Title: Explorations in Cosmology-13

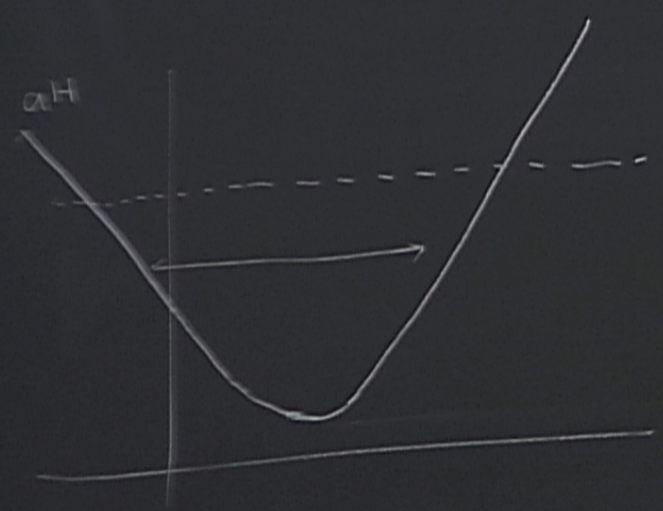
Date: Apr 22, 2015 10:15 AM

URL: http://pirsa.org/15040041

Abstract:

GOAL: WEINBERG'S THEOREM

WEINBERG, ASTRO-PH/0302326



SVT DECOMPOSITION

CONSIDER A SYSTEM LINEAR DIFF EQS WITH ROT+TRANS SYMMETRY

SCALARS $A(x), B(x), \ldots$

3-VECTORS $V_i(x), \ldots$

SYMM. 3-TENSORS $S_{ij}(x), \ldots$

TRANSLATION INVARIANCE $\Rightarrow$ k-M

$\partial_i \to ik_i$

E.G.

$$\dot{A} = \partial^2 B + \partial_i V_i + \partial_i \partial_j S_{ij}$$

$$\dot{V}_i = \partial_i B + \partial_j S_{ij}$$

$$\dot{S}_{ij} = \partial_i V_j + \partial_j V_i$$

ETC.

S WITH ROT+TRANS SYMMETRY

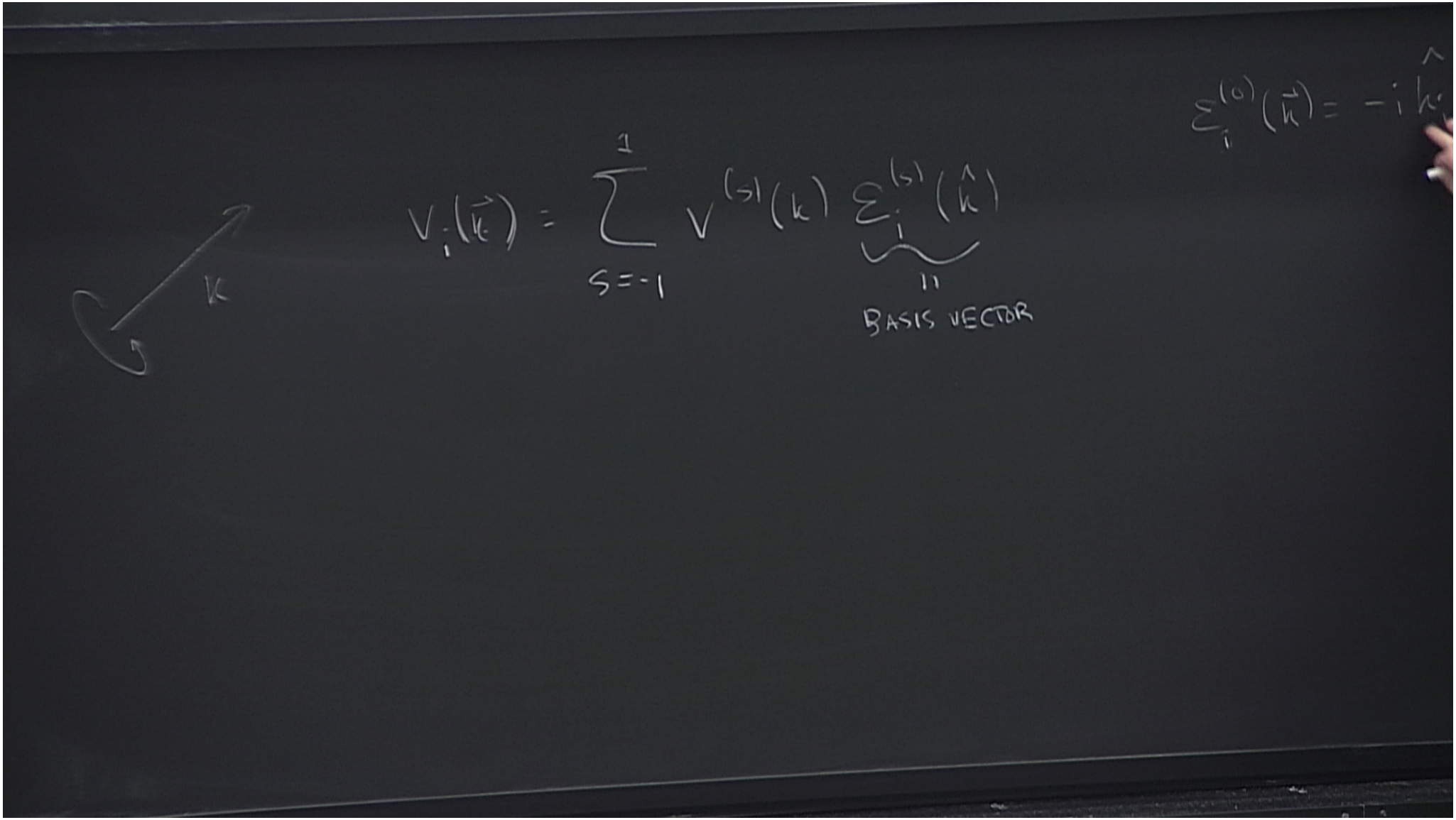
TRANSLATION INVARIANCE ⇒ k-MODES EVOLVES INDEPENDENTLY

