Title: Non-Equilibrium Dynamics of Topological Systems

Date: Apr 21, 2015 02:00 PM

URL: http://pirsa.org/15040072

Abstract: <p>In this talk I will discuss the non-equilibrium response of Chern insulators [1]. Focusing on the Haldane model, we study the dynamics induced by quantum quenches between topological and non-topological phases. A notable feature is that the Chern number, calculated for an infinite system, is unchanged under the dynamics following such a quench. However, in finite geometries, the initial and final Hamiltonians are distinguished by the presence or absence of edge modes. We study the edge excitations and describe their impact on the experimentally-observable edge currents and magnetization. We show that, following a quantum quench, the edge currents relax towards new equilibrium values, and that there is light-cone spreading of the currents into the interior of the sample. I will briefly comment on a complementary project to understand non-equilibrium charge transport using gauge-gravity duality and hydrodynamics.

[1] M. D. Caio, N. R. Cooper, M. J. Bhaseen, Quantum Quenches in Chern Insulators, arXiv:1504.01910</p>

Non-Equilibrium Dynamics of Topological Systems

Joe Bhaseen

TSCM Group
King's College London

Perimeter Institute for Theoretical Physics
Waterloo, Canada
21st April 2015

Outline

- Motivation from condensed matter
- Haldane Model
- Experimental realization in cold atoms
- Quenches between topological and non-topological phases
- Dynamics of bulk and boundary properties

M. D. Caio, N. R. Cooper and M. J. Bhaseen
"Quantum Quenches in Chern Insulators"

arXiv:1504.01910

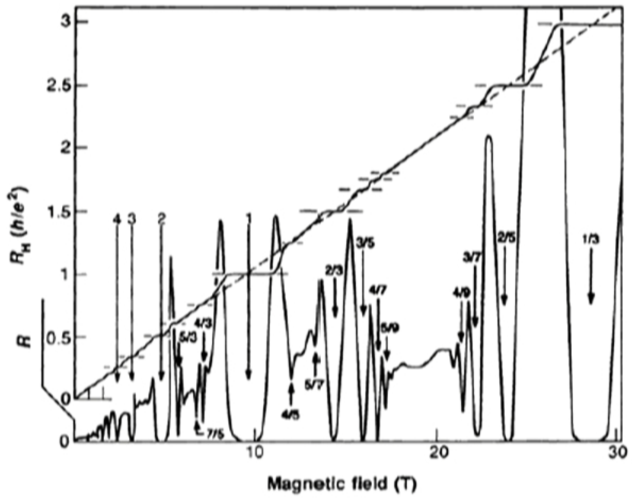
Work in progress

Non-equilibrium charge transport

MJB, Benjamin Doyon, Andy Lucas and Koenraad Schalm

Quantum Hall Effect

2D electrons in a magnetic field



http://www.nobelprize.org/nobel_prizes/physics/laureates/1998/press.html

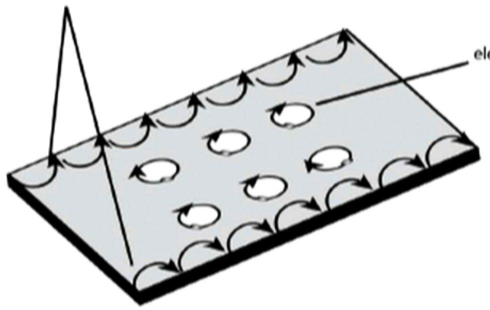
Robust Quantization

IQHE Disorder FQHE Interactions

Topological

Novel Properties

Fractional charges, non-Abelian statistics, gapless edge states



http://pfc.umd.edu/news/reports/floquet-topological-insulator-semiconductor-quantum-wells

Chiral Luttinger Liquid

X. G. Wen, *"Chiral Luttinger liquid and the edge excitations in the fractional quantum Hall states"*, Phys. Rev. B **41**, 12838 (1990)

Bulk–Boundary Correspondence

Bulk wavefunctions $\leftrightarrow$ conformal blocks of the CFT

Non-Equilibrium Dynamics

Time-dependent manipulations on topological systems

Some protocols for making topological systems involve driving

What happens to topological systems far from equilibrium?

What happens if you quench between topological and non-topological phases?

What happens to the edge currents?

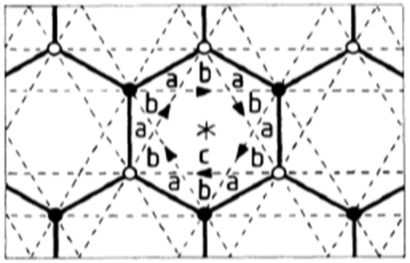
How do topological properties influence the dynamics?

Haldane Model

F. D. M. Haldane, *Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"*, Phys. Rev. Lett. **61**, 2015 (1988)

Spinless fermions on a honeycomb lattice

No interactions or disorder



Onsite potential breaks inversion symmetry and opens gap

Complex 2nd neighbor hopping breaks time-reversal and opens gap

Aharonov–Bohm phases from staggered fluxes $\phi_b = -\phi_a \quad \phi_c = 0$

QHE without a net magnetic field

Topological Insulators: Kane–Mele model = two Haldane models

Hamiltonian

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

$$\hat{H} = t_1 \sum_{\langle i,j \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) + t_2 \sum_{\langle\langle i,j \rangle\rangle} \left(e^{i\varphi_{ij}} \hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) + M \sum_{i \in A} \hat{n}_i - M \sum_{i \in B} \hat{n}_i$$

$$\{\hat{c}_j, \hat{c}_j^\dagger\} = \delta_{ij} \qquad \hat{n}_i \equiv \hat{c}_i^\dagger \hat{c}_i$$

$\langle i, j \rangle$ and $\langle\langle i, j \rangle\rangle$ summation over 1st and 2nd neighbors

A, B label two sub-lattices.

