Title: Modified Gravity, Dark Matter and Black Hole Shadows

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Abstract: <p>A modified gravity (MOG) theory has been developed over the past decade that can potentially fit all the available data in cosmology and the present universe. The basic ingredients of the theory are described by an action principle determined by the Einstein-Hilbert metric tensor and curvature tensor. An additional massive vector field Ï†Âµ is sourced by a gravitational charge $Q=\sqrt{\alpha G_N}M$, where α is a parameter, G_N is Newton's gravitational constant and M is the mass of a body. In addition, two scalar fields $G(x)$ and $\mu(x)$ are added to the action principle, where $G(x)$ describes a variable G_N and μ is the effective mass-range parameter of the the vector field ϕ_μ. For a slow moving test particle in a weak gravitational field, a modified Newtonian acceleration law is derived. This acceleration law reduces to the Newtonian acceleration law for distance scales $d << \mu^{-1}$. This acceleration law is applied to predict the rotation curves of galaxies and globular clusters with excellent fits to data for $\alpha=8.89\pm 0.34$ and $\mu=0.042\pm 0.004\,{\rm kpc}^{-1}$ without dark matter. An application to galaxy clusters with the same values of α and μ results in fits to cluster dynamics data without dark matter. The colliding clusters "bullet cluster" is also explained without dark matter. The vector field ϕ_μ coupling to standard model particles is of gravitational strength and the mass of the vector field is $m_\phi=2.6\times 10^{-28}\,eV$. This makes the vector field undetectable in the present universe. </p>

<p>For early universe cosmology before the formation of stars and galaxies, the mass m_ϕ, determined by the scalar field $\mu(x)$, is bigger, $m_\phi >> 2.6\times 10^{-28}\,eV$, and can act as an ultralight cold dark matter photon with gravitational strength coupling to matter. The modified gravity can fit the cosmological data up to the epoch when stars and galaxies are first formed, and when the ϕ_μ field density ρ_ϕ is significantly diluted compared to the baryon density ρ_b. After the commencement of the star and galaxy formation epoch, the modified gravity without dark matter takes over. A prediction of the matter power spectrum is made of the first formation of galaxies that do not have dark matter halos. The lack of detectability of the gravitationally sourced dark photon can explain why no convincing detection has so far been made of dark matter particles in laboratory and satellite experiments.</p>

<p>The modified gravity theory vacuum field equations with a smooth vector field source energy-momentum tensor are solved to produce a black hole that differs from the Schwarzschild, Kerr and Reissner-Norstr\"om black holes when the parameter $\alpha\neq 0$. A modified gravity solution is also obtained from a nonlinear regular vector field solution that is regular at $r=0$. This solution can describe a black hole with two horizons as well as a no black hole solution with no horizon. The black hole MOG solutions possess a photosphere and they cast a shadow against a bright background. The sizes and deformations of these shadows can be detected by the VLBI and Event Horizon (EHT) project. These observations will be able to test general relativity for strong gravitational fields. A traversable wormhole can be constructed using the modified gravity theory with a wormhole throat stabilized by the gravitationally sourced repulsive vector field. </p>

<p> </p>

Modified Gravity, Dark Matter and Black Hole Shadows

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Talk given at Perimeter Institute, April 2, 2015

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CONTENTS

1. Introduction

A strong motivation for modifying General Relativity (GR) is the problem with lack of luminous matter in the dynamics of galaxies and clusters of galaxies (Rubin & Ford, 1974, Zwicky 1933). The standard explanation is the existence of dark matter (DM). Some kind of cold DM is required to explain the Cosmic Microwave Background (CMB) data in the early universe cosmology epoch.

So far, no DM particle candidates have been detected in laboratory experiments and satellite missions.

An alternative resolution of the problem of explaining galaxy and galaxy cluster dynamics in the present universe epoch is to modify Newtonian and GR gravity.

A physically consistent modification of GR is Scalar-Tensor-Vector-Gravity (STVG) also call Modified Gravity (MOG). This is a fully covariant gravity theory with additional scalar fields and a massive vector field.


THE DARK MATTER - DARK ENERGY
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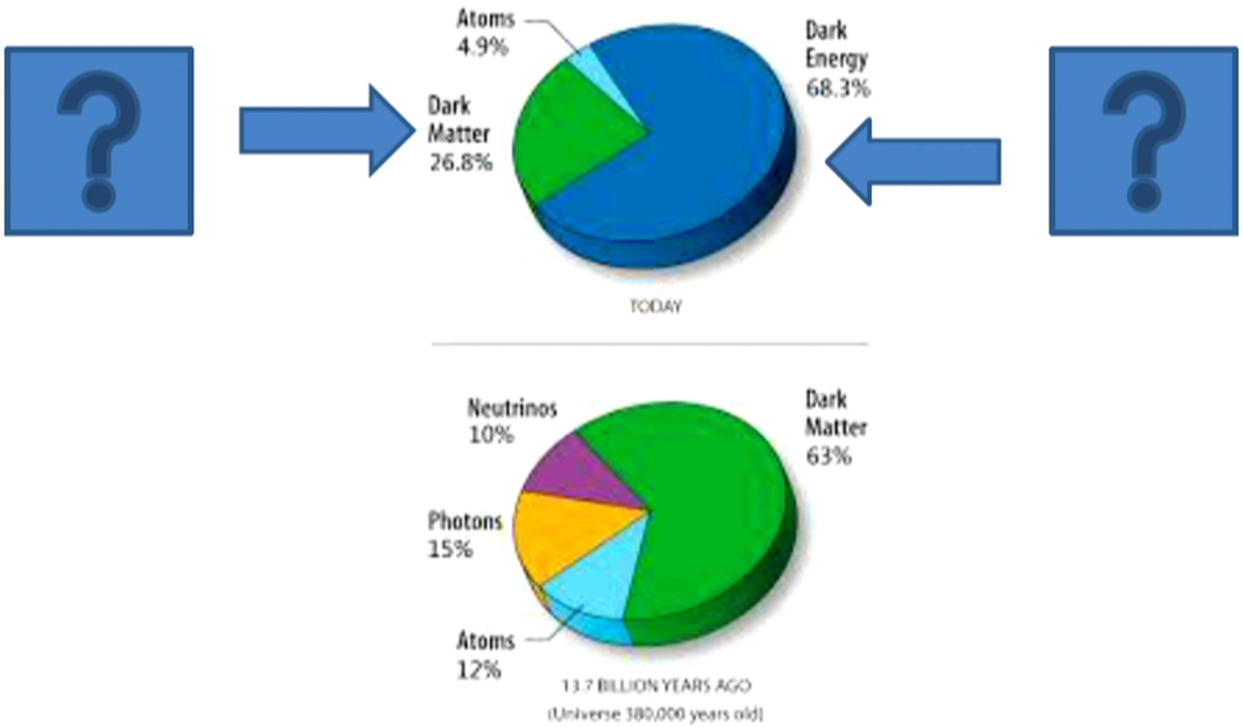
Atoms
4.9%
Dark
Energy
68.3%
Dark
Matter
26.8%
TODAY
Neutrinos
10%
Dark
Matter
63%
Photons
15%
Atoms
12%
13.7 BILLION YEARS AGO
(Universe 380,000 years old)

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Fact or Fiction?: Dark Matter Killed the Dinosaurs

A new out-of-this-world theory links mass extinctions with exotic astrophysics and galactic architecture

Scientific American, March 25, 2015 by Lee Billings

Experimental data that must be explained and fitted by MOG:

1. Planck and WMAP cosmic microwave background (CMB) data:
   Structure growth (stars and galaxies)
   Angular acoustical power spectrum
   Matter power spectrum
   Accelerated expansion of the universe.

2. Galaxy rotation curves and galaxy evolution and stability.
3. Galactic cluster dynamics.

4. Bullet Cluster 1E0657-558 .
5. Abell 520 cluster "train wreck" collision.

5. Gravitational lensing in cosmology.
6. Binary pulsar timing (PSR 1913+16 – Hulse-Taylor binary pulsar).

7. Solar system experiments:
   Weak equivalence experiments, light deflection by Sun, Shapiro time delay (Cassini probe), planetary orbits.

8. Strong gravity: Event Horizon Telescope and black holes.

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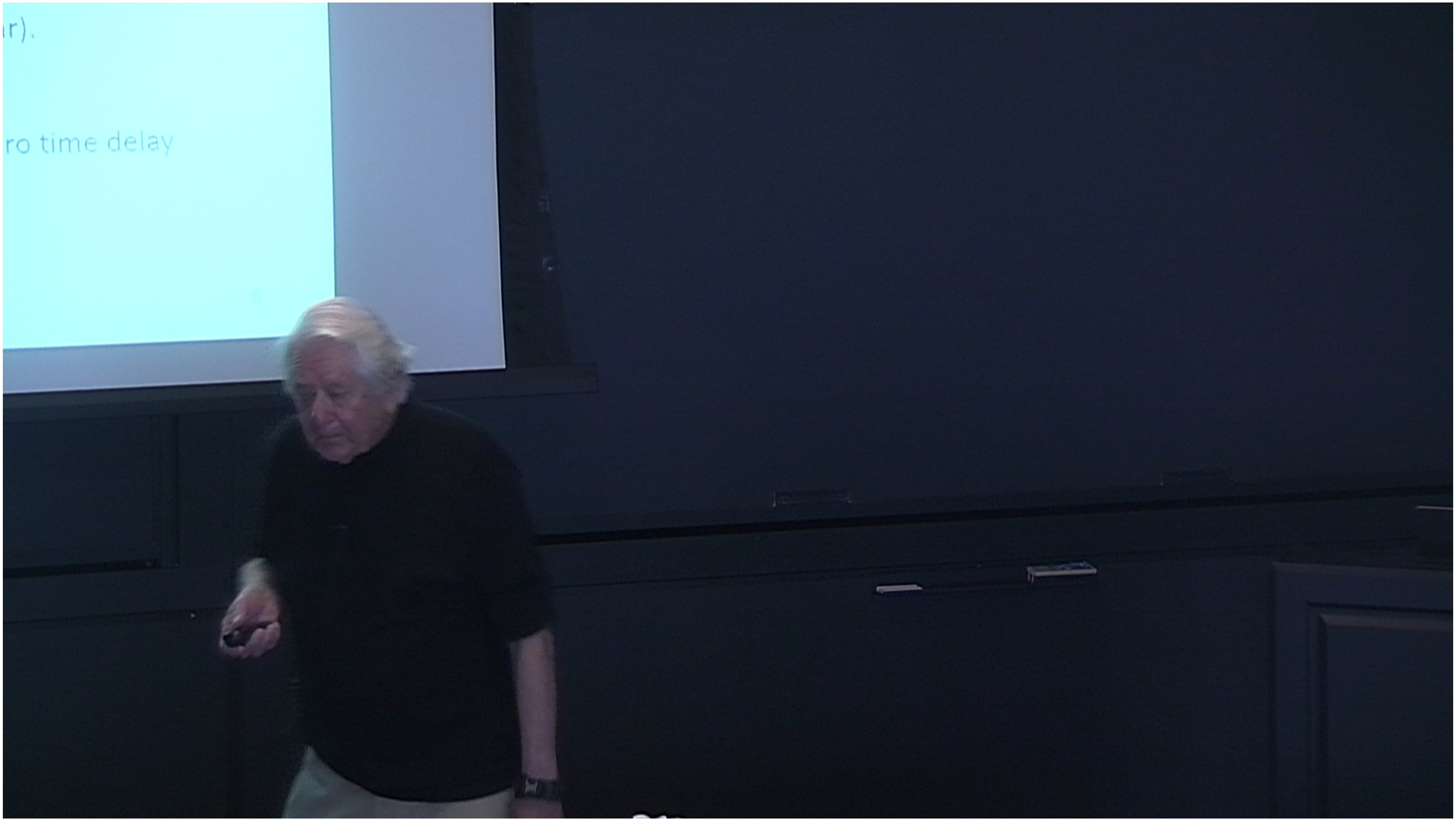
r).
ro time delay

2. MOG Field Equations (JCAP 0603 004 (2006), arXiv:gr-qc/0506021)

The MOG action is given by (STVG: JWM, JCAP 0603 004 (2006), arXiv:0506021 [gr-qc])

$$S = S_G + S_\phi + S_S + S_M$$

where

$$S_G = -\frac{1}{16\pi} \int \frac{1}{G} (R + 2\Lambda) \sqrt{-g}\; d^4x, \qquad (2)$$

$$S_\phi = -\omega \int \left[\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{2}\mu^2 \phi_\mu \phi^\mu + V_\phi(\phi)\right] \sqrt{-g}\; d^4x, \qquad (3)$$

$$S_S = -\int \frac{1}{G}\left[\frac{1}{2} g^{\mu\nu} \left(\frac{\nabla_\mu G \nabla_\nu G}{G^2} + \frac{\nabla_\mu \mu \nabla_\nu \mu}{\mu^2}\right) + \frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2}\right] \sqrt{-g}\; d^4x, \qquad (4)$$

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The action for pressureless dust can be written as:

$$S_M = \int (-\rho\sqrt{u^\mu u_\mu} - Qu^\mu \phi_\mu)\sqrt{-g}d^4x$$

Here, ρ is the density of matter and Q is the gravitational source charge:

$$Q = \sqrt{\alpha G_N}M$$

We choose the positive root for the gravitational charge, Q > 0 (gravitational repulsion.)

