Title: Explorations in Cosmology-12

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Abstract:

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4} \right)$$

$$\phi^3 \quad \phi^2 \varphi$$

$$\zeta = -\frac{H}{\dot\phi} \varphi$$

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$$(\partial_\mu \phi)^2 = -\dot\phi_0^2 - 2\dot\phi_0 \dot\varphi + (\partial_\mu \varphi)^2$$

$$\frac{1}{c_s^2} = \frac{1 + \frac{3}{2}\frac{\dot\phi_0^2}{\Lambda^4}}{1 + \frac{\dot\phi_0^2}{2\Lambda^4}}$$

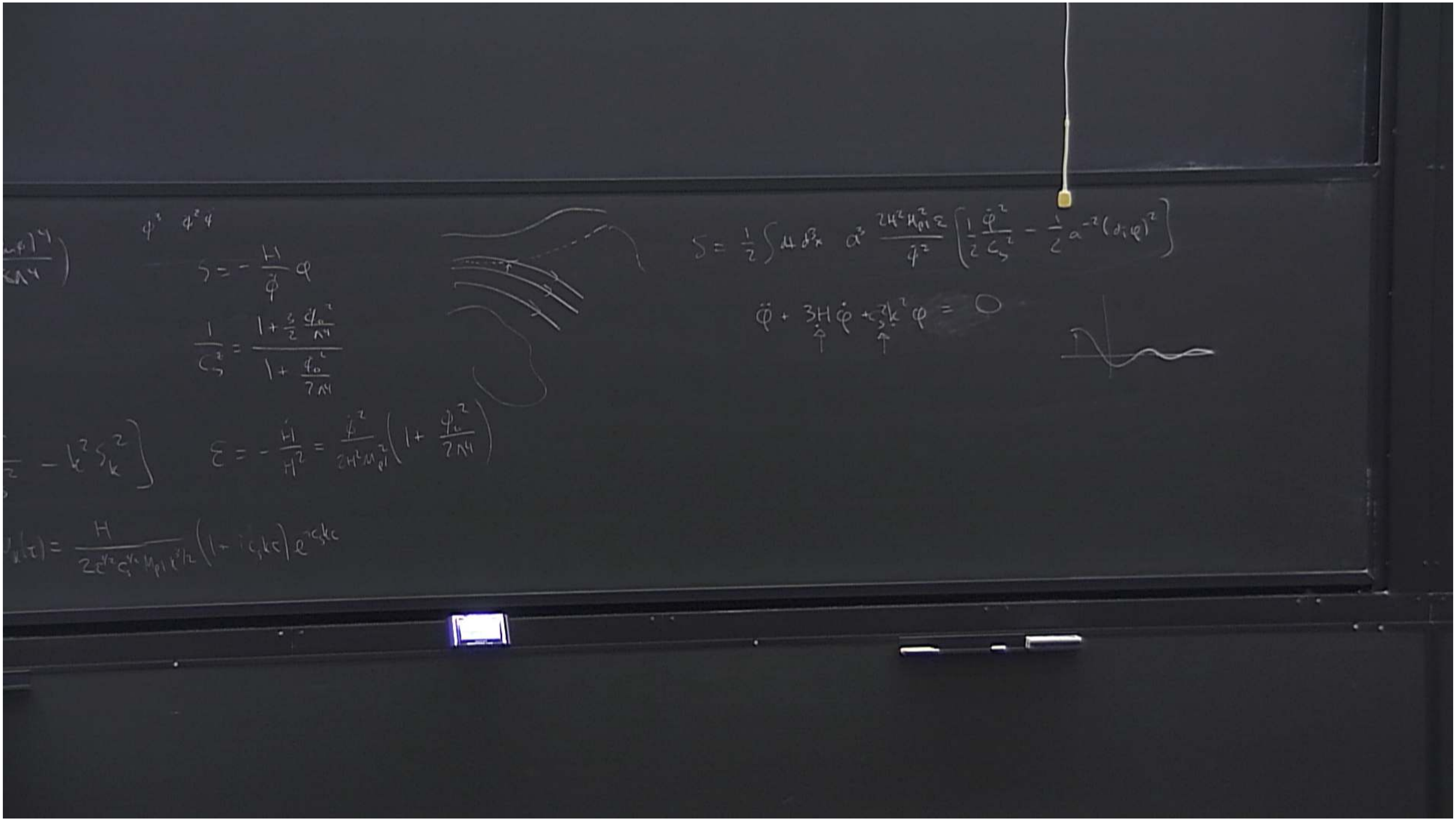
$$S = S_0 + S_1 + S_2 + S_3 + S_4$$

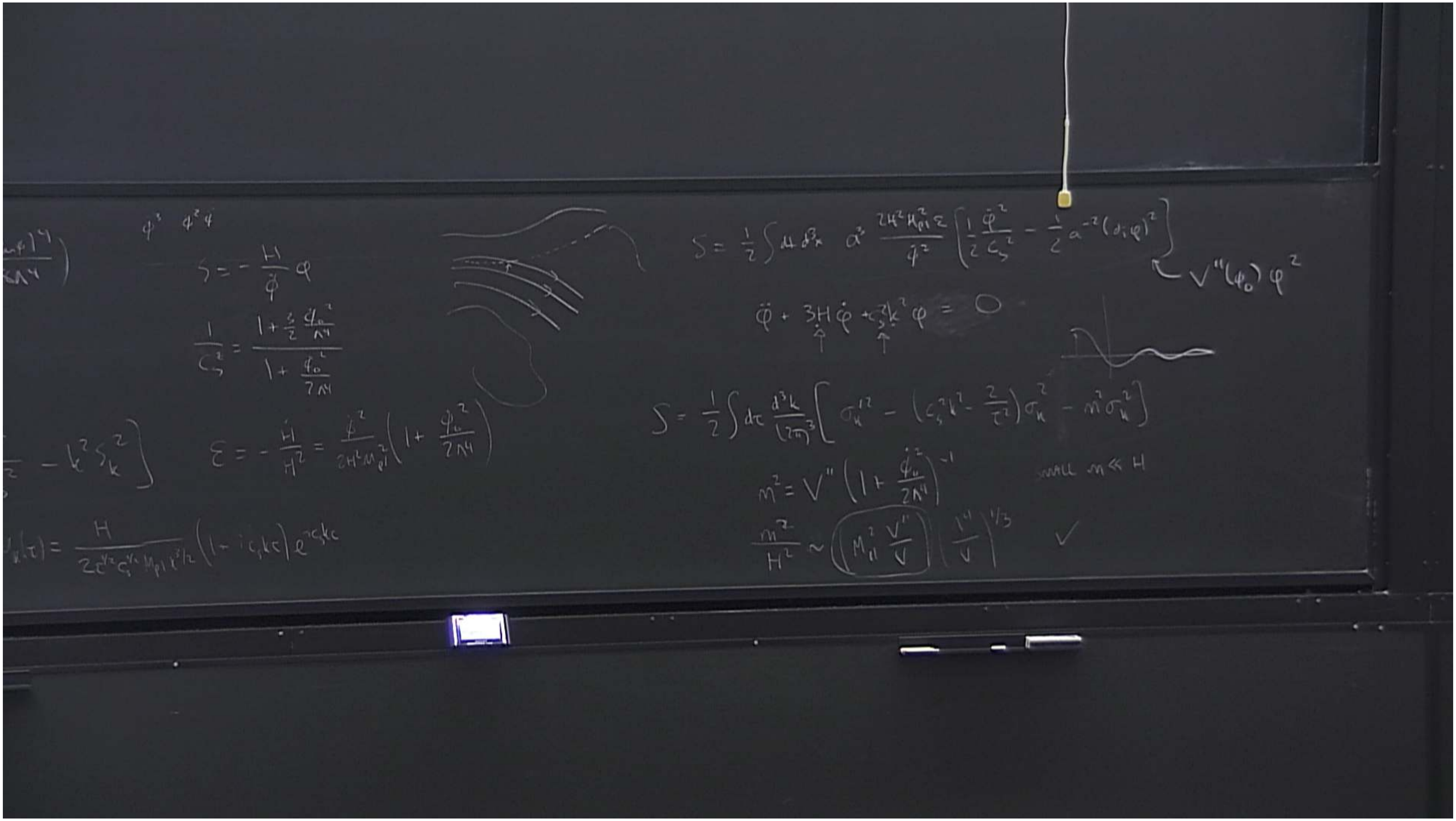
$$S_2 = \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} a^2 (2 M_{pl}^2 \varepsilon) \left[\frac{\zeta_k'^2}{c_s^2} - k^2 \zeta_k^2 \right]$$

$$\varepsilon = -\frac{\dot H}{H^2} = \frac{\dot\phi^2}{2H^2 M_{pl}^2} \left(1 + \frac{\dot\phi_0^2}{2\Lambda^4} \right)$$

$$\hat\zeta_k(\tau) = u_k(\tau) a_k + u_k^*(\tau) a_{-k}^\dagger$$

$$u_k(\tau) = \frac{H}{2\varepsilon^{1/2} c_s^{1/4} M_{pl} k^{3/2}} (1 - i c_s k \tau) e^{-i c_s k \tau}$$


$\phi^3 \quad \phi^2 4$
$\zeta = -\frac{H}{\dot\phi}\varphi$
$\frac{1}{c_s^2} = \frac{1+\frac{3}{2}\frac{\dot\phi_0^2}{\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$
$\epsilon = -\frac{\dot H}{H^2} = \frac{\dot\phi^2}{2H^2M_{pl}^2}\left(1+\frac{\dot\phi_0^2}{2\Lambda^4}\right)$
