Title: Interacting surface states of topological insulators and superfluids

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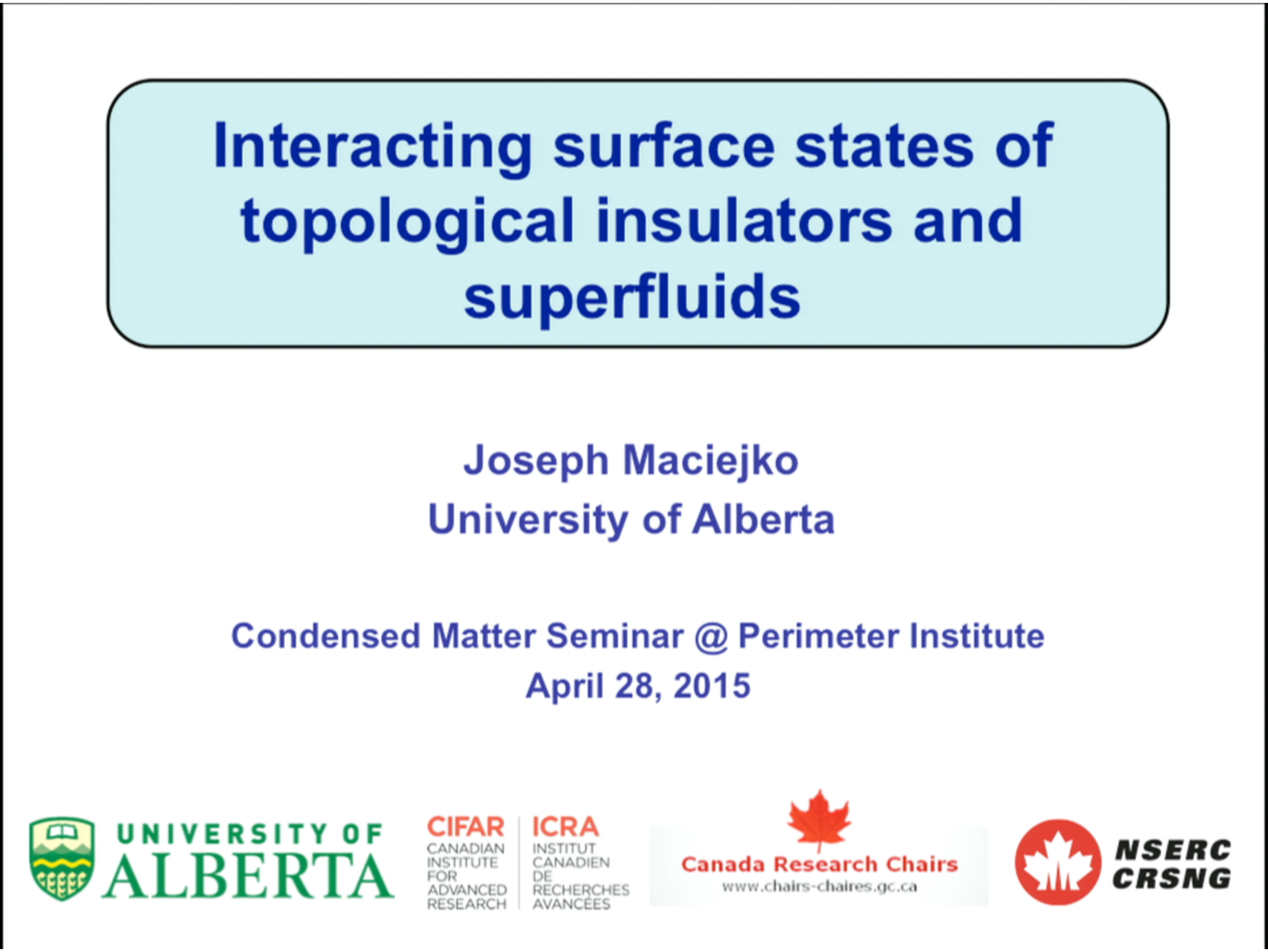
Abstract: <p>The standard theory of topological insulators and superfluids (or superconductors) assumes that the fermionic elementary excitations in these systems â€“ electrons in the insulator and Bogoliubov quasiparticles in the superfluid â€“ do not interact with one another. In this talk I will discuss extensions of this theory to include the effects of interparticle interactions on the topological surface states of 3D topological insulators and superfluids. First, I will argue that bulk quantum fluctuations of the superfluid order parameter in the only 3D topological superfluid known to date, 3He-B, can generate an effective two-body interaction between the surface Majorana fermions.</p>

<p>Possible consequences of this interaction will be discussed. Second, I will propose an extension of the phenomenological Landau theory of Fermi liquids to the surface states of 3D topological insulators, leading to the concept of helical Fermi liquid.</p>



Interacting surface states of topological insulators and superfluids

Joseph Maciejko
University of Alberta

Condensed Matter Seminar @ Perimeter Institute
April 28, 2015

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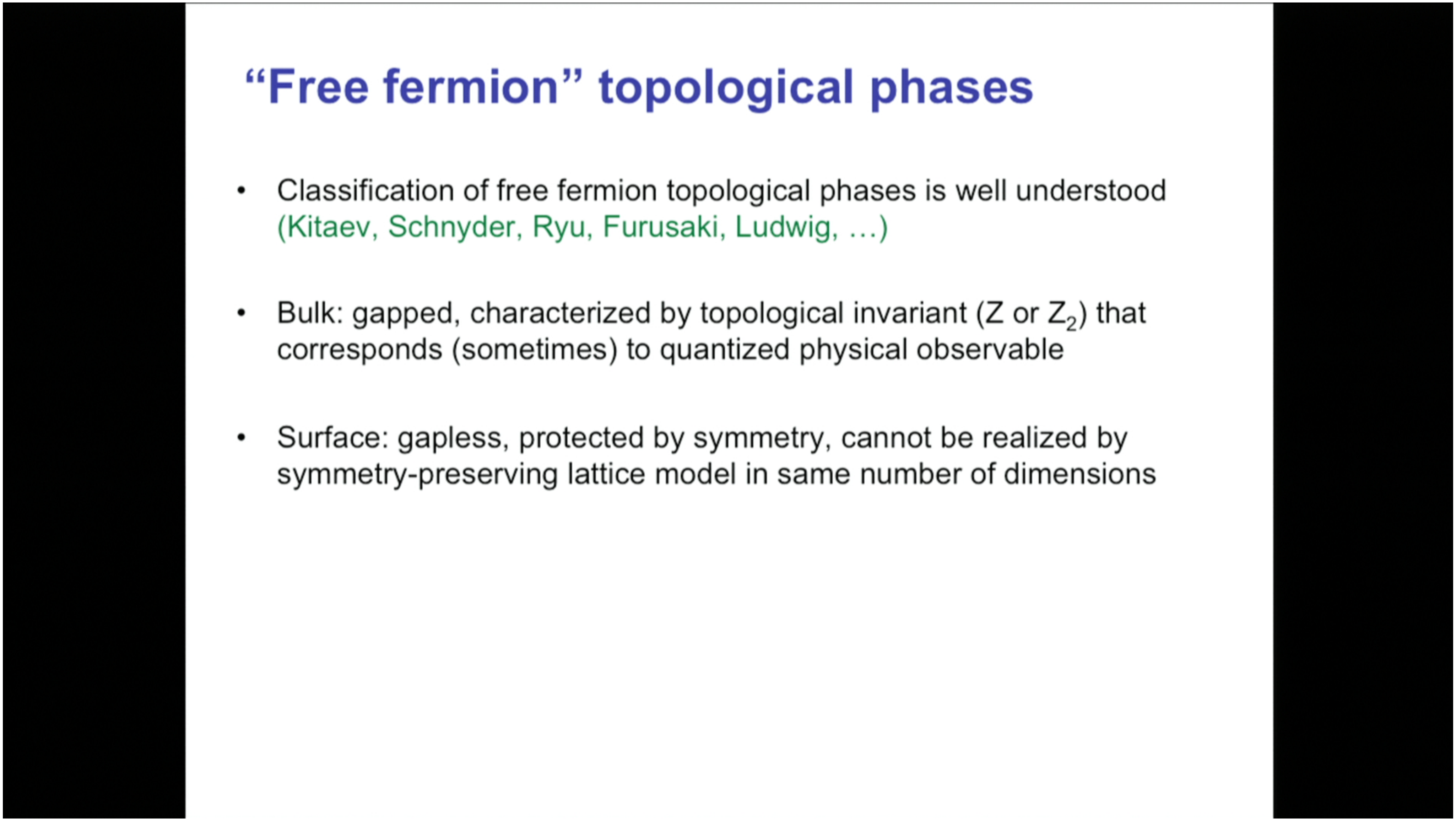
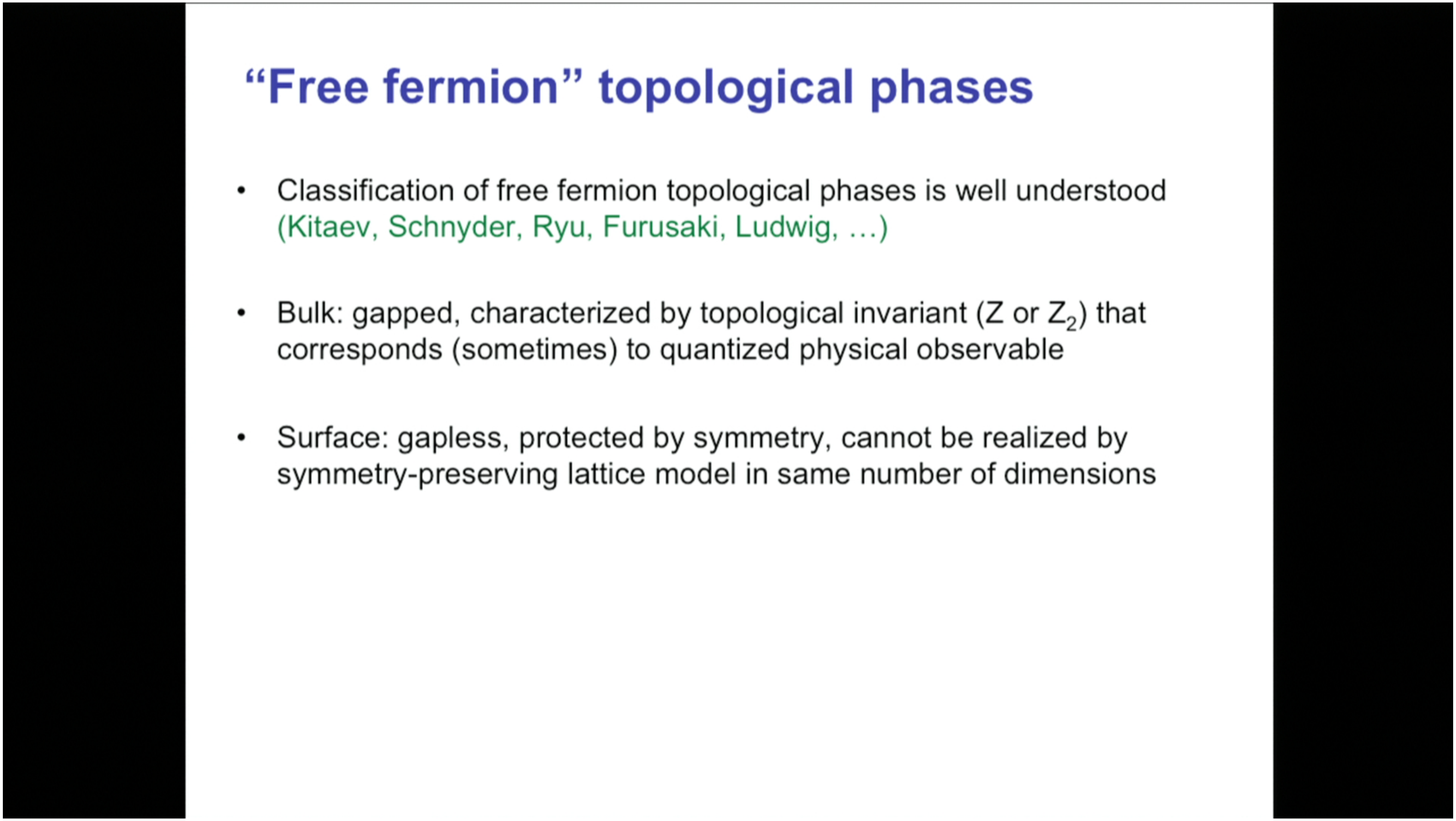


“Free fermion” topological phases

- Classification of free fermion topological phases is well understood (Kitaev, Schnyder, Ryu, Furusaki, Ludwig, …)
- Bulk: gapped, characterized by topological invariant (Z or Z_2) that corresponds (sometimes) to quantized physical observable
- Surface: gapless, protected by symmetry, cannot be realized by symmetry-preserving lattice model in same number of dimensions



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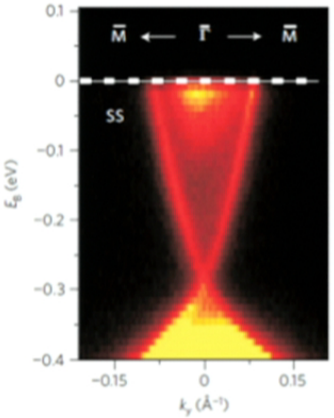
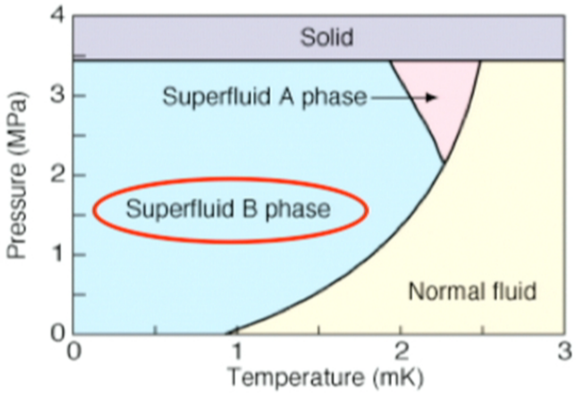
“Free fermion” topological phases

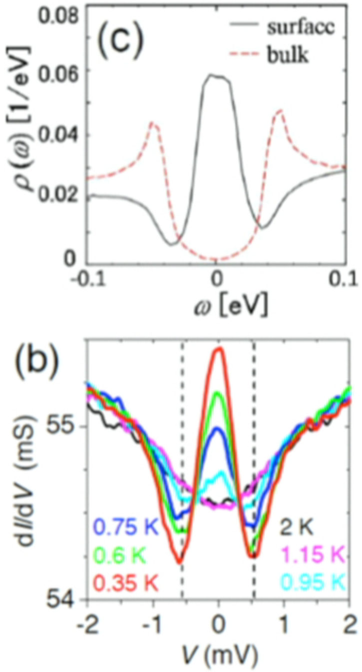
- 3D topological phases discovered experimentally via detection of 2D surface states



Bi$_2$Se$_3$ *(Xia et al., Nat. Phys. 2009)*



^{3}He-B? *(Murakawa et al., JPSJ 2011)*



Cu$_x$Bi$_2$Se$_3$? *(Sasaki et al., PRL 2011)*

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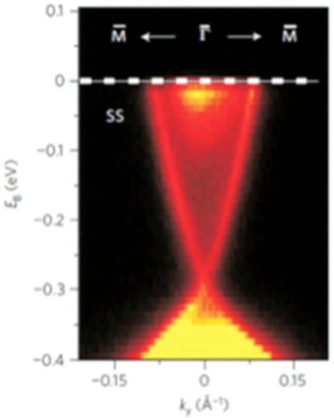

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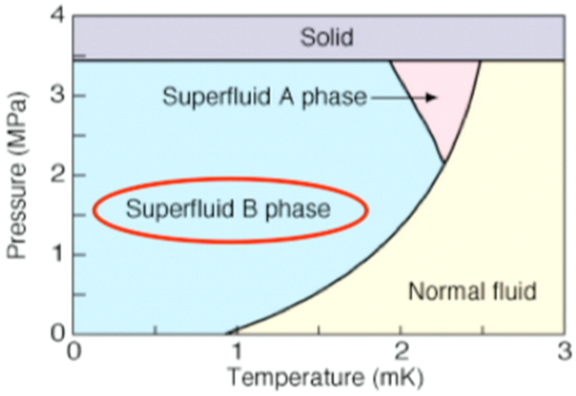

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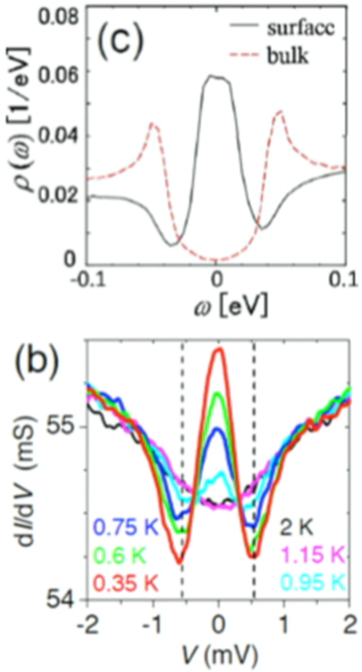

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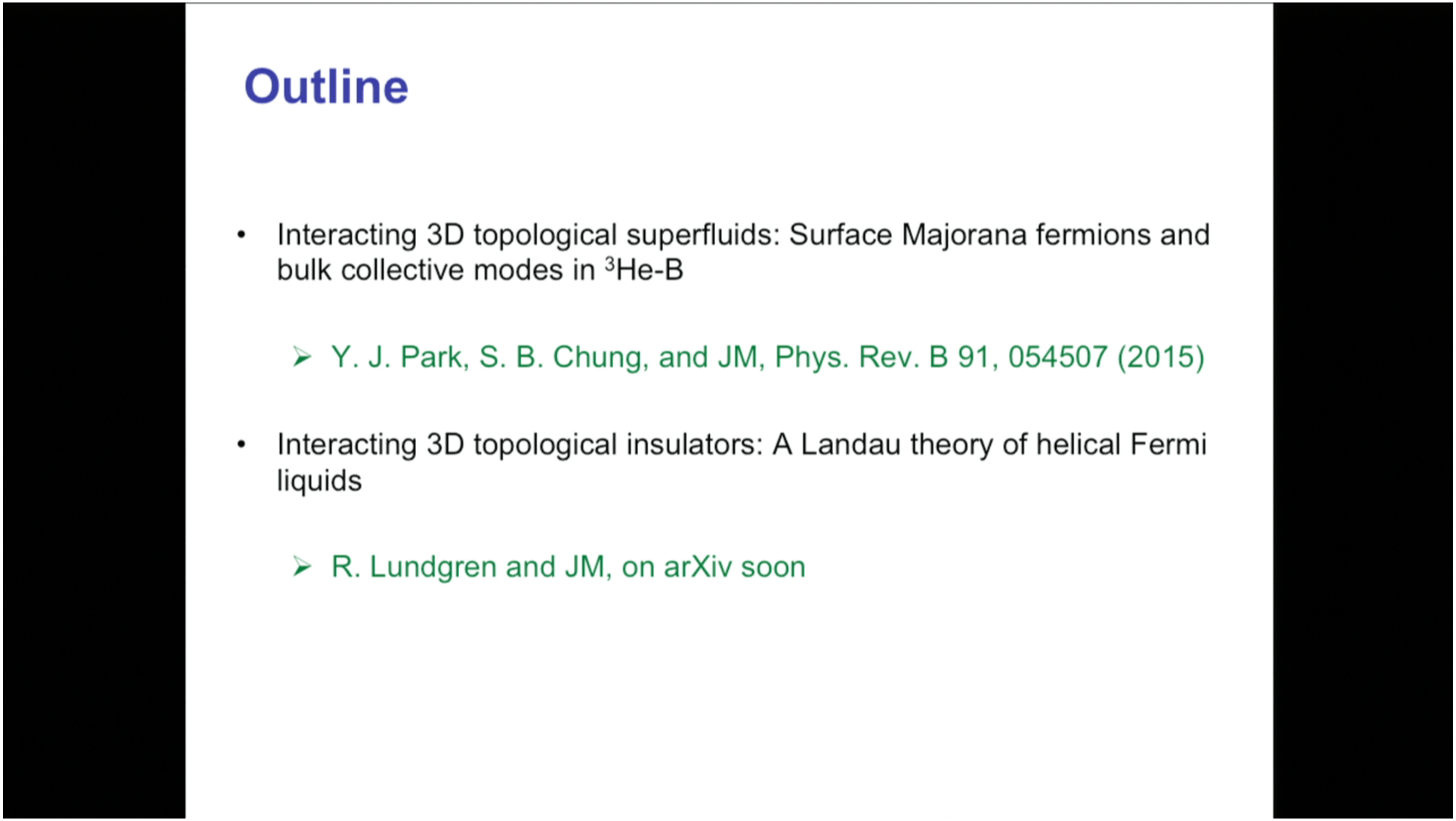
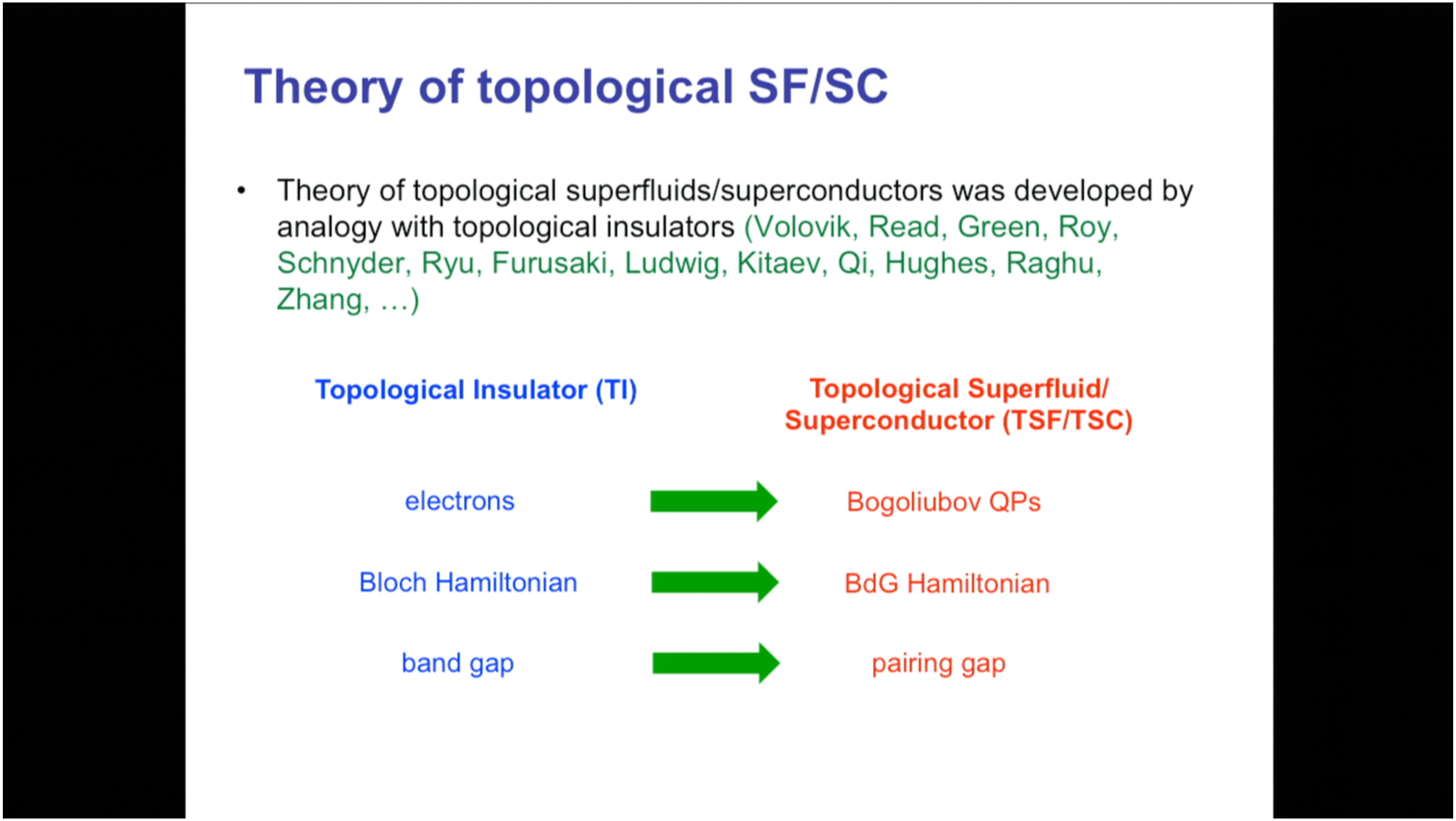


