Title: Braiding statistics and symmetry-protected topological phases

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Abstract: <p>Symmetry-protected topological (SPT) phases can be thought of as generalizations of topological insulators. Just as topological insulators have robust boundary modes protected by time reversal and charge conservation symmetry, SPT phases have boundary modes protected by more general symmetries. In this talk, I will describe a method for analyzing 2D and 3D SPT phases using braiding statistics. More specifically, I will show that 2D and 3D SPT phases can be characterized by gauging their symmetries and studying the braiding statistics of their gauge flux excitations. The 3D case is of particular interest as it involves a generalization of quasiparticle braiding statistics to three dimensions.</p>

Braiding statistics and symmetry-protected topological phases

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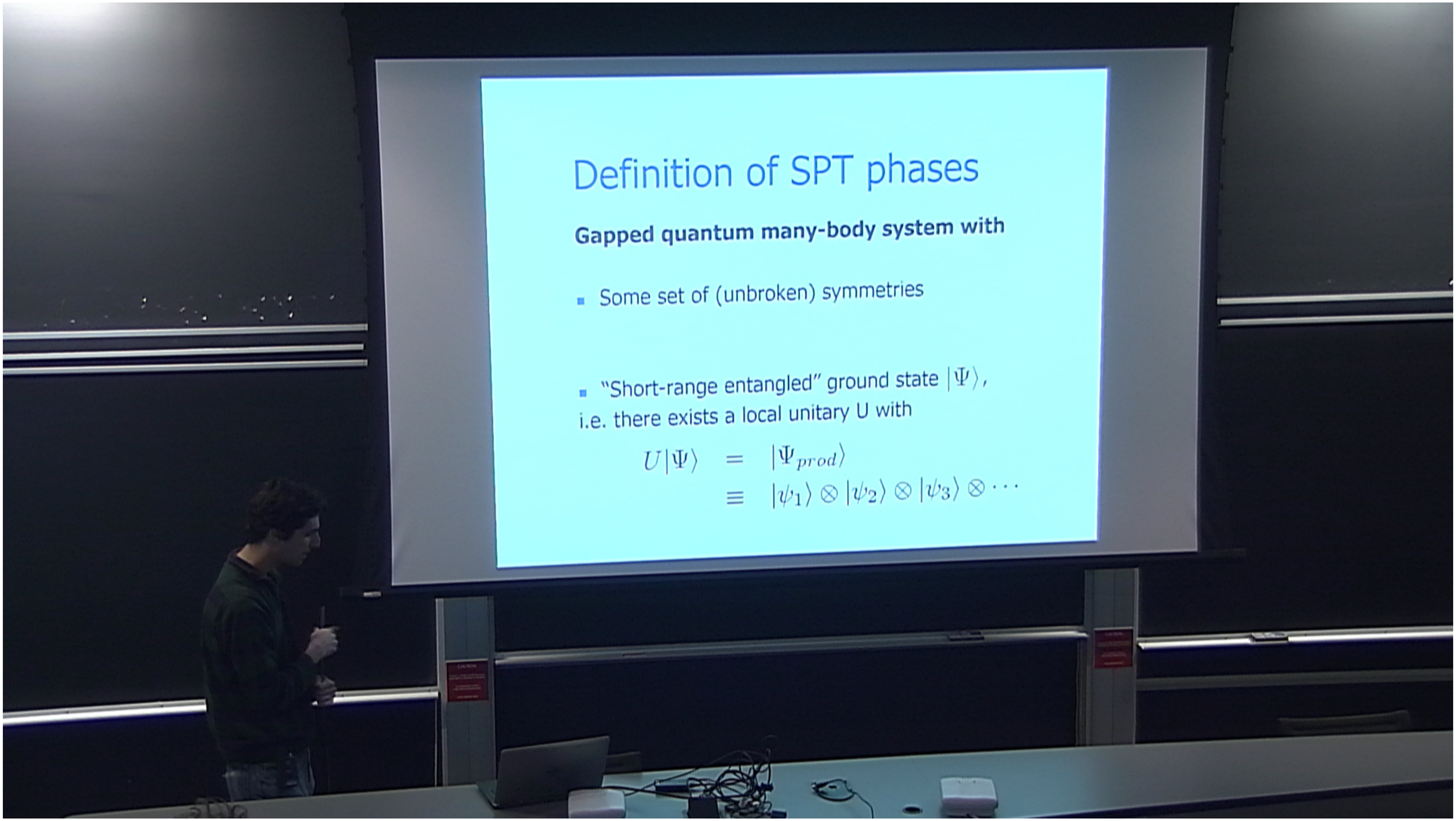
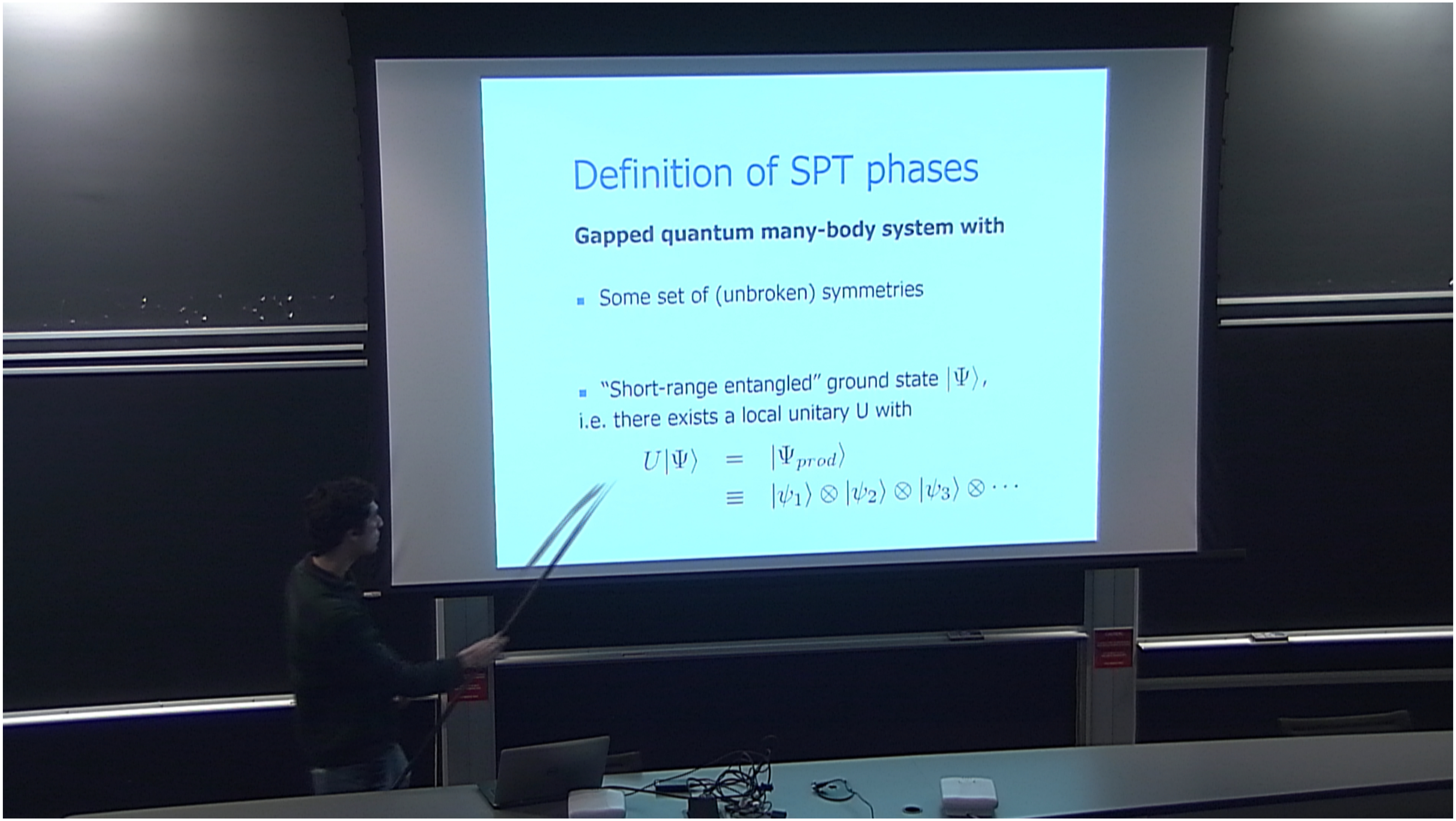