Phase factor $\varphi_{ij} = \pm\varphi$ breaks time-reversal and opens gap

Positive for anticlockwise 2nd neighbor hopping

Energy off-set $\pm M$ breaks spatial inversion and opens gap

Assume that $|t_2/t_1| \leq 1/3$ so that bands may touch, but not overlap

Low-Energy Description

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

Two species of Dirac fermions $(\alpha = \pm 1)$

For $t_2, M \ll t_1$, the Hamiltonian has a linear dispersion near six corners of hexagonal Brillouin zone, but only two are inequivalent

Close to half-filling, $\hat{H} = \hat{H}_+ + \hat{H}_-$ where

$$\hat{H}_\alpha = \begin{pmatrix} m_\alpha c^2 & -c\,k\,e^{\mathrm{i}\alpha\theta} \\ -c\,k\,e^{-\mathrm{i}\alpha\theta} & -m_\alpha c^2 \end{pmatrix}$$

$ke^{i\theta}$ parameterizes 2D momentum (k_x, k_y)

Effective speed of light $c = 3t_1/2\hbar$

Effective masses $m_\alpha = (M - 3\sqrt{3}\alpha t_2 \sin\varphi)/c^2$

Non-zero M or φ generically gaps the spectrum

Break inversion: normal **Break time-reversal: topological**

Low-Energy Description

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

Two species of Dirac fermions $(\alpha = \pm 1)$

For $t_2, M \ll t_1$, the Hamiltonian has a linear dispersion near six corners of hexagonal Brillouin zone, but only two are inequivalent

Close to half-filling, $\hat{H} = \hat{H}_+ + \hat{H}_-$ where

$$\hat{H}_\alpha = \begin{pmatrix} m_\alpha c^2 & -c\,k\,e^{\mathrm{i}\alpha\theta} \\ -c\,k\,e^{-\mathrm{i}\alpha\theta} & -m_\alpha c^2 \end{pmatrix}$$

$ke^{i\theta}$ parameterizes 2D momentum (k_x, k_y)

Effective speed of light $c = 3t_1/2\hbar$

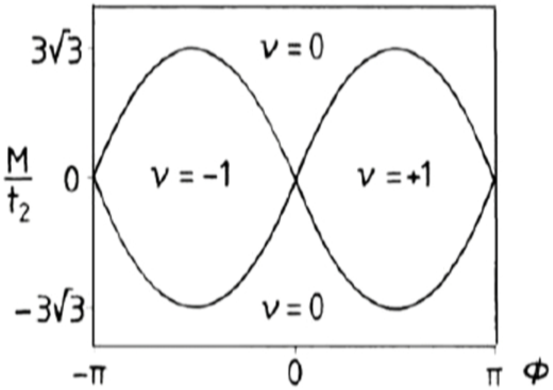
Effective masses $m_\alpha = (M - 3\sqrt{3}\alpha t_2 \sin\varphi)/c^2$

Non-zero M or φ generically gaps the spectrum

Break inversion: normal **Break time-reversal: topological**

Phase Diagram

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)



Boundaries correspond to vanishing of single effective Dirac mass

Connection to "Parity Anomaly"

$m_\alpha = (M - 3\sqrt{3}\alpha t_2 \sin\varphi)/c^2 = 0$ $M/t_2 = \pm 3\sqrt{3}\sin\varphi$

Non-topological $(\nu = 0)$ **Topological** $(\nu = \pm 1)$

Topological phases have a non-vanishing Chern number ν

Chern Number

For a state $|\psi\rangle$ this is defined by the integral of the Berry curvature over the 2D Brillouin zone

$$\nu = \frac{1}{2\pi} \int dk_x dk_y \, \Omega_{k_x k_y}$$

Berry curvature $\Omega_{k_x k_y} = \partial_{k_x} A_{k_y} - \partial_{k_y} A_{k_x}$

Berry connection $A_{k_\mu} = i\langle\psi|\partial_{k_\mu}|\psi\rangle$

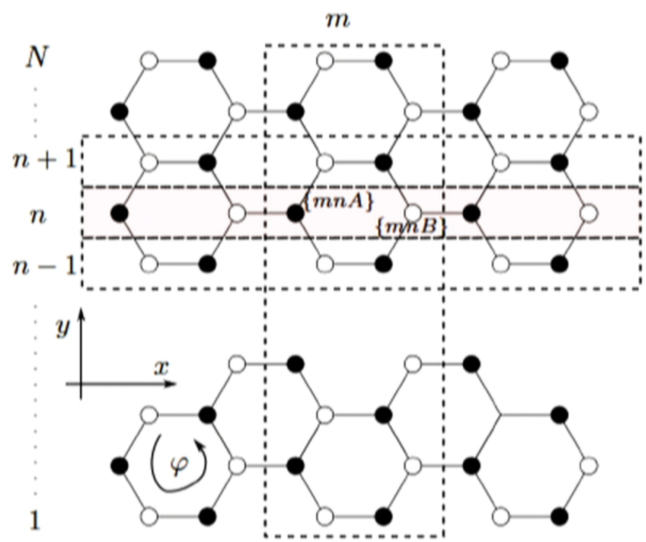
For the ground state of the Haldane model $\nu \in \pm 1, 0$

