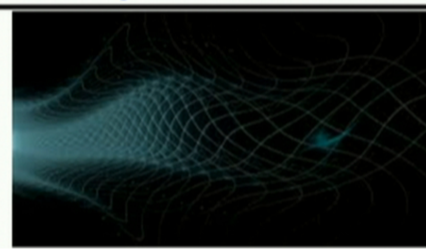


gravity = space time geometry

General Relativity



Quantum Space Time



What is Quantum Space Time?

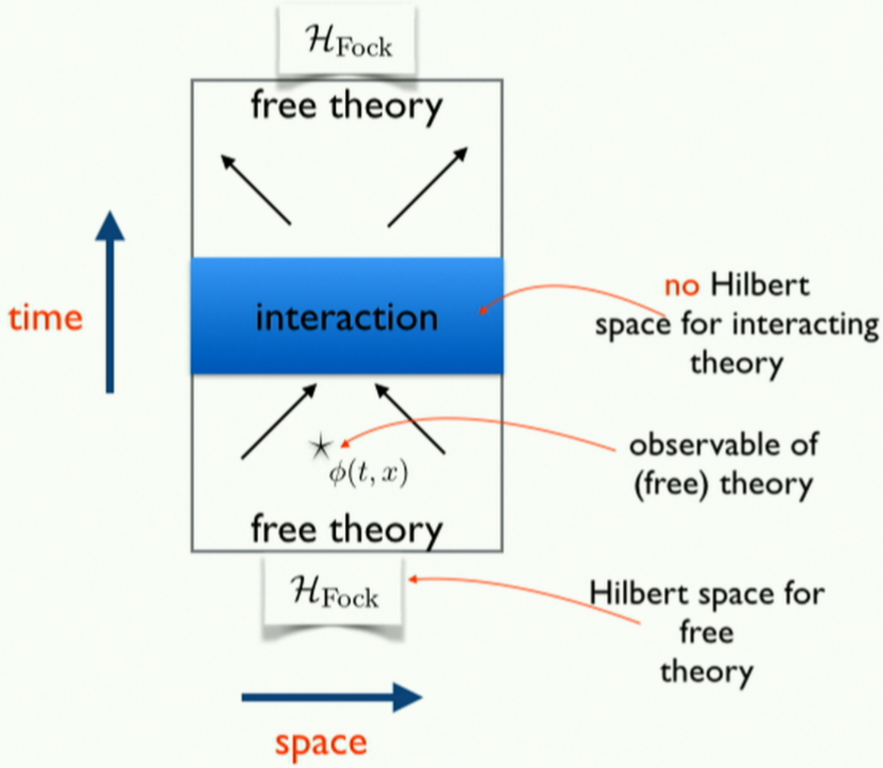
Overview

How to do quantum field theory without space time?

How to construct quantum geometry?

How to construct quantum space time?

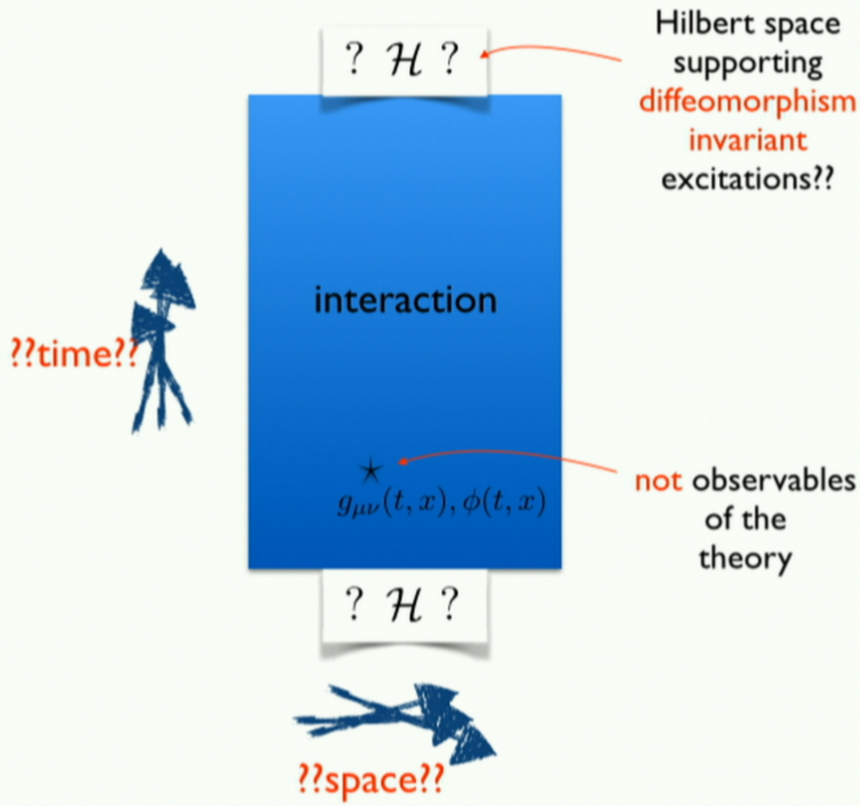
(Perturbative) Quantum Field Theory



Perturbative quantum gravity fails: non-renormalizable.

And does not answer crucial questions (eg big bang).

Quantum gravity



Space time coordinates have no physical significance. Need to implement diffeomorphisms invariance. This avoids assigning unphysical quantum fluctuations to choice of coordinates.

Aim:

Construct Hilbert space
supporting diffeomorphism invariant excitations
and operators to extract quantum geometry.

Examples: loop quantum gravity* ,
causal dynamical
triangulations, group field theories, …

* approach which most explicitly constructs
such a Hilbert space and quantum geometry

Progress

- 1990's: Ashtekar, Isham, Lewandowski:

  First construction of a spatially diffeomorphism invariant Hilbert space

  supporting the (kinematical) observable algebra of general relativity and matter.

  Based on a no-spatial-geometry vacuum. 2005: F-LOST uniqueness theorem.

- 2007: Koslowski (Sahlmann, Varadarajan, …):

  Condensation vacuum with background spatial geometry. Diffeomorphism co-variant.

- 2014: BD, Geiller (2015: Bahr, BD, Geiller):

  Construction of an alternative spatially diff invariant Hilbert space with no-curvature vacuum.

- 2013: BD, Steinhaus:

  Topological field theories (topological phases) give rise to spatially diff invariant Hilbert space.

- 2012/14: BD

  Strategy to obtain space-time diffeomorphism invariant theory:

  solving the full dynamics in an approximation scheme

Progress

- 1990's: Ashtekar, Isham, Lewandowski:

  First construction of a spatially diffeomorphism invariant Hilbert space supporting the (kinematical) observable algebra of general relativity and matter.

  Based on a no-spatial-geometry vacuum. 2005: F-LOST uniqueness theorem.

  Fleischhack - Lewandowski, Okolov, Sahlmann, Thiemann

There is only one Hilbert space representation, so we have to stick with it . It has to be the correct one. Even if it is a pain.

- 2014: BD, Geiller (2015: Bahr, BD, Geiller):

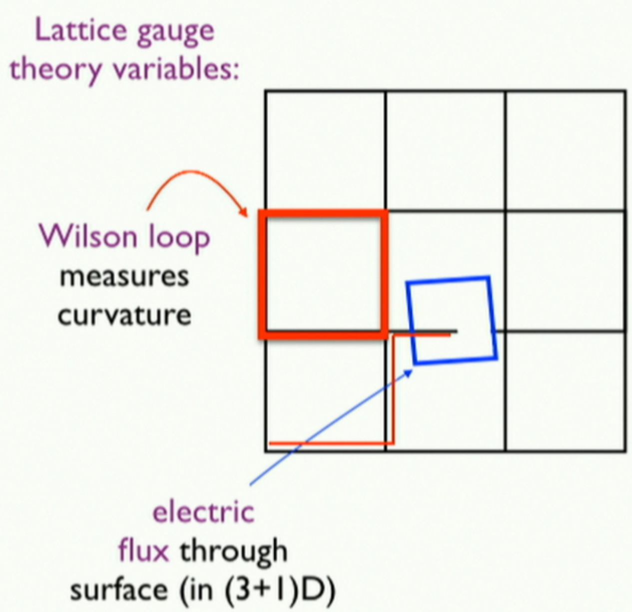
  Construction of an alternative spatially diffeomorphism invariant Hilbert space based on a no-curvature vacuum.