$S = \frac{1}{2}\int dt\, d^3x\, a^3 \frac{2H^2 M_{pl}^2 \epsilon}{\dot\phi^2}\left[\frac{1}{2}\frac{\dot\varphi^2}{c_s^2} - \frac{1}{2}a^{-2}(\partial_i\varphi)^2\right]$
$\ddot\varphi + 3H\dot\varphi + c_s^2 k^2 \varphi = 0$



$\varphi^3 \quad \varphi^2\varphi$
$\zeta = -\frac{H}{\dot\varphi}\varphi$
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$V''(\varphi_0)\varphi^2$
$\ddot\varphi + 3H\dot\varphi + c_s^2k^2\varphi = 0$
$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\sigma_k'^2 - \left(c_s^2k^2 - \frac{2}{\tau^2}\right)\sigma_k^2 - m^2\sigma_k^2\right]$
$m^2 = V''\left(1+\frac{\dot\varphi_0^2}{2\Lambda^4}\right)^{-1}$
small $m \ll H$
$\frac{m^2}{H^2} \sim \left(M_{pl}^2\frac{V''}{V}\right)\left(\frac{\Lambda^4}{V}\right)^{1/3}$
$- k^2\zeta_k^2$
$\zeta_k(\tau) = \frac{H}{2\varepsilon^{1/2}c_s^{1/2}M_{pl}k^{3/2}}(1+ic_sk\tau)e^{-ic_sk\tau}$

$\phi^3 \quad \phi^2\dot{\phi}$

$$\zeta = -\frac{H}{\dot{\phi}}\varphi$$

$$\frac{1}{c_s^2} = \frac{1+\frac{3}{2}\frac{\dot{\phi}_0^2}{\Lambda^4}}{1+\frac{\dot{\phi}_0^2}{2\Lambda^4}}$$

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$$S = \frac{1}{2}\int dt\, d^3x\; a^3\,\frac{2H^2M_{pl}^2\epsilon}{\dot{\phi}^2}\left[\frac{1}{2}\frac{\dot{\varphi}^2}{c_s^2} - \frac{1}{2}a^{-2}(\partial_i\varphi)^2\right]$$

