Title: From Black Holes to Quantum Transport

Date: Apr 22, 2015 02:00 PM

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Abstract: <p>In the last few years there has been significant interest in the possible applications of gauge-gravity duality to condensed matter systems. In this talk I will discuss recent applications of these holographic techniques to strongly correlated systems out of equilibrium. I will argue that insights from general relativity, hydrodynamics and quantum field theory may be combined to yield quantitative predictions for quantum transport.</p>

From Black Holes to Quantum Transport

Joe Bhaseen

TSCM Group
King's College London

Perimeter Institute
Waterloo, Canada
22nd April 2015


From Black Holes to Quantum Transport
PERIMETER
INSTITUTE

Strings, Cosmology & Condensed Matter

Benjamin Doyon **Koenraad Schalm** **Andy Lucas**



Ben Simons **Julian Sonner**

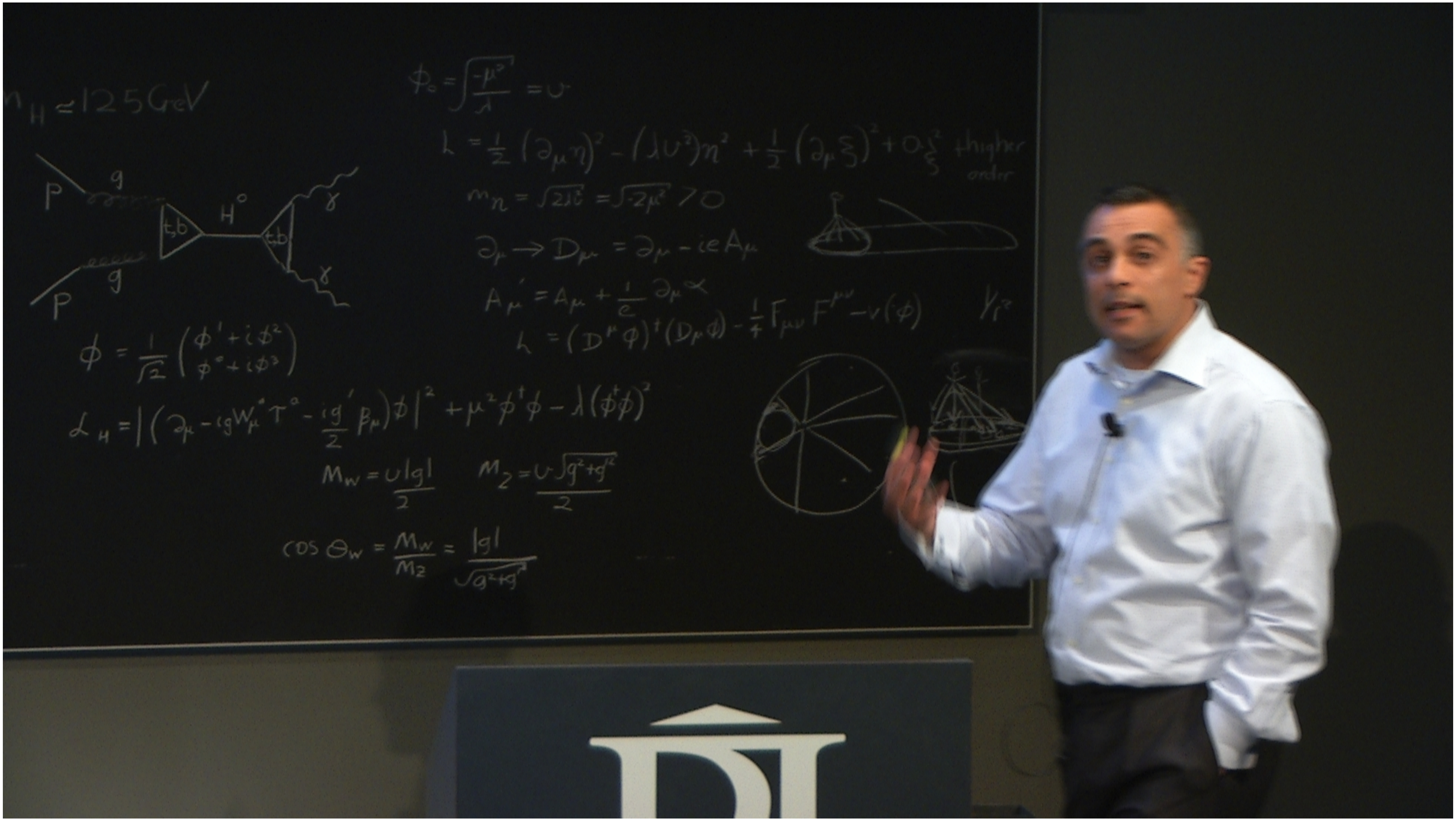

Jerome Gauntlett **Toby Wiseman**