Definition of SPT phases
Gapped quantum many-body system with
Some set of (unbroken) symmetries
"Short-range entangled" ground state $|\Psi\rangle$,
i.e. there exists a local unitary U with
$U|\Psi\rangle = |\Psi_{prod}\rangle$
$\equiv |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \otimes \cdots$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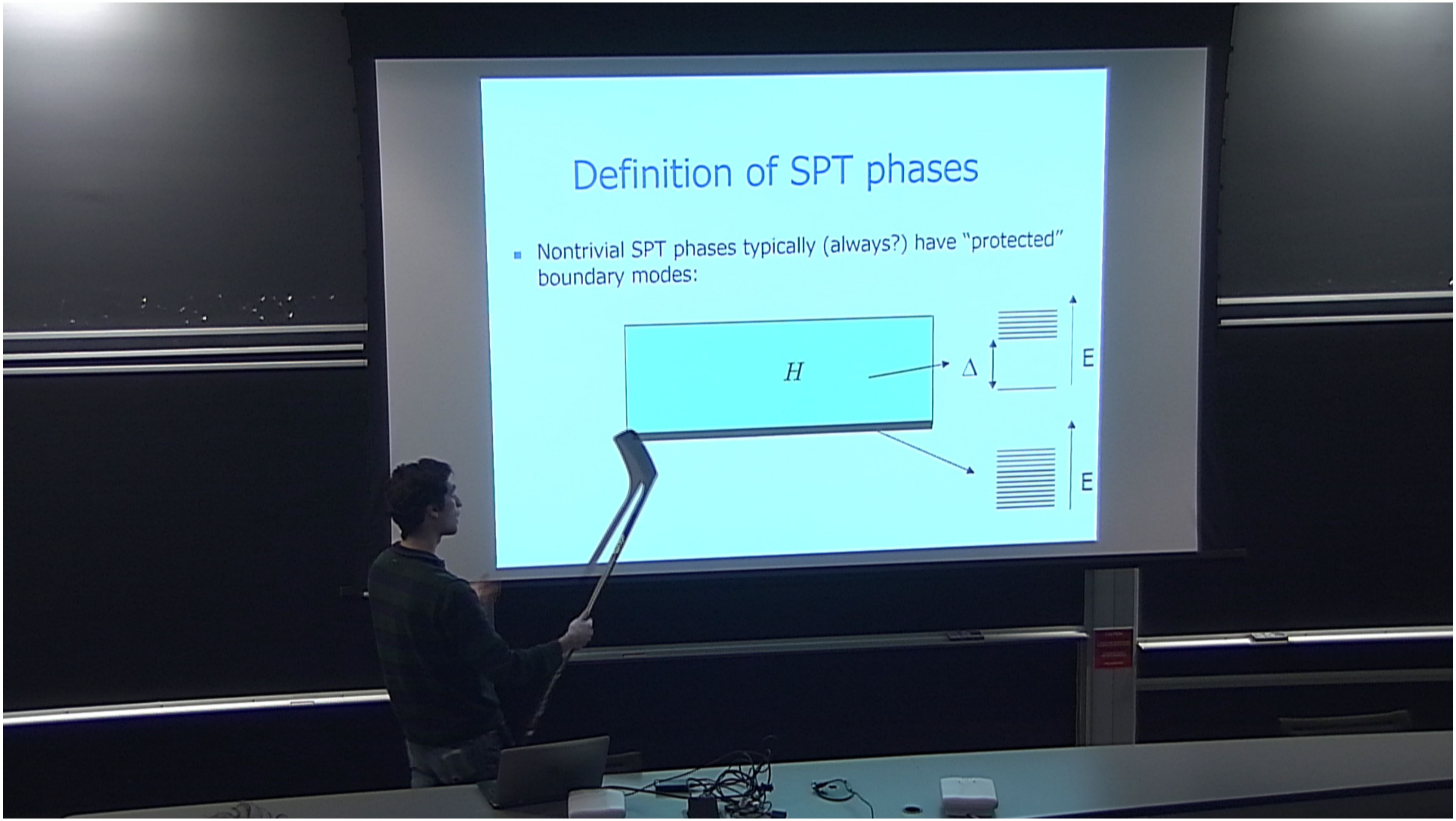
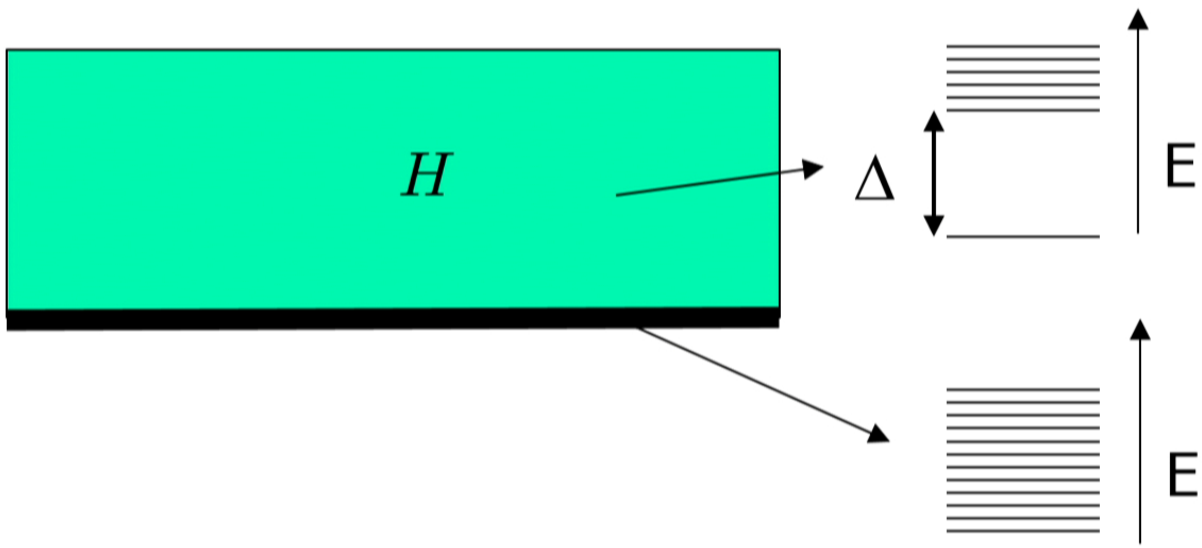
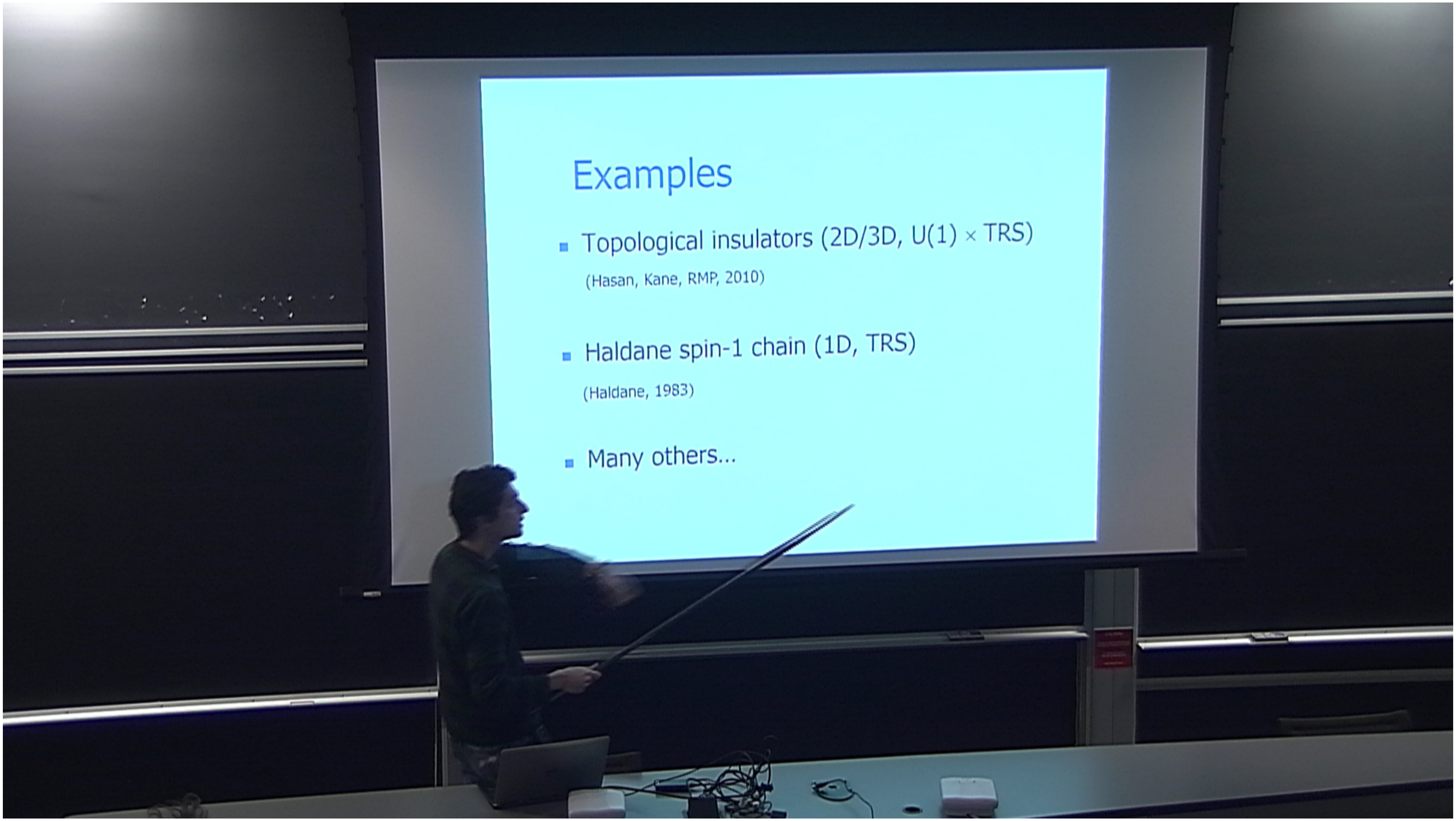

Definition of SPT phases
Nontrivial SPT phases typically (always?) have "protected" boundary modes:
H
Δ
E
E

Definition of SPT phases

- Nontrivial SPT phases typically (always?) have "protected" boundary modes:




Examples
Topological insulators (2D/3D, U(1) × TRS)
(Hasan, Kane, RMP, 2010)
Haldane spin-1 chain (1D, TRS)
(Haldane, 1983)
Many others...

Basic questions about SPT phases

- **Classification**: For each symmetry group and spatial dimension, how many SPT phases are there?

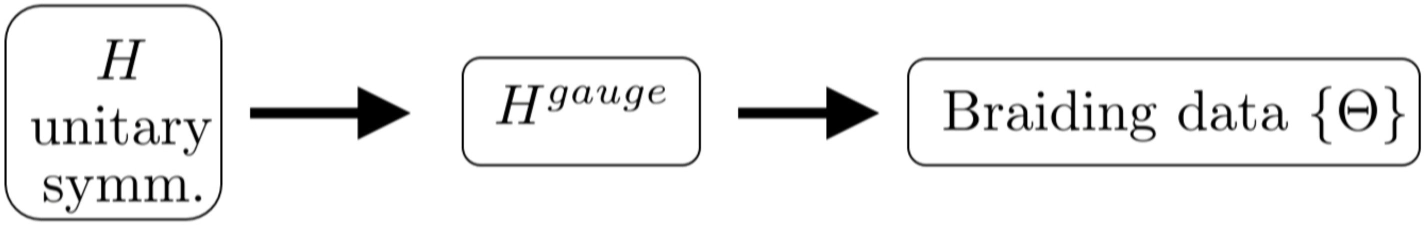
  - Non-interacting fermions (Schnyder et al, Kitaev, 2008)

  - General boson systems (Chen, Gu, Liu, Wen, 2011)

- **Characterization**: How can we determine whether a microscopic model belongs to a specific SPT phase?

