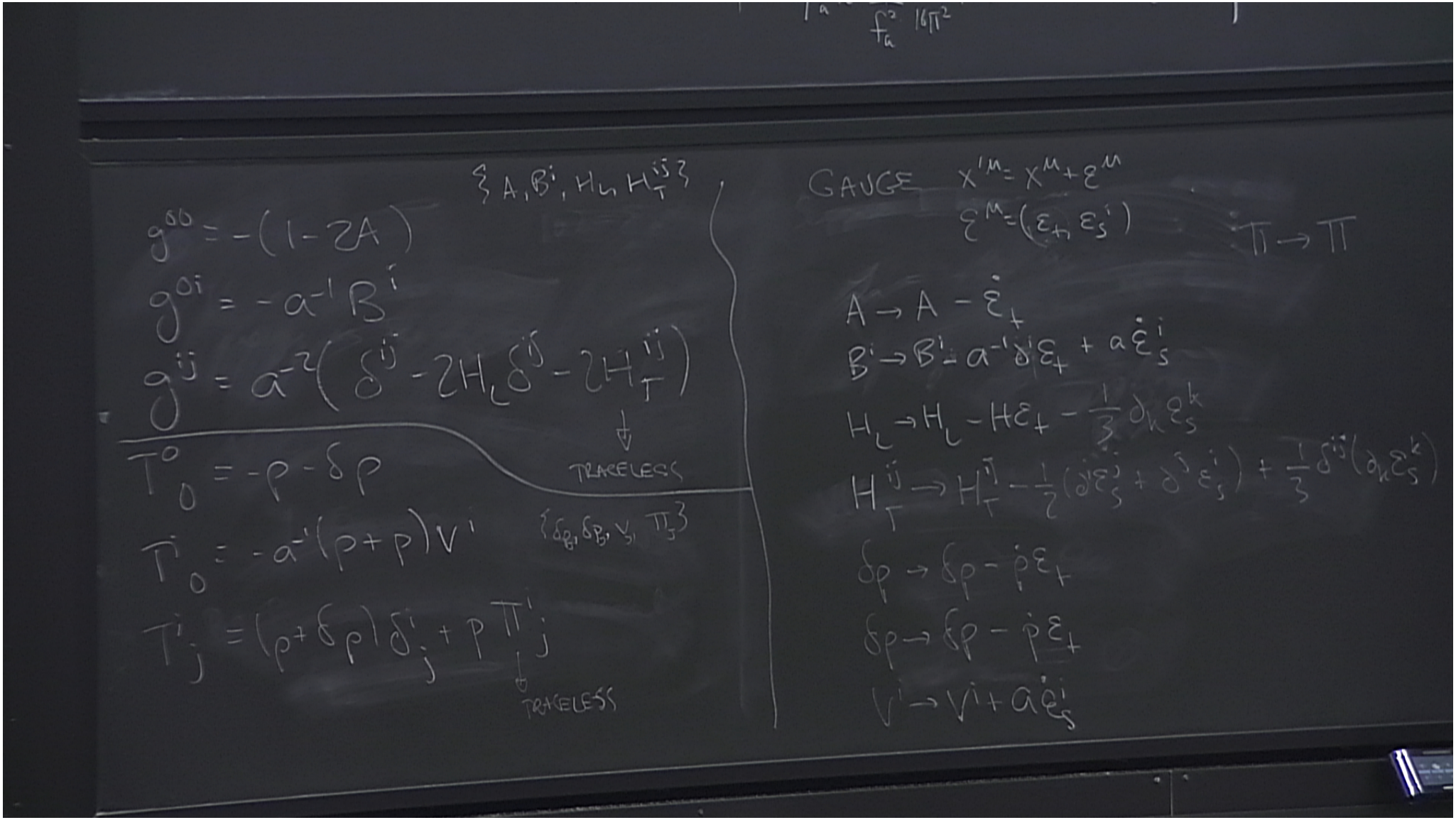