Outline

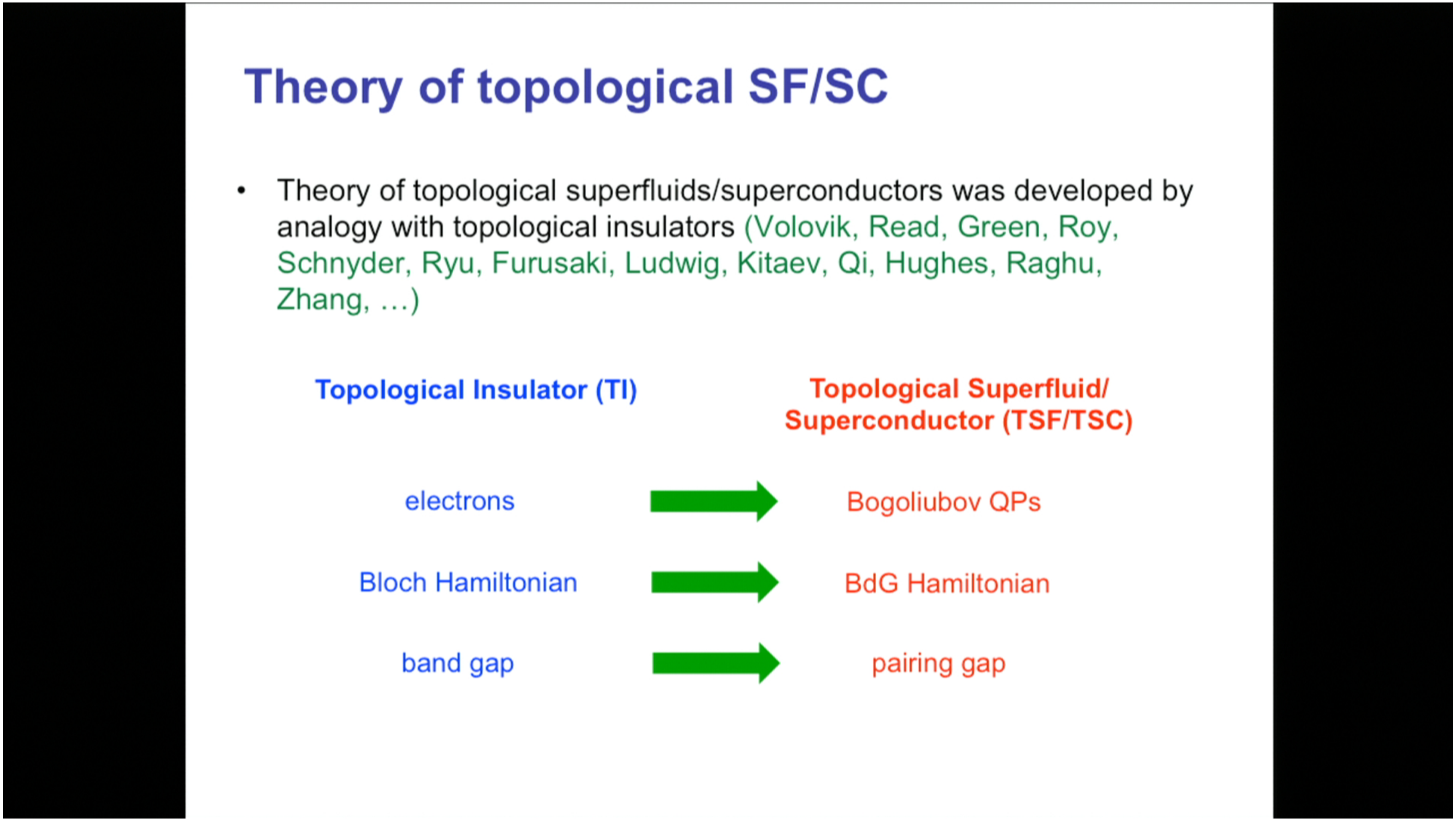
- Interacting 3D topological superfluids: Surface Majorana fermions and bulk collective modes in ^{3}He-B
  - ➢ Y. J. Park, S. B. Chung, and JM, Phys. Rev. B 91, 054507 (2015)
- Interacting 3D topological insulators: A Landau theory of helical Fermi liquids
  - ➢ R. Lundgren and JM, on arXiv soon



Theory of topological SF/SC

- Theory of topological superfluids/superconductors was developed by analogy with topological insulators (Volovik, Read, Green, Roy, Schnyder, Ryu, Furusaki, Ludwig, Kitaev, Qi, Hughes, Raghu, Zhang, …)

Topological Insulator (TI)		**Topological Superfluid/ Superconductor (TSF/TSC)**
electrons	→	Bogoliubov QPs
Bloch Hamiltonian	→	BdG Hamiltonian
band gap	→	pairing gap



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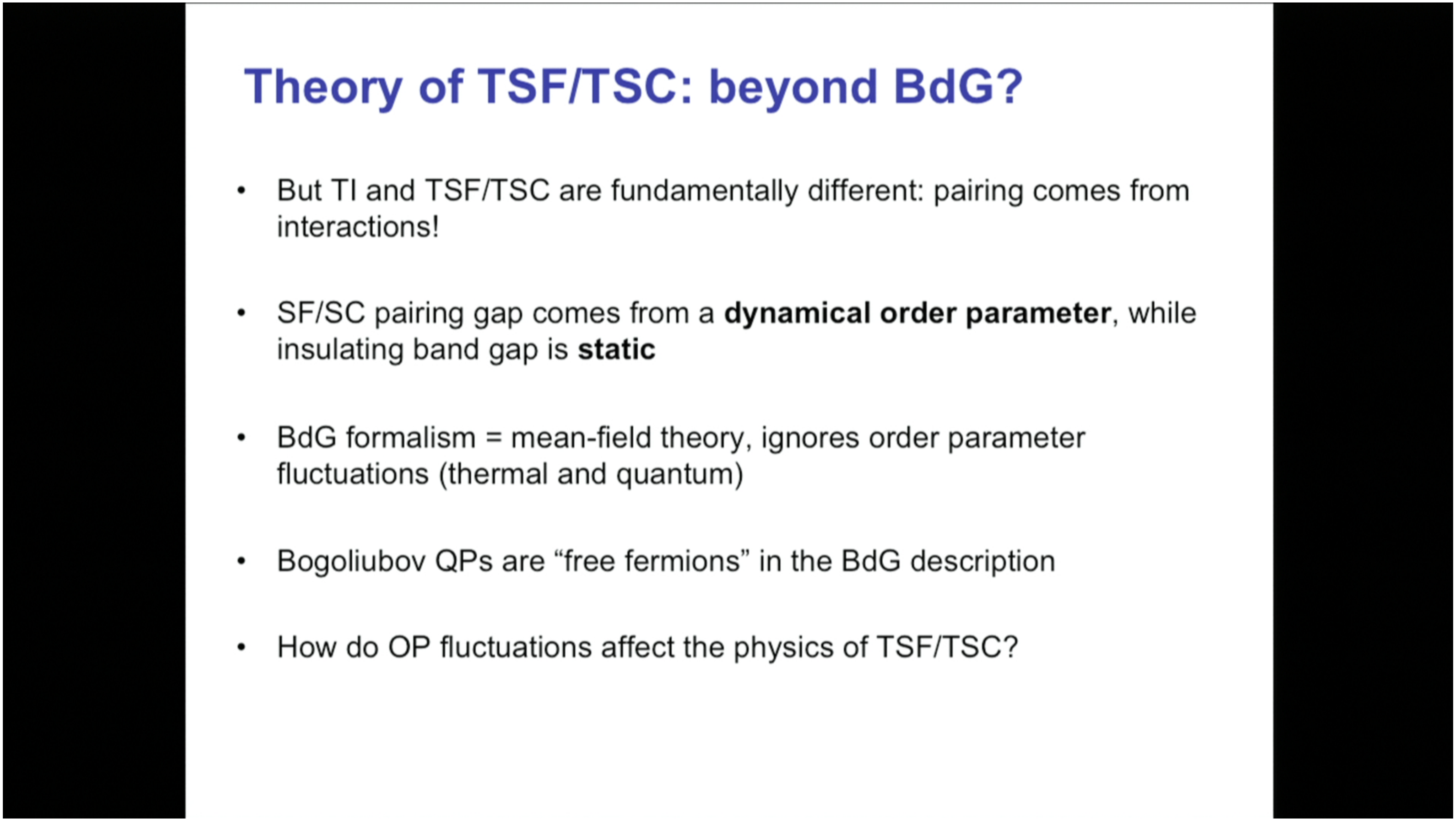
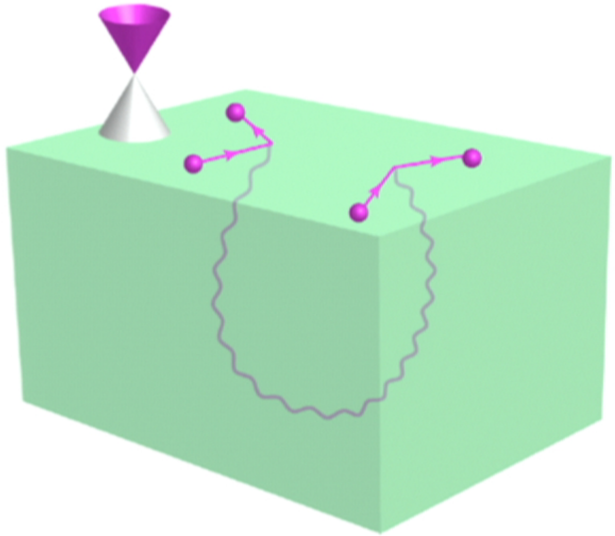


Theory of TSF/TSC: beyond BdG?

- But TI and TSF/TSC are fundamentally different: pairing comes from interactions!
- SF/SC pairing gap comes from a **dynamical order parameter**, while insulating band gap is **static**
- BdG formalism = mean-field theory, ignores order parameter fluctuations (thermal and quantum)
- Bogoliubov QPs are “free fermions” in the BdG description
- How do OP fluctuations affect the physics of TSF/TSC?

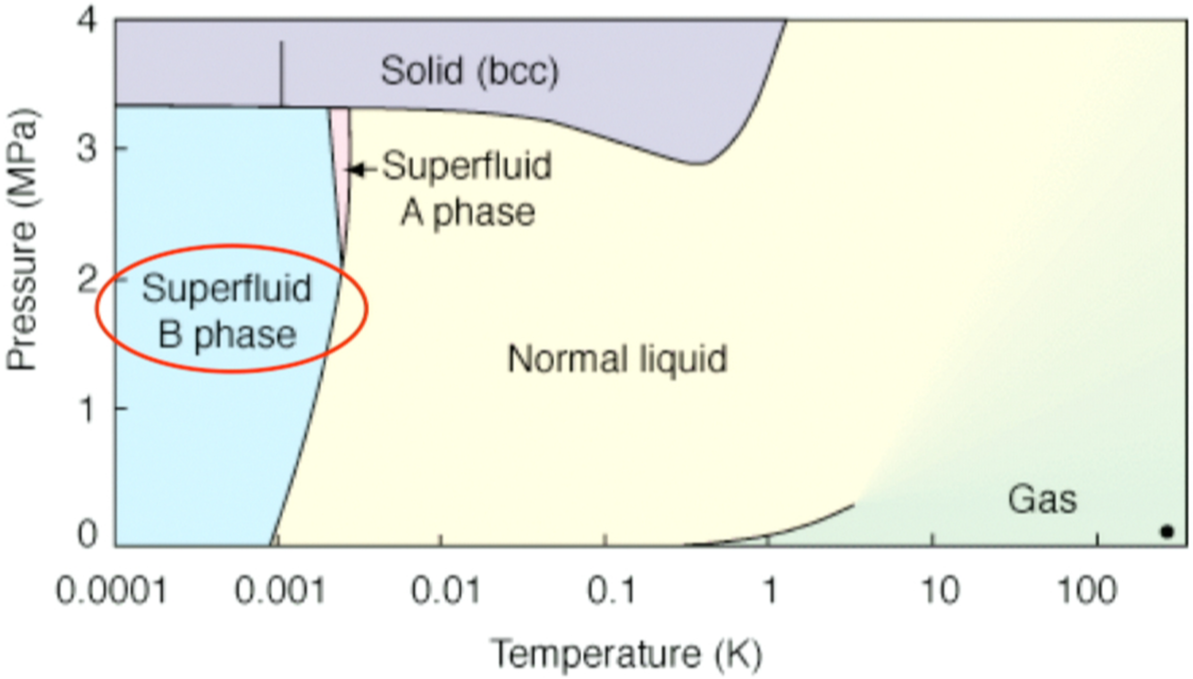
Theory of TSF/TSC: beyond BdG?

- Main message: bulk OP fluctuations can induce **interactions** among boundary Majorana fermions

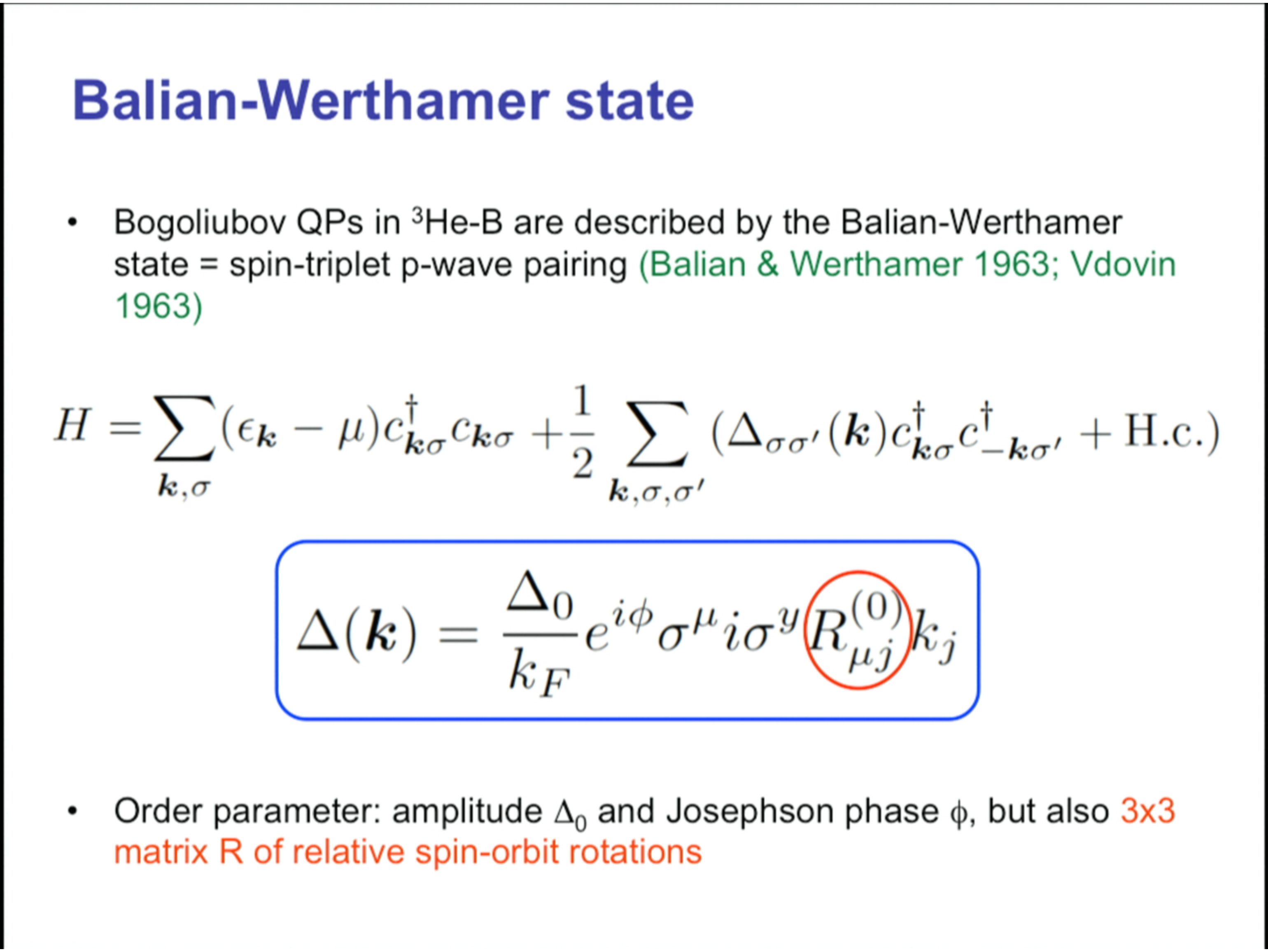


- Focus on quantum (T=0) OP fluctuations in ^{3}He-B (class DIII TSF with ν=1)

^{3}He phase diagram



(credit: Aalto University)



Balian-Werthamer state

- Bogoliubov QPs in ^{3}He-B are described by the Balian-Werthamer state = spin-triplet p-wave pairing (Balian & Werthamer 1963; Vdovin 1963)

$$H = \sum_{\boldsymbol{k},\sigma} (\epsilon_{\boldsymbol{k}} - \mu) c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} + \frac{1}{2} \sum_{\boldsymbol{k},\sigma,\sigma'} (\Delta_{\sigma\sigma'}(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}\sigma} c^\dagger_{-\boldsymbol{k}\sigma'} + \mathrm{H.c.})$$

$$\boxed{\Delta(\boldsymbol{k}) = \frac{\Delta_0}{k_F} e^{i\phi} \sigma^\mu i\sigma^y R^{(0)}_{\mu j} k_j}$$

- Order parameter: amplitude Δ_0 and Josephson phase ϕ, but also 3x3 matrix R of relative spin-orbit rotations

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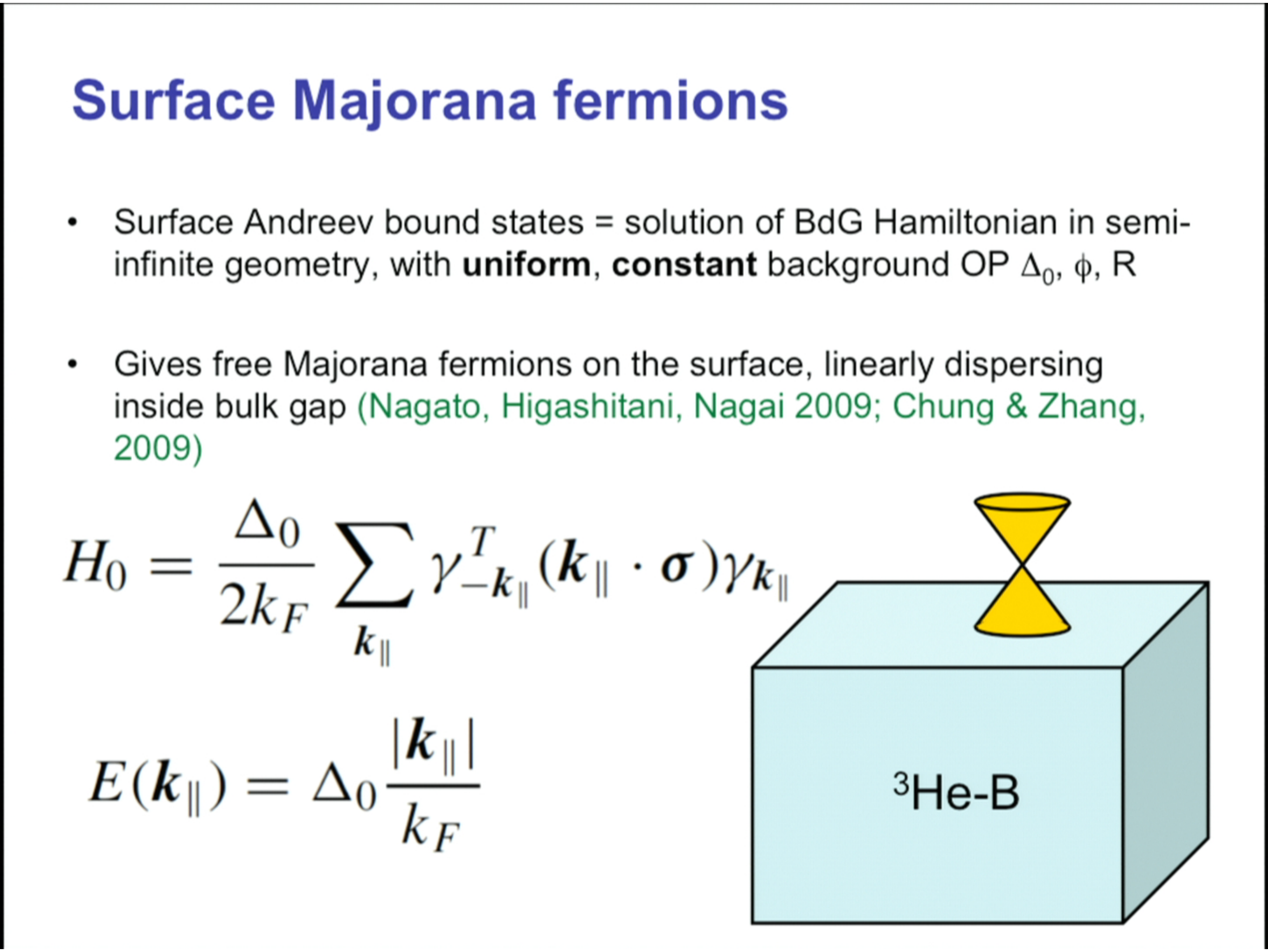

Surface Majorana fermions

- Surface Andreev bound states = solution of BdG Hamiltonian in semi-infinite geometry, with **uniform**, **constant** background OP Δ_0, ϕ, R
- Gives free Majorana fermions on the surface, linearly dispersing inside bulk gap (Nagato, Higashitani, Nagai 2009; Chung & Zhang, 2009)

$$H_0 = \frac{\Delta_0}{2k_F}\sum_{\boldsymbol{k}_\parallel}\gamma^T_{-\boldsymbol{k}_\parallel}(\boldsymbol{k}_\parallel \cdot \boldsymbol{\sigma})\gamma_{\boldsymbol{k}_\parallel}$$

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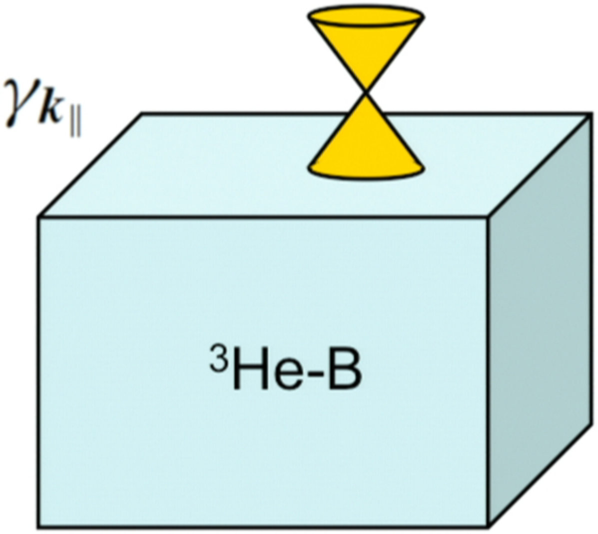

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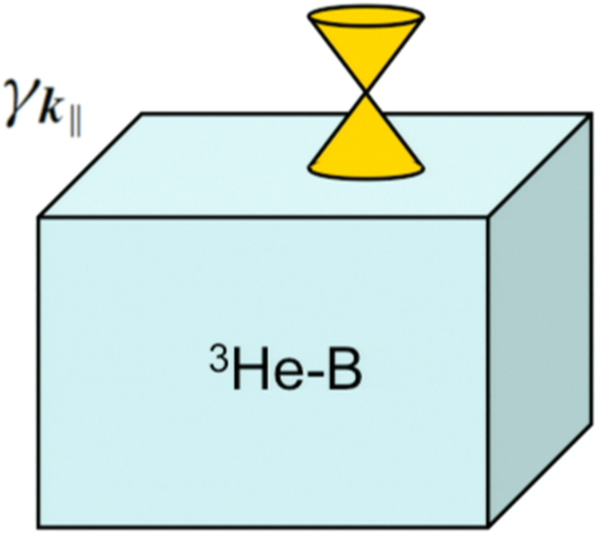

