

Title: Explorations in Cosmology-12

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Abstract:

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \psi)^2}{4\Lambda^4} \right)$$

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$$(\partial_\mu \phi)^2 = -\dot{\phi}_0^2 - 2\dot{\phi}_0 \dot{\varphi} + (\partial_\mu \varphi)^2$$

$$S = S_0 + S_1 + S_2 + S_3 + S_4$$

$$S_2 = \frac{1}{2} \int d^3k \frac{d^3k'}{(2\pi)^3} a^2 (2M_{\text{pl}}^2 \epsilon) \left[\frac{S_h^{12}}{c_s^2} - k^2 S_k^2 \right]$$

$$\hat{\chi}_k(\tau) = u_k(\tau) a_k + u_k^*(\tau) a_{-k}^+$$

$$u_k(\tau) = \frac{H}{2c_s^2 c_s^2 M_{\text{pl}}^2 k^2} (1 + i c_s k \tau) e^{-i c_s k \tau}$$

$\phi^3 \phi^2 \psi$

$$S = -\frac{H}{\phi} \varphi$$

$$\frac{1}{c_s^2} = \frac{1 + \frac{c}{2} \frac{\phi_0^2}{\Lambda^4}}{1 + \frac{\phi_0^2}{2M^2}}$$

$$\epsilon = -\frac{H}{H^2} = \frac{k^2}{2H^2 M_{\text{pl}}^2} \left(1 + \frac{\phi_0^2}{2M^2} \right)$$



$\frac{2\pi^2}{8\pi^4}$

$\psi^2 \psi^2$

$$\gamma = -\frac{H}{\dot{\phi}} \phi$$

$$\frac{1}{c_s^2} = \frac{1 + \frac{5}{2} \frac{d\ln^2}{d\ln}}{1 + \frac{4\alpha}{7M^2}}$$



$$S = \frac{1}{2} \int 4d^3x \alpha^2 \frac{24^2 M_{pl}^2 \Sigma}{\dot{\phi}^2} \left[\frac{1}{2} \frac{\dot{\phi}^2}{c_s^2} - \frac{1}{2} \alpha^{-2} (\partial_i \phi)^2 \right]$$

$$\ddot{\phi} + 3H \dot{\phi} + \frac{3}{2} k^2 \phi = 0$$



$\frac{1}{2} - k^2 S_k^2$

$$\epsilon = -\frac{H}{H^2} = \frac{\dot{\phi}^2}{24^2 M_{pl}^2} \left(1 + \frac{4\alpha}{7M^2} \right)$$

$$\psi(k, \tau) = \frac{H}{2c^{\frac{3}{2}} c_s^{\frac{3}{2}} M_{pl}^{\frac{3}{2}} \tau^{\frac{3}{2}}} (1 + i c_s k \tau) e^{-i c_s k \tau}$$

$$\frac{H}{2M^2} \left(\frac{H}{2M^2} \right)$$

$$\phi^2 \phi^2$$

$$S = -\frac{H}{\phi} \phi$$

$$\frac{1}{c_s^2} = \frac{1 + \frac{c}{2} \frac{\phi_0^2}{M^4}}{1 + \frac{\phi_0^2}{2M^4}}$$



$$S = \frac{1}{2} \int d^3x \alpha^2 \frac{2H^2 M_{pl}^2 \Sigma}{\dot{\phi}^2} \left\{ \frac{1}{2} \frac{\dot{\phi}^2}{c_s^2} - \frac{1}{2} a^{-2} (\partial_i \phi)^2 \right\}$$

$\sqrt{V''(\phi_0)} \phi^2$

$$\ddot{\phi} + 3H\dot{\phi} + c_s^2 k^2 \phi = 0$$



$$\left[-k^2 S_k \right]$$

$$\epsilon = -\frac{H}{H^2} = \frac{k^2}{2H^2 M_{pl}^2} \left(1 + \frac{\phi_0^2}{2M^4} \right)$$

$$S = \frac{1}{2} \int dt \frac{d^3k}{(2\pi)^3} \left[\sigma_k'^2 - (c_s^2 k^2 - \frac{2}{t^2}) \sigma_k^2 - m^2 \sigma_k^2 \right]$$

$$M^2 = V'' \left(1 + \frac{\phi_0^2}{2M^4} \right)^{-1}$$

small $m \ll H$

$$h_k(\tau) = \frac{H}{2c_s^{3/2} M_{pl}^{3/2}} (1 - i c_s k \tau) e^{-i c_s k \tau}$$

$$\frac{m^2}{H^2} \sim \left(\frac{M_{pl}^2 V''}{V} \right) \left(\frac{1}{V} \right)^{1/3}$$