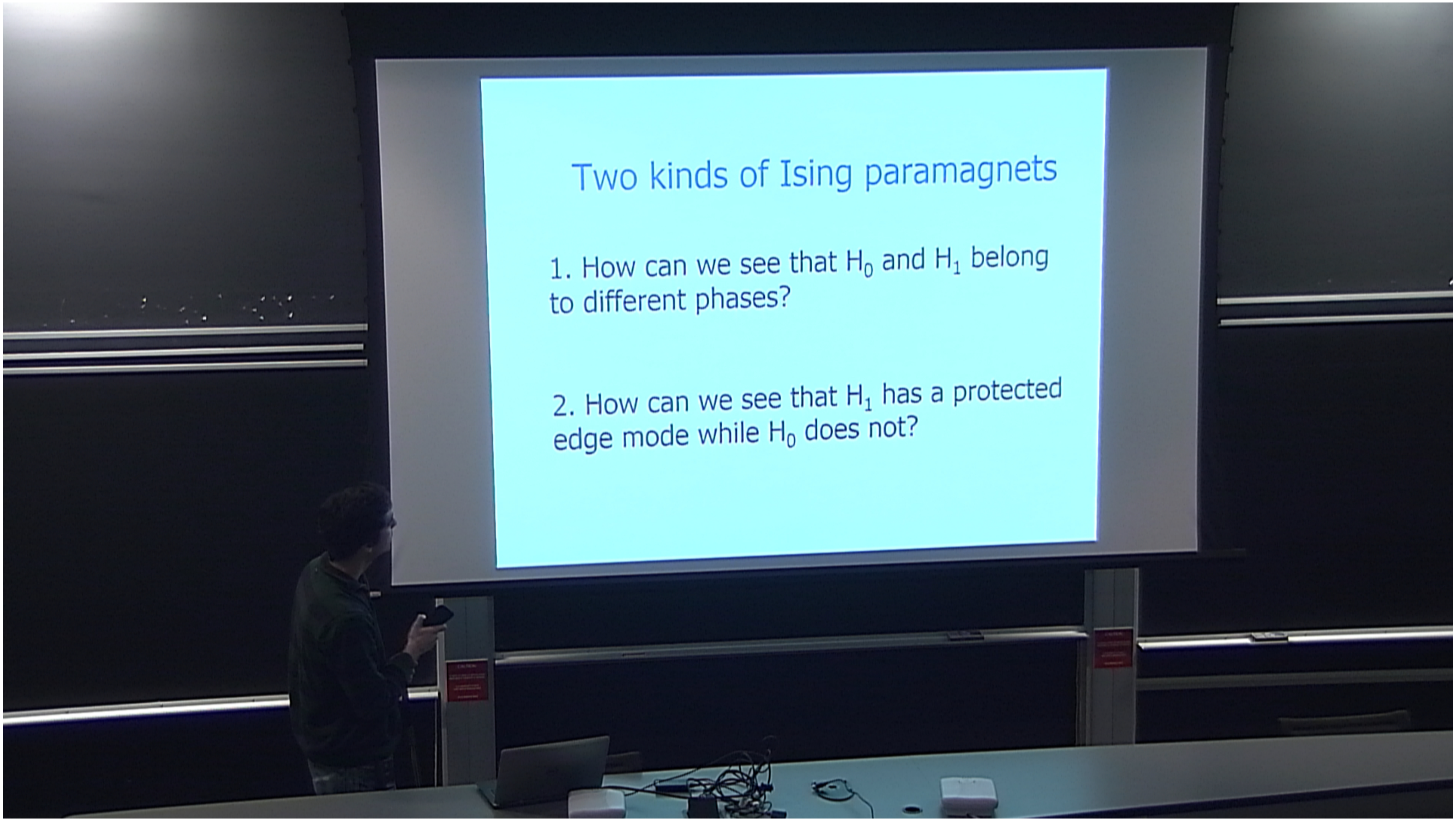
(ML, Z. Gu, 2012)
(C. Wang, ML, 2014)
(Jiang, Mesaros, Ran, 2014)
(C.-H. Lin, ML, in preparation)