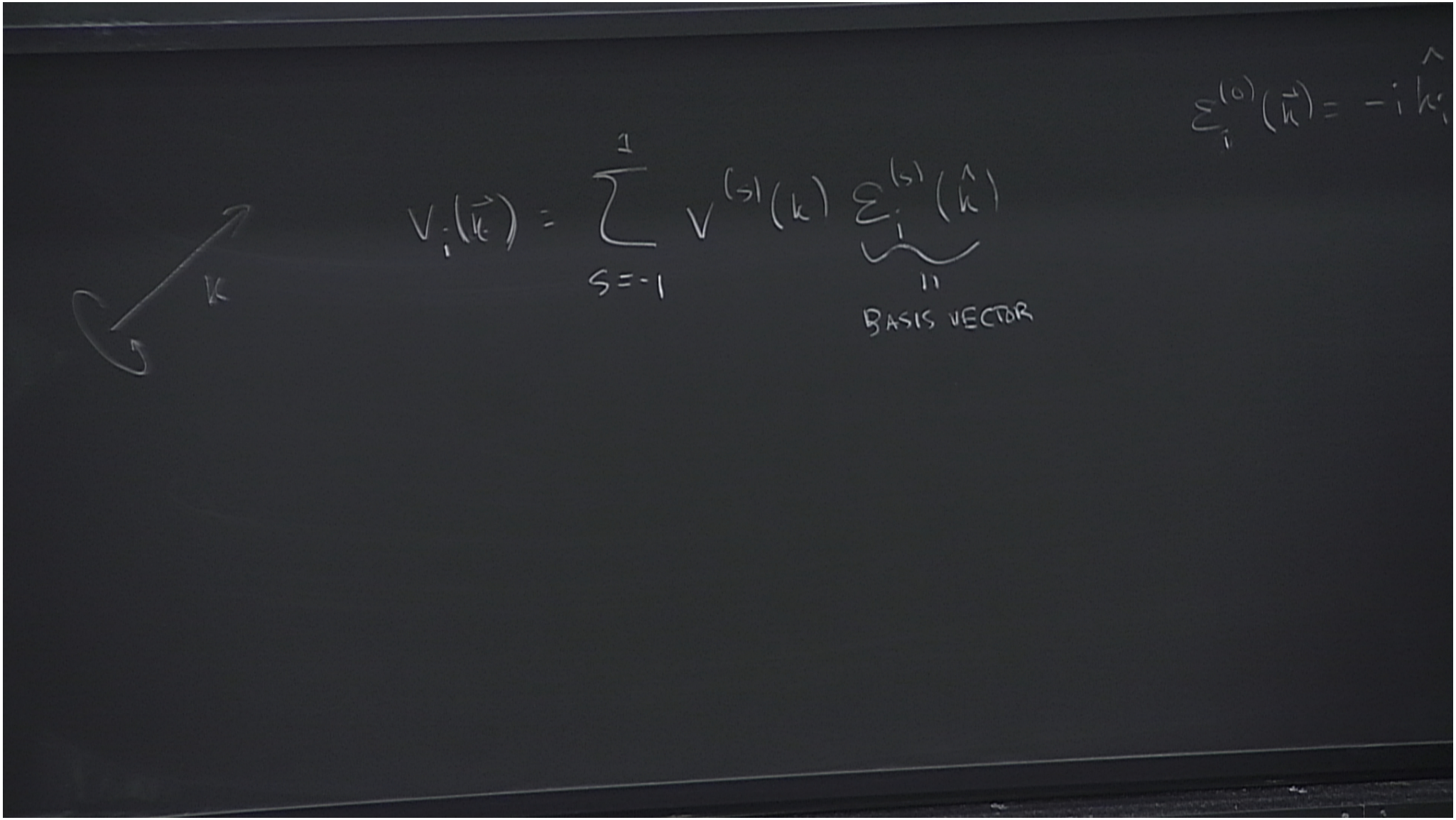
$\partial_i \to ik_i$

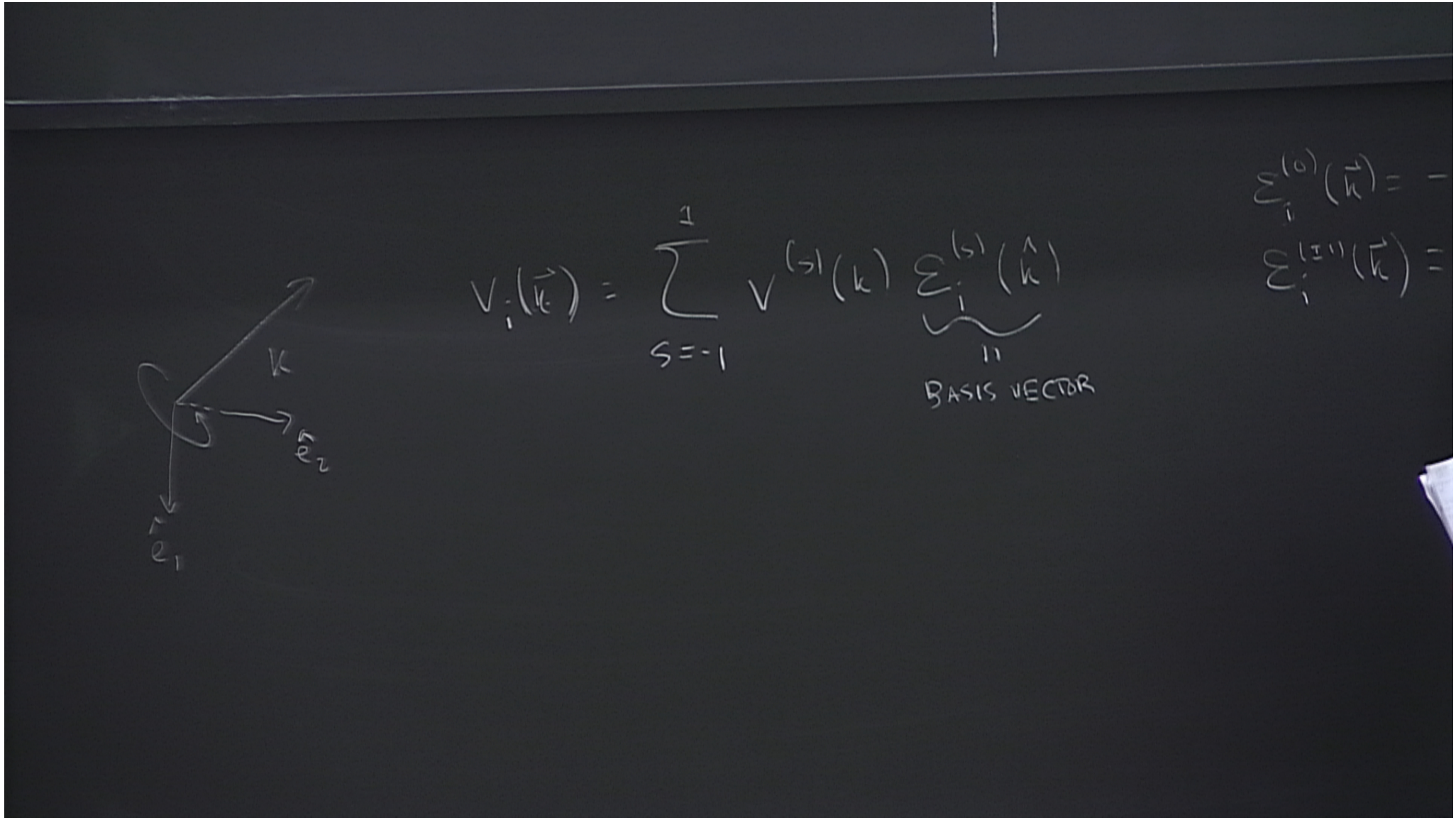
$$\dot{A}(\vec{k}) = -k^2 B(\vec{k}) + ik_i v_i(\vec{k}) - k_i k_j S_{ij}(\vec{k})$$

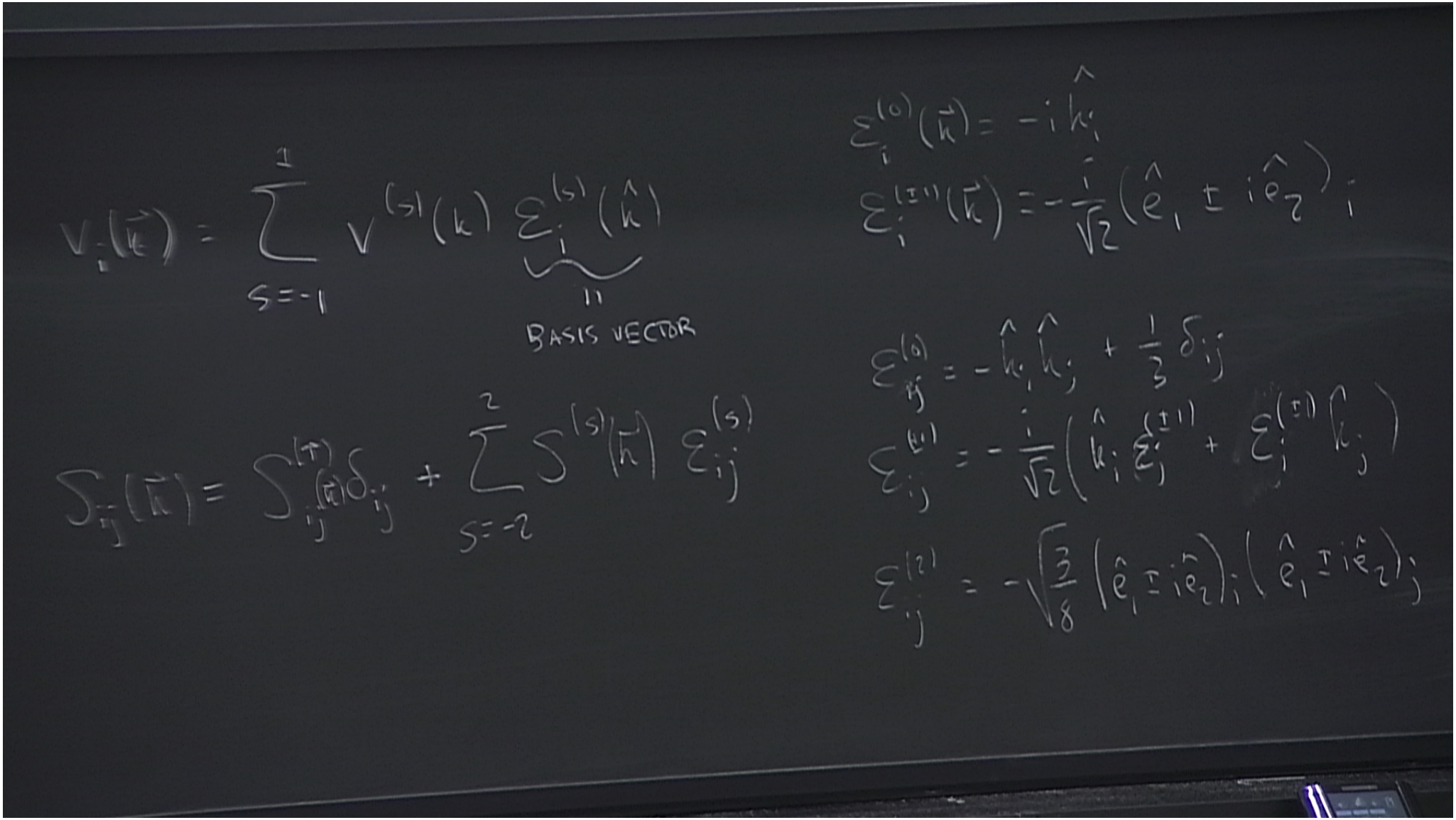
$$\dot{V}_i = ik_i B + ik_j S_{ij}$$

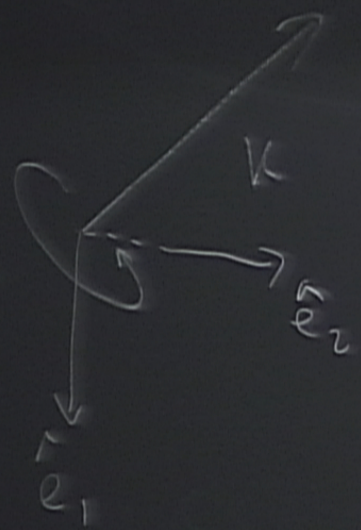
$$\dot{S}_{ij} = ik_i v_j + ik v_j$$


$V_i(\vec{k}) = \sum_{s=-1}^{1} V^{(s)}(k)\, \varepsilon_i^{(s)}(\hat{k})$
$\varepsilon_i^{(0)}(\vec{k}) = -i\hat{k}$
k
BASIS VECTOR


$\varepsilon_i^{(\sigma)}(\vec{k}) = -i\hat{k}_i$
$V_i(\vec{k}) = \sum_{s=-1}^{1} V^{(s)}(k)\, \varepsilon_i^{(s)}(\hat{k})$
k
BASIS VECTOR

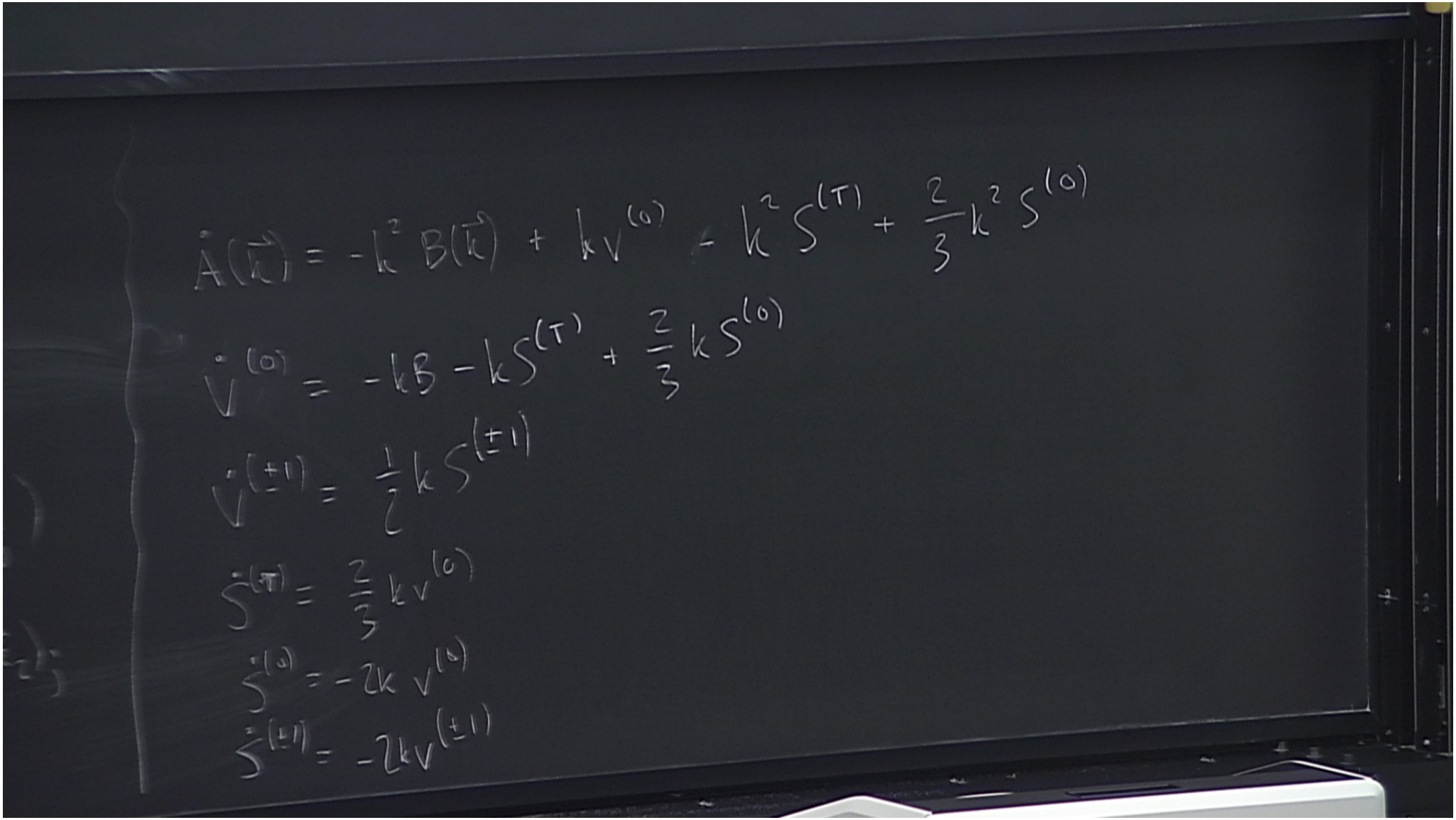

$V_i(\vec{k}) = \sum_{s=-1}^{1} V^{(s)}(k)\, \underbrace{\varepsilon_i^{(s)}(\hat{k})}_{\text{BASIS VECTOR}}$
$\varepsilon_i^{(0)}(\vec{k}) =$
$\varepsilon_i^{(\pm 1)}(\vec{k}) =$
k
$\hat{e}_2$
$\hat{e}_1$