✓

$\phi^2 \quad \phi^2 \phi$
 $S = -\frac{H}{\phi} \phi$
 $\frac{1}{c_s^2} = \frac{1 + \frac{5}{2} \frac{\phi_0^2}{M^4}}{1 + \frac{\phi_0^2}{2M^4}}$
 $\mathcal{E} = -\frac{H}{H^2} = \frac{k^2}{2H^2 M_{pl}^2} \left(1 + \frac{\phi_0^2}{2M^4} \right)$
 $\psi(\tau) = \frac{H}{2c_s^2 \phi_0 M_{pl}^2} (1 + i c_s k) e^{-i c_s k \tau}$



$$S = \frac{1}{2} \int d^3x \, a^3 \frac{2H^2 M_{pl}^2 \epsilon}{\dot{\phi}^2} \left[\frac{1}{2} \frac{\dot{\phi}^2}{c_s^2} - \frac{1}{2} a^{-2} (\partial_i \phi)^2 \right]$$

$\sqrt{(\phi_0)} \phi^2$

$$\ddot{\phi} + 3H\dot{\phi} + c_s^2 k^2 \phi = 0$$



$$S = \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \left[\sigma_k'^2 - (c_s^2 k^2 - \frac{2}{\tau^2}) \sigma_k^2 - m^2 \sigma_k^2 \right]$$

$$M^2 = V'' \left(1 + \frac{\phi_0^2}{2M^4} \right)^{-1} \quad \text{with } m \ll H$$

$$\frac{m^2}{H^2} \sim \left(\frac{M_{pl}^2 V''}{V} \right) \left(\frac{1}{V} \right)^{1/3} \quad \checkmark$$

$\psi^2 \phi^2 \psi$
 $S = -\frac{H}{\phi} \psi$
 $\frac{1}{c_s^2} = \frac{1 + \frac{3}{2} \frac{\phi_0^2}{M^2}}{1 + \frac{\phi_0^2}{2M^2}}$
 $\mathcal{E} = -\frac{H}{H^2} = \frac{\dot{\phi}^2}{2H^2 M_{pl}^2} \left(1 + \frac{\phi_0^2}{2M^2} \right)$
 $\psi(k, \tau) = \frac{H}{2c_s^2 a^2 M_{pl}^2 \tau^2} (1 + i c_s k \tau) e^{-i c_s k \tau}$



$$S = \frac{1}{2} \int d^3x \alpha^2 \frac{2H^2 M_{pl}^2 \Sigma}{\dot{\phi}^2} \left[\frac{1}{2} \frac{\dot{\phi}^2}{c_s^2} - \frac{1}{2} a^{-2} (\partial_i \phi)^2 \right] \sqrt{V(\phi_0)} \phi^2$$

$$\ddot{\phi} + 3H\dot{\phi} + c_s^2 k^2 \phi = 0$$



$$S = \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \left[\sigma_k'^2 - (c_s^2 k^2 - \frac{2}{\tau^2}) \sigma_k^2 - m^2 \sigma_k^2 \right]$$

$$M^2 = V'' \left(1 + \frac{\phi_0^2}{2M^2} \right)^{-1} \quad \text{with } m \ll H$$

$$\frac{M^2}{H^2} \sim \left(\frac{M_{pl}^2 V''}{V} \right) \left(\frac{V'}{V} \right)^{1/3} \quad \checkmark$$

CUBIC ACTION

$$\begin{aligned} S_3 &= \int d^4x \sqrt{g} \frac{1}{8\Lambda^4} (-4\dot{\phi}_0 \dot{\phi}) (\partial_\mu \phi)^2 \\ &= \int dt d^3x a \left(-\frac{\dot{\phi}_0}{2\Lambda^4} \right) [\dot{\phi} (\partial_i \phi)^2 - \dot{\phi}^3] \\ &= \int dt d^3x a \left(\frac{\dot{\phi}_0^4}{2H^3 \Lambda^4} \right) [S'(\partial_i S)^2 - S'^3] \end{aligned}$$