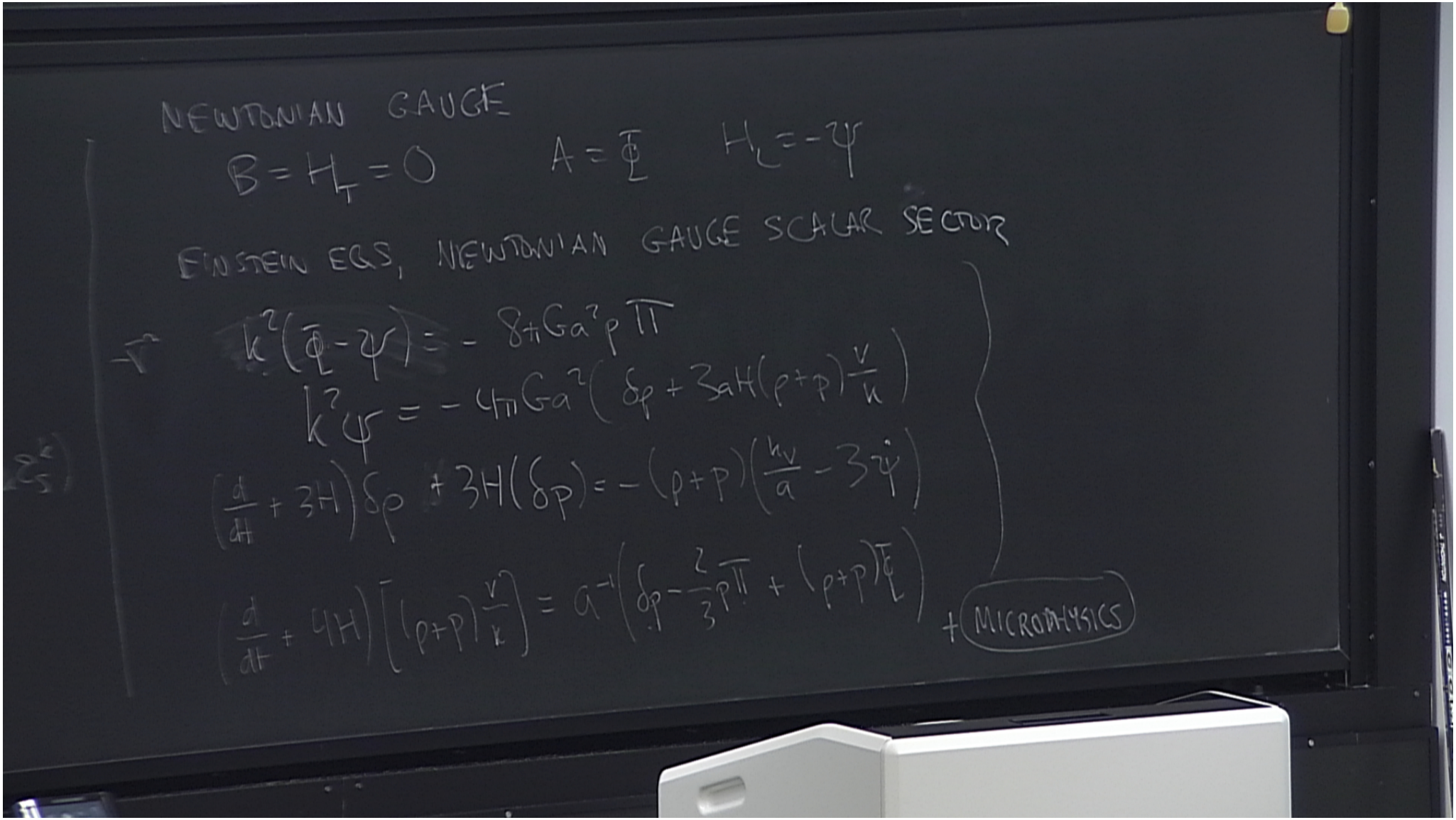
Title: Explorations in Cosmology-14