King's, Leiden, Harvard, Cambridge, Imperial

Outline

- Motivation from condensed matter
- Gauge-gravity duality
- Far from equilibrium dynamics
- Recent work on energy flow
- Einstein equations, hydrodynamics, transport
- Current status and future developments

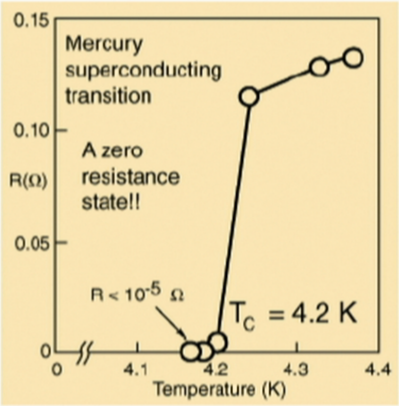
MJB, Benjamin Doyon, Andrew Lucas, Koenraad Schalm
"Far from equilibrium energy flow in quantum critical systems"
`arXiv:1311.3655`

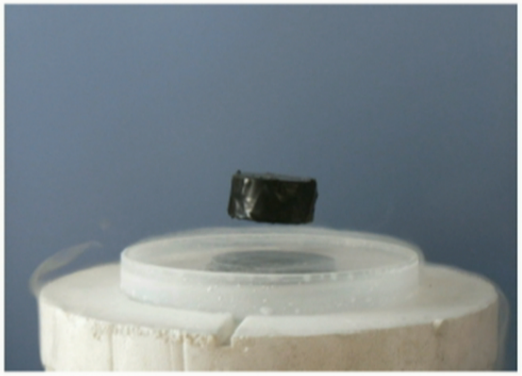

mH ≃ 125 GeV
H°
t,b
Mw = v|g| / 2
cos θw = Mw / Mz
higher order
PI

Condensed Matter Theory

Novel states of matter

Superconductor





http://hyperphysics.phy-astr.gsu.edu/hbase/solids/scex.html#c1

http://en.wikipedia.org/wiki/Superconductivity

Zero Resistance Kamerlingh Onnes (1911) $-269°C$

Phase Transition

"More is different"

10^{23}

$100,000,000,000,000,000,000,000$

Rich cooperative behaviour

Emergence

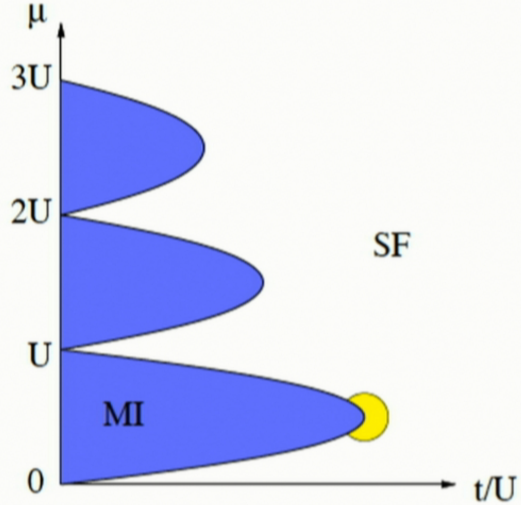
http://en.wikipedia.org/wiki/Philip_Warren_Anderson