Decomposed into contributions from two Dirac points $\nu = \nu_+ + \nu_-$

$$\nu_\alpha = -\frac{\alpha}{2} \, \mathrm{sign}(m_\alpha) \in \pm 1/2 \qquad \nu = -\frac{1}{2} \left[\mathrm{sign}(m_+) - \mathrm{sign}(m_-)\right]$$

Quenching between different phases corresponds to changing the sign of one or both of the masses m_α

Finite Strip

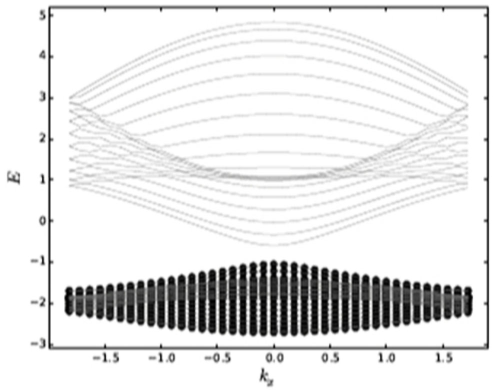


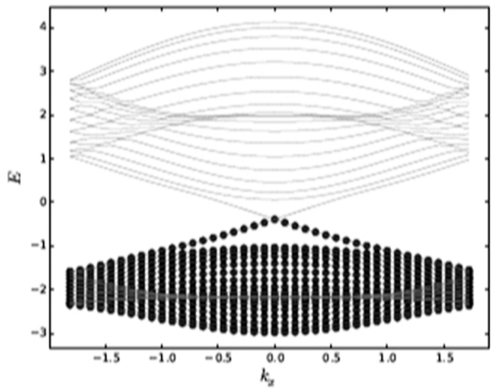
Armchair strip with N rows of lattice sites

Energy Spectrum

$t_1 = 1,\ t_2 = 1/3,\ M = 1,\ N = 20$

Non-topological ($\varphi = \pi/6$)   Topological ($\varphi = \pi/3$)



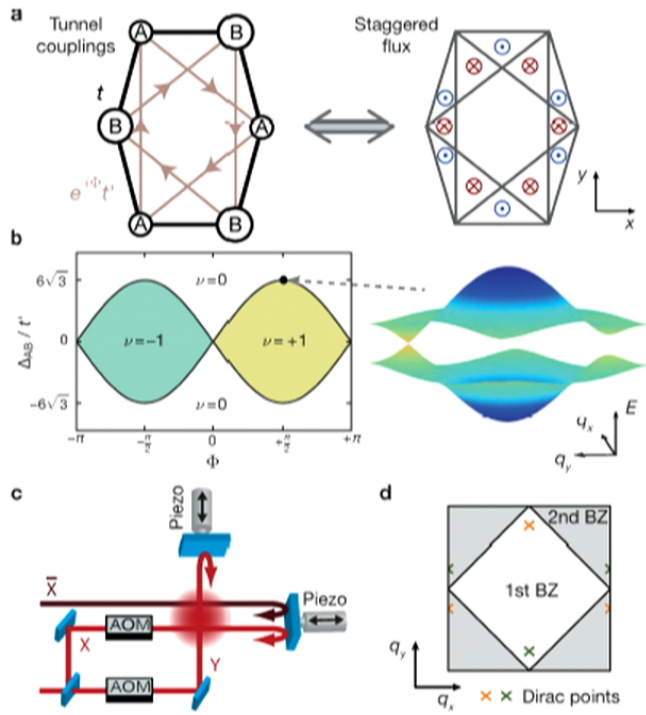


Topological phases have edge states

N. Hao *et al*, Phys. Rev. B 78, 075438 (2008)

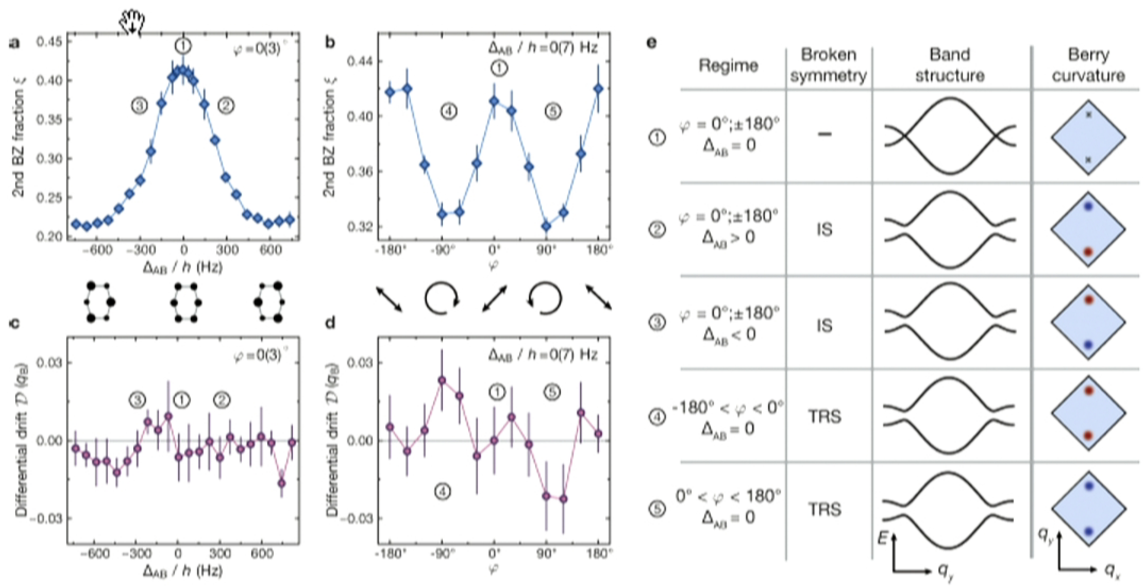
Experimental Realization

Jotzu, Messer, Desbuquois, Lebrat, Uehlinger, Greif, Esslinger,
Experimental realisation of the topological Haldane model,
Nature **515**, 237-240 (2014)



