Title: Explorations in Cosmology-11

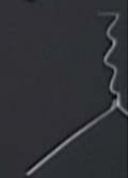
Date: Apr 20, 2015 10:15 AM

URL: http://pirsa.org/15040039

Abstract:

$$(2.2\times 10^{-9})^{1/2}$$

$$N^{-1/2} \gtrsim \frac{S}{N} = \frac{\langle \zeta\zeta\zeta\rangle}{\langle \zeta\zeta\rangle^{3/2}} \sim \frac{H\varepsilon^{-2}\left[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)\right]}{H^{3}\varepsilon^{-3/2}} = \Delta_{\zeta}\left[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)\right]$$



$([\zeta\zeta\gamma])$

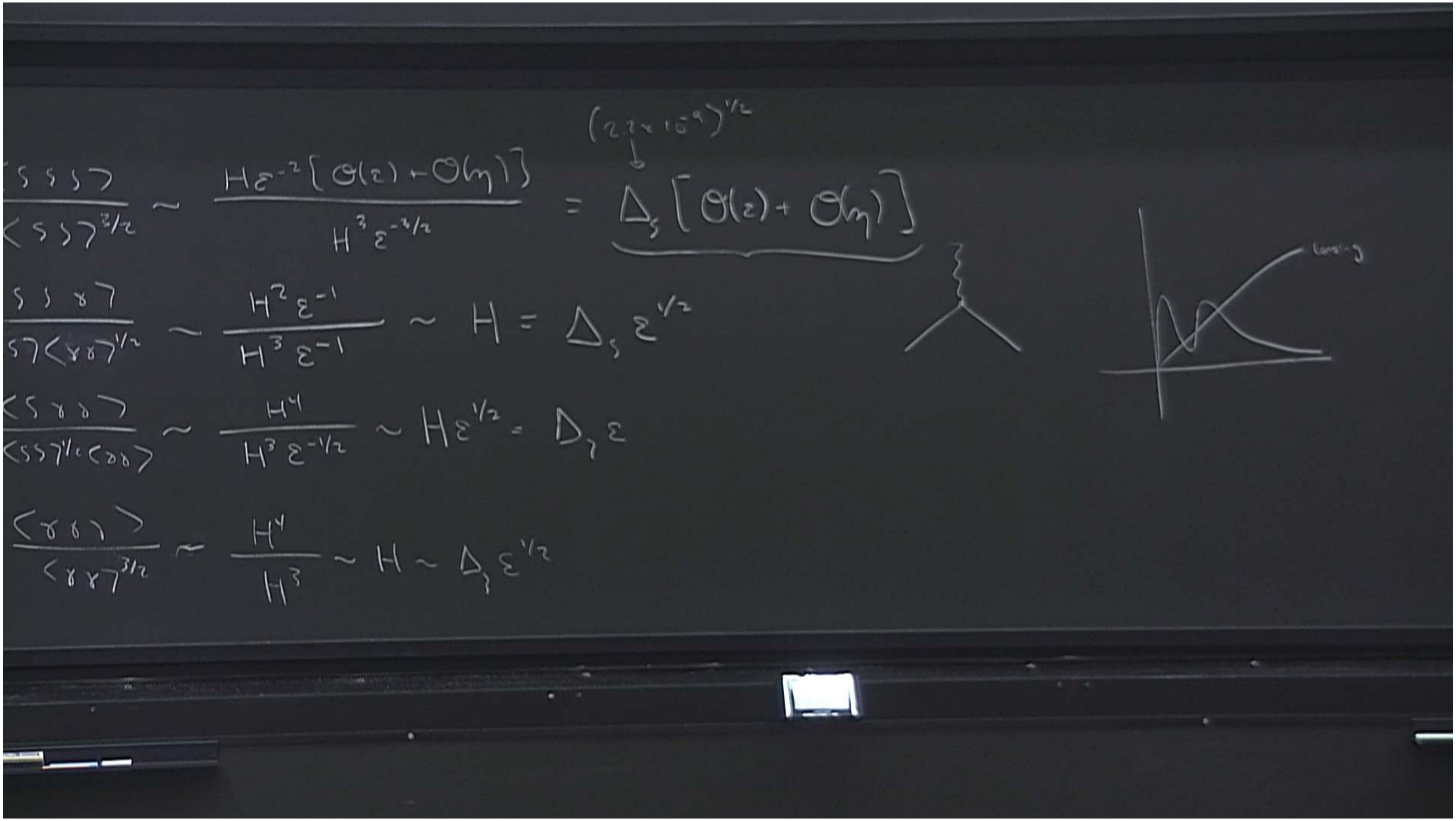
$$\frac{S}{N} = \frac{\langle \zeta\zeta\gamma\rangle}{\langle \zeta\zeta\rangle\langle\gamma\gamma\rangle^{1/2}} \sim \frac{H^{2}\varepsilon^{-1}}{H^{3}\varepsilon^{-1}} \sim H = \Delta_{\zeta}\,\varepsilon^{1/2}$$

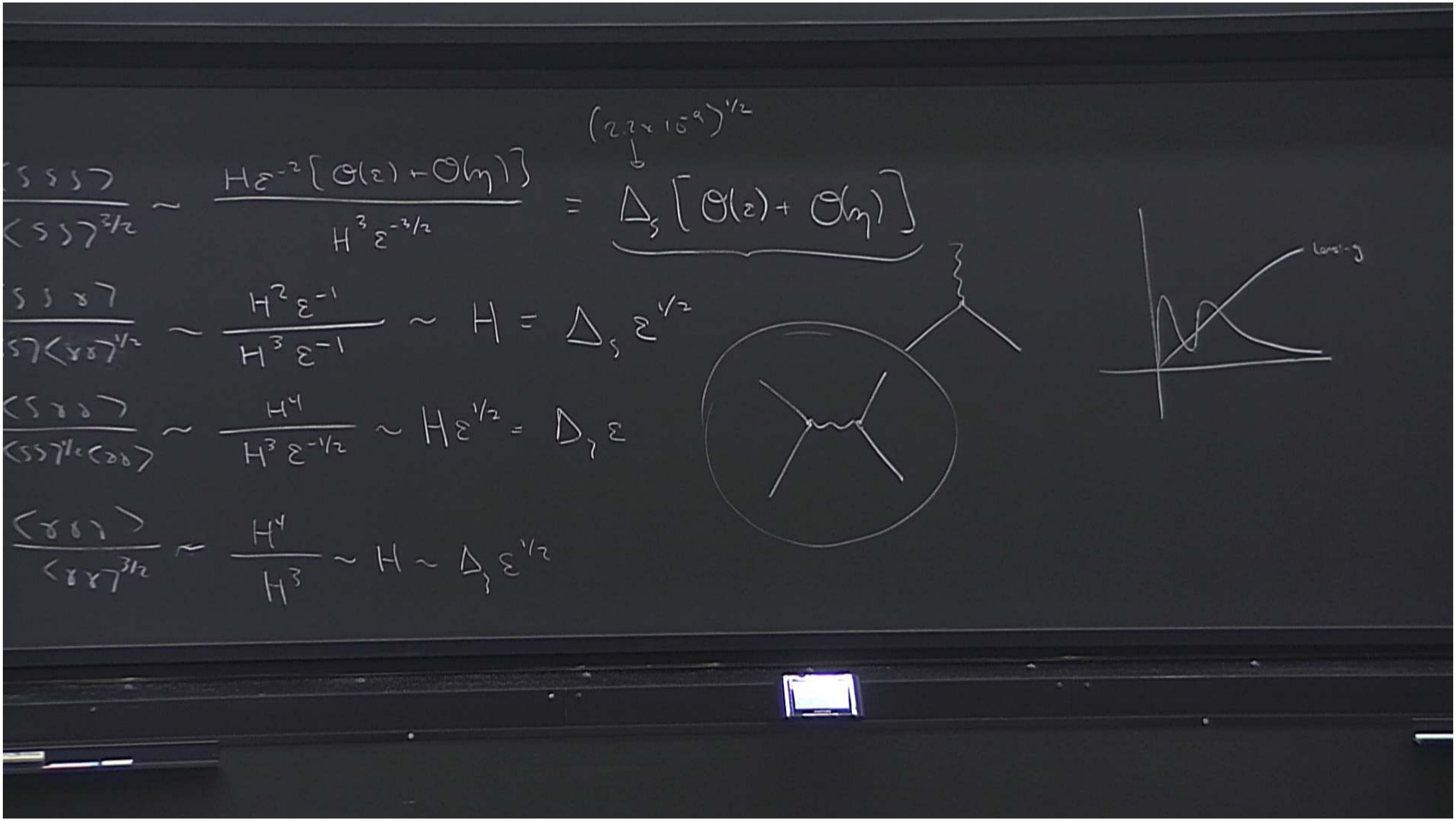
$[\zeta\gamma\gamma]$

$$\frac{S}{N} = \frac{\langle \zeta\gamma\gamma\rangle}{\langle \zeta\zeta\rangle^{1/2}\langle\gamma\gamma\rangle} \sim \frac{H^{4}}{H^{3}\varepsilon^{-1/2}} \sim H\varepsilon^{1/2} = \Delta_{\zeta}\,\varepsilon$$

$[\gamma\gamma\gamma]$

$$\frac{S}{N} = \frac{\langle \gamma\gamma\gamma\rangle}{\langle \gamma\gamma\rangle^{3/2}} \sim \frac{H^{4}}{H^{3}} \sim H \sim \Delta_{\zeta}\,\varepsilon^{1/2}$$


$\frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} \sim \frac{H\varepsilon^{-2}[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)]}{H^3\varepsilon^{-3/2}} = \Delta_\zeta[\mathcal{O}(\varepsilon)+\mathcal{O}(\eta)]$
$(2.2\times10^{-9})^{1/2}$
$\frac{\langle \zeta\zeta\gamma \rangle}{\langle \zeta\zeta \rangle\langle \gamma\gamma \rangle^{1/2}} \sim \frac{H^2\varepsilon^{-1}}{H^3\varepsilon^{-1}} \sim H = \Delta_\zeta\varepsilon^{1/2}$
$\frac{\langle \zeta\gamma\gamma \rangle}{\langle \zeta\zeta \rangle^{1/2}\langle \gamma\gamma \rangle} \sim \frac{H^4}{H^3\varepsilon^{-1/2}} \sim H\varepsilon^{1/2} = \Delta_\zeta\varepsilon$
$\frac{\langle \gamma\gamma\gamma \rangle}{\langle \gamma\gamma \rangle^{3/2}} \sim \frac{H^4}{H^3} \sim H \sim \Delta_\zeta\varepsilon^{1/2}$


