Title: Universal Dynamics and Black Hole Thermality from Conformal Field Theory

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Abstract: <p>I will discuss recent work studying universal properties of gravity in anti-de Sitter (AdS) spacetime from the perspective of conformal field theory (CFT). After reviewing relevant aspects of the AdS/CFT correspondence, I will demonstrate that all CFTs in three or more dimensions possess a spectrum of operators consistent with long-distance locality and Newtonian gravity in AdS. In generalizing these results to two-dimensional CFTs, I will then show that operators with large scaling dimension create classical backgrounds in the limit of large central charge. This result can be directly related to eigenstate thermalization in 2d CFTs and black hole geometries in 3d AdS. I will conclude by discussing methods for going beyond this large central charge limit in 2d and for studying black holes in higher dimensions.</p>

Universal Dynamics and Black Hole Thermality from Conformal Field Theory

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AdS/CFT from the "Outside-In"

Gravity in Anti-de Sitter $\leftrightarrow$ Conformal Field Theory
(AdS_{d+1}) (CFT_d)

(Macroscopic) Long-distance AdS physics is:

- Universal
- "Weakly Coupled"

Long-distance locality in AdS provides universal **perturbative expansion** for CFT

Use **AdS intuition** to guide **CFT derivations**

$\rightarrow$ **No assumptions** about AdS description

Punchline

- For $d \geq 3$ ($\mathrm{AdS}_{\geq 4}/\mathrm{CFT}_{\geq 3}$) :

All CFTs have universal large-spin structure consistent with macro-locality in AdS

- For $d = 2$ ($\mathrm{AdS}_3/\mathrm{CFT}_2$) with large central charge:

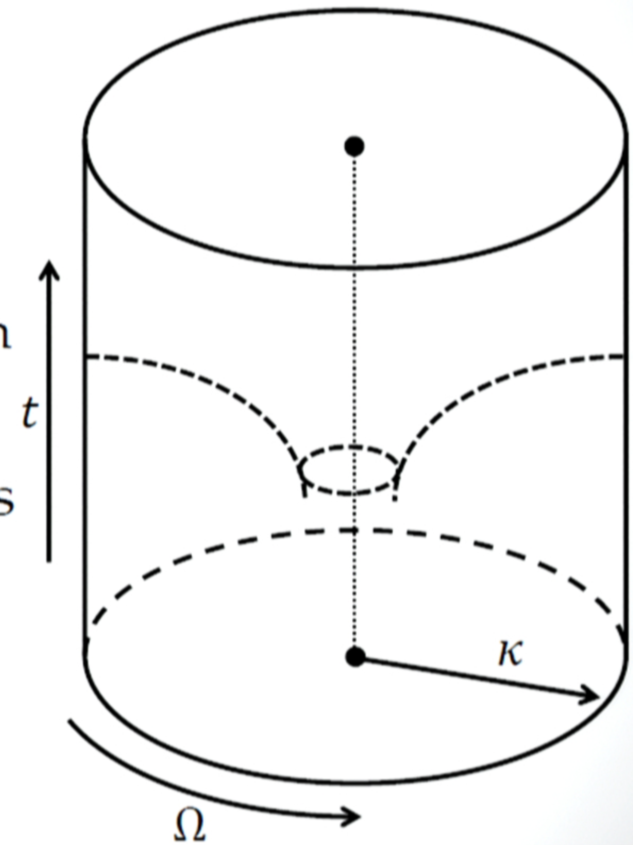
Operators with large scaling dimension create classical backgrounds for operators with small dimension

Setup: AdS_{d+1}

- "Gravity in a box"
- Curvature scale $R_{AdS} = 1$
- Gravitational well towards center
- Hamiltonian → Time translation

$$H|\psi\rangle = E|\psi\rangle$$

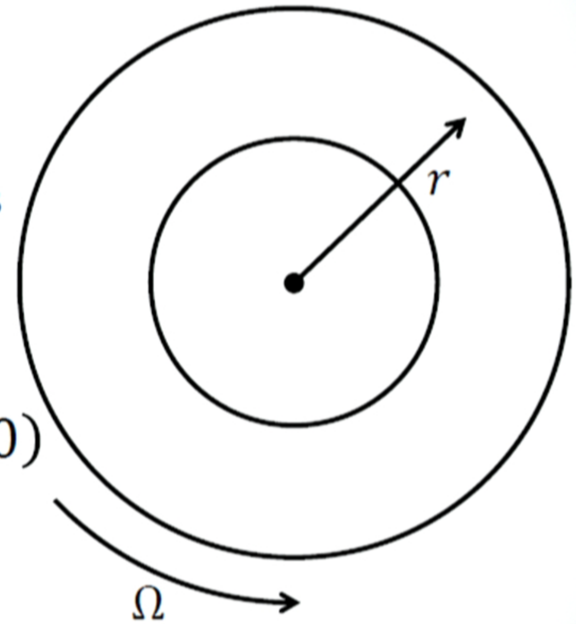
"Object" → Energy Eigenstates



Setup: CFT$_d$

- Flat spacetime
- Operators ↔ States $\mathcal{O}_i|0\rangle = |\psi\rangle$
- Dilatation → Scale transformations

$$D|\psi\rangle = \Delta|\psi\rangle$$



Operator Product Expansion:

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_i C_{12i}\, f_{12i}(x,\partial)\mathcal{O}_i(0)$$

Theory Dependent (C_{12i})