There are several Hilbert spaces. Can choose the most convenient one.

- 2013: BD, Steinhaus:

  Topological field theories (topological phases) give rise to spatially diffeomorphism invariant Hilbert spaces.

Non-perturbative qft: lattice qft and condensed matter systems



Using a lattice allows formulation of non-perturbative physics.

Problem: breaks diffeomorphism symmetry.

Key: use the same variables but do not restrict to the lattice.

Challenge: specifying a diffeomorphism invariant vacuum state 'everywhere'.

Building (loop) quantum geometry

=

representation of operators encoding

geometry on Hilbert space

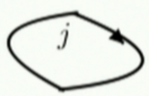
Quantum geometry operators

Ashtekar variables (1986)
Geometric variables (metric, curvature) can be encoded into variables of electro-magnetism (generalized to SU(2) Yang Mills).

magnetic field observable:
Wilson loop operator associated to a curve.

measures magnetic field integrated over enclosed surface
generates an electrical flux line

h^j_{curve}



gravity context:
measures (extrinsic) curvature,
generates 'quanta of spatial geometry'

electric field observable:
Electric flux associated to a surface.

measures electric field flux through surface
generates magnetic field

E_{surf}

gravity context:
measures (spatial) areas, angles, volumes
generates (extrinsic) curvature

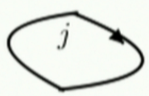
Quantum geometry operators

Ashtekar variables (1986)
Geometric variables (metric, curvature) can be encoded into variables of electro-magnetism (generalized to SU(2) Yang Mills).

magnetic field observable:
Wilson loop operator associated to a curve.

measures magnetic field integrated over enclosed surface
generates an electrical flux line

h^j_{curve}



gravity context:
measures (extrinsic) curvature,
generates 'quanta of spatial geometry'

electric field observable:
Electric flux associated to a surface.

measures electric field flux through surface
generates magnetic field

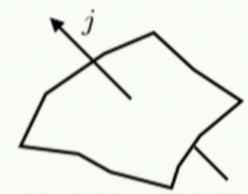
E_{surf}

gravity context:
measures (spatial) areas, angles, volumes
generates (extrinsic) curvature

Algebra of quantum geometry operators

$$[E_{\mathrm{surf}}, h^j_{\mathrm{curve}}] = h^j_{\mathrm{curve}} \circ (\tau)^j$$

Lie algebra generator



Only non-vanishing if holonomy curve (electric flux line) cuts through surface.

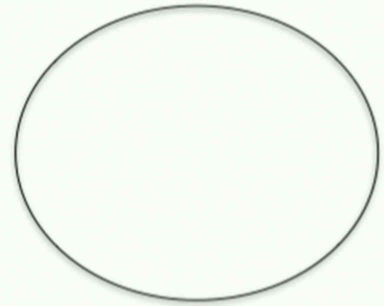
Is of topological nature.
(Does not need background metric.)

Building quantum geometry states: version I

[Ashtekar-Lewandowski-Isham representation, 90's]

the vacuum state:
all flux operators have vanishing expectation values and vanishing fluctuations

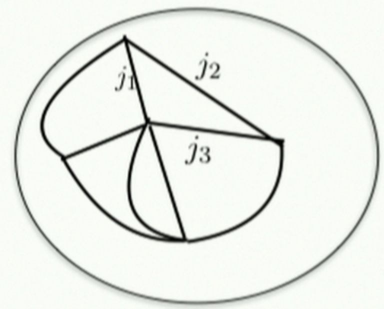
$$|0\rangle$$



excited states:
by applying Wilson loop operators,
some fluxes get non-vanishing expectation values.

encode spatial geometry!

$$h^{j_1}_{\mathrm{curve}_1} h^{j_2}_{\mathrm{curve}_2} h^{j_3}_{\mathrm{curve}_3} |0\rangle$$



First rigorous realization of quantum geometry.

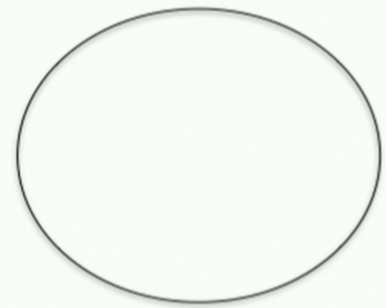
[Ashtekar, Rovelli, Smolin, Jacobson, Lewandowski, Isham, ……..]

Building quantum geometry states: version 1

[Ashtekar-Lewandowski-Isham representation, 90's]

the vacuum state:
all flux operators have vanishing expectation values and vanishing fluctuations

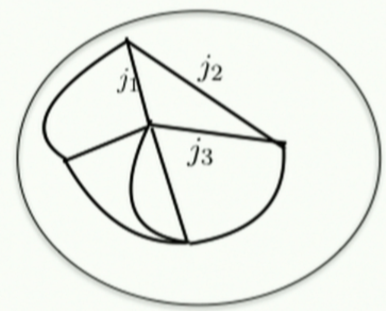
$$|0\rangle$$



excited states:
by applying Wilson loop operators,
some fluxes get non-vanishing expectation values.

encode spatial geometry!

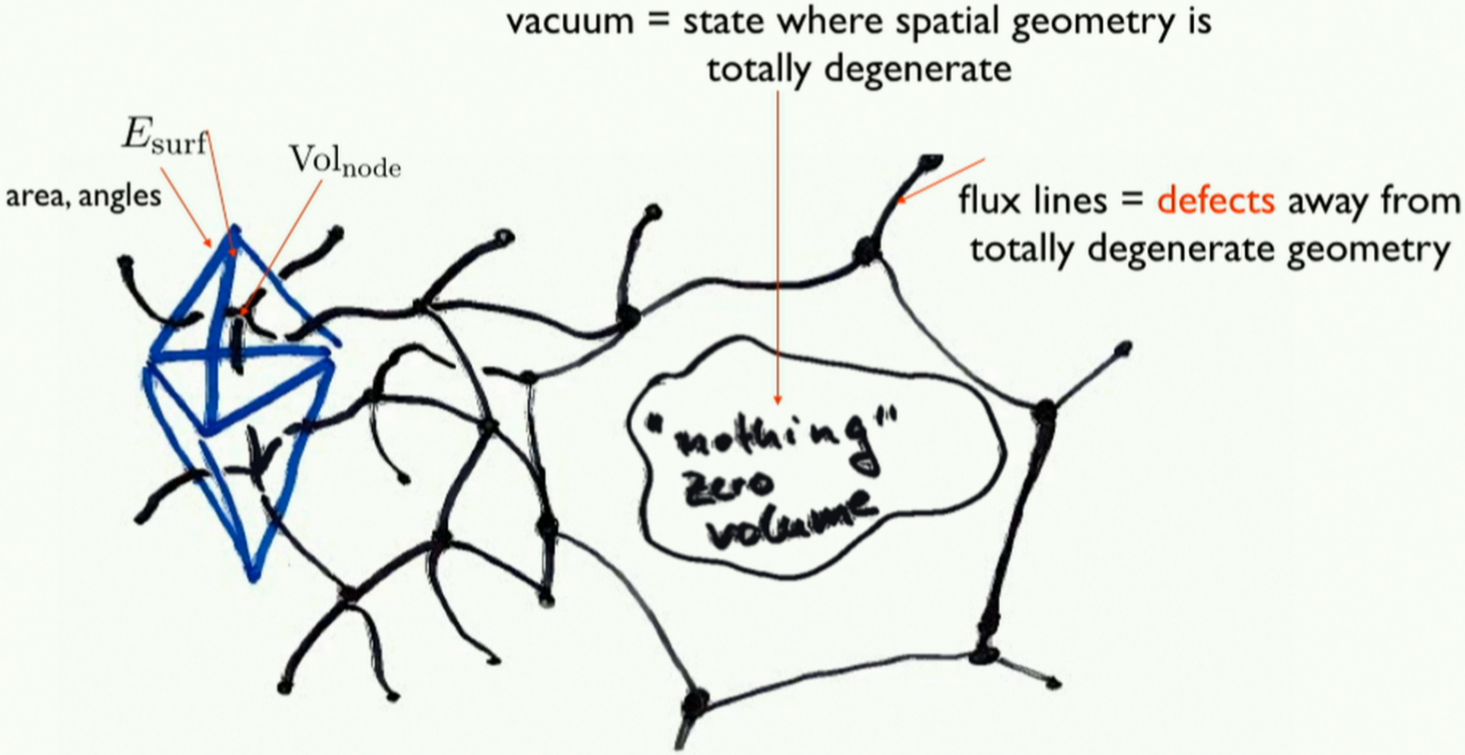
$$h^{j_1}_{\mathrm{curve}_1} h^{j_2}_{\mathrm{curve}_2} h^{j_3}_{\mathrm{curve}_3} |0\rangle$$



First rigorous realization of quantum geometry.