Varying the action with respect to the fields results in the MOG field equations. The variation of the actions S_M , S_ϕ and S_S and S_{EM} with respect to the metric yields the energy-momentum tensor:

$$T_{\mu\nu} = T_{M\mu\nu} + T_{\phi\mu\nu} + T_{S\mu\nu} + T_{EM}$$

$$T_{X\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_X}{\delta g^{\mu\nu}}, \quad (X = [M, \phi, S, EM]).$$

A test particle obeys the modified weak field Newtonian acceleration law:

$$\ddot{r} = -\frac{G_N M}{r^2}\left[1 + \alpha - \alpha(1 + \mu r)e^{-\mu r}\right]$$

$$\Phi(r) = -\frac{G_N M}{r}\left[1 + \alpha - \alpha e^{-\mu r}\right]$$

For an extended distribution of matter:

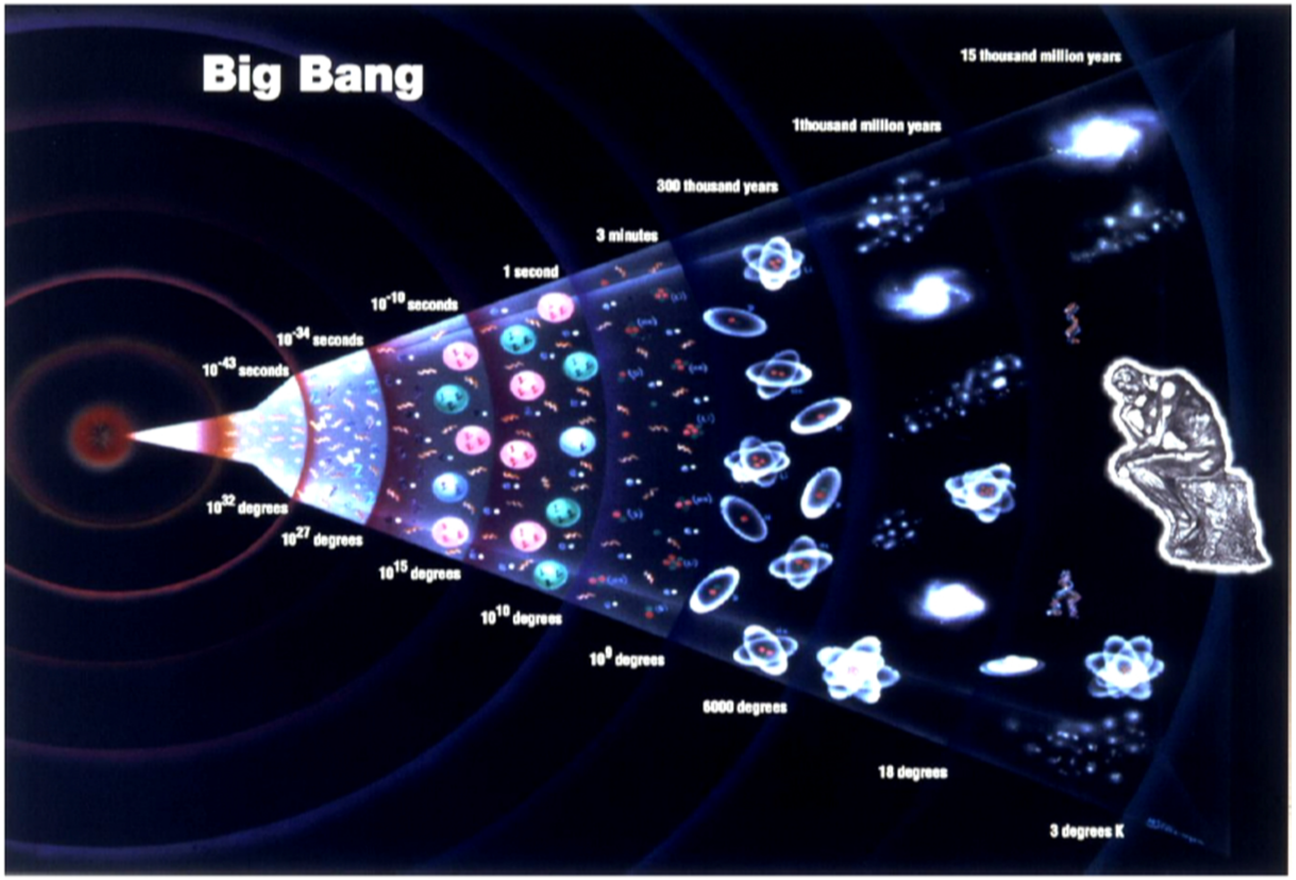
$$\Phi_{\rm eff}(\boldsymbol{x}) = -G_{\rm N}(1+\alpha)\int \frac{\rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}{\rm d}^3x' + G_{\rm N}\alpha\int \frac{\rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}{\rm e}^{-\mu|\boldsymbol{x}-\boldsymbol{x}'|}{\rm d}^3x'$$

A photon follows a null-geodesic path:

$$\frac{dk^\mu}{d\lambda} + \Gamma^\nu{}_{\alpha\beta}k^\alpha k^\beta = 0$$

k^μ = photon momentum

Big bang at t = 0 followed by either inflationary expansion period or by variable speed of light (VSL) with $c > c_0$ (c_0 = measured speed of light today, JWM, arXiv:1404.5567 [astro-ph.CO])



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3. MOG Cosmology (JWM, arXiv:1409.0853 [astro-ph.CO]).

We base our cosmology on the homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) background metric:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

We use the energy-momentum tensor of a perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \qquad \rho = \rho_M + \rho_\phi + \rho_S$$

The MOG Friedmann equations are given by:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G\rho}{3} + \frac{\dot{a}}{a}\frac{\dot{G}}{G} + \frac{\Lambda}{3},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{2}\left(\frac{\ddot{G}}{G} - \frac{2\dot{G}^2}{G^2} + \frac{\dot{a}\dot{G}}{aG}\right) + \frac{\Lambda}{3}$$

In the following, we assume a spatially flat universe $K = 0$.

$\dot{G} \sim 0$, and we also assume that $\dot{\mu} \sim 0$. $G = G_N(1 + \alpha)$.

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \qquad \dot{\rho} + 3\frac{d\ln a}{dt}(\rho + p) = 0$$

$$\delta\rho(\mathbf{r}, \mathbf{t}) = \frac{1}{(2\pi)^{3/2}}\bar{\rho}\int d^3k\delta_{\mathbf{k}}(t)\exp(i(a_0/a)\mathbf{k}\cdot\mathbf{r})$$

$$\delta p(\mathbf{r}, \mathbf{t}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\delta p_{\mathbf{k}}(t)\exp(i(a_0/a)\mathbf{k}\cdot\mathbf{r}) \qquad k^2\Phi_{\mathbf{k}} = -4\pi G\left(\frac{a}{a_0}\right)^2\bar{\rho}\delta_{\mathbf{k}}.$$

$$\Phi(\mathbf{r}, \mathbf{t}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\Phi_{\mathbf{k}}(t)\exp(i(a_0/a)\mathbf{k}\cdot\mathbf{r}),$$

where $\delta = \delta\rho/\bar{\rho}$ is the relative density perturbation contrast.

In the present scenario there are five components to the energy density:

1. Neutral pressureless phion particles (γ')
2. Baryon matter
3. Photons
4. Neutrinos
5. Dark energy (Λ).

Here, ρ_r, ρ_d, ρ_γ and ρ_ν denote the radiation, dark energy, photon and neutrino densities, respectively. Thus, we have

$$\rho = \rho_m + \rho_r + \rho_d \qquad \rho_m = \rho_b + \rho_\phi + \rho_S$$

$$\rho_r = \rho_\gamma + \rho_\nu$$

We assume that when ρ_ϕ dominates in the early universe before decoupling $\alpha < 1$.

$$G \sim G_\infty \sim G_N$$

At the time of big-bang-nucleosynthesis (BBN), we have $G \sim G_N$, guaranteeing that the production of elements agrees with observation. At horizon entry and before decoupling $\rho_\phi \gg \rho_b$.

After decoupling $\rho_\phi \sim \rho_b$ until stellar and galaxy formation when $\rho_\phi \ll \rho_b$, and the MOG non-relativistic acceleration law sets in to explain the rotation curves of galaxies and the dynamics of clusters without detectable dark matter.