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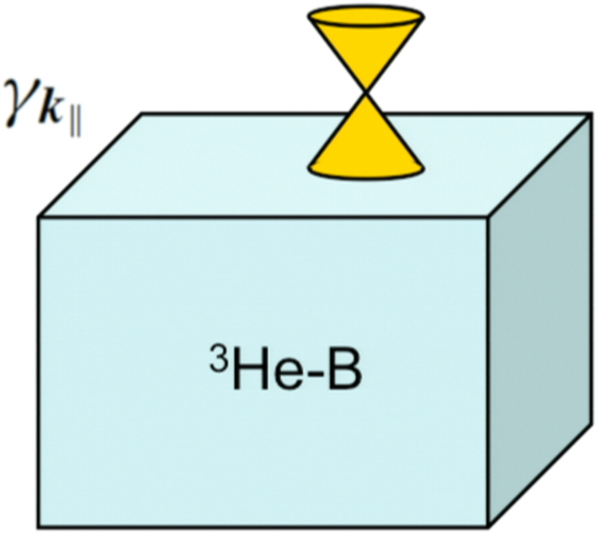
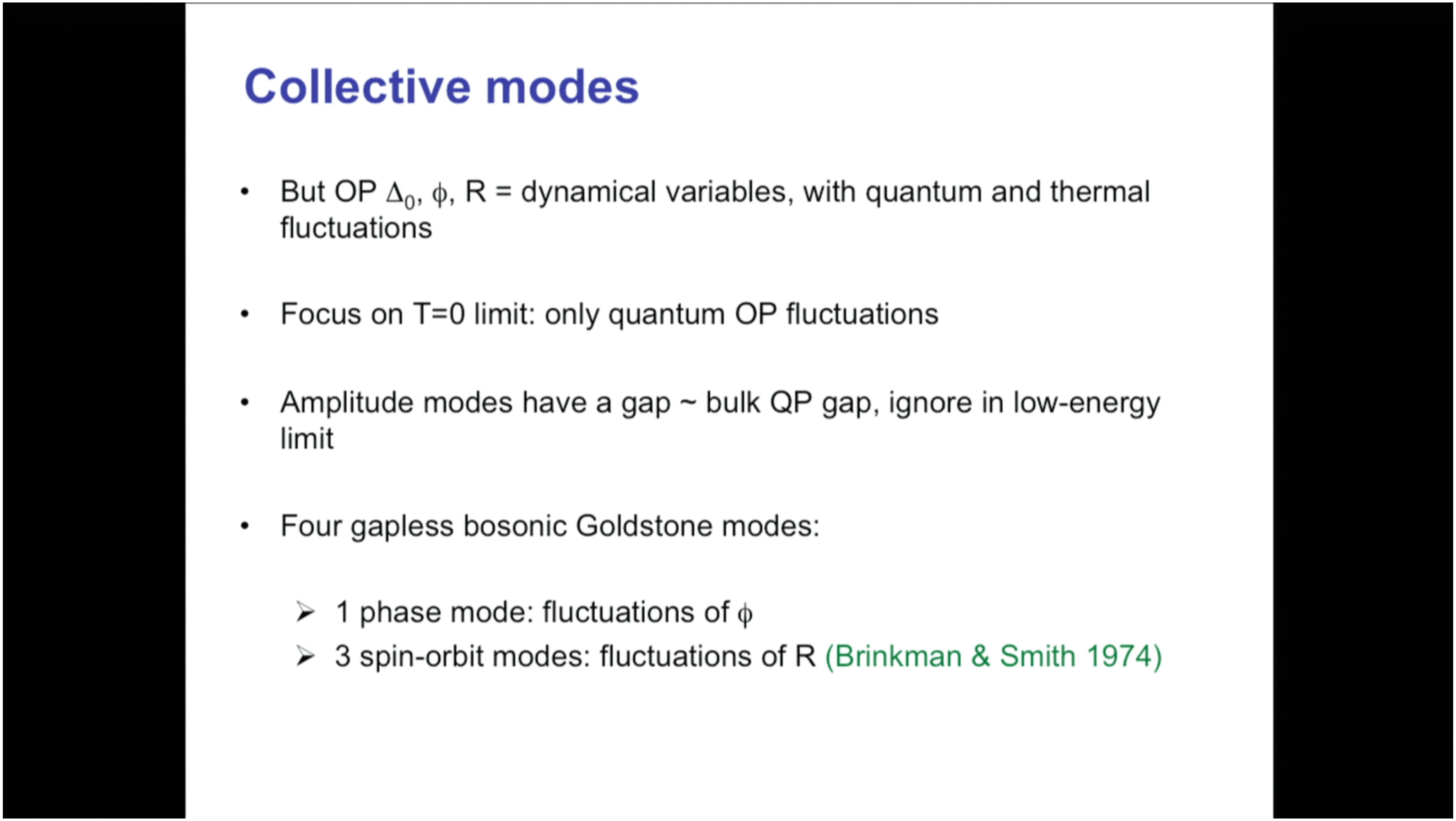
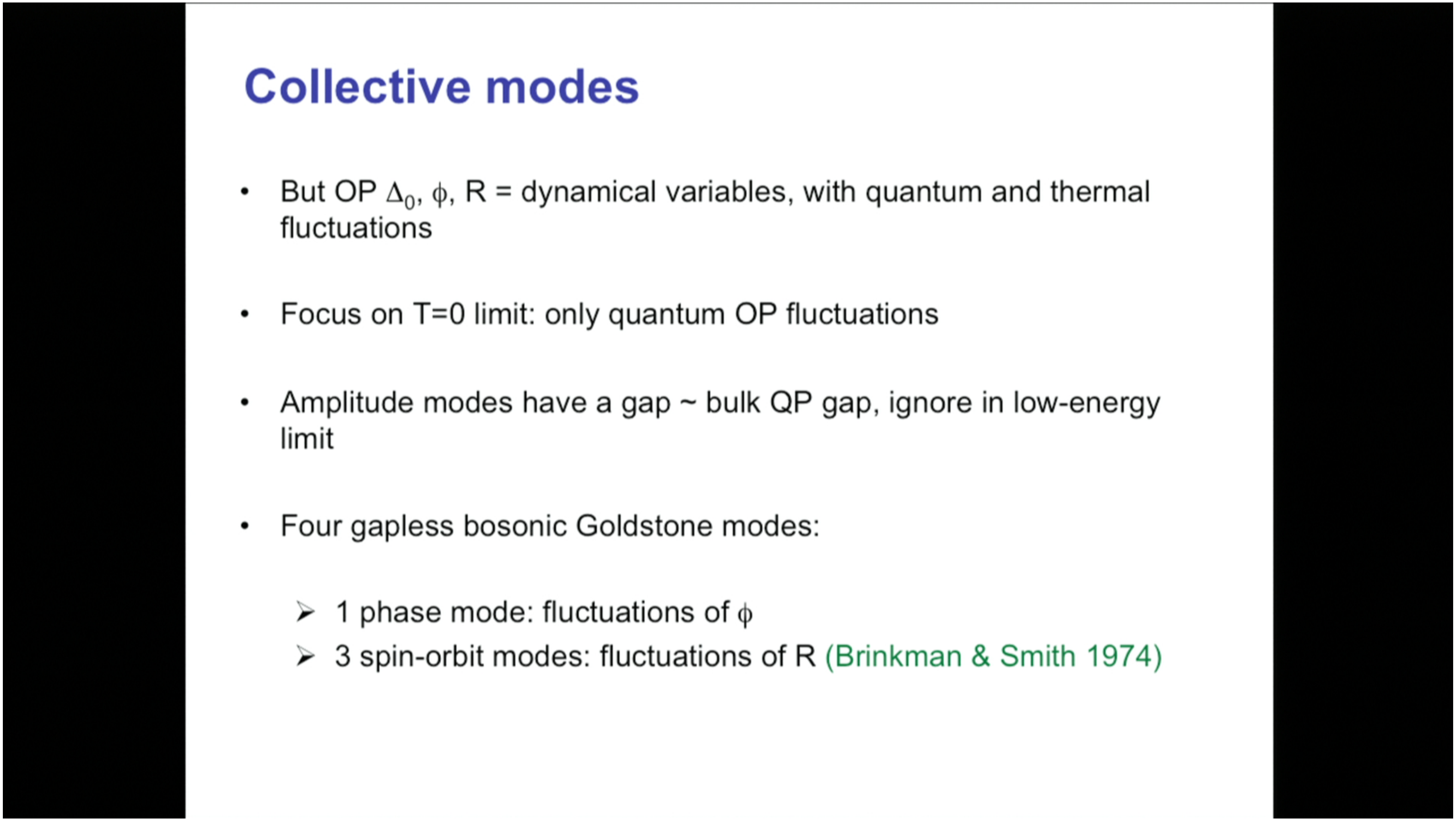




Collective modes

- But OP Δ_0, ϕ, R = dynamical variables, with quantum and thermal fluctuations
- Focus on T=0 limit: only quantum OP fluctuations
- Amplitude modes have a gap ~ bulk QP gap, ignore in low-energy limit
- Four gapless bosonic Goldstone modes:
  - 1 phase mode: fluctuations of ϕ
  - 3 spin-orbit modes: fluctuations of R (Brinkman & Smith 1974)



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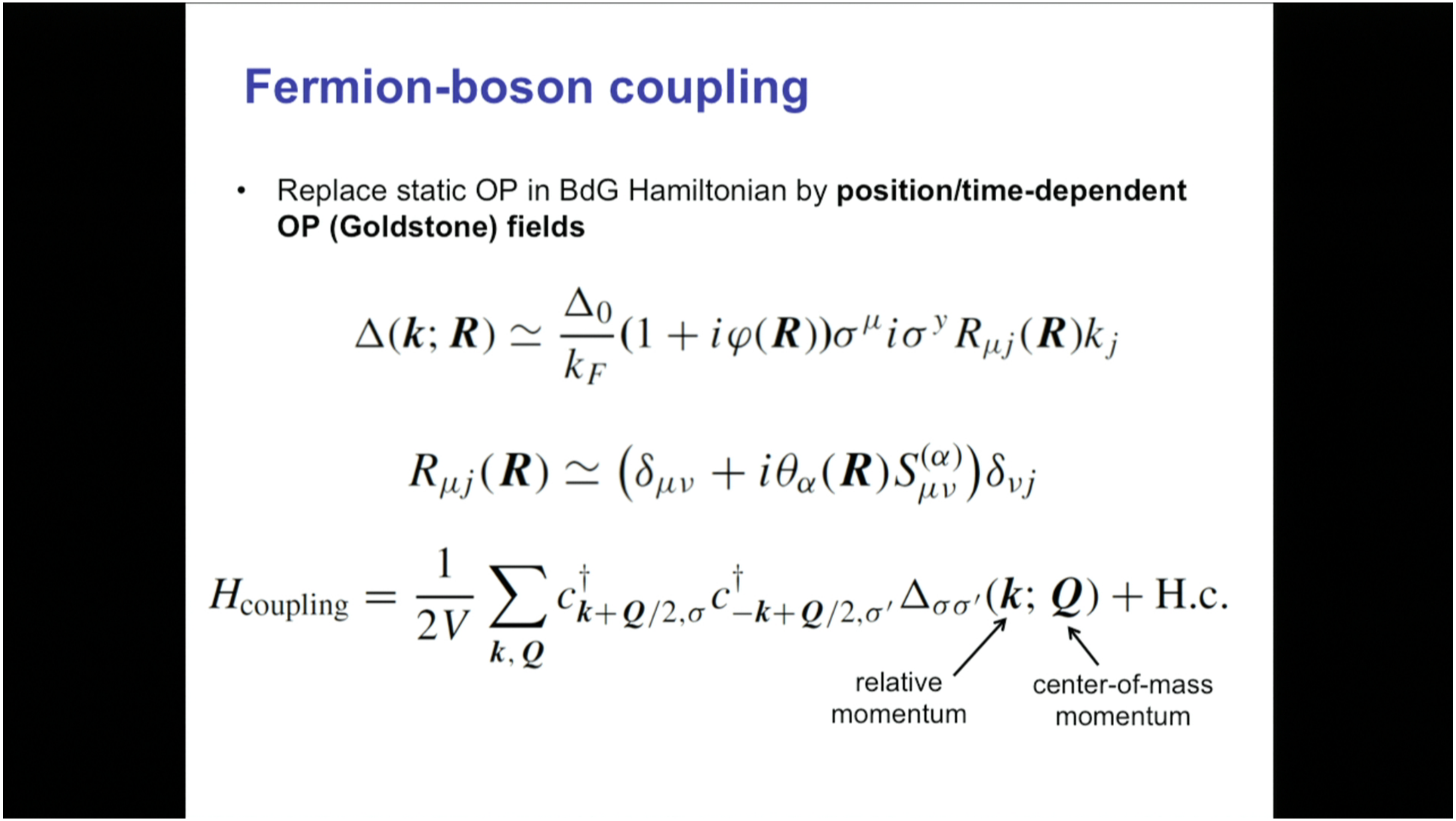

Fermion-boson coupling

- Replace static OP in BdG Hamiltonian by **position/time-dependent OP (Goldstone) fields**

$$\Delta(\boldsymbol{k};\boldsymbol{R}) \simeq \frac{\Delta_0}{k_F}(1+i\varphi(\boldsymbol{R}))\sigma^{\mu} i\sigma^{y} R_{\mu j}(\boldsymbol{R})k_j$$

$$R_{\mu j}(\boldsymbol{R}) \simeq \left(\delta_{\mu\nu} + i\theta_{\alpha}(\boldsymbol{R})S^{(\alpha)}_{\mu\nu}\right)\delta_{\nu j}$$

$$H_{\text{coupling}} = \frac{1}{2V}\sum_{\boldsymbol{k},\boldsymbol{Q}} c^{\dagger}_{\boldsymbol{k}+\boldsymbol{Q}/2,\sigma} c^{\dagger}_{-\boldsymbol{k}+\boldsymbol{Q}/2,\sigma'} \Delta_{\sigma\sigma'}(\underset{\substack{\nearrow \\ \text{relative} \\ \text{momentum}}}{\boldsymbol{k}};\ \underset{\substack{\nwarrow \\ \text{center-of-mass} \\ \text{momentum}}}{\boldsymbol{Q}}) + \text{H.c.}$$

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relative momentum

center-of-mass momentum

Surface-bulk coupling

- In a semi-infinite geometry, this implies a coupling between **surface Majorana fermions** and **bulk Goldstone modes**

- Of the phase mode ϕ and the spin-orbit modes θ_x, θ_y, θ_z, only θ_x couples to the Majorana fermions

$$H_{\text{coupling}} = \frac{\Delta_0}{V}\sum_{\boldsymbol{Q}} \theta_x(-\boldsymbol{Q})\rho(\boldsymbol{Q})$$

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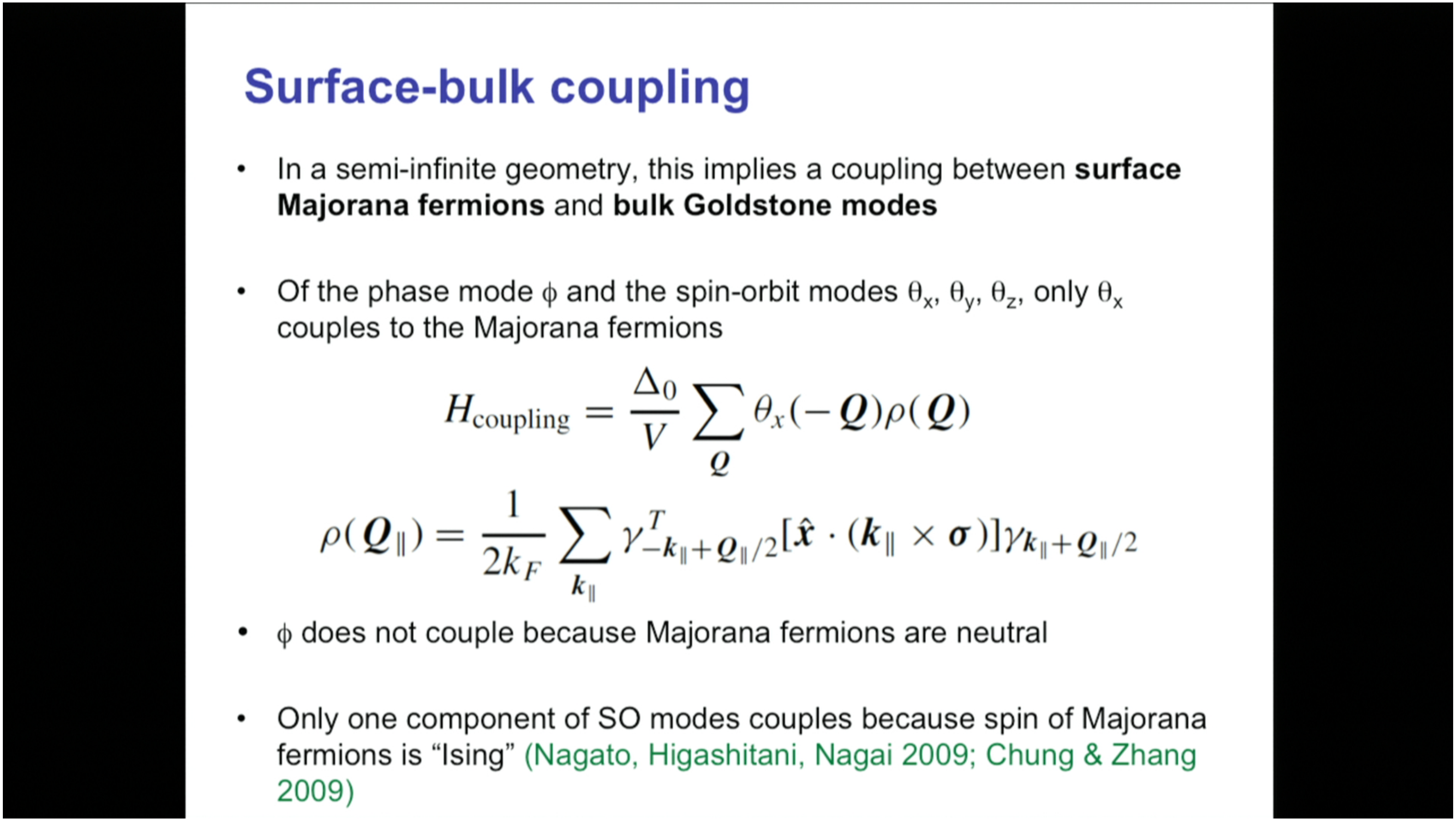
- ϕ does not couple because Majorana fermions are neutral

- Only one component of SO modes couples because spin of Majorana fermions is “Ising” (Nagato, Higashitani, Nagai 2009; Chung & Zhang 2009)



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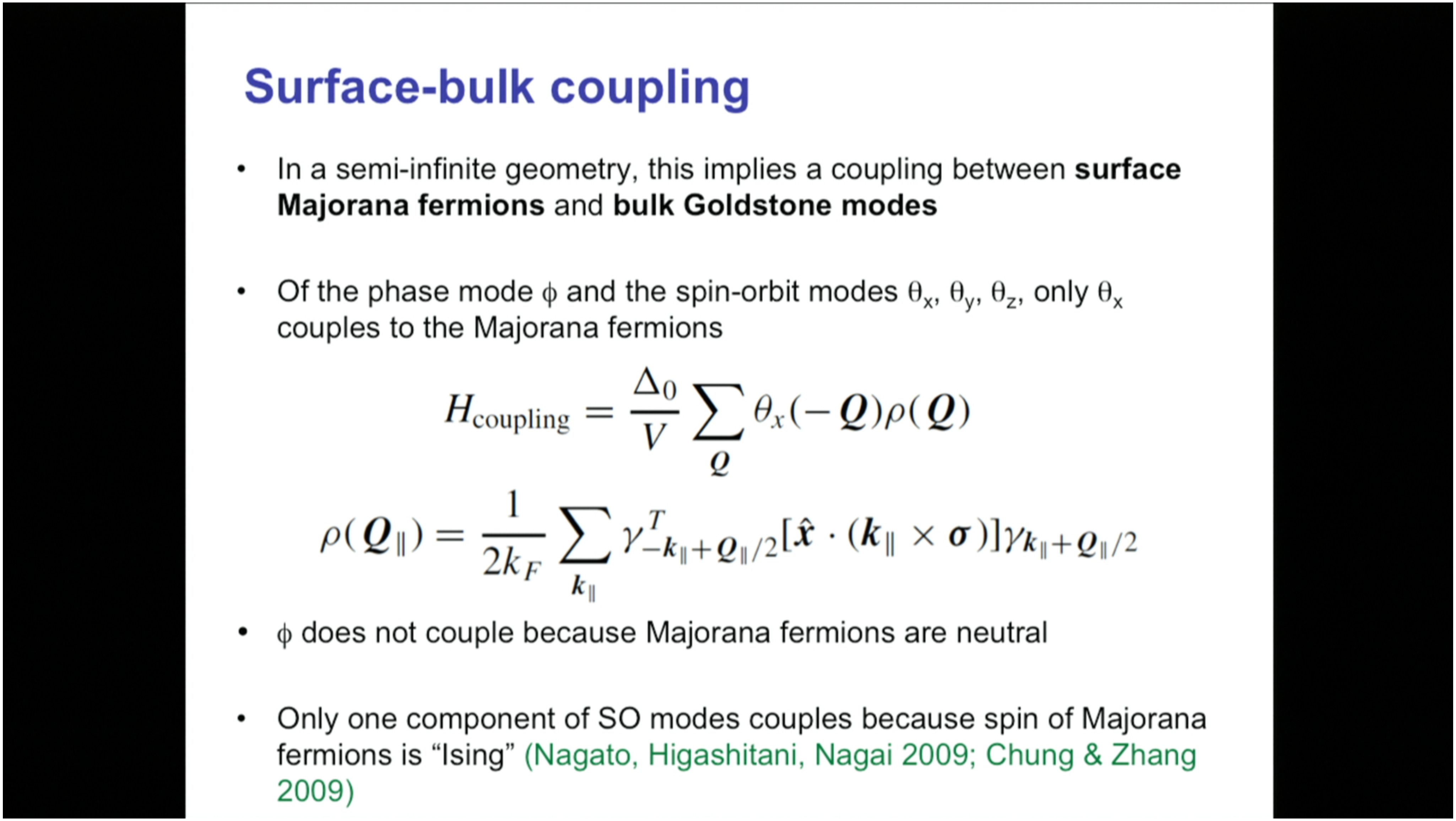

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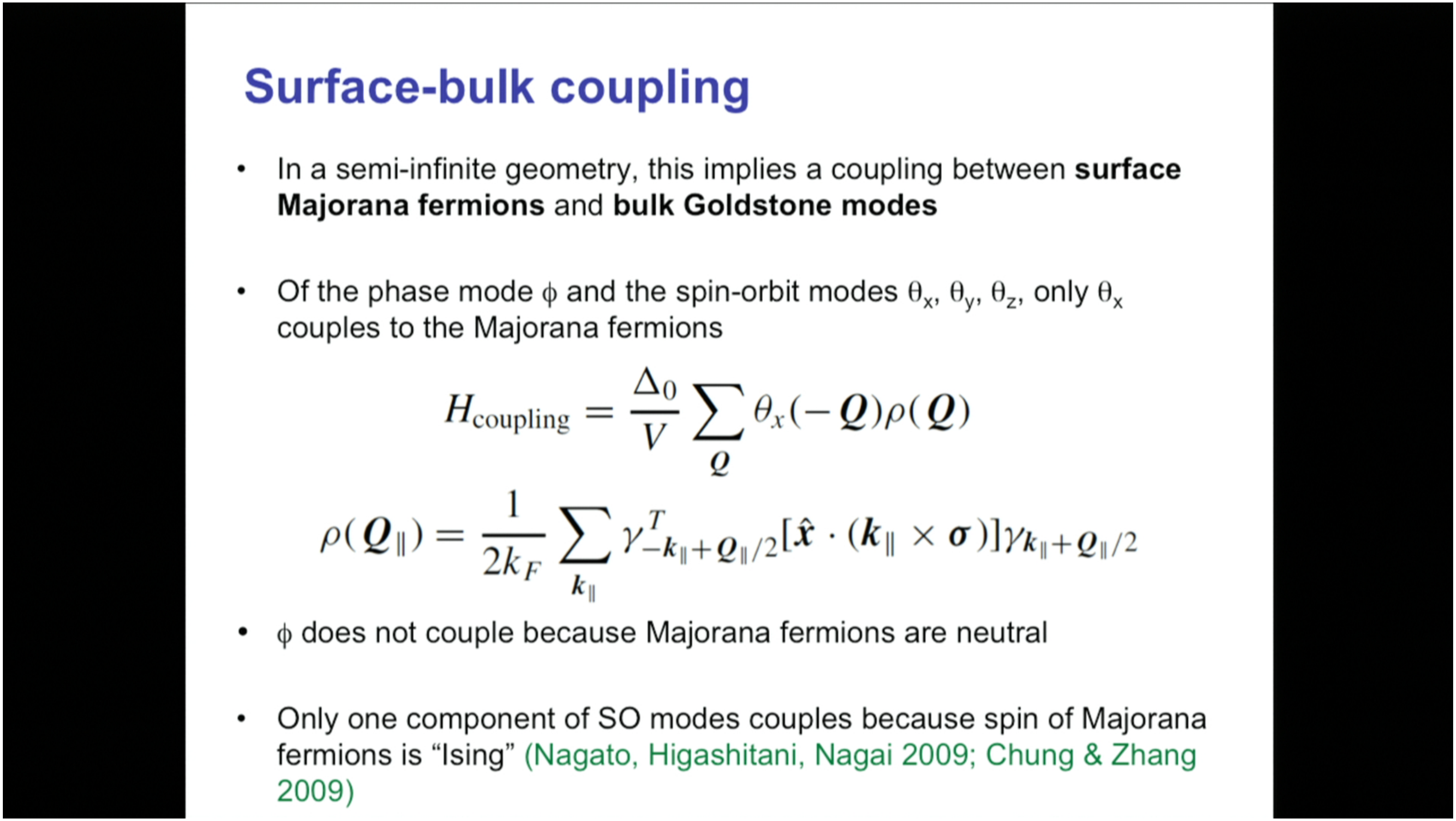