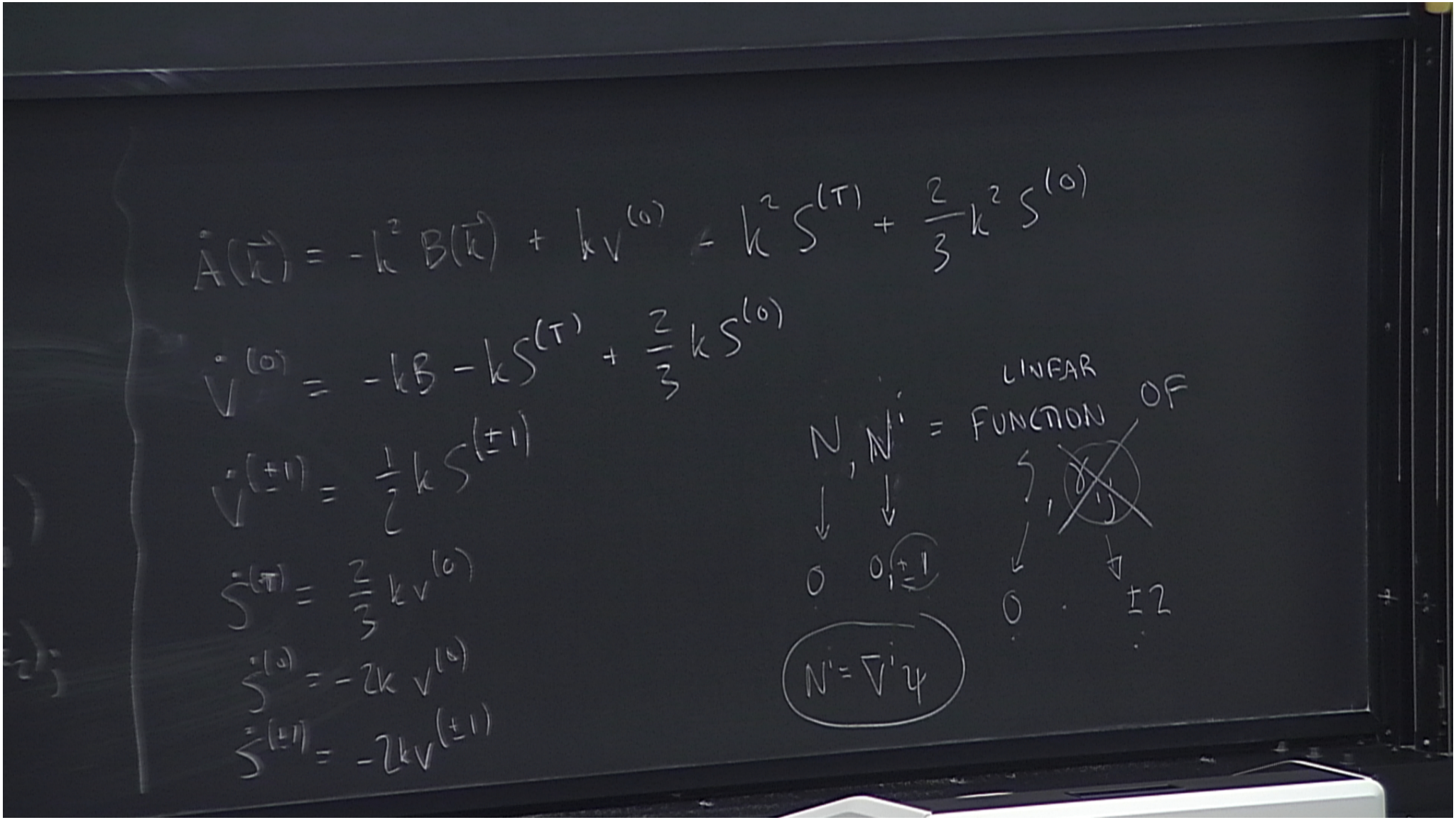

$V_i(\vec{k}) = \sum_{s=-1}^{1} V^{(s)}(k)\, \varepsilon_i^{(s)}(\hat{k})$
BASIS VECTOR
$S_{ij}(\vec{k}) = S^{(T)}(k)\delta_{ij} + \sum_{s=-2}^{2} S^{(s)}(\vec{k})\, \varepsilon_{ij}^{(s)}$
$\varepsilon_i^{(0)}(\vec{k}) = -i\hat{k}_i$
$\varepsilon_i^{(\pm 1)}(\vec{k}) = -\frac{1}{\sqrt{2}}(\hat{e}_1 \pm i\hat{e}_2)_i$
$\varepsilon_{ij}^{(0)} = -\hat{k}_i\hat{k}_j + \frac{1}{3}\delta_{ij}$
$\varepsilon_{ij}^{(\pm 1)} = -\frac{1}{\sqrt{2}}\left(\hat{k}_i\, \varepsilon_j^{(\pm 1)} + \varepsilon_i^{(\pm 1)}\hat{k}_j\right)$
$\varepsilon_{ij}^{(2)} = -\sqrt{\frac{3}{8}}(\hat{e}_1 \pm i\hat{e}_2)_i(\hat{e}_1 \pm i\hat{e}_2)_j$

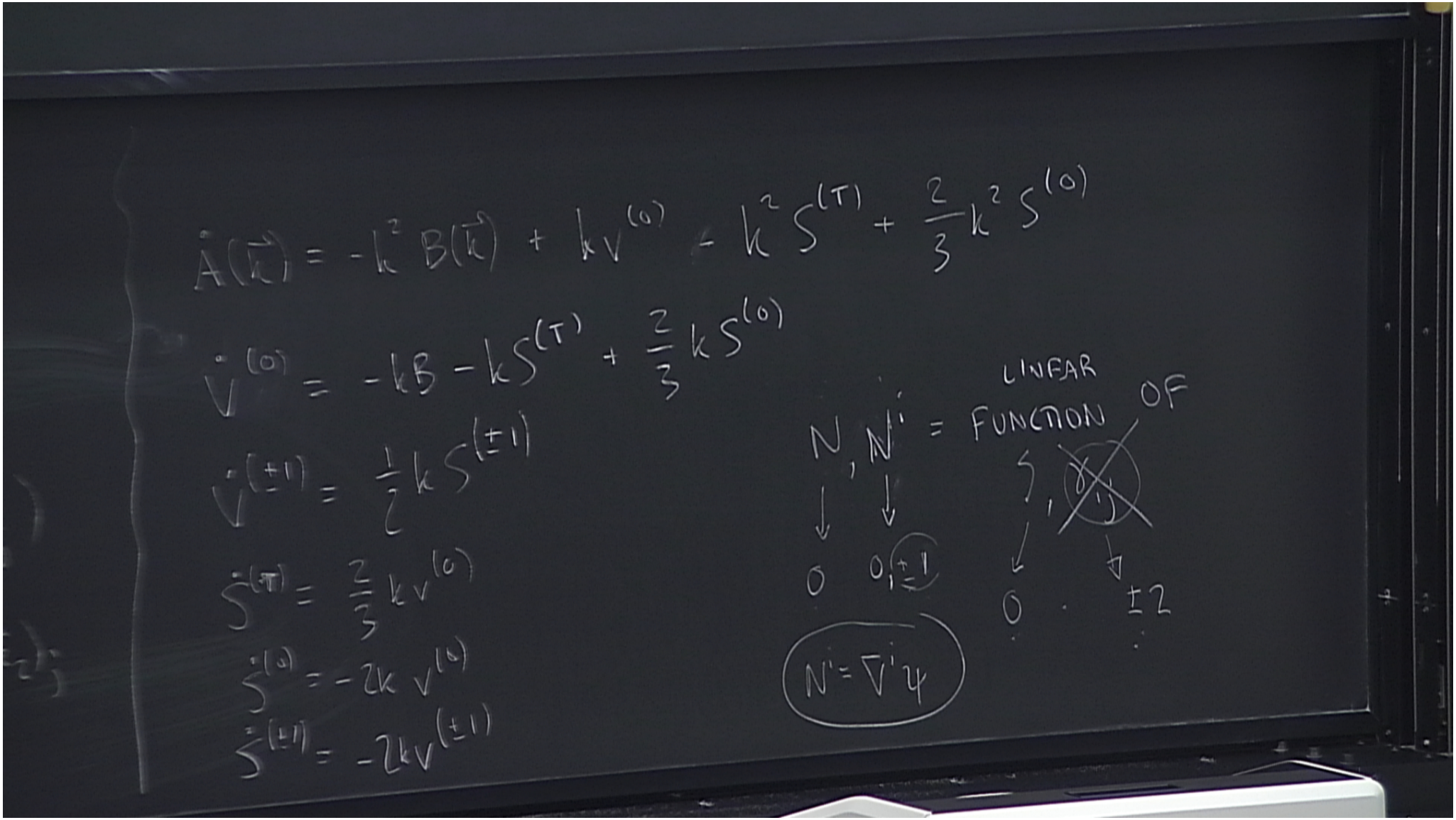


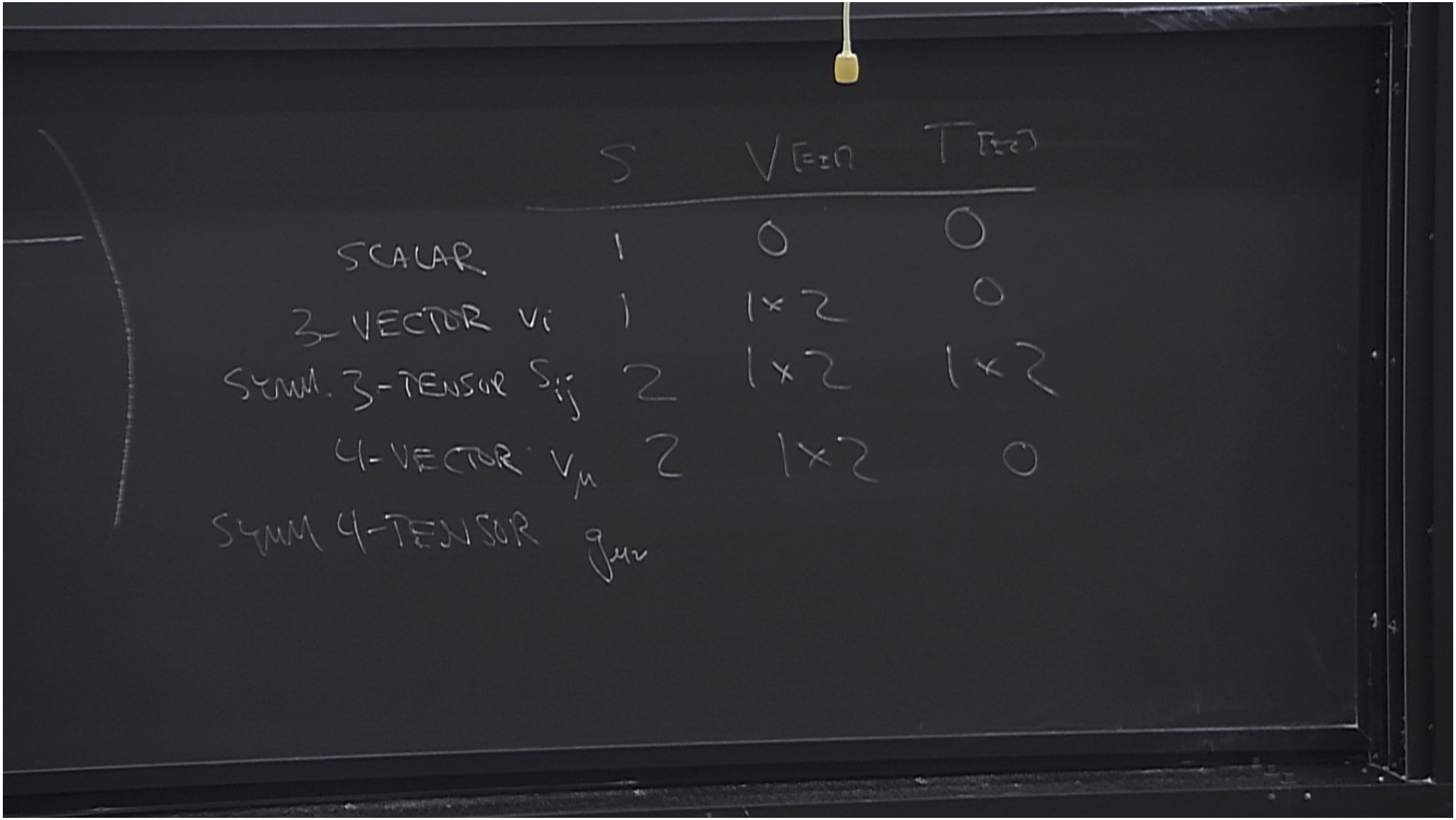
$$V_i(\vec{k}) = \sum_{s=-1}^{1} v^{(s)}(k)\, \underbrace{\varepsilon_i^{(s)}(\hat{k})}_{\text{BASIS VECTOR}}$$