The first Friedman equation becomes for $\rho_\phi >> \rho_b$:

$$H^2 = \frac{8\pi G_N \rho_\phi}{3} + \frac{\Lambda}{3} \qquad \rho_\phi = \frac{1}{2}\mu^2\phi_0^2$$

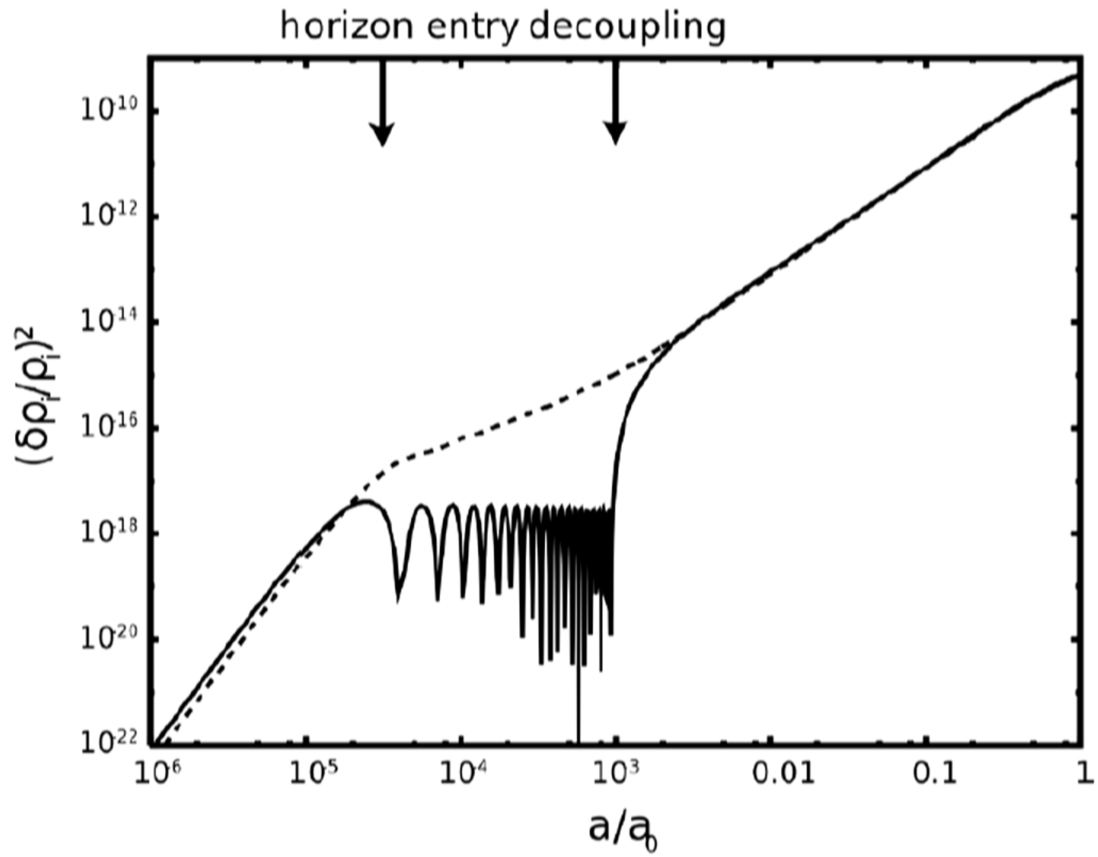
The Jeans equation for density perturbations is

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} + \left(\frac{c_s^2 a_0^2 k^2}{a^2} - 4\pi G_N \bar{\rho}\right)\delta_{\mathbf{k}} = 0 \qquad c_s = \sqrt{\frac{dp}{d\rho}}$$

For the "dark photon" ϕ_μ the speed of sound c_s = 0 and

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - 4\pi G_N \bar{\rho}\delta_{\mathbf{k}} = 0$$

Evolution of structure growth, density perturbations and baryon-photon acoustic oscillations in MOG for



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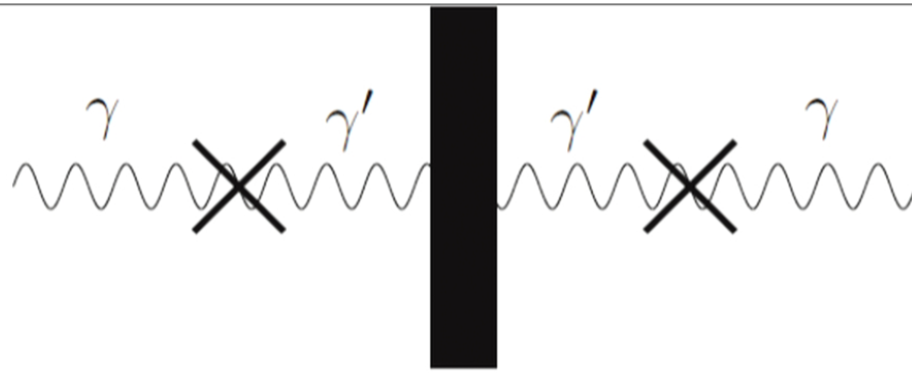

Schematic picture of a "light shining through a wall" experiment. The crosses denote the nondiagonal mass terms that convert photons into paraphotons. The photon γ oscillates into the paraphoton γ′ and, after the wall, back into the photon γ which can then be detected.

Consider a U(1) neutral ϕ_μ hidden photon, in addition to the usual $U(1)_{QED}$ photon. The most general renormalizable Lagrangian for the vector field ϕ_μ sector and for weak gravitational fields is

$$\mathcal{L}_\phi = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\chi F^{\mu\nu}B_{\mu\nu} + \frac{1}{2}m^2_{\gamma'}\phi_\mu\phi^\mu$$

where $F_{\mu\nu}$ is the electromagnetic field strength (J. Jaeckel and A. Ringwald, Phys. Lett. B659: 509 (2008), arXiv:0707.3063 [hep-ph]).

The acoustical angular power spectrum at the CMB can be calculated in MOG.

$$\Delta^2 = Ak^3T^2(k)P_0(k). \qquad T(k) = T_b(k) + T_\phi(k).$$

For a constant non-zero value of α , we have in the present universe $\rho_\phi \ll \rho_b$:

$$(G_N\rho)_{\Lambda CDM} = (G_N(1+\alpha)\rho)_{\rm MOG}. \quad \rho_{\Lambda CDM} = \rho_b + \rho_{CDM} \qquad \rho_{\rm MOG} = \rho_b$$

Now, red-shifting towards the CMB, ρ_ϕ becomes smoothly bigger than ρ_b and $\alpha < 1$:

$$(G_N\rho)_{\Lambda\rm CDM} = (G_N\rho_\phi)_{\rm MOG}$$

It follows that the angular acoustical power spectrum calculation can be duplicated in MOG using the Planck 2013 best-fit values:

$$\Omega_b h^2 = 0.022199. \quad \Omega_c h^2 = 0.11847, \; n_s = 0.9624, \; H_0 = 67.94\,{\rm km\,sec^{-1}\,Mpc^{-1}}$$

$\Omega_\Lambda = 0.6939$, $\sigma_8 = 0.8271$

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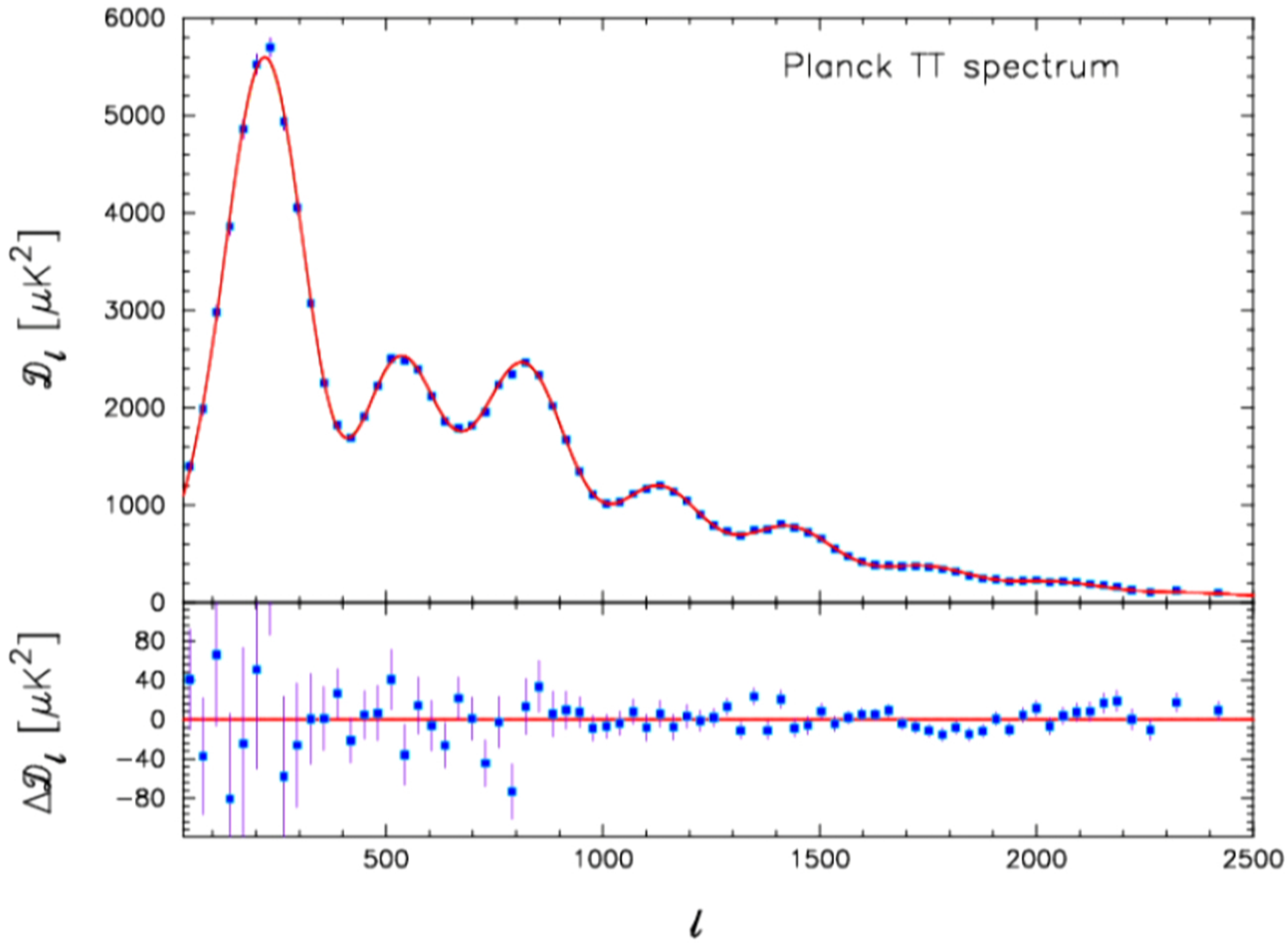
Planck TT spectrum
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell\ [\mu K^2]$
80
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0
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$\Delta\mathcal{D}_\ell\ [\mu K^2]$
500
1000
1500
2000
2500
ℓ

As the universe expands beyond the time of decoupling, the gravitational attraction between baryons increases and $G = G_\infty = G_N(1+\alpha)$. Eventually, as the large scale structures form $\rho_\phi < \rho_b$ and the baryon dominated MOG takes over. The galaxy rotation curves and the galactic cluster dynamics are determined without dark matter. The best fit values for α and μ are:

$$\alpha = 8.89 \pm 0.34 \qquad\qquad \mu = 0.04 \pm 0.004\,\mathrm{kpc}^{-1}$$

The phion mass parameter is a scalar field which evolves with time as the universe expands. After horizon entry $m_\phi \gg 10^{-28}\,\mathrm{eV}$ and the phion particle behaves like cold dark matter (CDM). When the earliest stars and galaxies form, the phion mass undergoes a significant decrease. In the present universe from the best fit value $\mu = 0.04\,\mathrm{kpc}^{-1}$ we get $m_\phi = 2.6\times 10^{-28}\,\mathrm{eV}$. The phion (hidden photon) mass becomes ultra-light and cannot contribute to the dynamics of galaxies and galactic clusters.

We conclude that dark matter particles cannot be detected in the present universe, either by laboratory experiments or in astrophysical observations (Pamela, AMS, gamma ray bursts).