WRITE $\frac{\dot{\phi}_0^2}{2\Lambda^4}$

$$\begin{aligned}
&= \int d^4x \sqrt{g} \frac{1}{8\Lambda^4} (-4\dot{\phi}_0 \dot{\phi}) (\partial_\mu \phi)^2 \\
&= \int dt d^3x a \left(-\frac{\dot{\phi}_0}{2\Lambda^4} \right) [\dot{\phi} (\partial_i \phi)^2 - \phi'^3] \\
&= \int dt d^3x a \left(\frac{\dot{\phi}_0^4}{2H^3 \Lambda^4} \right) [s' (\partial_i s)^2 - s'^3] \\
&= \int dt d^3x a \frac{\epsilon_{\text{Mpl}}^2}{H} \left(\frac{1}{c_s^2} - 1 \right) [s' (\partial_i s)^2 - s'^3]
\end{aligned}$$

WRITE $\frac{\dot{\phi}_0^2}{2H^3 \Lambda^4} = \frac{\epsilon_{\text{Mpl}}^2}{H} \left(\frac{1}{c_s^2} - 1 \right)$

$$\langle sss \rangle = \langle sss \rangle_{s'(\partial_i s)^2} + \langle sss \rangle_{s'^3}$$

$$= \int dt d^3x a \left(\frac{\dot{\phi}_0^4}{2H^3 M_{pl}^4} \right) [s'(0, \eta)^2 - s'^3]$$

$$= \int dt d^3x a \frac{\epsilon_{M_{pl}}^2}{H} \left(\frac{1}{c_s^2} - 1 \right) [s'(0, \eta)^2 - s'^3]$$

$$\langle sss \rangle_{s'^3} = -i \int \langle 0 | [s_{k_1}(0) s_{k_2}(0) s_{k_3}(0), H_E(\tau)] | 0 \rangle$$

$$= -i \int_{-\infty}^0 dt u_{k_1}(0) u_{k_2}(0) u_{k_3}(0) \frac{1}{(-H\tau)} \frac{\epsilon_{M_{pl}}^2}{H} \left(\frac{1}{c_s^2} - 1 \right) (6 u_{k_1}'(\tau)^* u_{k_2}'(\tau)^* u_{k_3}'(\tau)^* + c.c.)$$

$$= \dots$$

$$= - \frac{3H^4}{8M_{pl}^4 \epsilon^2} \left(\frac{1}{c_s^2} - 1 \right) \frac{1}{k_1 k_2 k_3 (\sum k_i)^3}$$

LIKELIHOOD $\langle sss \rangle_{s'(0, \eta)^2} = \frac{H^4}{16\epsilon c_s^2 M_{pl}^4} \left(\frac{1}{c_s^2} - 1 \right) \frac{k_2 \cdot k_3}{k_1 k_2 k_3} \left(\frac{1}{\sum k_i} + \frac{k_2 + k_3}{(\sum k_i)^2} + 2 \frac{k_2 k_3}{(\sum k_i)^3} \right) + 2 \text{ PERM.}$

$$= \dots$$

$$= -\frac{3H^4}{8M_{pl}^4 \epsilon^2} \left(\frac{1}{c_s^2} - 1 \right) \frac{1}{k_1 k_2 k_3 (\sum k_i)^3}$$

LIKEWISE $\langle sss \rangle_{s(\text{os})}^2 = \frac{H^4}{16\epsilon^2 c_s^2 M_{pl}^4} \left(\frac{1}{c_s^2} - 1 \right) \frac{k_2 k_3}{k_1 k_2 k_3} \left(\frac{1}{\sum k_i} + \frac{k_2 + k_3}{(\sum k_i)^2} + 2 \frac{k_2 k_3}{(\sum k_i)^3} \right) + 2 \text{ PERM.}$

$$\Rightarrow \langle ss \rangle = \frac{H^2}{4M_{pl}^2 \epsilon c_s} \frac{1}{k^3} \quad \langle ssss \rangle = \mathcal{O} \left(\frac{H^4}{\epsilon^2 M_{pl}^4} \right)$$

$$\frac{s}{N} \text{ PER MODE} \sim \frac{\langle ssss \rangle}{\langle ss \rangle^{3/2}} = \frac{H^4 \epsilon^{-2} M_{pl}^{-4}}{H^3 \epsilon^{-3/2} M_{pl}^{-3}} = H \epsilon^{-1/2} M_{pl}^{-1} \sim \Delta_s = [2 \times 10^{-9}]^{1/2} \sim [FEW \times 10^{-5}]$$