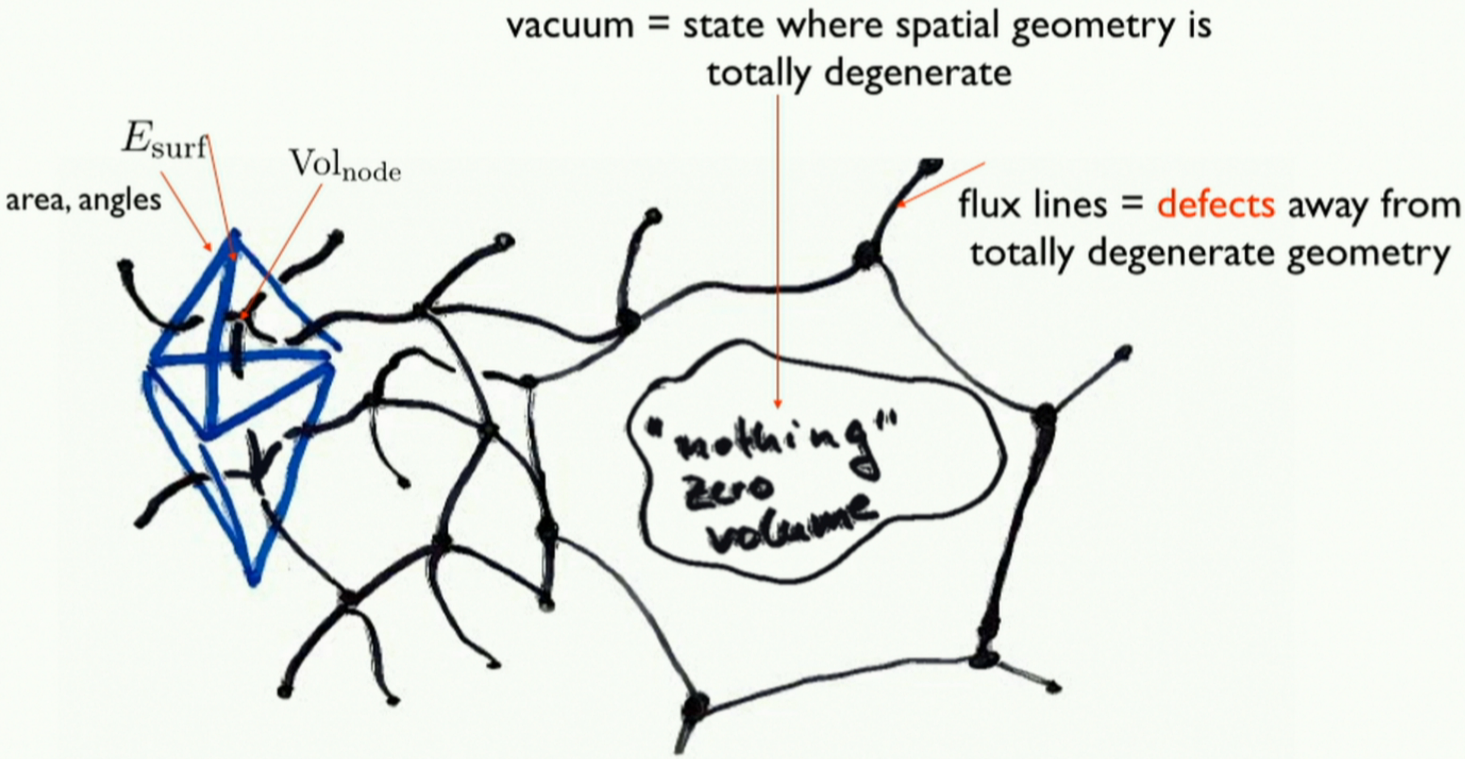
[Ashtekar, Rovelli, Smolin, Jacobson, Lewandowski, Isham, ……..]

Building quantum geometry states: version 1



Quantum state determines quantum geometry
(in a spatial diffeomorphism invariant way).

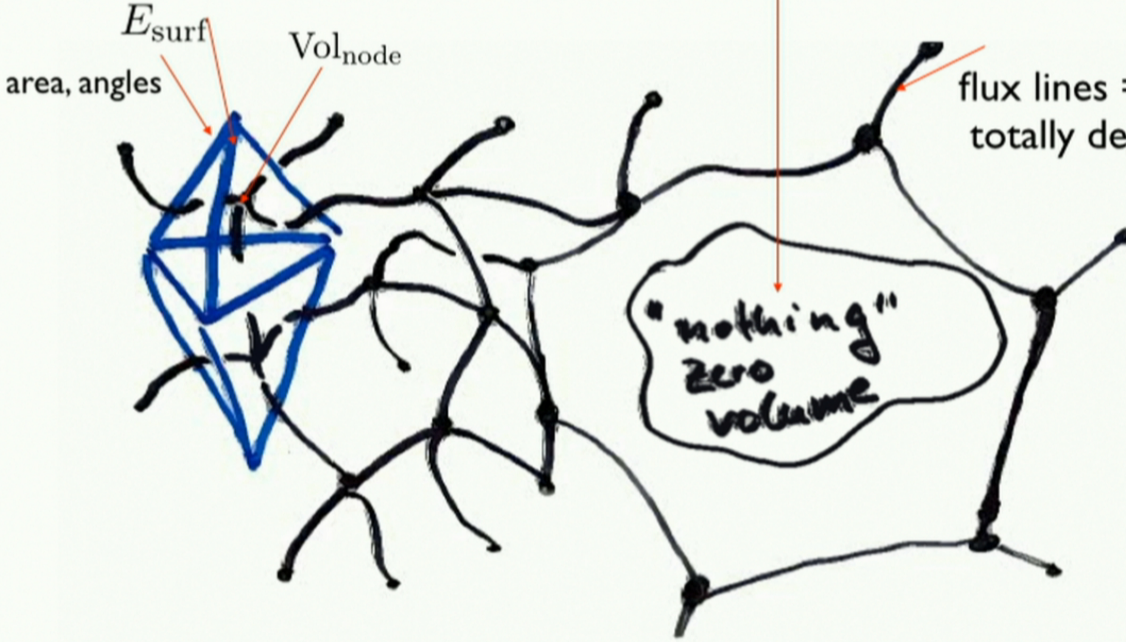
Building quantum geometry states: version 1



Quantum state determines quantum geometry
(in a spatial diffeomorphism invariant way).

Building quantum geometry states: version 1

vacuum = state where spatial geometry is totally degenerate



flux lines = defects away from totally degenerate geometry

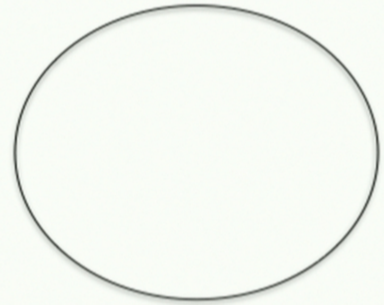
Quantum state determines quantum geometry
(in a spatial diffeomorphism invariant way).