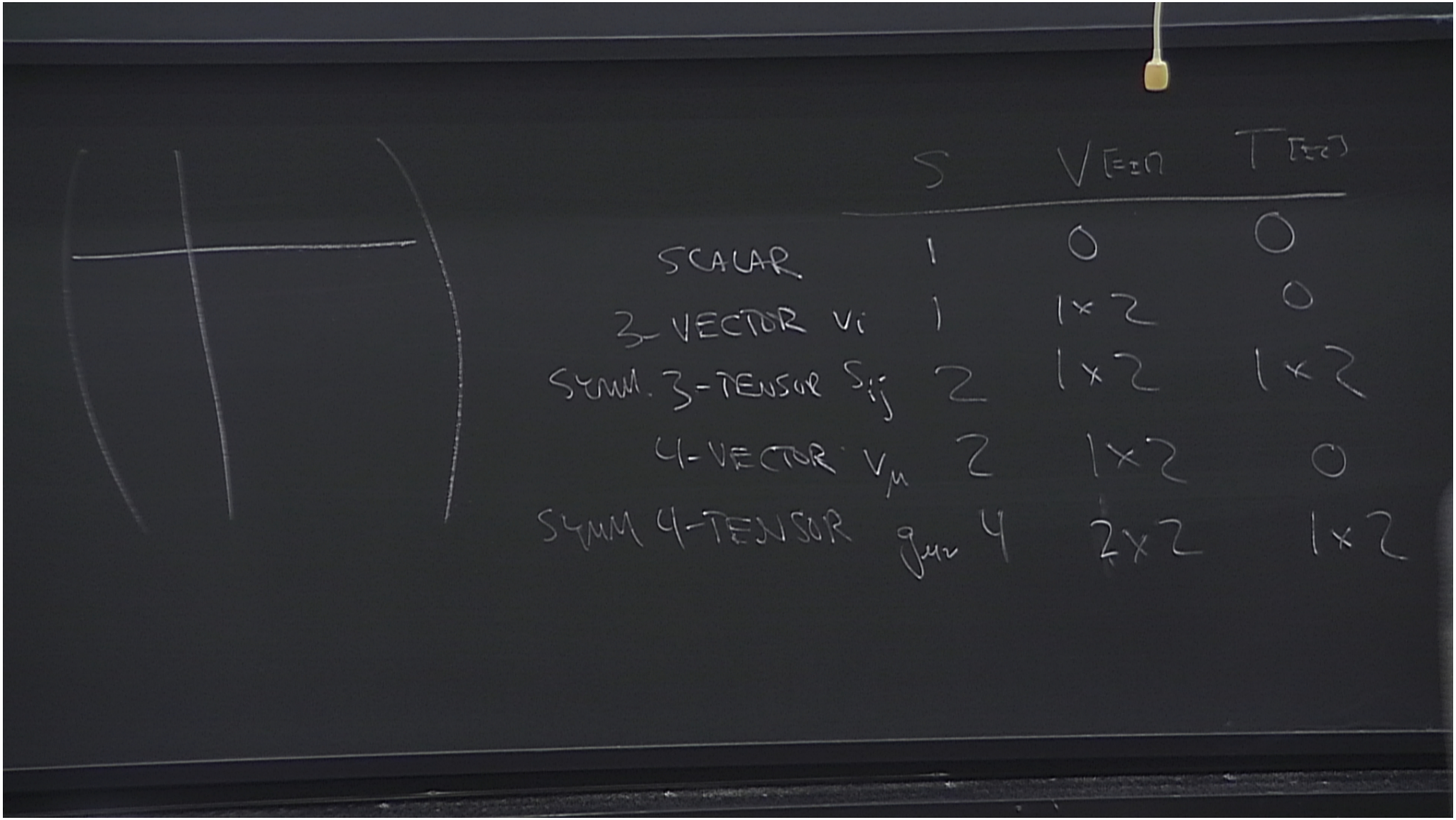
$$S_{ij}(\vec{k}) = S_{ij}^{(T)}(k)\delta_{ij} + \sum_{s=-2}^{2} S^{(s)}(\vec{k})\, \varepsilon_{ij}^{(s)}$$

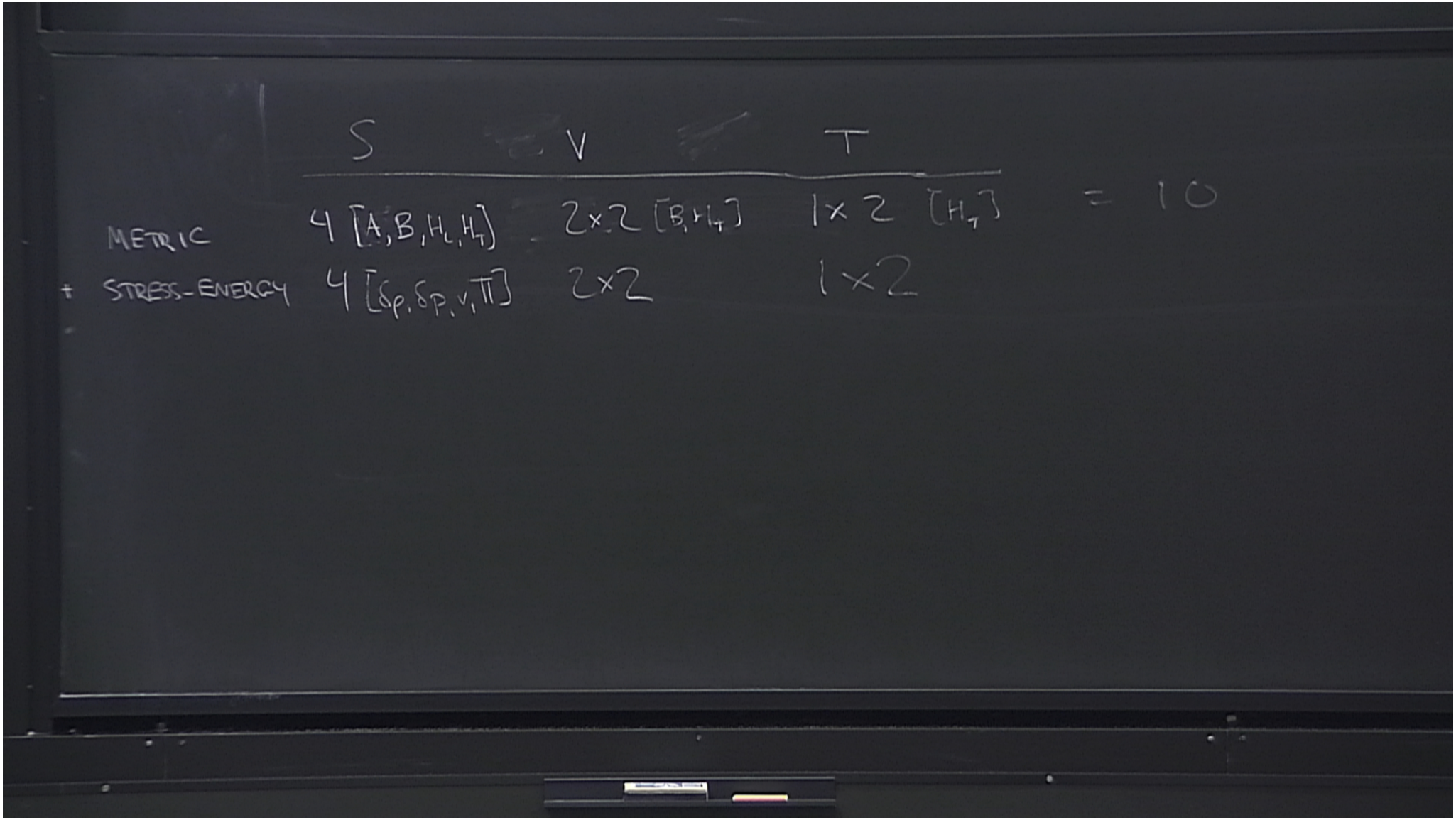

$\dot{A}(\vec{k}) = -k^2 B(\vec{k}) + kV^{(0)} - k^2 S^{(T)} + \frac{2}{3}k^2 S^{(0)}$
$\dot{V}^{(0)} = -kB - kS^{(T)} + \frac{2}{3}kS^{(0)}$
$\dot{V}^{(\pm 1)} = \frac{1}{2}kS^{(\pm 1)}$
$\dot{S}^{(T)} = \frac{2}{3}kV^{(0)}$
$\dot{S}^{(0)} = -2kV^{(0)}$
$\dot{S}^{(\pm 1)} = -2kV^{(\pm 1)}$