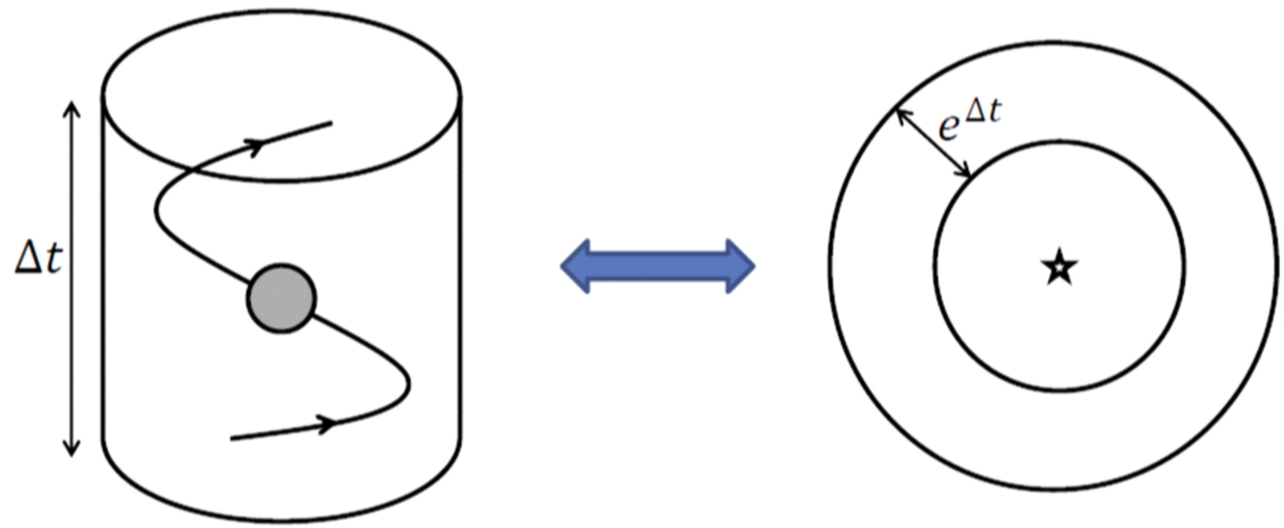
Fixed by Symmetry (f_{12i})

AdS/CFT

$$\mathcal{H}_{AdS} = \mathcal{H}_{CFT}$$

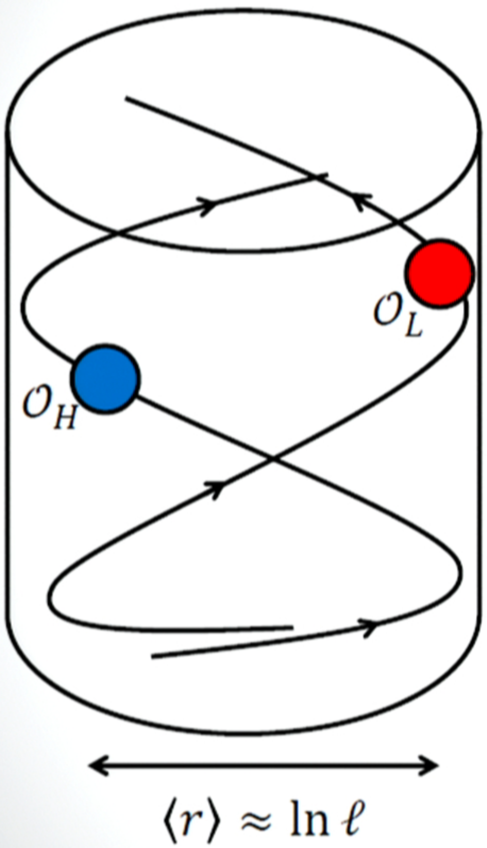
- Distinct descriptions/interpretations of same **Hilbert space**
- Correspondence of **states:** $|\psi\rangle_{AdS} = |\psi\rangle_{CFT}$

AdS/CFT



AdS_{d+1}	CFT_d
• Object, $\|\psi\rangle$	• Operator, $\mathcal{O}_i$
• Energy, E	• Scaling Dimension, Δ
• Angular Momentum, ℓ	• Spin, ℓ

Long Distance = Large Spin



"Two-object" states/operators:

$$[\mathcal{O}_H \mathcal{O}_L]_{n,\ell} \sim \mathcal{O}_H \partial^{2n} \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}_L$$

$$\Delta = \Delta_H + \Delta_L + 2n + \ell + \gamma(n,\ell)$$

Rest Mass ($\Delta_H + \Delta_L$) **Kinetic Energy** ($2n + \ell$) **Binding Energy** ($\gamma(n,\ell)$)

$$\gamma(n,\ell) \to 0 \text{ as } \ell \to \infty$$

Cluster Decomposition

For **any** operators $\mathcal{O}_H$, $\mathcal{O}_L$ (in $d \geq 3$)

Additional operators $\mathcal{O}_H(x)\mathcal{O}_L(0) \supset [\mathcal{O}_H\mathcal{O}_L]_{n,\ell}$

with $\Delta = \Delta_H + \Delta_L + 2n + \ell + \gamma(n, \ell)$

$$\gamma(n, \ell) \to 0 \text{ as } \ell \to \infty$$

"Anomalous Dimension" ↔ Long-Range Forces

All CFTs have AdS "Macro-Locality"!

Callan, Gross; Alday, Maldacena; Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Komargodski, Zhiboedov

How to Study $\gamma(n, \ell)$?

AdS scattering $\rightarrow$ Binding energy
Long-distance $\leftrightarrow$ Light object exchange

Study operators in **both:**
$\mathcal{O}_H(x)\mathcal{O}_H(0)$ and $\mathcal{O}_L(x)\mathcal{O}_L(0)$

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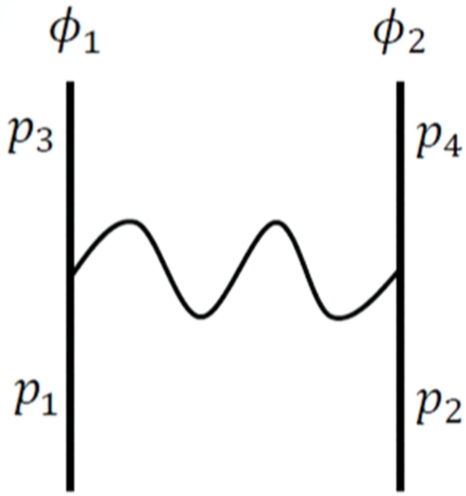
Contribution to observable
$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle$
determines $\gamma(n, \ell)$