Building quantum geometry states: version II

[BD, Geiller 14a,14b; Bahr, BD, Geiller to appear 15]

the vacuum state:
all curvature operators have vanishing
expectation values and vanishing fluctuations

$|0\rangle$

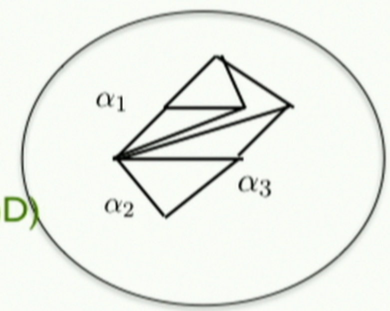


excited states:
by applying flux operators, some curvature operators get non-vanishing
expectation values.

only exponentiated
fluxes exist as operators

$\exp(\alpha_3 i E_{\mathrm{surf}_3}) \exp(\alpha_2 i E_{\mathrm{surf}_2}) \exp(\alpha_1 i E_{\mathrm{surf}_1}) |0\rangle$

curvature defects
along curves (in (3+1)D)

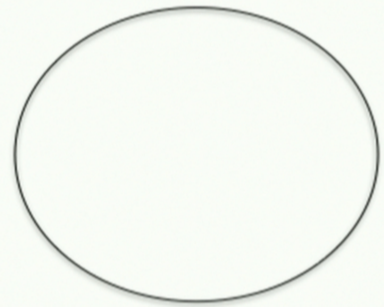


Building quantum geometry states: version II

[BD, Geiller 14a,14b; Bahr, BD, Geiller to appear 15]

the vacuum state:
all curvature operators have vanishing expectation values and vanishing fluctuations

$|0\rangle$

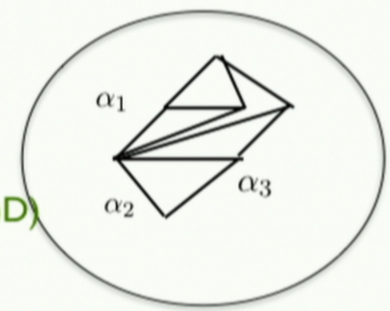


excited states:
by applying flux operators, some curvature operators get non-vanishing expectation values.

only exponentiated fluxes exist as operators

$$\exp(\alpha_3 i E_{\mathrm{surf}_3})\exp(\alpha_2 i E_{\mathrm{surf}_2})\exp(\alpha_1 i E_{\mathrm{surf}_1})\,|0\rangle$$

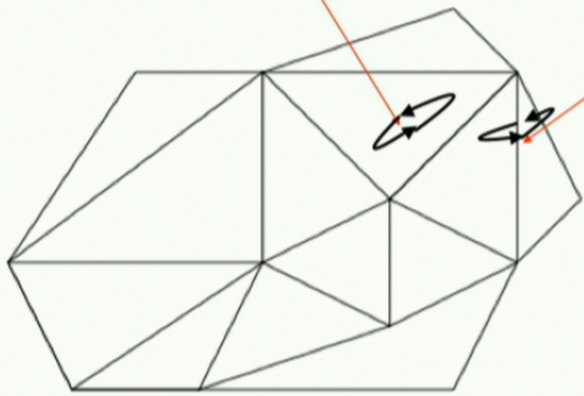
curvature defects along curves (in (3+1)D)



Building quantum geometry states: version II

[BD, Geiller 14a,14b; Bahr, BD, Geiller to appear 15]

vacuum peaked on vanishing curvature,
flux variables (spatial geometry) maximally uncertain



lines = curvature defects

state of
BF topological theory
with defects (= excitations)

Remark: Gives solution of
(2+1)D gravity
(with point particles).

Quantum state determines a (very different) quantum geometry
(in a spatial diffeomorphism invariant way).

Two vacua (and quantum-geometry representations)

[BD, Geiller 14a, 14b]

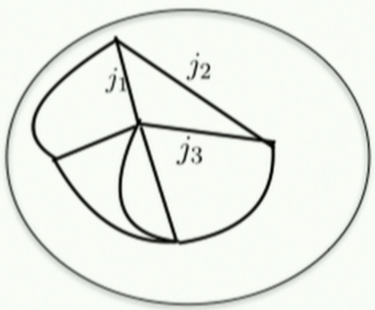
Phase transition

Ashtekar - Lewandowski - Isham vacuum (90's)

$$\psi_{\mathrm{vac}}(\{h_{\mathrm{curve}}\}) = 1$$

peaked on degenerate (spatial) geometry
maximal uncertainty in
(extrinsic) curvature

excitations:
spin network states supported on graphs
describing spatial geometry defects

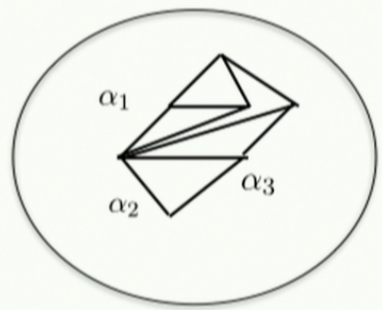


BF (topological) theory vacuum

$$\psi_{\mathrm{vac}}(\{h_{\mathrm{loops}}\}) = \prod_{\mathrm{loops}} \delta(h_{\mathrm{loops}})$$

peaked on vanishing
(Ashtekar connection) curvature
maximal uncertainty in spatial geometry

excitations:
curvature defects on edge network
(triangulation)



Quantum geometry dynamics?

All quantum geometry states describe 4D quantum geometry (histories), however (almost) all of these describe "virtual" (non-dynamical) quantum histories.

Need physical states, which solve the quantum equations of motion of the theory.

Why different (kinematical) vacua?

In standard qft: needed to describe symmetry breaking / condensation processes.

In q-gravity: need states satisfying the quantum equations of motions (physical states).

This is like asking for the energy eigenstates of an interacting quantum field theory: solving the theory.

Such states will not be (normalizable) in the Hilbert space we started with (kinematical Hilbert space).

Nevertheless some kinematical vacua might give easier access to physical states than other kinematical vacua.

Indeed BF theory is the starting point for spin foams, encoding the dynamics of (loop) quantum gravity.

What is a good vacuum (physical) state?

Should be adjusted to the dynamics of the system.

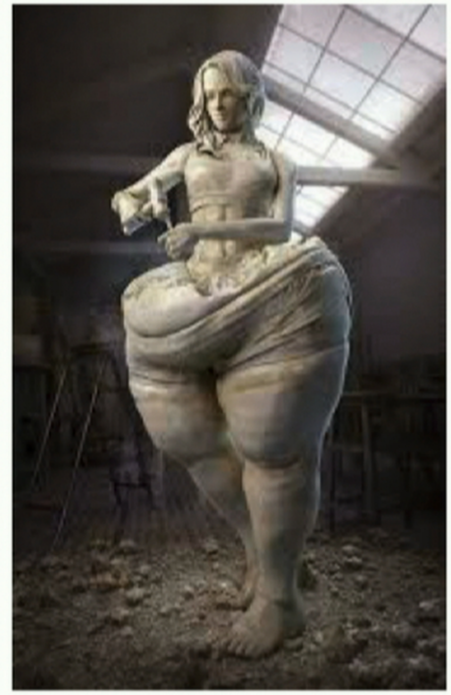
Time-evolution = applying path integral.

Usually:
Vacuum state should be invariant under time evolution.

In diff-invariant systems:
All physical states should be invariant under time evolution.

Path integral is a projector onto physical states.

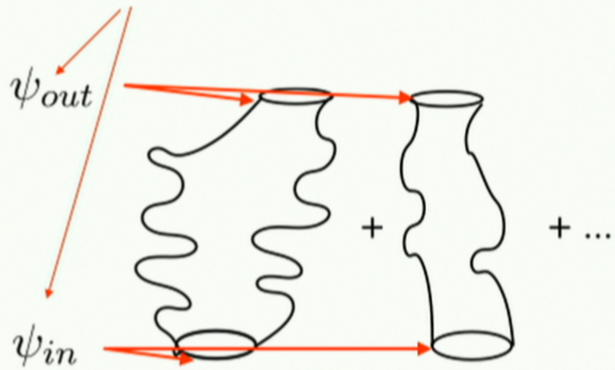
Need to construct the gravitational path integral.



Changing coupling constants and thus adjusting the the dynamics.