(2.2 × 10^-9)^1/2
⟨ζζζ⟩ / ⟨ζζ⟩^3/2 ~ Hε^-2 [O(ε) + O(η)] / H^3 ε^-3/2 = Δ_ζ [O(ε) + O(η)]
⟨ζζγ⟩ / ⟨ζζ⟩⟨γγ⟩^1/2 ~ H^2 ε^-1 / H^3 ε^-1 ~ H = Δ_ζ ε^1/2
⟨ζγγ⟩ / ⟨ζζ⟩^1/2 ⟨γγ⟩ ~ H^4 / H^3 ε^-1/2 ~ Hε^1/2 = Δ_γ ε
⟨γγγ⟩ / ⟨γγ⟩^3/2 ~ H^4 / H^3 ~ H ~ Δ_γ ε^1/2
Lensing

A NON-GAUSSIAN "WARM-UP" MODEL

BAUMANN SEC 5.3.2

CREMINELLI astro-ph/03

$$S = \int d^4 \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4}\right] \qquad g^{\mu\nu},\ \phi$$

$$\boxed{\delta S = -2\int \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}} \qquad \delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}$$

$$= -2\int d^4x \left[-\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}\right]\left[-\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4}\right]$$

$$+ \int d^4 \sqrt{-g}\,\Big[$$

A NON-GAUSSIAN "WARM-UP" MODEL

BAUMANN SEC 5.3.2

CREMINELLI astro-ph/03061..

$$-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$$

$$S = \int d^4 \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) + \frac{(\partial_\mu\phi)^4}{8\Lambda^4}\right] \qquad g^{\mu\nu},\ \phi$$

$$\delta S = -2\int \sqrt{-g}\, T_{\mu\nu}\,\delta g^{\mu\nu} \qquad \delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\,\delta g^{\mu\nu}$$

$$= -2\int d^4x\left[-\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\,\delta g^{\mu\nu}\right]\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) + \frac{(\partial_\mu\phi)^4}{8\Lambda^4}\right]$$

$$+ \int d^4\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu}\right.$$

## A NON-GAUSSIAN "WARM-UP" MODEL

BAUMANN SEC 5.3.2

CREMINELLI astro-ph/03061

$$-\frac{1}{2} g^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi) = -\frac{1}{2}(\partial_\mu \phi)^2$$

$$S = \int d^4 \sqrt{-g} \left[ -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4} \right] \qquad g^{\mu\nu},\ \phi$$

$$\boxed{\delta S = -2 \int \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}} \qquad \delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}$$

$$= -2 \int d^4x \left[ -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu} \right] \left[ -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4} \right]$$

$$+ \int d^4 \sqrt{-g} \left[ -\frac{1}{2}(\partial_\mu \phi)(\partial_\nu \phi)\, \delta g^{\mu\nu} + \frac{1}{4\Lambda^2}(\partial_\rho \phi)(\partial_\mu \phi)(\partial_\nu \phi)\, \delta g^{\mu\nu} \right]$$

A NON-GAUSSIAN "WARM-UP" MODEL

BAUMANN SEC 5.3.2

CREMINELLI astro-ph/030

$$-\frac{1}{2} g^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi)$$

$$S = \int d^4 \sqrt{-g} \left[ -\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4} \right]$$

$g^{\mu\nu}, \phi$

$$\boxed{\delta S = -2 \int \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}}$$

$$\delta \sqrt{-g} = -\frac{1}{2} \sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}$$

$$= -2 \int d^4x \left[ -\frac{1}{2} \sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu} \right] \left[ -\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) + \frac{(\partial_\mu \phi)^4}{8\Lambda^4} \right]$$

$$+ \int d^4 \sqrt{-g} \left[ -\frac{1}{2} (\partial_\mu \phi)(\partial_\nu \phi)\, \delta g^{\mu\nu} + \frac{1}{4\Lambda^2} (\partial_\rho \phi)^2 (\partial_\mu \phi)(\partial_\nu \phi)\, \delta g^{\mu\nu} \right]$$

§3.2

astro-ph/0306122

$a(t)$
$\uparrow$
$g^{\mu\nu}, \phi$

$$\delta g_{\mu\nu} = - g_{\mu\lambda} g_{\nu\rho} \delta g^{\rho\sigma}$$

$$T_{\mu\nu} = \left(-\frac{1}{2}(\partial_\rho\phi)^2 - V(\phi) + \frac{(\partial_\rho\phi)^4}{8\Lambda^4}\right) g_{\mu\nu} + \left(1 - \frac{(\partial_\rho\phi)^2}{2\Lambda^4}\right)$$

$$\left.\begin{aligned} \rho = T_{00} &= \frac{1}{2}\dot\phi^2 + V(\phi) + \frac{3}{8\Lambda^4}\dot\phi^4 \\ p &= \frac{1}{2}\dot\phi^2 - V(\phi) + \frac{1}{8\Lambda^4}\dot\phi^4 \end{aligned}\right\} \Rightarrow \left( H^2 = \frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\right.\right.$$

$\delta g^{\mu\nu}$

$\frac{(\partial_\mu\phi)^4}{8\Lambda^4}\Big]$

$\phi)\, \delta g^{\mu\nu}\Big]$

$$\ldots g_{\lambda\rho}\,\delta g^{\rho\sigma}$$

$$\ldots (\partial_\rho\phi)^2 - V(\phi) + \frac{(\partial_\rho\phi)^4}{8\Lambda^4}\Big)\, g_{\mu\nu} + \left(1 - \frac{(\partial_\rho\phi)^2}{2\Lambda^4}\right)\partial_\mu\phi\,\partial_\nu\phi$$

$$\left.\begin{array}{l} \ldots\ \frac{1}{2}\dot\phi^2 + V(\phi) + \frac{3}{8\Lambda^4}\dot\phi^4 \\ \ldots\ - V(\phi) + \frac{1}{8\Lambda^4}\dot\phi^4 \end{array}\right\} \Rightarrow \boxed{H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot\phi^2 + V(\phi) + \frac{3}{8\Lambda^4}\dot\phi^4\right)}$$

EOM FOR $\phi$:

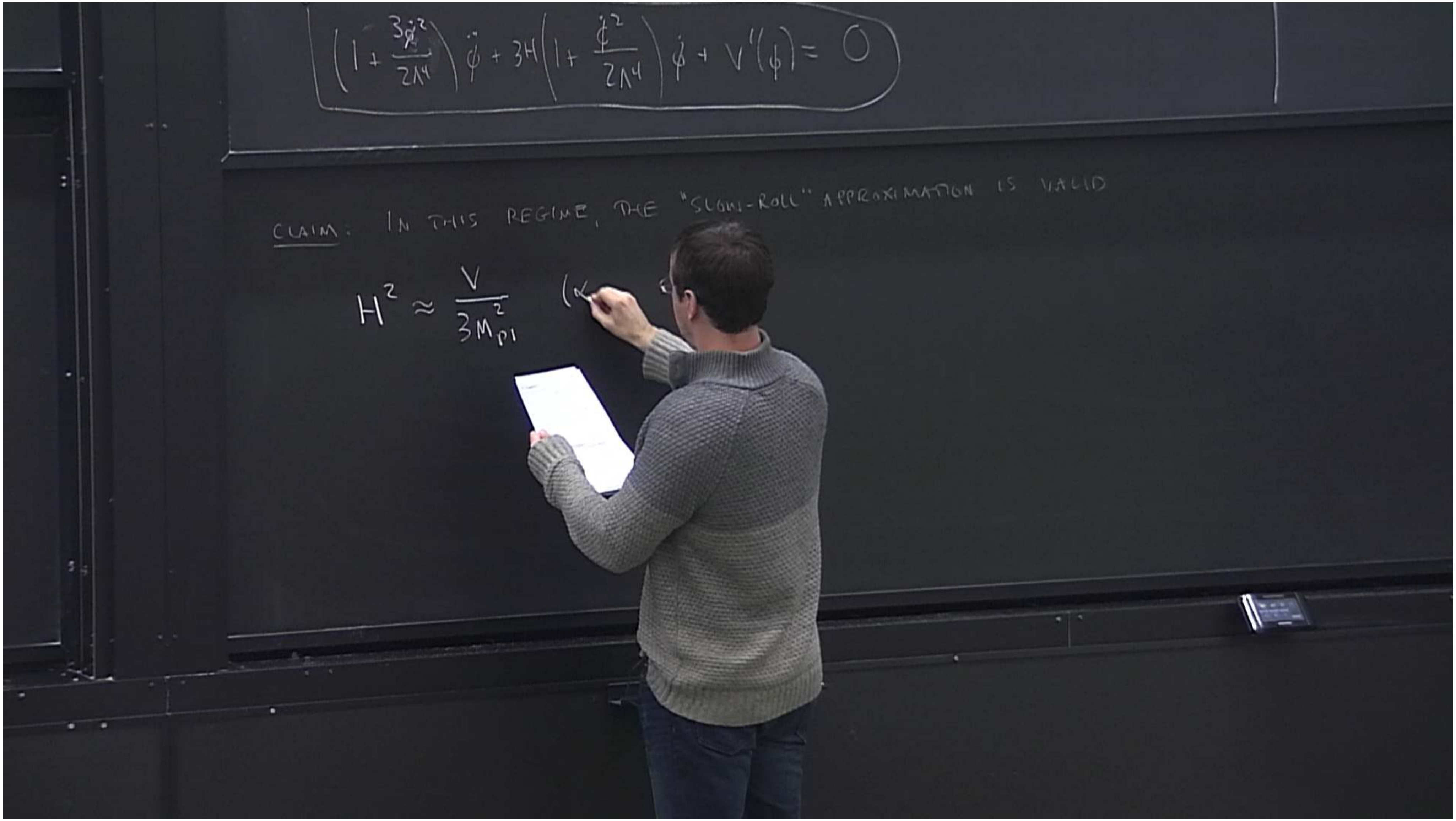
$$0 = \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial_\mu \phi} \right) - \frac{\partial \mathcal{L}}{\partial \phi}$$