Date: Apr 23, 2015 10:15 AM

URL: http://pirsa.org/15040042

Abstract:


$\frac{1}{f_a^2}\frac{1}{16\pi^2}$
$\{A, B^i, H_L, H_T^{ij}\}$
$g^{00} = -(1-2A)$
$g^{0i} = -a^{-1}B^i$
$g^{ij} = a^{-2}(\delta^{ij} - 2H_L\delta^{ij} - 2H_T^{ij})$
TRACELESS
$T^0_{\ 0} = -\rho - \delta\rho$
$T^i_{\ 0} = -a^{-1}(p+\rho)v^i$
$T^i_{\ j} = (p+\delta p)\delta^i_{\ j} + p\,\Pi^i_{\ j}$
TRACELESS
$\{\delta_B, \delta_B, v_s, \Pi_s\}$
GAUGE $x'^\mu = x^\mu + \varepsilon^\mu$
$\varepsilon^\mu = (\varepsilon_+, \varepsilon_s^i)$
$\Pi \to \Pi$
$A \to A - \dot{\varepsilon}_+$
$B^i \to B^i - a^{-1}\partial^i\varepsilon_+ + a\dot{\varepsilon}_s^i$
$H_L \to H_L - H\varepsilon_+ - \frac{1}{3}\partial_k\varepsilon_s^k$
$H_T^{ij} \to H_T^{ij} - \frac{1}{2}(\partial^i\varepsilon_s^j + \partial^j\varepsilon_s^i) + \frac{1}{3}\delta^{ij}(\partial_k\varepsilon_s^k)$
$\delta\rho \to \delta\rho - \dot{\rho}\varepsilon_+$
$\delta\rho \to \delta\rho - \dot{\rho}\varepsilon_+$
$v^i \to v^i + a\dot{\varepsilon}_s^i$


NEWTONIAN GAUGE
$B = H_T = 0$ $A = \Phi_L$ $H_L = -\psi$
EINSTEIN EQS, NEWTONIAN GAUGE SCALAR SECTOR
$k^2(\Phi_L - \psi) = -8\pi G a^2 \rho \Pi$
$k^2\psi = -4\pi G a^2 \left(\delta\rho + 3aH(\rho+p)\frac{v}{k}\right)$
$\left(\frac{d}{dt} + 3H\right)\delta\rho + 3H(\delta p) = -(\rho+p)\left(\frac{kv}{a} - 3\dot{\psi}\right)$
$\left(\frac{d}{dt} + 4H\right)\left[(\rho+p)\frac{v}{k}\right] = a^{-1}\left(\delta p - \frac{2}{3}p\Pi + (\rho+p)\Gamma_L\right)$
+ MICROPHYSICS

WEINBERG'S THEOREM: FOR ANY "REASONABLE" EQUATIONS OF MOTION, THE SCALAR SECTOR EQS HAVE A "ADIABATIC" SOLUTION

$$\left(\delta\bar{q}_I\right)_k = -\left(1-\alpha(t)\right)\lambda_k + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\alpha(t) = \frac{H(t)}{a(t)}\int_{t_0}^{t} dt'\, a(t') \approx \frac{2}{5+3w}$$

i) LET'S LOOK FOR A GAUGE SYMMETRY (ξ^t, ξ^i_s)

$\delta H_T^{ij} = \delta B = 0$ (NEWTONIAN GAUGE PRESERVED)

SHIFTS IN $\Phi, \Psi, \delta\rho, \delta p, v, \Pi$ ARE FUNCTIONS OF t [AND x]

CLAIM: THE MOST GENERAL SUCH GAUGE TRANSFORMATION IS

$$\xi^t = \xi^t(t)$$

$$\xi^i_s = -\lambda x^i + \underbrace{\omega^{ij}x^j + c^i}_{\text{"UNINTERESTING"}} \qquad [\omega^{ij} \text{ ANTISYMMETRIC}]$$