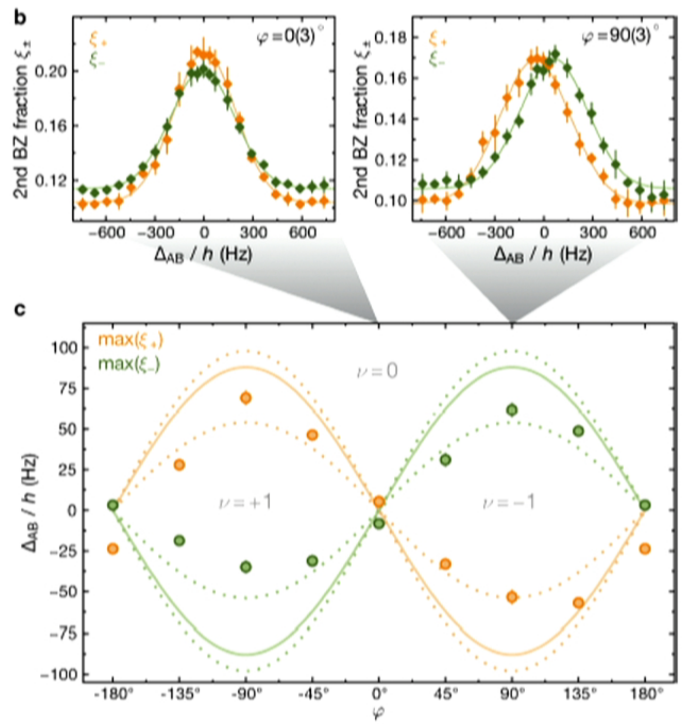
Measure Gaps

Jotzu *et al*, Nature **515**, 237-240 (2014)



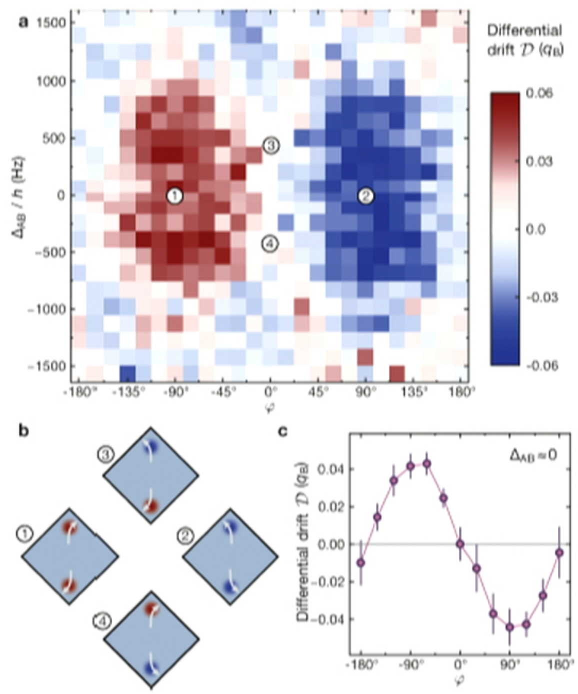
Mapping Out Transition Lines

Jotzu *et al*, Nature **515**, 237-240 (2014)



Measure Drift

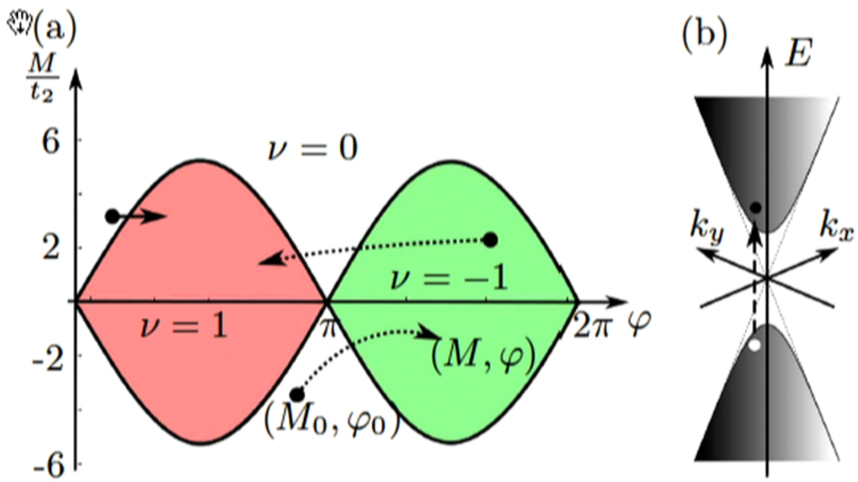
Jotzu *et al*, Nature **515**, 237-240 (2014)