Path integral = sum over spacetime geometries

'boundary states' encode (actually 4D) quantum geometry



'sum' over quantum space time geometries

need to define how to sum

quantum amplitude

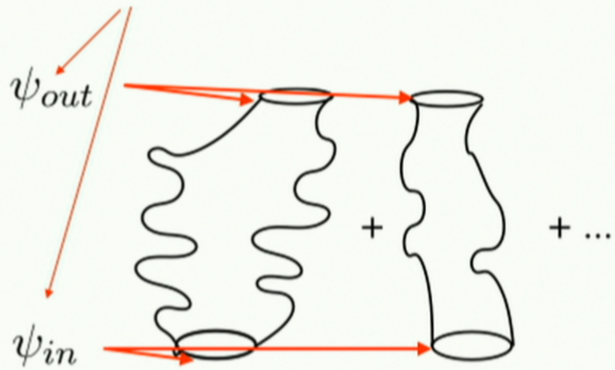
$$\langle \psi_{out} | \mathcal{P} | \psi_{in} \rangle = \overline{\psi_{out}}(\partial_{out}\mathrm{conf}\,1)\,\exp(\frac{i}{\hbar}S(\mathrm{conf}\,1))\,\psi_{in}(\partial_{in}\mathrm{conf}\,1) + \overline{\psi_{out}}(\partial_{out}\mathrm{conf}\,2)\,\exp(\frac{i}{\hbar}S(\mathrm{conf}\,2))\,\psi_{in}(\partial_{in}\mathrm{conf}\,2) + \ldots$$

path integral matrix element

Physical states: $\psi = \mathcal{P}\psi$

Path integral = sum over spacetime geometries

'boundary states' encode (actually 4D) quantum geometry



'sum' over quantum space time geometries

need to define how to sum

quantum amplitude

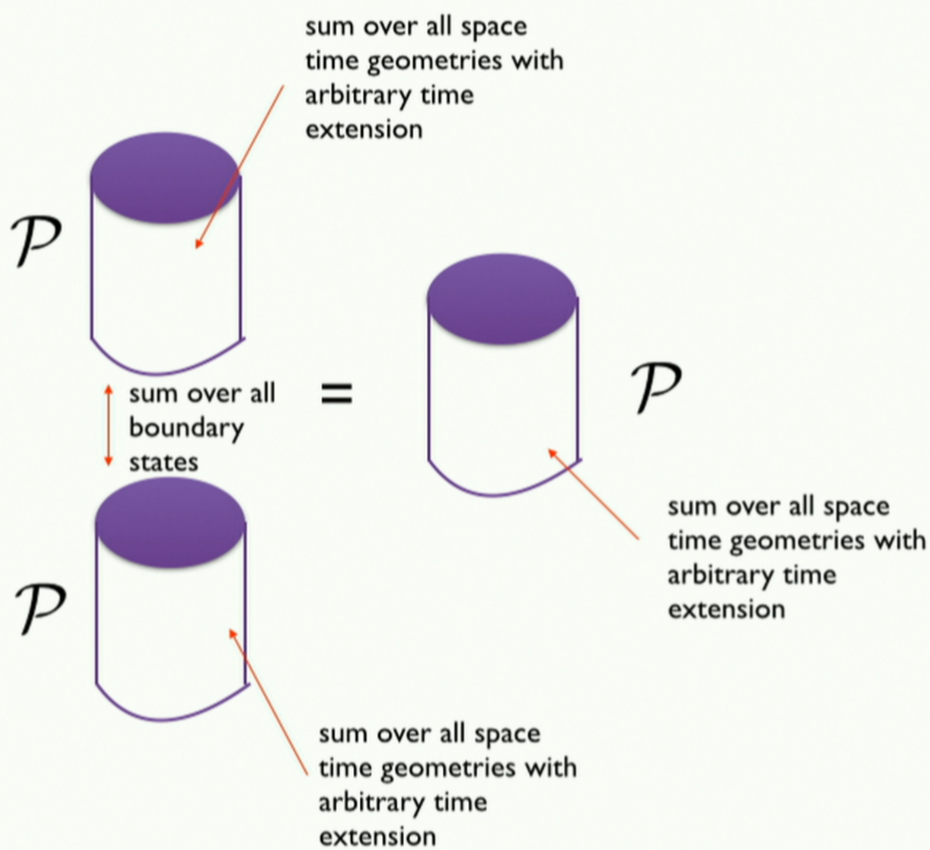
$$\langle \psi_{out} | \mathcal{P} | \psi_{in} \rangle \;=\; \overline{\psi_{out}}(\partial_{out}\mathrm{conf}\,1)\;\exp(\frac{i}{\hbar}S(\mathrm{conf}\,1))\;\psi_{in}(\partial_{in}\mathrm{conf}\,1)\;+$$
$$\overline{\psi_{out}}(\partial_{out}\mathrm{conf}\,2)\;\exp(\frac{i}{\hbar}S(\mathrm{conf}\,2))\;\psi_{in}(\partial_{in}\mathrm{conf}\,2)\;+\ldots$$

path integral matrix element

Physical states: $\psi \;=\; \mathcal{P}\psi$

Path integral is a projector

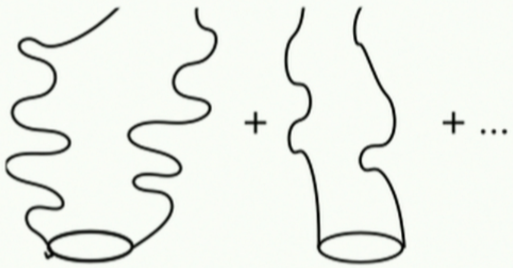
[Halliwell, Hartle 91]

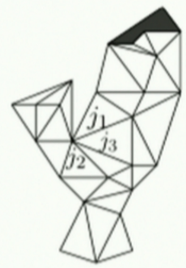


$$\mathcal{P} \circ \mathcal{P} = \mathcal{P}$$

projector property

Discretization and spin foam models





sum over geometries = sum over labels associated to the triangulation

construction of amplitudes from GR action → spin foam model

[Reisenberger, Rovelli, Barrett, Crane, Freidel, Krasnov, Livine, Speziale...]

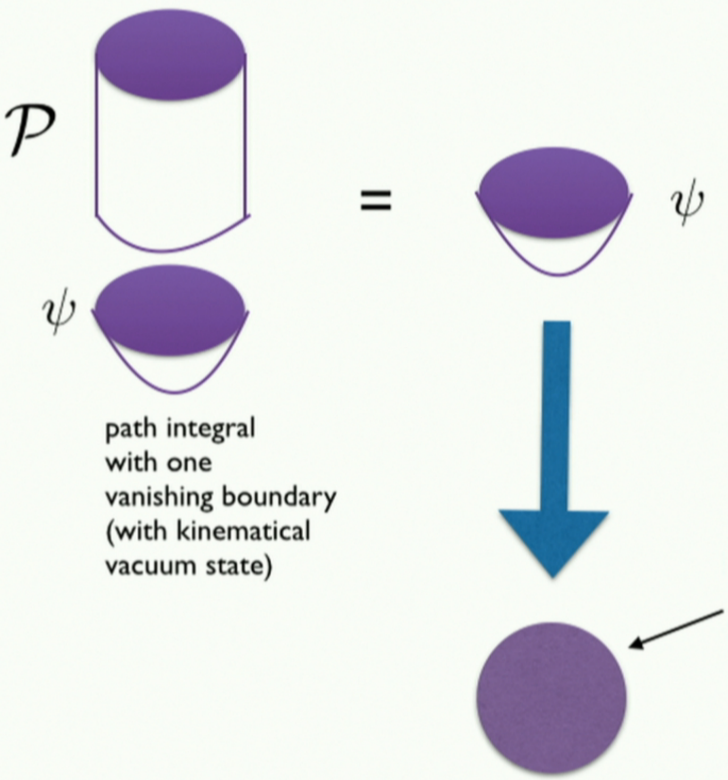
However the projector property can be expected to hold only in the refinement limit.

[Bahr, BD, Steinhaus 09 ...11]

Do we know states with $\psi = \mathcal{P}\psi$?

- In 3D: yes, the BF vacuum state
- In 4D: not yet for 'gravitational' spin foam models (it is actually the key problem)

How can we construct physical states?