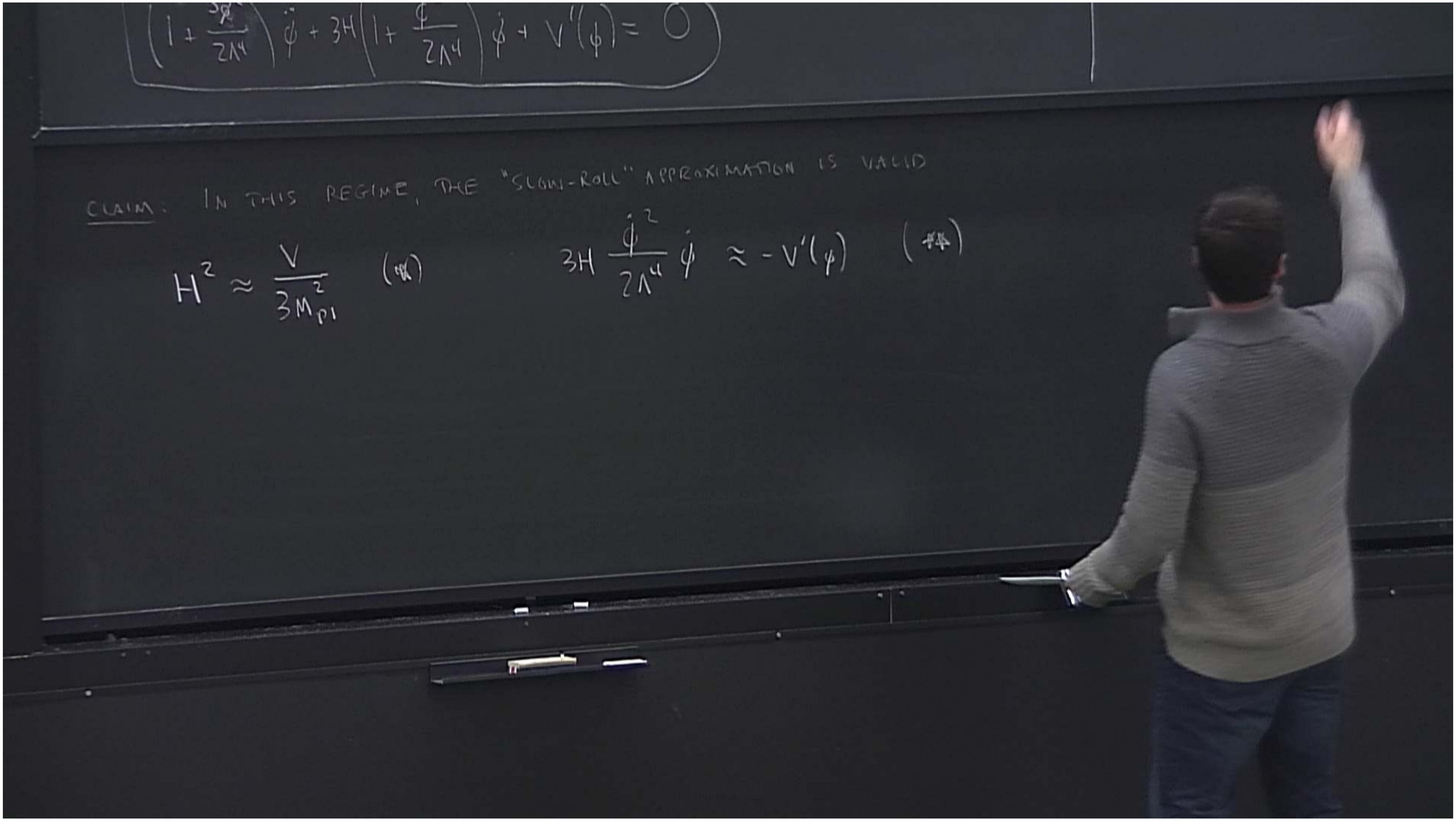
$$= \partial_\mu \left( -\sqrt{-g}\, \partial^\mu \phi + \frac{1}{2\Lambda^4} \sqrt{-g}\, (\partial_\rho \phi)^2 \partial^\mu \phi \right) + \sqrt{-g}\, V'(\phi)$$

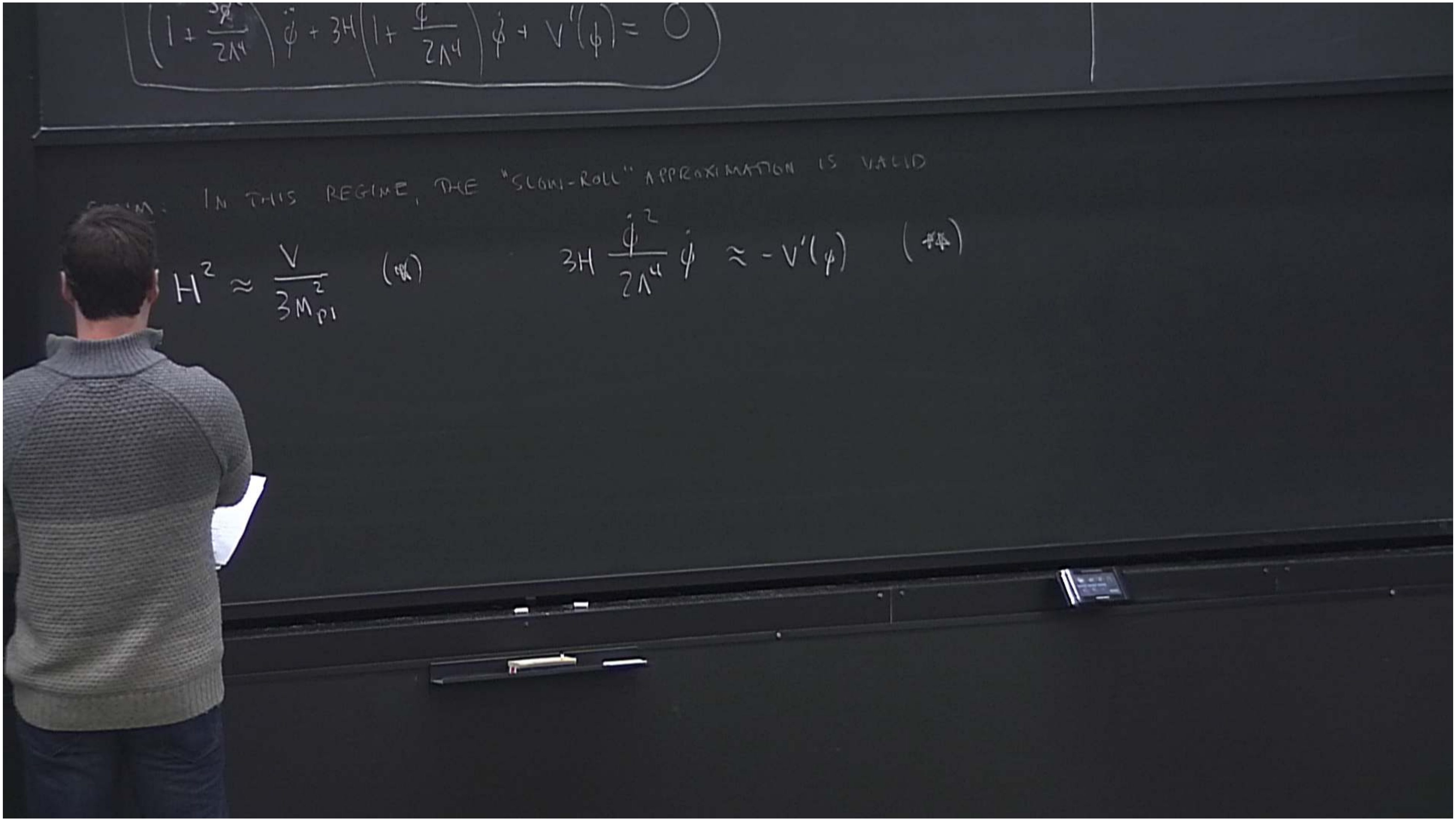
$$= \frac{d}{dt}\left[ -a^3 \dot\phi + \frac{1}{2\Lambda^4} a^3 \dot\phi^2 \dot\phi \right] + a^3 V'(\phi)$$

$$= a^3 \left( \ddot\phi + \frac{3}{2\Lambda^4} \dot\phi^2 \ddot\phi \right) + 3 a^3 H \left( \dot\phi + \frac{1}{2\Lambda^4} \dot\phi^3 \right) + a^3 V'(\phi)$$

$$\boxed{\left(1 + \frac{3\dot\phi^2}{2\Lambda^4}\right) \ddot\phi + 3H\left(1 + \frac{\dot\phi^2}{2\Lambda^4}\right)\dot\phi + V'(\phi) = 0}$$


(1 + 3φ̇²/2Λ⁴) φ̈ + 3H (1 + φ̇²/2Λ⁴) φ̇ + V'(φ) = 0
CLAIM: IN THIS REGIME, THE "SLOW-ROLL" APPROXIMATION IS VALID
H² ≈ V / 3M²pl


$\left(1 + \frac{3\dot{\phi}^2}{2\Lambda^4}\right)\ddot{\phi} + 3H\left(1 + \frac{\dot{\phi}^2}{2\Lambda^4}\right)\dot{\phi} + V'(\phi) = 0$
CLAIM: IN THIS REGIME, THE "SLOW-ROLL" APPROXIMATION IS VALID
$H^2 \approx \frac{V}{3M_{Pl}^2}$ (*)
$3H \frac{\dot{\phi}^2}{2\Lambda^4}\dot{\phi} \approx -V'(\phi)$ (**)