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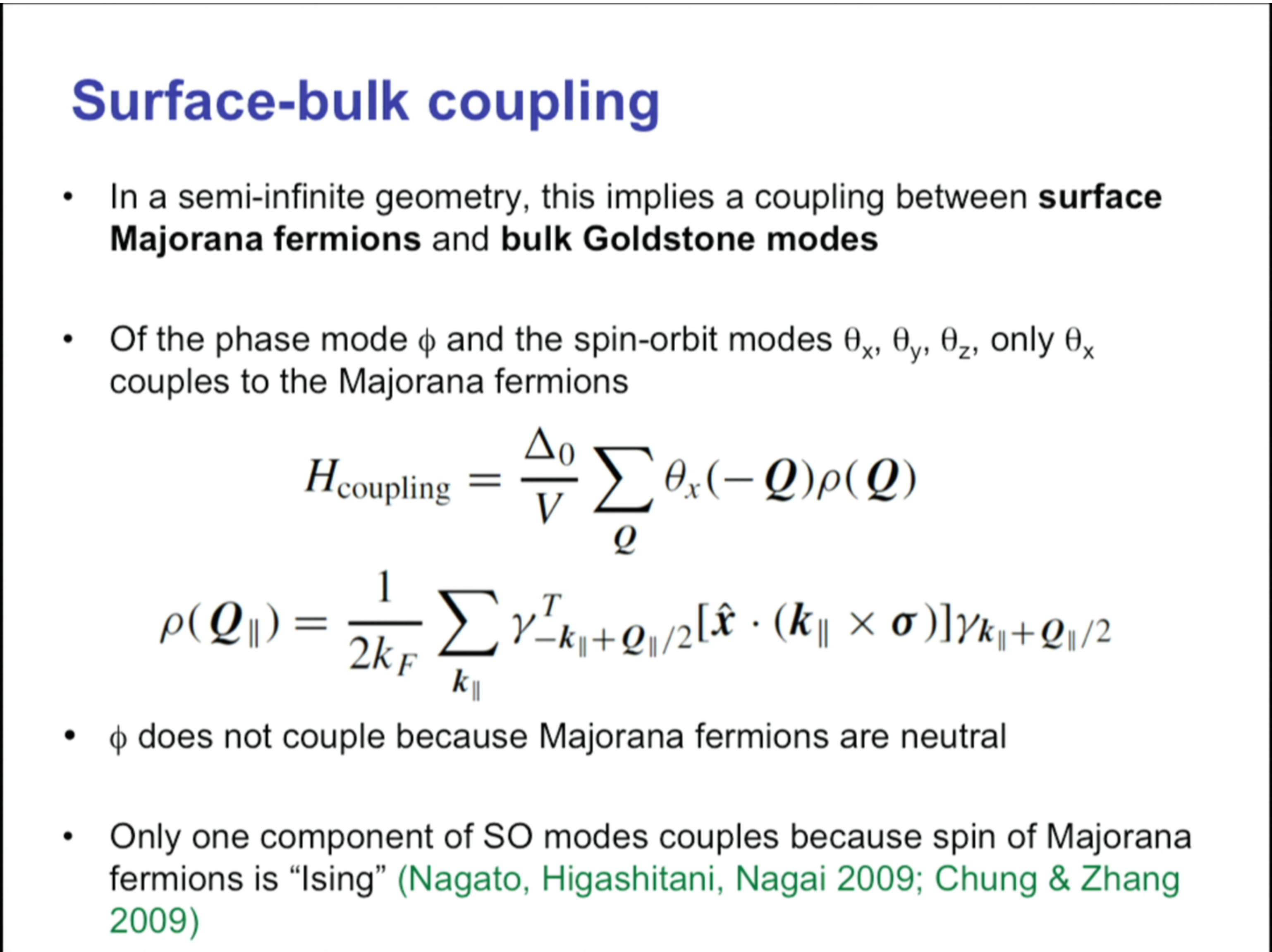

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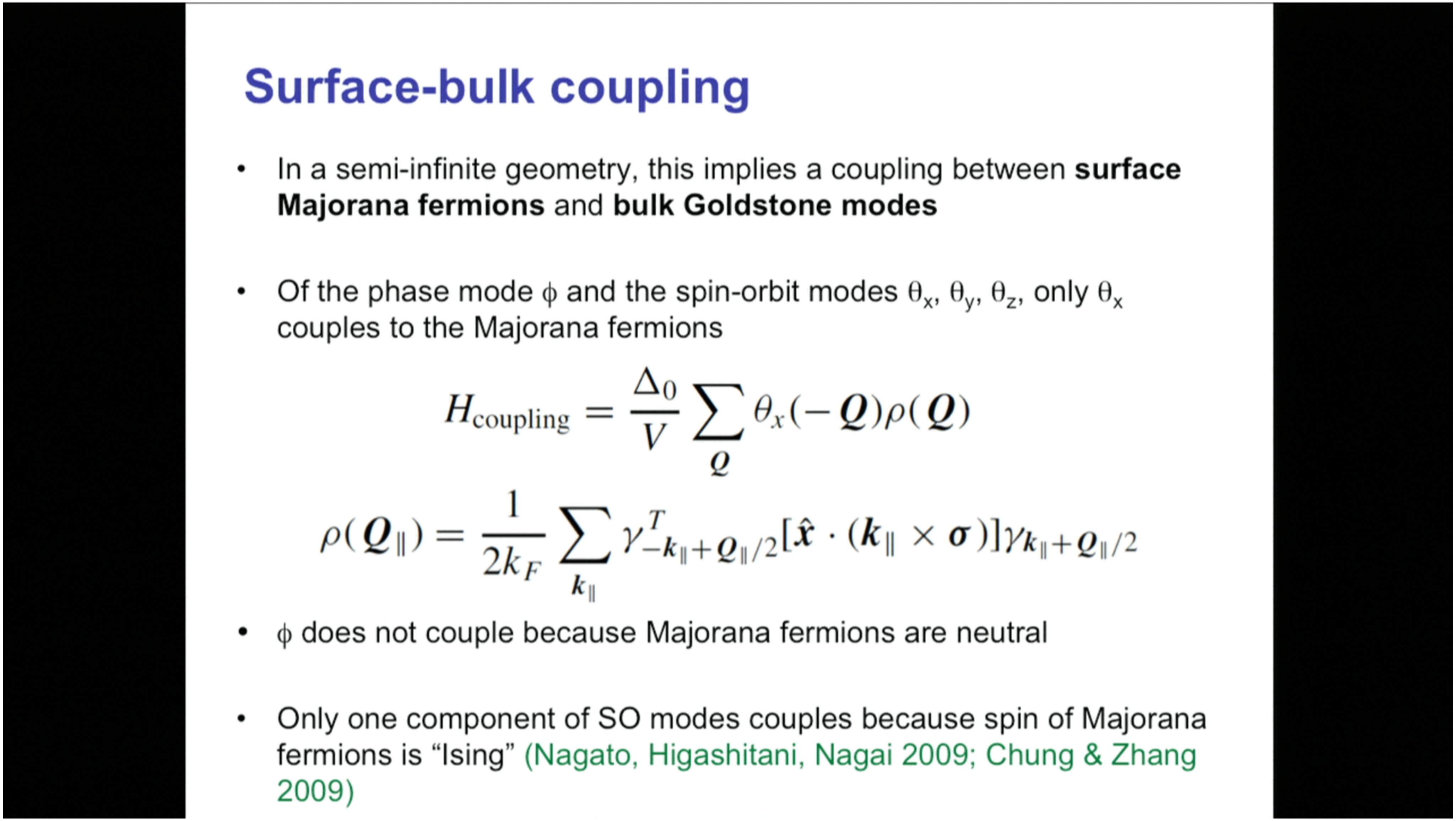

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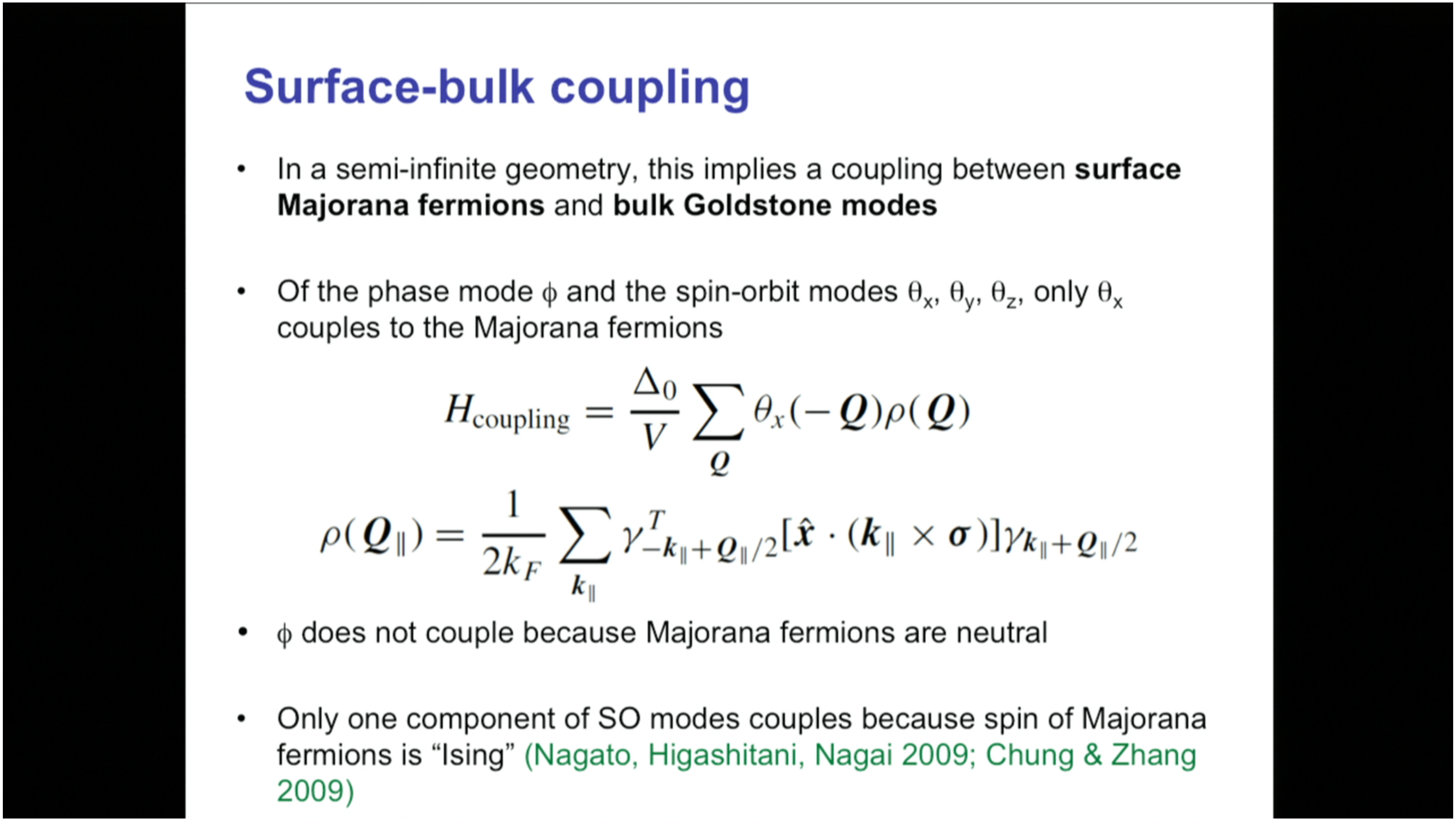
$$\rho(\boldsymbol{Q}_\parallel) = \frac{1}{2k_F}\sum_{\boldsymbol{k}_\parallel} \gamma^T_{-\boldsymbol{k}_\parallel+\boldsymbol{Q}_\parallel/2}[\hat{\boldsymbol{x}}\cdot(\boldsymbol{k}_\parallel\times\boldsymbol{\sigma})]\gamma_{\boldsymbol{k}_\parallel+\boldsymbol{Q}_\parallel/2}$$

- ϕ does not couple because Majorana fermions are neutral
- Only one component of SO modes couples because spin of Majorana fermions is "Ising" (Nagato, Higashitani, Nagai 2009; Chung & Zhang 2009)



Surface-bulk coupling

- In a semi-infinite geometry, this implies a coupling between **surface Majorana fermions** and **bulk Goldstone modes**

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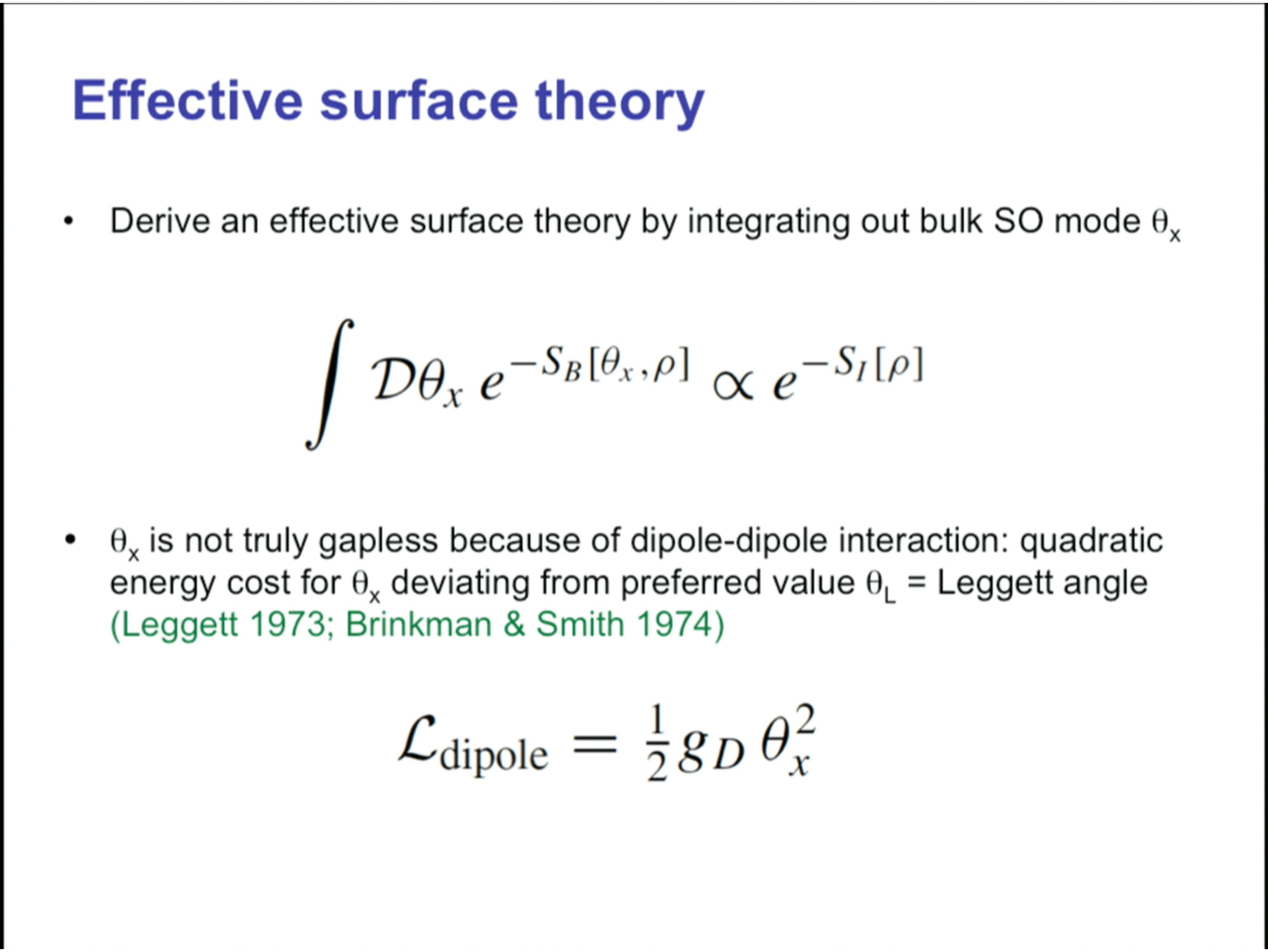
Effective surface theory

- Derive an effective surface theory by integrating out bulk SO mode θ_x

$$\int \mathcal{D}\theta_x \, e^{-S_B[\theta_x,\rho]} \propto e^{-S_I[\rho]}$$

- θ_x is not truly gapless because of dipole-dipole interaction: quadratic energy cost for θ_x deviating from preferred value θ_L = Leggett angle (Leggett 1973; Brinkman & Smith 1974)

$$\mathcal{L}_{\text{dipole}} = \tfrac{1}{2} g_D \, \theta_x^2$$



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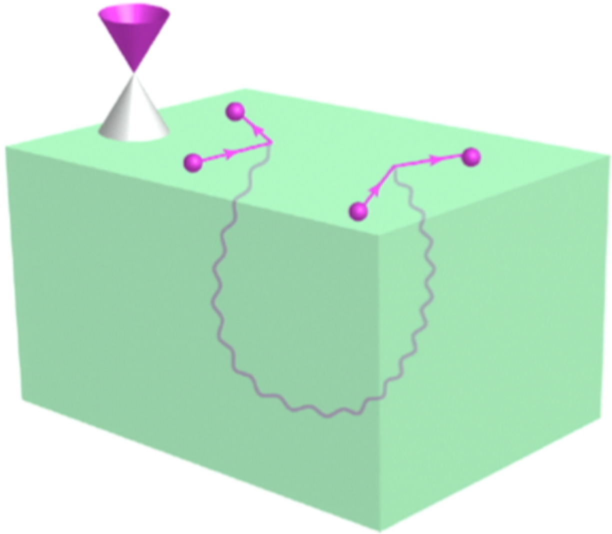
Effective surface interaction

- Integrating out θ_x yields effective surface interaction = interaction between Majorana fermions mediated by exchange of bulk (quasi-)Goldstone bosons

$$H = \frac{v}{2}\sum_{\boldsymbol{k}} \gamma^T_{-\boldsymbol{k}}(\boldsymbol{k}\cdot\boldsymbol{\sigma})\gamma_{\boldsymbol{k}} - \frac{g_0}{2}\sum_{\boldsymbol{Q}} \rho(-\boldsymbol{Q})\rho(\boldsymbol{Q})$$



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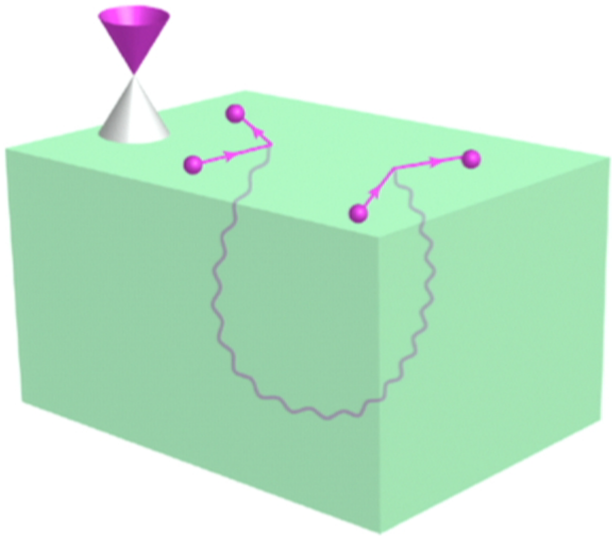

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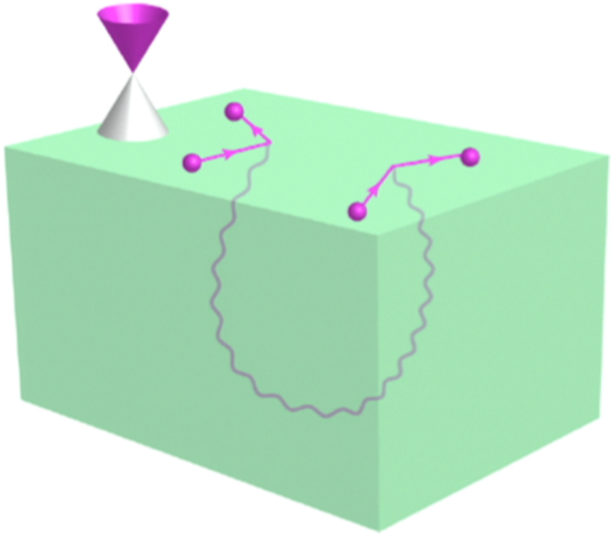

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Possible broken symmetries

- Interaction is perturbatively irrelevant at the free Majorana fermion fixed point, but here coupling constant is finite

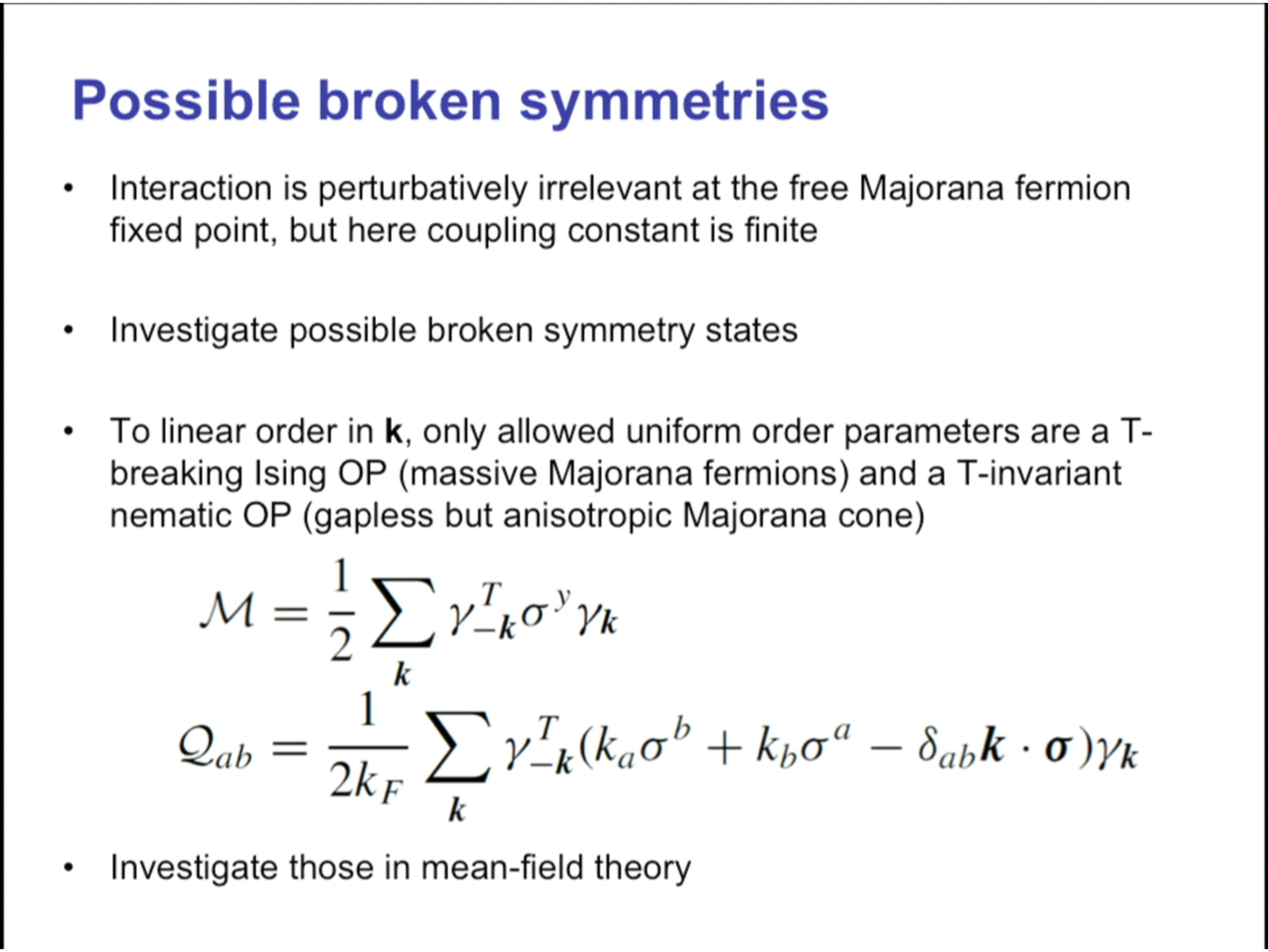
- Investigate possible broken symmetry states

- To linear order in **k**, only allowed uniform order parameters are a T-breaking Ising OP (massive Majorana fermions) and a T-invariant nematic OP (gapless but anisotropic Majorana cone)

$$\mathcal{M} = \frac{1}{2}\sum_{\boldsymbol{k}} \gamma_{-\boldsymbol{k}}^T \sigma^y \gamma_{\boldsymbol{k}}$$

$$\mathcal{Q}_{ab} = \frac{1}{2k_F}\sum_{\boldsymbol{k}} \gamma_{-\boldsymbol{k}}^T (k_a\sigma^b + k_b\sigma^a - \delta_{ab}\boldsymbol{k}\cdot\boldsymbol{\sigma})\gamma_{\boldsymbol{k}}$$