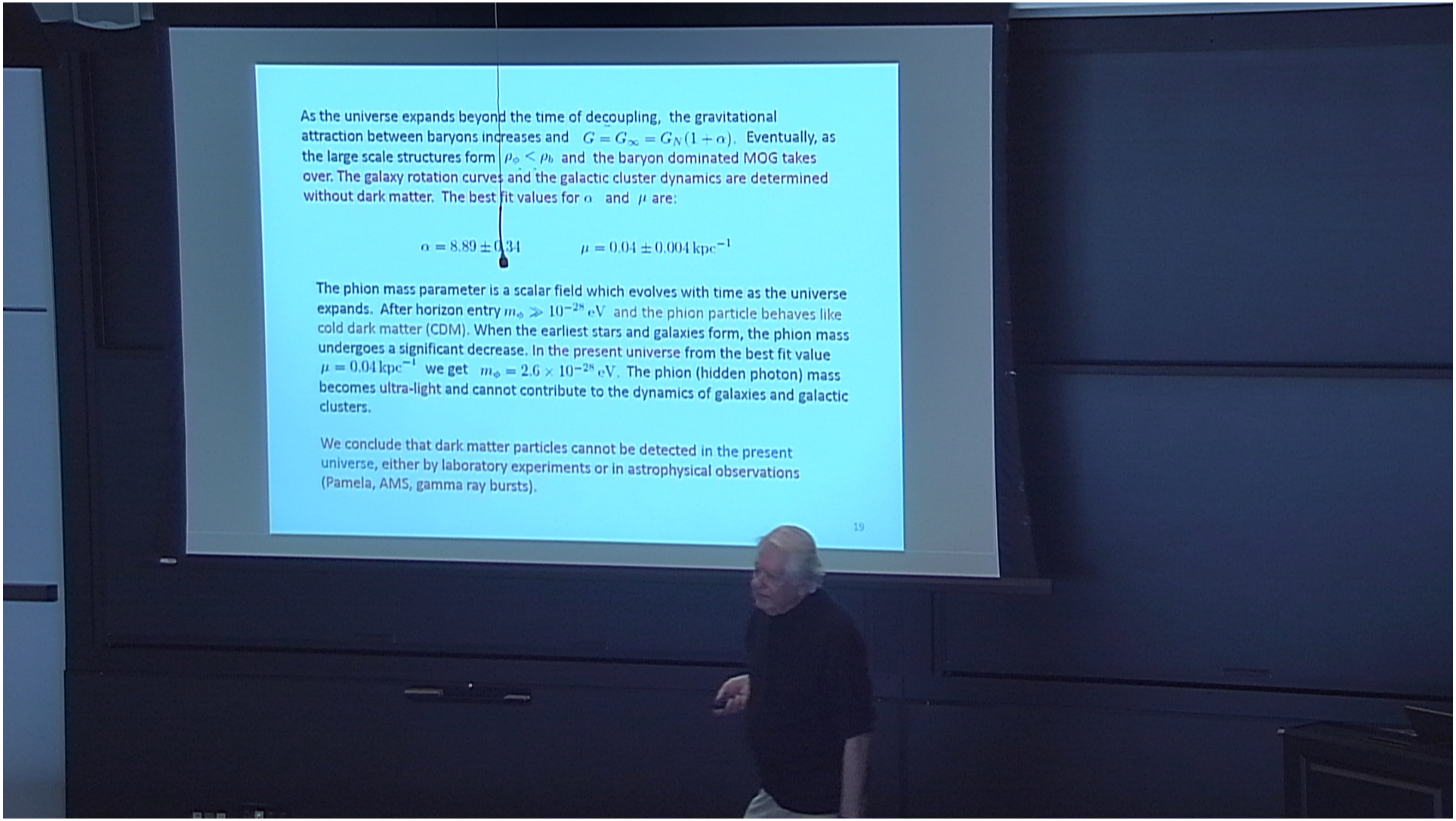
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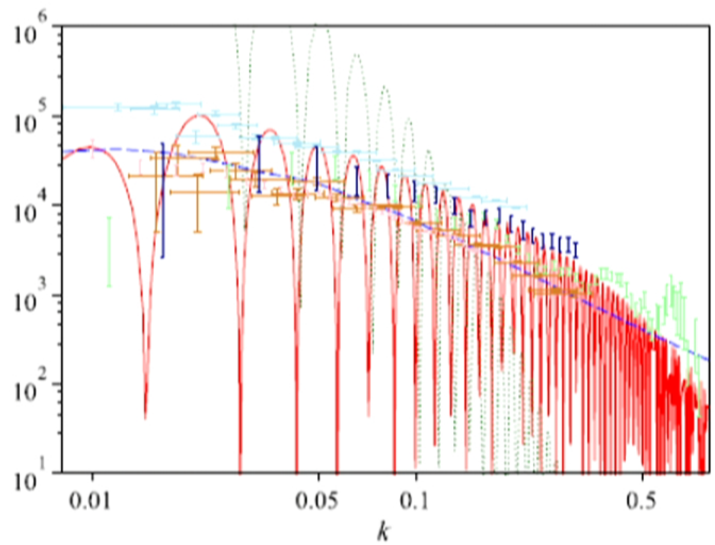
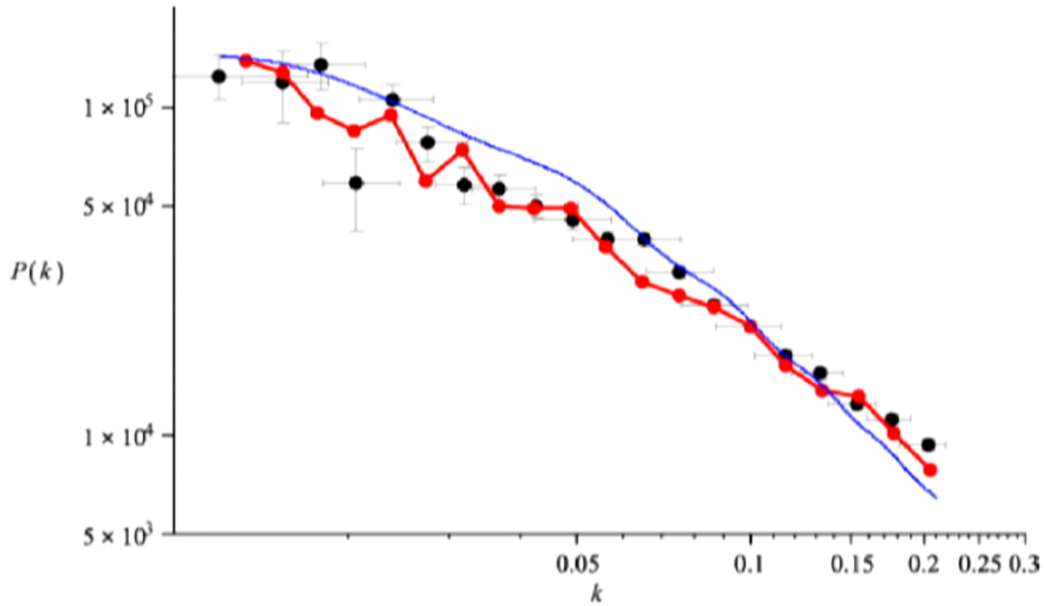
Figure 1. The matter power spectrum. Three models are compared against five data sets (see text): Λ-cold dark matter (Λ-CDM) (dashed blue line, $\Omega_b = 0.035$, $\Omega_c = 0.245, \Omega_\Lambda = 0.72$, $H = 71$ km/s/Mpc), a baryon-only model (dotted green line, $\Omega_b = 0.035$, $H = 71$ km/s/Mpc) and modified gravity (MOG) (solid red line, $\alpha = 19$, $\mu = 5$ h Mpc^{-1}, $\Omega_b = 0.035$, $H = 71$ km/s/Mpc), Data points are colored light blue [Sloan Digital Sky Survey (SDSS) 2006], gold (SDSS 2004), pink [Two-degree-Field (2dF)], light green [UK Schmidt Telescope (UKST)] and dark blue (CfA).

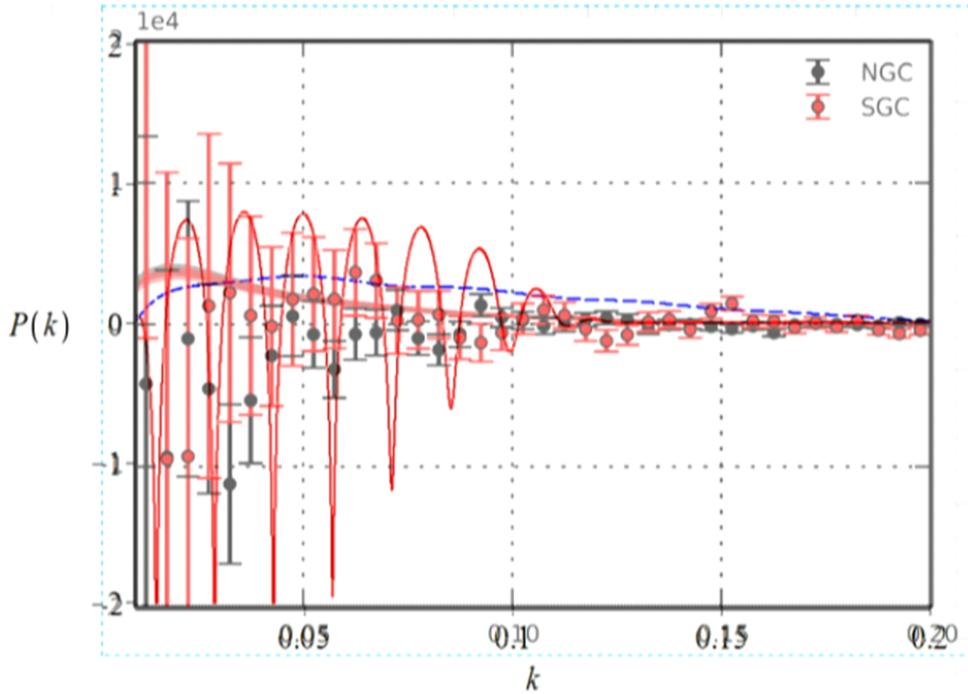


The matter power spectrum determined by the distribution of matter obtained from large scale galaxy surveys can also be predicted by MOG. A suitable window function and an initial scale invariant power spectrum P_0 are chosen to determine P(k). Baryon unit oscillations are greatly dampened by the window function.



With a sufficiently large survey of galaxies, the unit baryon oscillations will begin to be observed and distinguish between MOG, without detectable dark matter, and the standard $\Lambda\mathrm{CDM}$ model without unit baryon oscillations. This is a generic test that can distinguish MOG from dark matter models.

A prediction for the matter power spectrum that can distinguish MOG, without dark matter, from the standard ΛCDM model. Data from Battye, Charnock and Moss (arXiv:1409.2769).

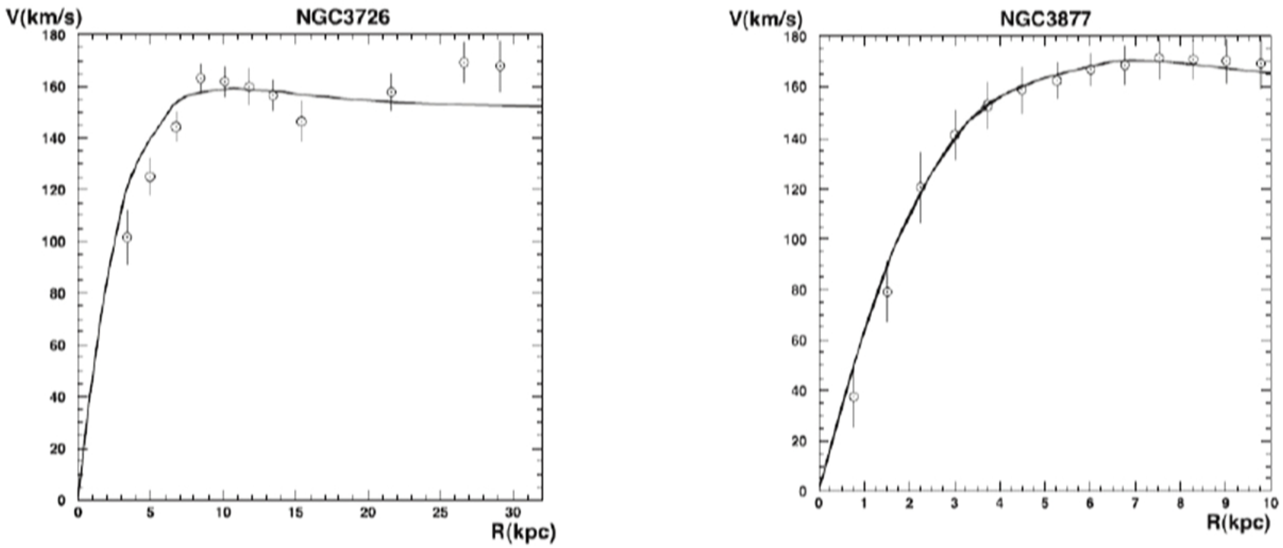


4. ROTATION CURVES OF GALAXIES (JWM & S. Rahvar, MNRAS 436, 1439 (2013), arXiv: 1306.6383 [astro-ph]).

Recent applications of MOG to galaxy dynamics is based on continuous distributions of baryon matter and realistic models of galaxy bulges and disks.