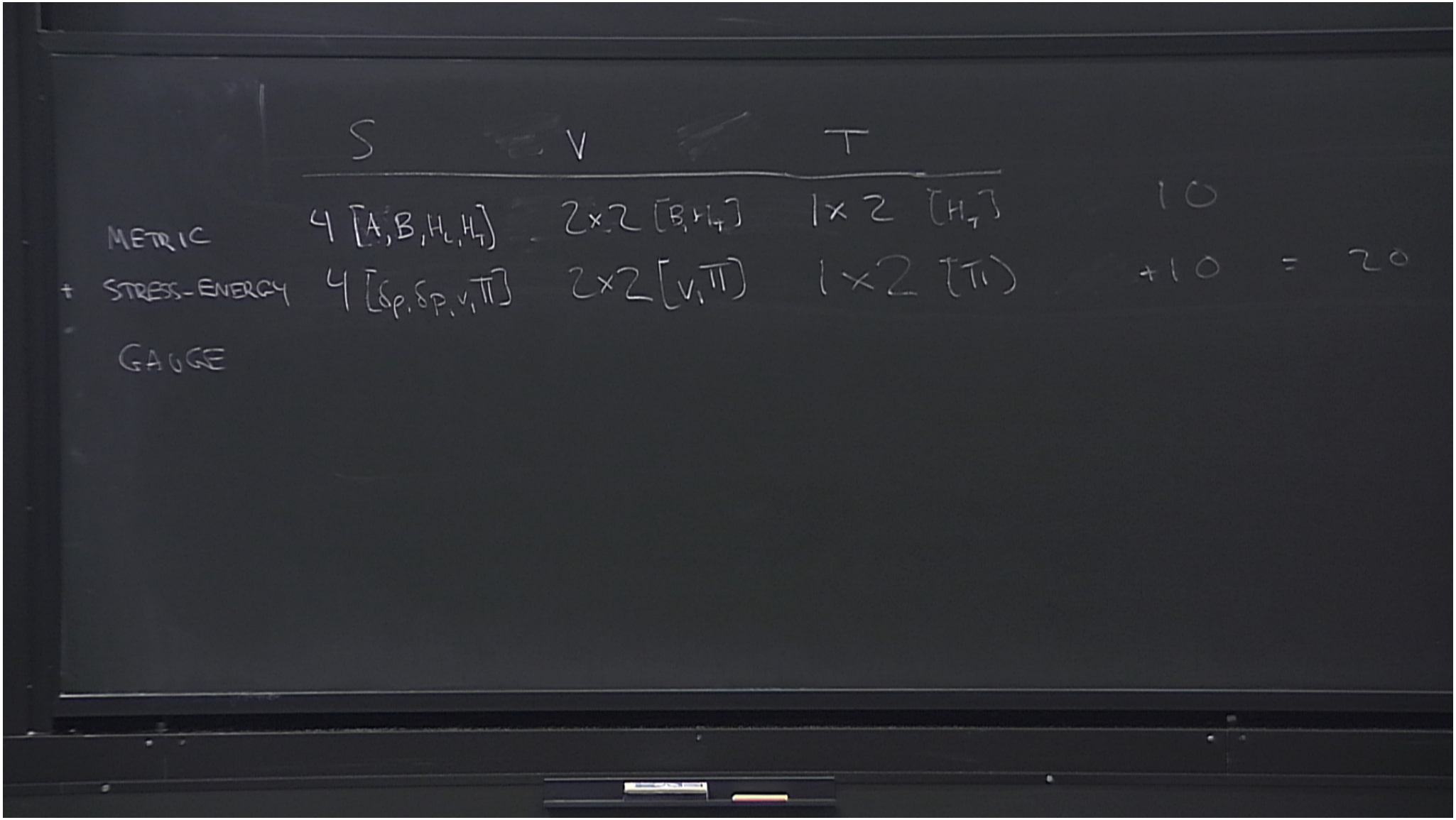

$\dot{A}(\vec{k}) = -k^2 B(\vec{k}) + k v^{(0)} - k^2 S^{(T)} + \frac{2}{3} k^2 S^{(0)}$
$\dot{v}^{(0)} = -kB - kS^{(T)} + \frac{2}{3} kS^{(0)}$
$\dot{v}^{(\pm 1)} = \frac{1}{2} k S^{(\pm 1)}$
$\dot{S}^{(T)} = \frac{2}{3} k v^{(0)}$
$\dot{S}^{(0)} = -2k v^{(0)}$
$\dot{S}^{(\pm 1)} = -2k v^{(\pm 1)}$
LINEAR
N, N' = FUNCTION OF
0 0, ±1
S, (0, 1)
0 ±2
N' = ∇' 4

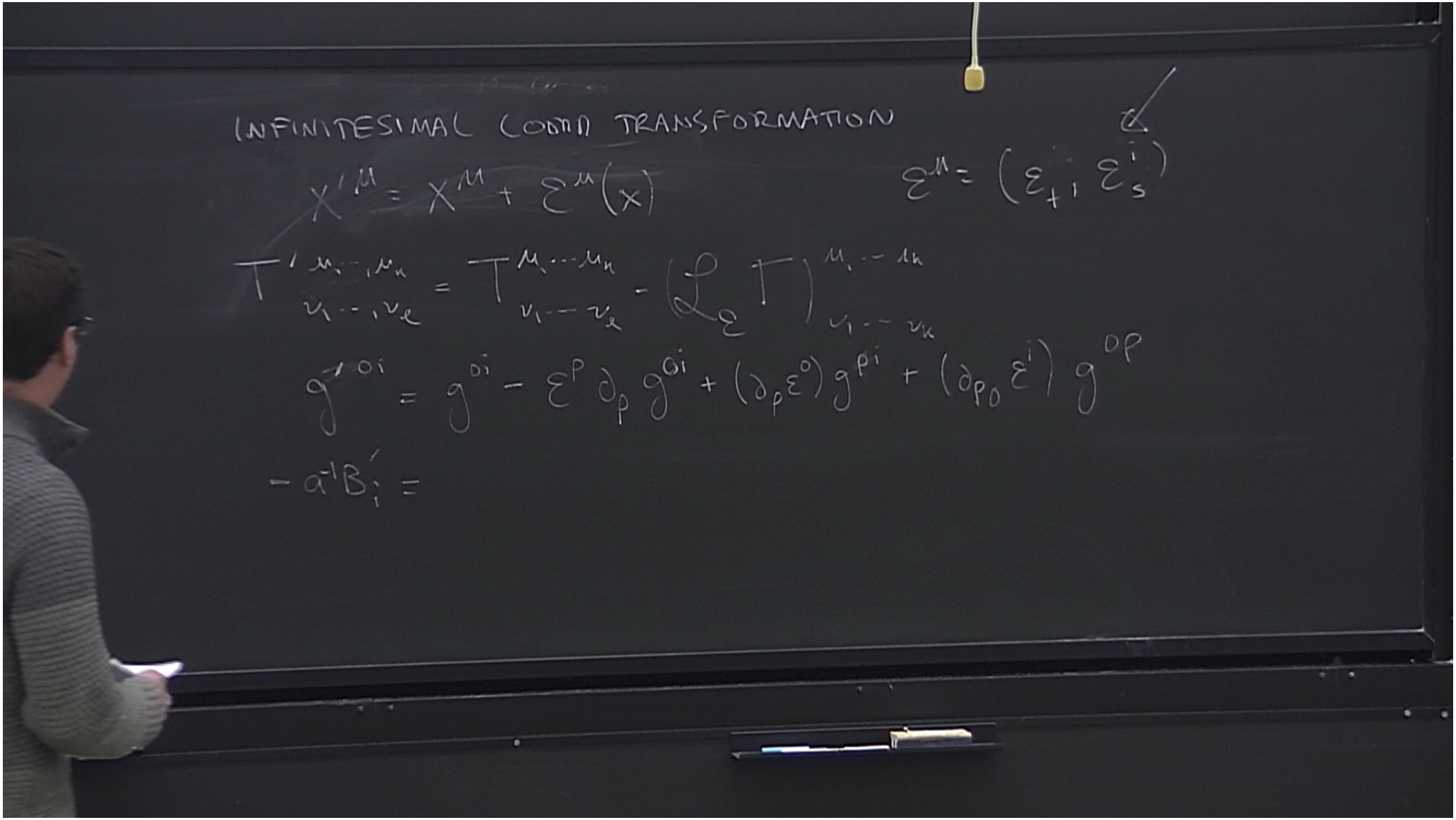

$\dot{A}(\vec{k}) = -k^2 B(\vec{k}) + kv^{(0)} - k^2 S^{(T)} + \frac{2}{3}k^2 S^{(0)}$
$\dot{v}^{(0)} = -kB - kS^{(T)} + \frac{2}{3}kS^{(0)}$
$\dot{v}^{(\pm 1)} = \frac{1}{2}kS^{(\pm 1)}$
$\dot{S}^{(T)} = \frac{2}{3}kv^{(0)}$
$\dot{S}^{(0)} = -2kv^{(0)}$
$\dot{S}^{(\pm 1)} = -2kv^{(\pm 1)}$
LINEAR
$N, N' =$ FUNCTION OF
$S, \cancel{S}, \cancel{(1)}$
$0 \quad 0, \pm 1$
$0 \quad \pm 2$
$N' = \nabla' \psi$


S
V[E=1]
T[E=2]
SCALAR 1 0 0
3-VECTOR V_i 1 1×2 0
SYMM. 3-TENSOR S_{ij} 2 1×2 1×2
4-VECTOR V_μ 2 1×2 0
SYMM 4-TENSOR $g_{\mu\nu}$


S
V [E=1]
T [E=2]
SCALAR 1 0 0
3-VECTOR V_i 1 1×2 0
SYMM. 3-TENSOR S_{ij} 2 1×2 1×2
4-VECTOR V_μ 2 1×2 0
SYMM 4-TENSOR $g_{\mu\nu}$ 4 2×2 1×2


S V T
METRIC 4 [A, B, H_L, H_T] 2×2 [B, H_T] 1×2 [H_T] = 10
+ STRESS-ENERGY 4 [δρ, δp, v, Π] 2×2 1×2


S V T
METRIC 4 [A, B, H_L, H_T] 2×2 [B, H_T] 1×2 [H_T] 10
+ STRESS-ENERGY 4 [δρ, δp, v, Π] 2×2 [v, Π] 1×2 [T_1] +10 = 20
GAUGE


INFINITESIMAL COORD TRANSFORMATION
$x'^{\mu} = x^{\mu} + \varepsilon^{\mu}(x)$
$\varepsilon^{\mu} = (\varepsilon_t, \varepsilon^i_s)$
$T'^{\mu_1 \cdots \mu_n}_{\nu_1 \cdots \nu_\ell} = T^{\mu_1 \cdots \mu_n}_{\nu_1 \cdots \nu_\ell} - (\mathcal{L}_\varepsilon T)^{\mu_1 \cdots \mu_n}_{\nu_1 \cdots \nu_k}$
$g'^{0i} = g^{0i} - \varepsilon^p \partial_p g^{0i} + (\partial_p \varepsilon^0) g^{pi} + (\partial_{p_0} \varepsilon^i) g^{0p}$
$-a^{-1} B'_i =$

INFINITESIMAL COORD TRANSFORMATION

$$x'^{\mu} = x^{\mu} + \varepsilon^{\mu}(x) \qquad \varepsilon^{\mu} = (\varepsilon_t, \varepsilon^i_s)$$

$$T'^{\mu_1 \cdots \mu_n}_{\nu_1 \cdots \nu_k} = T^{\mu_1 \cdots \mu_n}_{\nu_1 \cdots \nu_k} - (\mathcal{L}_\varepsilon T)^{\mu_1 \cdots \mu_n}_{\nu_1 \cdots \nu_k}$$

$$g'^{0i} = g^{0i} - \varepsilon^p \partial_p g^{0i} + (\partial_p \varepsilon^0) g^{pi} + (\partial_p \varepsilon^i) g^{0p}$$

$$-a^{-1} B'_i = -a^{-1} B + (\partial_i \varepsilon_t) a^{-2} + (\partial_0 \varepsilon^i_s)(-1)$$

$$\boxed{B'_i = B_i - a^{-1}(\partial_i \varepsilon_t) + a \partial_0 \varepsilon^i_s}$$

	S	V	T	
METRIC	4 $[A, B, H_L, H_T]$	2×2 $[B, H_T]$	1×2 $[H_T]$	10
+ STRESS-ENERGY	4 $[\delta\rho, \delta p, v, \Pi]$	2×2 $[v, \Pi]$	1×2 $[\Pi]$	$+10=20$
− GAUGE	-2 $[\varepsilon_t, \varepsilon_s]$	-1×2 $[\varepsilon_S]$		$-4=16$