- Investigate those in mean-field theory



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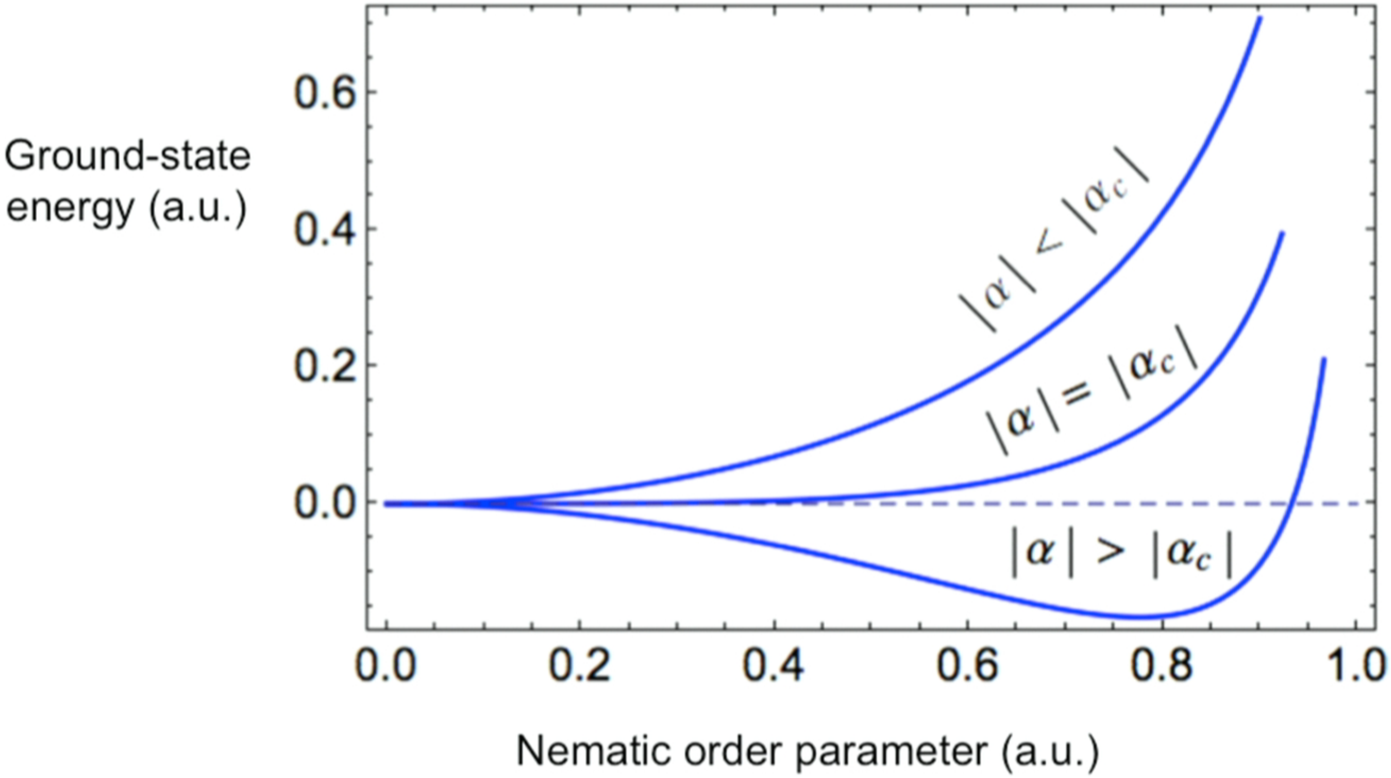
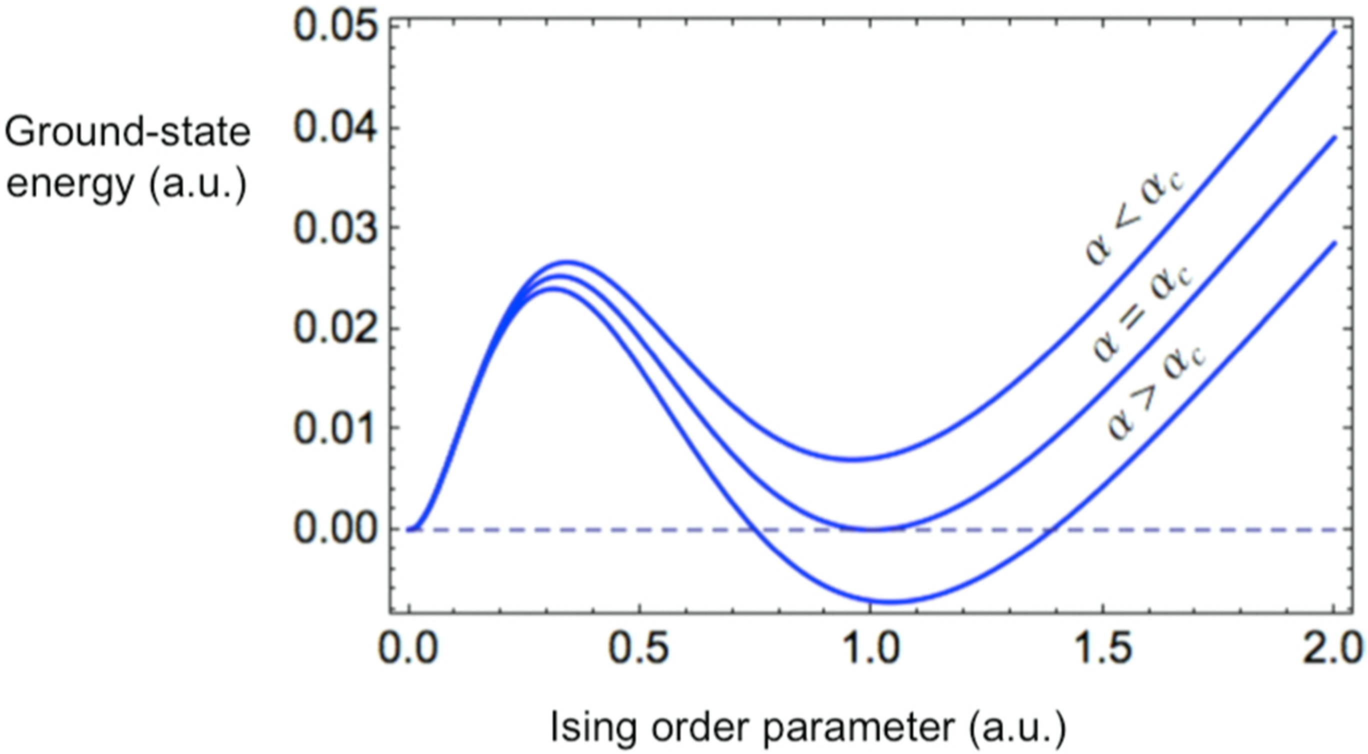
Nematic order

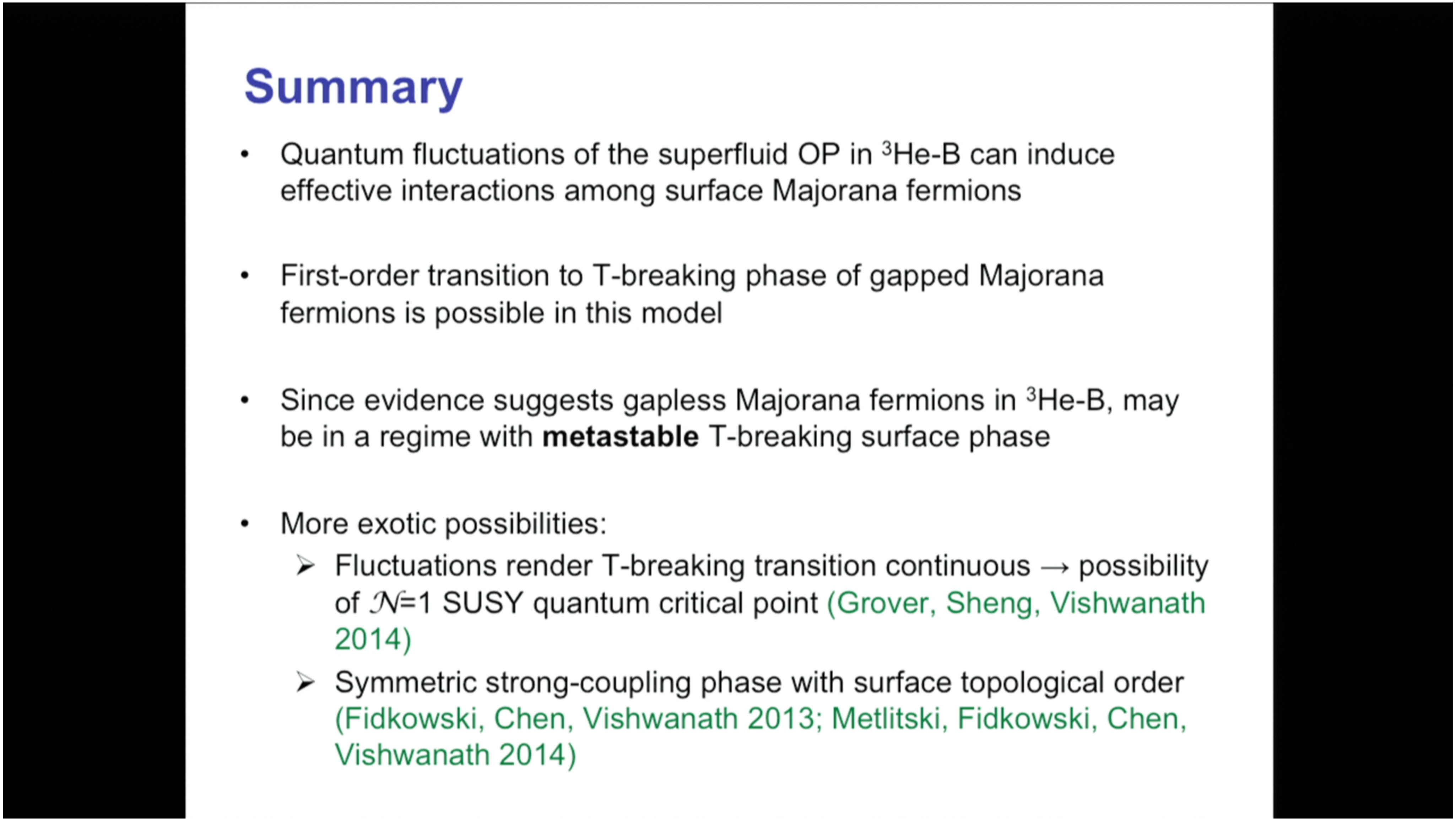


- Continuous nematic transition is possible for negative couplings, but here coupling is positive

Ising order



- First-order transition to T-breaking phase with gapped Majorana fermions is possible in principle



Summary

- Quantum fluctuations of the superfluid OP in ^{3}He-B can induce effective interactions among surface Majorana fermions
- First-order transition to T-breaking phase of gapped Majorana fermions is possible in this model
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- More exotic possibilities:
  - Fluctuations render T-breaking transition continuous → possibility of $\mathcal{N}$=1 SUSY quantum critical point (Grover, Sheng, Vishwanath 2014)
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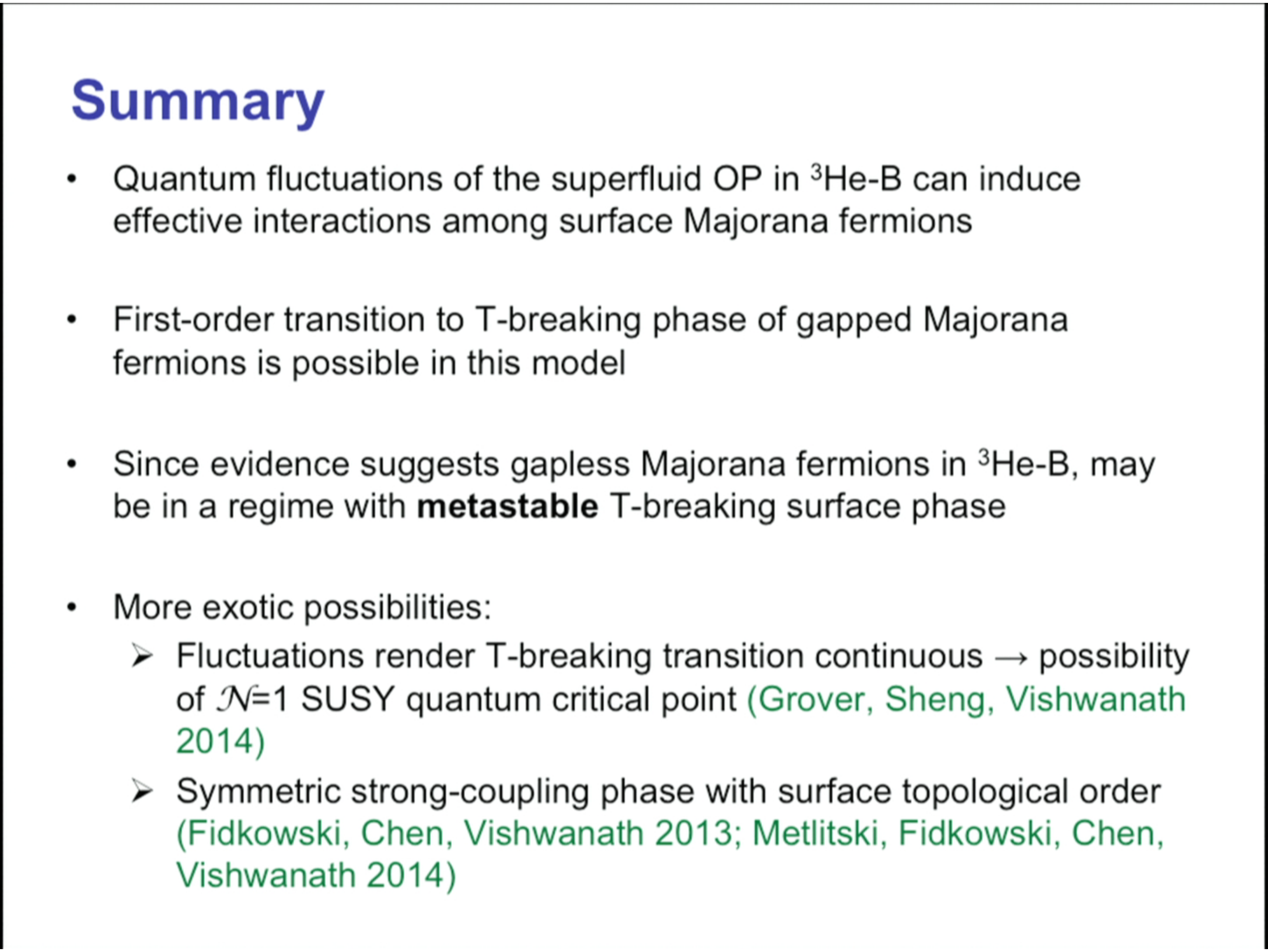

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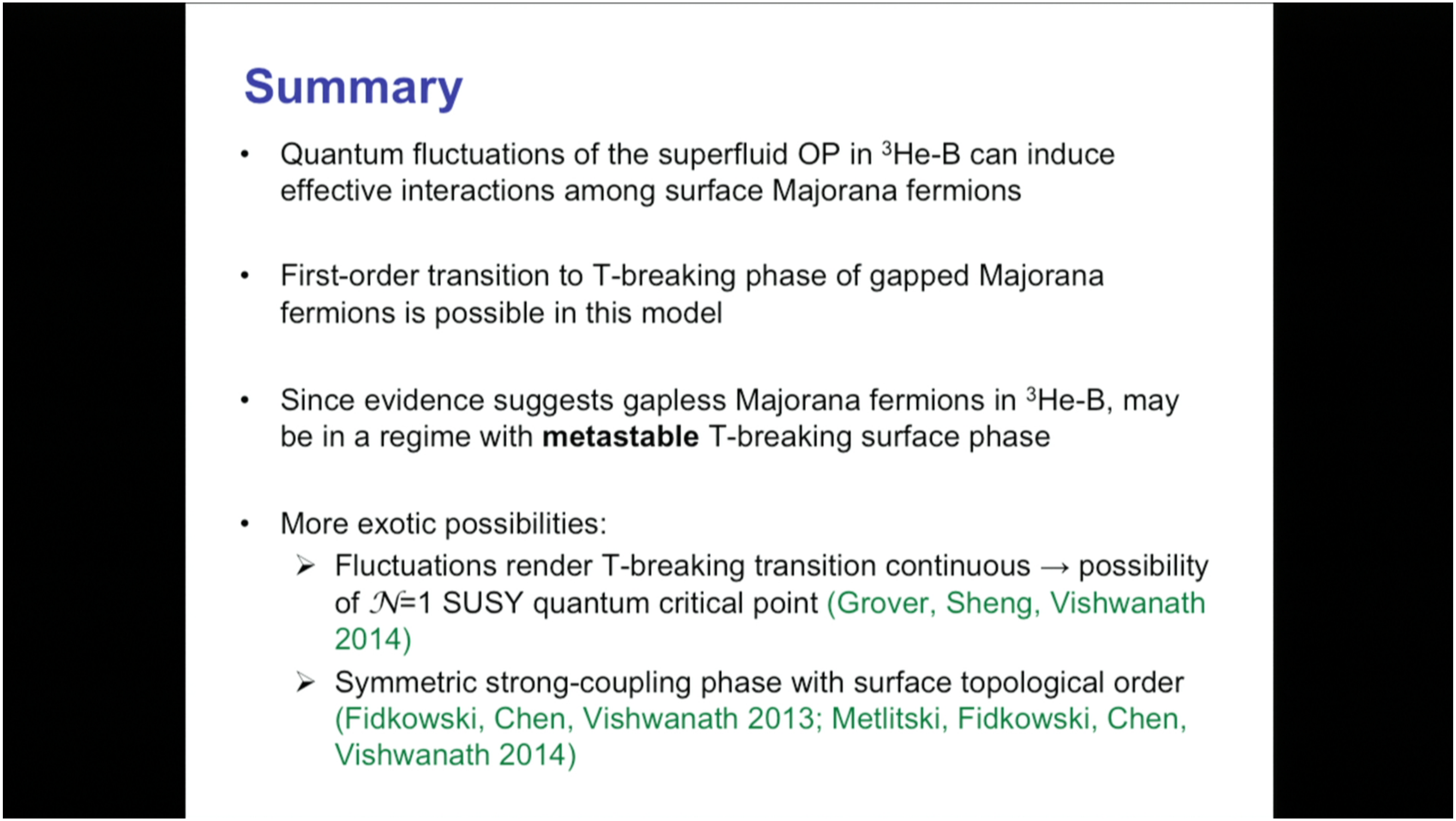

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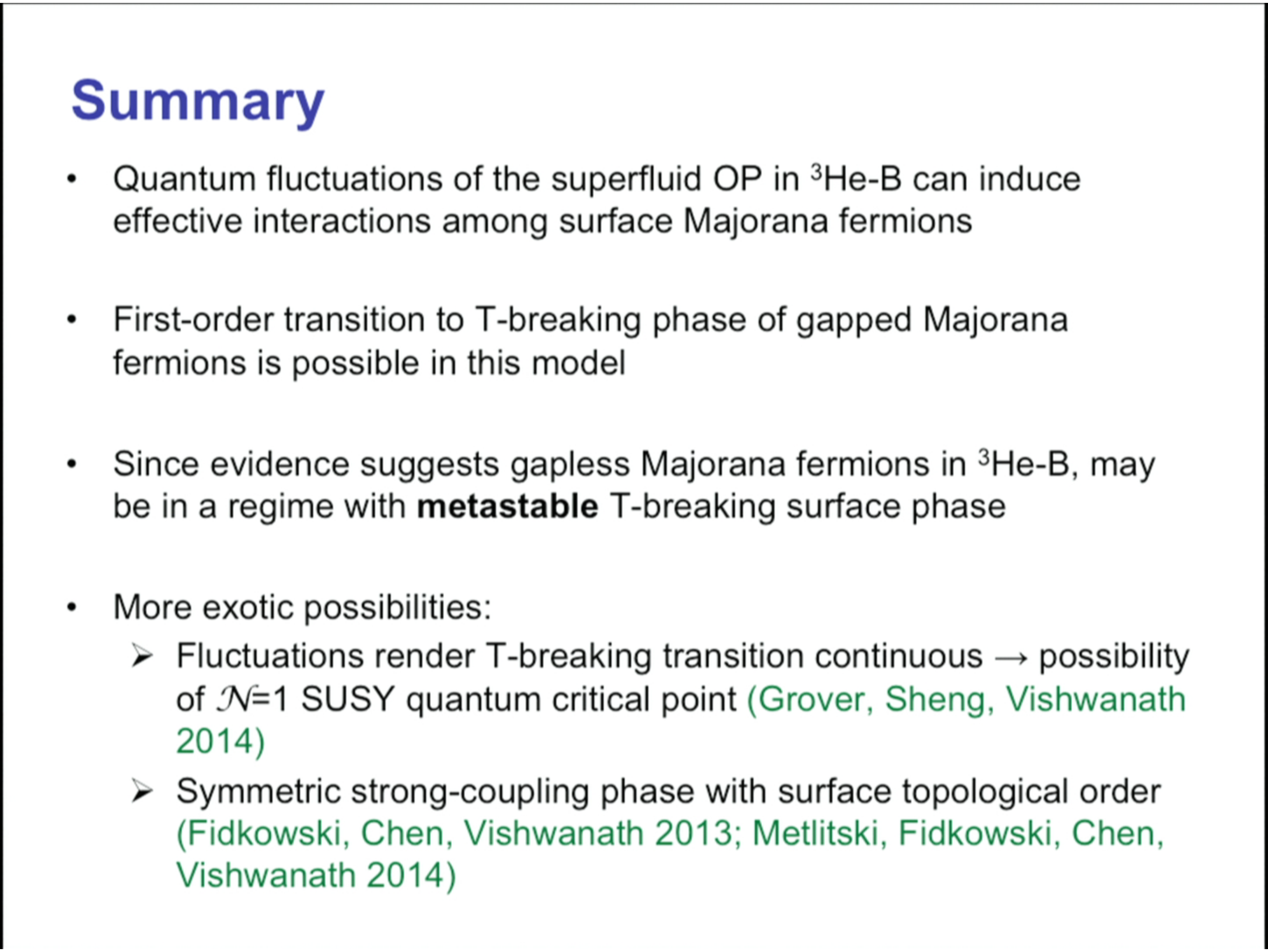

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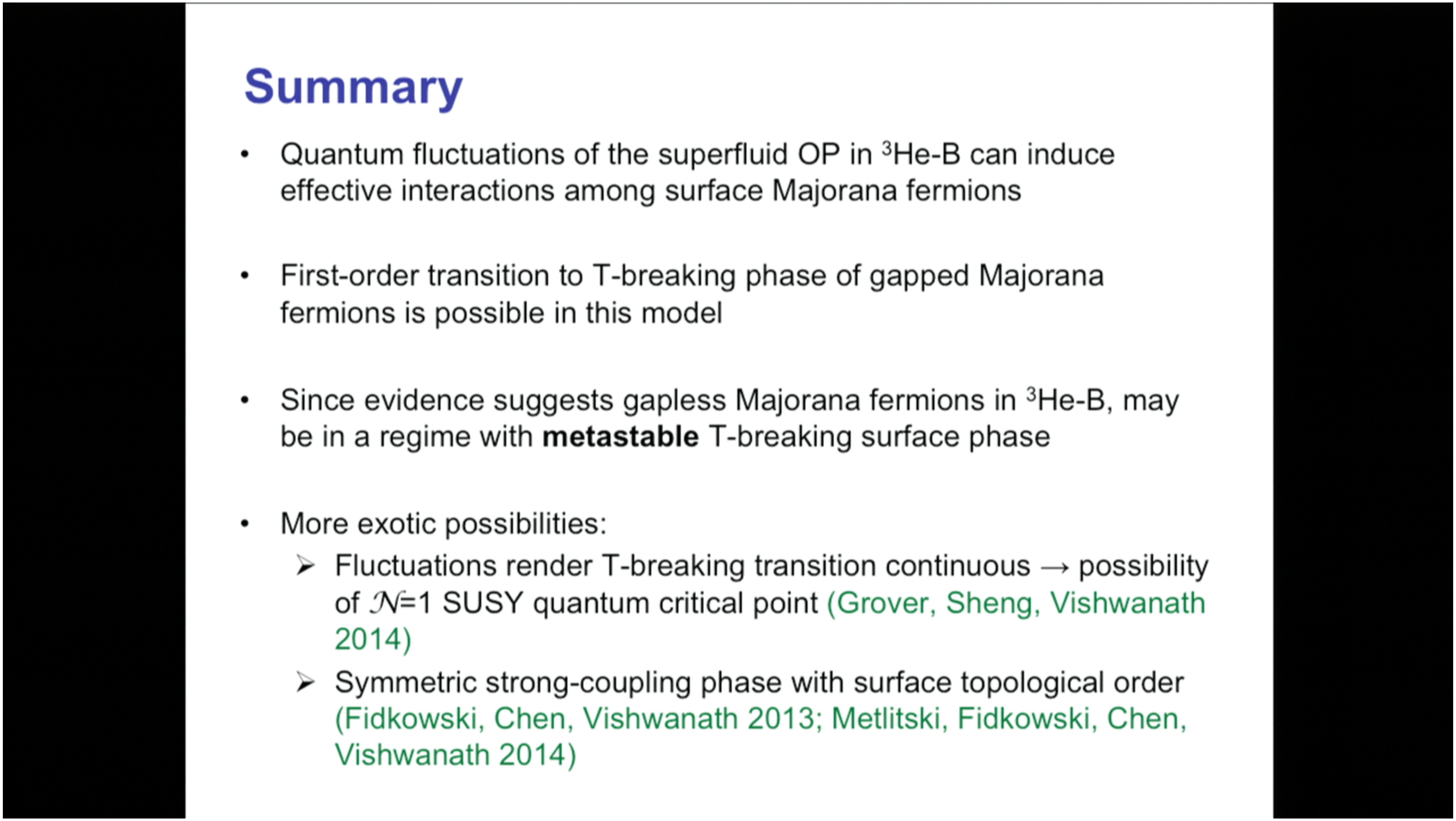