Cooper Pairs **Fractional Charges** **No Quasi-Particles**

Strongly Correlated Systems

The Bose–Hubbard Model

$$H = -t\sum_{\langle ij\rangle}(a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2}\sum_i n_i(n_i-1) - \mu\sum_i n_i$$



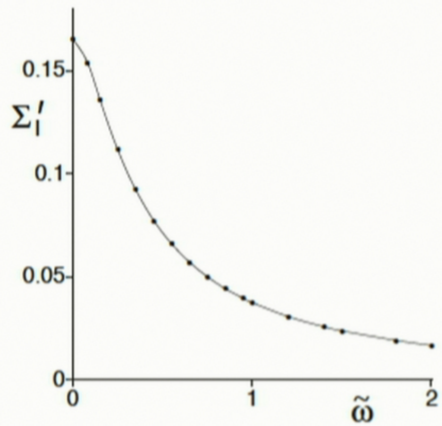
Fisher, Weichman, Grinstein and Fisher, PRB **40**, 546 (1989)

$$L = \int d^Dx\, |\partial_\mu \Phi|^2 - m^2|\Phi|^2 - \frac{u_0}{3}|\Phi|^4$$

Universal Transport

Damle and Sachdev, PRB **56**, 8714 (1997)

$$\sigma(\omega) = \frac{(2e)^2 T^{d-2}}{\epsilon^2} \Sigma\left(\frac{\omega}{\epsilon^2 T}\right) \qquad d = 3 - \epsilon$$



Two spatial dimensions $\sigma(0) \approx 1.037 \left(\frac{4e^2}{h}\right)$

Crossover between hydrodynamic and collisionless regimes

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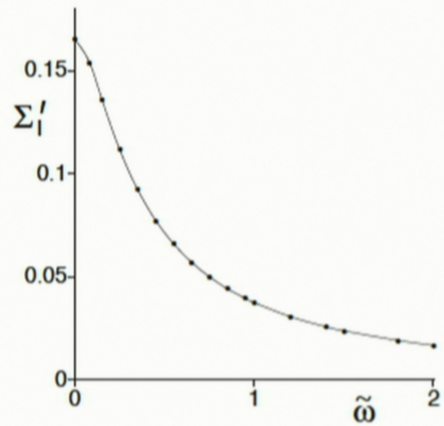

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Transport near Quantum Critical Points

Linear response for SF-MI transition in Bose–Hubbard

Damle and Sachdev, PRB **56**, 8714 (1997)

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MJB, Green and Sondhi, PRL **98**, 166801 (2007)

$$\alpha_{xy} = \frac{\mathcal{S}}{B} \qquad \bar{\kappa}_{xx} \simeq g\epsilon^2 \frac{T^{d+3}}{(2e)^2 B^2} \qquad g \approx 5.5$$

Hartnoll, Kovtun, Müller and Sachdev, PRB **76**, 144502 (2007)

$$\bar{\kappa}_{xx}(B) = \frac{TS^2}{B^2 \sigma_{xx}(0)}$$

Relativistic hydrodynamics & AdS/CFT link all coeffs

QBE   Müller *et al*, PRB (2008)   MJB *et al*, PRB (2009)

Viscosity/Entropy | **Quark Gluon Plasma** | **Graphene**

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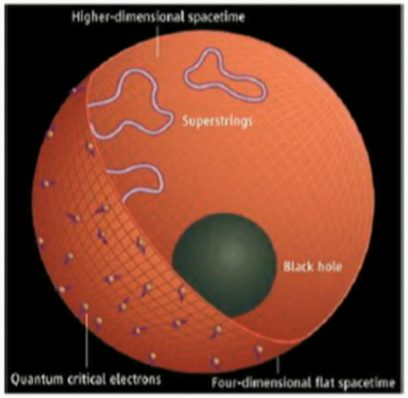
Gauge-Gravity Duality