Measure topological properties **Interactions & spin**

Quenches in the Haldane Model

Caio, Cooper & Bhaseen, arXiv:1504.01910



D'Alessio, Rigol, arXiv:1409.6319

Time-Evolution of ν

Within the low-energy Dirac representation

$$|\psi_\alpha(k)\rangle = a_\alpha(k)e^{-\mathrm{i}E^l_\alpha(k)t}\,|l_\alpha(k)\rangle + b_\alpha(k)e^{-\mathrm{i}E^u_\alpha(k)t}\,|u_\alpha(k)\rangle$$

Chern number is formally given by

$$\nu_\alpha(t) = -\alpha\,\mathrm{sign}\,m_\alpha\left(\frac{1}{2} - |b_\alpha(0)|^2\right) - |b_\alpha(\infty)||a_\alpha(\infty)|\cos[(E^u_\alpha(\infty) - E^l_\alpha(\infty))t + \delta]$$

$a_\alpha(k), b_\alpha(k) \in \mathbb{C} \qquad \delta = \arg(a_\alpha(\infty)) - \arg(b_\alpha(\infty))$

$E^{l,u}_\alpha(k)$ are energies in lower and upper bands

In general, $\nu_\alpha(t)$ is time-dependent, and differs from $\pm 1/2$

However, time-dependence only enters via coeffs at $k=\infty$

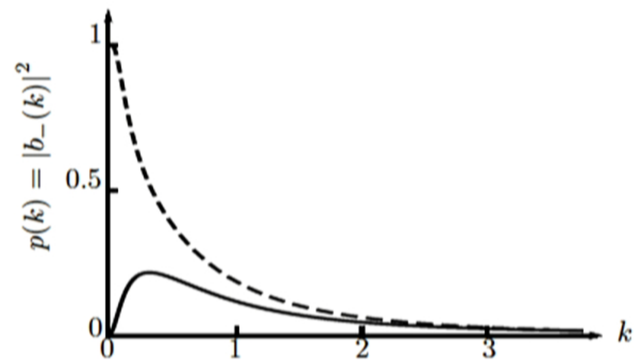
Following a quantum quench $b_\alpha(\infty) = 0$

Probability of Occupying Upper Band

Mass quench for a single Dirac point

Sign-preserving $m_- = -1 \to m'_- = -0.1$ (solid)

Sign-changing $m_- = -1 \to m'_- = 0.1$ (dashed)



$b_\alpha(\infty) = 0$

In addition, $b_\alpha(0) = 0, \pm 1$, so potential modification of ν_α is compensated by change in sign of m_α

ν_α time-independent & does not change from initial value

Similar results for linear sweep $m_\alpha(t) = t/\tau$

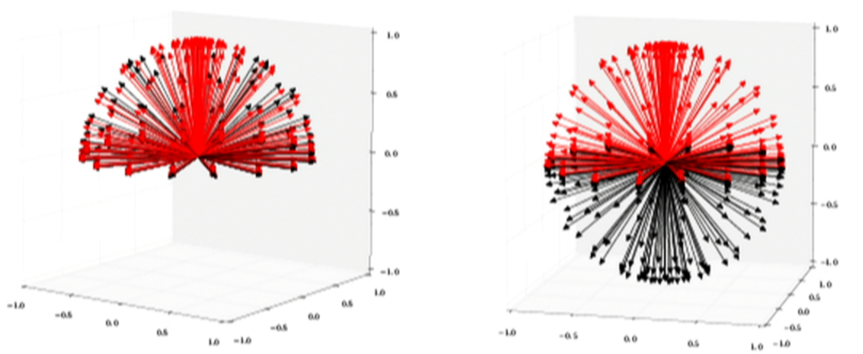
Preservation of Chern Number

Spin-textures in momentum space

Dirac Hamiltonian describes a spin in an effective magnetic field

$$\hat{H}_\alpha(\mathbf{k}) = -\mathbf{h}_\alpha(\mathbf{k}).\frac{\hat{\boldsymbol{\sigma}}}{2}$$

Meron spin configurations



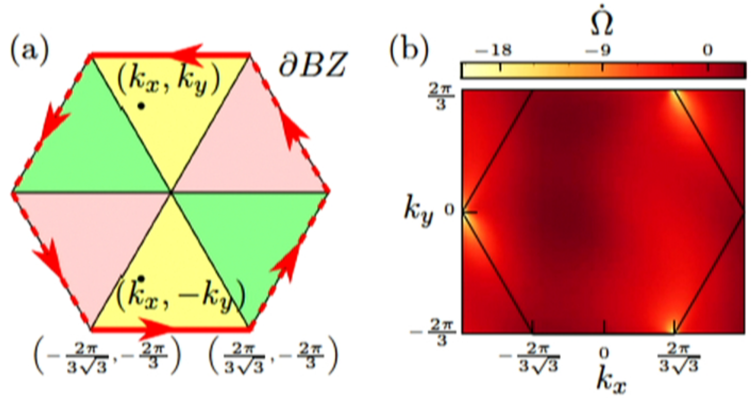
$\nu_\alpha = \pm 1/2$ configurations wind on upper and lower half-sphere

Following a quantum quench, spins precess in new magnetic field, but preserve topological characteristics