" $\Delta_s [\theta(\epsilon) + \theta(\eta)]$

$$N \approx \left(\frac{s}{N} \right)^{-2} \sim 10^9$$

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) + \underbrace{\frac{(\partial_\mu \phi)^4}{8\Lambda^4}}_{\phi^3 \phi^2 \phi} \right)$$

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$$(\partial_\mu \phi)^2 = -\dot{\phi}_0^2 - 2\dot{\phi}_0 \dot{\varphi} + (\partial_\mu \varphi)^2 \quad \begin{matrix} \dot{\varphi} \sim V \Lambda \\ \dot{\varphi}^4 \Rightarrow \Lambda^4 \end{matrix}$$

$$S = -\frac{H}{\dot{\phi}} \varphi$$

$$\frac{1}{c_s^2} = \frac{1 + \frac{3}{2} \frac{\dot{\phi}_0^2}{\Lambda^4}}{1 + \frac{\dot{\phi}_0^2}{2\Lambda^4}}$$

$$S = S_0 + S_1 + S_2 + S_3 + S_4$$

$$S_2 = \frac{1}{2} \int d^3k \frac{d^3k'}{(2\pi)^3} a^2 (2M_{pl})^2 \epsilon \left[\frac{S_{h12}}{c_s^2} - k^2 S_k^2 \right]$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2 M_{pl}^2} \left(1 + \frac{\dot{\phi}_0^2}{2\Lambda^4} \right)$$

$$\hat{\chi}_k(\tau) = u_k(\tau) a_k + u_k^*(\tau) a_{-k}^+$$

$$u_k(\tau) = \frac{H}{2\epsilon^{1/2} c_s^{1/2} M_{pl} k^{3/2}} (1 + i c_s k \tau) e^{i c_s k \tau}$$

DBI INFLATION

2003-2004

SILVERSTEIN ET AL

$$S = \int d^4x \sqrt{-g} \left[-\Lambda^4 \sqrt{1 - \frac{\dot{\chi}}{\Lambda^4}} - V(\phi) \right]$$

DBI INFLATION

2003-2004

SILVERSTEIN ET AL

$$S = \int d^4x \sqrt{-g} \left[-\Lambda^4 \sqrt{1 - \frac{\dot{\varphi}^2}{\Lambda^4}} - V(\varphi) \right]$$

DBI INFLATION

2003-2004

SILVERSTEIN ET AL

$$S = \int d^4x \sqrt{-g} \left[-\Lambda^4 \sqrt{1 - \frac{x}{\Lambda^4}} - V(\phi) \right] \quad X = -\frac{1}{2}(\partial_\mu \phi)^2$$

WE'LL ASSUME $\Lambda = \text{CONST.}$ BUT IT CAN A FUNCTION OF ϕ

DEFINE $F(X, \phi) = -\Lambda^4 \sqrt{1 - \frac{x}{\Lambda^4}} - V(\phi)$

MANY OF OUR CALCULATIONS WILL APPLY TO

SILVERSTEIN ET AL

$$\left[\frac{\chi}{\Lambda^4} - V(\phi) \right]$$

$$X = -\frac{1}{2}(\partial_\mu \phi)^2$$



$$M_{pl} \frac{V'}{V} \sim \mathcal{O}(1)$$

IT CAN BE A FUNCTION OF ϕ

$$-V(\phi)$$

MANY OF OUR CALCULATIONS WILL APPLY TO $S = \int d^4x \sqrt{-g} F$

LIKEWISE $\langle \dots \rangle_{\delta}^2 = \frac{H^4}{16 \epsilon^2 c_s^2 M_{pl}^4} \left(\frac{1}{c_s^2} - 1 \right) \frac{k_2 \cdot k_3}{k_1 k_2 k_3} \left(\frac{1}{\sum k_i} + \frac{2}{(\sum k_i)^2} + \frac{2}{(\sum k_i)^3} \right)$

E.G. BACKGROUND STRESS-ENERGY

$a(t)$ $\phi(t)$

$$\rho = 2X F_X - F$$

$$p = F$$

$$F_{XX} = \frac{\partial^2 F}{\partial X^2}$$

FRIEDMANN: $H^2 = \frac{\rho}{3M_{pl}^2}$

$$\dot{H} = -\frac{1}{2M_{pl}^2}(\rho + p) = -\frac{1}{M_{pl}^2} X F_X$$

$$\Leftrightarrow \epsilon = -\frac{\dot{H}}{H^2} = \frac{X F_X}{H^2 M_{pl}^2}$$

ϕ EQ OF MOTION $(2X F_{XX} + F_X) \ddot{\phi} + 3H F_X \dot{\phi} - F_{\phi} + 2X F_{X\phi} = 0$