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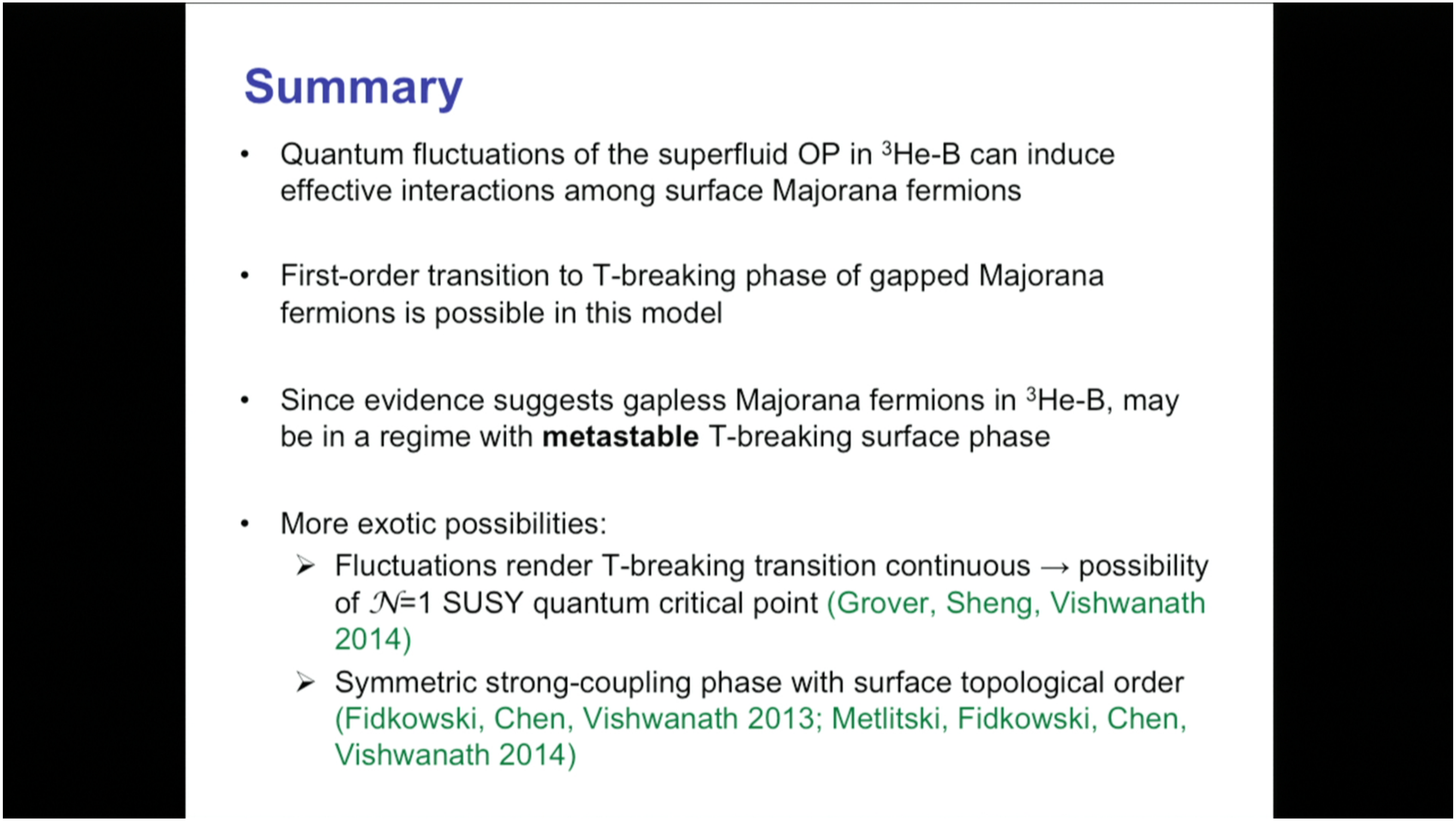

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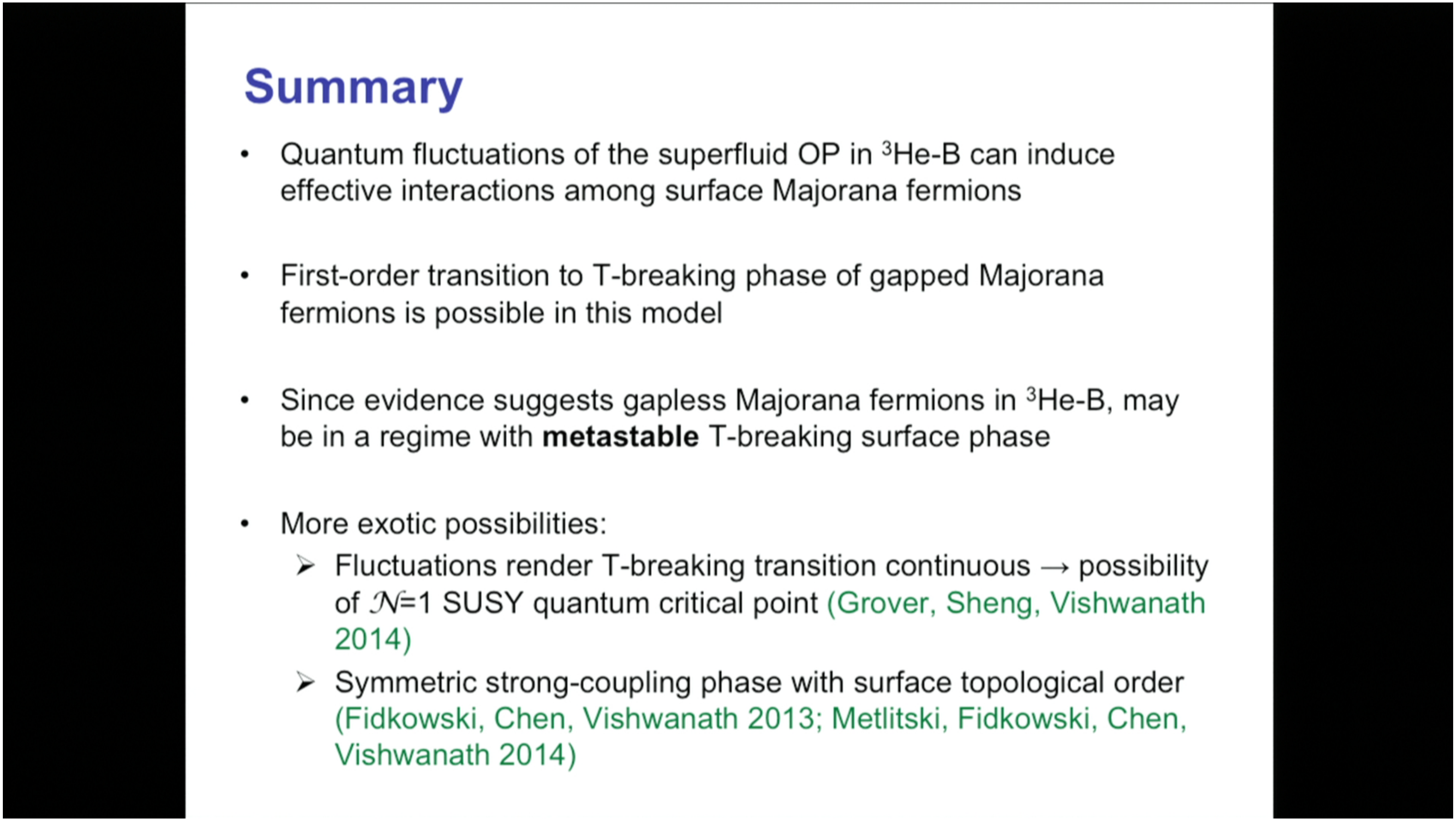

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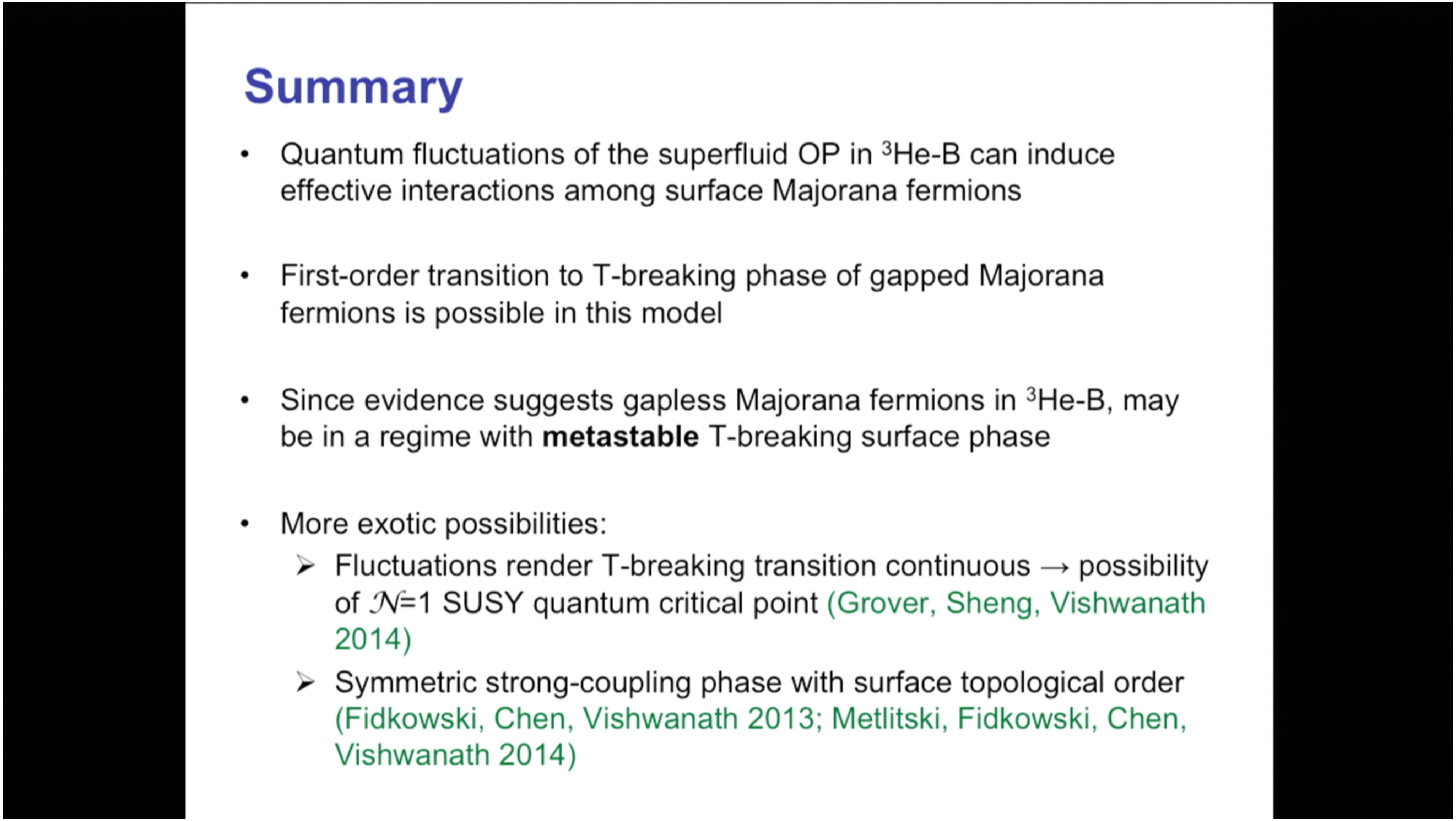

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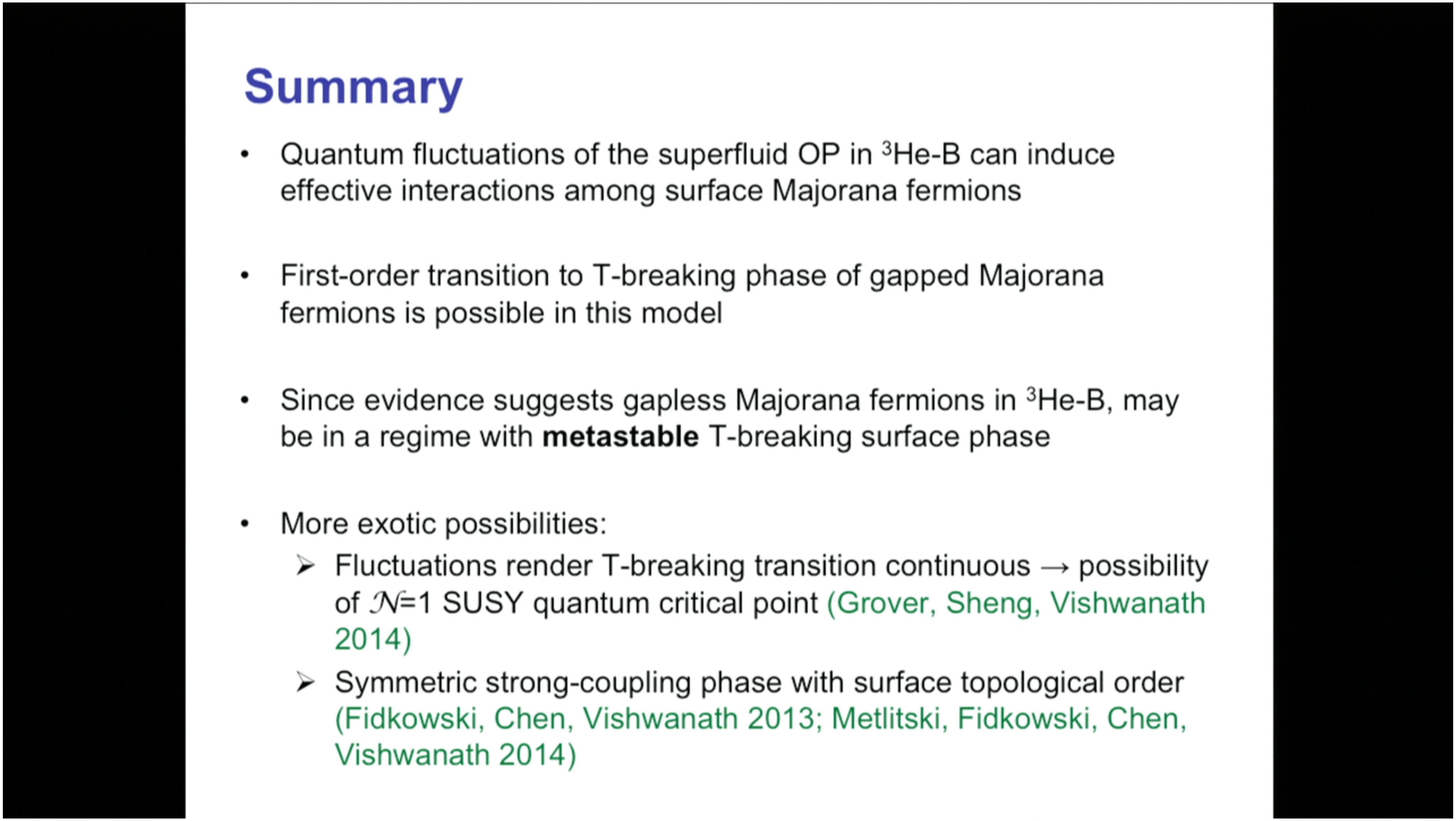

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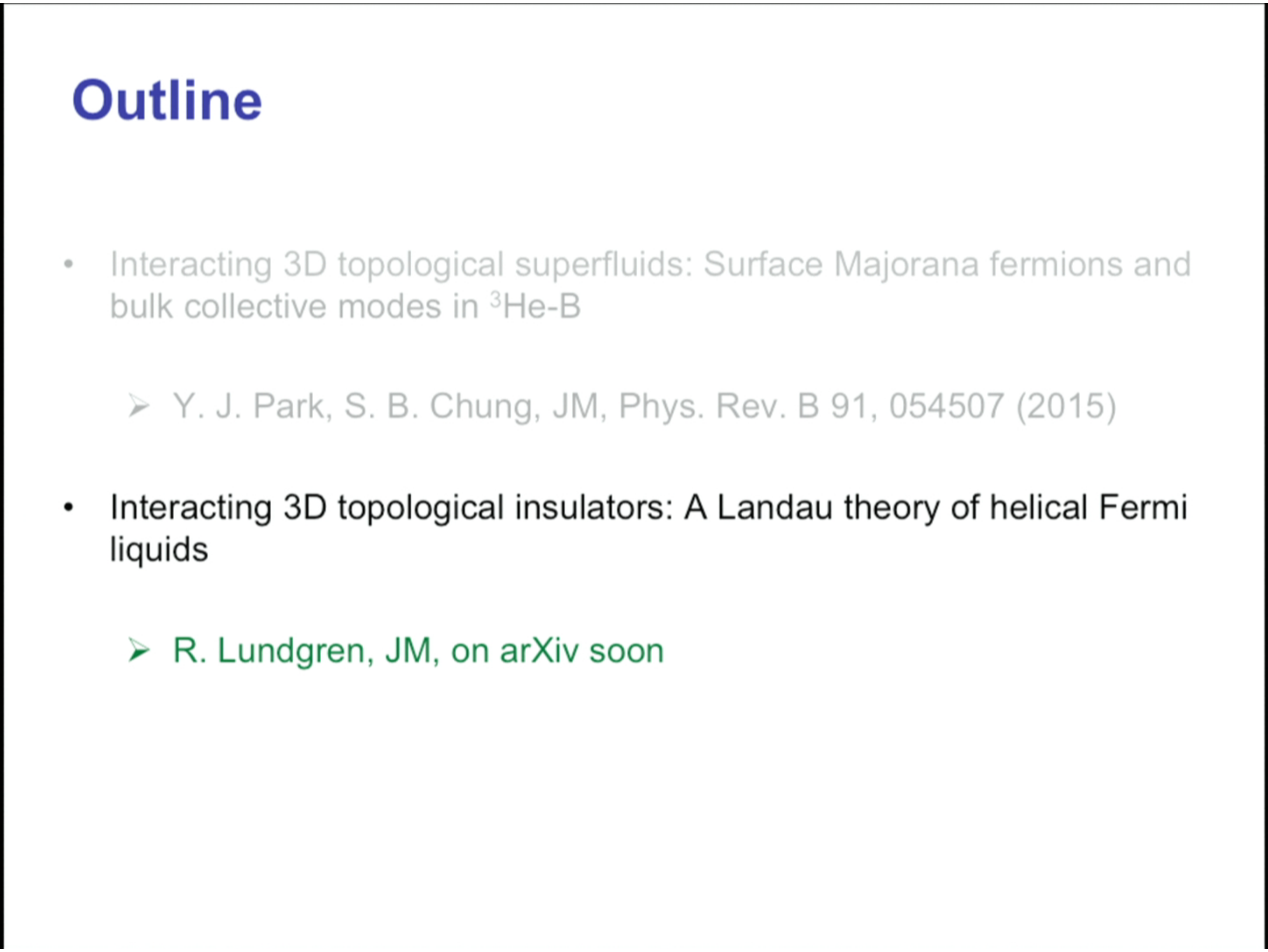
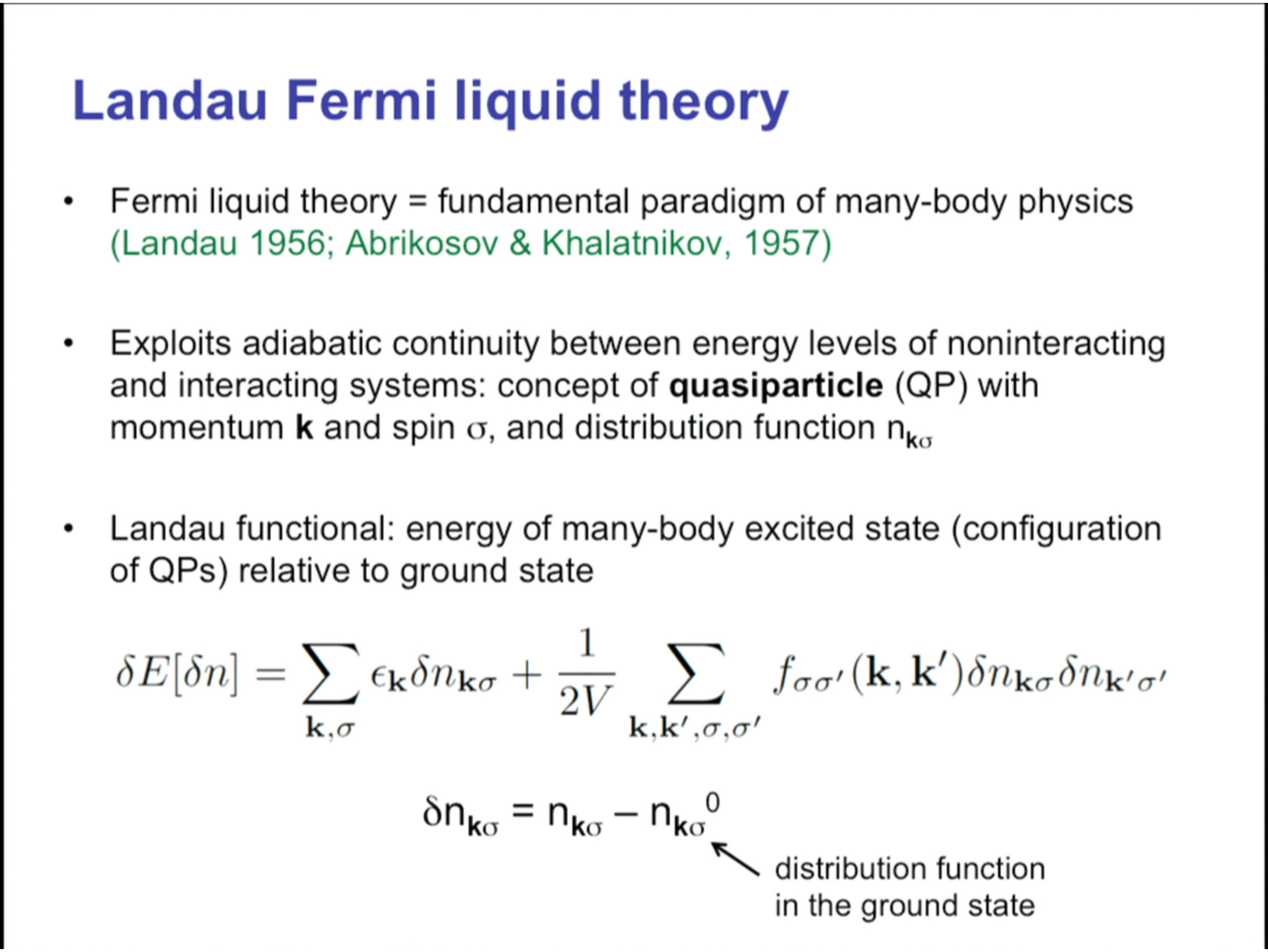


Outline

- Interacting 3D topological superfluids: Surface Majorana fermions and bulk collective modes in ^{3}He-B
  - ➢ Y. J. Park, S. B. Chung, JM, Phys. Rev. B 91, 054507 (2015)
- Interacting 3D topological insulators: A Landau theory of helical Fermi liquids
  - ➢ R. Lundgren, JM, on arXiv soon



Landau Fermi liquid theory

- Fermi liquid theory = fundamental paradigm of many-body physics (Landau 1956; Abrikosov & Khalatnikov, 1957)

- Exploits adiabatic continuity between energy levels of noninteracting and interacting systems: concept of **quasiparticle** (QP) with momentum **k** and spin σ, and distribution function $n_{\mathbf{k}\sigma}$

- Landau functional: energy of many-body excited state (configuration of QPs) relative to ground state

$$\delta E[\delta n] = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \delta n_{\mathbf{k}\sigma} + \frac{1}{2V} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'}$$

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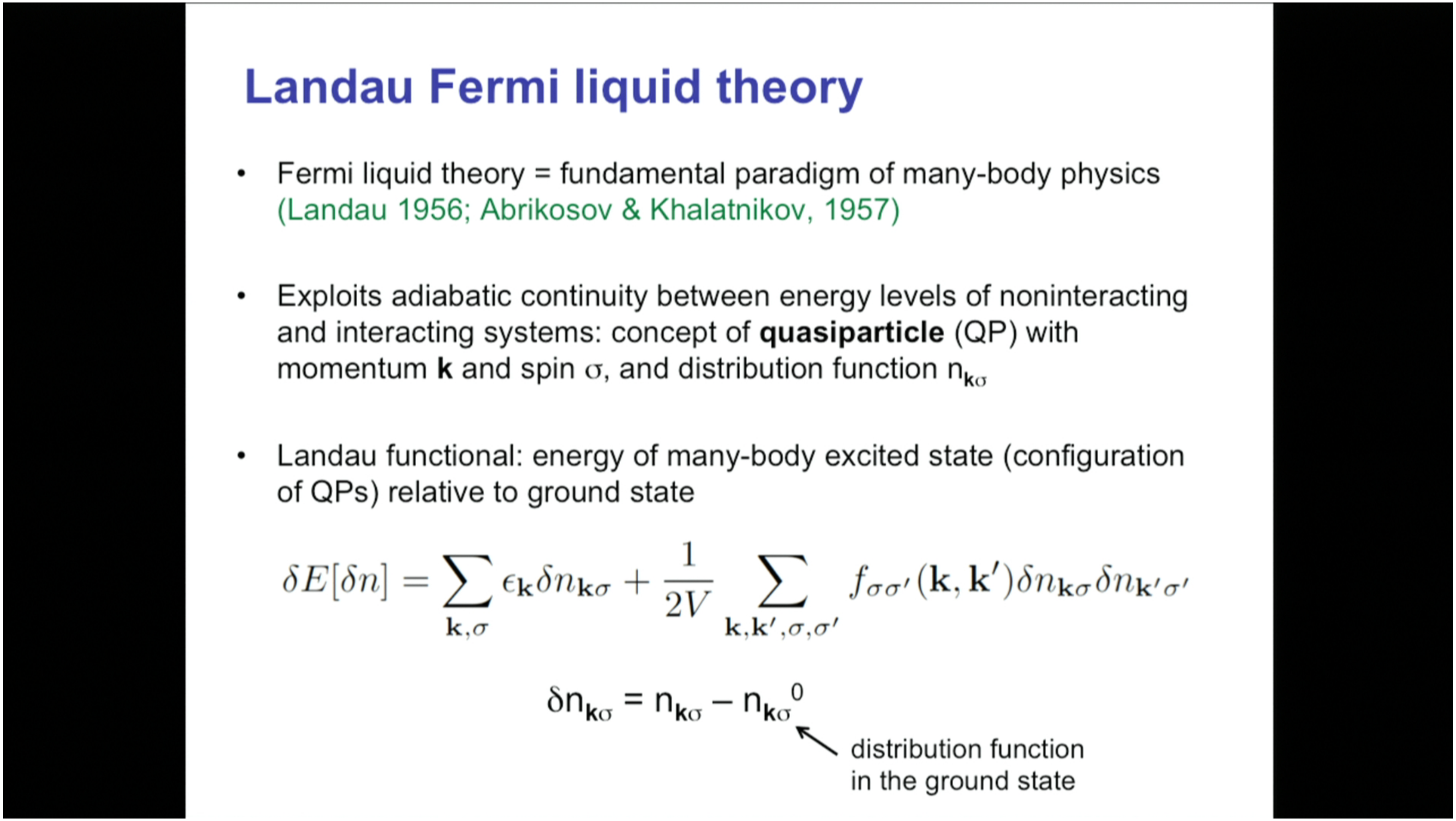
distribution function
in the ground state



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distribution function in the ground state

Landau Fermi liquid theory

- Renormalization of physical properties due to interactions is expressed in terms of Landau parameters, e.g.:

effective mass $$\frac{m^*}{m} = 1 + \tfrac{1}{3} F_1^s$$ Galilean invariance

specific heat ($c_v = \gamma T$) $$\frac{\gamma}{\gamma_0} = \frac{m^*}{m}$$

compressibility $$\frac{\kappa}{\kappa_0} = \frac{m^*}{m} \frac{1}{1 + F_0^s}$$

spin susceptibility $$\frac{\chi}{\chi_0} = \frac{m^*}{m} \frac{1}{1 + F_0^a}$$

A theory of helical Fermi liquids?