Useful Analogy

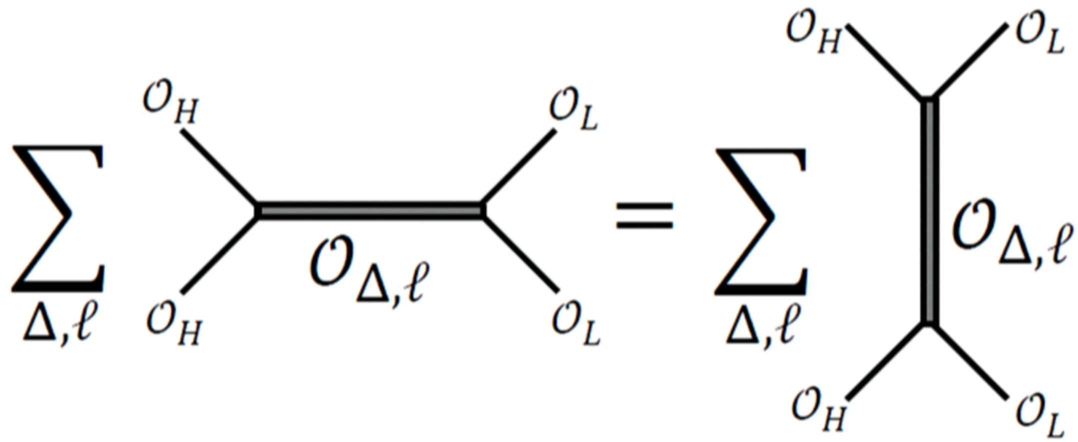


Partial Waves

$$= \sum_{\ell} C_\ell \, P_\ell(\cos\theta) \sim \frac{1}{1-\cos\theta}$$

Divergence as $\theta \to 0 \Longleftrightarrow \ell \to \infty$ Coefficients

CFT Bootstrap



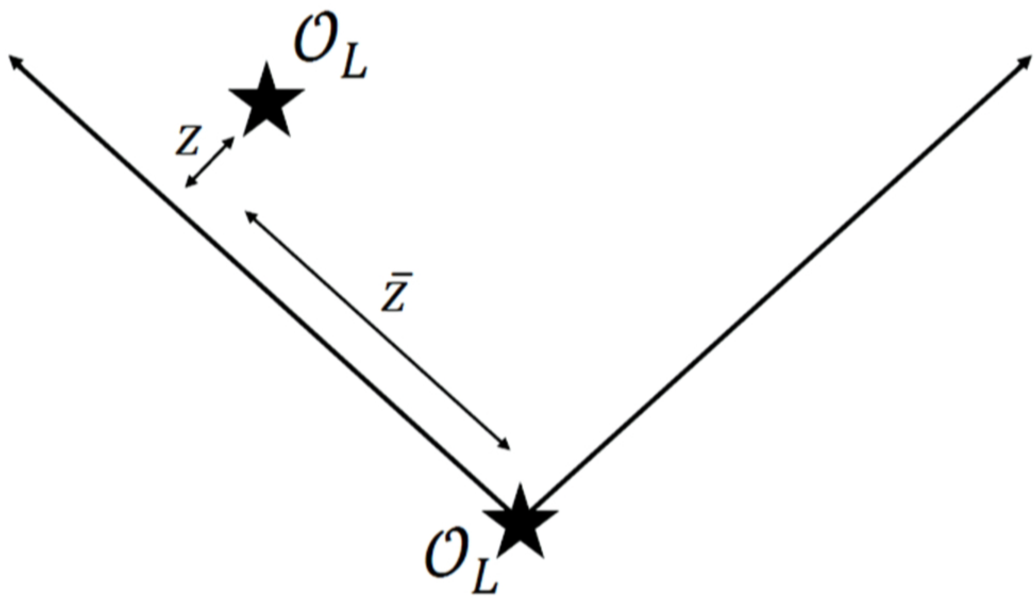
$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle = \sum_{\Delta,\ell} P^{HH,LL}_{\Delta,\ell}\, g_{\Delta,\ell}(z,\bar{z}) = \sum_{\Delta,\ell} P^{HL,HL}_{\Delta,\ell}\, g_{\Delta,\ell}(1-z, 1-\bar{z})$$

Conformal Blocks / Partial Waves

$$z\bar{z} = \left(\frac{x_{12}x_{34}}{x_{13}x_{24}}\right)^2 \qquad (1-z)(1-\bar{z}) = \left(\frac{x_{14}x_{23}}{x_{13}x_{24}}\right)^2$$

Lightcone OPE Limit

$$z \to 0, \bar{z} \to 1$$



$(\mathcal{O}_H, \mathcal{O}_H$ fixed$)$

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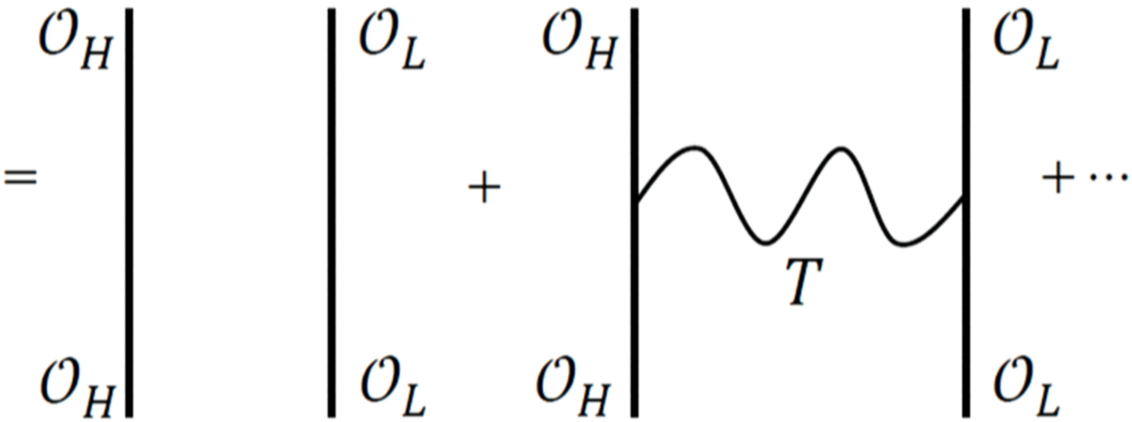
Lightcone OPE = Low Twist