$\left(1+\frac{3\dot\phi^2}{2\Lambda^4}\right)\ddot\phi + 3H\left(1+\frac{\dot\phi^2}{2\Lambda^4}\right)\dot\phi + V'(\phi) = 0$
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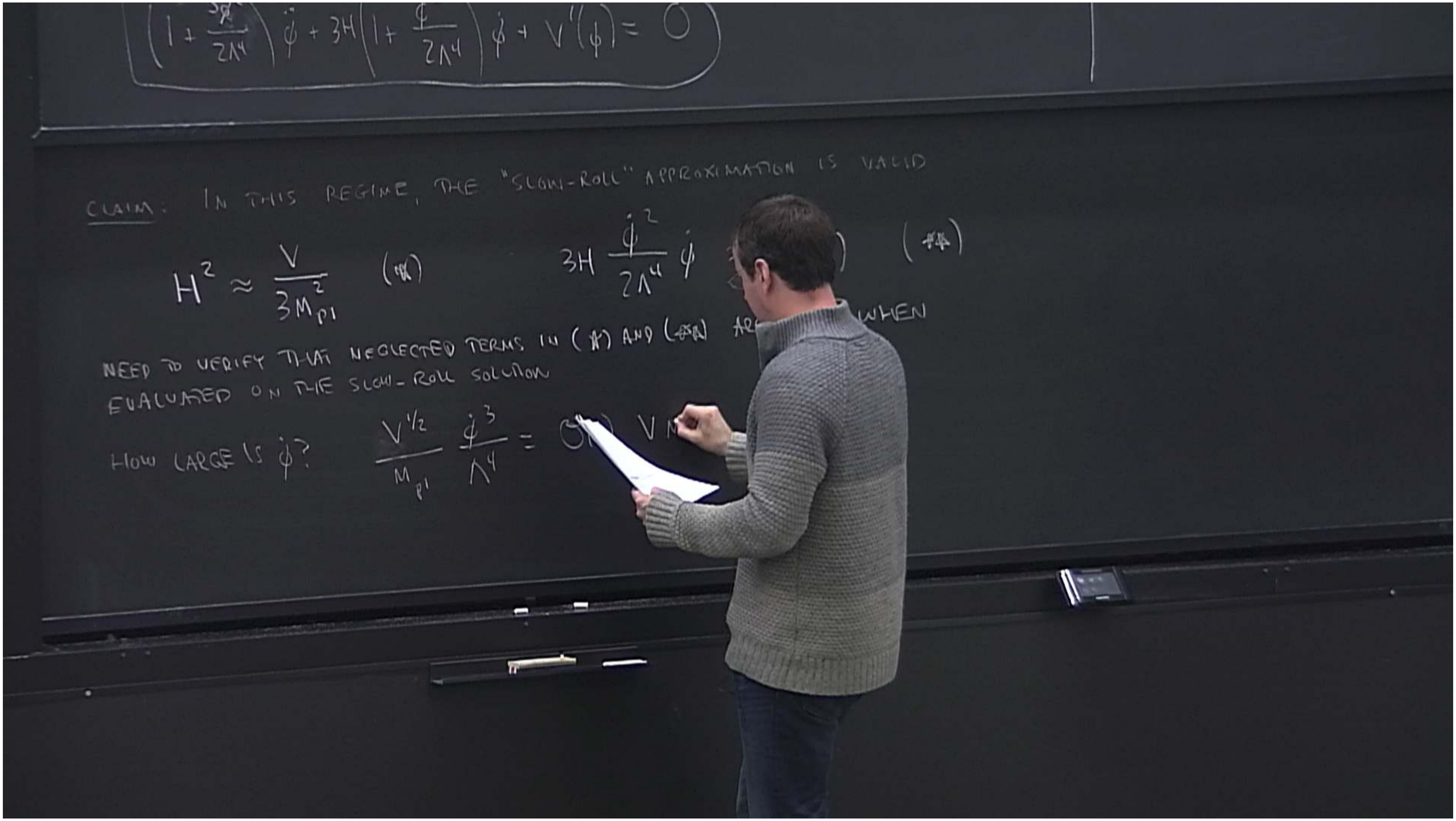
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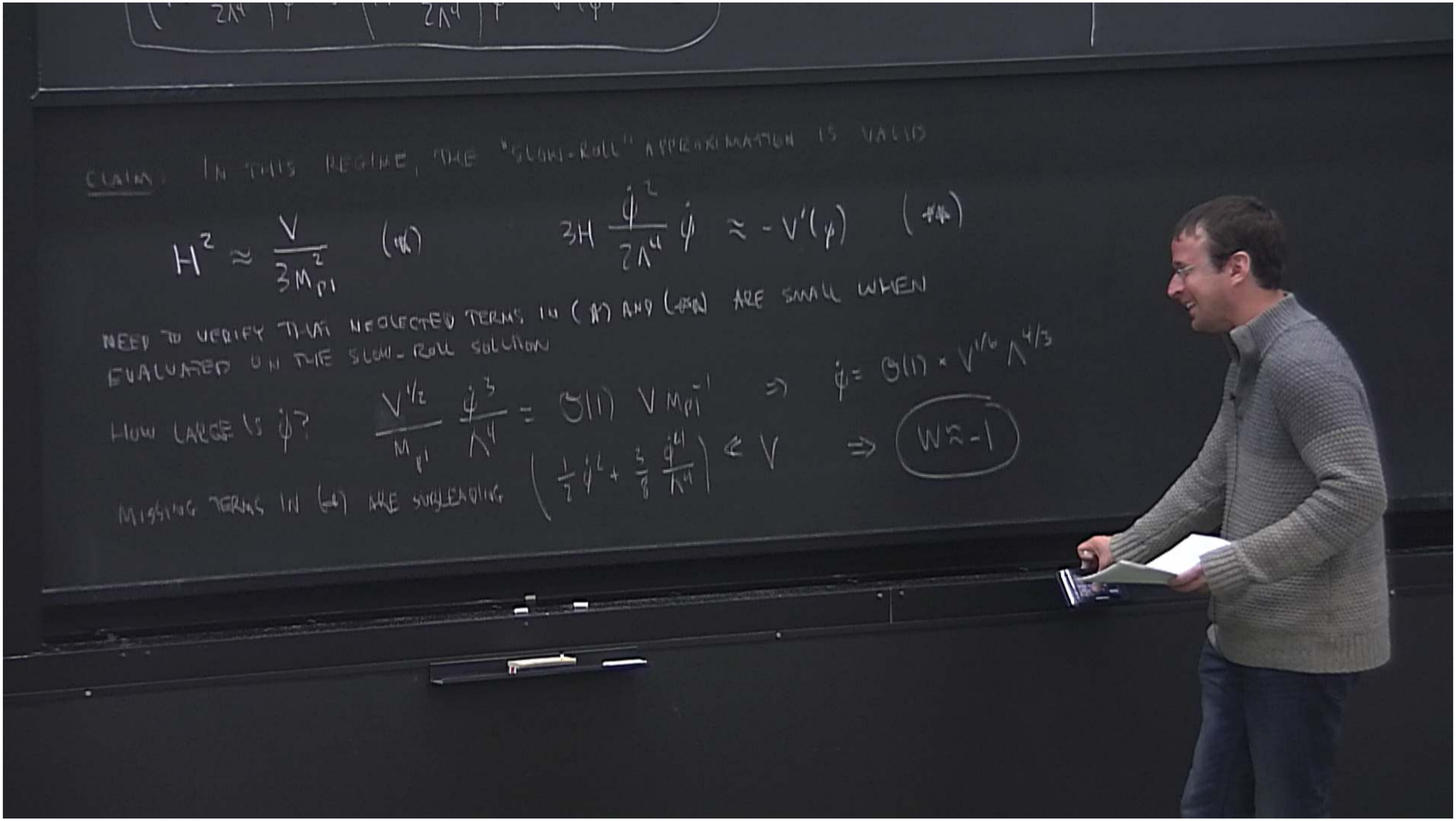
CLAIM: IN THIS REGIME, THE "SLOW-ROLL" APPROXIMATION IS VALID
H² ≈ V/3M_pl² (*)
3H φ̇²/2Λ⁴ φ̇
(**)
NEED TO VERIFY THAT NEGLECTED TERMS IN (*) AND (**) AR
WHEN
EVALUATED ON THE SLOW-ROLL SOLUTION
HOW LARGE IS φ̇?
V^½/M_pl φ̇³/Λ⁴ =
V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$H^2 \approx \frac{V}{3M_{pl}^2}$ (*)
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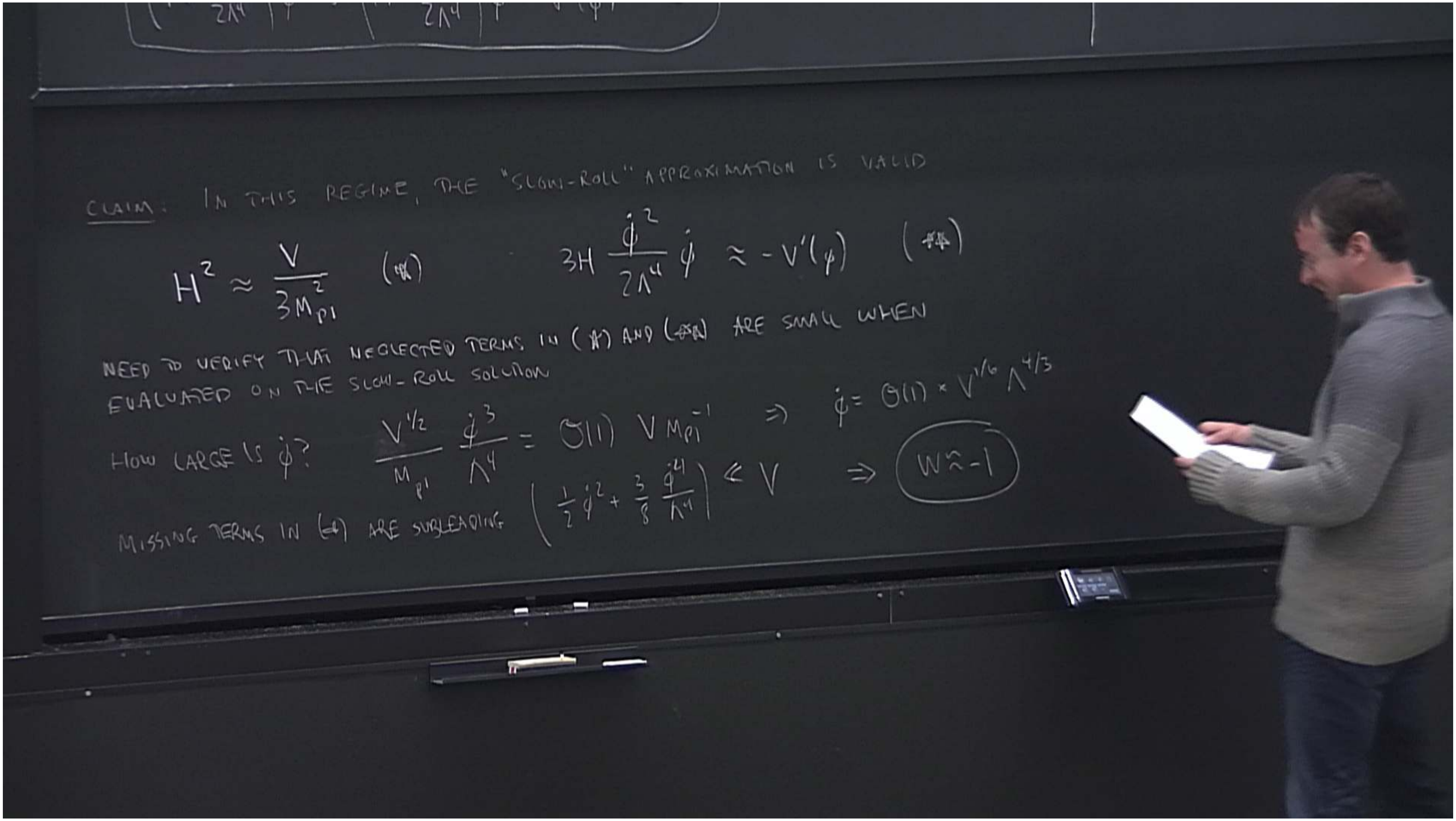
NEED TO VERIFY THAT NEGLECTED TERMS IN (*) AND (**) ARE SMALL WHEN
EVALUATED ON THE SLOW-ROLL SOLUTION
HOW LARGE IS $\dot\phi$?
$\frac{V^{1/2}}{M_{pl}} \frac{\dot\phi^3}{\Lambda^4} = \mathcal{O}(1)\ V M_{pl}^{-1}$ ⇒ $\dot\phi = \mathcal{O}(1) \times V^{1/6}\Lambda^{4/3}$
$\left(\frac{1}{2}\dot\phi^2 + \frac{3}{8}\frac{\dot\phi^4}{\Lambda^4}\right) \ll V$ ⇒ $w \approx -1$
MISSING TERMS IN (**) ARE SUBLEADING