AdS/CFT correspondence (Anti-de Sitter/Conformal Field Theory)

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)



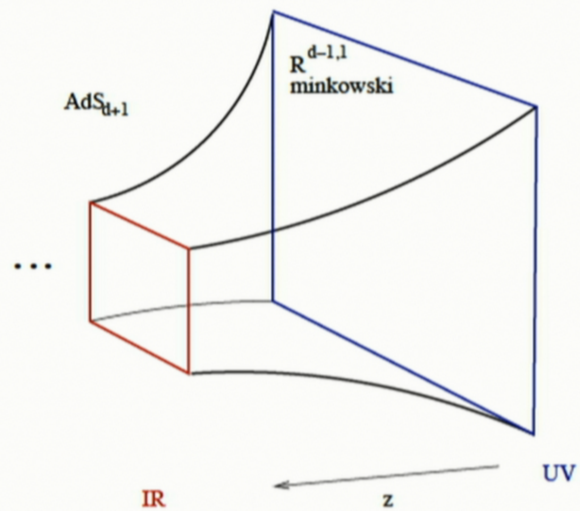
S. Hartnoll, Science **322**, 1639 (2008)

For an overview see for example John McGreevy, *Holographic duality with a view toward many body physics*, arXiv:0909.0518

Strong-Weak Duality

A different way of looking at things

AdS/CFT Correspondence



Gubser–Klebanov–Polyakov–Witten

$$Z[\phi_0]_{\mathrm{CFT}} \simeq e^{-S_{\mathrm{AdS}}[\phi]}|_{\phi\sim\phi_0 \; at \; z=0}$$

$$\phi(z) \sim z^{d-\Delta}\phi_0(1+\dots) + z^{\Delta}\phi_1(1+\dots)$$

Fields in AdS $\leftrightarrow$ operators in dual CFT $\quad \phi \leftrightarrow \mathcal{O}$

Progress in AdS/CMT

Anti-de Sitter/Condensed Matter Theory

Transport Coefficients

Viscosity, Conductivity, Hydrodynamics, Bose–Hubbard, Graphene

Strange Metals

Non-Fermi liquids, instabilities, cuprates

Holographic Duals

Superfluids, Fermi Liquid, $O(N)$, Luttinger Liquid

Equilibrium or close to equilibrium

Non-Equilibrium High Energy Physics

Thermalization in Strongly Coupled Gauge Theories

Chesler & Yaffe (2009), de Boer & Keski Vakkuri (2011), ...

Buchel, Lehner, & Myers, *"Thermal quenches in $N = 2^*$ plasmas"*, JHEP 1208 (2012)

Buchel, Heller, & Myers, *"Equilibration rates in a strongly coupled nonconformal quark-gluon plasma"*, `arXiv:1503.07114`

Quantum Quenches

Aparício & López (2011), Basu & Das (2012), ...

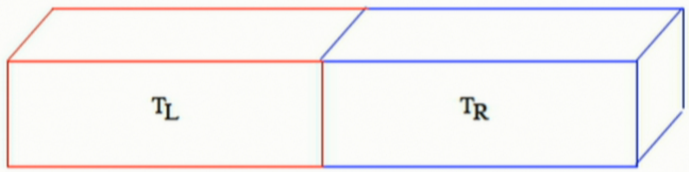
Buchel, Lehner, Myers, & van Niekerk, *"Quantum quenches of holographic plasmas"*, `arXiv:1302.2924`

Das, Galante, & Myers *"Universality in fast quantum quenches"*, `arXiv:1411.7710`