DBI INFLATION $F = -\Lambda^4 \sqrt{1 - \frac{\dot{\chi}^2}{\Lambda^4}} - V(\phi)$

$$1) M_{pl} \frac{V'}{V} = \mathcal{O}(1)$$

$$2) M_{pl}^2 \frac{V''}{V} \lesssim \mathcal{O}(1)$$

$$3) \frac{\Lambda^4}{V} \ll 1$$

\Rightarrow SLOW-ROLL REGIME

$$H^2 \approx \frac{V}{3M_{pl}^2}$$

$$3HF_{\chi} \dot{\chi} \approx -V'(\phi)$$

$$\Rightarrow \epsilon \ll 1$$

$$\Rightarrow \chi \approx \Lambda^4 \left(1 - \frac{\dot{\chi}^2}{\Lambda^4}\right) \approx \left(\frac{\Lambda^4}{V}\right)$$

$$\phi(x, t) = \phi_0(t) + \varphi(x, t)$$

$$X = -\frac{1}{2} (\partial_\mu \phi)^2$$

$$= \frac{1}{2} \dot{\phi}^2 + \phi \dot{\phi} - \frac{1}{2} (\partial_\mu \phi)^2$$

$$S = \int d^4x \left[\bar{F} + \bar{F}_x \delta X + \frac{1}{2} \bar{F}_{xx} (\delta X)^2 + \frac{1}{6} \bar{F}_{xxx} (\delta X)^3 + \dots \right]$$

$$S_2 = \frac{1}{2} \int dt d^3x a^2 \left[(F_x + 2X F_{xx}) \varphi'^2 - F_x (\partial_i \varphi)^2 \right]$$

DEFINE $\frac{1}{c_s^2} =$

$$\begin{aligned}
 & (\delta \varphi)^2 \\
 & \left[\frac{1}{2} \bar{F}_{xx} (\delta X)^2 + \frac{1}{6} \bar{F}_{xxx} (\delta X)^3 + \dots \right] \\
 & \left[(X F_{xx}) \rho^{1/2} - F_x (\delta \varphi)^2 \right]
 \end{aligned}$$

DEFINE $\frac{1}{C_s^2} = \frac{F_x + 2X F_{xx}}{F_x}$

$$\begin{aligned}
 & = 1 + 2 \frac{X F_{xx}}{F_x} \\
 & = \mathcal{O}\left(\frac{v}{\lambda^4}\right)
 \end{aligned}$$

$$\phi(x, t) = \phi_0(t) + \varphi(x, t)$$

$$X = -\frac{1}{2} (\partial_\mu \varphi)^2$$

$$= \frac{1}{2} \dot{\varphi}^2 + \varphi \ddot{\varphi} - \frac{1}{2} (\partial_\mu \varphi)^2$$

$$S = \int d^4x \left[\bar{F} + \bar{F}_X \delta X + \frac{1}{2} \bar{F}_{XX} (\delta X)^2 + \frac{1}{6} \bar{F}_{XXX} (\delta X)^3 + \dots \right]$$

$$S_2 = \frac{1}{2} \int dt d^3x a^2 \left[(F_X + 2X F_{XX}) \varphi'^2 - F_X (\partial_i \varphi)^2 \right]$$

$$= \frac{1}{2} \int dt d^3x a^2 F_X \left[\frac{1}{c_s^2} \varphi'^2 - (\partial_i \varphi)^2 \right]$$

DEFINE $\frac{1}{c_s^2}$

$$\xi = \frac{H}{\phi} \varphi$$

$$\xi = \frac{X F_X}{H^2 M_{pl}^2}$$

$$\xi_2 = \frac{1}{2} \int d\tau d^3x a^2 (2z M_{pl}^2) \left[\frac{1}{c_s^2} \dot{\xi}^2 - (\partial_i \xi)^2 \right]$$