CLAIM: IN THIS REGIME, THE "SLOW-ROLL" APPROXIMATION IS VALID
$H^2 \approx \frac{V}{3M_{Pl}^2}$ (*)
$3H \frac{\dot\phi^2}{2\Lambda^4}\dot\phi \approx -V'(\phi)$ (**)
NEED TO VERIFY THAT NEGLECTED TERMS IN (*) AND (**) ARE SMALL WHEN EVALUATED ON THE SLOW-ROLL SOLUTION
HOW LARGE IS $\dot\phi$?
$\frac{V^{1/2}}{M_{Pl}} \frac{\dot\phi^3}{\Lambda^4} = \mathcal{O}(1)\, V M_{Pl}^{-1}$ ⇒ $\dot\phi = \mathcal{O}(1) \times V^{1/6} \Lambda^{4/3}$
$\left(\frac{1}{2}\dot\phi^2 + \frac{3}{8}\frac{\dot\phi^4}{\Lambda^4}\right) \ll V$ ⇒ $w \approx -1$
MISSING TERMS IN (*) ARE SUBLEADING

$\ddot{\phi} \lesssim V^{1/3}\Lambda^{8/3}M_{pl}^{-1}$

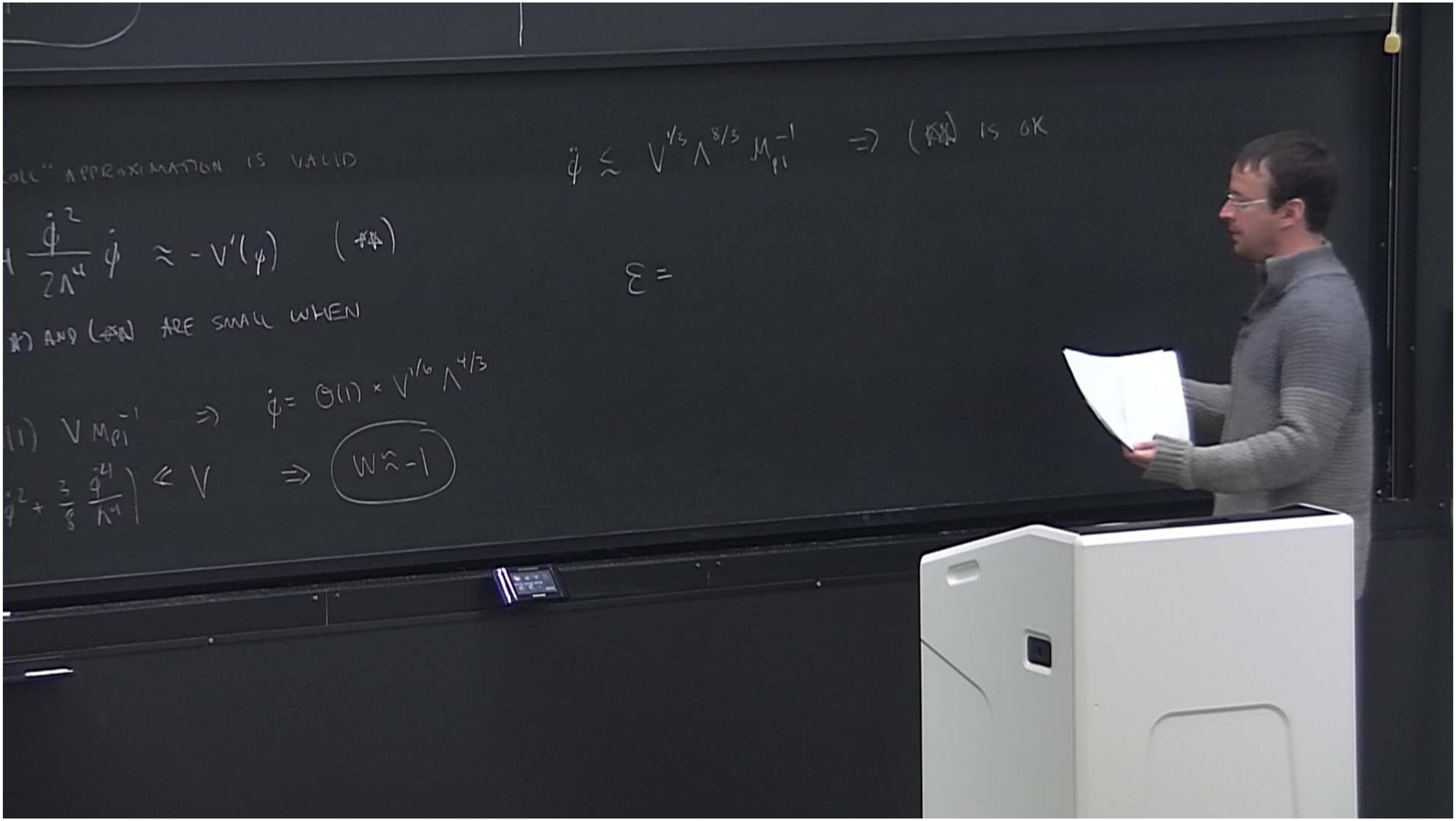
CLAIM: IN THIS REGIME, THE "SLOW-ROLL" APPROXIMATION IS VALID

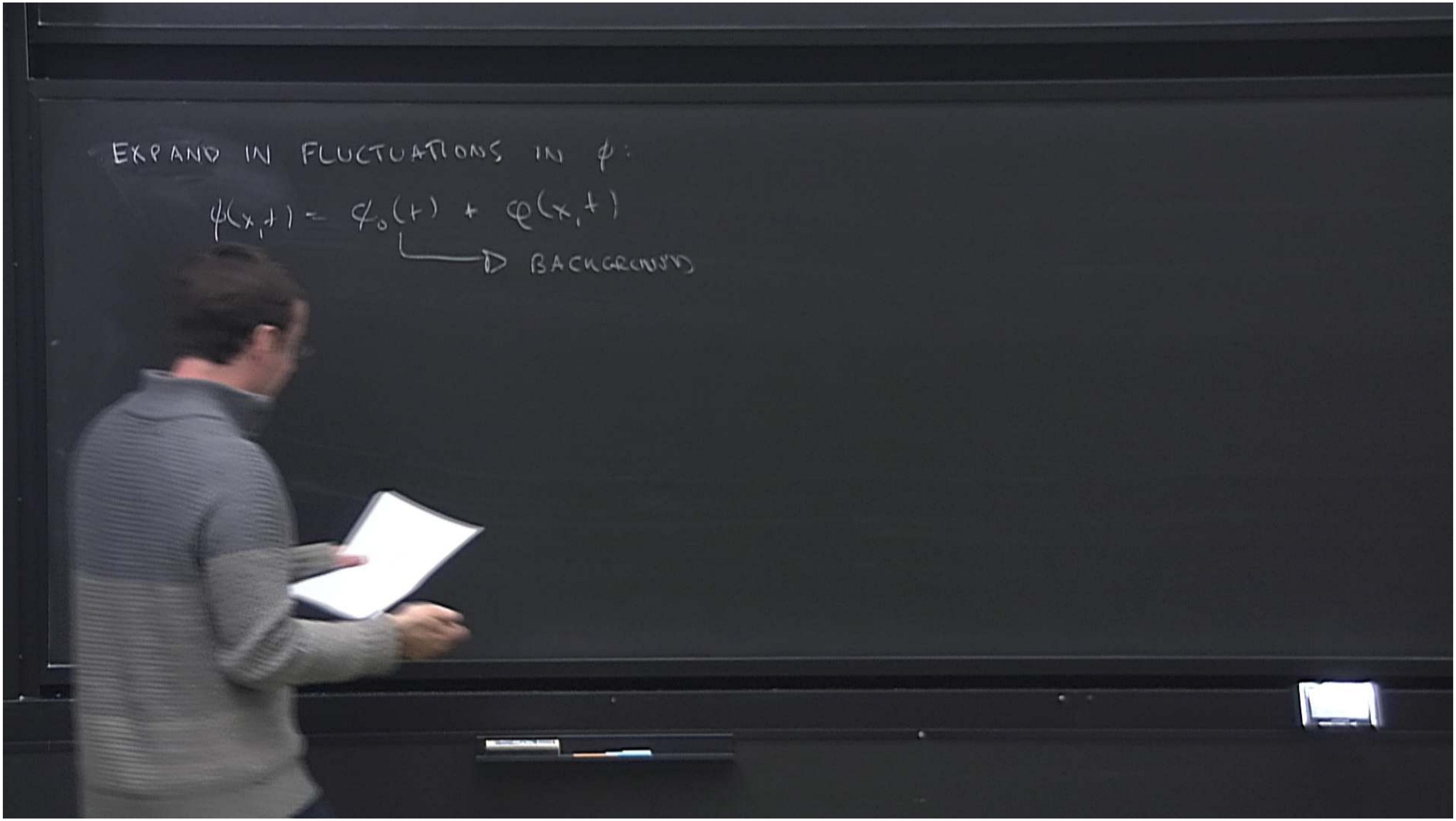
$$H^2 \approx \frac{V}{3M_{pl}^2} \quad (*) \qquad 3H\frac{\dot{\phi}^2}{2\Lambda^4}\dot{\phi} \approx -V'(\phi) \quad (**)$$