$V''(\phi_0)\varphi^2$

$$\ddot{\varphi} + 3H\dot{\varphi} + c_s^2k^2\varphi = 0$$

$$S = -\frac{1}{2}\int d\tau\,\frac{d^3k}{(2\pi)^3}\left[\sigma_k'^2 - \left(c_s^2k^2 - \frac{2}{\tau^2}\right)\sigma_k^2 - m^2\sigma_k^2\right]$$

$$m^2 = V''\left(1+\frac{\dot{\phi}_0^2}{2\Lambda^4}\right)^{-1}$$

small $m \ll H$

$$\frac{m^2}{H^2} \sim \left(M_{pl}^2\frac{V''}{V}\right)\left(\frac{\Lambda^4}{V}\right)^{1/3}$$

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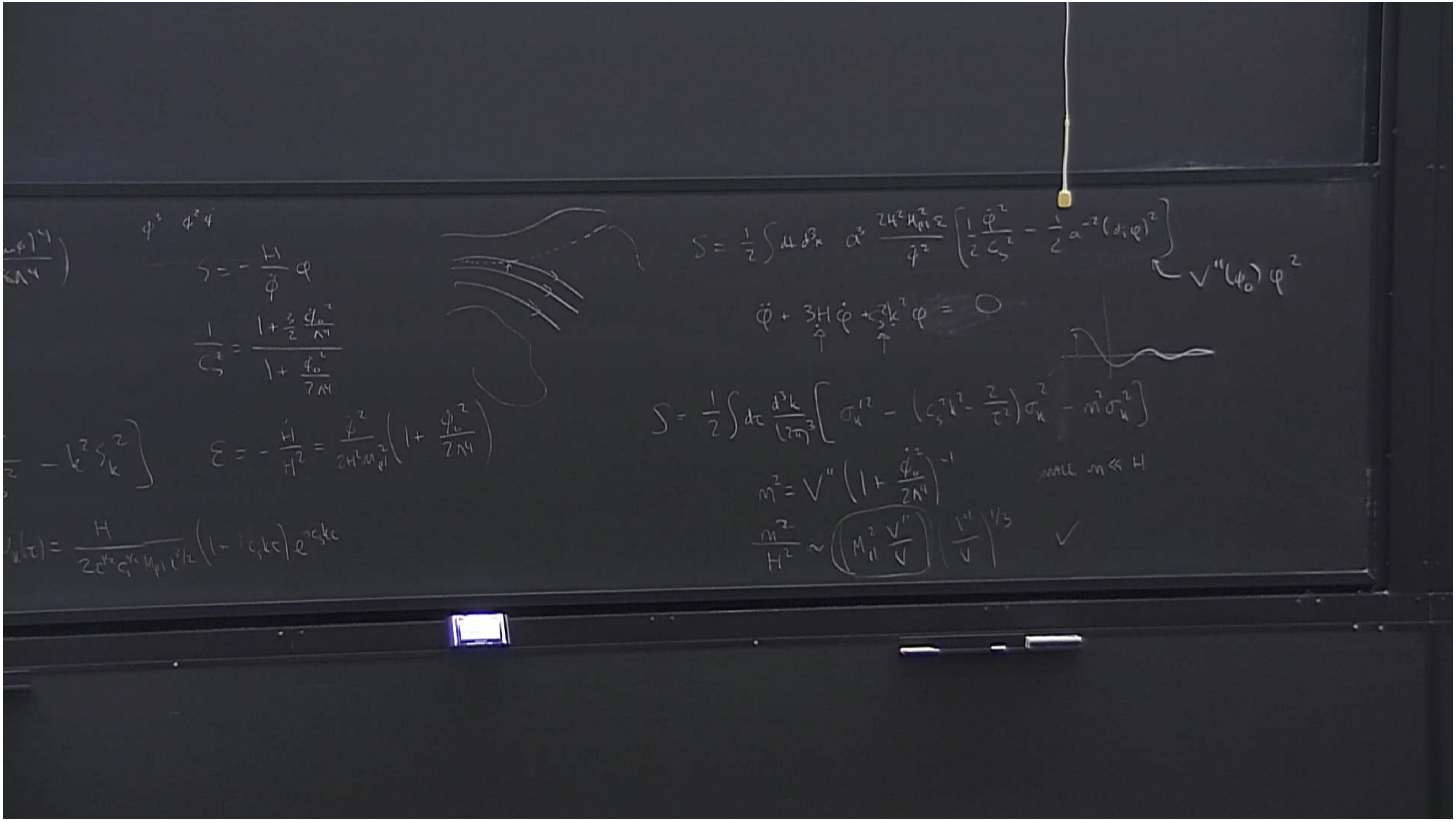