- Surface states of actual 3D TI materials are interacting: should be “helical Fermi liquids”
- Can we construct a phenomenological Landau theory for them?
- Expect qualitative differences due to spin-orbit (SO) coupling
- FL theory for non-topological 2D SO coupled 2DEG recently constructed (Ashrafi, Rashba, Maslov, PRB 2013), but complicated due to presence of two Fermi surfaces
- Here due to topology, helical FL has a single Fermi surface: will look like effectively spinless FL theory!

Landau Fermi liquid theory

- Interactions between QPs near the Fermi surface described by dimensionless **Landau parameters F_ℓ^s, F_ℓ^a**: parameterization of general short-range interactions consistent with symmetries (TRS, spatial SO(d) rotations, spin SU(2) rotations)

$$\delta E[\delta n] = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \delta n_{\mathbf{k}\sigma} + \frac{1}{2V} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'}$$

$$f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') = f_{\sigma\sigma'}(\mathbf{k}_F,\mathbf{k}'_F) = f^s(\theta) + \sigma\sigma' f^a(\theta)$$

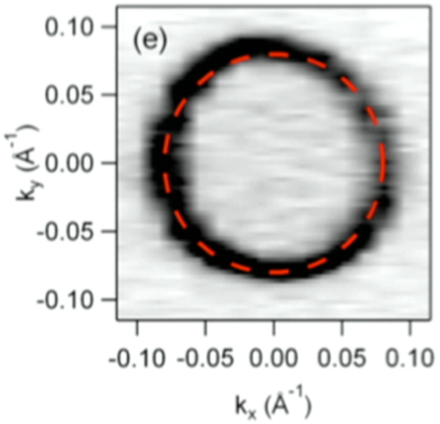
$$f_l^{s,a} = (2l+1) \int_0^\pi \frac{d\Omega}{4\pi} f^{s,a}(\theta) P_l(\cos\theta)$$

$$F_l^{s,a} = 2N^*(0) f_l^{s,a}$$

Symmetries of the helical FL

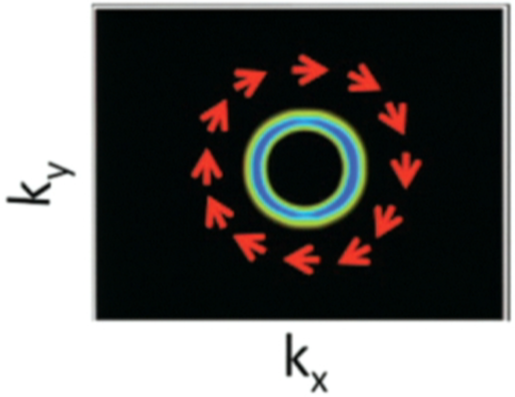
- TRS = protecting symmetry of 3D TI
- Rotation symmetry: will focus on materials with (almost) perfectly circular Fermi surface, e.g.:

Bi_2Se_3



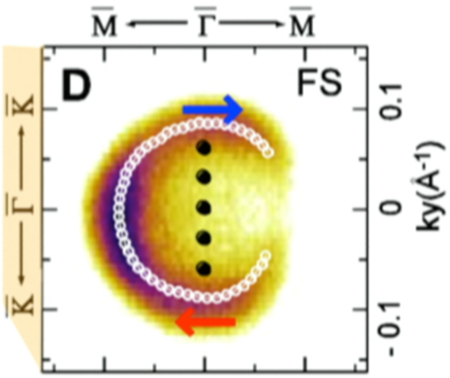
Pan et al., PRL 2011

Bi_2Te_2Se



Neupane et al., PRB 2013

$TlBiSe_2$



Kuroda et al., arXiv 2013

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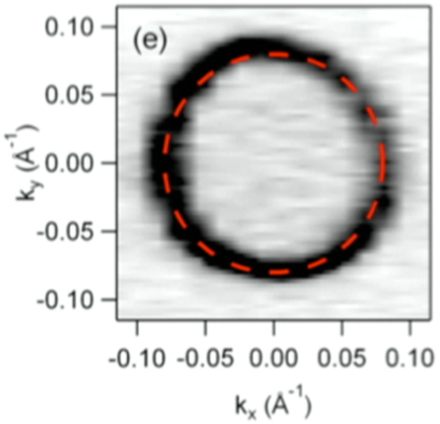

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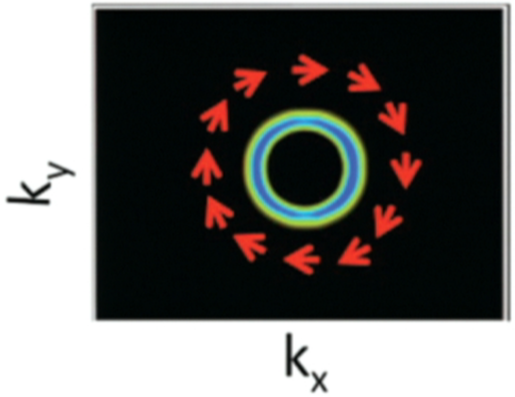

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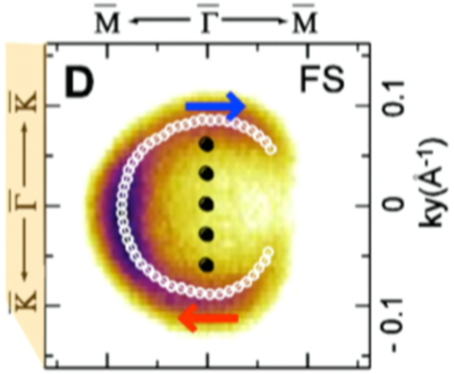

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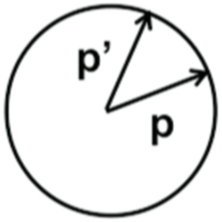
Landau functional

- Due to SO coupling, QP distribution function is in a general a 2x2 matrix in spin space

$$\delta n_{\boldsymbol{p}}^{\alpha\beta} \equiv n_{\boldsymbol{p}}^{\alpha\beta} - n_{\boldsymbol{p}}^{(0)\alpha\beta}$$

- Can construct Landau functional:



$$\delta E[\delta n_{\boldsymbol{p}}] = \int đp \, h_{\alpha\beta}(\boldsymbol{p}) \delta n_{\boldsymbol{p}}^{\alpha\beta} + \frac{1}{2} \int đp \, đp' \, V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}') \delta n_{\boldsymbol{p}}^{\alpha\beta} \delta n_{\boldsymbol{p}'}^{\gamma\delta}$$

$$h(\boldsymbol{p}) = v_F \, \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

Dirac Hamiltonian
(spin-momentum locking)

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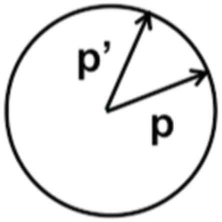
$$\delta n_{\boldsymbol{p}}^{\alpha\beta} \equiv n_{\boldsymbol{p}}^{\alpha\beta} - n_{\boldsymbol{p}}^{(0)\alpha\beta}$$

- Can construct Landau functional:



$$\delta E[\delta n_{\boldsymbol{p}}] = \int \text{đ}p\, h_{\alpha\beta}(\boldsymbol{p})\delta n_{\boldsymbol{p}}^{\alpha\beta} + \frac{1}{2}\int \text{đ}p\, \text{đ}p'\, V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}')\delta n_{\boldsymbol{p}}^{\alpha\beta}\delta n_{\boldsymbol{p}'}^{\gamma\delta}$$

$$h(\boldsymbol{p}) = v_F\, \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

Dirac Hamiltonian
(spin-momentum locking)

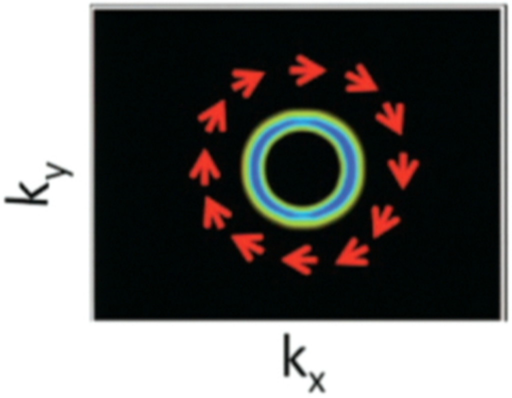
Spin-orbit rotation symmetry

- Rotation symmetry: L_z and S_z are not good quantum numbers, only $J_z=L_z+S_z$ is (spin-momentum locking)

$$h(\boldsymbol{p}) = v_F \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

$$[J_z, h(\boldsymbol{p})] = 0$$

$$J_z = -i\frac{\partial}{\partial \theta_{\boldsymbol{p}}} + \frac{1}{2}\sigma^z$$



- Determine most general interaction invariant under J_z rotations and TRS

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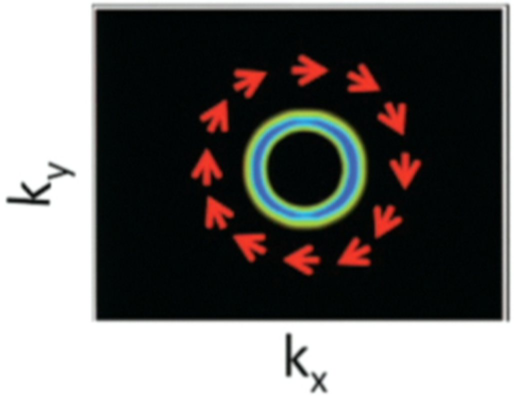
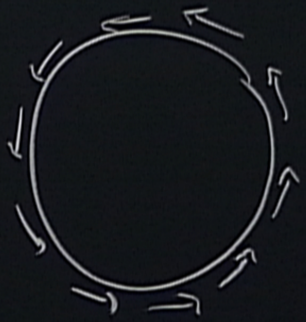


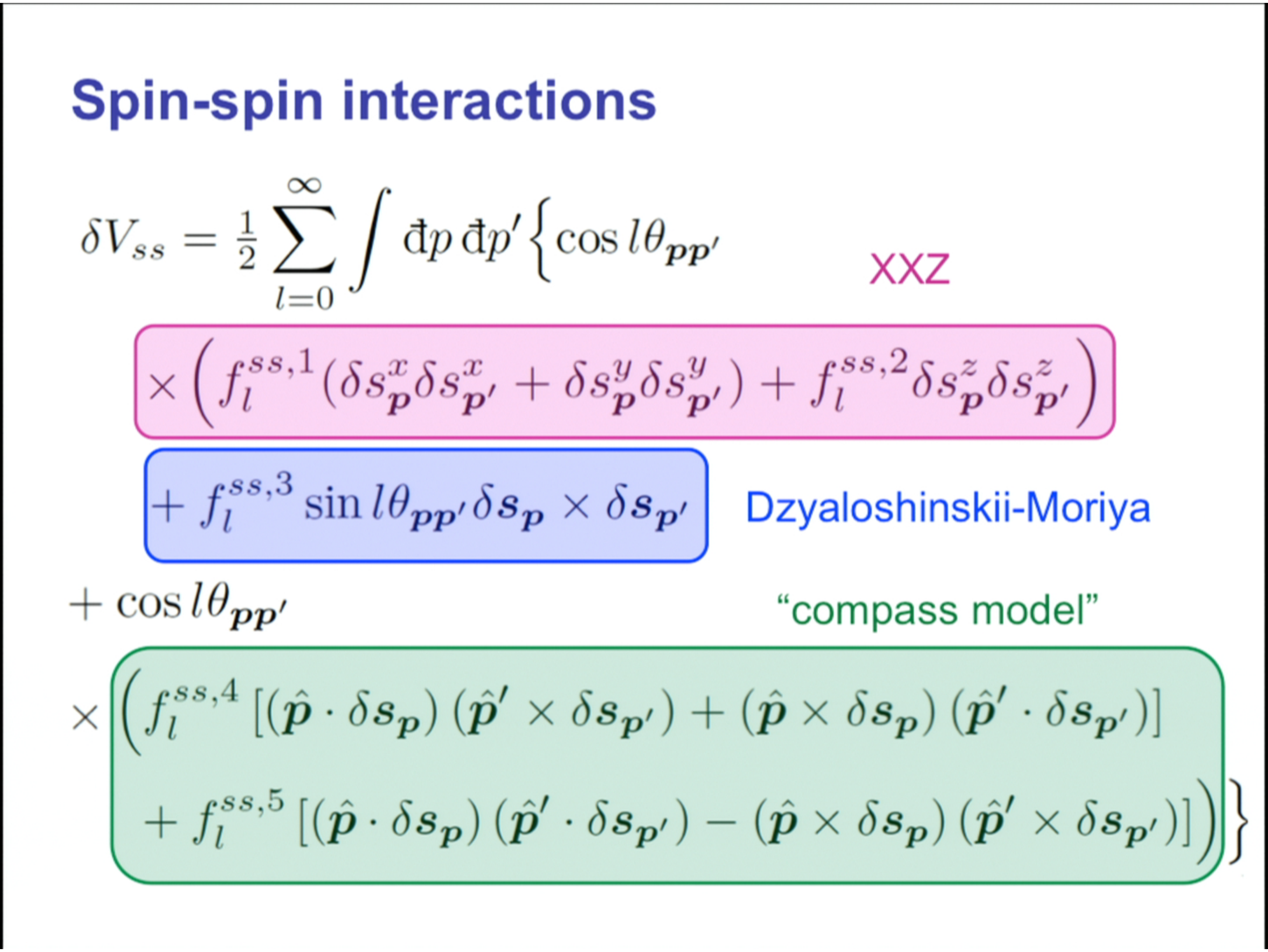
- Determine most general interaction invariant under J_z rotations and TRS

$$\delta n_p^{\alpha\beta}$$

$$\delta \rho_p = \delta_{\alpha\beta}\,\delta n_p^{\alpha\beta}$$

$$\delta \vec{S}_p = \vec{\sigma}_{\alpha\beta}\,\delta n_p^{\alpha\beta}$$





Spin-spin interactions

$$\delta V_{ss} = \tfrac{1}{2}\sum_{l=0}^{\infty}\int \text{đ}p\,\text{đ}p'\Big\{\cos l\theta_{\boldsymbol{pp'}}$$

XXZ

$$\boxed{\times\Big(f_l^{ss,1}(\delta s^x_{\boldsymbol{p}}\delta s^x_{\boldsymbol{p'}} + \delta s^y_{\boldsymbol{p}}\delta s^y_{\boldsymbol{p'}}) + f_l^{ss,2}\delta s^z_{\boldsymbol{p}}\delta s^z_{\boldsymbol{p'}}\Big)}$$

$$\boxed{+ f_l^{ss,3}\sin l\theta_{\boldsymbol{pp'}}\delta\boldsymbol{s}_{\boldsymbol{p}} \times \delta\boldsymbol{s}_{\boldsymbol{p'}}}$$

Dzyaloshinskii-Moriya

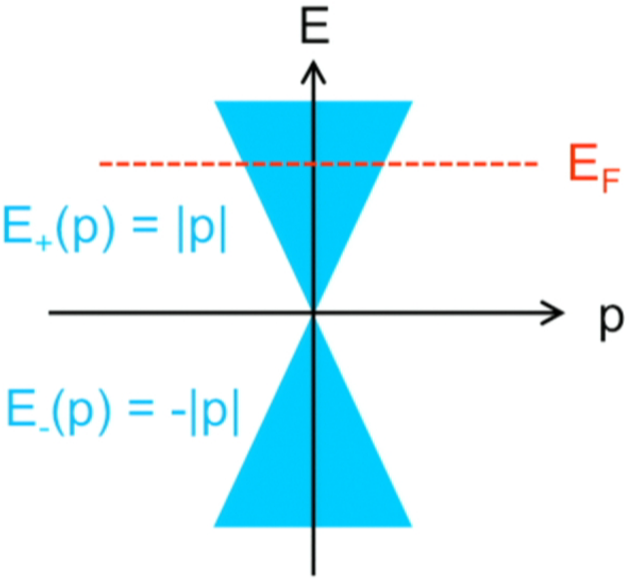
$$+ \cos l\theta_{\boldsymbol{pp'}}$$

“compass model”

$$\times\boxed{\Big(f_l^{ss,4}\left[(\hat{\boldsymbol{p}}\cdot\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\boldsymbol{p}}'\times\delta\boldsymbol{s}_{\boldsymbol{p'}}) + (\hat{\boldsymbol{p}}\times\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\boldsymbol{p}}'\cdot\delta\boldsymbol{s}_{\boldsymbol{p'}})\right] + f_l^{ss,5}\left[(\hat{\boldsymbol{p}}\cdot\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\boldsymbol{p}}'\cdot\delta\boldsymbol{s}_{\boldsymbol{p'}}) - (\hat{\boldsymbol{p}}\times\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\boldsymbol{p}}'\times\delta\boldsymbol{s}_{\boldsymbol{p'}})\right]\Big)}\Big\}$$

Projected Fermi liquid theory

- FL theory: only keep degrees of freedom near Fermi surface



$$c_{\boldsymbol{p}\uparrow} = \frac{ie^{-i\theta_{\boldsymbol{p}}}}{\sqrt{2}}(\psi_{\boldsymbol{p}+} + \psi_{\boldsymbol{p}-})$$

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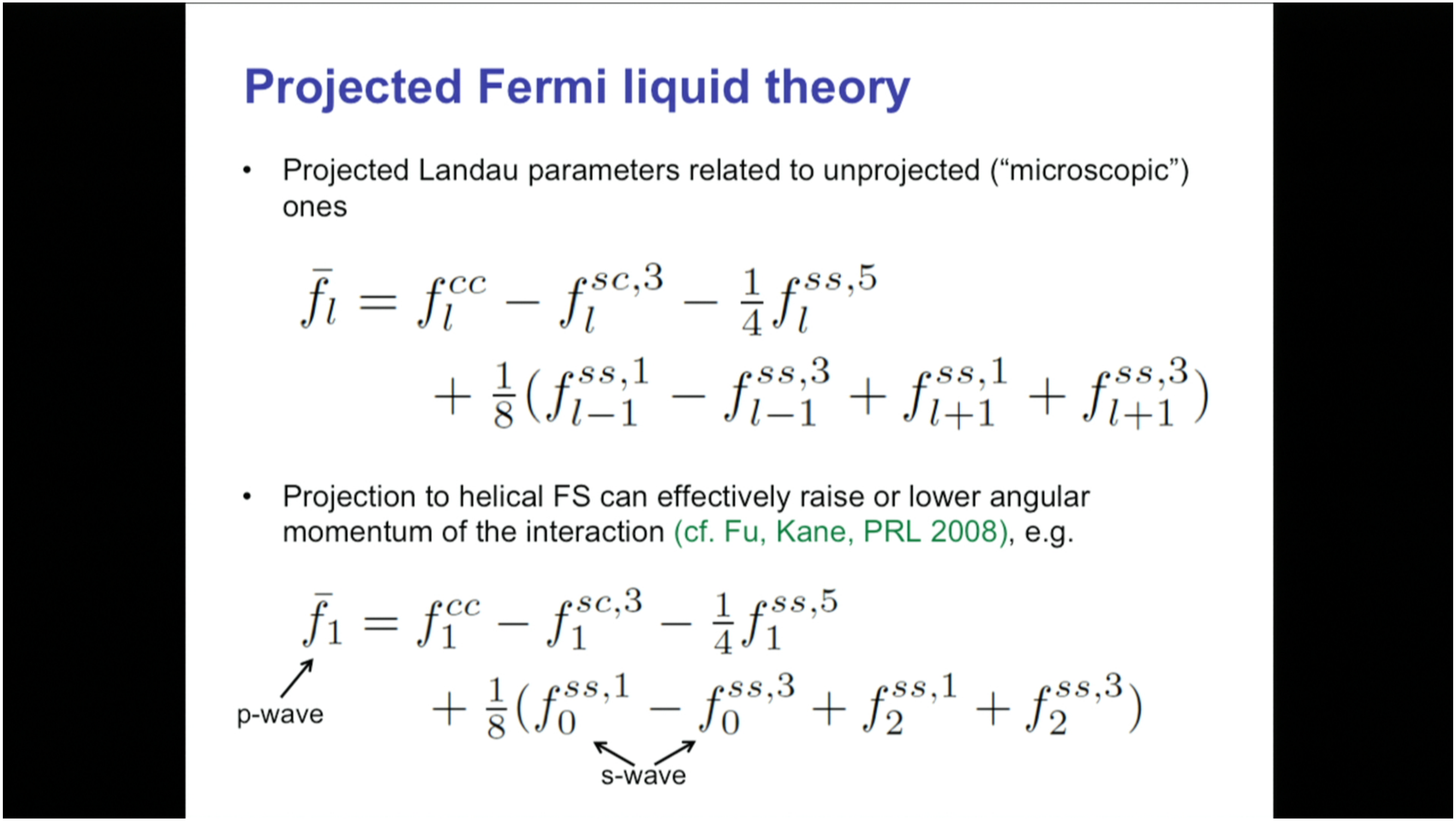