Hydrodynamics

Thermalization

Condensed matter and high energy physics

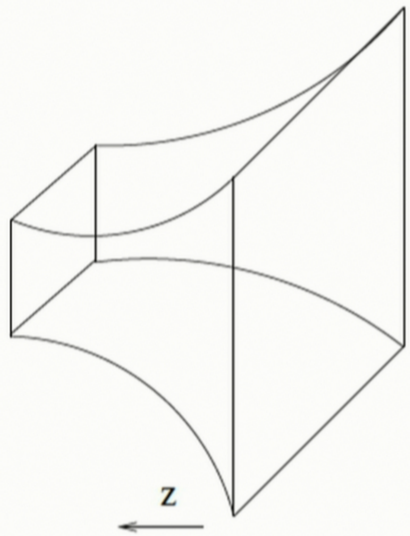


Why not connect two strongly correlated systems together and see what happens?

AdS/CFT

Energy flow may be studied within pure Einstein gravity

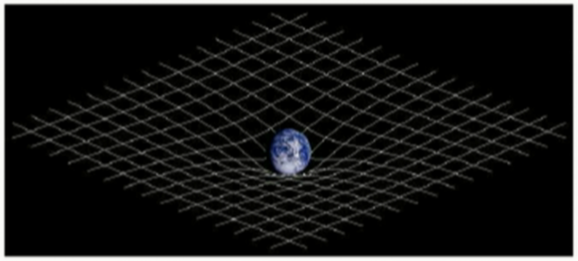
$$S = \frac{1}{16\pi G}\int d^{d+2}x\,\sqrt{-g}(R-2\Lambda)$$



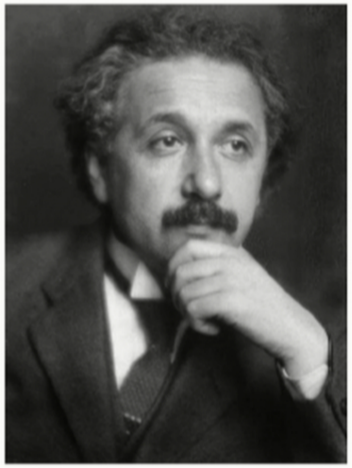
$$g_{\mu\nu} \leftrightarrow T_{\mu\nu}$$

Einstein Centenary

The Field Equations of General Relativity (1915)



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$



http://en.wikipedia.org/wiki/Einstein_field_equations

$g_{\mu\nu}$ metric $R_{\mu\nu}$ Ricci curvature R scalar curvature

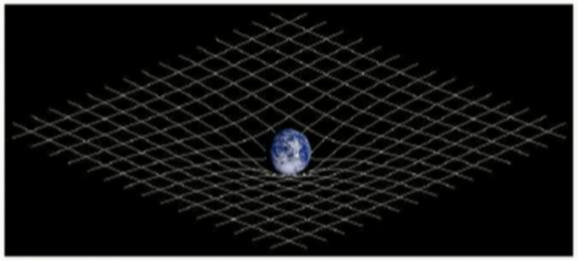
Λ cosmological constant $T_{\mu\nu}$ energy-momentum tensor

Coupled Nonlinear PDEs

Schwarzchild Solution (1916) $R_{\mathrm{S}} = \frac{2MG}{c^2}$ **Black Holes**

Einstein Centenary

The Field Equations of General Relativity (1915)



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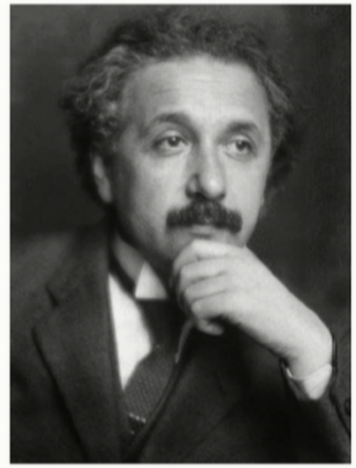