(version of)

Hartle Hawking

no boundary wave function.

Is a physical state.

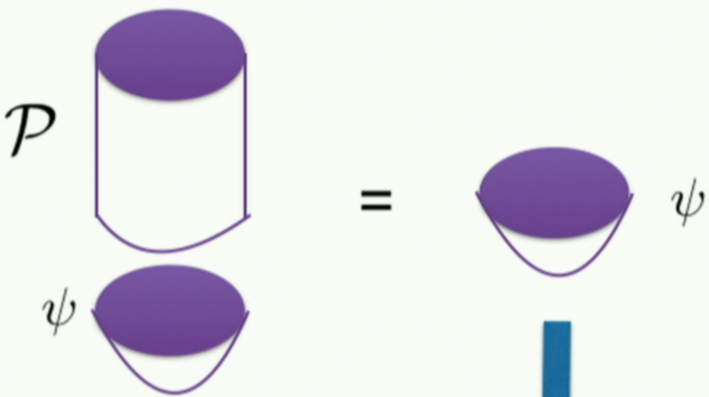
Path integral over a disk gives

vacuum functional for boundary wave functions

$$\mathcal{A}_{vac}(\psi_{out}) = \langle \psi_{out} | \mathcal{P} | \emptyset \rangle$$

<u>encodes (continuum) dynamics.</u>

How can we construct physical states?



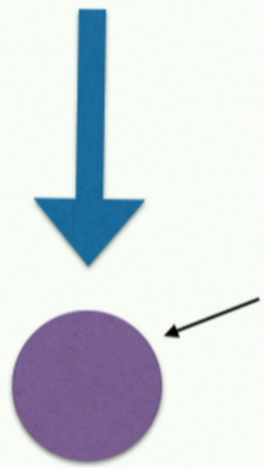
path integral
with one
vanishing boundary
(with kinematical
vacuum state)

(version of)

Hartle Hawking

no boundary wave function.

Is a physical state.



Path integral over a disk gives

vacuum functional for boundary wave functions

$$\mathcal{A}_{vac}(\psi_{out}) = \langle \psi_{out} | \mathcal{P} | \emptyset \rangle$$

<u>encodes (continuum) dynamics.</u>

Need to compute the path integral in the refinement limit.

Problem: Extremely difficult for 4D (gravitational) spin foams.

- cannot apply Monte Carlo simulations, due to complex amplitudes
- additional difficulties: infinite summations and (emerging) divergencies due to diffeomorphism symmetry
- so far no real space coarse graining method for 4D spin foam models available [BD, Mizera, Steinhaus14] (but under tensor network method are under development)

Devised 2D 'analogue models' capturing key dynamical ingredients of spin foams.

- mimics a 2D-4D duality of lattice gauge theory to spin systems
- hope that phase structure is similar [BD, Eckert, Martin-Benito, Schnetter, Steinhaus, 11-13]
- path integral / refinement limit can be computed via tensor network renormalization [Vidal, Levin-Nave, Gu-Wen, …]

Need to compute the path integral in the refinement limit.

Problem: Extremely difficult for 4D (gravitational) spin foams.

- cannot apply Monte Carlo simulations, due to complex amplitudes
- additional difficulties: infinite summations and (emerging) divergencies due to diffeomorphism symmetry
- so far no real space coarse graining method for 4D spin foam models available (but under tensor network method are under development) [BD, Mizera, Steinhaus14]

Devised 2D 'analogue models' capturing key dynamical ingredients of spin foams.

- mimics a 2D-4D duality of lattice gauge theory to spin systems
- hope that phase structure is similar [BD, Eckert, Martin-Benito, Schnetter, Steinhaus, 11-13]
- path integral / refinement limit can be computed via tensor network renormalization [Vidal, Levin-Nave, Gu-Wen, …]

Need to compute the path integral in the refinement limit.

Problem: Extremely difficult for 4D (gravitational) spin foams.

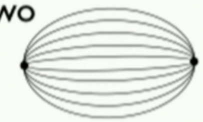
- cannot apply Monte Carlo simulations, due to complex amplitudes
- additional difficulties: infinite summations and (emerging) divergencies due to diffeomorphism symmetry
- so far no real space coarse graining method for 4D spin foam models available [BD, Mizera, Steinhaus14] (but under tensor network method are under development)

Devised 2D 'analogue models' capturing key dynamical ingredients of spin foams.

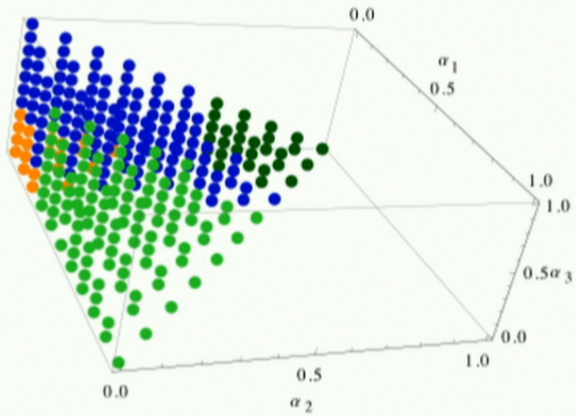
- mimics a 2D-4D duality of lattice gauge theory to spin systems
- hope that phase structure is similar [BD, Eckert, Martin-Benito, Schnetter, Steinhaus, 11-13]
- path integral / refinement limit can be computed via tensor network renormalization [Vidal, Levin-Nave, Gu-Wen, …]

Phase diagram for spin foam analogues

- models are similar to anyonic spin chains [Feiguin et al 06]
- but can be also interpreted as particular spin foams describing the gluing of two space time atoms



- changing certain parameters in initial model: changes how the atoms glue (technically: changes implication of simplicity constraints)
- anyonic spin chains support very rich phase structure, classification in [BD, Kaminski 13 and to appear]



Interpretation: different phases describe uncoupled space time atoms (green) and coupled space time atoms (orange,blue).

Positive indication for finding a geometric phase in spin foams.

[BD, Martin-Benito, Schnetter NJP 13]

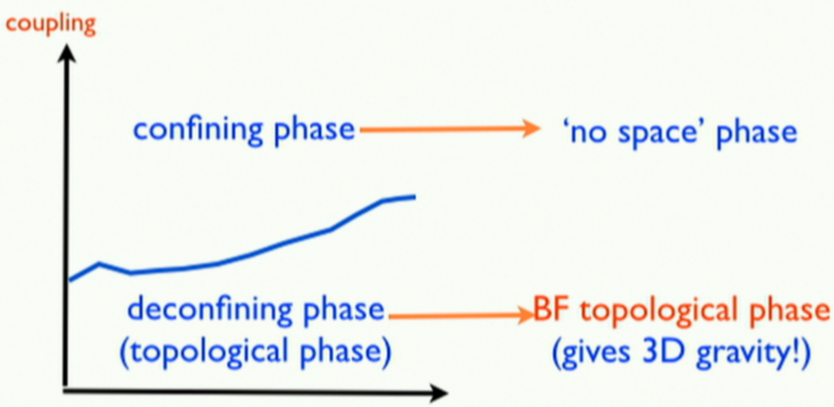
BD, Martin-Benito, Steinhaus PRD 13]

Phase diagram for spin foams ?

- need to develop (tensor network) coarse graining algorithms for

  spin foams = generalized lattice gauge theories
- first algorithm for 3D Abelian lattice gauge theories: decorated tensor networks [BD, Mizera, Steinhaus 14]
- 3D Non-Abelian lattice gauge theories [Delcamp, BD to appear]

Phases in lattice gauge theory



Are there more phases in spin foams?

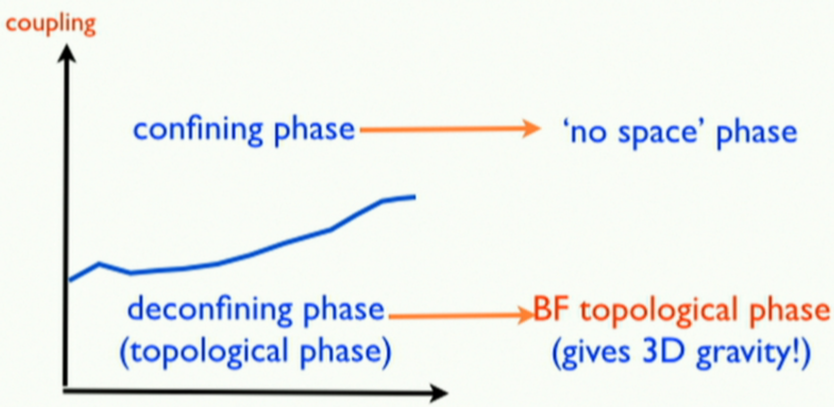
Positive indication from

2D analogue models.