λ: CONSTANT

$\frac{1}{f_a^2}\frac{1}{16\pi^2}$

$\{A, B^i, H_L, H_T^{ij}\}$

$g^{00} = -(1-2A)$

$g^{0i} = -a^{-1}B^i$

$g^{ij} = a^{-2}(\delta^{ij} - 2H_L\delta^{ij} - 2H_T^{ij})$

↓ TRACELESS

$T^0_{\ 0} = -\rho - \delta\rho$

$T^i_{\ 0} = -a^{-1}(\rho+p)V^i$ $\quad \{\delta_B, \delta_B, V_S, \Pi_S\}$

$T^i_{\ j} = (p+\delta p)\delta^i_{\ j} + p\,\Pi^i_{\ j}$

↓ TRACELESS

GAUGE $x'^\mu = x^\mu + \varepsilon^\mu$

$\varepsilon^\mu = (\varepsilon_t, \varepsilon_S^i)$ $\quad \Pi \to \Pi$

$A \to A - \dot{\varepsilon}_t$

$B^i \to B^i - a^{-1}\partial^i\varepsilon_t + a\dot{\varepsilon}_S^i$

$H_L \to H_L - H\varepsilon_t - \frac{1}{3}\partial_k\varepsilon_S^k$

$H_T^{ij} \to H_T^{ij} - \frac{1}{2}(\partial^i\varepsilon_S^j + \partial^j\varepsilon_S^i) + \frac{1}{3}\delta^{ij}(\partial_k\varepsilon_S^k)$

$\delta\rho \to \delta\rho - \dot{\rho}\varepsilon_t$

$\delta\rho \to \delta\rho - \dot{\rho}\varepsilon_t$

$V^i \to V^i + a\dot{\varepsilon}_S^i$

H_T EQ:

$$\partial^j \varepsilon_S^k + \partial^k \varepsilon_S^j - \frac{2}{3}\delta^{jk}(\partial_k \varepsilon_S^k) = 0$$

$$\underbrace{\partial^i \partial^j \varepsilon_S^k}_{A^{ijk}} + \partial^i \partial^k \varepsilon_S^j = 0$$

$$A^{ijk} = -A^{ijk}$$

$\xi_+ = \xi_+(t)$ FROM δ_ρ [OR δ_P]

H_T EQ:

$\Rightarrow \xi^i_S = \xi^i_S(x)$ FROM B

$\Rightarrow \partial_k \xi^k_S = \text{CONST.}$ FROM H_L

$$g_{ke}\Gamma^{e}_{ij} = \frac{1}{2}\partial_i g_{jk} + \frac{1}{2}\partial_j g_{ik} - \frac{1}{2}\partial_k g_{ij}$$

$$A_{kij} + A_{ikj} = \partial_j g_{ik}$$

H_T EQ: $\partial^j \varepsilon_S^k + \partial^k \varepsilon_S^j - \frac{2}{3}\delta^{ij}(\partial_k \varepsilon_S^k) = 0$

$\underbrace{\partial^i \partial^j \varepsilon_S^k}_{A^{ijk}} + \partial^i \partial^k \varepsilon_S^j = 0$

$\partial^i \partial^j \varepsilon_S^k = 0 \Rightarrow \varepsilon_S^i = A^{ij} x^j + C^i$ → CONSTANT

PLUG-INTO (★): $A_{ij} + A_{ji} - \frac{2}{3}\delta_{ij} A_{kk} = 0$

$\Rightarrow A_{ij} = \omega_{ij} + \lambda \delta_{ij}$ ω_{ij} A...

$\xi_+ = \xi_+(t)$ FROM $\delta\rho$ [OR δp]

H_T EQ:

$\Rightarrow \xi^i_S = \xi^i_S(x)$ FROM B

$\Rightarrow \partial_k \xi_S^k = \text{CONST.}$ FROM H_L

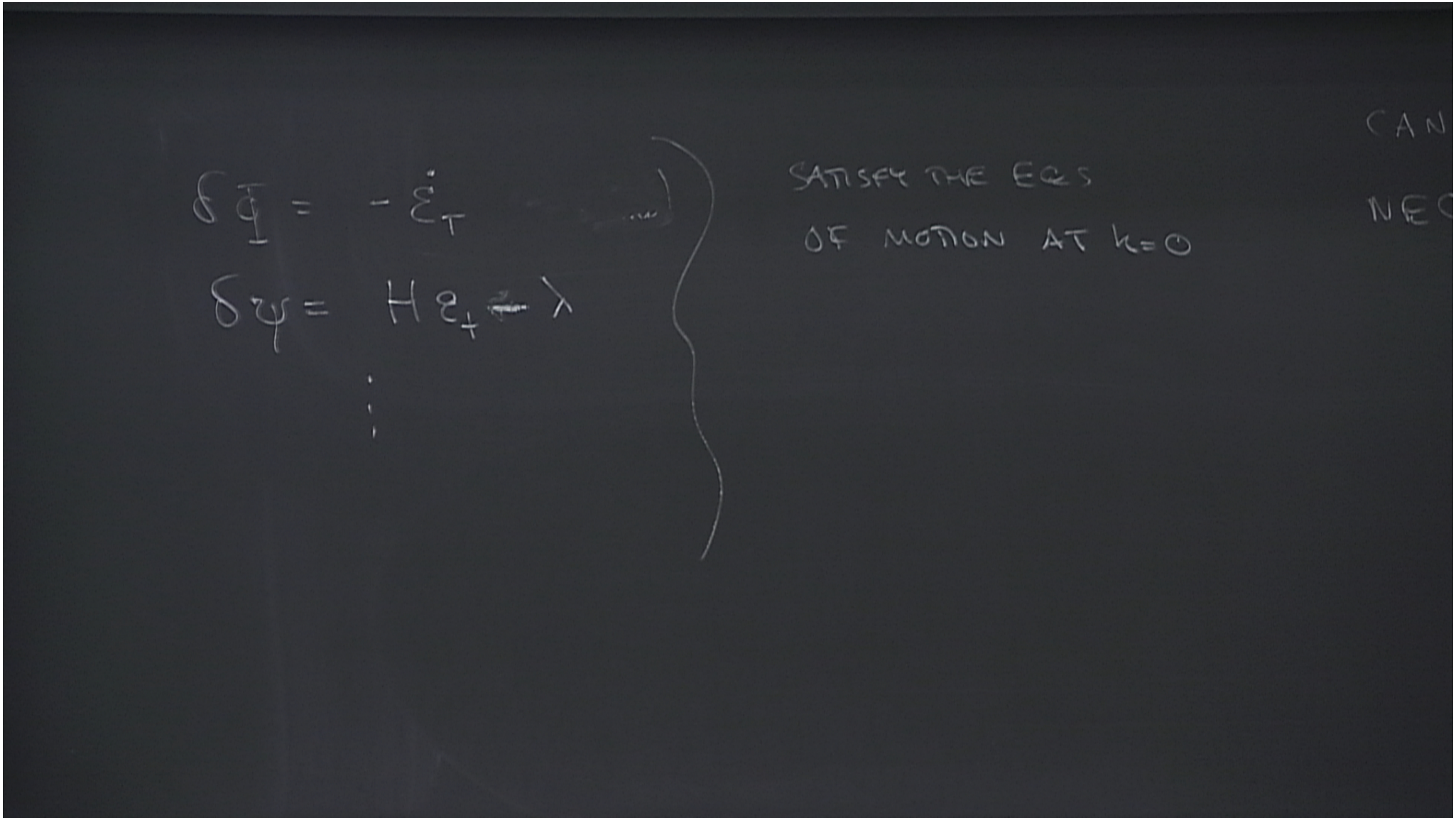
$\Rightarrow \xi^i_S = \omega_{ij} x^j = \lambda x^i + c^i$ ∂^i

$$\delta \tilde{q} = -\dot{\xi}_T \quad \ldots)$$

$$\delta\psi = H\xi_+ + \lambda$$

$$\vdots$$

SATISFY THE EQS OF MOTION AT $k=0$


$\delta\tilde{q} = -\dot{\xi}_T$ ...
$\delta\psi = H\xi_t - \lambda$
:
SATISFY THE EQS
OF MOTION AT $k=0$
CAN
NE

CAN WE EXTEND TO $k \neq 0$?

NECESSARY CONDITION: $\Phi = \Psi$ TO LOWEST ORDER IN $\frac{k^2}{a(t)^2}$

$$\dot{\xi}_+ + H \xi_+ = \lambda$$

$$a^{-1} \frac{d}{dt}\left(a \xi_+\right) = \lambda$$

$$\xi_+(t) = \frac{\lambda}{a(t)} \int_{-\infty}^{t} dt'\, a(t') = \frac{\lambda}{H(t)}\, \alpha(t)$$

ORDER IN $\frac{k^2}{aH^2}$

$$\delta\bar{q} = -(1-\alpha(t))\,\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta\varphi = -(1-\alpha(t))\,\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta\rho_J = -\dot{\rho}_J \frac{\alpha(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta p_J = -\dot{p}_J \frac{\alpha(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta v_J = 0$$

?