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle =$$



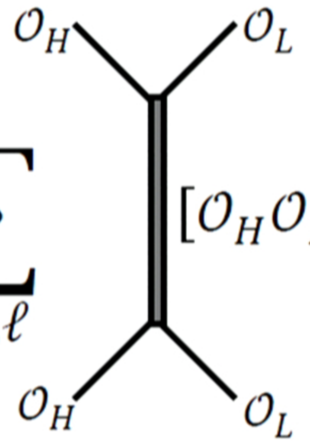
$$\approx \frac{1}{z^{\frac{1}{2}(\Delta_1+\Delta_2)}}\left(1 \quad + \quad z^{\frac{1}{2}(\Delta_T-\ell_T)}\log(1-\bar{z}) \; + \cdots \right)$$

Wick Contraction / "Identity Block"

"Twist" $\tau \equiv \Delta - \ell \;\geq d-2$

Lightcone OPE = Large Spin

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle = \sum_{n,\ell}$$

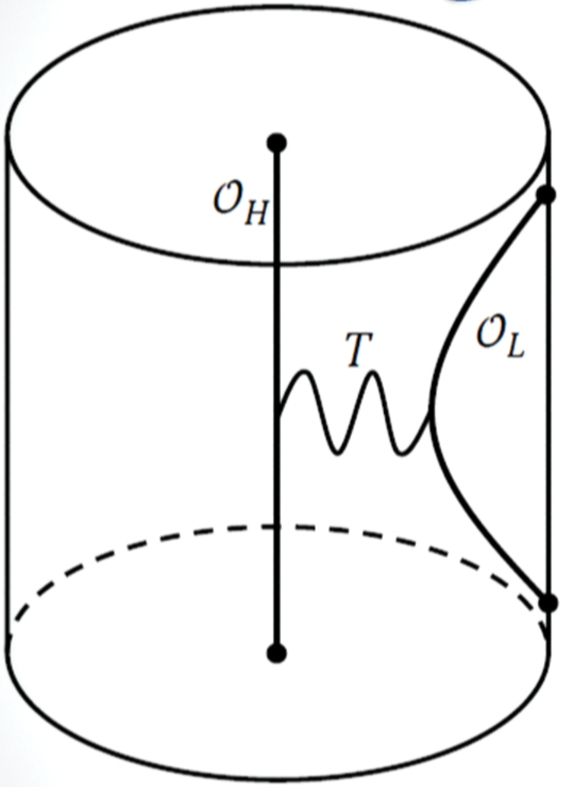


$$[\mathcal{O}_H \mathcal{O}_L]_{n,\ell} + \cdots$$

$$\approx \sum_{n,\ell} (1-\bar{z})^{\frac{1}{2}\gamma(n,\ell)} \log(z) + \cdots \sim \frac{1}{z^{\frac{1}{2}(\Delta_H + \Delta_L)}}$$

Divergence as $z \to 0$ ⟺ $\ell \to \infty$ Coefficients

Long-Range Forces



$$\Longleftrightarrow \quad P_T\, g_T(z, \bar{z})$$

$$\gamma(n, \ell) \approx \frac{P_T}{\ell^{\tau_T}} \;(\ell \to \infty)$$

Example:

Gravity $\leftrightarrow$ Stress-Energy Tensor

$$\gamma(n, \ell) \approx \frac{\Delta_H \Delta_L}{c} \frac{1}{\ell^{d-2}} \sim \frac{G M_H M_L}{r^{d-2}}$$

Central Charge $c \sim \frac{1}{G}$

Why Is $d = 2$ Special?

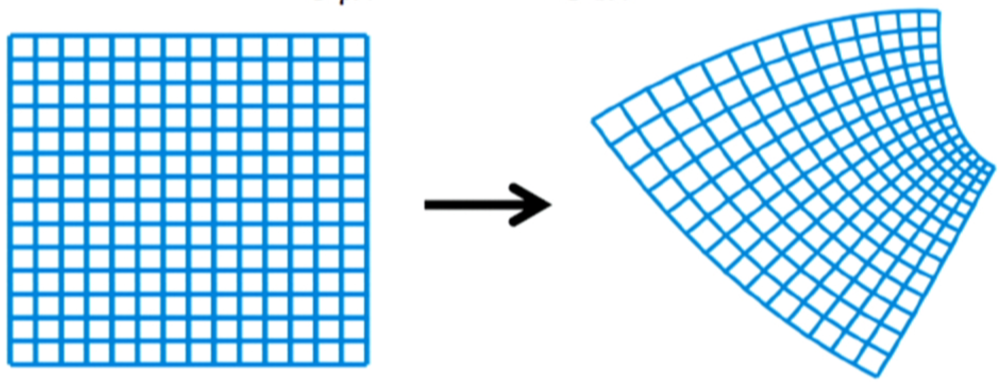
For $d \geq 3$: **All** nontrivial operators have $\tau > 0$
$\rightarrow$ Finite number of leading contributions to $\gamma(n, \ell)$

For $d = 2$: **Infinitely many** operators with $\tau = 0$
$\rightarrow$ Cannot isolate Wick contraction / $\gamma(n, \ell) \not\to 0$

Virasoro Symmetry

Conformal transformations preserve *angles*:

$$g_{\mu\nu} \rightarrow \Omega(x) g_{\mu\nu}$$

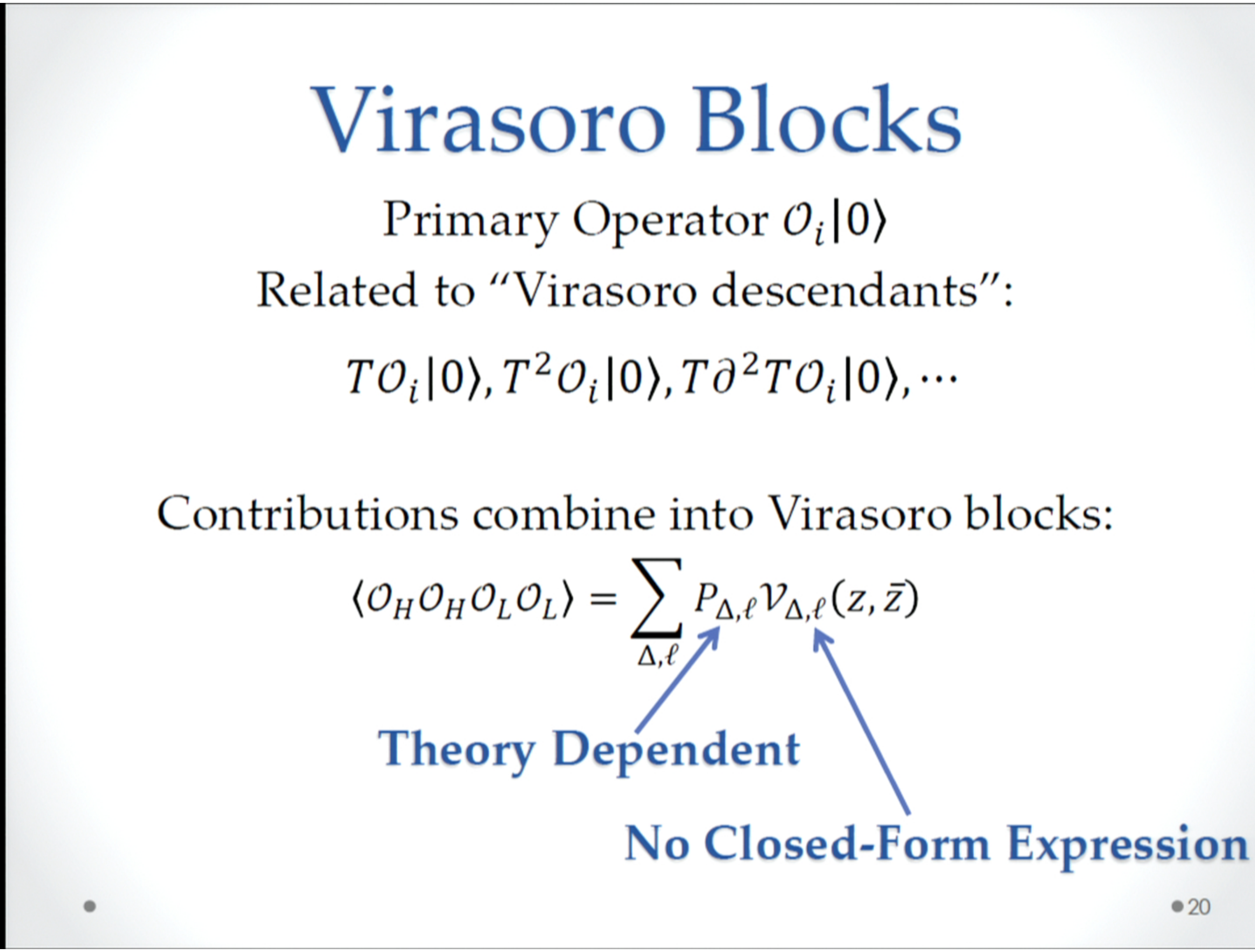


$$ds^2 = dz\, d\bar{z}$$

$$z \rightarrow f(z) \quad \bar{z} \rightarrow \bar{f}(\bar{z})$$

$$dz\, d\bar{z} \rightarrow f'(z)\bar{f}'(\bar{z}) dz d\bar{z}$$

2d CFT → Infinite-dimensional Virasoro symmetry


Virasoro Blocks
Primary Operator $\mathcal{O}_i|0\rangle$
Related to "Virasoro descendants":
$T\mathcal{O}_i|0\rangle, T^2\mathcal{O}_i|0\rangle, T\partial^2 T\mathcal{O}_i|0\rangle, \cdots$
Contributions combine into Virasoro blocks:
$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle = \sum_{\Delta,\ell} P_{\Delta,\ell} \mathcal{V}_{\Delta,\ell}(z,\bar{z})$
Theory Dependent
No Closed-Form Expression
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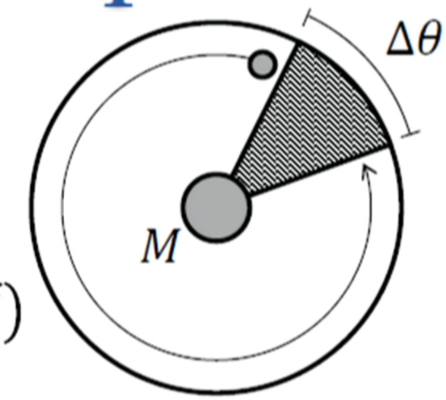
What Do We Expect?

Deficit Angles



- $M < \frac{1}{8G}$
- Space locally AdS, with slice removed, $\Delta\theta = 2\pi\left(1 - \sqrt{1 - 8GM}\right)$

BTZ Black Holes

- $M > \frac{1}{8G}$
- Hawking radiation with approximately blackbody spectrum

$$T_H = \frac{\sqrt{8GM - 1}}{2\pi}$$

Semi-Classical Limit

Need Virasoro blocks to study bootstrap in $d = 2$

Determine form in limit:

$$c \to \infty \text{ with } \frac{\Delta_H}{c}, \Delta_L \text{ fixed}$$

Heavy "Source" **Light "Probe"**

Equivalent to $G \to 0$

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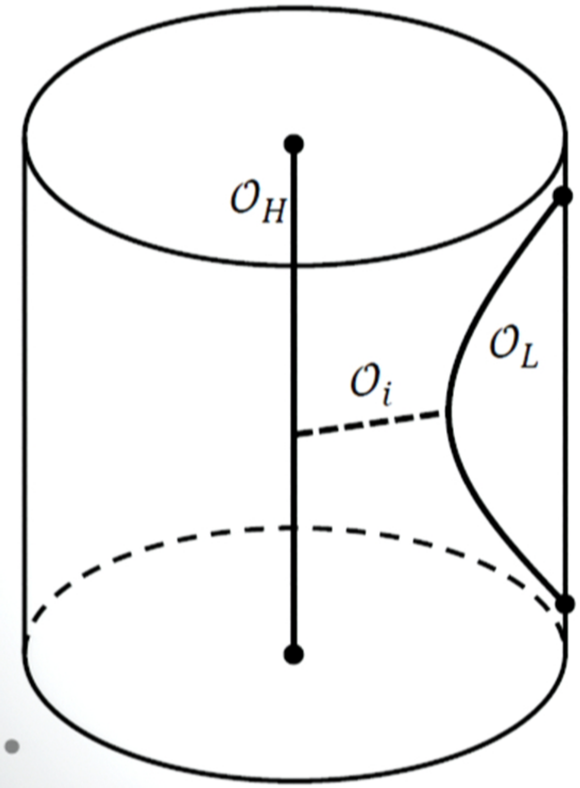
Global Conformal Blocks