$\zeta = -\frac{H}{\dot\varphi}\varphi$
$\frac{1}{c_s^2} = \frac{1+\frac{3}{2}\frac{\dot\varphi_0^2}{\Lambda^4}}{1+\frac{\dot\varphi_0^2}{2\Lambda^4}}$
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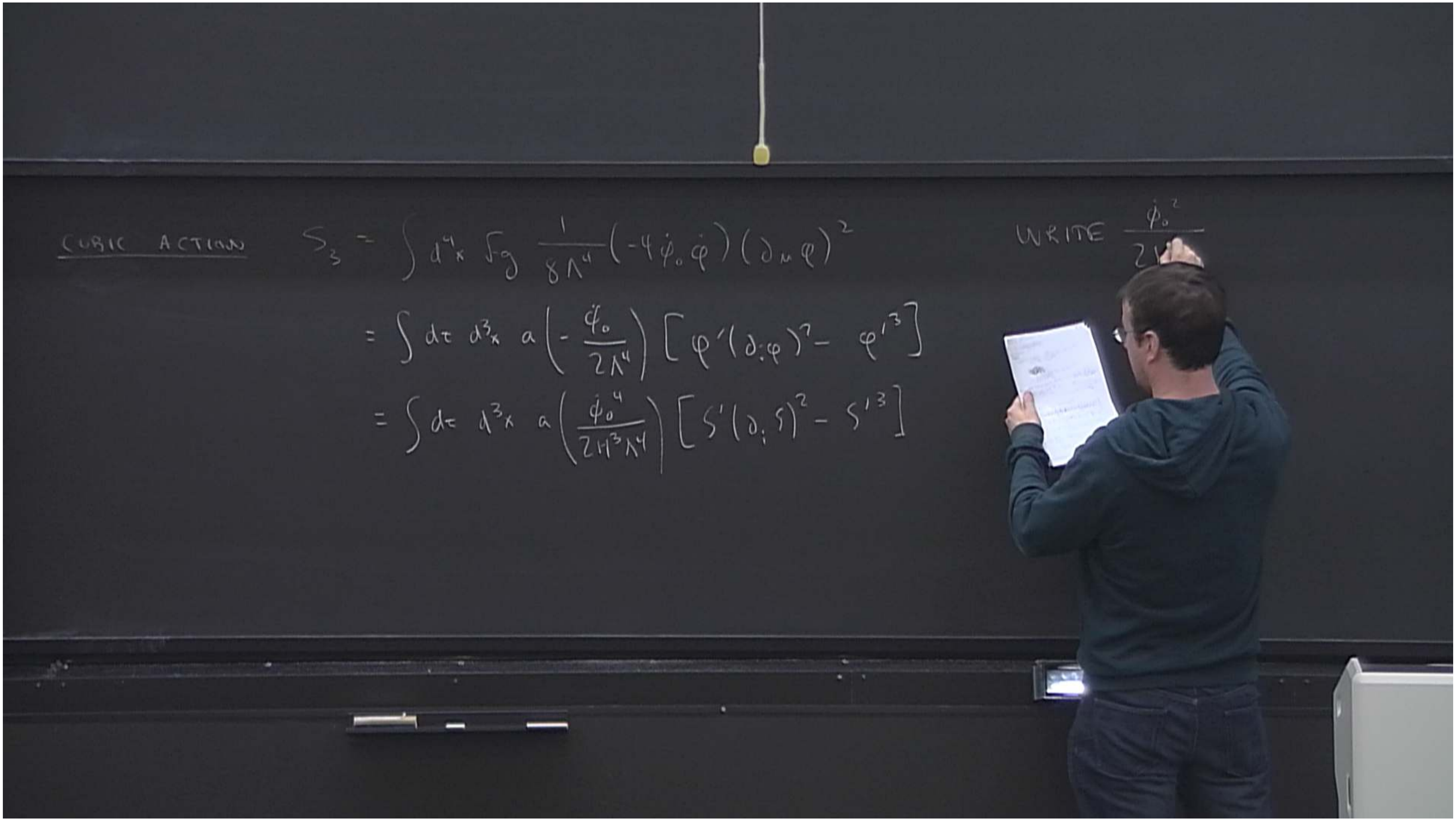
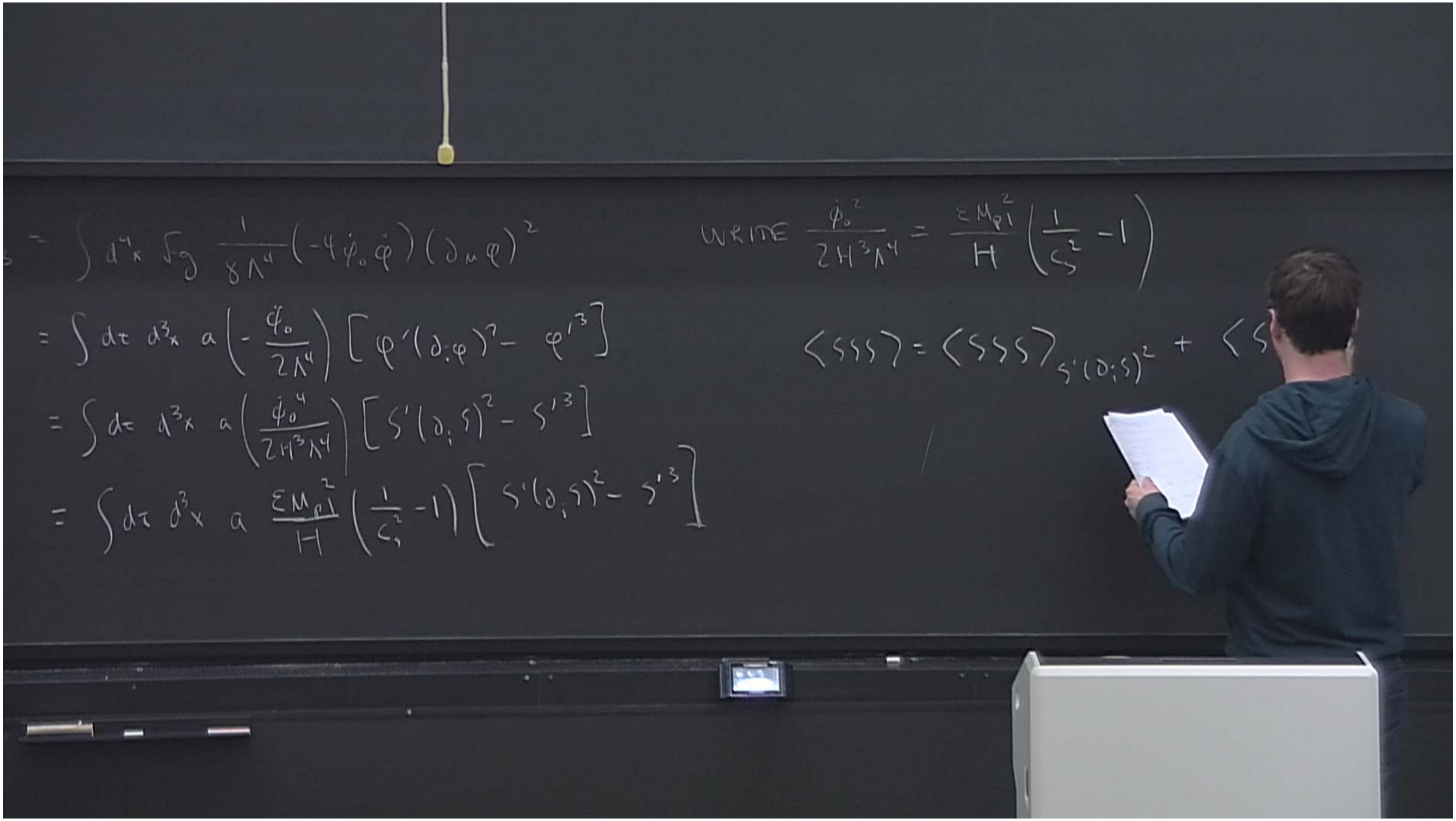