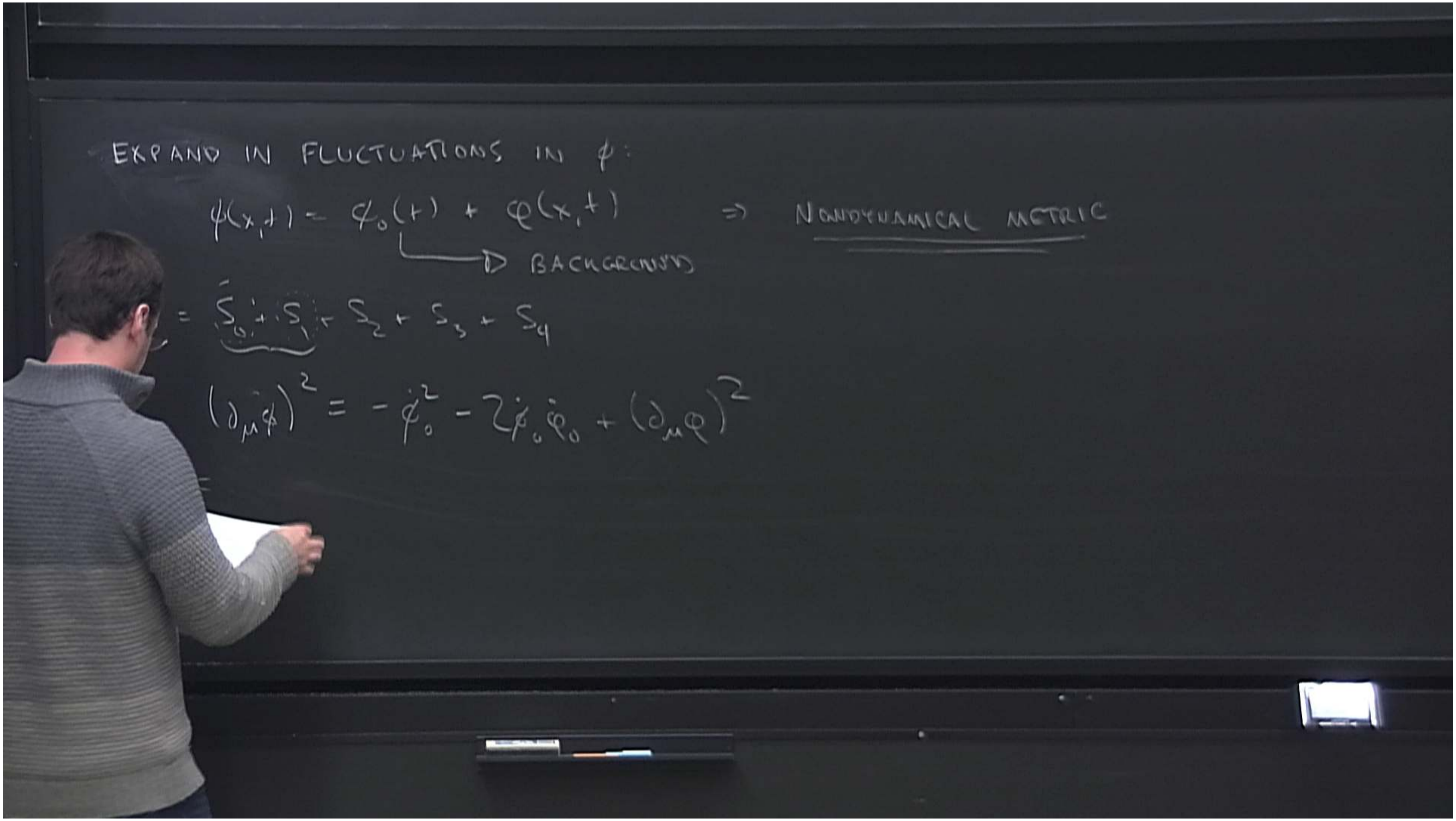
NEED TO VERIFY THAT NEGLECTED TERMS IN (*) AND (**) ARE SMALL WHEN EVALUATED ON THE SLOW-ROLL SOLUTION

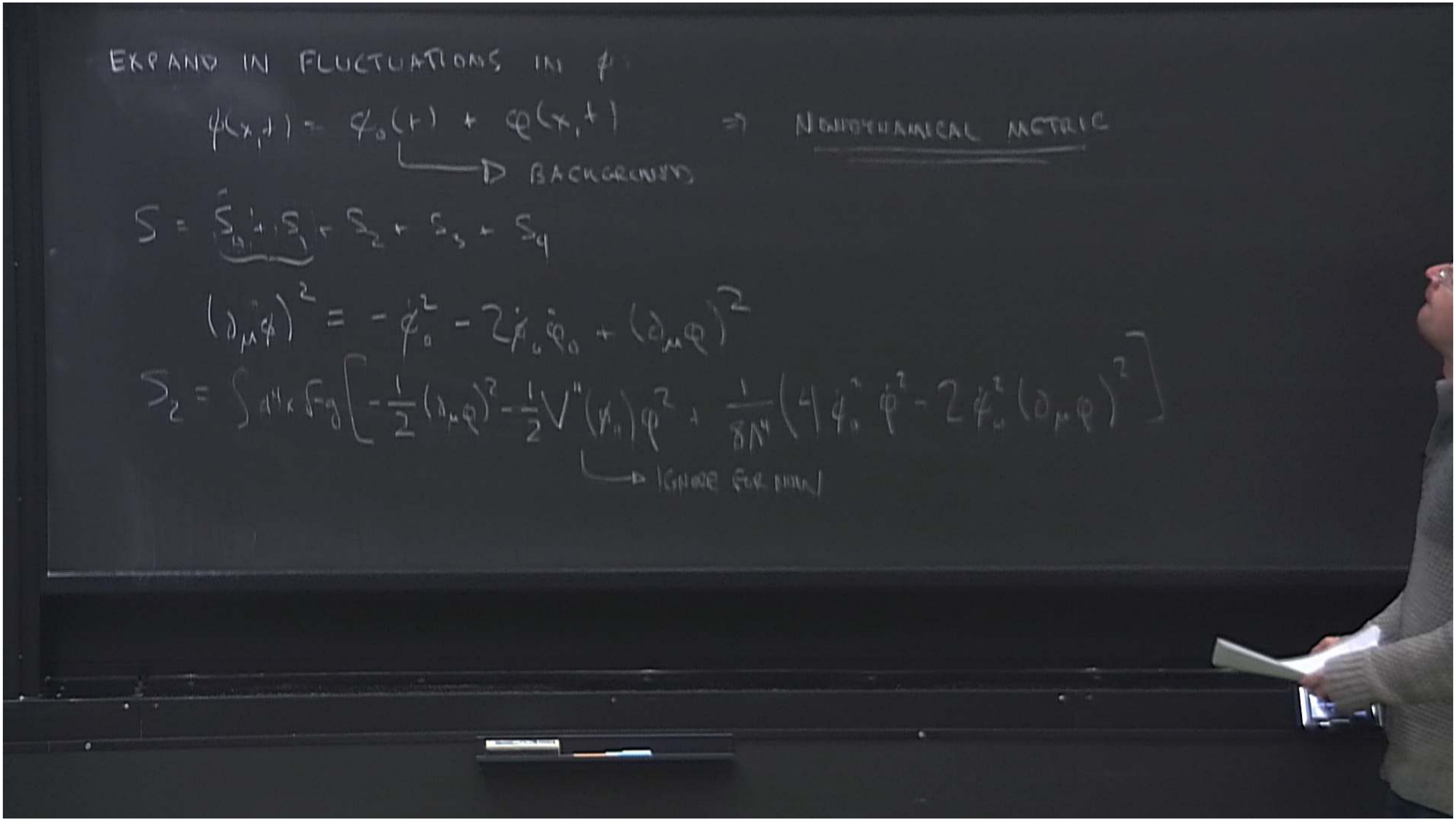
HOW LARGE IS $\dot{\phi}$? $\quad \frac{V^{1/2}}{M_{pl}}\frac{\dot{\phi}^3}{\Lambda^4} = O(1)\, V M_{pl}^{-1} \Rightarrow \dot{\phi} = O(1) \times V^{1/6}\Lambda^{4/3}$

MISSING TERMS IN (*) ARE SUBLEADING $\left(\frac{1}{2}\dot{\phi}^2 + \frac{3}{8}\frac{\dot{\phi}^4}{\Lambda^4}\right) \ll V \Rightarrow w \approx -1$