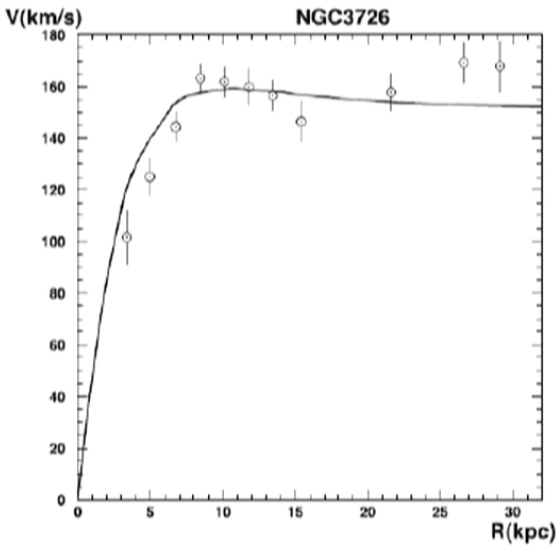
We choose a subsample of nearby galaxies from the THINGS catalogue with high-resolution measurements of velocity and density of hydrogen profile (de Blok et al. 2008). For this set of galaxies, we adopt in the weak field approximation μ and α as well as the stellar mass-to-light ratio M/L when fitting the rotation curves of galaxies to the data. We then find the best values of α and μ and fix these two parameters. Then, we fit the observed rotation curves of the larger Ursa Major sample of galaxies, letting the stellar mass-to-light ratio M/L be the only free parameter.

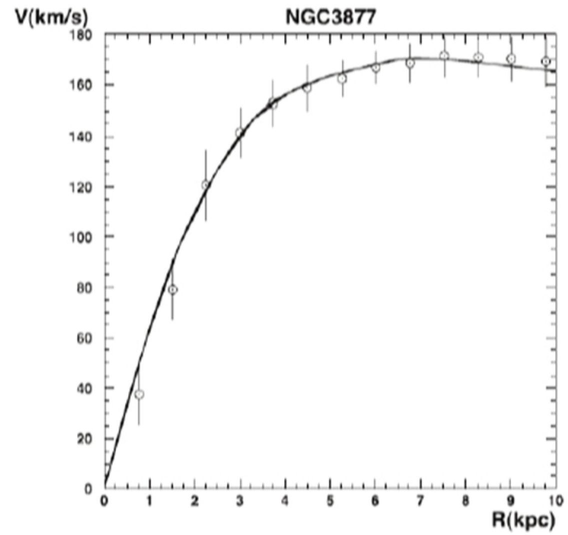
We adopt the best-fitting values of α and μ. and let the stellar-to-mass ratio M/L be the only free parameter and obtain fits to the Ursa Major catalogue of galaxies. The average value of χ^2 for all the galaxies is $\overline{\chi^2} = 1.07$.



The universal values of the parameters : $\alpha = 8.89 \pm 0.34$ and $\mu = 0.042 \pm 0.004$ kpc^{-1}.

We adopt the best-fitting values of α and μ. and let the stellar-to-mass ratio M/L be the only free parameter and obtain fits to the Ursa Major catalogue of galaxies. The average value of χ^2 for all the galaxies is $\overline{\chi^2} = 1.07$.





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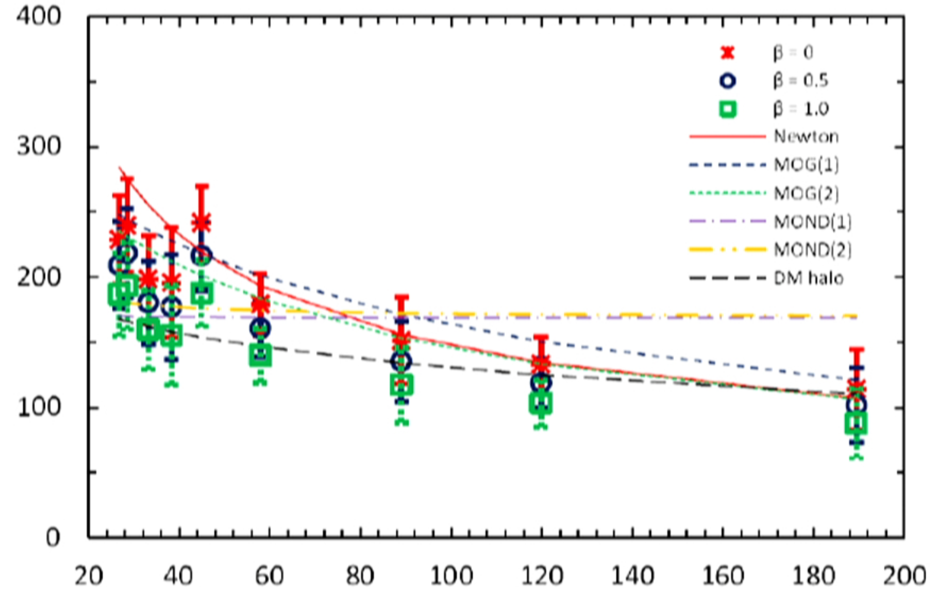
Pijushpani Bhattacharjee[1,2,3], Soumini Chaudhury[2,4], and Susmita Kundu[2,5]

Ap. J. 785, 63 (2014). J. W. Moffat and V. T. Toth, Phys. Rev. D91, 043004 (2015).

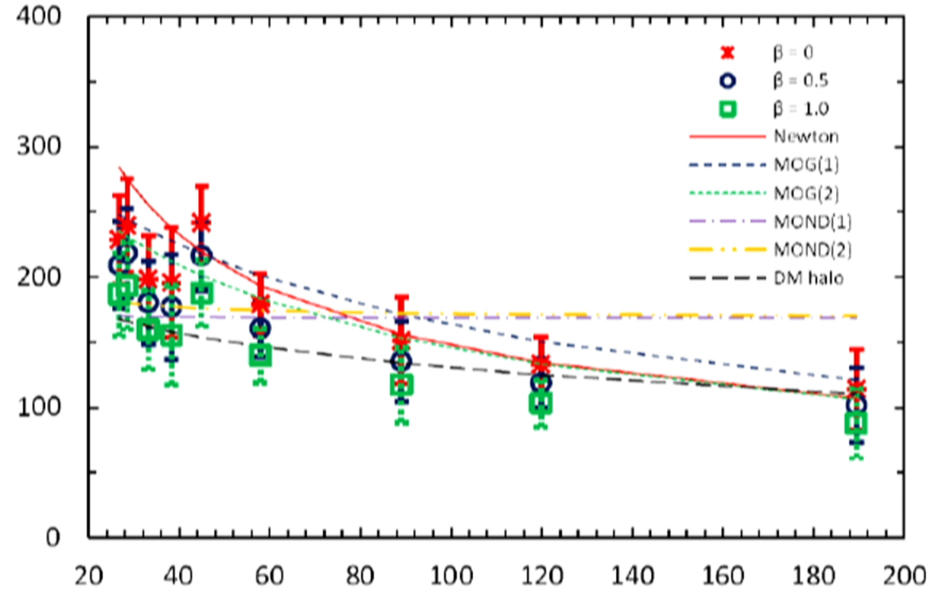


The solid red line is the Newtonian fit with a mass $M = 5 \times 10^{11} M_{\odot}$. The blue medium dashed and green short dashed lines correspond to MOG using the values $M = 4 \times 10^{10} M_{\odot}$, $\alpha = 15.01$, $\mu = 0.0313\ \mathrm{kpc}^{-1}$, and $M = 5 \times 10^{10} M_{\odot}$, $\alpha = 8.89$, $\mu = 0.04\ \mathrm{kpc}^{-1}$, respectively. The purple dash-dotted line is MOND with $M = 5 \times 10^{10} M_{\odot}$, $a_0 = 1.21 \times 10^{-8}\ \mathrm{cm/s}^2$ The black long - dashed line is the dark matter halo prediction.

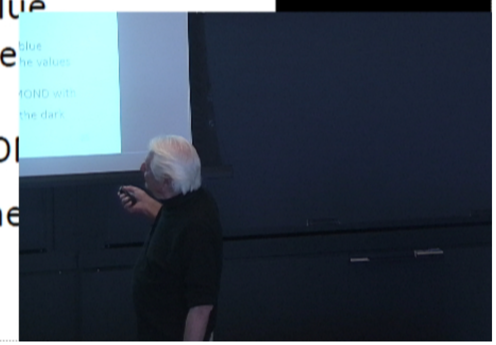
Rotation Curve of the Milky Way out to ~200 kpc

Pijushpani Bhattacharjee[1,2,3], Soumini Chaudhury[2,4], and Susmita Kundu[2,5]

Ap. J. 785, 63 (2014). J. W. Moffat and V. T. Toth, Phys. Rev. D91, 043004 (2015).



The solid red line is the Newtonian fit with a mass $M = 5 \times 10^{11} M_{\odot}$. The blue medium dashed and green short dashed lines correspond to MOG using the $M = 4 \times 10^{10} M_{\odot},\ \alpha = 15.01,\ \mu = 0.0313\ \mathrm{kpc}^{-1}$, and $M = 5 \times 10^{10} M_{\odot}$, $\alpha = 8.89,\ \mu = 0.04\ \mathrm{kpc}^{-1}$, respectively. The purple dash-dotted line is MOI $M = 5 \times 10^{10} M_{\odot},\ a_0 = 1.21 \times 10^{-8}\ \mathrm{cm/s^2}$ The black long - dashed line is the matter halo prediction.



5. CLUSTER DYNAMICS (JWM & S. Rahvar, MNRAS, 441, 3724 (2014), arXiv:1309.5077 [astro-ph]).

We have used the observations of the nearby cluster of galaxies obtained by the Chandra telescope to examine MOG.