CUBIC ACTION
$S_3 = \int d^4x \sqrt{g}\, \frac{1}{8\Lambda^4} (-4\dot{\varphi}_0 \dot{\varphi}) (\partial_\mu \varphi)^2$
$= \int d\tau\, d^3x\, a \left(-\frac{\dot{\varphi}_0}{2\Lambda^4}\right) \left[\varphi' (\partial_i \varphi)^2 - \varphi'^3\right]$
$= \int d\tau\, d^3x\, a \left(\frac{\dot{\varphi}_0^4}{2H^3\Lambda^4}\right) \left[\zeta' (\partial_i \zeta)^2 - \zeta'^3\right]$
WRITE $\frac{\dot{\varphi}_0^2}{2}$


$S = \int d^4x \sqrt{g} \frac{1}{8\Lambda^4}(-4\dot{\phi}_0\dot{\varphi})(\partial_\mu\varphi)^2$
$= \int d\tau\, d^3x\, a\left(-\frac{\dot{\phi}_0}{2\Lambda^4}\right)\left[\varphi'(\partial_i\varphi)^2 - \varphi'^3\right]$
$= \int d\tau\, d^3x\, a\left(\frac{\dot{\phi}_0^4}{2H^3\Lambda^4}\right)\left[\zeta'(\partial_i\zeta)^2 - \zeta'^3\right]$
$= \int d\tau\, d^3x\, a\, \frac{\varepsilon M_{pl}^2}{H}\left(\frac{1}{c_s^2} - 1\right)\left[\zeta'(\partial_i\zeta)^2 - \zeta'^3\right]$
WRITE $\frac{\dot{\phi}_0^2}{2H^3\Lambda^4} = \frac{\varepsilon M_{pl}^2}{H}\left(\frac{1}{c_s^2} - 1\right)$
$\langle\zeta\zeta\zeta\rangle = \langle\zeta\zeta\zeta\rangle_{\zeta'(\partial_i\zeta)^2} + \langle\zeta$

$$= \int d\tau\, d^3x\; a\left(\frac{\dot\phi_0^{\,4}}{2H^3\Lambda^4}\right)\left[\zeta'(\partial_i\zeta)^2 - \zeta'^3\right]$$

$$= \int d\tau\, d^3x\; a\,\frac{\varepsilon M_{pl}^2}{H}\left(\frac{1}{c_s^2}-1\right)\left[\zeta'(\partial_i\zeta)^2 - \zeta'^3\right]$$

$$\langle\zeta\zeta\zeta\rangle_{\zeta'^3} = -i\int \langle 0|\left[\zeta_{k_1}(0)\,\zeta_{k_2}(0)\,\zeta_{k_3}(0),\, H_I(\tau)\right]|0\rangle$$

$$= -i\int_{-\infty}^{0} d\tau\, u_{k_1}(0)\,u_{k_2}(0)\,u_{k_3}(0)\,\frac{1}{(-H\tau)}\,\frac{\varepsilon M_{pl}^2}{H}\left(\frac{1}{c_s^2}-1\right) 6\, u'_{k_1}(\tau)^*\, u'_{k_2}(\tau)^*\, u'_{k_3}(\tau)^* + c.c.$$

$$= \cdots$$

$$= -\frac{3H^4}{8M_{pl}^4\varepsilon^2}\left(\frac{1}{c_s^2}-1\right)\frac{1}{k_1k_2k_3(\Sigma k_i)^3}$$

LIKEWISE $$\langle\zeta\zeta\zeta\rangle_{\zeta'(\partial\zeta)^2} = \frac{H^4}{16\varepsilon c_s^2 M_{pl}^4}\left(\frac{1}{c_s^2}-1\right)\frac{k_2\cdot k_3}{k_1k_2^2k_3^2}\left(\frac{1}{\Sigma k_i} + \frac{k_2+k_3}{(\Sigma k_i)^2} + 2\frac{k_2k_3}{(\Sigma k_i)^3}\right) + 2\text{ PERM.}$$

$$= \cdots$$

$$= -\frac{3H^4}{8M_{pl}^4\varepsilon^2}\left(\frac{1}{c_s^2}-1\right)\frac{1}{k_1k_2k_3(\Sigma k_i)^3}$$

LIKEWISE $\langle \zeta\zeta\zeta\rangle'_{\dot\zeta(\partial\zeta)^2} = \frac{H^4}{16\varepsilon^2 c_s^2 M_{pl}^4}\left(\frac{1}{c_s^2}-1\right)\frac{\vec k_2\cdot\vec k_3}{k_1k_2^2k_3^2}\left(\frac{1}{\Sigma k_i}+\frac{k_2+k_3}{(\Sigma k_i)^2}+2\frac{k_2k_3}{(\Sigma k_i)^3}\right)$ + 2 PERM.