GAUGE

$A \to A - \dot{\varepsilon}_t$

$B_i \to B_i - a^{-1}\partial_i\varepsilon_t + a\dot{\varepsilon}_{S\,i}$

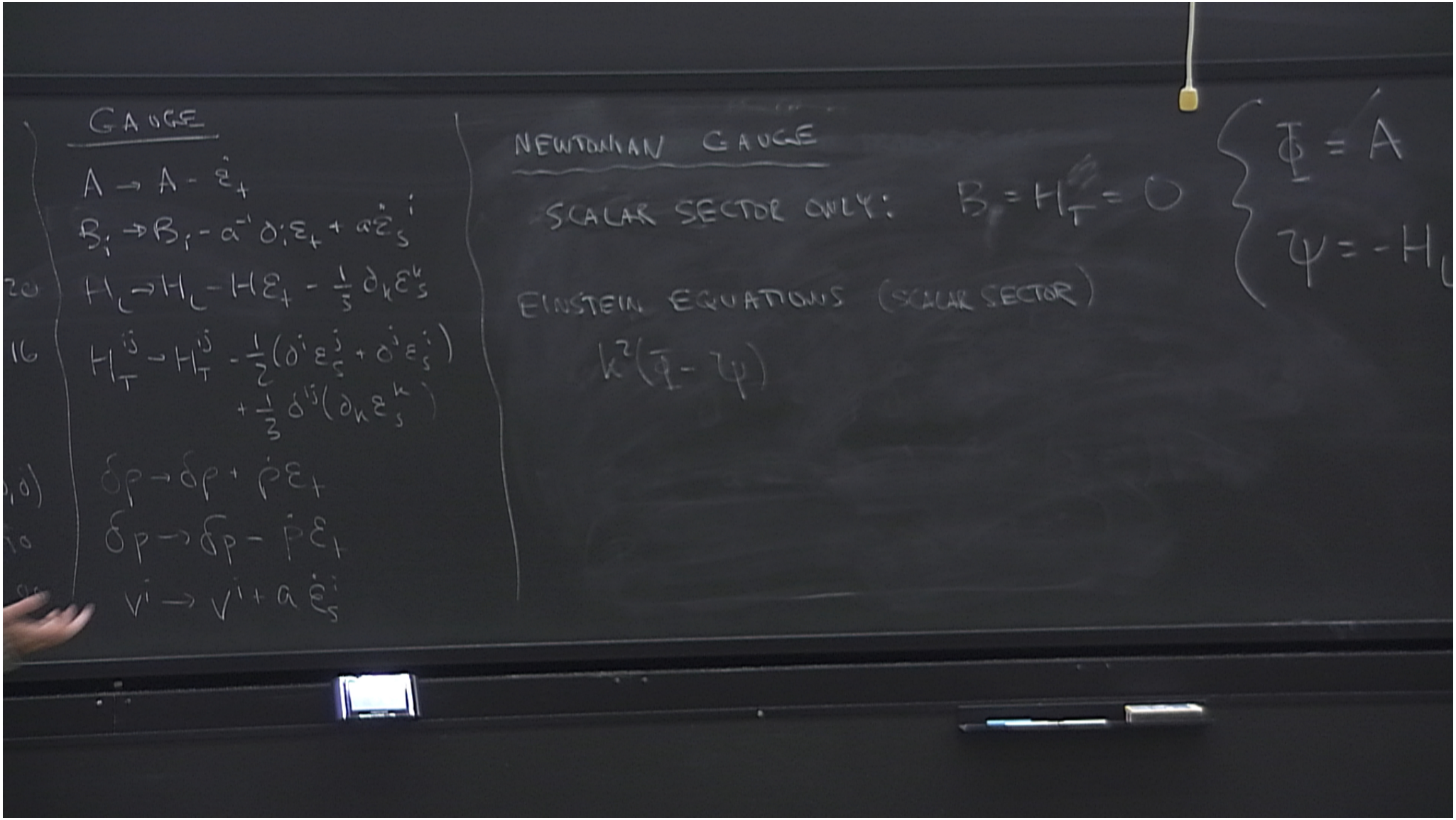
$H_L \to H_L - H\varepsilon_t - \frac{1}{3}\partial_k\varepsilon^k_S$

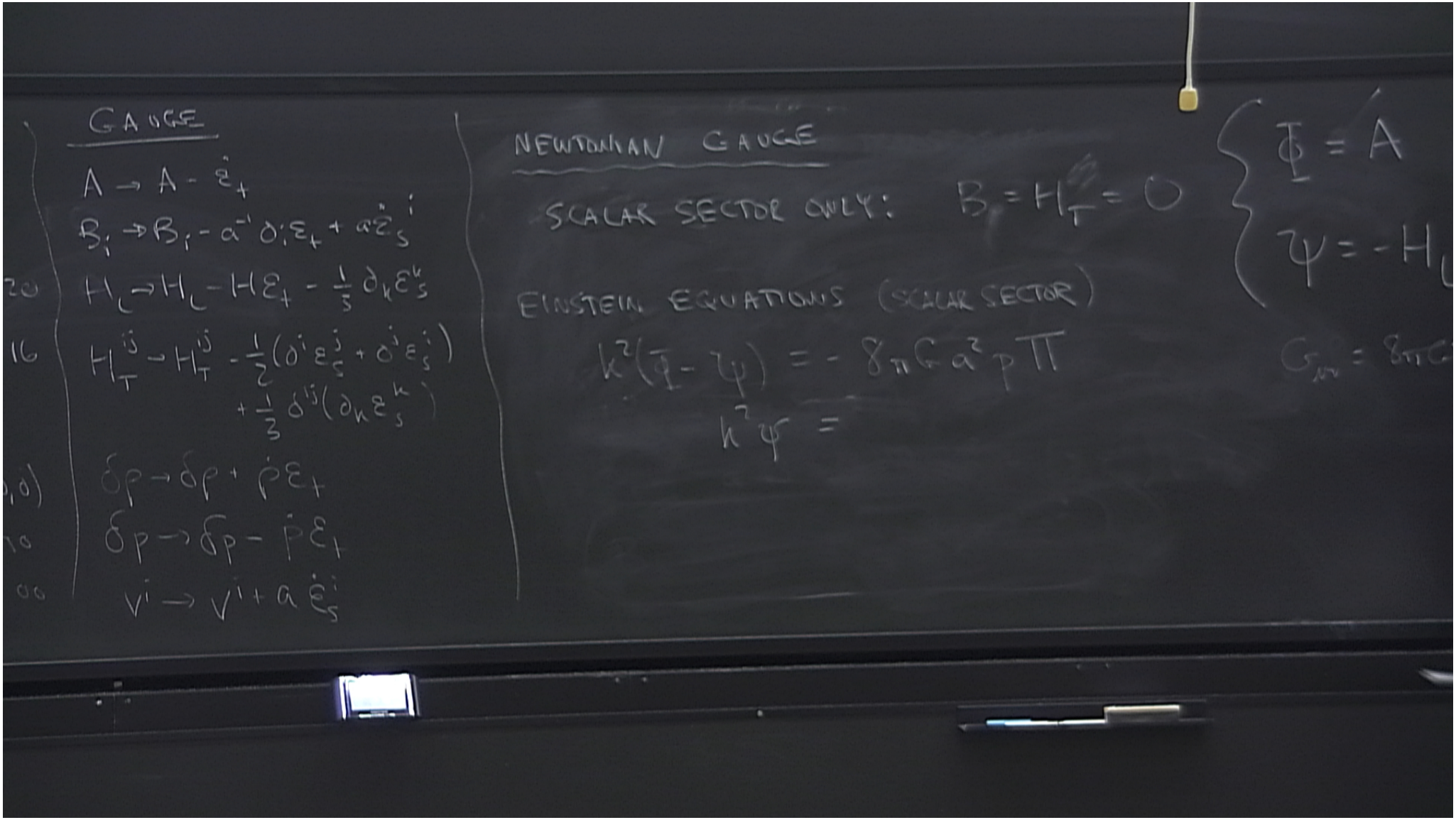
$H^{ij}_T \to H^{ij}_T - \frac{1}{2}(\partial^i\varepsilon^j_S + \partial^j\varepsilon^i_S) + \frac{1}{3}\delta^{ij}(\partial_k\varepsilon^k_S)$

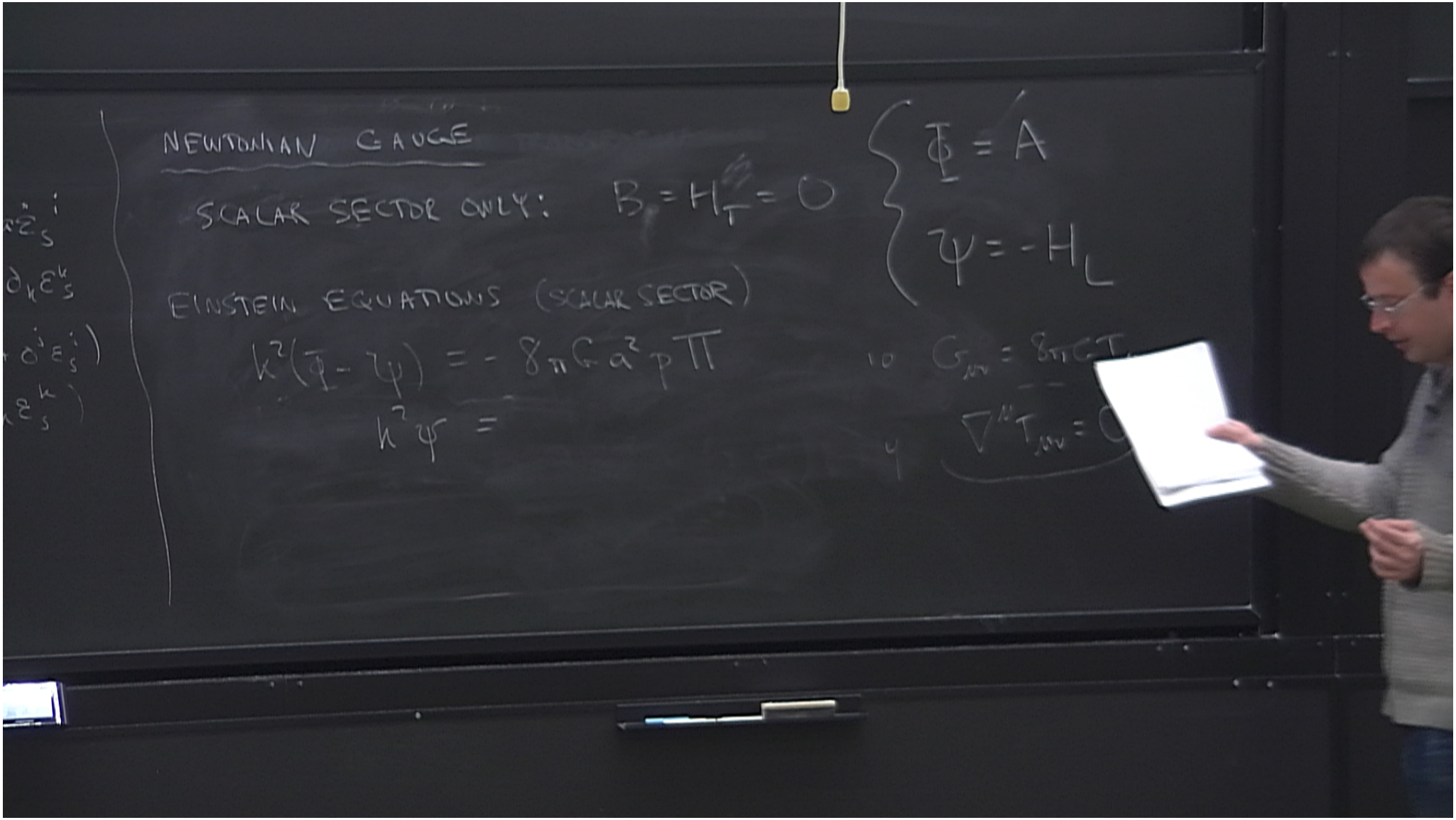
$\delta\rho \to \delta\rho + \dot{\rho}\varepsilon_t$