ROLL" APPROXIMATION IS VALID
(✱✱) AND (✱✱) ARE SMALL WHEN
W ≈ -1
(✱✱) IS OK
ε =


EXPAND IN FLUCTUATIONS IN $\phi$:
$\phi(x,t) = \phi_0(t) + \varphi(x,t)$
BACKGROUND


EXPAND IN FLUCTUATIONS IN $\phi$:
$\phi(x,t) = \phi_0(t) + \varphi(x,t)$
$\Rightarrow$ NONDYNAMICAL METRIC
BACKGROUND
$S = S_0 + S_1 + S_2 + S_3 + S_4$
$(\partial_\mu \phi)^2 = -\dot{\phi}_0^2 - 2\dot{\phi}_0\dot{\varphi}_0 + (\partial_\mu \varphi)^2$


EXPAND IN FLUCTUATIONS IN $\phi$:
$\phi(x,t) = \phi_0(t) + \varphi(x,t)$
BACKGROUND
$\Rightarrow$ NONDYNAMICAL METRIC
$S = S_0 + S_1 + S_2 + S_3 + S_4$
$(\partial_\mu \phi)^2 = -\dot{\phi}_0^2 - 2\dot{\phi}_0 \dot{\varphi}_0 + (\partial_\mu \varphi)^2$
$S_2 = \int d^4x \sqrt{-g} \left[ -\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} V''(\phi_0)\varphi^2 + \frac{1}{8\Lambda^4}\left(4\dot{\phi}_0^2 \dot{\varphi}^2 - 2\dot{\phi}_0^2 (\partial_\mu \varphi)^2\right) \right]$
IGNORE FOR NOW

EXPAND IN FLUCTUATIONS IN $\phi$:

$$\phi(x,t) = \phi_0(t) + \varphi(x,t)$$

$\phi_0(t)$ → BACKGROUND

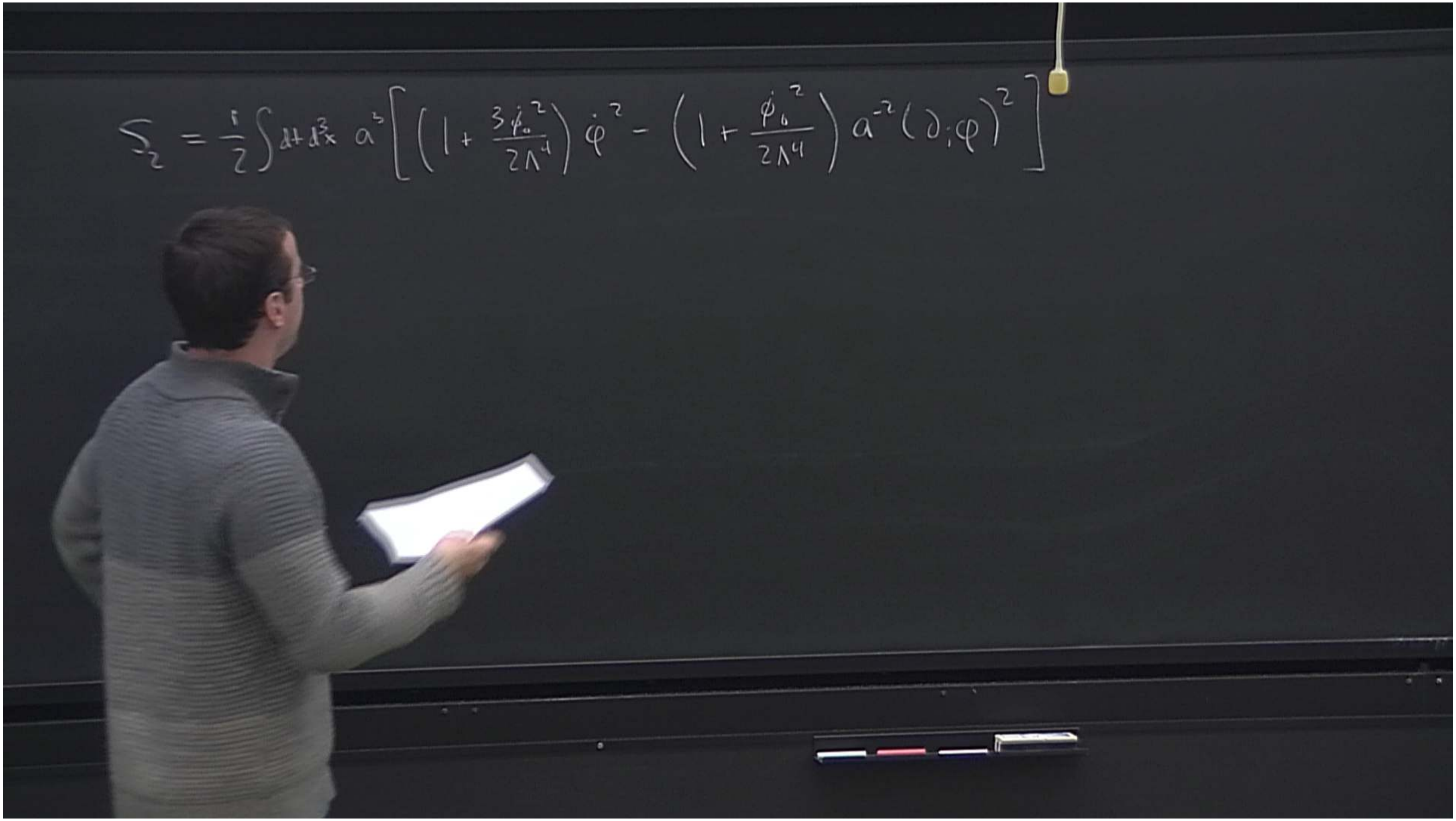
⇒ NONDYNAMICAL METRIC

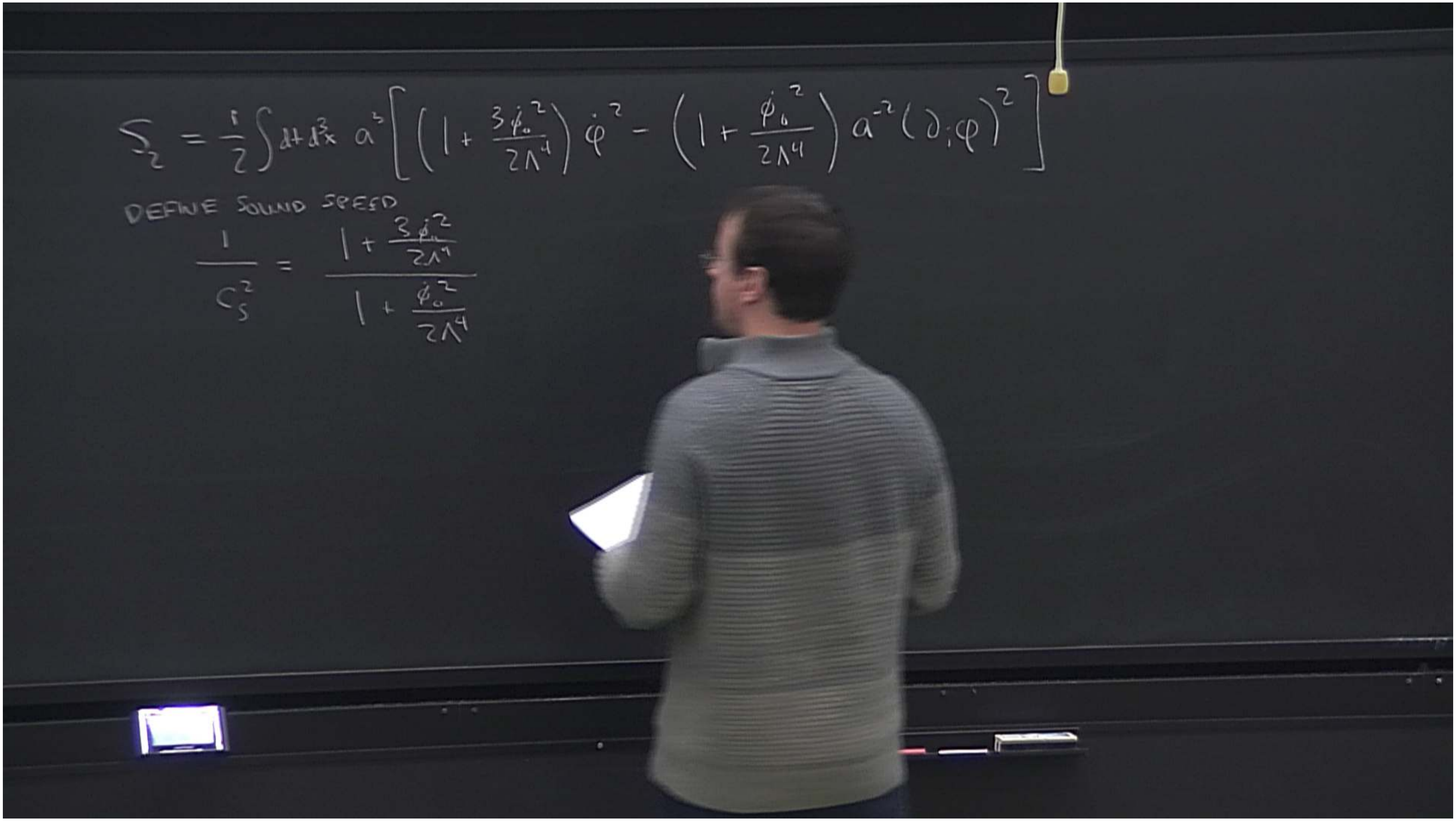
$$S = \underbrace{S_0 + S_1}_{} + S_2 + S_3 + S_4$$

$$(\partial_\mu \phi)^2 = -\dot{\phi}_0^2 - 2\dot{\phi}_0\dot{\varphi}_0 + (\partial_\mu \varphi)^2$$

$$S_2 = \int d^4x \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}V''(\phi_0)\varphi^2 + \frac{1}{8\Lambda^4}\left(4\dot{\phi}_0^2\dot{\varphi}^2 - 2\dot{\phi}_0^2(\partial_\mu\varphi)^2\right)\right]$$

$V''(\phi_0)\varphi^2$ → IGNORE FOR NOW


$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\left[\left(1+\frac{3\dot{\phi}_0^2}{2\Lambda^4}\right)\dot{\varphi}^2 - \left(1+\frac{\dot{\phi}_0^2}{2\Lambda^4}\right)a^{-2}(\partial_i\varphi)^2\right]$