$$\Rightarrow \langle\zeta\zeta\rangle = \frac{H^2}{4M_{pl}^2\varepsilon c_s}\frac{1}{k^3} \qquad \langle\zeta\zeta\zeta\rangle = \mathcal{O}\left(\frac{H^4}{\varepsilon^2 M_{pl}^4}\right)$$

$\frac{S}{N}$ PER MODE $\sim \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^{3/2}} = \frac{H^4\varepsilon^{-2}M_{pl}^{-4}}{H^3\varepsilon^{-3/2}M_{pl}^{-3}} = H\varepsilon^{-1/2}M_{pl}^{-1} \sim \Delta_\zeta = [2\times10^{-9}]^{1/2} \sim [\text{FEW}\times10^{-5}]$

$$\Delta_\zeta\,[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)]$$

$$N \approx \left(\frac{S}{N}\right)^{-2} \sim 10^9$$

$$S = \int d^4x \sqrt{-g}\left(-\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) + \underbrace{\frac{(\partial_\mu\phi)^4}{8\Lambda^4}}\right)$$

$\phi^3 \quad \phi^2\phi$

$$\zeta = -\frac{H}{\dot\phi}\varphi$$

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$$\dot\phi \sim V \quad \Lambda$$

$$\dot\phi^4 \gg \Lambda^4$$

$$\frac{1}{c_s^2} = \frac{1+\frac{3}{2}\frac{\dot\phi_0^2}{\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$$

$$(\partial_\mu\phi)^2 = -\dot\phi_0^2 - 2\dot\phi_0\dot\varphi + (\partial_\mu\varphi)^2$$

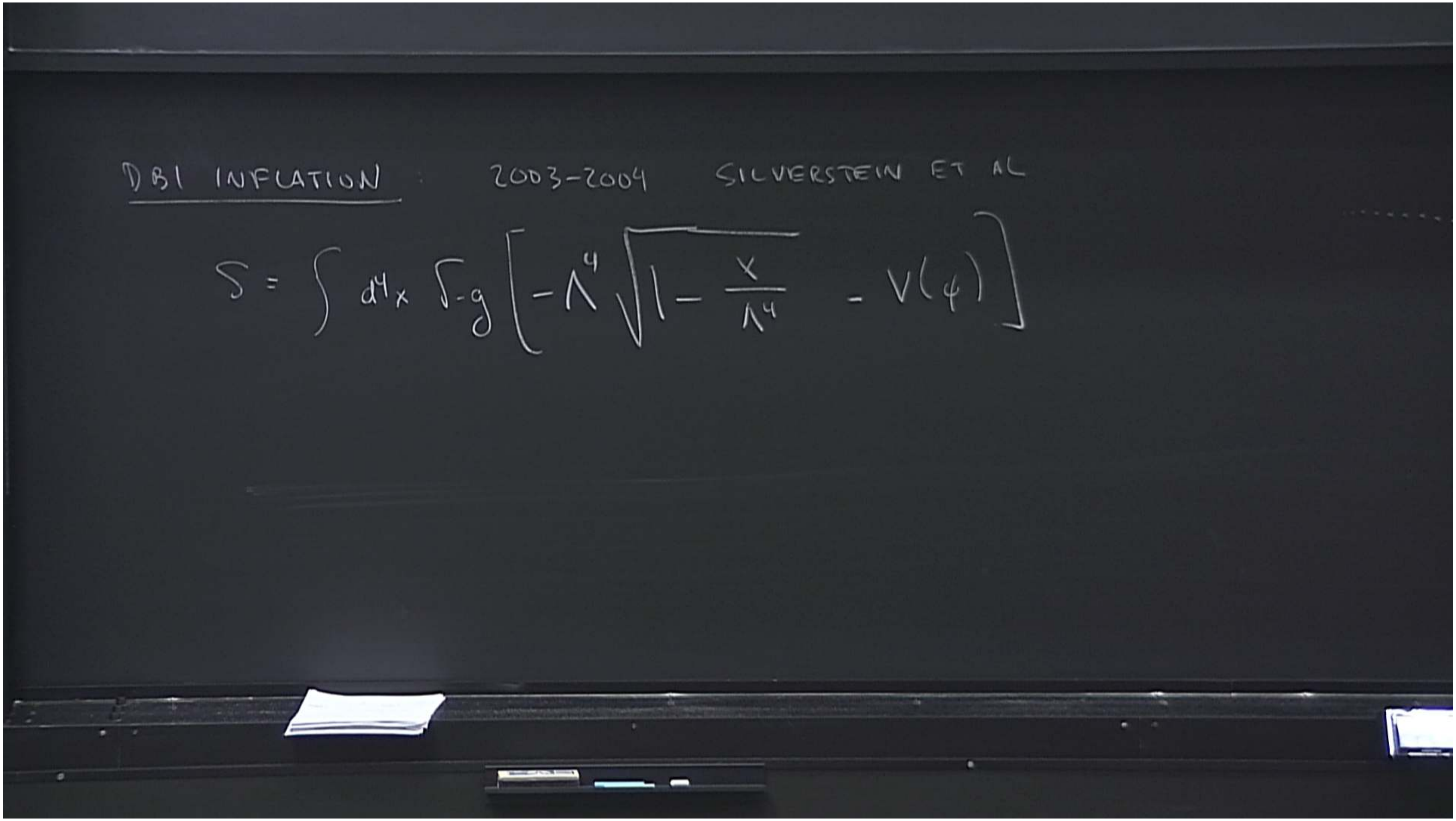
$$S = S_0 + S_1 + S_2 + S_3 + S_4$$

$$S_2 = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} a^2 (2M_{pl}^2\varepsilon)\left[\frac{\zeta_k'^2}{c_s^2} - k^2\zeta_k^2\right]$$

$$\varepsilon = -\frac{\dot H}{H^2} = \frac{\dot\phi^2}{2H^2M_{pl}^2}\left(1+\frac{\dot\phi_0^2}{2\Lambda^4}\right)$$

$$\hat\zeta_k(\tau) = u_k(\tau)a_k + u_k^*(\tau)a_{-k}^\dagger$$

$$u_k(\tau) = \frac{H}{2\varepsilon^{1/2}c_s^{1/2}M_{pl}k^{3/2}}(1 - ic_sk\tau)e^{ic_sk\tau}$$