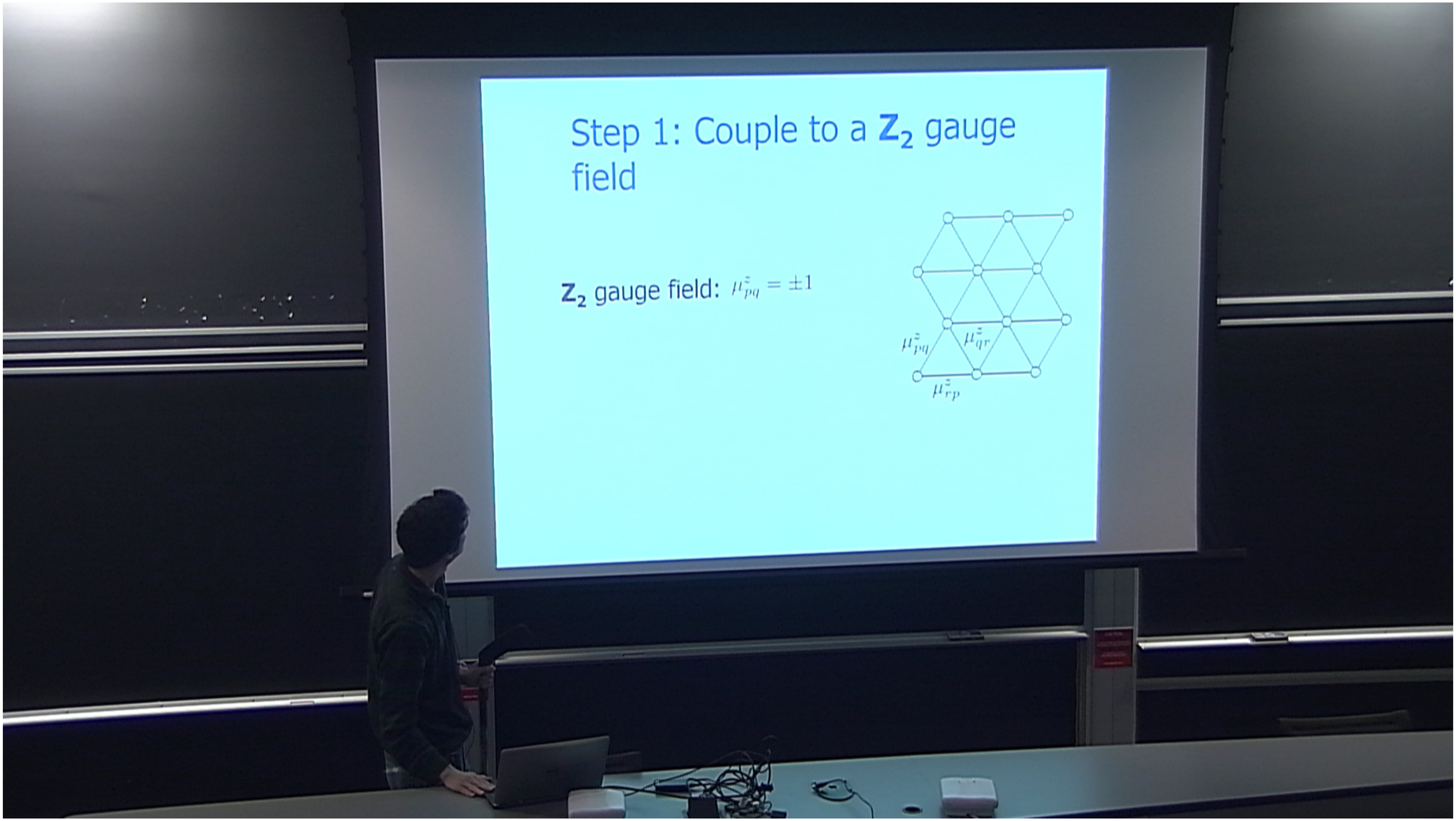
Main results

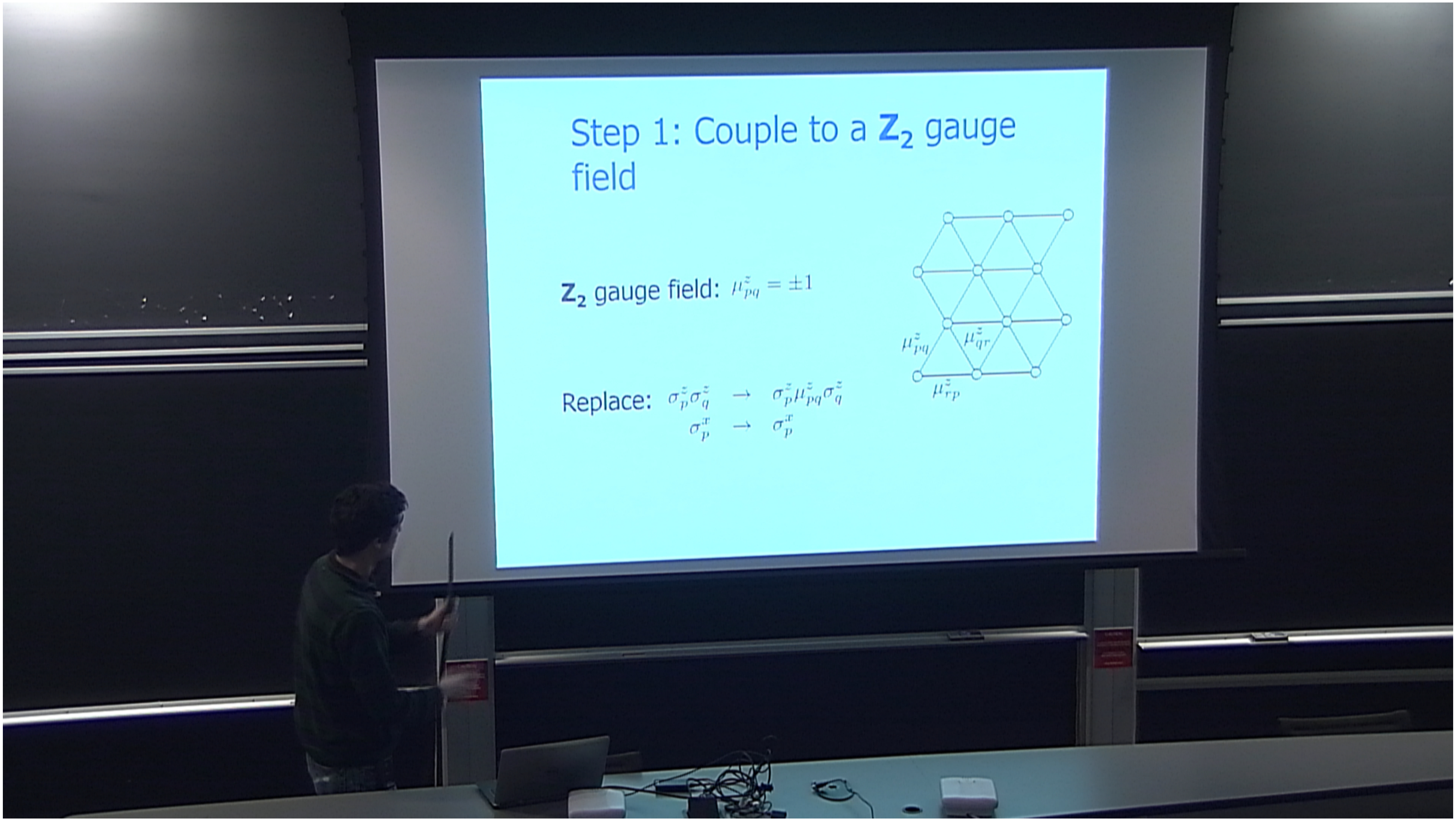


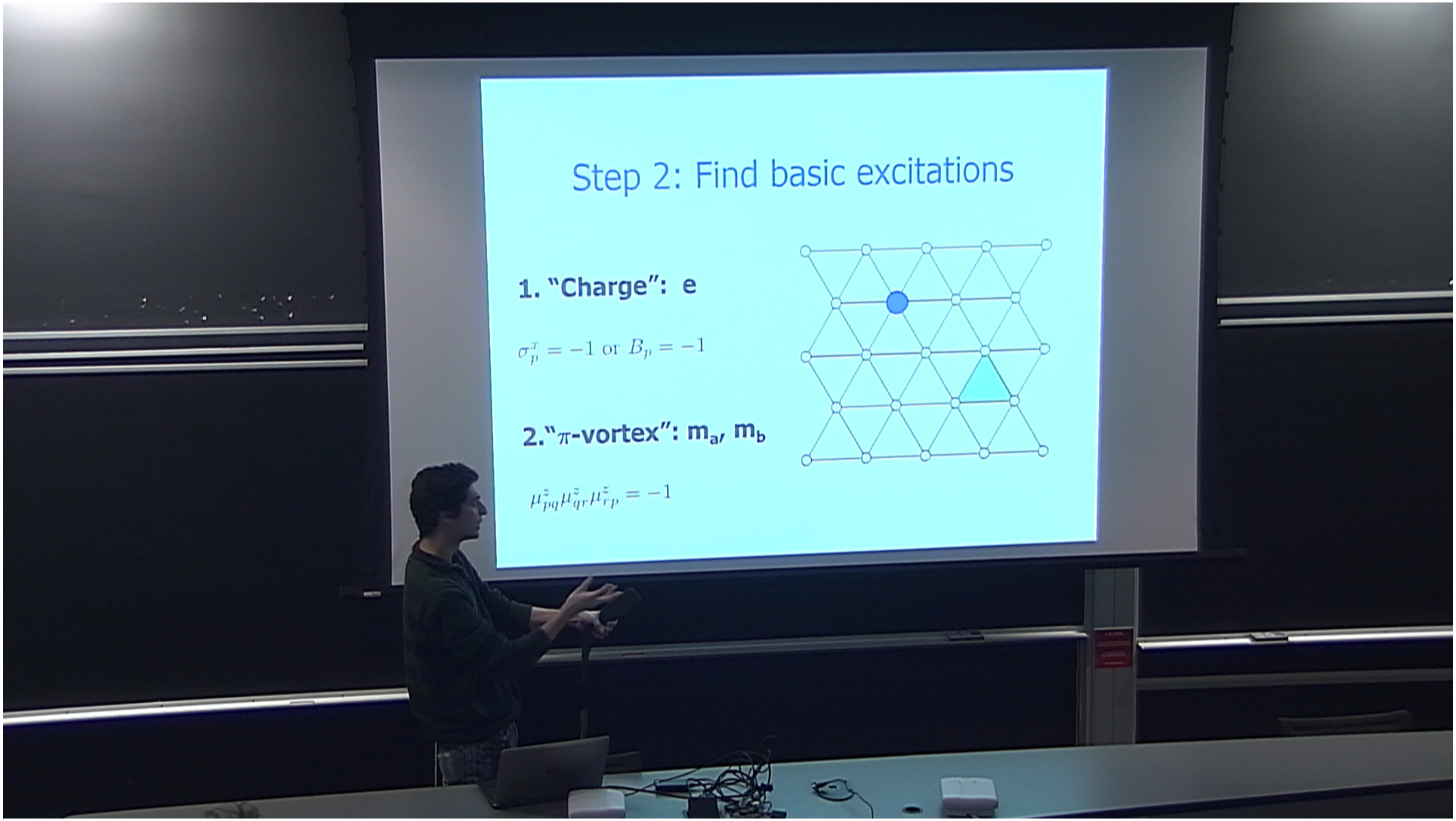
1. Braiding data distinguishes “many” SPT phases

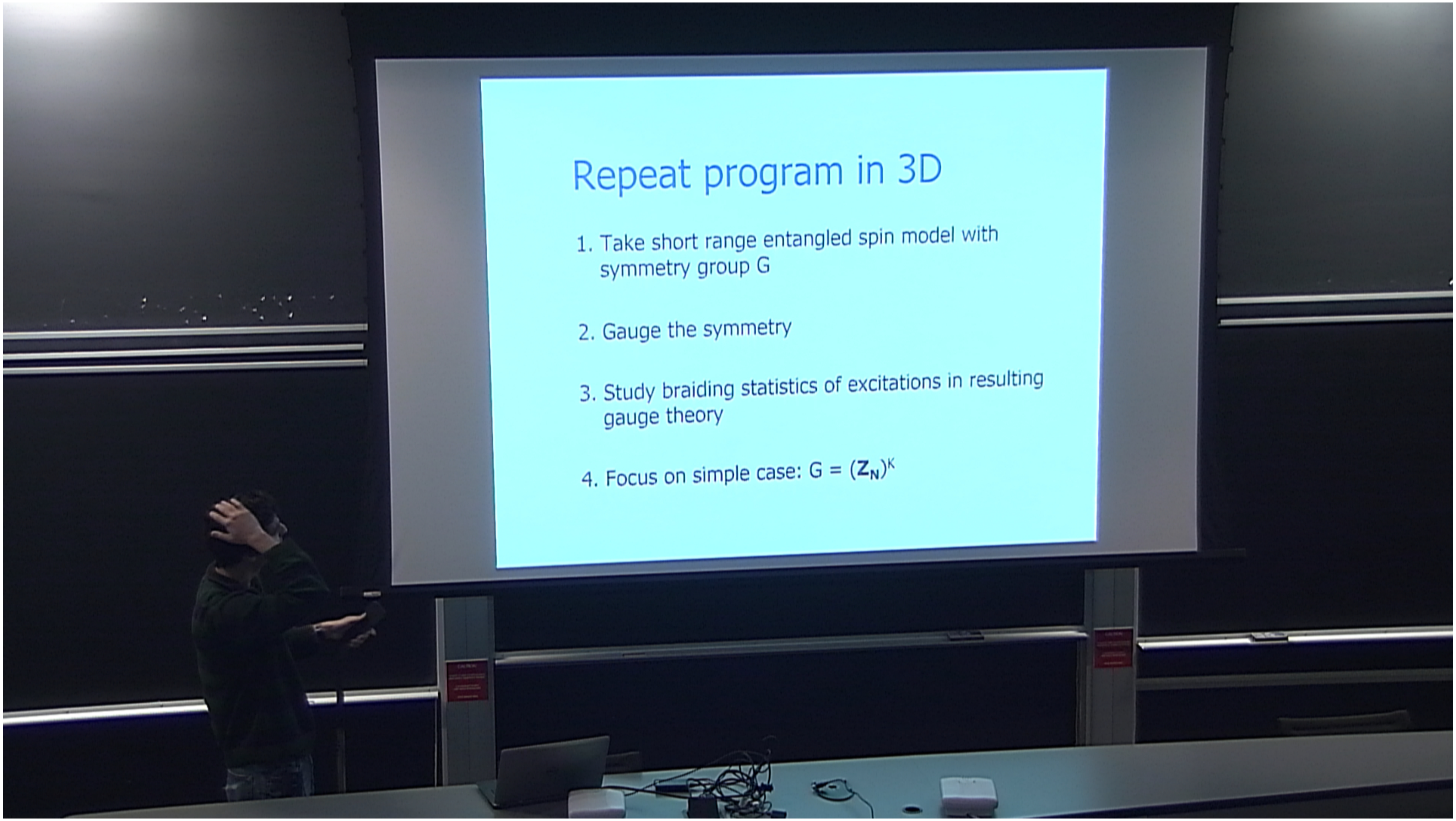
2. “Nontrivial” braiding data implies protected boundary modes.


Two kinds of Ising paramagnets
1. How can we see that H0 and H1 belong to different phases?
2. How can we see that H1 has a protected edge mode while H0 does not?