Using the Virial theorem or a relaxed spherically symmetric cluster, we can relate the temperature and gas profile to the internal acceleration of the cluster

$$\frac{k_B T(r)}{\mu_p m_p r}\left(\frac{d\ln\rho_g(r)}{d\ln r}+\frac{d\ln T(r)}{d\ln r}\right)=g(r)$$

The left-hand side of this equation is given by data, which has to be consistent with the dynamical mass obtained from the above equation:

$$M_{\rm dyn}(r) = -3.68\times 10^{10}\frac{rT(r)}{1+\alpha}\left(\frac{d\ln\rho_g(r)}{d\ln r}+\frac{d\ln T(r)}{d\ln r}\right)$$

We write the overall mass in Newtonian gravity in terms of the MOG dynamical mass:

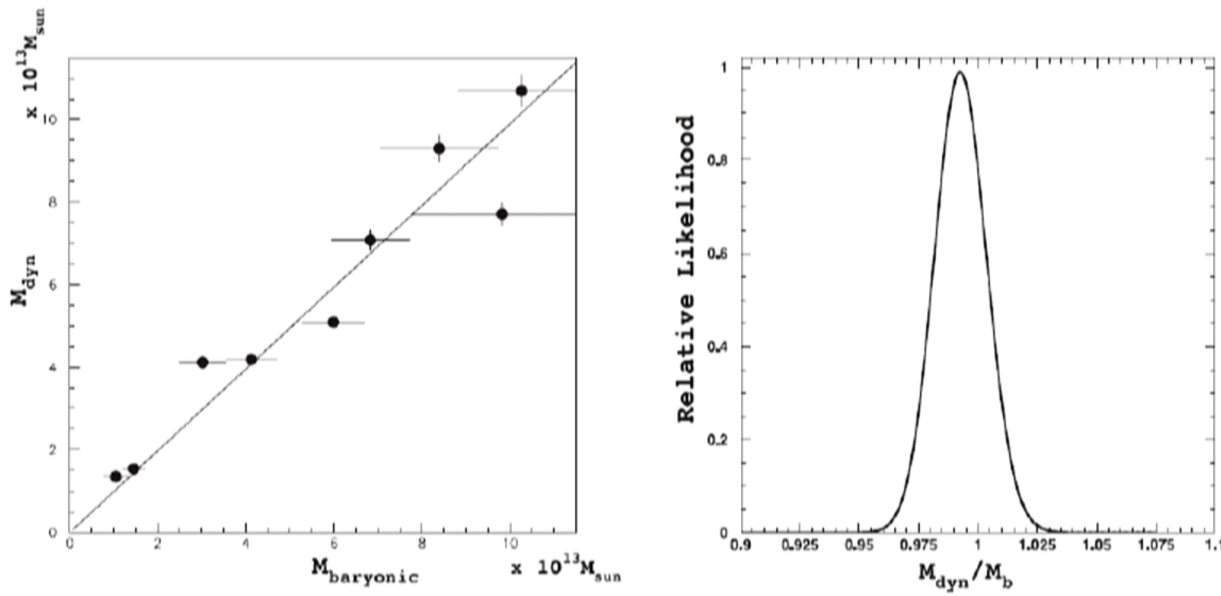
$$M_{\rm dyn} = \frac{M_N}{1+\alpha}$$

Here, α is already fixed by the fits to the galaxy rotation curves α = 8.89.

The majority of the baryonic cluster mass is gas. For MOG to be consistent with the data, the MOG dynamical mass has to be identically equal to the Newtonian baryonic mass.

We find for the best fit χ^2 value $\beta = M_{\rm dyn}/\check{M}_{\rm b} = 0.98.$

This yields with a $1\,\sigma$ level of confidence the best value $\beta = 0.98^{+0.02}_{-0.02}$.



Comparison of the dynamical mass in MOG versus the baryonic mass for clusters. The baryonic mass is composed of gas and stars. The filled circles indicate the corresponding masses up to r_{500} with the corresponding error bars. The solid line shows the best fit to the linear relation $M_{\rm dyn} = \beta_{\rm cl} M_{\rm bar}$ between the two masses with the best fit value of $\beta = 0.99$. The likelihood function for this fit is given in the right-hand panel.

6. Bullet Cluster 1E0657-558 and Abell 520 Cluster Collisions (J. R. Brownstein and JWM, MNRAS, 382, 29 (2007), arXiv:0702146 [astro-ph]). N. Israel and J. W. Moffat, (in preparation).

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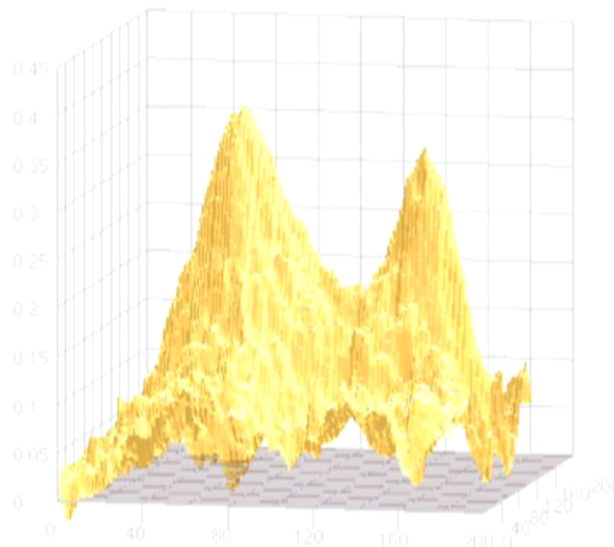
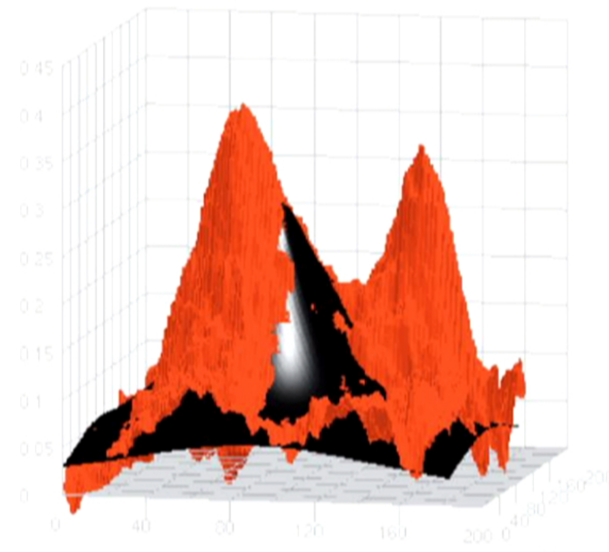


Convergence κ – map data
for Bullet Cluster (Clowe et al 2006).



MOG prediction for κ – map convergence.

The fit to the Bullet Cluster data requires no non-baryonic dark matter!

The MOG predictions for the A520 cluster collision are presently being investigated (Norman Israel and JWM).

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7. MOG Black Holes

JWM, arXiv:1412.5424 [gr-qc] and JWM, Eur. Phys. J. C (2015) 75:130, arXiv:1502.01677 [gr-qc]

The STVG vacuum field equations are ($T_{M\mu\nu} = 0$) :

$$R_{\mu\nu} = -8\pi G T_{\phi\mu\nu}$$

We assume that $\partial_t G = 0$ and we ignore the small mass $m_\phi \sim 0$:

$$m_\phi = 2.6 \times 10^{-28}\ \mathrm{eV}$$

$$T_{\phi\mu\nu} = -\frac{1}{4\pi}(B_\mu{}^\alpha B_{\nu\alpha} - \frac{1}{4}g_{\mu\nu}B^{\alpha\beta}B_{\alpha\beta}) \qquad B_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$$

$$\nabla_\nu B^{\mu\nu} = \frac{1}{\sqrt{-g}}\partial_\nu(\sqrt{-g}B^{\mu\nu}) = 0. \qquad \nabla_\sigma B_{\mu\nu} + \nabla_\mu B_{\nu\sigma} + \nabla_\nu B_{\sigma\mu} = 0$$

The exact solution of the field equations yields the metric:

$$ds^2 = \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)dt^2 - \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

where $G = G_N(1+\alpha)$, the gravitational source charge is $Q = \pm\sqrt{\alpha G_N} M$ and Q > 0 and α > 0, so that the gravitational vector field is always a repulsive force field (this is required to fit galaxy and cluster data without detectable dark matter).

As for the Reissner-Nordstrom electrically charge solution, there are two horizons (In principle, stationary black holes can also have an electric charge. However, in astrophysical settings, in which ions are available in copious quantities, gravitationally relevant charges will neutralize rapidly):

$$r_\pm = GM \pm \sqrt{G^2M^2 - GQ^2} = G_N M \left(1 + \alpha \pm \sqrt{1+\alpha}\right)$$

We obtain for α = 0 the Schwarzschild event horizon: $r_+ = r_s = 2G_N M$.
For α > 0 there is no horizon-free solution with a naked singularity.

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(In collaboration with Jonas Mureiko and Mir Faizal 2015)

The temperature of the MOG black hole can be calculated from the surface gravity:

$$T = \frac{\kappa}{2\pi} \quad , \quad \kappa = \frac{1}{2}\frac{dg_{00}}{dr}\,(r = r_+)$$

$$T = \frac{1}{2\pi G_N M} \cdot \frac{1}{(1+\sqrt{1+\alpha})(1+\alpha+\sqrt{1+\alpha})}$$

When $\alpha = 0$ the usual Schwarzschild black hole temperature is obtained:

$$T = \frac{1}{8\pi G_N M}$$

The radiative power of the black hole is

$$\frac{dM}{dt} = 4\pi R_+^2 \sigma T^4$$

The lifetime of the black hole is

$$\tau_\alpha = -\int_{M_{\rm BH}}^{0} \frac{dM}{4\pi\sigma R_+^2 T^4}$$

$$= \frac{4\pi^3}{3\sigma}(1+\sqrt{1+\alpha})^4(1+\alpha+\sqrt{1+\alpha})^2 G_N^2 M_{\rm BH}^3.$$

A solution regular at r = 0 can be obtained from the MOG field equations based on an electrically charged regular vacuum solution :
(Ayón-Beato and Garcia (Phys. Rev. Lett., **80**, 5056 (1998))).

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GMr^2}{(r^2+\alpha G_N GM^2)^{3/2}} + \frac{\alpha G_N GM^2 r^2}{(r^2+\alpha G_N GM^2)^2}\right)dt^2 \\ &- \left(1 - \frac{2GMr^2}{(r^2+\alpha G_N GM^2)^{3/2}} + \frac{\alpha G_N GM^2 r^2}{(r^2+\alpha G_N GM^2)^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \end{aligned}$$

The associated gravitational B_{0r} field is given by

$$B_{0r} = (\sqrt{\alpha G_N})r^4\left(\frac{r^2 - 5\alpha G_N GM^2}{(r^2+\alpha G_N GM^2)^4} + \frac{15}{2}\frac{GM}{(r^2+\alpha G_N GM^2)^{7/2}}\right)$$

$$g_{00} = 1 - \frac{2GM}{r} + \frac{\alpha G_N GM^2}{r^2} + \mathcal{O}(1/r^3) \qquad B_{0r} = \frac{\sqrt{\alpha G_N}}{r^2} + \mathcal{O}(1/r^3)$$

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For small r the metric behaves as

$$ds^2 = (1 - \frac{1}{3}\Lambda r^2)dt^2 - (1 - \frac{1}{3}\Lambda r^2)^{-1}dr^2 - r^2 d\Omega^2$$

where the cosmological constant Λ is given by

$$\Lambda = \frac{3}{G_N^2 M^2}\left(\frac{\alpha^{1/2} - 2}{\alpha^{3/2}(1+\alpha)}\right)$$

The interior of the regular solution is a deSitter/anti-deSitter vacuum solution.

For $\alpha < \alpha_{crit}$, where $\alpha = \alpha_{crit} \approx 0.673$, there are two horizons, and for $r > \alpha_{crit}$, there is no horizon and no naked singularity.