DBI INFLATION : 2003-2004 SILVERSTEIN ET AL
$S = \int d^4x \sqrt{-g} \left[-\Lambda^4 \sqrt{1 - \frac{X}{\Lambda^4}} - V(\phi) \right]$

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$$S = \int d^4x \sqrt{-g} \left[-\Lambda^4 \sqrt{1 - \frac{X}{\Lambda^4}} - V(\phi) \right]$$

DBI INFLATION : 2003-2004 SILVERSTEIN ET AL

$$S = \int d^4x \sqrt{-g}\left[-\Lambda^4\sqrt{1-\frac{X}{\Lambda^4}} - V(\phi)\right] \qquad X = -\frac{1}{2}(\partial_\mu\phi)^2$$

WE'LL ASSUME $\Lambda = $ CONST. BUT IT CAN A FUNCTION OF ϕ

DEFINE $F(X,\phi) = -\Lambda^4\sqrt{1-\frac{X}{\Lambda^4}} - V(\phi)$

MANY OF OUR CALCULATIONS WILL APPLY T

SILVERSTEIN ET AL

$$\frac{X}{\Lambda^4} - V(\phi) \Big] \qquad X = -\frac{1}{2}(\partial_\mu \phi)^2$$

$$M_{pl} \frac{V'}{V} \sim \mathcal{O}(1)$$

UT IT CAN A FUNCTION OF ϕ

MANY OF OUR CALCULATIONS WILL APPLY TO $S = \int d^4x \sqrt{-g}\, F$

$- V(\phi)$

LIKEWISE $\langle \zeta\zeta\zeta \rangle_{\dot{\zeta}(\partial\zeta)^2} = \frac{H^4}{16\varepsilon^2 c_s^2 M_{pl}^4}\left(\frac{1}{c_s^2}-1\right)\frac{k_2\cdot k_3}{k_1 k_2^2 k_3^2}\left(\frac{\ldots}{\sum k_i} + \frac{\ldots}{(\sum k_i)^2} + 2\frac{\ldots}{(\sum k_i)^3}\right)$

E.G. BACKGROUND STRESS-ENERGY $\quad \rho = 2XF_X - F$

$a(t) \qquad \phi(t) \qquad\qquad p = F$

$$F_{XX} = \frac{\partial^2 F}{\partial X^2}$$

FRIEDMANN: $\quad H^2 = \frac{\rho}{3M_{pl}^2}$

$$\dot{H} = -\frac{1}{2M_{pl}^2}(\rho + p) = -\frac{1}{M_{pl}^2}XF_X \iff \varepsilon = -\frac{\dot{H}}{H^2} = \frac{XF_X}{H^2M_{pl}^2}$$

ϕ EQ OF MOTION $\quad (2XF_{XX} + F_X)\ddot{\phi} + 3HF_X\dot{\phi} - F_\phi + 2XF_{X\phi} = 0$

DBI INFLATION $F = -\Lambda^4 \sqrt{1 - \frac{X}{\Lambda^4}} - V(\phi)$

1) $M_{pl} \frac{V'}{V} = \mathcal{O}(1)$

2) $M_{pl}^2 \frac{V''}{V} \lesssim \mathcal{O}(1)$

3) $\frac{\Lambda^4}{V} \ll 1$

$\Rightarrow$ SLOW-ROLL REGIME

$H^2 \approx \frac{V}{3M_{pl}^2}$

$3HF_X \dot{\phi} \approx -V'(\phi)$

$\Rightarrow \varepsilon \ll 1$

$\Rightarrow X \approx \Lambda^4 \quad \left(1 - \frac{X}{\Lambda^4}\right) \approx \left(\frac{\Lambda^4}{V}\right)$

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$$X = -\frac{1}{2}(\partial_\mu \phi)^2$$

$$= \frac{1}{2}\dot{\phi}^2 + \dot{\phi}\dot{\varphi} - \frac{1}{2}(\partial_\mu \varphi)^2$$

$$S = \int d^4x \left[\bar{F} + \bar{F}_X \delta X + \frac{1}{2}\bar{F}_{XX}(\delta X)^2 + \frac{1}{6}\bar{F}_{XXX}(\delta X)^3 + \cdots \right]$$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, a^2 \left[(F_X + 2X F_{XX})\varphi'^2 - F_X (\partial_i \varphi)^2 \right]$$

DEFINE $\frac{1}{c_s^2} =$

$(\partial_\mu \varphi)^2$

$\frac{1}{2}\bar{F}_{XX}(\delta X)^2 + \frac{1}{6}\bar{F}_{XXX}(\delta X)^3 + \cdots \Big]$

$2XF_{XX})\,\varphi'^2 - F_X(\partial_i\varphi)^2\Big]$

DEFINE $\frac{1}{c_s^2} = \frac{F_X + 2XF_{XX}}{F_X}$

$= 1 + 2\frac{XF_{XX}}{F_X}$

$= \mathcal{O}\left(\frac{V}{\Lambda^4}\right)$

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$$X = -\frac{1}{2}(\partial_\mu \phi)^2$$

$$= \frac{1}{2}\dot{\phi}^2 + \dot{\phi}\dot{\varphi} - \frac{1}{2}(\partial_\mu \varphi)^2$$

$$S = \int d^4x \left[\bar{F} + \bar{F}_X \delta X + \frac{1}{2} \bar{F}_{XX} (\delta X)^2 + \frac{1}{6} \bar{F}_{XXX} (\delta X)^3 + \cdots \right]$$

$$S_2 = \frac{1}{2} \int d\tau\, d^3x\, a^2 \left[(F_X + 2X F_{XX}) \varphi'^2 - F_X (\partial_i \varphi)^2 \right]$$