D'Alessio & Rigol, arXiv:1409.6319 **"No-Go Theorem"**

Preservation of Chern Number

First Brillouin zone of the Haldane model

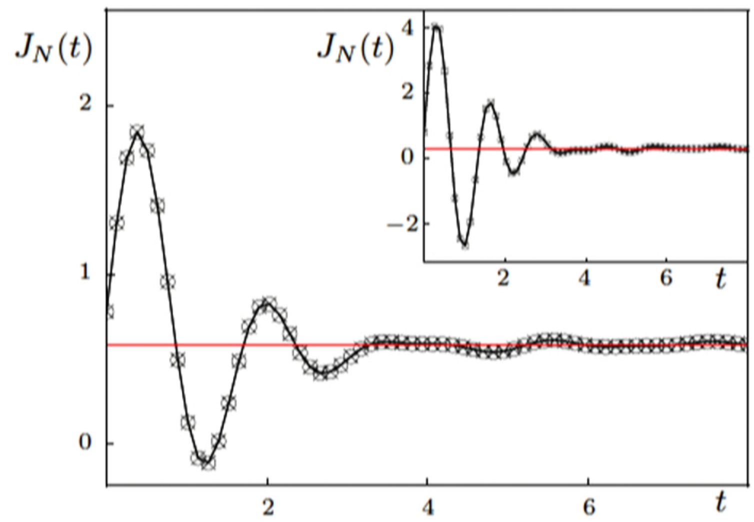


(a) In each of the triangles the Berry connection and curvature are time-dependent. However, ν is given by line integral of Berry connection along the zone boundary. Following a quench, the time-dependent contributions to ν from opposite sides of boundary cancel.

(b) Time-derivative of Berry curvature for $M = 1$, $t_1 = 1$, $t_2 = 1/3$ following quench from non-topological ($\varphi = \pi/6$) to topological phase ($\varphi = \pi/3$). Although $\dot{\Omega} \neq 0$, numerical integration confirms $\dot{\nu} = 0$.

Dynamics of Edge Currents

Quench from topological to non-topological phase



Red Line: Ground state current of <u>final</u> Hamiltonian

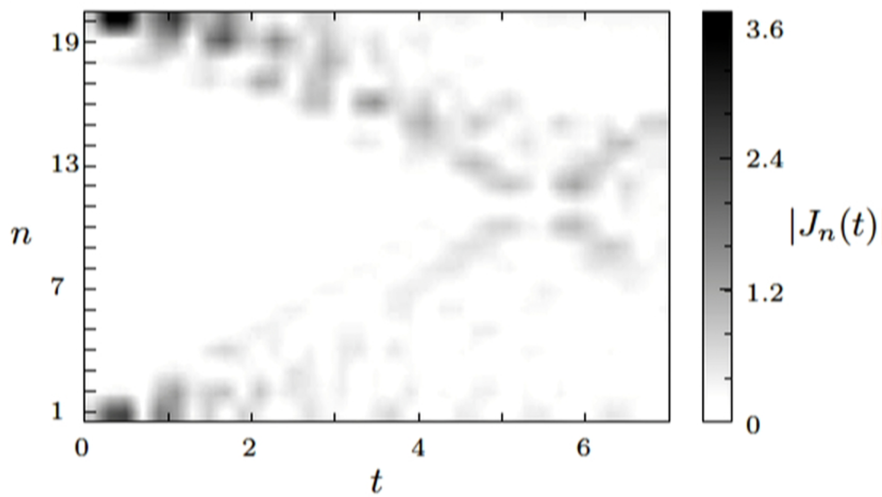
Agreement between $N = 30$ (circles) and $N = 40$ (crosses)

Intrinsic dynamics of edge currents

Light-Cone Spreading

Dynamics of the currents $|J_n^x(t)|$ following a quench from the topological to non-topological phase

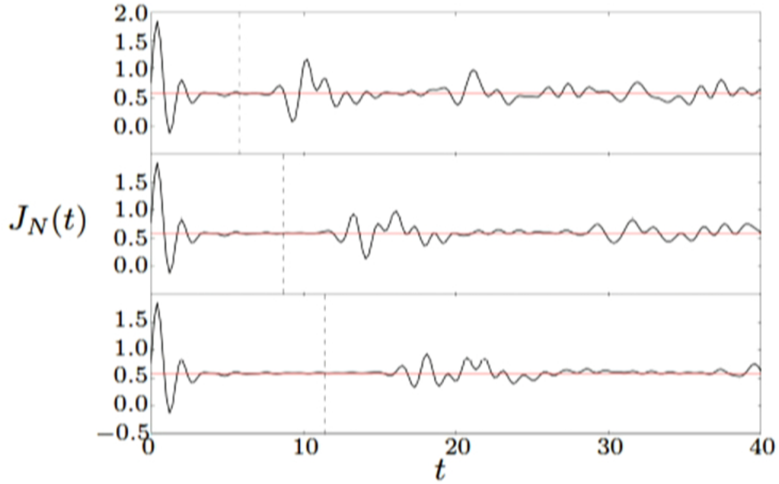
$$t_1 = 1,\ t_2 = 1/3,\ \varphi = \pi/3,\ M = 1.4 \to 2.2$$



Effective speed of light of Haldane model $c = 3t_2/2\hbar = 3/2$

Resurgent Oscillations

Quench from topological to non-topological with $N = 20,\ 30,\ 40$



Dashed lines show time-scale at which signals from two edges meet

$$t = (N/2)\sqrt{3}/2c$$

Questions

- Interpretation of ν out of equilibrium?
- Impact on Hall conductivity out of equilibrium?
- GGE for edge states?
- Decoherence in non-unitary settings?
- Interactions? Theory of dynamics?
- Dynamics of other models? e.g. Kitaev model
- Dynamics of topological insulators?
- Other dimensionalities?