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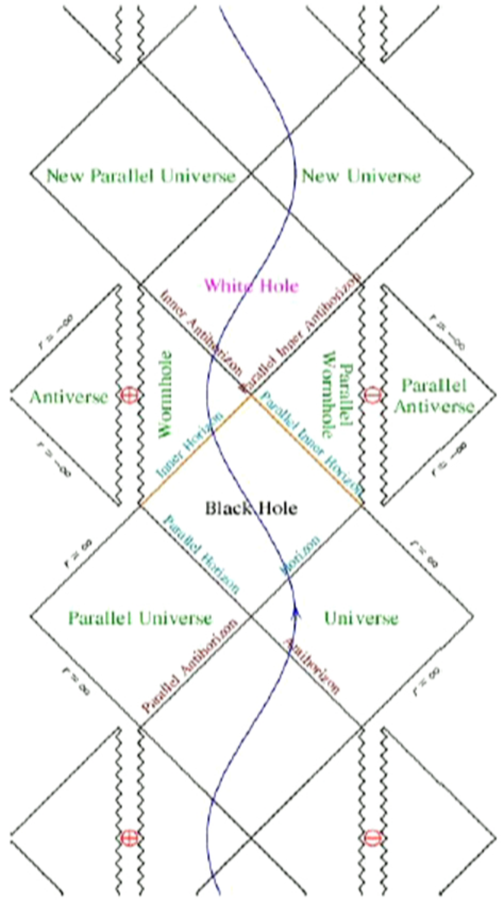
Penrose diagram for the singular MOG black hole (image by Andrew Hamilton)



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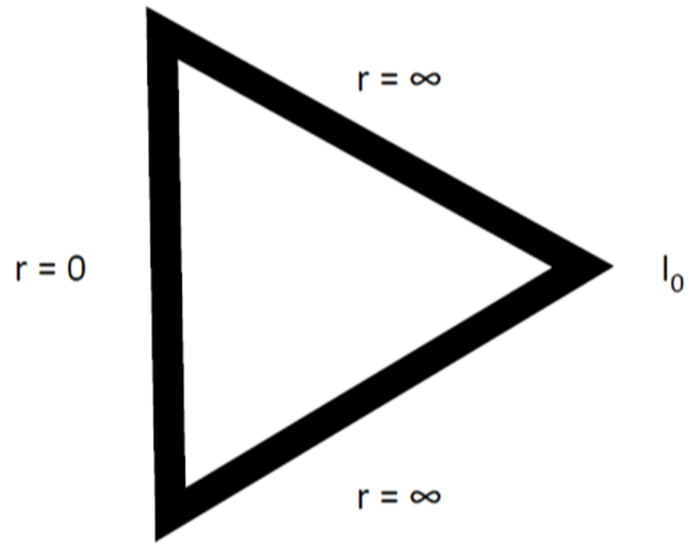

No black hole Penrose diagram regular at r = 0

The metric solution for a rotating black hole called the Kerr-MOG black hole is

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta\, d\phi\right)^2$$

$$-\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)d\phi - adt\right]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2$$

$$\Delta = r^2 - 2G_N(1+\alpha)Mr + a^2 + \alpha G_N^2(1+\alpha)M^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta.$$

The horizons are

$$r_\pm = G_N(1+\alpha)M\left[1 \pm \sqrt{1 - \frac{a^2}{G_N^2(1+\alpha)^2M^2} - \frac{\alpha}{1+\alpha}}\right] \quad \Delta = 0$$

$$r_E = G_N(1+\alpha)M\left[1 + \sqrt{1 - \frac{a^2\cos^2\theta}{G_N^2(1+\alpha)^2M^2} - \frac{\alpha}{1+\alpha}}\right] \quad g_{00} = 0$$

8. Black Hole Shadows (Silhouettes)

We shall take it as given that our modified gravity–Schwarzschild and modified gravity–Kerr black holes are characterized by only the two parameters mass M and angular momentum J. They are stationary and asymptotically flat solutions and they satisfy the "no-hair" theorem. An interesting consequence of these properties of the solution is that the shadow outline created by the black hole is determined by M and $a = J/M$ and the relative position of an asymptotic observer.

The black hole casts a shadow in front of an illuminated background in the asymptotically flat region and the shadow is determined by a set of closed photon orbits. A photon moving on a closed orbit with radius r in our modified gravity–Kerr spacetime with nonzero a and M has the apparent position in the (x,y) reference frame of a distant observer located in the angle of latitude θ

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The size of the photosphere is determined by

$$r_\gamma = r = \frac{3}{2} G_N (1+\alpha) M \left(1 + \sqrt{1 - \frac{8\alpha}{9(1+\alpha)}}\right)$$

$r_{\rm shad}$

$$= \frac{\left[3(1+\alpha) \pm \sqrt{(9+\alpha)(1+\alpha)}\right]^2}{\left\{4\left[(1+\alpha) \pm \sqrt{(9+\alpha)(1+\alpha)}\right]^2 - 16(1+\alpha)\right\}^{1/2}} G_N M .$$

The non-rotating shadow radius can be approximated by

$$r_{\rm shad} \sim (2.59 + 2\alpha) r_s$$

$$x = \frac{r\Delta + r\alpha G_N^2(1+\alpha)M^2 - G_N(1+\alpha)M(r^2 - a^2)}{a[r - G_N(1+\alpha)M]\sin\theta}$$

$$y = \left\{ \frac{4r^2\Delta}{[r - G_N(1+\alpha)M]^2} - (x + a\sin\theta)^2 \right\}^{1/2},$$

$$\Delta = r^2 - 2G_N(1+\alpha)Mr + a^2 + \alpha G_N^2(1+\alpha)M^2$$

For the non-rotating case $a = 0$ we have

$$\Delta = r^2 - 2G_N(1+\alpha)Mr + \alpha G_N^2(1+\alpha)M^2$$

and

$$x^2 + y^2 = \frac{r^4}{\Delta}.$$

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and

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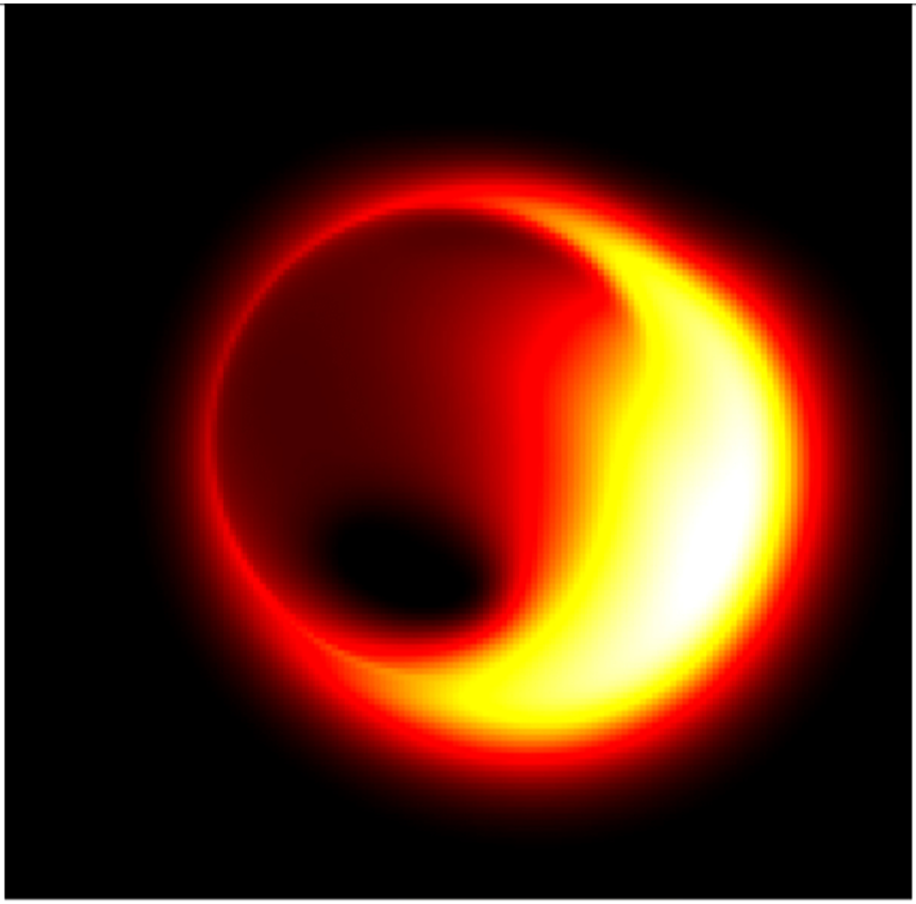
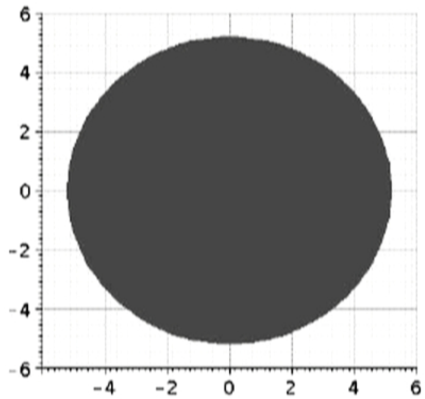
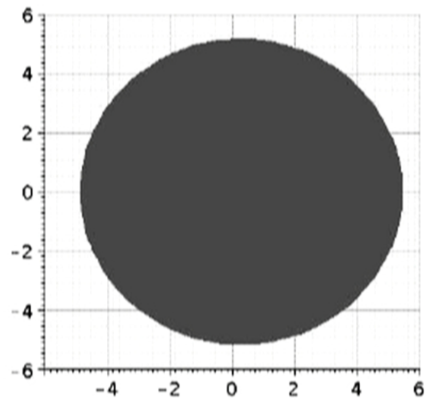
The South Pole Telescope. Credit: Daniel Luong-Van, National Science Foundation



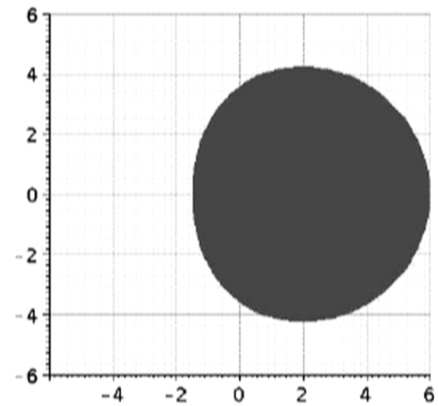
Computer-generated model of the Sagittarius A* black hole shadow. Image courtesy Avery Broderick at the Perimeter Institute for Theoretical Physics and the University of Waterloo



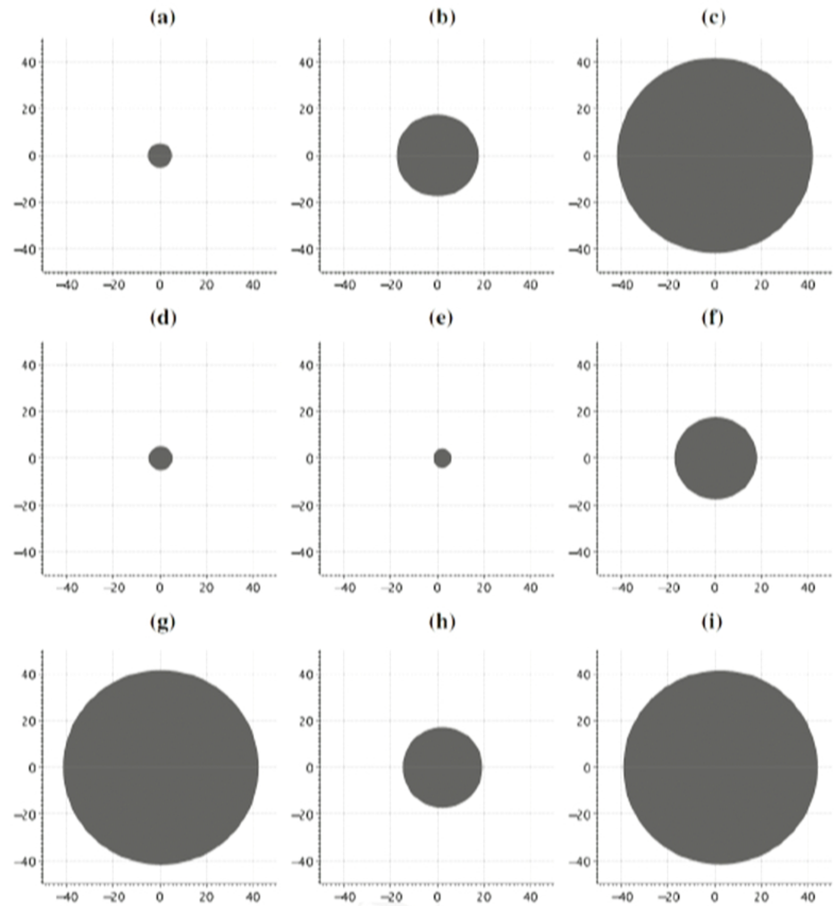
Schwarzschild BH a=0



Kerr BH with a=0.16



Kerr BH with a=0.95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9. Orbit of S2/SO-2 Star in Sagittarius