EST ORDER IN $\frac{k^2}{a^2H^2}$

$$\delta\bar{q} = -(1-\alpha(t))\,\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta\psi = -(1-\alpha(t))\,\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$\alpha(t)$

$$\delta\rho_J = -\dot{\rho}_J \frac{\alpha(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta\rho_J = -\dot{\rho}_J \frac{\alpha(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$v_J = -\frac{\delta\rho_J}{3(\rho_J+\rho_{J-1})}\frac{k}{aH} = \frac{\dot{\rho}_J}{3(\rho_J+\rho_J)}\frac{\alpha(t)}{H(t)}\left(\frac{k}{aH}\right) + \mathcal{O}\left(\frac{k^3}{a^3H^3}\right)$$

?

ORDER IN $\frac{k^2}{a^2H^2}$

$$\delta\bar{q} = -(1-\alpha(t))\,\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta\psi = -(1-\alpha(t))\,\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta\rho_J = -\dot{\rho}_J \frac{\alpha(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$\delta p_J = -\dot{p}_J \frac{\alpha(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$v_J = -\frac{\delta p_J}{3(\rho_J+p_J)}\frac{k}{aH} = \frac{\dot{p}_J}{3(\rho_J+p_J)}\frac{\alpha(t)}{H(t)}\left(\frac{k}{aH}\right)\lambda + \mathcal{O}\left(\frac{k^3}{a^3H^3}\right)$$

?

ST ORDER IN $\frac{k^2}{a^2H^2}$

$$(\delta q)_k = -(1-\alpha(t))\,\zeta_k + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$(\delta\psi)_k = -(1-\alpha(t))\,\zeta_k + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$\alpha(t)$

$$(\delta\rho_J)_k = -\dot\rho_J \frac{\alpha(t)}{H(t)}\zeta_k + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$$(\delta\rho_J)_k = -\dot\rho_J \frac{\alpha(t)}{H(t)}\zeta_k + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

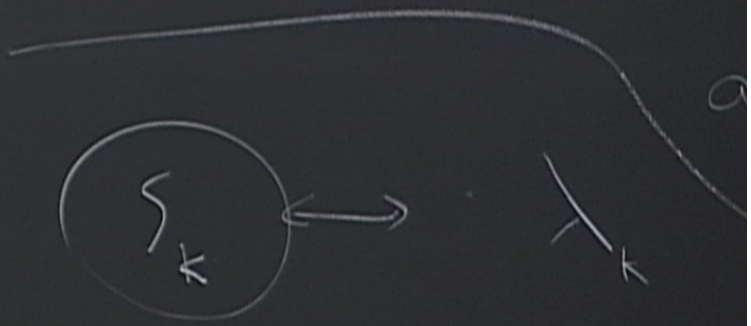
$$(v_J)_k = -\frac{\delta\rho_J}{3(\rho_J+\rho_{J-1})}\frac{k}{aH} = \frac{\dot\rho_J}{3(\rho_J+\rho_J)}\frac{\alpha(t)}{H(t)}\left(\frac{k}{aH}\right)\zeta_k + \mathcal{O}\left(\frac{k^3}{a^3H^3}\right)$$

?

$\delta q = -\dot{\varepsilon}_T$ ...

$\delta\psi = H\varepsilon_T - \lambda$

SATISFY THE EQS OF MOTION AT $k=0$

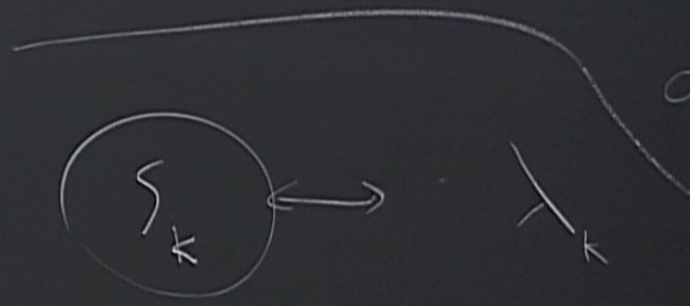


$\delta\bar{q} = -\dot{\xi}_T$ ...