Sending $G \to 0 \leftrightarrow$ No gravitational interactions (no contributions from T)



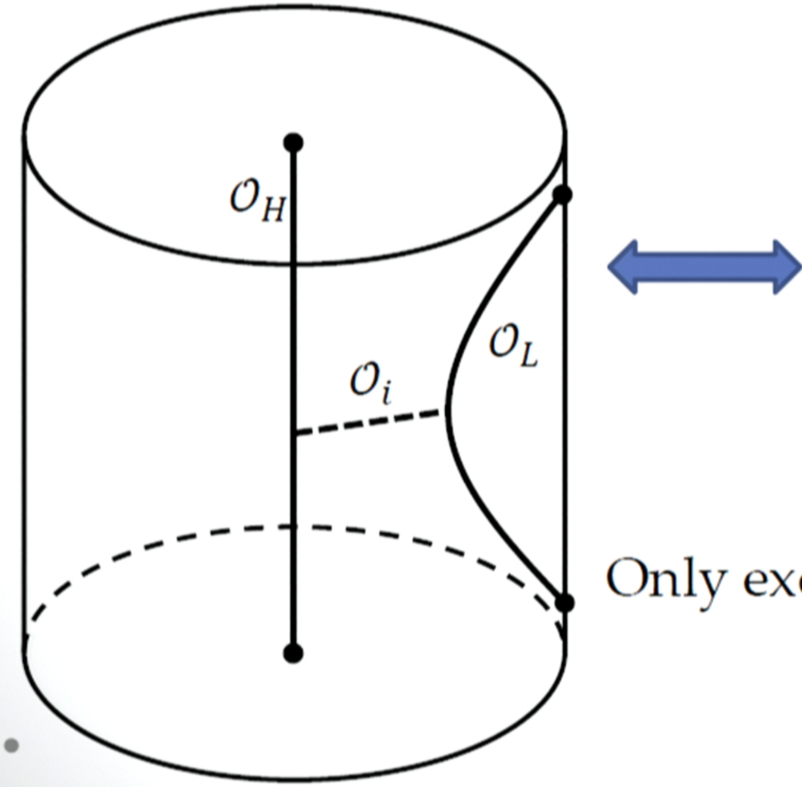
$\Leftrightarrow$ $g_i(z, \bar{z})$

Known Form

Only exchange of single "object"

Global Conformal Blocks

Sending $G \to 0 \leftrightarrow$ No gravitational interactions (no contributions from T)



$g_i(z, \bar{z})$

Known Form

Only exchange of single "object"

Change of Coordinates

- Consider correlation function (completely fixed by symmetry):

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_H(0)T(z)\rangle = \frac{\Delta_H}{z^2}$$

- Under coordinate transformation $z \to w(z)$:

$$T(z) \to T(w) \equiv \left(\frac{dz}{dw}\right)^2 T(z) + \langle T(z)\rangle_w$$

Primary Transformation

Schwarzian Derivative

Change of Coordinates

- Correlation function becomes:

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_H(0)T(w)\rangle = \left(\frac{dz}{dw}\right)^2 \frac{\Delta_H}{z^2} + \langle T(z)\rangle_w$$

- Can choose $w(z)$ such that these cancel!

$$w(z) = z^\alpha \qquad \alpha = \sqrt{1 - 12\frac{\Delta_H}{c}}$$

$$\rightarrow \boxed{\langle \mathcal{O}_H(\infty)\mathcal{O}_H(0)T(w)\rangle = 0}$$

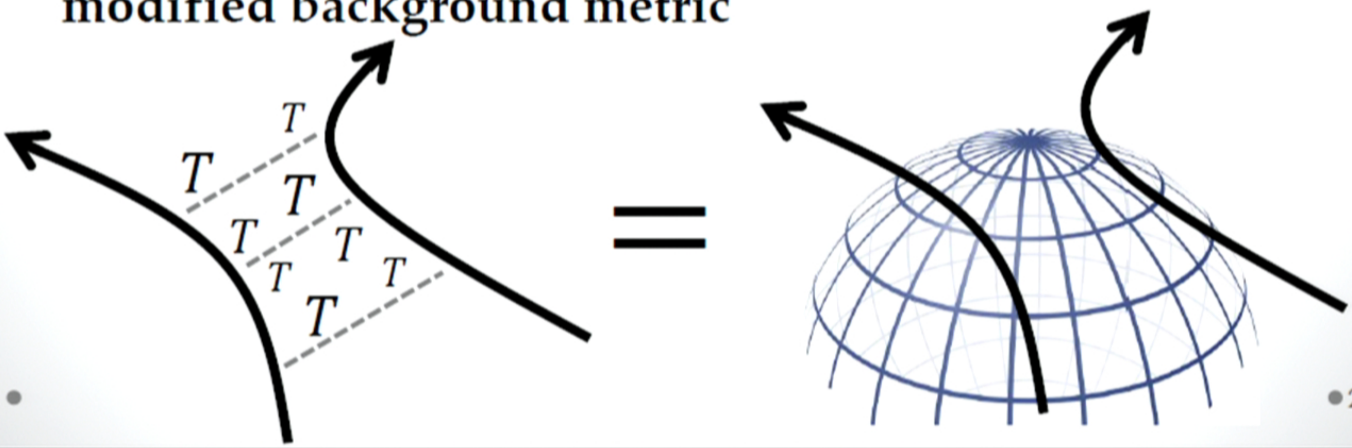
- Virasoro symmetry ensures that **all** correlation functions with T also vanish as $c \to \infty$
- Trivialize interactions with T by altering background metric

Classical Background

- For $c \to \infty$ with $\frac{\Delta_H}{c}, \Delta_L, \tau_i$ fixed:

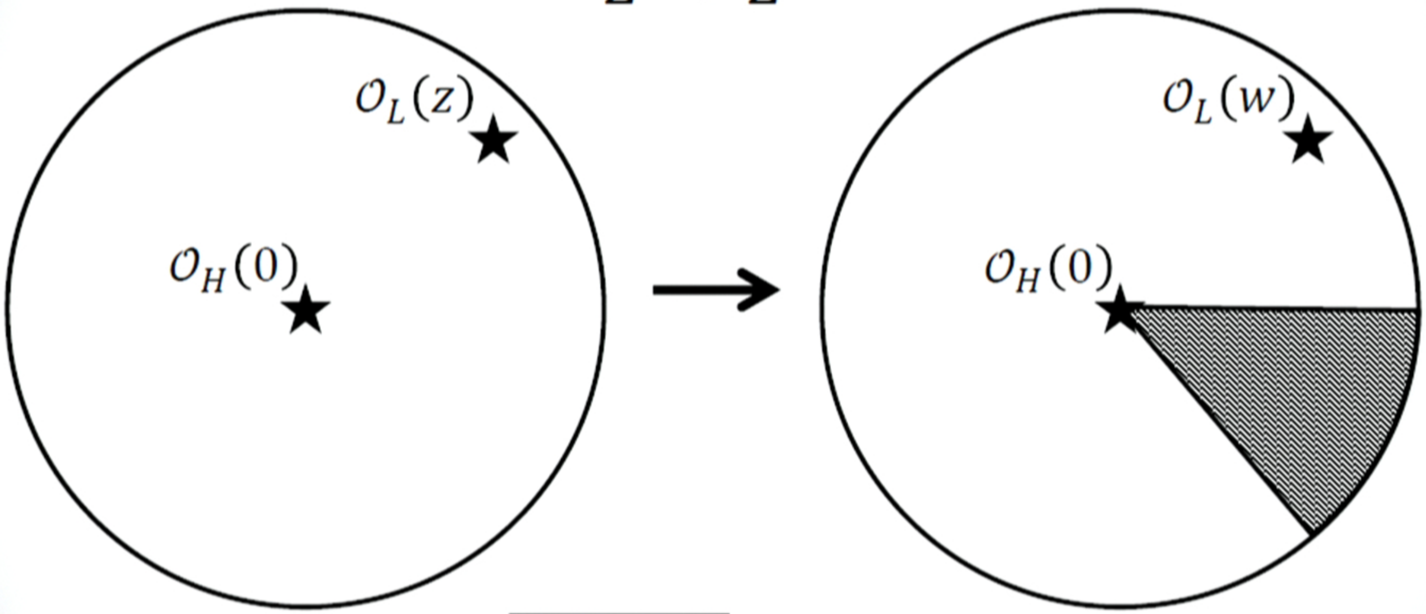
$$\mathcal{V}_i(z) \approx g_i(w) = (1-w)^{\frac{\tau_i}{2}} \, {}_2F_1\left(\frac{\tau_i}{2}, \frac{\tau_i}{2}; \tau_i; 1-w\right)$$

- Virasoro blocks are simply global conformal blocks in a **modified background metric**



Deficit Angles

$$z \to z^{\alpha}$$



$$\alpha = \sqrt{1 - 12\frac{\Delta_H}{c}} = \sqrt{1 - 8GM_H}$$

Identical to AdS!

BTZ Black Holes

For $\Delta_H \geq \frac{c}{12}: \alpha \to i|\alpha|$

$$z \sim e^t \to z^\alpha \sim e^{i|\alpha|t}$$

Periodic in Euclidean time t!

Identity block: $\mathcal{V}_0(z) \approx \left(\frac{\alpha/2}{\sinh \alpha z/2}\right)^{\Delta_L} \approx \langle \mathcal{O}_L(z)\mathcal{O}_L(0)\rangle_{T_H}$

$$T_H = \frac{\sqrt{12\frac{\Delta_H}{c} - 1}}{2\pi} = \frac{\sqrt{8GM_H - 1}}{2\pi}$$

Thermal 2-pt function at BTZ temperature

Black Hole Thermality

Lightcone OPE Limit: $z \to 0$

$$\langle \mathcal{O}_H | \mathcal{O}_L(\bar{z}) \mathcal{O}_L(0) | \mathcal{O}_H \rangle \approx \mathcal{V}_0(\bar{z}) \approx \langle \mathcal{O}_L(\bar{z}) \mathcal{O}_L(0) \rangle_T$$

At long distances:

- **All** heavy operators mimic thermal background for light operators
- **All** BTZ black holes are approximately thermal

What about other Virasoro blocks?

Black Hole Thermality

- *If* full correlation function is periodic
  $\rightarrow$ Eigenstate thermalization
- Analytic in $w(z) \rightarrow$ Periodic in t
- Single Virasoro blocks with $\tau_i \ll c$ analytic
- Need sum over all blocks to be well-behaved
- Similar issue in periodicity of global conformal blocks about $z = 0$

Beyond the Semi-Classical Limit

- Study $\frac{1}{c}$ corrections to thermality with recursion relation:

$$\mathcal{V}(c, h_i, h, z) = \mathcal{V}(\infty, h_i, h, z) + \sum_{m,n} \frac{R_{m,n}}{c - c_{m,n}} \mathcal{V}(c_{m,n}, h_i, h + mn, z)$$

- Use $c = \infty$ "seed" to derive *exact* Virasoro block as series expansion in z
- Improve by replacing the global conformal block "seed" with semi-classical Virasoro block
- Recursively derive exact Virasoro block as series in w