Step 1: Couple to a **Z₂** gauge field
Z₂ gauge field: $\mu^z_{pq} = \pm 1$
μ^z_{pq}
μ^z_{qr}
μ^z_{rp}


Step 1: Couple to a $\mathbf{Z_2}$ gauge field
$\mathbf{Z_2}$ gauge field: $\mu^z_{pq} = \pm 1$
Replace: $\sigma^z_p \sigma^z_q \rightarrow \sigma^z_p \mu^z_{pq} \sigma^z_q$
$\sigma^x_p \rightarrow \sigma^x_p$
μ^z_{pq}
μ^z_{qr}
μ^z_{rp}


Step 2: Find basic excitations
1. "Charge": e
$\sigma_p^x = -1$ or $B_p = -1$
2."π-vortex": m_a, m_b
$\mu_{pq}^z \mu_{qr}^z \mu_{rp}^z = -1$


Repeat program in 3D
1. Take short range entangled spin model with symmetry group G
2. Gauge the symmetry
3. Study braiding statistics of excitations in resulting gauge theory
4. Focus on simple case: G = $(\mathbf{Z_N})^K$

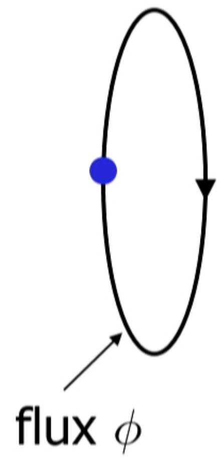
Excitations in $(\mathbf{Z_N})^K$ gauge theories

1. "Charges"

- Characterized by gauge charge:

  $q = (q_1,\ldots,q_K)$, q_i = integer (mod N)

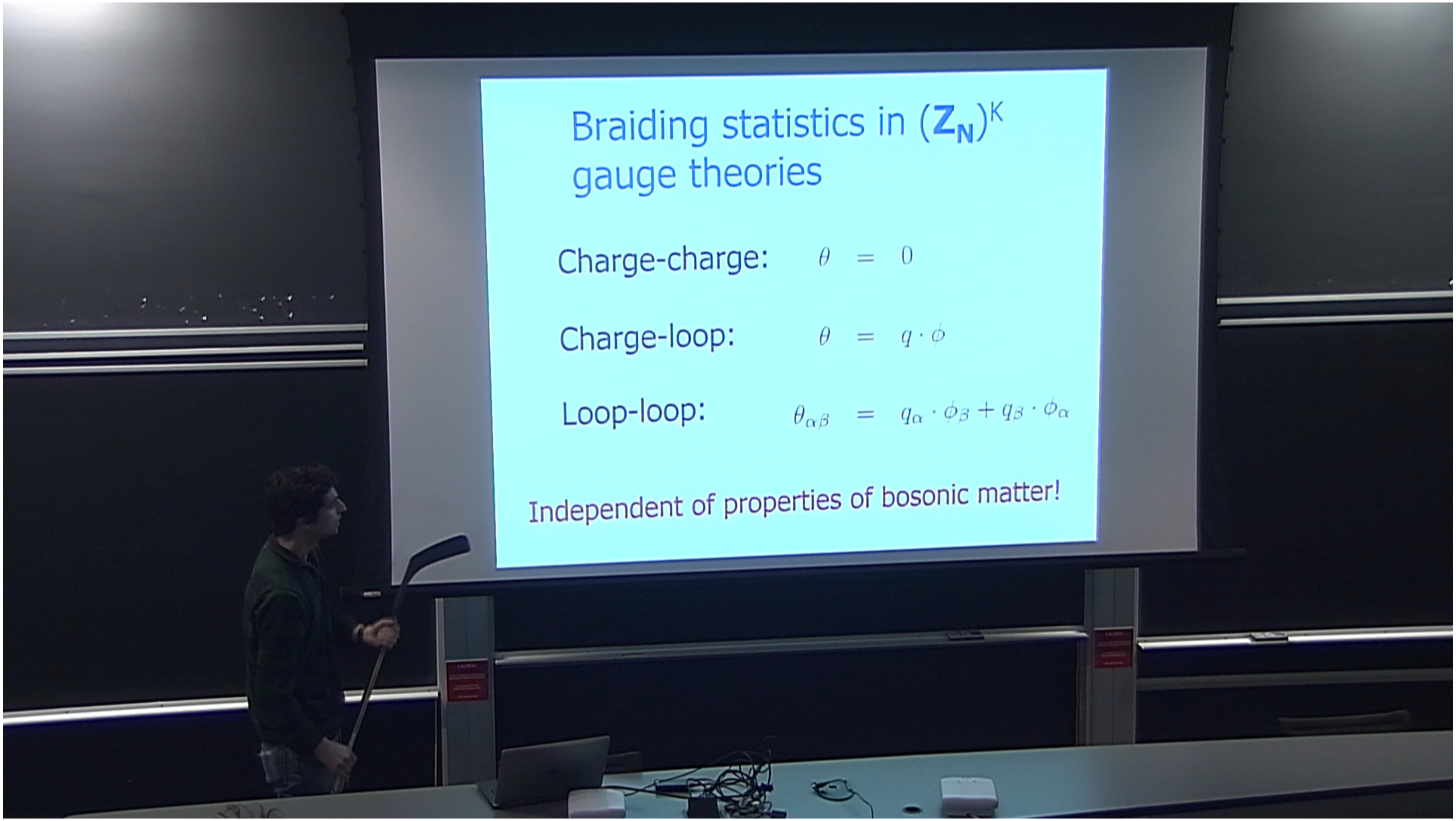
2. "Vortex loops"

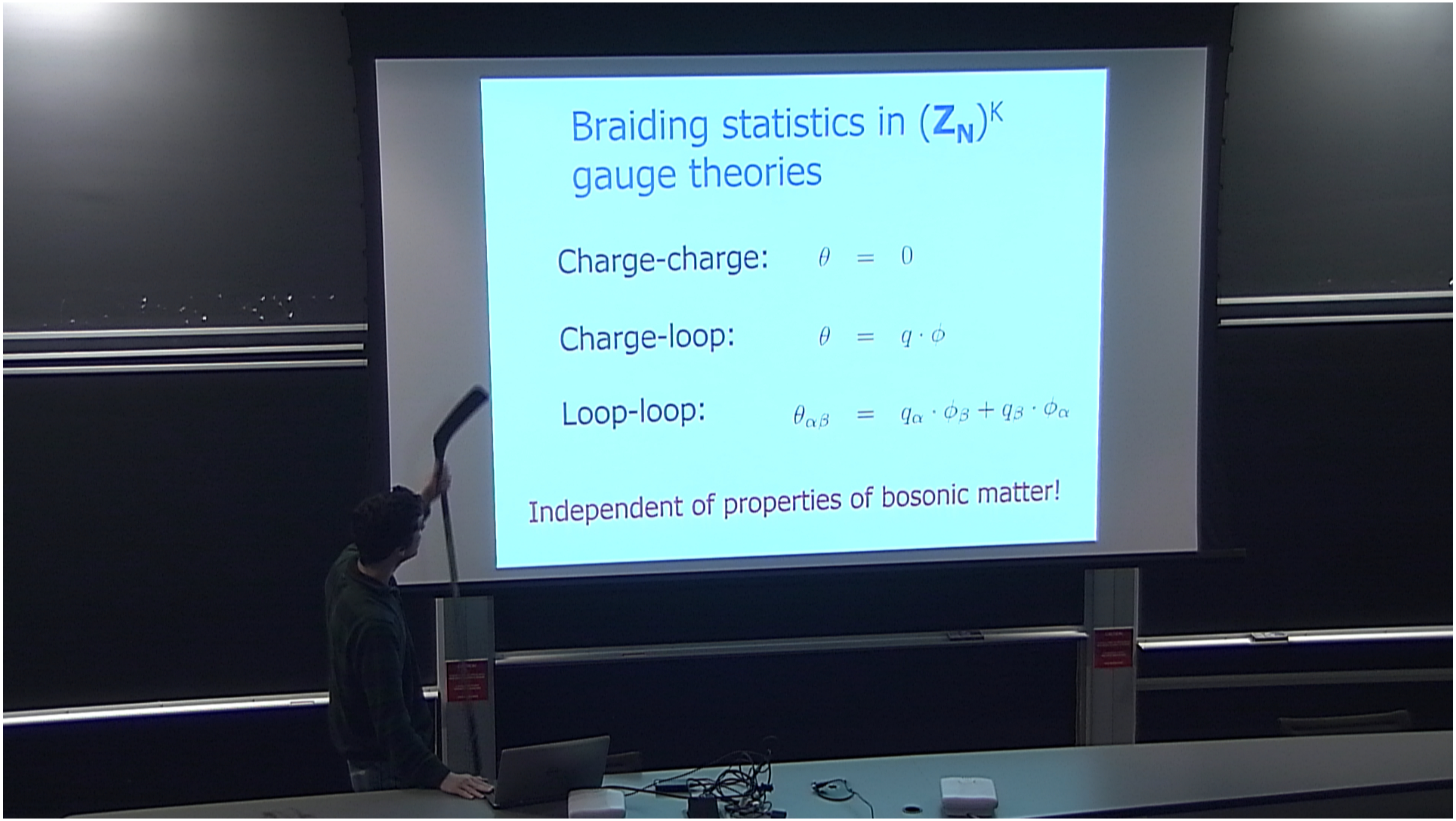


- Characterized by gauge flux:

  $\phi = (\phi_1,\ldots,\phi_K)$, $\phi_i = (2\pi/N) \cdot$ integer

- Vortex loops can also carry gauge charge


Braiding statistics in $(\mathbf{Z_N})^K$ gauge theories
Charge-charge: $\theta = 0$
Charge-loop: $\theta = q \cdot \phi$
Loop-loop: $\theta_{\alpha\beta} = q_\alpha \cdot \phi_\beta + q_\beta \cdot \phi_\alpha$
Independent of properties of bosonic matter!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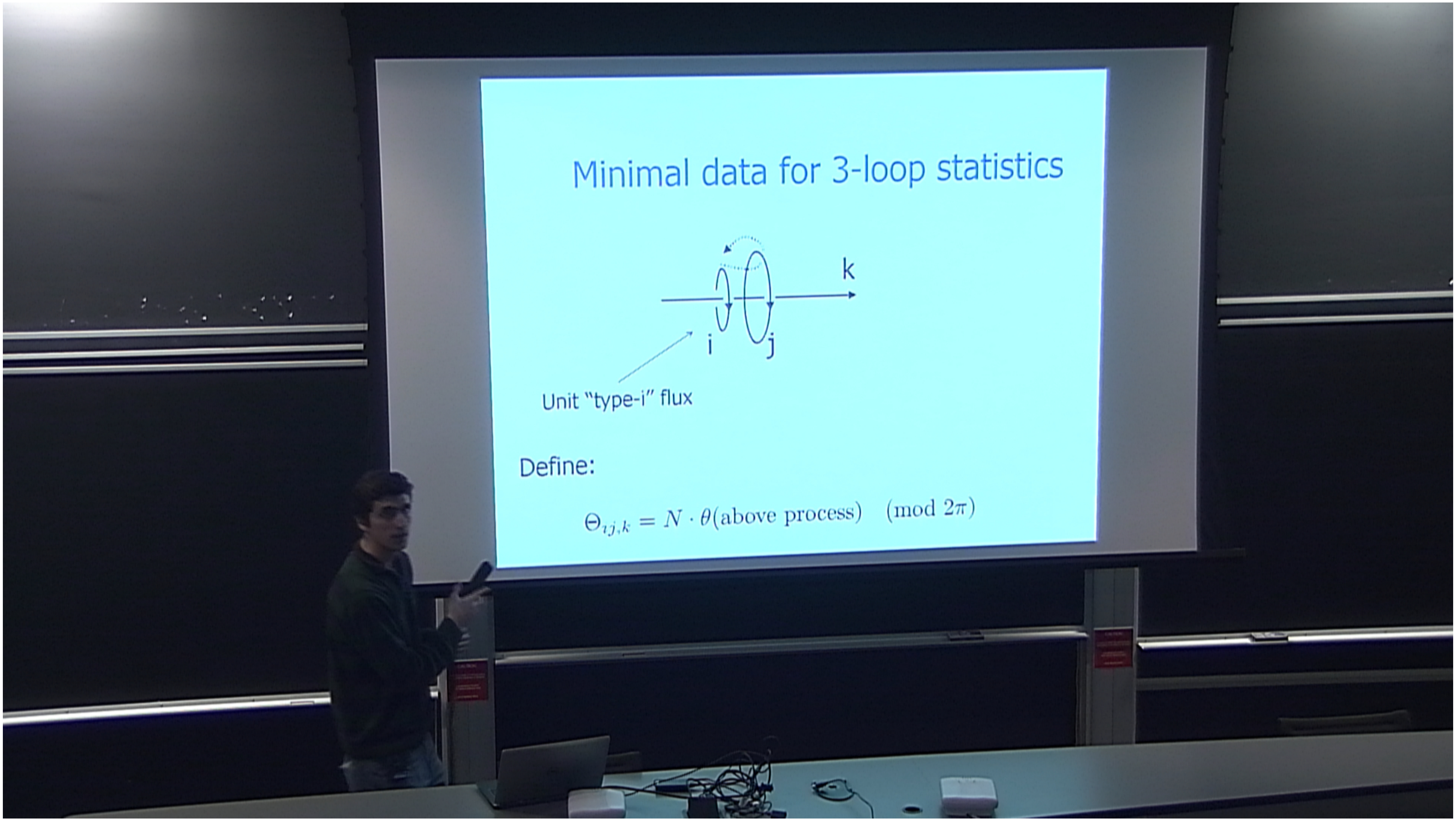
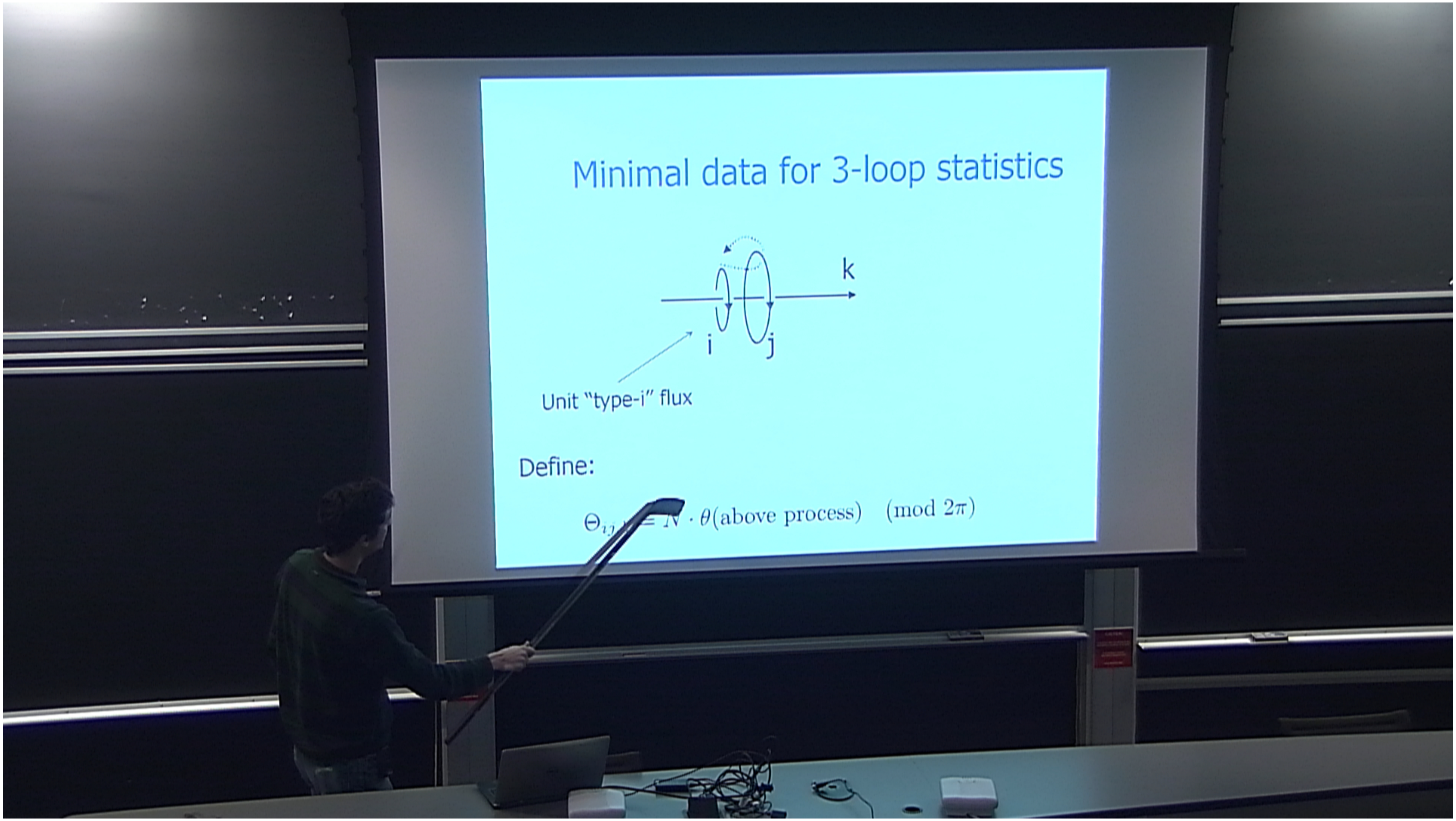