AdS/CFT

Gravity $g_{\mu\nu}$ Topological $\epsilon_{\mu\nu}$

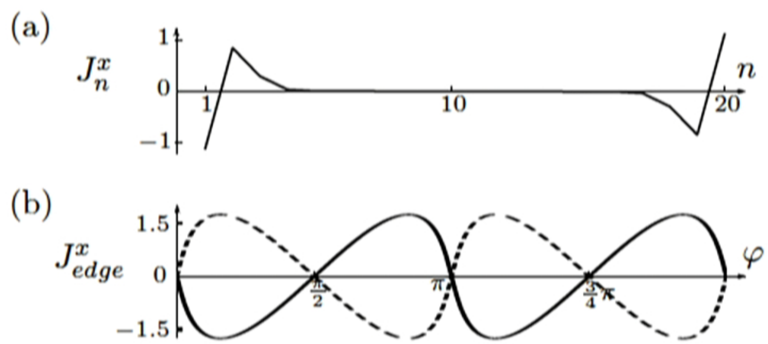
Non-equilibrium dynamics of gapless surface states

Currents in a Finite Width Strip

Topological Phase

(a) Total longitudinal current J_n^x for $N = 20$, $M = 0$ and $\varphi = \pi/3$

(b) Edge currents for $n = 1$ (solid) and $n = 20$ (dashed) for $M = 0$



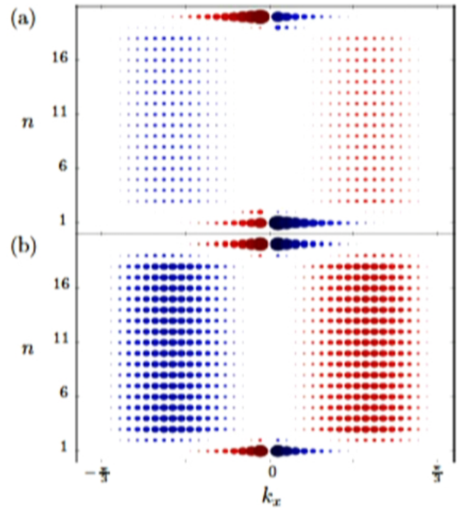
Edge currents can vanish within topological phases

Edge currents have π-periodicity in φ rather than 2π

Counter-propagating contributions

k-space contributions to equilibrium currents along strip $\langle \hat{J}^x_n(k_x) \rangle$

$N = 20,\ t_1 = 1,\ t_2 = 1/3,\ M = 0$



Size of dots proportional to $\langle \hat{J}^x_n \rangle^4$

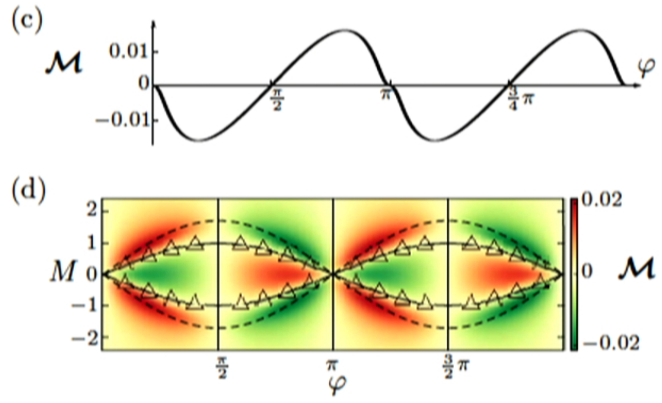
Blue negative **Red** positive

(a) $\varphi = \pi/3$ (b) $\varphi = \pi/2$ left- and right- movers cancel

Orbital Magnetization

$$\boldsymbol{\mathcal{M}} = \frac{1}{2\mathcal{A}} \int d^2r\, \mathbf{r} \times \langle \hat{\mathbf{J}}(\mathbf{r}) \rangle$$

$\hat{\mathbf{J}}(\mathbf{r})$ is the local current density operator and $\mathcal{A}$ is the area.



Magnetization vanishes within topological phases

[1] Thonhauser, Ceresoli, Vanderbilt & Resta, PRL **95**, 137205 (2005)

[2] Ceresoli, Thonhauser, Vanderbilt, & Resta, PRB **74**, 024408 (2006)

$\mathcal{M}$ vanishes on sinusoidal locus within topological phases

Questions

- Interpretation of ν out of equilibrium?
- Impact on Hall conductivity out of equilibrium?
- GGE for edge states?
- Decoherence in non-unitary settings?
- Interactions? Theory of dynamics?
- Dynamics of other models? e.g. Kitaev model
- Dynamics of topological insulators?
- Other dimensionalities?

AdS/CFT

Gravity $g_{\mu\nu}$ Topological $\epsilon_{\mu\nu}$

Non-equilibrium dynamics of gapless surface states

Full Counting Statistics

A large body of results in the mesoscopic literature

Free Fermions

Levitov & Lesovik, *"Charge Distribution in Quantum Shot Noise"*, JETP Lett. **58**, 230 (1993)

Levitov, Lee & Lesovik, *"Electron Counting Statistics and Coherent States of Electric Current"*, J. Math. Phys. **37**, 4845 (1996)

Luttinger Liquids and Quantum Hall Edge States

Kane & Fisher, *"Non-Equilibrium Noise and Fractional Charge in the Quantum Hall Effect"*, PRL **72**, 724 (1994)

Fendley, Ludwig & Saleur, *"Exact Nonequilibrium dc Shot Noise in Luttinger Liquids and Fractional Quantum Hall Devices"*, PRL (1995)