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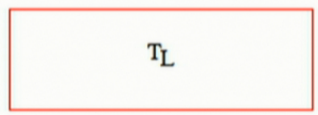
Schwarzchild Solution (1916) $R_S = \frac{2MG}{c^2}$ **Black Holes**

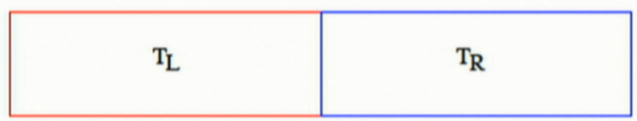
Non-Equilibrium CFT

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

Two critical 1D systems (central charge c) at temperatures T_L & T_R



Join the two systems together



Alternatively, take one critical system and impose a step profile

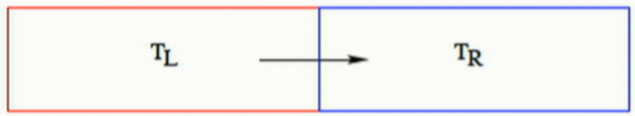
Local Quench

Steady State Energy Flow

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45** 362001 (2012)

If systems are very large ($L \gg vt$) they act like heat baths

For times $t \ll L/v$ a steady energy current flows



Non-equilibrium steady state

$$J = \frac{c\pi^2 k_B^2}{6h}(T_{\rm L}^2 - T_{\rm R}^2)$$

Universal result out of equilibrium

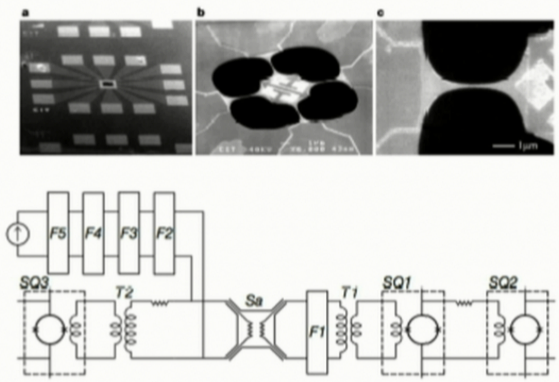
Direct way to measure central charge; velocity doesn't enter

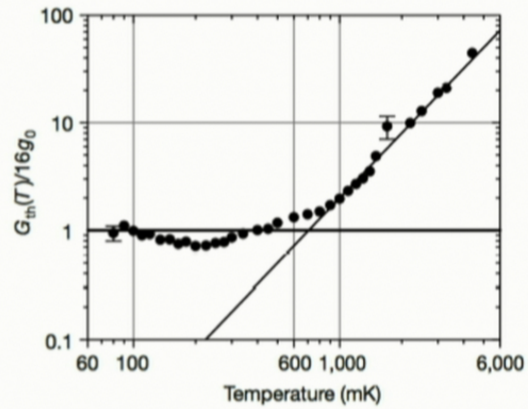
Sotiriadis and Cardy. J. Stat. Mech. (2008) P11003

c **doesn't have to be $\in \mathbb{Z}$** **Emergence** **Stefan–Boltzmann**

Experiment

Schwab, Henriksen, Worlock and Roukes, *Measurement of the quantum of thermal conductance*, Nature **404**, 974 (2000)





Quantum of Thermal Conductance

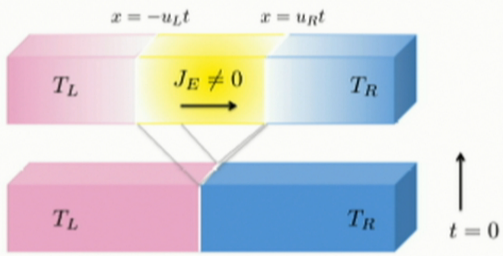
Numerical Simulations

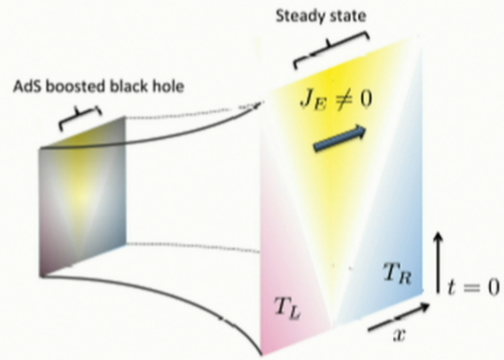
Time-dependent Density Matrix Renormalization Group (DMRG)