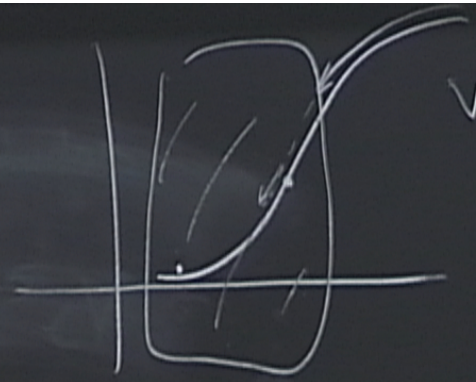
$\delta\psi = H\xi_T - \dot{\lambda}$

SATISFY THE EQS OF MOTION AT $k=0$





REHEATING



LET'S THINK ABOUT A FREE MASSIVE SCALAR FIELD IN FRW

$$S = \frac{1}{2}\int \sqrt{-g}\left(-\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2\right)$$

WITH NO SPATIAL FLUCTUATIONS: $\phi = \phi(t)$

$a = a(t)$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

EOM: $\ddot{\phi} + 3H\dot{\phi} + m^2\phi^2$

$$H^2 = H^2_{\text{OTHER STUFF}} + \underbrace{\frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right)}_{H_\phi^2}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$$

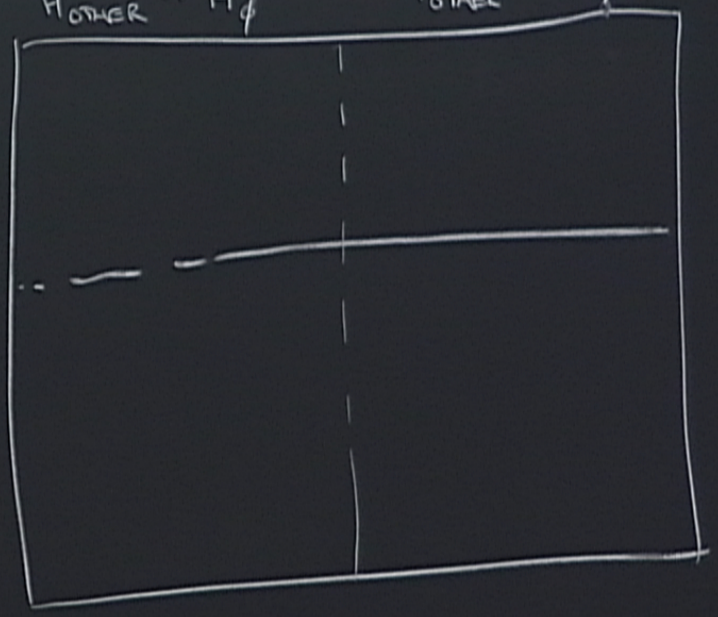
$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

EOM: $\ddot{\phi} + 3H\dot{\phi} + m^2\phi^2$

$$H^2 = H^2_{\text{OTHER STUFF}} + \underbrace{\frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right)}_{H_\phi^2}$$