MODE FUNCTIONS

$$\hat{\xi}_k(\tau) = u_k(\tau) a_k + u_k^*(\tau) a_{-k}^{\dagger}$$

$$u_k(\tau) = \frac{H}{2z^{1/2} c_s^{1/2} M_{pl}^3 / c} (1 + i k c_s \tau) e^{-i k c_s \tau}$$

IN DBI, $\frac{1}{c_s^2} = \mathcal{O}\left(\frac{v^2}{\Lambda^4}\right) \gg 1$

$$S_3 = \int dt d^3x a^3 \left[\frac{1}{2} F_{xx} (-\dot{\phi}) (\partial_m \phi)^2 + \frac{1}{6} F_{xxx} \dot{\phi}^3 \phi'^3 \right]$$

$$= \int dt d^3x a \left[-\frac{\dot{\phi}^2}{2} F_{xx} \phi' (\partial_i \phi)^2 + \left(\frac{\dot{\phi}^2}{2} F_{xx} + \frac{\dot{\phi}^3}{6} F_{xxx} \right) \phi'^3 \right]$$

$$\zeta = -\frac{H}{\dot{\phi}} \phi$$

WRITE $F_{xx} = \frac{2zH^2 M_{pl}^2}{\dot{\phi}^4} \left(\frac{1}{c_s^2} - 1 \right)$ $\delta \stackrel{\text{DEF}}{=} \frac{\lambda F_{xxx}}{F_{xx}}$

$$S_3 = \int dt d^3x a \left(\frac{\epsilon M_{pl}^2}{H} \right) \left(\frac{1}{c_s^2} - 1 \right) \left[\zeta' (\partial_i \zeta)^2 - \left(1 + \frac{2}{3} \delta \right) \zeta'^3 \right]$$

$$\langle SSSS \rangle = \frac{H^4}{16 \epsilon^2 c_s^2 M_{pl}^4} \left(\frac{1}{c_s^2} - 1 \right) \frac{k_2 \cdot k_3}{k_1 k_2^2 k_3^2} \left(\frac{1}{2k_1} + \frac{k_2 + k_3}{(2k_1)^2} + 2 \frac{k_2 k_3}{(2k_1)^3} \right) + 2 \text{PERM.}$$

$$- \frac{3H^4}{8 M_{pl}^4 \epsilon^2} \left(\frac{1}{c_s^2} - 1 \right) \left(1 + \frac{2}{3} \delta \right) \frac{1}{k_1 k_2 k_3 (2k_1)^3}$$

In DBI, $\delta \approx \mathcal{O}\left(\frac{1}{c_s^2}\right)$
 $\Delta_S [0(\eta) + 0(\eta)] \leftarrow \text{SF slow roll}$

$\Delta_S \leftarrow (0_{\text{ind}})^4$

$$\langle SSSS \rangle = \mathcal{O}\left(\frac{H^4}{\epsilon^2 c_s^4 M_{pl}^4}\right)$$

$$\frac{S}{N} = \frac{\langle SSSS \rangle}{\langle SSS \rangle^{3/2}} = \frac{H^4 \epsilon^{-2} c_s^{-4} M_{pl}^{-4}}{H^3 \epsilon^{-3/2} c_s^{-3/2} M_{pl}^{-3}} = H \epsilon^{-1/2} c_s^{-5/2} M_{pl}^{-1} = \Delta_S c_s^{-2} \leftarrow \text{DBI}$$

$$-1) \frac{k_2 k_3}{k_1 k_2 k_3} \left(\frac{1}{\bar{z} k_1} + \frac{k_2 + k_3}{(\bar{z} k_1)^2} + 2 \frac{k_2 k_3}{(\bar{z} k_1)^3} \right) + 2 \text{ PERM.}$$

$$\frac{1}{c_s} - 1) \left(1 + \frac{2}{3} \delta \right) \frac{1}{k_1 k_2 k_3 (\bar{z} k_1)^3}$$

IN DBI, $\delta \approx \mathcal{O}\left(\frac{1}{c_s^2}\right)$

$\Delta_S [O(z) + O(\eta)] \leftarrow$ SF slow roll

$\Delta_S \leftarrow (a_{inf} c)^4$

$$\frac{H^4 \epsilon^{-2} c_s^{-4} M_{pl}^{-4}}{H^3 \epsilon^{-3/2} c_s^{-3/2} M_{pl}^{-3}} = H \epsilon^{-1/2} c_s^{-5/2} M_{pl}^{-1} = \Delta_S c_s^{-2} \leftarrow \text{DBI}$$