Phase diagram for spin foams ?

- need to develop (tensor network) coarse graining algorithms for
  spin foams = generalized lattice gauge theories
- first algorithm for 3D Abelian lattice gauge theories: decorated tensor networks
  [BD, Mizera, Steinhaus 14]
- 3D Non-Abelian lattice gauge theories [Delcamp, BD to appear]

Phases in lattice gauge theory

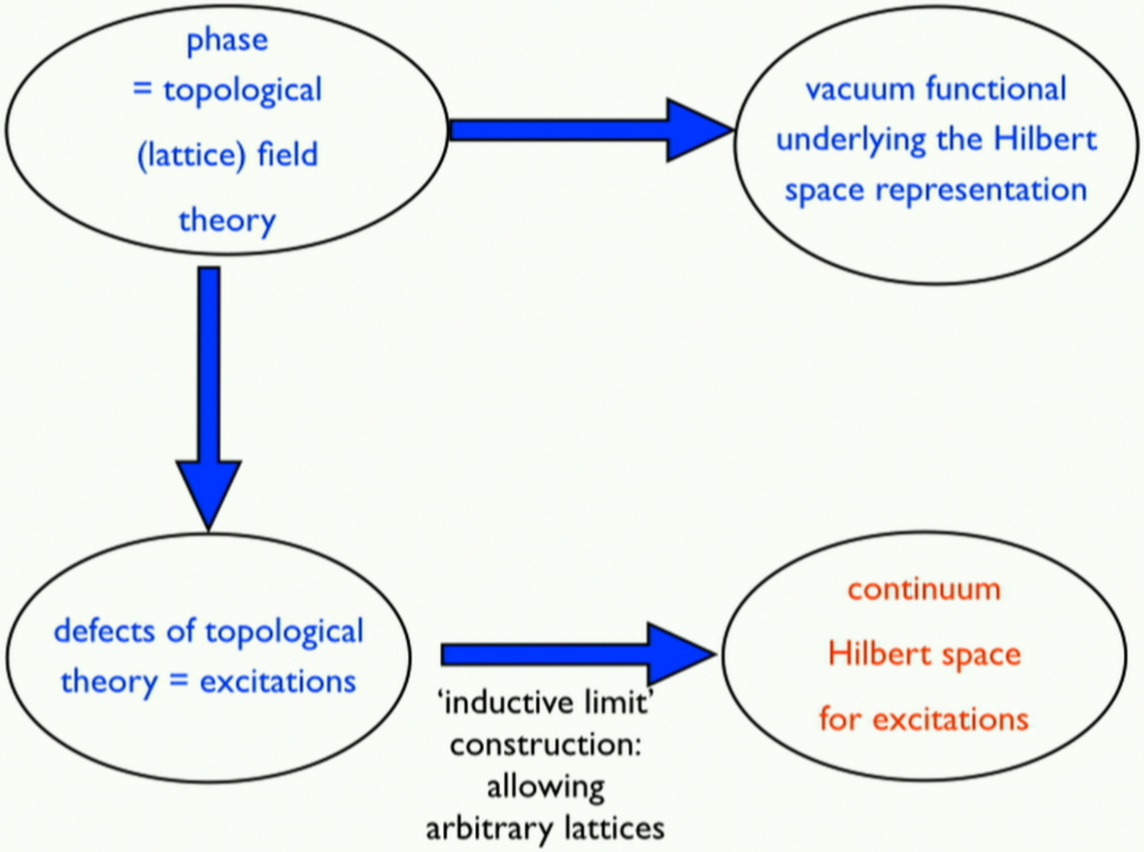


Are there more phases in spin foams?

Positive indication from 2D analogue models.

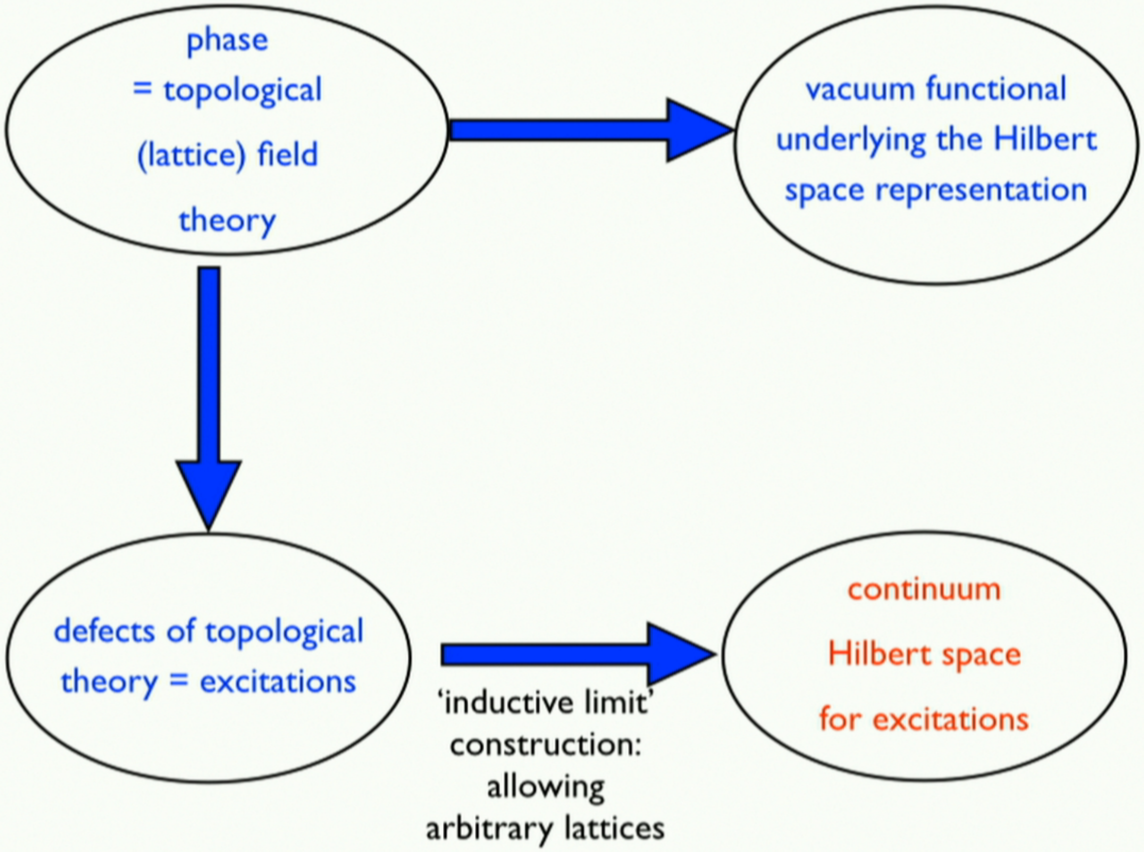
New phases give rise to new vacua and new quantum geometry realizations

[BD, Steinhaus NJP 13]