Minimal data for 3-loop statistics
k
i
j
Unit "type-i" flux
Define:
$\Theta_{ij,k} = N \cdot \theta(\text{above process}) \pmod{2\pi}$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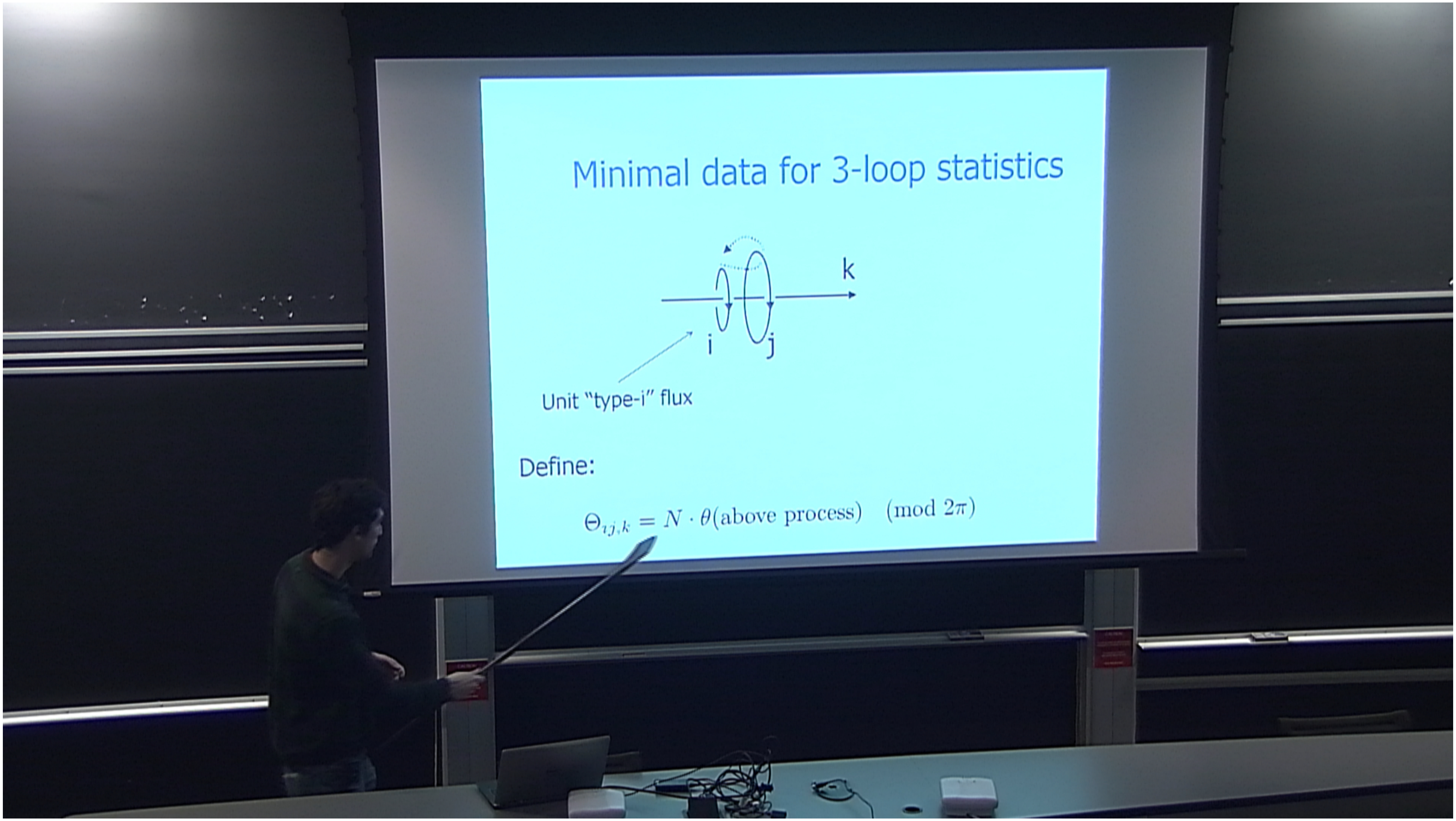
· θ(above process) (mod 2π)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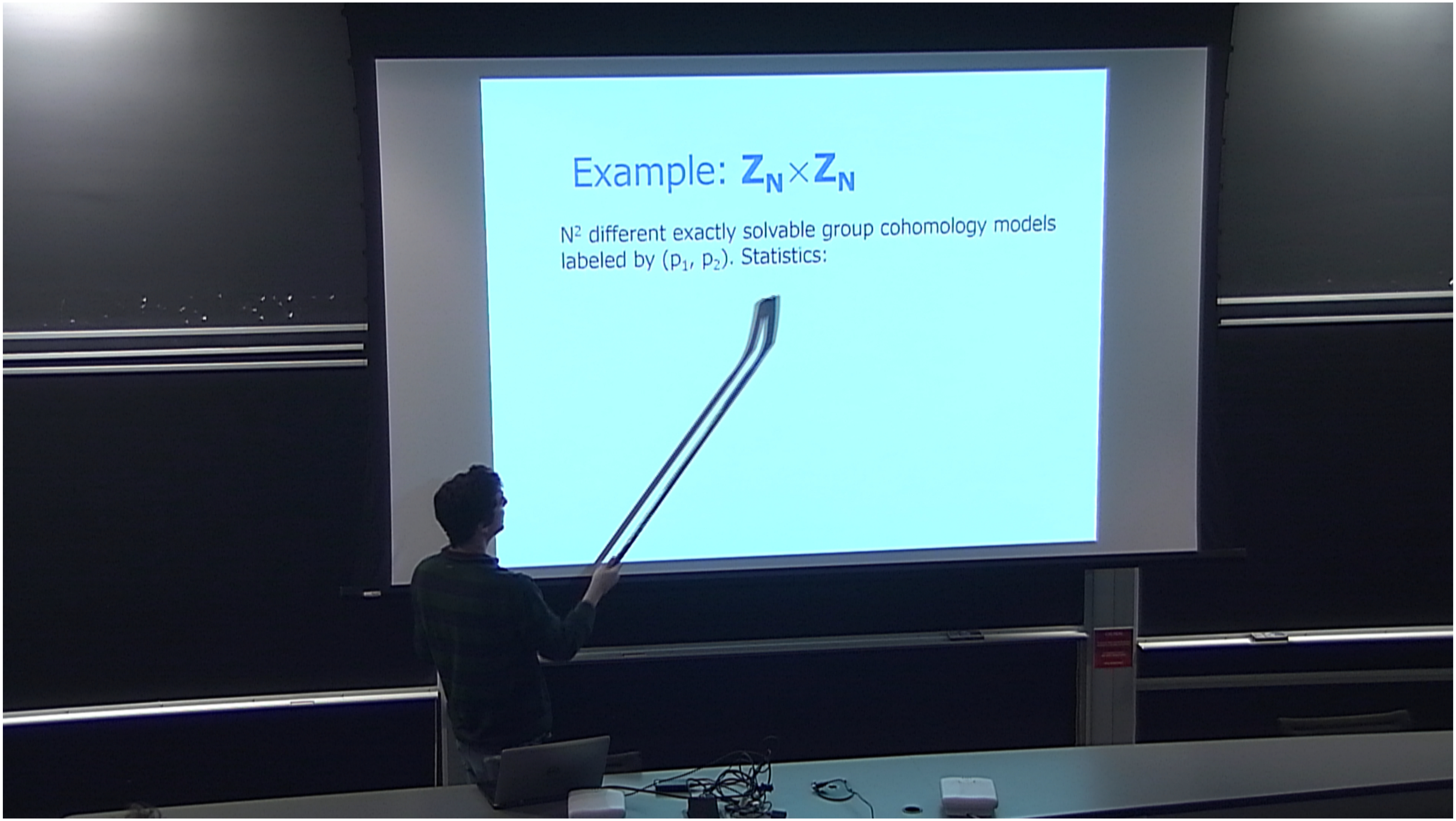
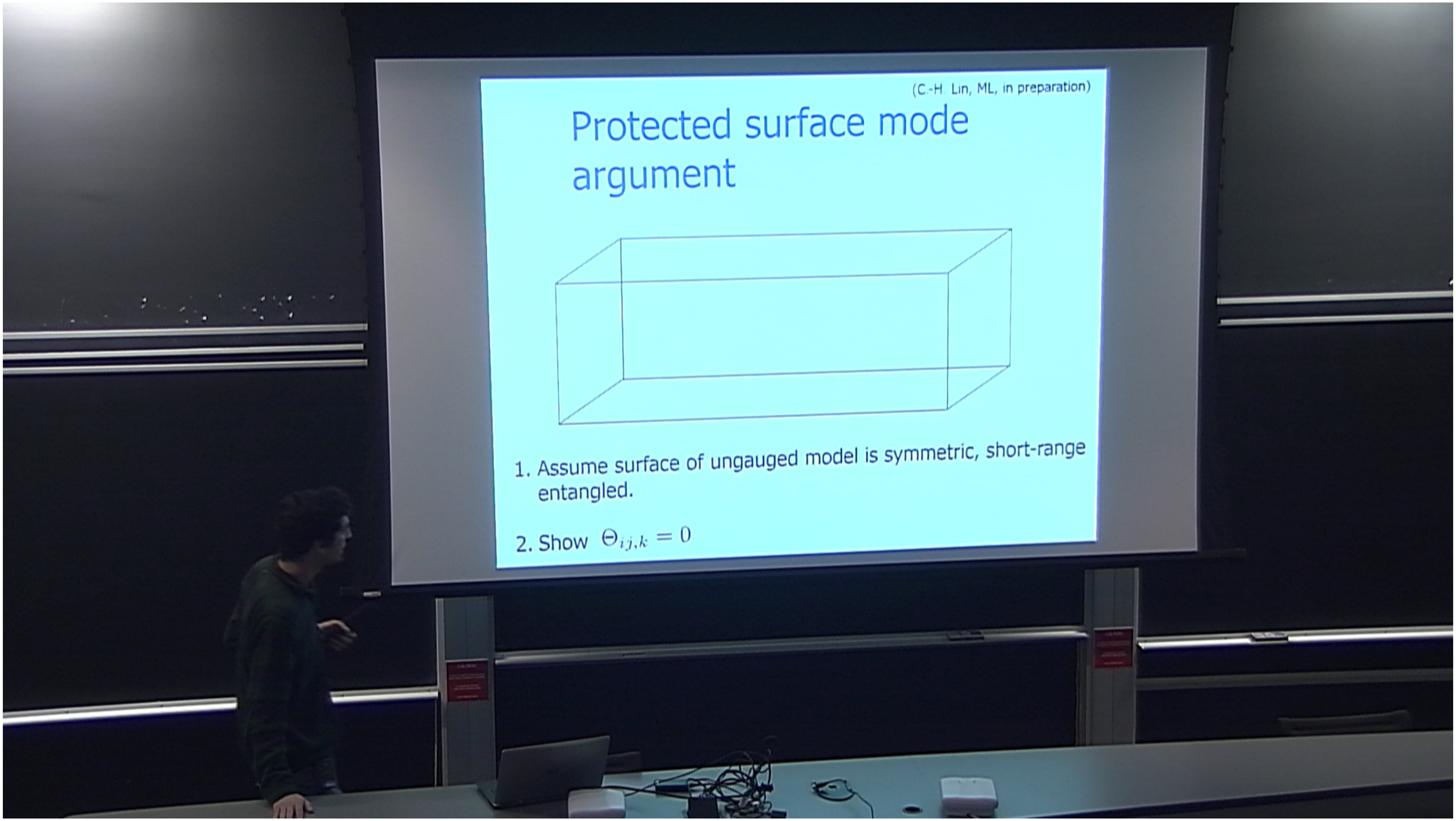
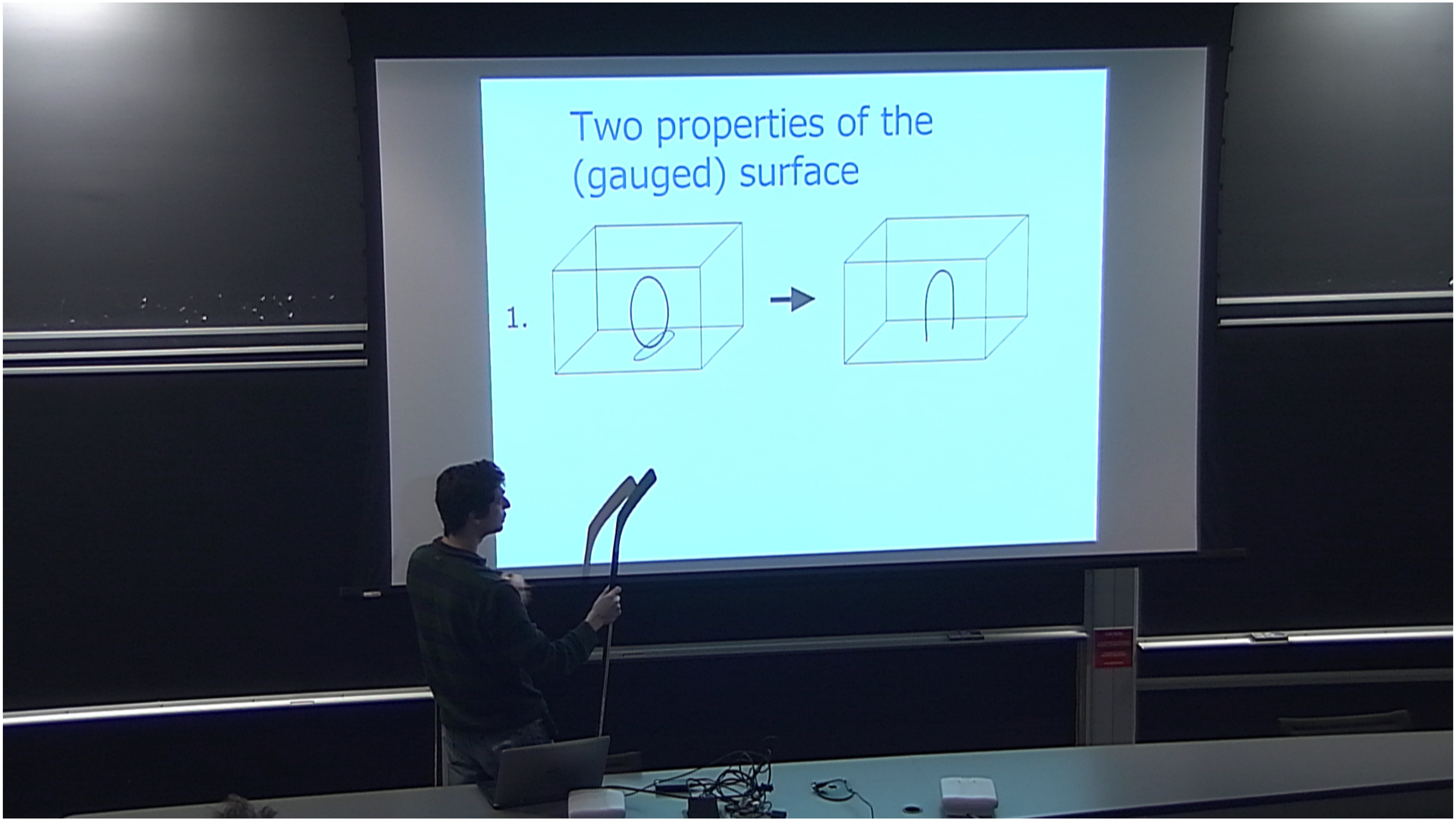
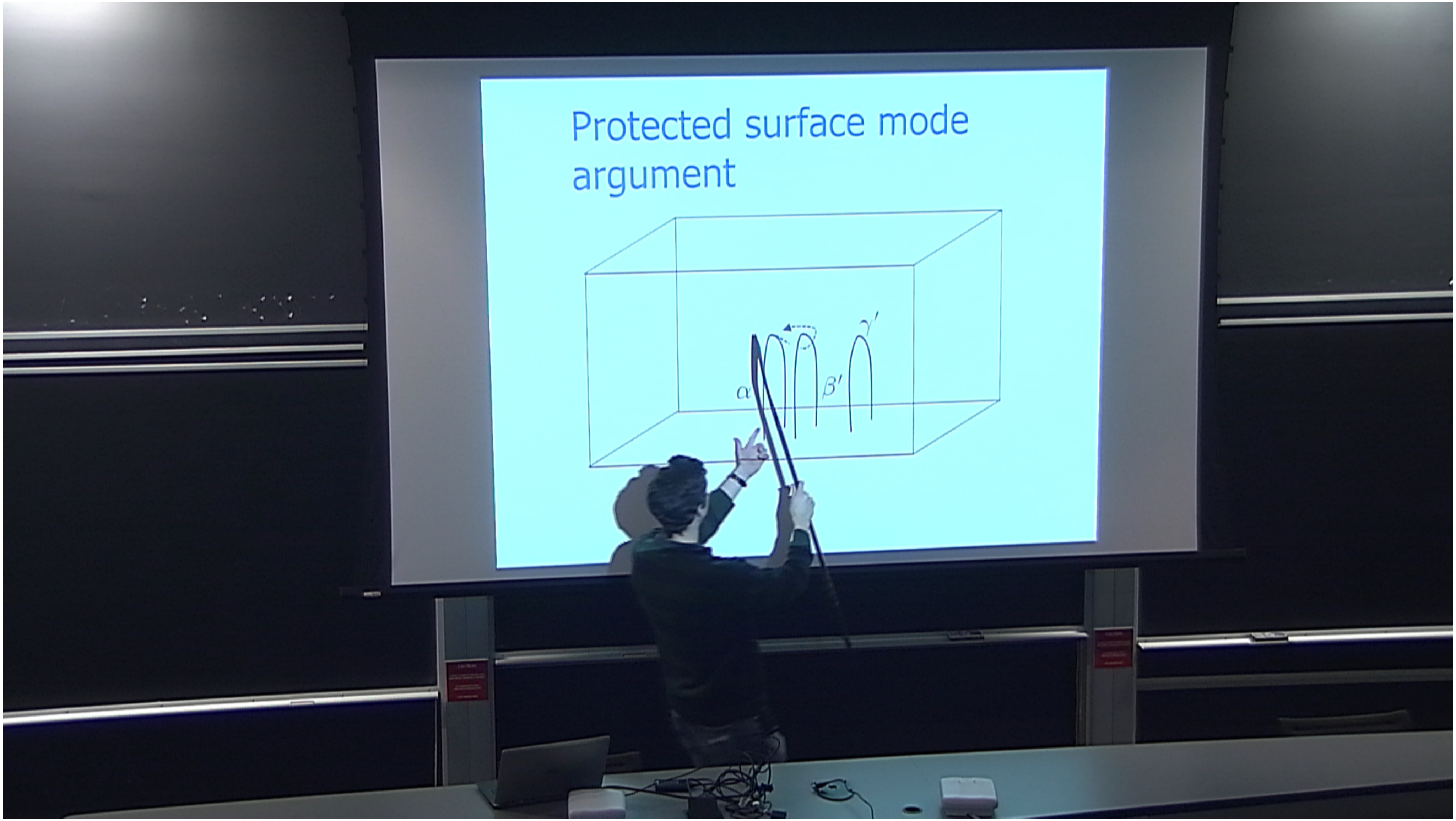
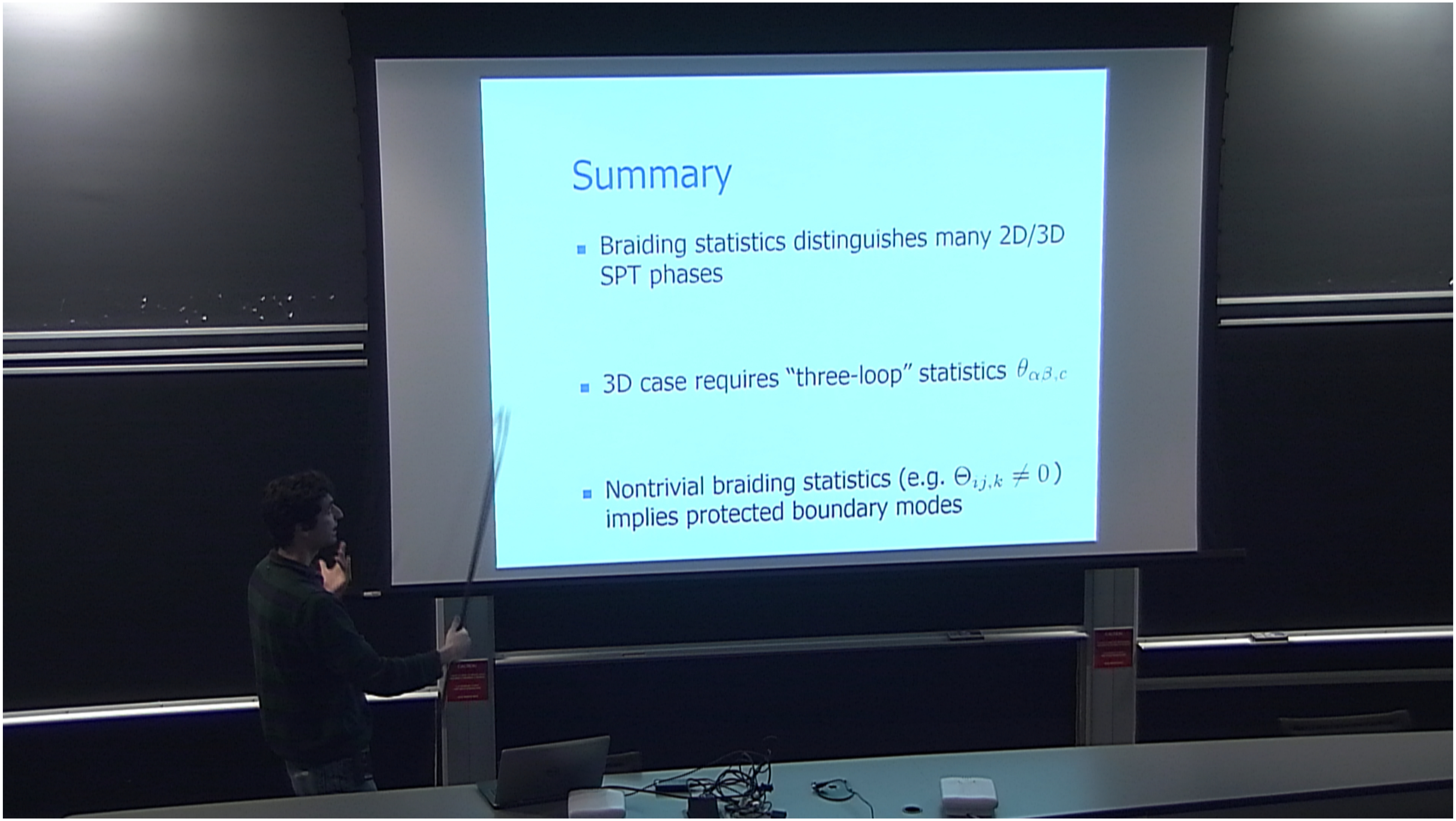

Example: Z_N × Z_N
N² different exactly solvable group cohomology models
labeled by (p_1, p_2). Statistics:


(C.-H. Lin, ML, in preparation)
Protected surface mode argument
1. Assume surface of ungauged model is symmetric, short-range entangled.
2. Show $\Theta_{ij,k} = 0$


Two properties of the
(gauged) surface
1.


Protected surface mode
argument
α
β'
γ'


Summary
Braiding statistics distinguishes many 2D/3D SPT phases
3D case requires "three-loop" statistics $\theta_{\alpha\beta,c}$
Nontrivial braiding statistics (e.g. $\Theta_{ij,k} \neq 0$) implies protected boundary modes