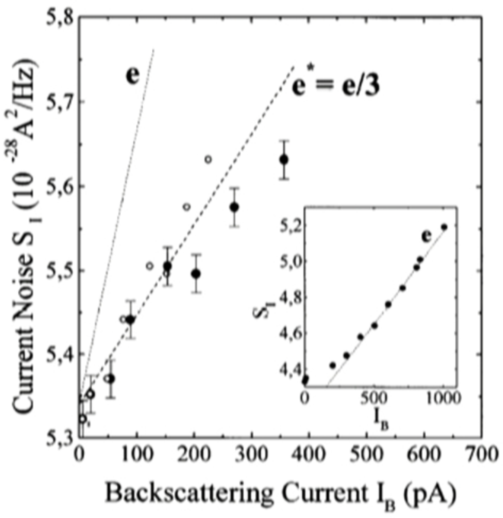
Quantum Impurity Problems

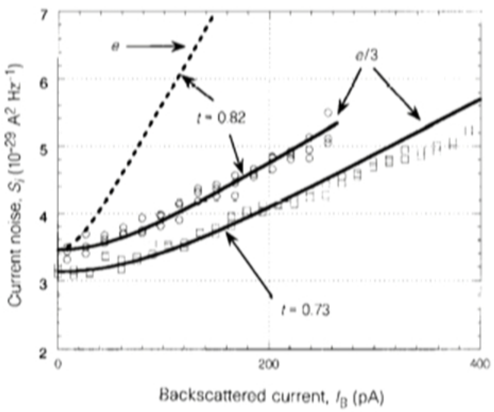
Komnik & Saleur, *"Quantum Fluctuation Theorem in an Interacting Setup: Point Contacts in Fractional Quantum Hall Edge State Devices"*, PRL **107**, 100601 (2011)

Shot Noise in the Quantum Hall Effect

Saminadayar *et al*, *"Observation of the e/3 Fractionally Charged Laughlin Quasiparticle"*, PRL **79**, 2526 (1997)

R. de-Picciotto *et al*, *"Direct observation of a fractional charge"*, Nature **389**, 162 (1997)



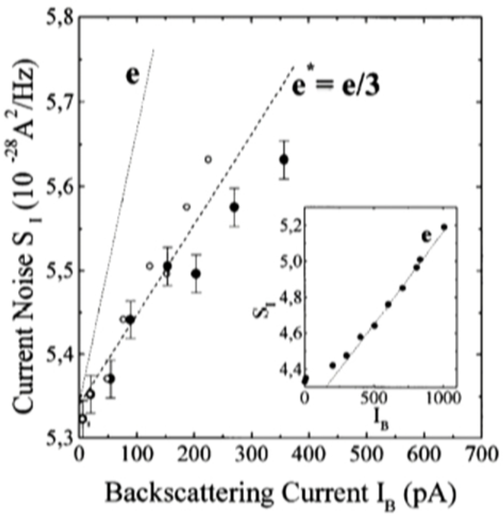


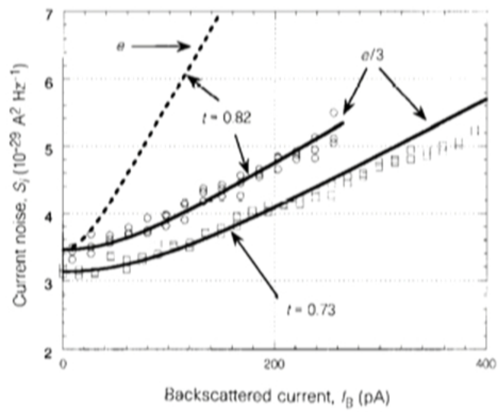
$S_I = 2e^* I_B$ $e^* = e/3$

Shot Noise in the Quantum Hall Effect

Saminadayar *et al*, *"Observation of the e/3 Fractionally Charged Laughlin Quasiparticle"*, PRL **79**, 2526 (1997)

R. de-Picciotto *et al*, *"Direct observation of a fractional charge"*, Nature **389**, 162 (1997)

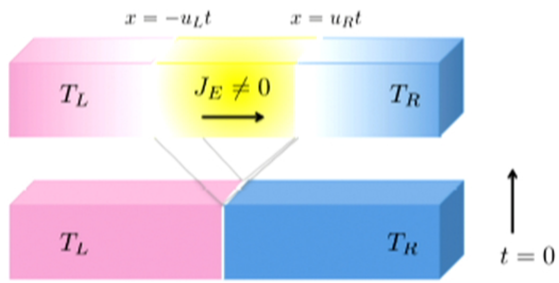




$S_I = 2e^* I_B$ $e^* = e/3$

Hydrodynamics

MJB, Benjamin Doyon, Andy Lucas, Koenraad Schalm *"Far from equilibrium energy flow in quantum critical systems"*, `arXiv:1311.3655`



Average energy flow $\langle J_E \rangle$ **Distribution of fluctuations**

Chemical Potentials $\mu_{\rm L}, \mu_{\rm R}$

Average charge flow $\langle J_Q \rangle$ **Distribution of fluctuations**

Full counting statistics for interacting quantum systems

Arbitrary dimension

Conclusions

Quantum quenches in Chern insulators

Dynamics of the Haldane model

Preservation of ν under unitary evolution

Final state retains a finger-print of initial Hamiltonian

Finite width strips

Dynamics of edge currents and light-cone propagation

Currents approach new equilibrium values

Depends on final Hamiltonian

Generalizations

Other models and dimensionalities

Charge noise, hydrodynamics, Einstein-Maxwell