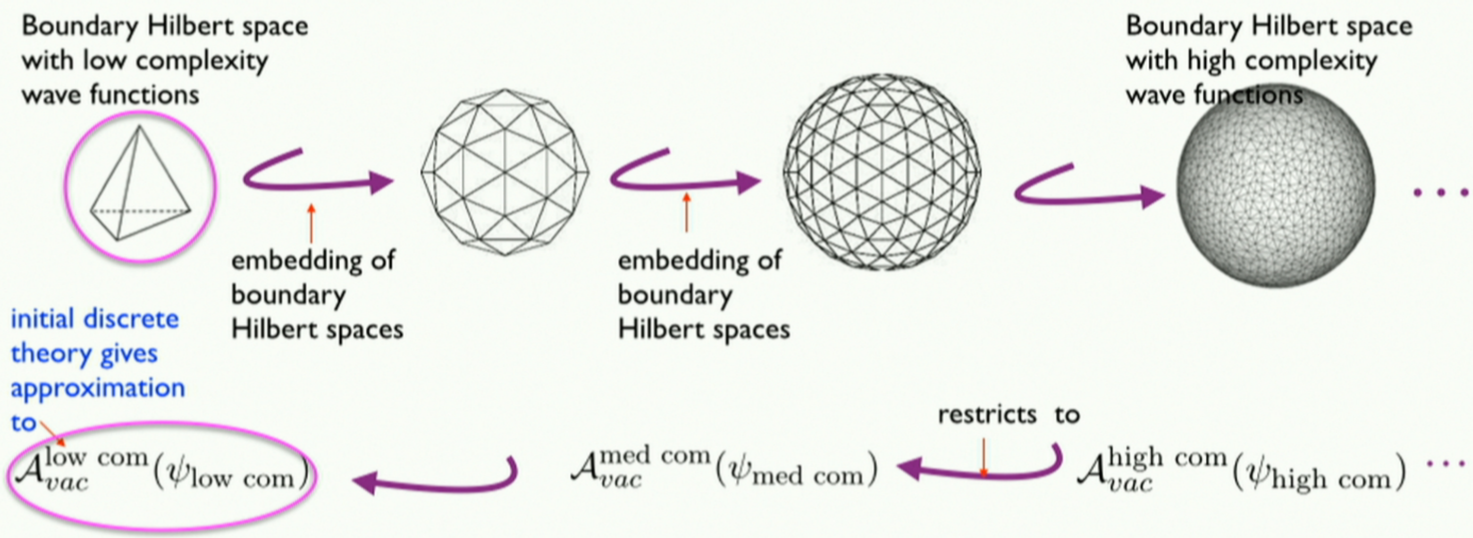


New phases give rise to new vacua and new quantum geometry realizations

[BD, Steinhaus NJP 13]



How to express the continuum dynamics [BD NJP 12, 14]



A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

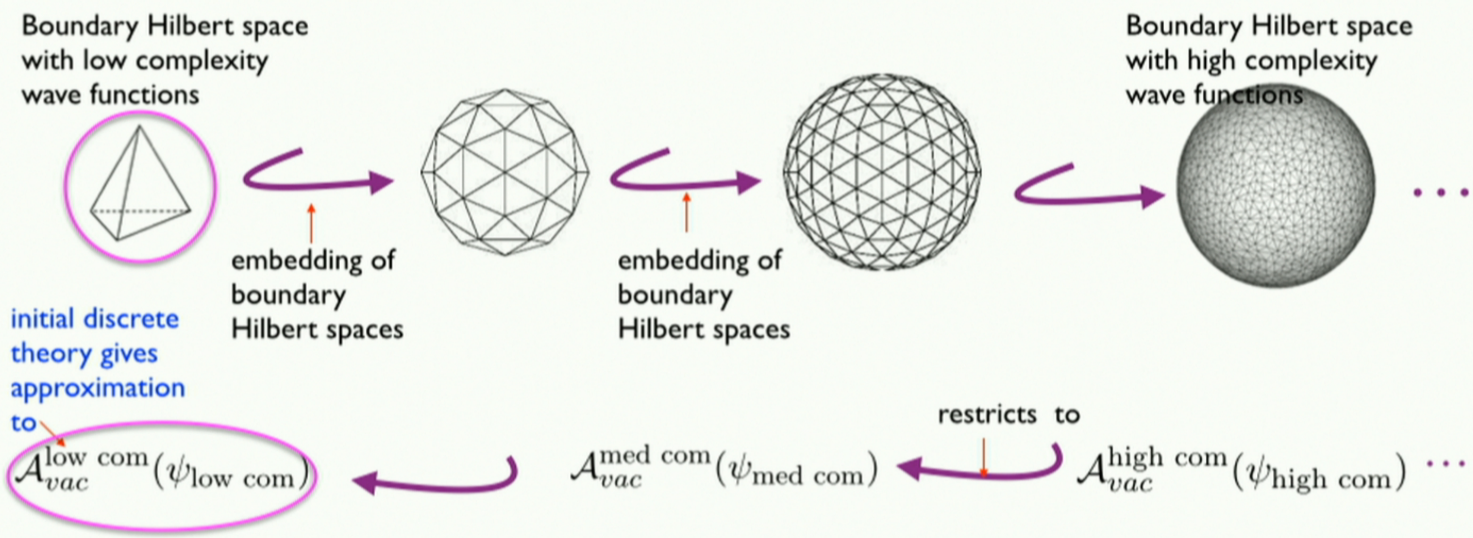
* Corresponds to a complete renormalization trajectory,
with scale given by complexity parameter.

Amplitudes can be computed iteratively in an approximation scheme.

Least effort necessary for low complexity = homogeneous configurations.

[BD NJP 12, 14]

How to express the continuum dynamics [BD NJP 12, 14]



A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

Amplitudes can be computed iteratively in an approximation scheme.

Least effort necessary for low complexity = homogeneous configurations.

[BD NJP 12, 14]

Summary

Quantum gravity models as many body system

- tensor network algos
- categorification

Identify phases and transitions

- (modified) inductive limit construction

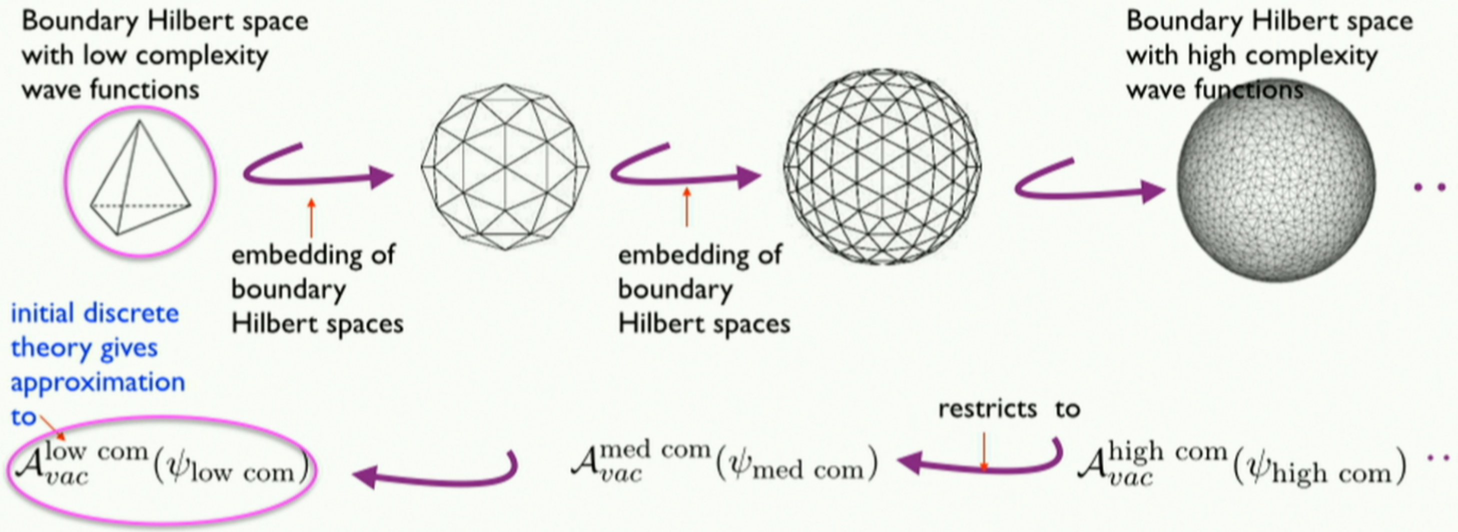
New quantum geometry realizations

- computing refinement limit with tensor network algos

continuum limit: consistent family of amplitudes

Quantum Space Time

How to express the continuum dynamics [BD NJP 12, 14]



A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

Amplitudes can be computed iteratively in an approximation scheme.

Least effort necessary for low complexity = homogeneous configurations.

[BD NJP 12, 14]


EXIT