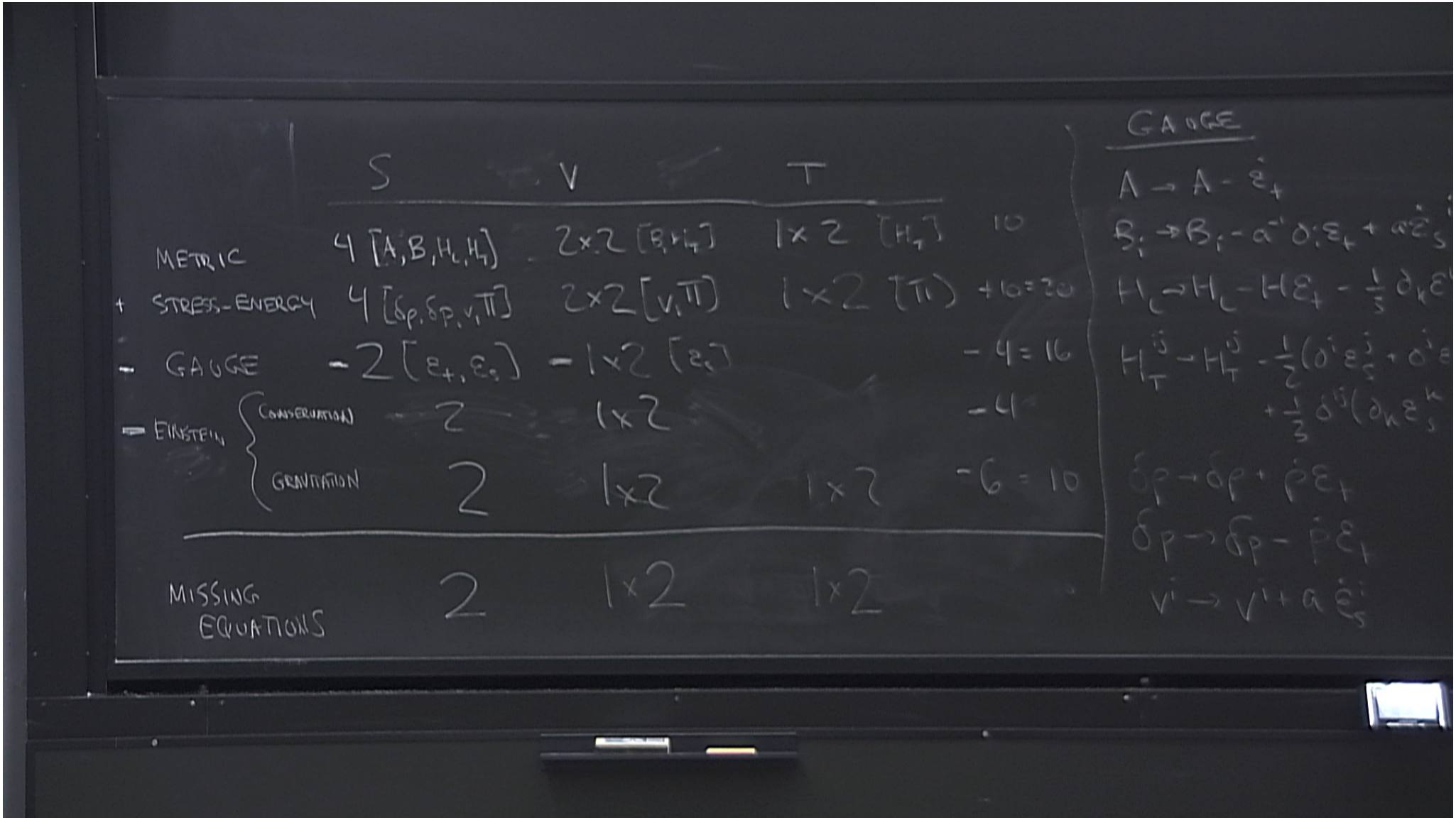
$\delta p \to \delta p - \dot{p}\varepsilon_t$

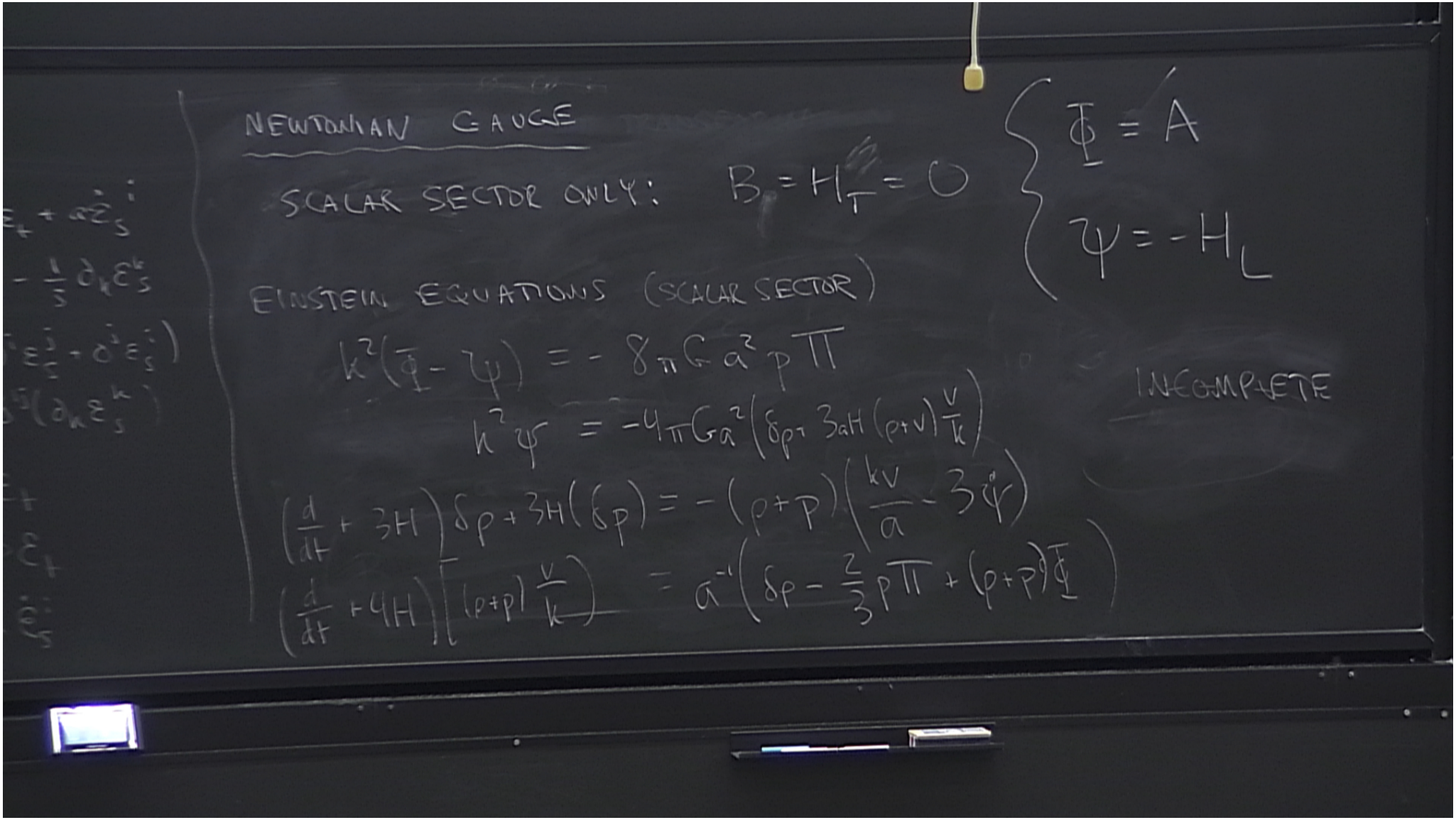
$v^i \to v^i + a\dot{\varepsilon}^i_S$

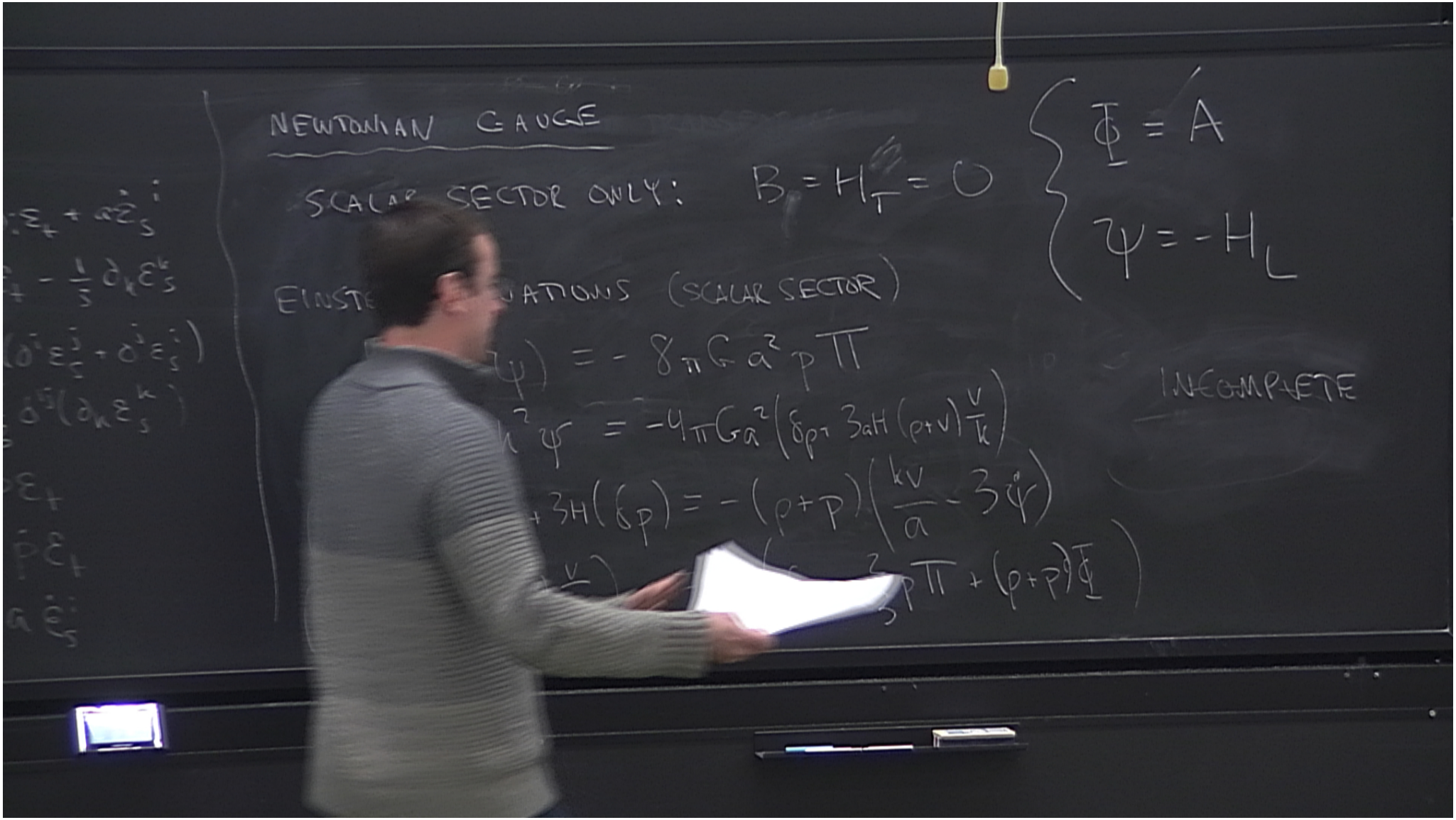

GAUGE
$A \to A - \dot{\varepsilon}_t$
$B_i \to B_i - a^{-1}\partial_i \varepsilon_t + a\dot{\varepsilon}_{Si}$
$H_L \to H_L - H\varepsilon_t - \frac{1}{3}\partial_k \varepsilon^k_S$
$H^{ij}_T \to H^{ij}_T - \frac{1}{2}(\partial^i \varepsilon^j_S + \partial^j \varepsilon^i_S) + \frac{1}{3}\delta^{ij}(\partial_k \varepsilon^k_S)$
$\delta\rho \to \delta\rho + \dot{\rho}\varepsilon_t$
$\delta\rho \to \delta\rho - \dot{\rho}\varepsilon_t$
$v^i \to v^i + a\dot{\varepsilon}^i_S$
NEWTONIAN GAUGE
SCALAR SECTOR ONLY: $B_i = H^{ij}_T = 0$
$\Phi = A$
$\Psi = -H_L$
EINSTEIN EQUATIONS (SCALAR SECTOR)
$k^2(\Phi - \Psi)$