Strong gravity field prediction for star S2/SO-2 orbiting black hole Sgr*.

semimajor axis = 5.5 light days

eccentricity = 0.881 ± 0.007

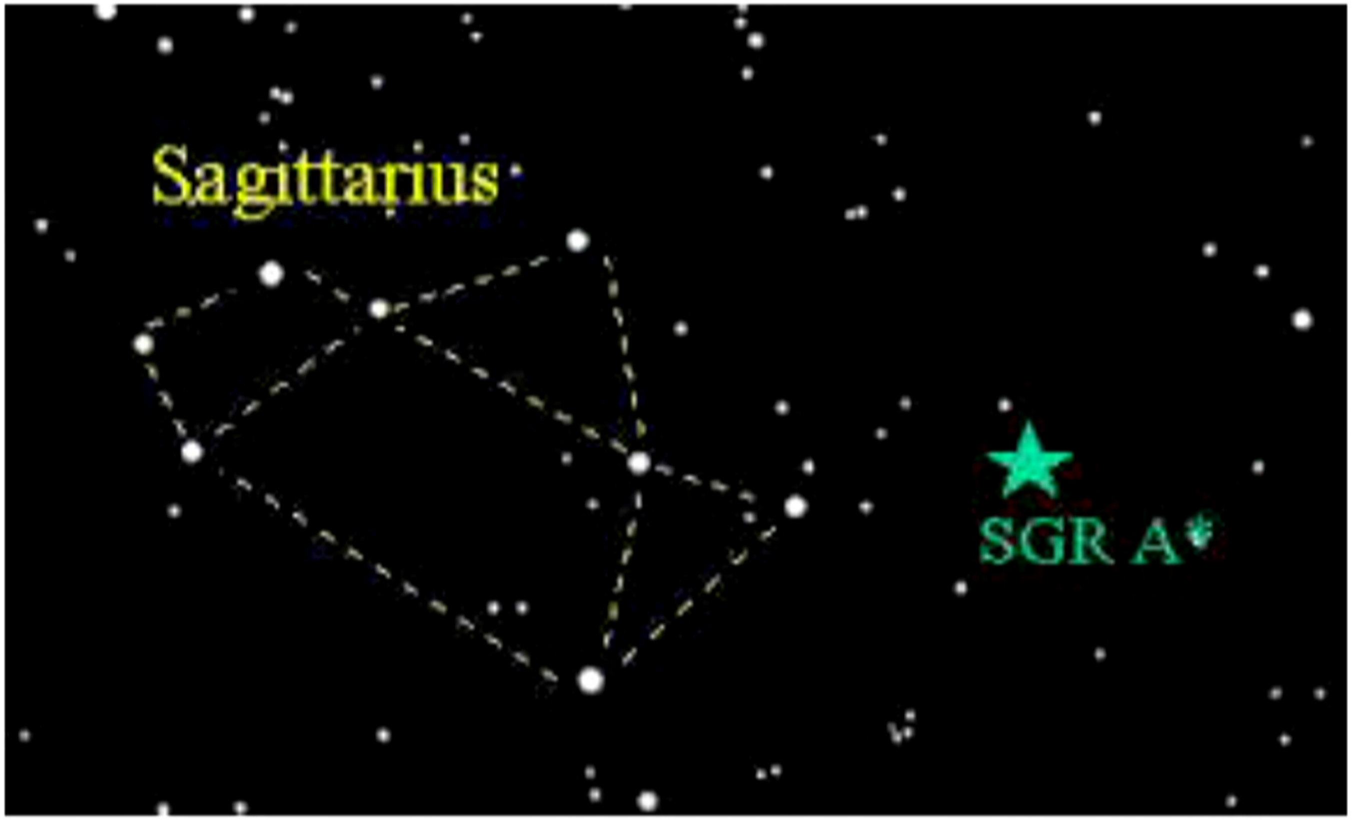
period = 15.56 ± 0.35 yr

Black Hole mass = $4.3 \times 10^6\, M_{SUN}$

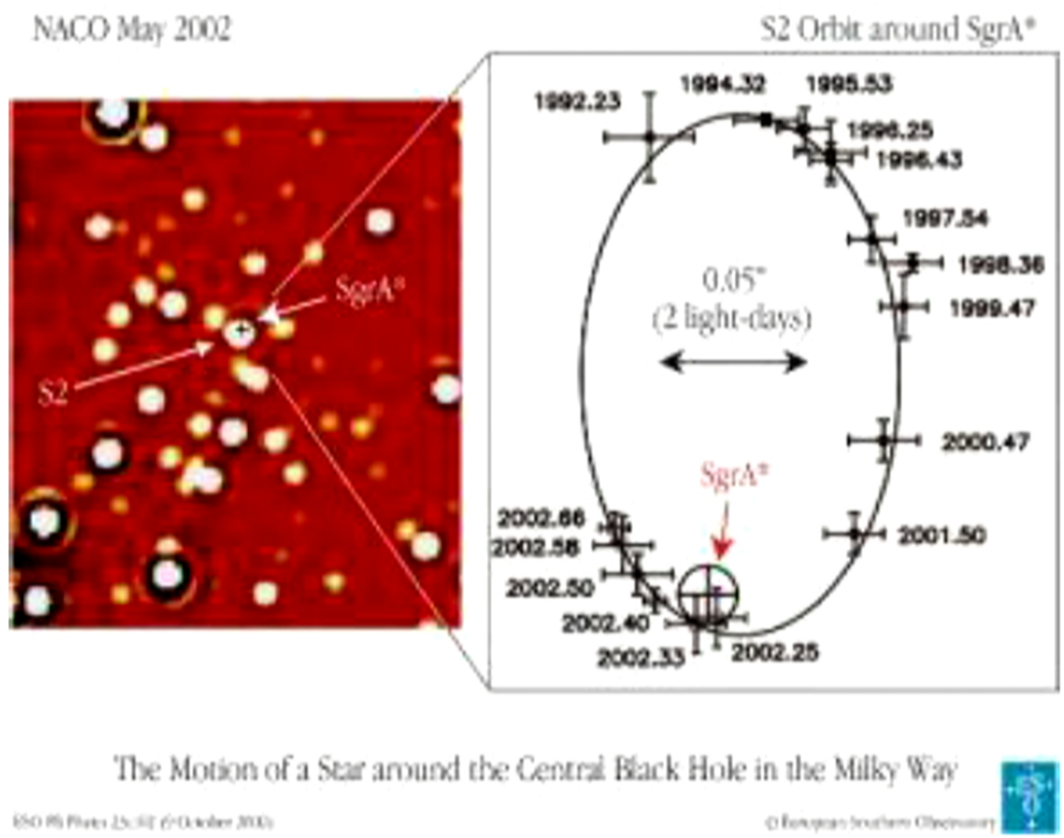
Periastron advance:

$\Delta\phi = (6\pi\, G_N\, M_{BH})/a(1-e^2) = 0.18^0/5$ yr

MOG predicts the same periastron advance as GR. The same holds true for the perihelion advance of Mercury: $\Delta\phi =$ 43 "/ century.



The center of the Milky Way sits just above the spout of the teapot.



The Motion of a Star around the Central Black Hole in the Milky Way

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10. MOG Wormhole Solution

A wormhole can be constructed using the Aichelberg-Schein solution:

F. Schein and P. C. Aichelburg, Phys. Rev. Lett., **77**, 4130 (1996), arXiv:gr-qc/9606069 (1996).

By using the MOG interior black hole metric with an outer horizon and the Majumda-Papapetrou spacetime metric, a traversable wormhole can be constructed. Because of the vector field repulsive gravitational force, the wormhole can be stable as a gateway to a distant part of the universe, or to another universe.

The field connected to two external spheres S1 and S2 is described by the Majumdar-Papapetrou metric. The MOG gravitational attraction balances the vector gravitational repulsion resulting in a stable wormhole throat, allowing an observer to traverse from one asymptotically flat region of spacetime to another distant one.

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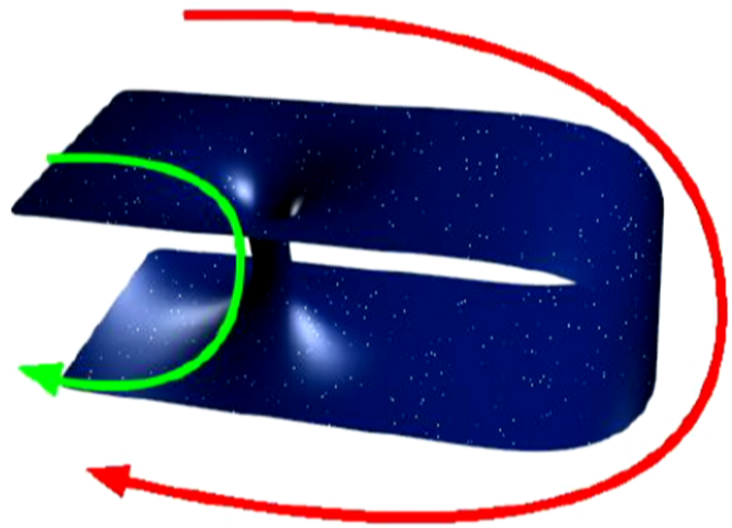

A MOG wormhole with a bridge connecting two asymptotically flat spacetimes (from Wikipedia).

1. Planck and WMAP cosmic microwave background (CMB) data:
   - Structure growth (stars and galaxies) ✓
   - Angular acoustical power spectrum ✓
   - Matter power spectrum. ✓
   - Accelerated expansion of the universe ✓
   - Galaxy rotation curves. Galactic cluster dynamics. ✓
2. Bullet Cluster 1E0657-558 . ✓
4. Abell 520 cluster "train wreck" collision. ?
5. Gravitational lensing in cosmology. ✓
6. Binary pulsar timing (PSR 1913+16). ✓
7. Solar system experiments. ✓
8. Strong gravity: Event Horizon Telescope and black holes. ?

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12. Conclusions

- The modified gravity (MOG) theory solves the problem of dark matter in early universe cosmology and late-time large scale structure galaxy and galaxy cluster dynamics. It provides a fully covariant gravitational action principle and field equations.
- The action principle contains, in addition to the metric Einstein-Hilbert action, a varying Newtonian constant G, a repulsive gravitational field ϕ_μ and an effective mass-range scalar field μ.
- The theory predicts structure growth, the expansion of the universe and the angular acoustical power spectrum in the CMB data. Galaxy and cluster dynamics are explained without detectable dark matter in the late-time universe. The predicted matter power spectrum can distinguish between MOG and ΛCDM. Galaxies do not possess a dark matter halo.
- Before recombination and before the formation of the first stars and galaxies, the gravitational strength vector field ϕ_μ is a cold dark matter ultralight paraphoton γ'. After this epoch, the mass decreases to $m_\phi = 2.6 \times 10^{-28}$ eV, modified gravity takes over and enhanced gravity explains the large scale structure of the late-time universe without dark matter.

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- The singular Schwarzschild-MOG and Kerr-MOG black holes have no naked singularity solutions for non-zero values of the parameter α. The sizes and shapes of the MOG black holes grow for increasing values of α.

- The VLBI and EHT astronomical observations with small enough micro arc - second resolution can distinguish between the Schwarzschild-MOG, Kerr-MOG black holes and their GR counterparts, testing GR for strong gravitational fields.

END