DEFINE

$$= \frac{1}{2} \int d\tau\, d^3x\, a^2 F_X \left[\frac{1}{c_s^2} \varphi'^2 - (\partial_i \varphi)^2 \right]$$

$$\zeta = \frac{H}{\dot\phi}\varphi \qquad \varepsilon = \frac{X F_X}{H^2 M_{pl}^2}$$

$$S_2 = \frac{1}{2}\int d\tau\, d^3x \; a^2 (2\varepsilon M_{pl}^2)\left[\frac{1}{c_s^2}\zeta'^2 - (\partial_i \zeta)^2\right]$$

MODE FUNCTIONS

$$\hat\zeta_k(\tau) = u_k(\tau)\, a_k + u_k^*(\tau)\, a_{-k}^\dagger$$

$$u_k(\tau) = \frac{H}{2\varepsilon^{1/2} c_s^{1/2} M_{pl} k^{3/2}}(1 + ik c_s \tau)\, e^{-ik c_s \tau}$$

IN DBI, $\frac{1}{c_s^2} = \mathcal{O}\left(\frac{V}{\Lambda^4}\right) \gg 1$

$$S_3 = \int dt\, d^3x\, a^3 \left[\frac{1}{2} F_{XX} \left(-\dot{\phi}\dot{\varphi}\right) \left(\partial_\mu \varphi\right)^2 + \frac{1}{6} F_{XXX}\, \dot{\phi}^3 \dot{\varphi}^3 \right]$$

$$= \int d\tau\, d^3x\, a \left[-\frac{\dot{\phi}^2}{2} F_{XX}\, \varphi' \left(\partial_i \varphi\right)^2 + \left(\frac{\dot{\phi}^2}{2} F_{XX} + \frac{\dot{\phi}^3}{6} F_{XXX} \right) \varphi'^3 \right]$$

$$\zeta = -\frac{H}{\dot{\phi}} \varphi$$

$$\gamma \overset{\text{DEF}}{=} \frac{X F_{XXX}}{F_{XX}}$$

WRITE $F_{XX} = \dfrac{2\epsilon H^2 M_{pl}^2}{\dot{\phi}^4} \left(\dfrac{1}{c_s^2} - 1 \right)$

$$S_3 = \int d\tau\, d^3x\, a \left(\frac{\epsilon M_{pl}^2}{H} \right) \left(\frac{1}{c_s^2} - 1 \right) \left[\zeta' \left(\partial_i \zeta\right)^2 - \underbrace{\left(1 + \frac{2}{3}\gamma\right)} \varphi'^3 \right]$$

$$\langle \zeta\zeta\zeta \rangle = \frac{H^4}{16\,\varepsilon^2 c_s^2 M_{pl}^4}\left(\frac{1}{c_s^2}-1\right)\frac{k_2\cdot k_3}{k_1 k_2^2 k_3^2}\left(\frac{1}{\Sigma k_i}+\frac{k_2+k_3}{(\Sigma k_i)^2}+2\frac{k_2 k_3}{(\Sigma k_i)^3}\right) + 2\ \text{PERM.}$$

$$-\ \frac{3H^4}{8 M_{pl}^4 \varepsilon^2}\left(\frac{1}{c_s^2}-1\right)\left(1+\frac{2}{3}\gamma\right)\frac{1}{k_1 k_2 k_3 (\Sigma k_i)^3}$$

IN DBI, $\gamma \approx \mathcal{O}\left(\frac{1}{c_s^2}\right)$

$\Delta_s[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)] \leftarrow$ SF SLOW ROLL

$\Delta_s \longleftarrow (\partial_\mu \phi)^4$

$$\langle \zeta\zeta\zeta \rangle = \mathcal{O}\left(\frac{H^4}{\varepsilon^2 c_s^4 M_{pl}^4}\right)$$

$$\frac{S}{N} = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} = \frac{H^4 \varepsilon^{-2} c_s^{-4} M_{pl}^{-4}}{H^3 \varepsilon^{-3/2} c_s^{-3/2} M_{pl}^{-3}} = H \varepsilon^{-1/2} c_s^{-5/2} M_{pl}^{-1} = \Delta_s c_s^{-2} \leftarrow \text{DBI}$$

$$-1\Big)\frac{k_2+k_3}{k_1k_2^2k_3^2}\left(\frac{1}{\Sigma k_i}+\frac{k_2+k_3}{(\Sigma k_i)^2}+2\frac{k_2k_3}{(\Sigma k_i)^3}\right)+\text{2 PERM.}$$

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$$\left(\frac{1}{c_s^2}-1\right)\left(1+\frac{2}{3}\gamma\right)\frac{1}{k_1k_2k_3(\Sigma k_i)^3}$$

IN DBI, $\gamma \approx \mathcal{O}\left(\frac{1}{c_s^2}\right)$

$\Delta_s\left[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)\right] \leftarrow$ SF SLOW ROLL

$\Delta_s \longleftarrow (\partial_\mu\phi)^4$

$$\frac{H^4\varepsilon^{-2}c_s^{-4}M_{pl}^{-4}}{H^3\varepsilon^{-3/2}c_s^{-3/2}M_{pl}^{-3}} = H\varepsilon^{-1/2}c_s^{-5/2}M_{pl}^{-1} = \Delta_s c_s^{-2} \leftarrow \text{DBI}$$