Karrasch, Ilan and Moore, Phys. Rev. B **88**, 195129 (2013)

Quantum Information and Entanglement

AdS/CFT





Steady State Region

Spatially Homogeneous

Boost Solution

Lorentz boosted stress tensor of a finite temperature CFT

Perfect fluid

$$\langle T^{\mu\nu}\rangle_{\rm s} = a_d T^{d+1}\left(\eta^{\mu\nu} + (d+1)u^\mu u^\nu\right)$$

$$\eta^{\mu\nu} = \mathrm{diag}(-1, 1, \cdots, 1) \qquad u^\mu = (\cosh\theta, \sinh\theta, 0, \ldots, 0)$$

One spatial dimension

$$a_1 = \frac{L\pi}{4G} \qquad c = \frac{3L}{2G}$$

$$T_{\rm L} = Te^{\theta} \qquad T_{\rm R} = Te^{-\theta} \qquad \langle T_{tx}\rangle = \frac{c\pi^2 k_B^2}{6h}(T_{\rm L}^2 - T_{\rm R}^2)$$

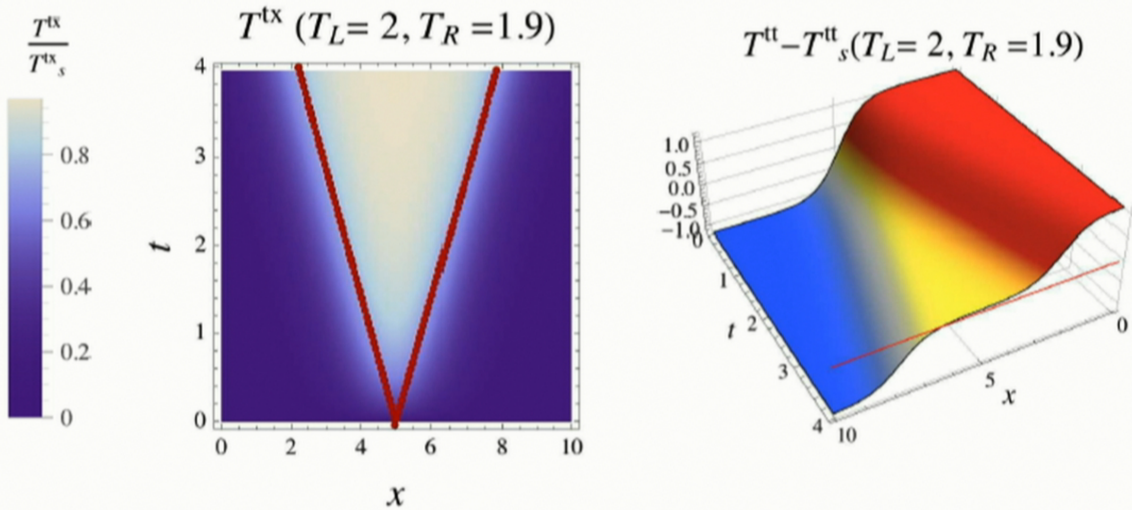
Can also obtain complete steady state density matrix

Describes all the cumulants of the energy transfer process

The non-equilibrium steady state (NESS)
is a Lorentz boosted thermal state

Run past a thermal state with temperature $T = \sqrt{T_L T_R}$

Numerics I



Excellent agreement with predictions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