$H \gg m$

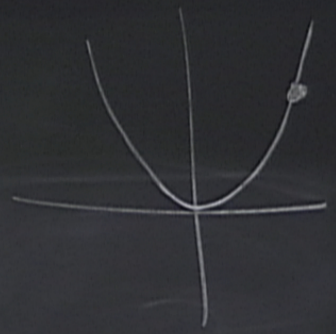
$H \ll m$



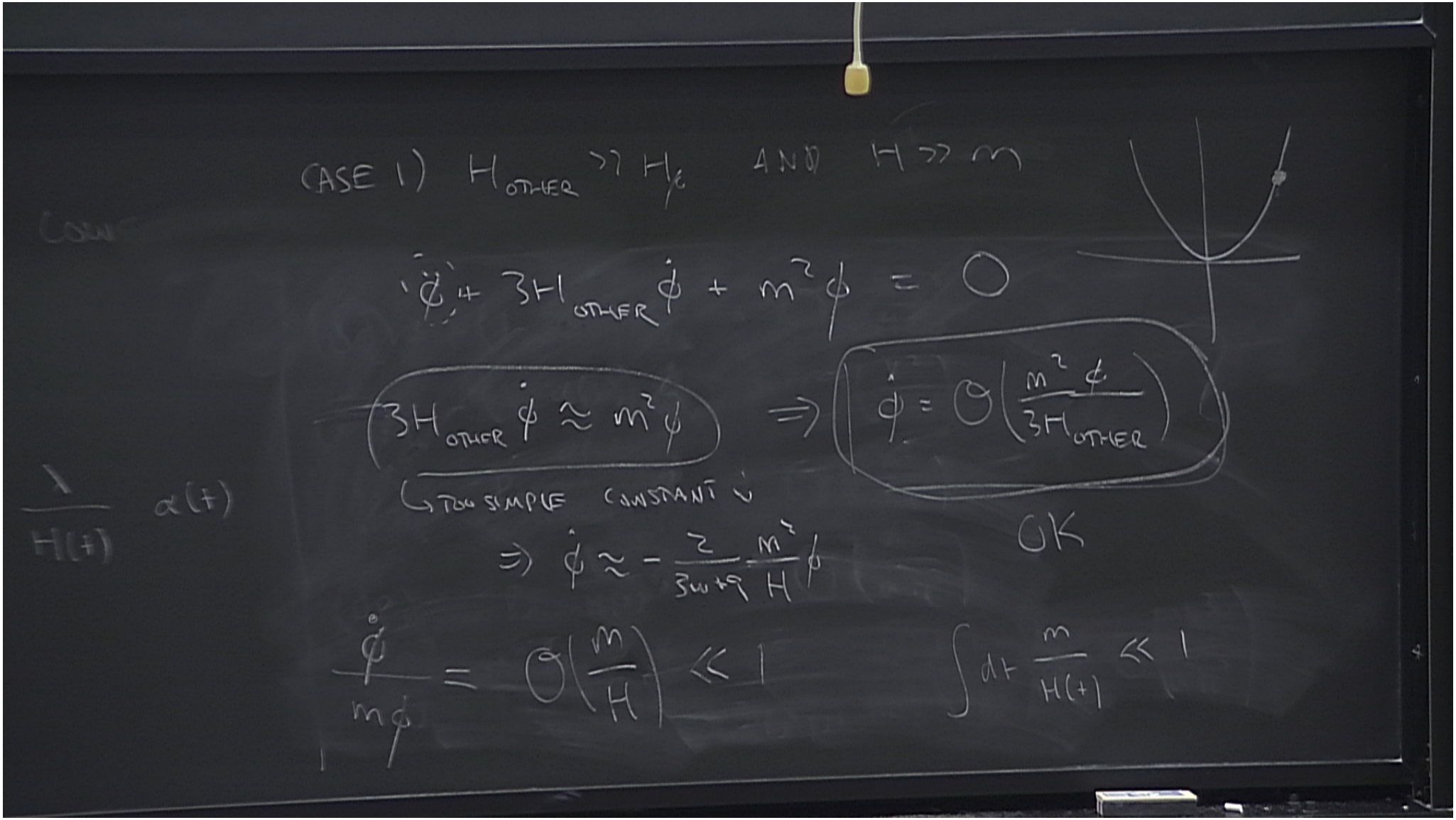
CASE 1) $H_{OTHER} \gg H_\phi$ AND $H \gg m$

$$\ddot{\phi} + 3H_{OTHER}\dot{\phi} + m^2\phi = 0$$

$$3H_{OTHER}\dot{\phi} \approx m^2\phi \quad \Rightarrow \quad \dot{\phi} = \mathcal{O}\left(\frac{m^2\phi}{3H_{OTHER}}\right)$$



$\frac{1}{H(t)}$ $a(t)$


CASE 1) $H_{OTHER} \gg H_\phi$ AND $H \gg m$
$\ddot{\phi} + 3H_{OTHER}\dot{\phi} + m^2\phi = 0$
$3H_{OTHER}\dot{\phi} \approx m^2\phi \Rightarrow \dot{\phi} = \mathcal{O}\left(\frac{m^2\phi}{3H_{OTHER}}\right)$
TOO SIMPLE CONSTANT
OK
$\Rightarrow \dot{\phi} \approx -\frac{2}{3w+9}\frac{m^2}{H}\phi$
$\frac{\dot{\phi}}{m\phi} = \mathcal{O}\left(\frac{m}{H}\right) \ll 1$
$\int dt \frac{m}{H(t)} \ll 1$
$\frac{1}{H(t)}$ $a(t)$