Projected Fermi liquid theory

- Projected Landau parameters related to unprojected ("microscopic") ones

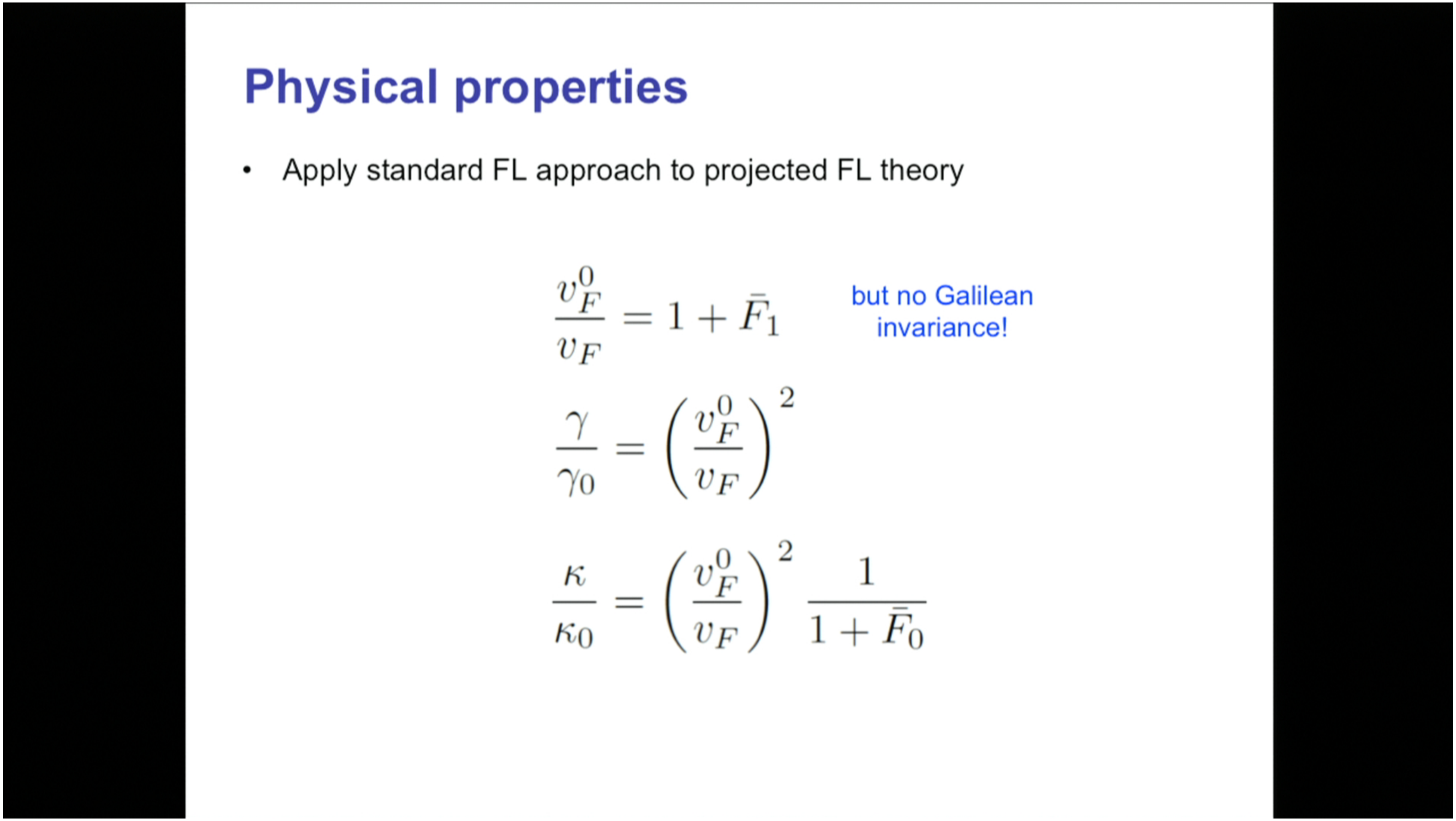
$$\bar{f}_l = f_l^{cc} - f_l^{sc,3} - \frac{1}{4} f_l^{ss,5} + \frac{1}{8}\left(f_{l-1}^{ss,1} - f_{l-1}^{ss,3} + f_{l+1}^{ss,1} + f_{l+1}^{ss,3}\right)$$

- Projection to helical FS can effectively raise or lower angular momentum of the interaction (cf. Fu, Kane, PRL 2008), e.g.

$$\bar{f}_1 = f_1^{cc} - f_1^{sc,3} - \frac{1}{4} f_1^{ss,5} + \frac{1}{8}\left(f_0^{ss,1} - f_0^{ss,3} + f_2^{ss,1} + f_2^{ss,3}\right)$$

p-wave

s-wave



Physical properties

- Apply standard FL approach to projected FL theory

$$\frac{v_F^0}{v_F} = 1 + \bar{F}_1$$

but no Galilean invariance!

$$\frac{\gamma}{\gamma_0} = \left(\frac{v_F^0}{v_F}\right)^2$$

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1 + \bar{F}_0}$$

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Pomeranchuk instabilities

- Instabilities towards spontaneous distortions of the Fermi surface (Pomeranchuk, JETP 1958)

$$p_F(\theta) - p_F = \sum_{l=-\infty}^{\infty} A_l e^{il\theta}$$

$$\delta \bar{E}\left[\delta \bar{n}_{\boldsymbol{p}}\right] = \frac{\epsilon_F}{2\pi\hbar^2} \sum_{l=0}^{\infty} (1 + \bar{F}_l)|A_l|^2$$

- Stability of Fermi surface (δE > 0) requires

$$\bar{F}_l > -1$$

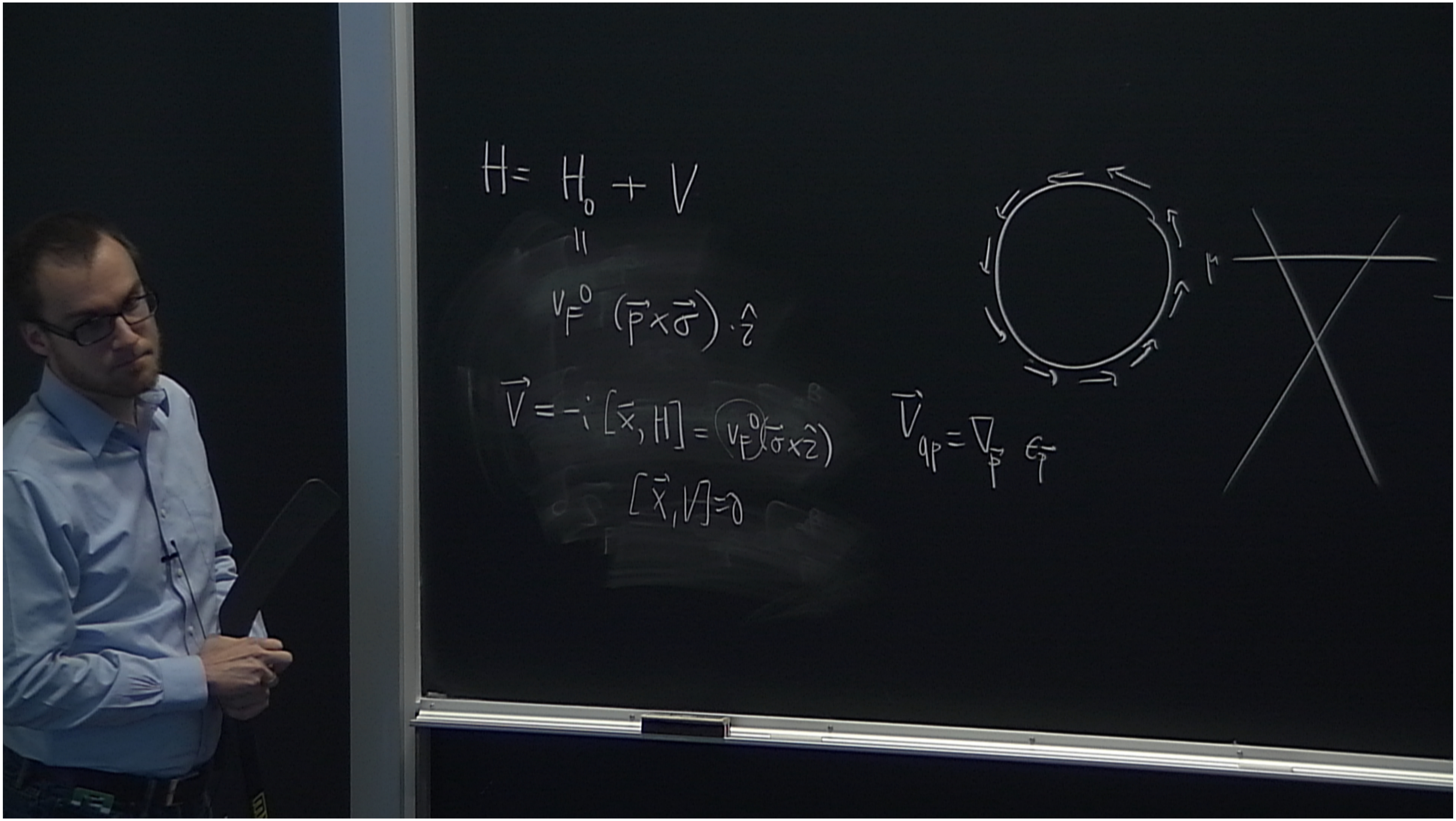
Pomeranchuk instabilities

- l=0: phase separation

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1+\bar{F}_0}$$

- l=1: in-plane ferromagnetism (Xu, PRB 2010)

$$\chi_{xx} = \tfrac{1}{8} g^2 \mu_B^2 \rho(\epsilon_F) \frac{1}{1+\bar{F}_1}$$



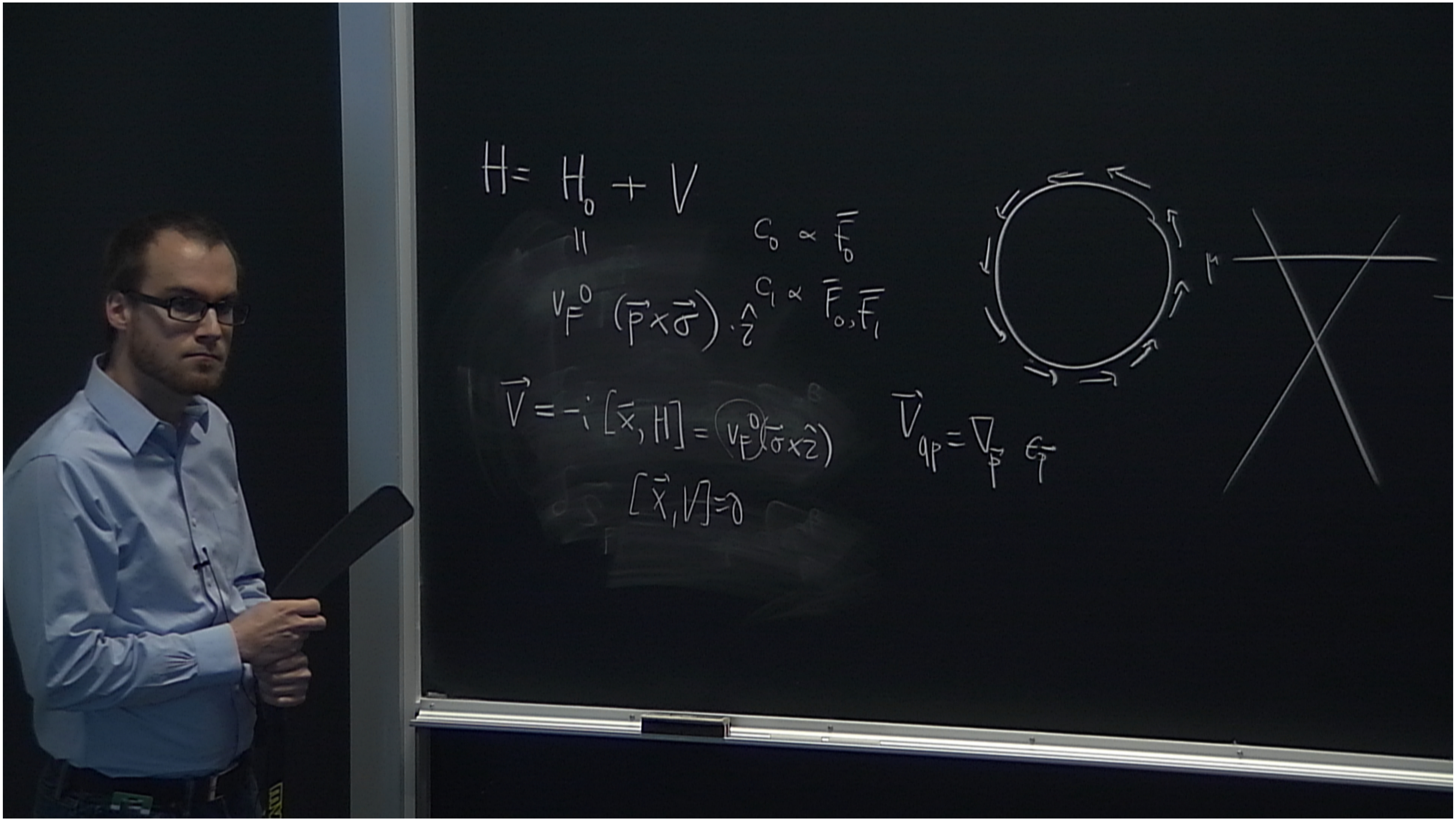
$$H = H_0 + V$$

$$H_0 = v_F^0 \, (\vec{p} \times \vec{\sigma}) \cdot \hat{z}$$

$$\vec{V} = -i[\vec{x}, H] = v_F^0 (\vec{\sigma} \times \hat{z})$$

$$[\vec{x}, V] = 0$$

$$\vec{V}_{qp} = \nabla_{\vec{p}} \, \epsilon_{\vec{p}}$$



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$$c_0 \propto \bar{F}_0$$

$$c_1 \propto \bar{F}_0, \bar{F}_1$$

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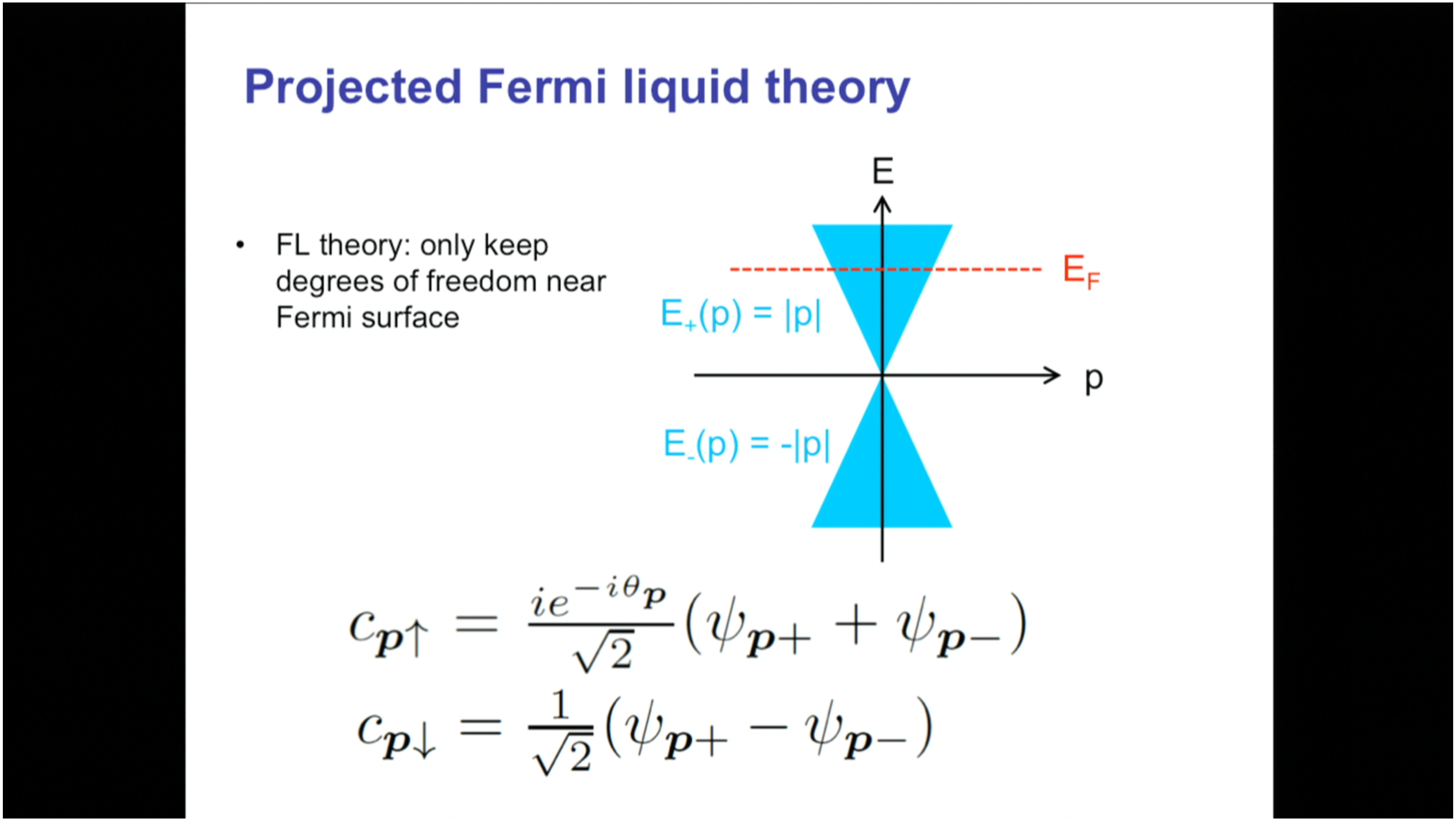
Dzyaloshinskii-Moriya

$$+ \cos l\theta_{\boldsymbol{pp}'}$$

“compass model”

$$\times\boxed{\begin{aligned}\Big(& f_l^{ss,4}\left[(\hat{\mathbf{p}}\cdot\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\mathbf{p}}'\times\delta\boldsymbol{s}_{\boldsymbol{p}'}) + (\hat{\mathbf{p}}\times\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\mathbf{p}}'\cdot\delta\boldsymbol{s}_{\boldsymbol{p}'})\right]\\ &+ f_l^{ss,5}\left[(\hat{\mathbf{p}}\cdot\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\mathbf{p}}'\cdot\delta\boldsymbol{s}_{\boldsymbol{p}'}) - (\hat{\mathbf{p}}\times\delta\boldsymbol{s}_{\boldsymbol{p}})(\hat{\mathbf{p}}'\times\delta\boldsymbol{s}_{\boldsymbol{p}'})\right]\Big)\end{aligned}}\Big\}$$



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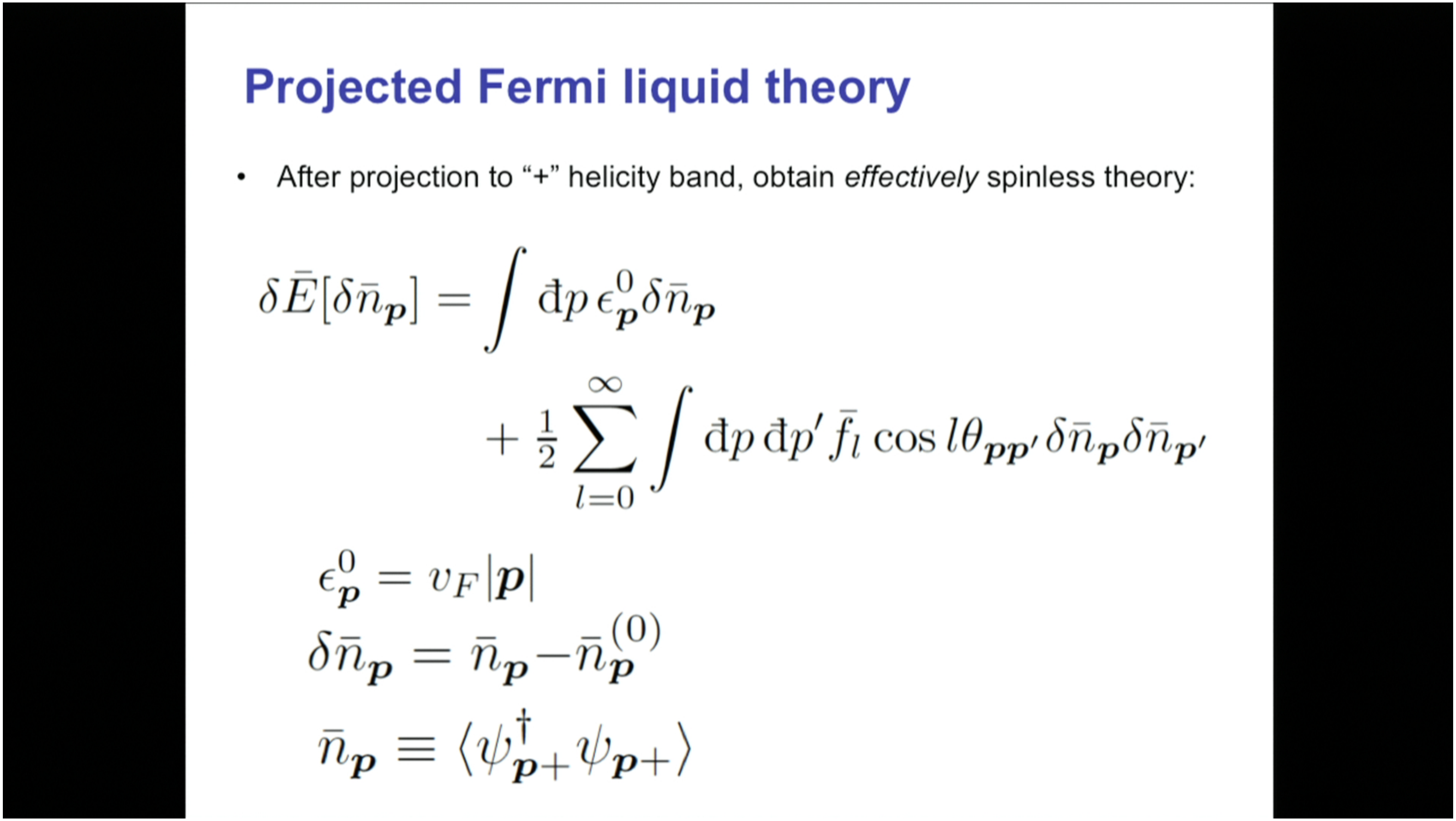


Projected Fermi liquid theory

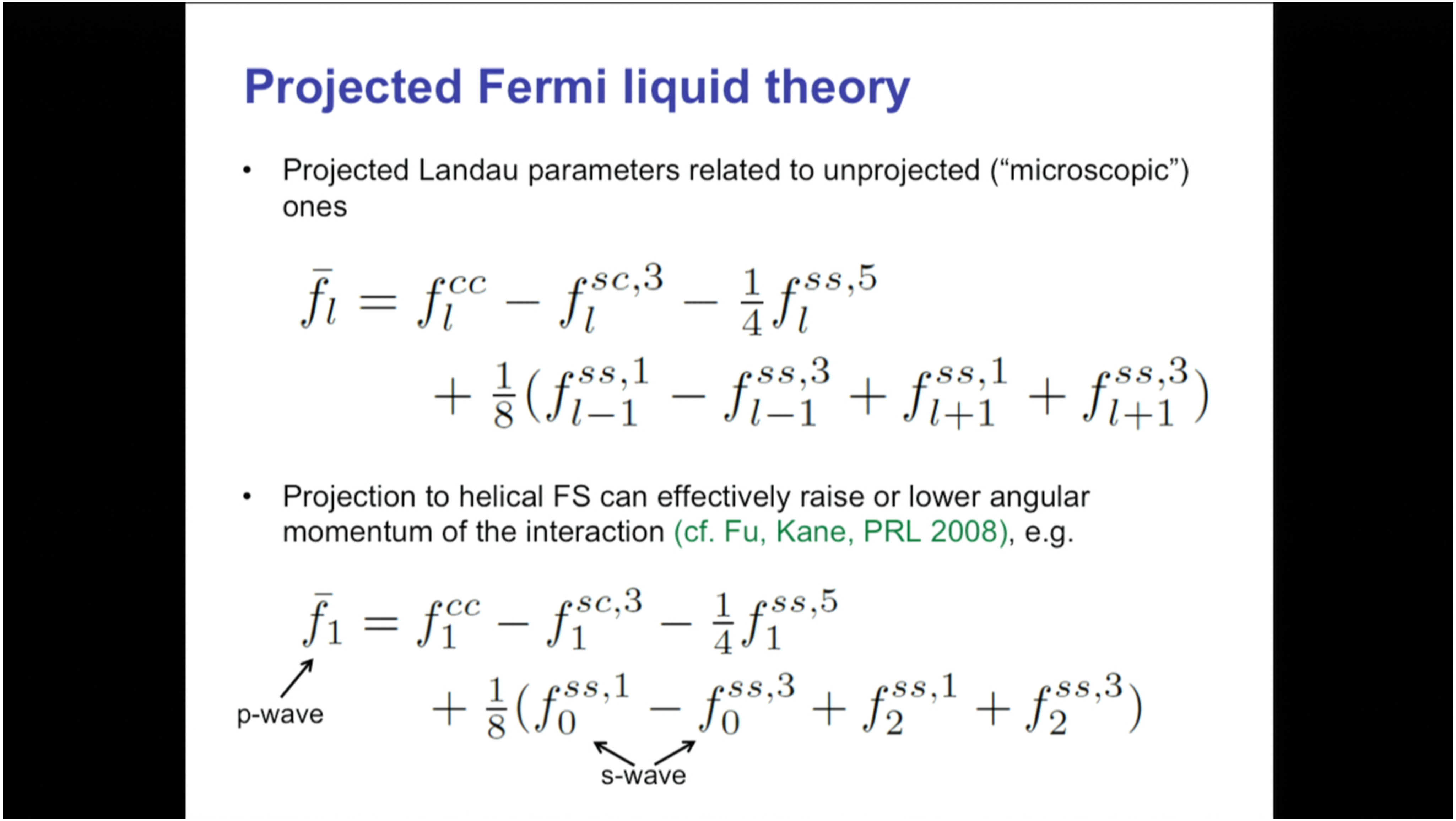
- After projection to "+" helicity band, obtain *effectively* spinless theory:

$$\delta\bar{E}[\delta\bar{n}_{\boldsymbol{p}}] = \int \text{đ}p\, \epsilon^0_{\boldsymbol{p}} \delta\bar{n}_{\boldsymbol{p}} + \frac{1}{2}\sum_{l=0}^{\infty}\int \text{đ}p\, \text{đ}p'\, \bar{f}_l \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'} \delta\bar{n}_{\boldsymbol{p}} \delta\bar{n}_{\boldsymbol{p}'}$$

$$\epsilon^0_{\boldsymbol{p}} = v_F|\boldsymbol{p}|$$

$$\delta\bar{n}_{\boldsymbol{p}} = \bar{n}_{\boldsymbol{p}} - \bar{n}^{(0)}_{\boldsymbol{p}}$$

$$\bar{n}_{\boldsymbol{p}} \equiv \langle \psi^\dagger_{\boldsymbol{p}+}\psi_{\boldsymbol{p}+}\rangle$$



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p-wave

s-wave