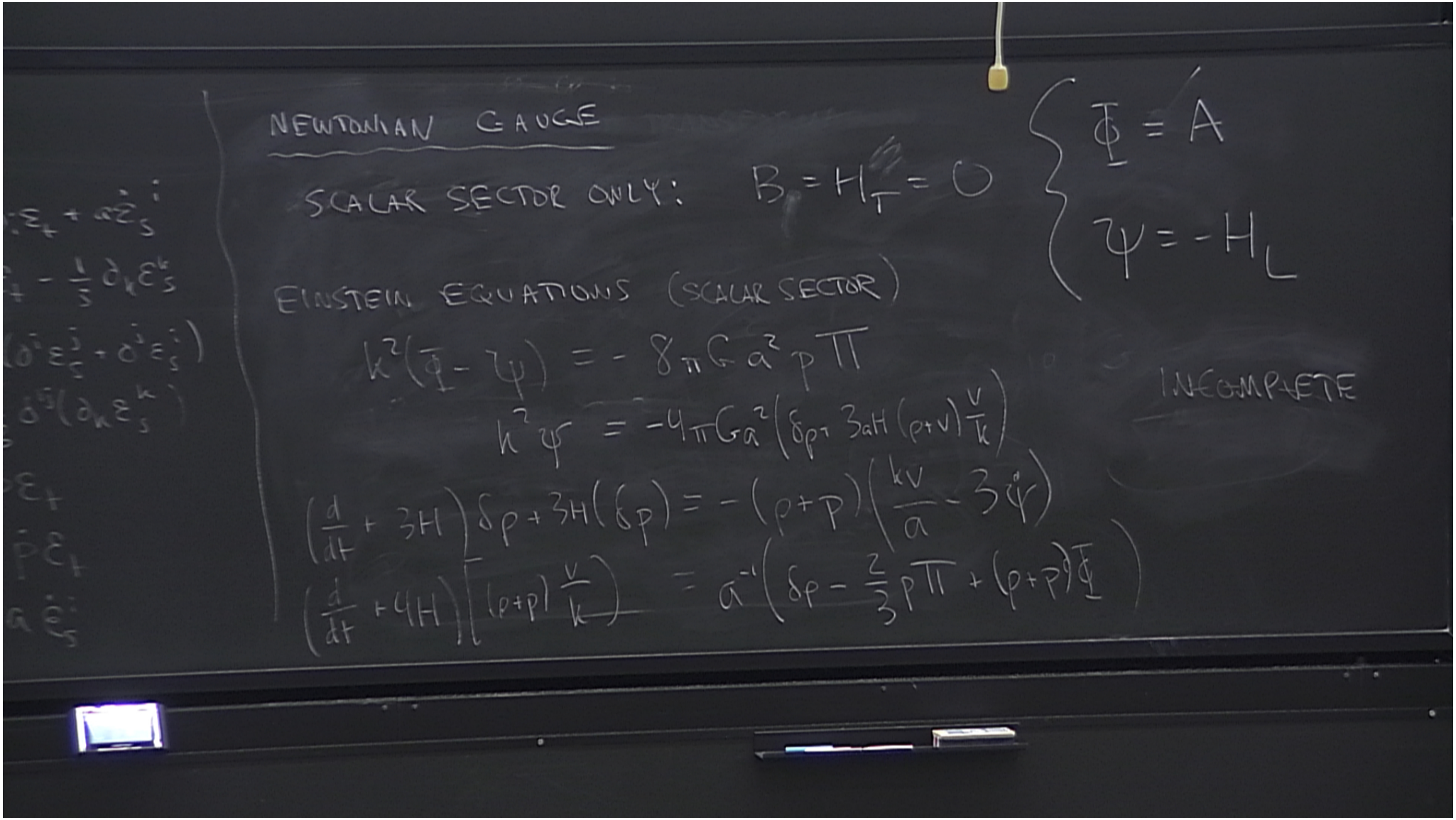

GAUGE
$A \to A - \dot{\varepsilon}_t$
$B_i \to B_i - a^{-1}\partial_i \varepsilon_t + a\dot{\varepsilon}_{Si}$
$H_L \to H_L - H\varepsilon_t - \frac{1}{3}\partial_k \varepsilon^k_S$
$H^{ij}_T \to H^{ij}_T - \frac{1}{2}(\partial^i \varepsilon^j_S + \partial^j \varepsilon^i_S) + \frac{1}{3}\delta^{ij}(\partial_k \varepsilon^k_S)$
$\delta\rho \to \delta\rho + \dot{\rho}\varepsilon_t$
$\delta p \to \delta p + \dot{p}\varepsilon_t$
$v^i \to v^i + a\dot{\varepsilon}^i_S$
NEWTONIAN GAUGE
SCALAR SECTOR ONLY: $B = H_T = 0$
$\Phi = A$
$\psi = -H_L$
EINSTEIN EQUATIONS (SCALAR SECTOR)
$k^2(\Phi - \psi) = -8\pi G a^2 \rho \Pi$
$k^2\psi =$
$G_{\mu\nu} = 8\pi G$


NEWTONIAN GAUGE
SCALAR SECTOR ONLY: $B = H_T = 0$
$\Phi = A$
$\psi = -H_L$
EINSTEIN EQUATIONS (SCALAR SECTOR)
$k^2(\Phi - \psi) = -8\pi G a^2 \rho \Pi$
$k^2\psi =$
$G_{\mu\nu} = 8\pi G T$
$\nabla^\mu T_{\mu\nu} = 0$


S
V
T
METRIC
4 [A, B, H_L, H_T]
2×2 [B, H_T]
1×2 [H_T]
10
+ STRESS-ENERGY
4 [δρ, δp, v, Π]
2×2 [v, Π]
1×2 [Π]
+10 = 20
- GAUGE
-2 [ε_+, ε_-]
-1×2 [ε_-]
-4 = 16
- EINSTEIN
CONSERVATION
2
1×2
-4
GRAVITATION
2
1×2
1×2
-6 = 10
MISSING EQUATIONS
2
1×2
1×2
GAUGE
A → A - ε̇_+
B_i → B_i - a⁻¹ ∂_i ε_+ + a ε̇_i
H_L → H_L - H ε_+ - (1/3) ∂_k ε^k
H_T^ij → H_T^ij - (1/2)(∂^i ε_T^j + ∂^j ε
+ (1/3) δ^ij (∂_k ε_T^k
δρ → δρ + ρ̇ ε_+
δp → δp - ṗ ε_+
v^i → v^i + a ε̇_i


NEWTONIAN GAUGE
SCALAR SECTOR ONLY: $B = H_T = 0$
$\Phi = A$
$\Psi = -H_L$
EINSTEIN EQUATIONS (SCALAR SECTOR)
$k^2(\Phi - \Psi) = -8\pi G a^2 \rho \Pi$
$k^2\Psi = -4\pi G a^2 (\delta\rho + 3aH(\rho+v)\frac{v}{k})$
$(\frac{d}{dt} + 3H)\delta\rho + 3H(\delta p) = -(\rho+p)(\frac{kv}{a} - 3\dot{\Psi})$
$(\frac{d}{dt} + 4H)[(\rho+p)\frac{v}{k}] = a^{-1}(\delta p - \frac{2}{3}\rho\Pi + (\rho+p)\Phi)$
INCOMPLETE


NEWTONIAN GAUGE
SCALAR SECTOR ONLY: $B = H_T = 0$
$\Phi = A$
$\psi = -H_L$
EINSTEIN EQUATIONS (SCALAR SECTOR)
$\psi) = -8\pi G a^2 p\Pi$
$k^2\psi = -4\pi G a^2(\delta\rho + 3aH(\rho+v)\frac{v}{k})$
$+3H(\delta p) = -(\rho+p)(\frac{kv}{a} - 3\dot{\psi})$
$p\Pi + (\rho+p)\Phi$
INCOMPLETE


NEWTONIAN GAUGE
SCALAR SECTOR ONLY: $B = H_T = 0$
$\Phi = A$
$\Psi = -H_L$
EINSTEIN EQUATIONS (SCALAR SECTOR)
$k^2(\dot{\Phi} - \Psi) = -8\pi G a^2 \rho \Pi$
$k^2\Psi = -4\pi G a^2 (\delta\rho + 3aH(\rho+v)\frac{v}{k})$
$(\frac{d}{dt} + 3H)\delta\rho + 3H(\delta p) = -(\rho+p)(\frac{kv}{a} - 3\dot{\Psi})$
$(\frac{d}{dt} + 4H)[(\rho+p)\frac{v}{k}] = a^{-1}(\delta p - \frac{2}{3}\rho\Pi + (\rho+p)\Phi)$
INCOMPLETE

$\dot{S}_{ij} = \partial_i v_j + \partial_j v_i$ ETC.

$$\alpha(t) = \frac{H(t)}{a(t)} \int_{-\infty}^{t} dt'\, a(t') \simeq \frac{2}{5+3w}$$ FOR w CONSTANT

WEINBERG'S THEOREM

REGARDLESS OF THE EQS OF MOTION, THE SCALAR SECTOR HAS A SOLUTION FOR $k \ll aH$ WITH:

$$\mathcal{R}(t) = -\left(1-\alpha(t)\right)\lambda + \mathcal{O}\left(\frac{k}{aH}\right)^2$$

$$\Psi = -\left(1-\alpha(t)\right)\lambda + \mathcal{O}\left(\frac{k}{aH}\right)^2$$

$$\delta\rho_J = -\dot{\rho}_J \frac{a(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k}{aH}\right)^2$$

$$\delta p_J = -\dot{p}_J \frac{a(t)}{H(t)}\lambda + \mathcal{O}\left(\frac{k}{aH}\right)^2$$

$$v_J = \frac{\lambda \dot{\rho}_J}{3(\rho_J + p_J)} \frac{a(t)}{H(t)} \left(\frac{k}{aH}\right) + \mathcal{O}\left(\frac{k}{aH}\right)^3$$