Perlmutter

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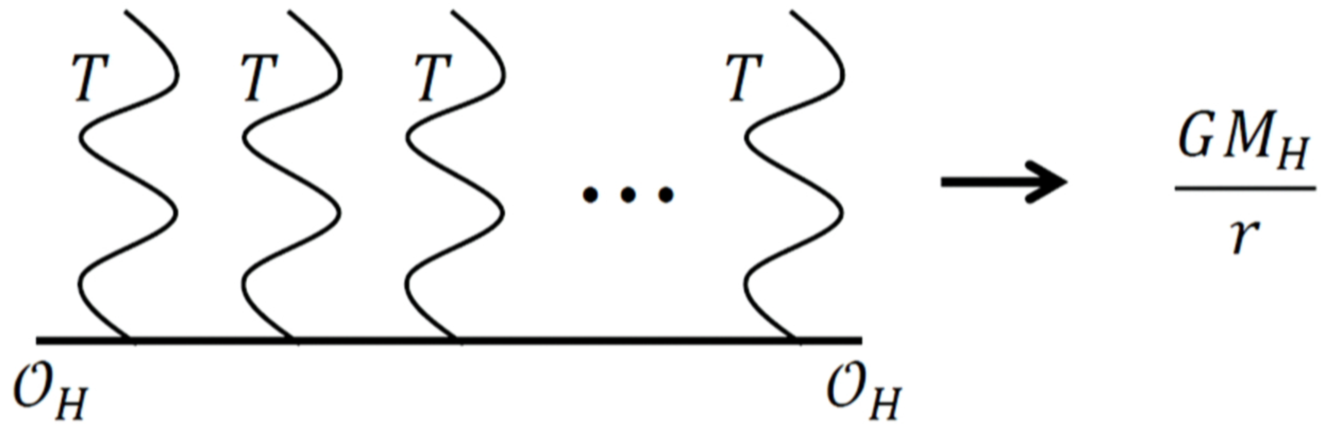
Towards Higher d

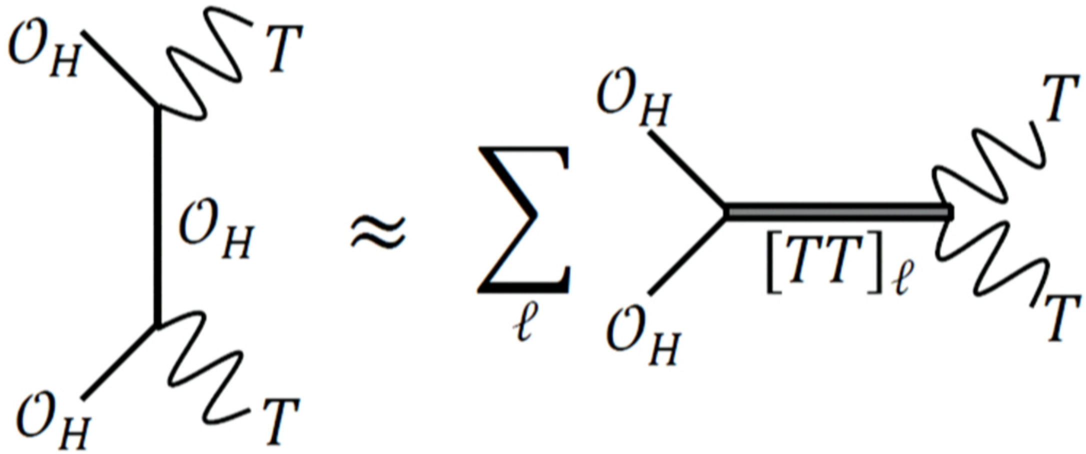
- Example: $d = 4$
$$\langle T^{\mu\nu} \rangle = \frac{a_4}{(4\pi)^2}\left[g^{\mu\nu}\left(\frac{R^2}{2} - R^{\alpha\beta}R_{\alpha\beta}\right) + 2R^{\mu\lambda}R^{\nu}{}_{\lambda} - \frac{4R}{3}R^{\mu\nu}\right]$$
- Choose coordinates to eliminate 3-pt function $\langle \mathcal{O}_H \mathcal{O}_H T \rangle$
- No Virasoro symmetry, so generally:
$$\langle \mathcal{O}_H \mathcal{O}_H T \cdots T \rangle \neq 0$$
- Need to study relation between T and $[TT]_\ell, [TTT]_\ell, \cdots$
- First step: Eikonal limit and exponentiation at large spin

Background "T" Field



Are there classical "background fields" in $d \geq 3$?

Eikonalization at Large Spin



Apply results to $\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle$ for $z \to 0, \bar{z} \to 1$:

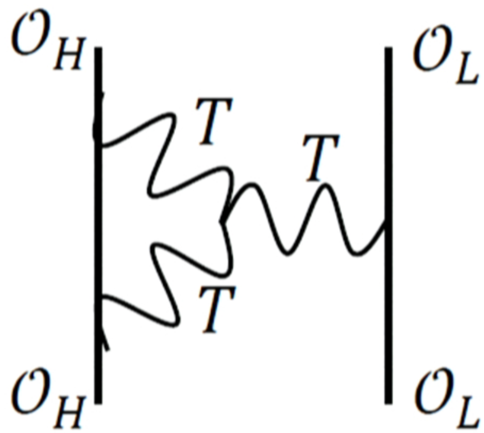
$$\sum_\ell P_{[TT]_\ell} g_{[TT]_\ell}(z, \bar{z}) \approx \frac{1}{2} \left(P_T g_T(z, \bar{z}) \right)^2$$

Black Holes in Higher d

Can generalize to multi-T exchange:

$$\sum_{\ell} P_{[T\cdots T]_\ell}\, g_{[T\cdots T]_\ell}(z,\bar{z}) \approx e^{P_T g_T(z,\bar{z})}$$

Need to understand effects from:



Summary

- All CFTs have universal "long-distance" structure at large spin ($d \geq 3$)
- Low twist operators $\rightarrow$ Long-distance interactions
- Heavy operators create background for light operators through exchange of $T^{\mu\nu}$ ($d = 2$)
- Semi-classical Virasoro blocks equivalent to global conformal blocks in modified metric
- Operators with $\Delta < \frac{c}{12}$ create deficit angle
- Operators with $\Delta > \frac{c}{12}$ create approximately thermal background
- Low twist operators eikonalize at large spin ($d \geq 3$)

Future Directions

- Eigenstate thermalization at large $c \rightarrow$ Transitions between distinct heavy states should be Boltzmann-suppressed
- Periodicity of full correlation function $\rightarrow$ Mellin space for w coordinates?
- Corrections to thermality $\rightarrow$ Use recursion relation to study nonperturbative gravity
- Combine constraints on $T^{\mu\nu}$ interactions and VEVs to study thermality in higher dimensions
- Add UV cutoff to CFT, derive background metric as function of cutoff $\rightarrow$ Bulk Einstein equations?
- Combine with modular invariance $\rightarrow$ Further constrain high-energy spectrum?
- Entanglement entropy from "twist" operators