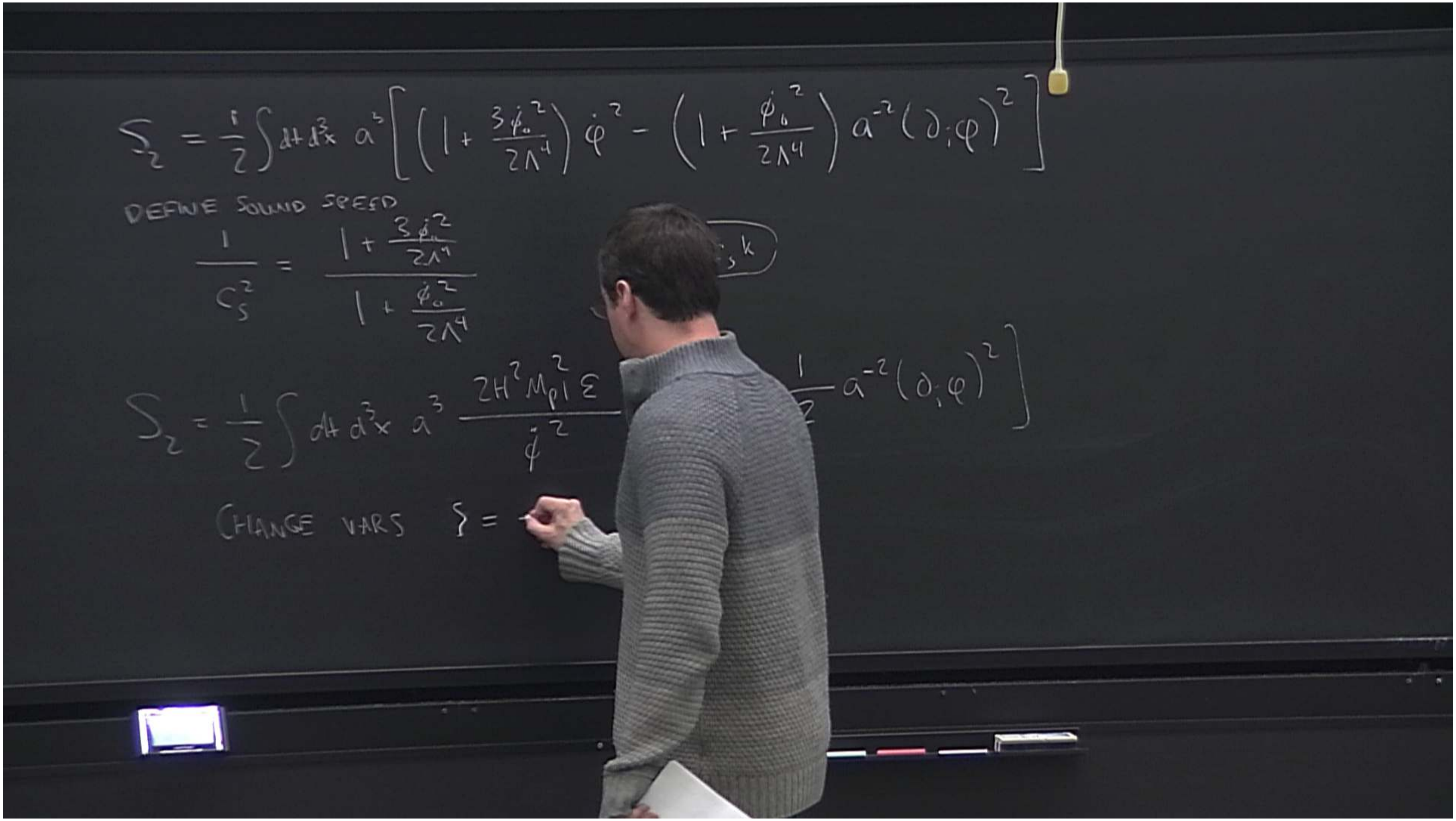

$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\left[\left(1+\frac{3\dot\phi_0^2}{2\Lambda^4}\right)\dot\varphi^2 - \left(1+\frac{\dot\phi_0^2}{2\Lambda^4}\right)a^{-2}(\partial_i\varphi)^2\right]$
DEFINE SOUND SPEED
$\frac{1}{c_s^2} = \frac{1+\frac{3\dot\phi_0^2}{2\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\left[\left(1+\frac{3\dot{\phi}_0^2}{2\Lambda^4}\right)\dot{\varphi}^2 - \left(1+\frac{\dot{\phi}_0^2}{2\Lambda^4}\right)a^{-2}(\partial_i\varphi)^2\right]$$

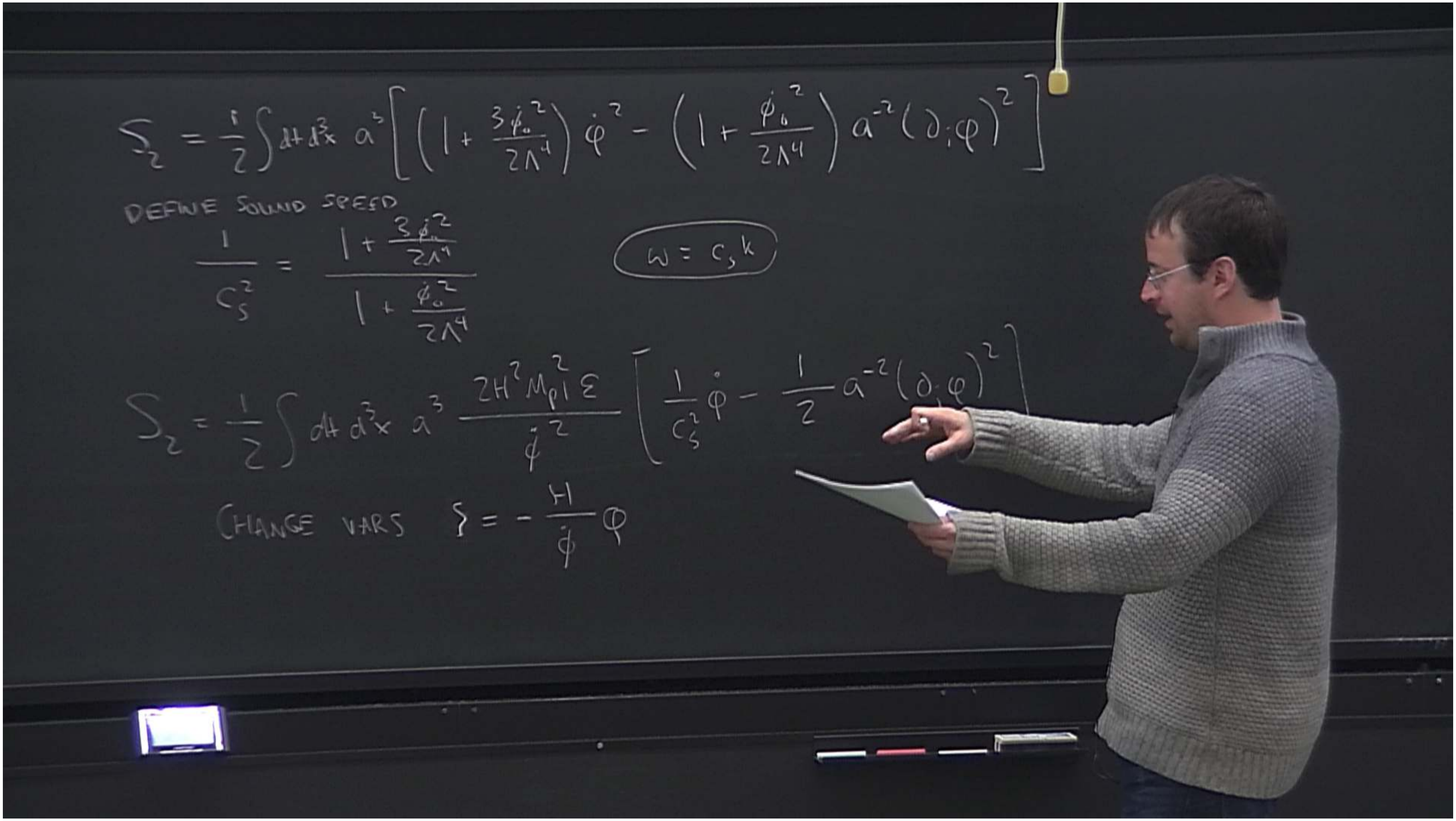
DEFINE SOUND SPEED

$$\frac{1}{c_s^2} = \frac{1+\frac{3\dot{\phi}_0^2}{2\Lambda^4}}{1+\frac{\dot{\phi}_0^2}{2\Lambda^4}}$$

$$\omega = c_s k$$



$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\left[\left(1+\frac{3\dot\phi_0^2}{2\Lambda^4}\right)\dot\varphi^2 - \left(1+\frac{\dot\phi_0^2}{2\Lambda^4}\right)a^{-2}(\partial_i\varphi)^2\right]$
DEFINE SOUND SPEED
$\frac{1}{c_s^2} = \frac{1+\frac{3\dot\phi_0^2}{2\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$
$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\, \frac{2H^2 M_{pl}^2 \varepsilon}{\dot\phi^2}$
$a^{-2}(\partial_i\varphi)^2\Big]$
CHANGE VARS $\zeta =$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DEFINE SOUND SPEED
$\frac{1}{c_s^2} = \frac{1+\frac{3\dot\phi_0^2}{2\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$
$\omega = c_s k$
$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\,\frac{2H^2M_{pl}^2\,\epsilon}{\dot\phi^2}\left[\frac{1}{c_s^2}\dot\varphi - \frac{1}{2}a^{-2}(\partial_i\varphi)^2\right]$
CHANGE VARS $\zeta = -\frac{H}{\dot\phi}\varphi$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\left[\left(1+\frac{3\dot\phi_0^2}{2\Lambda^4}\right)\dot\varphi^2 - \left(1+\frac{\dot\phi_0^2}{2\Lambda^4}\right)a^{-2}(\partial_i\varphi)^2\right]$$

DEFINE SOUND SPEED

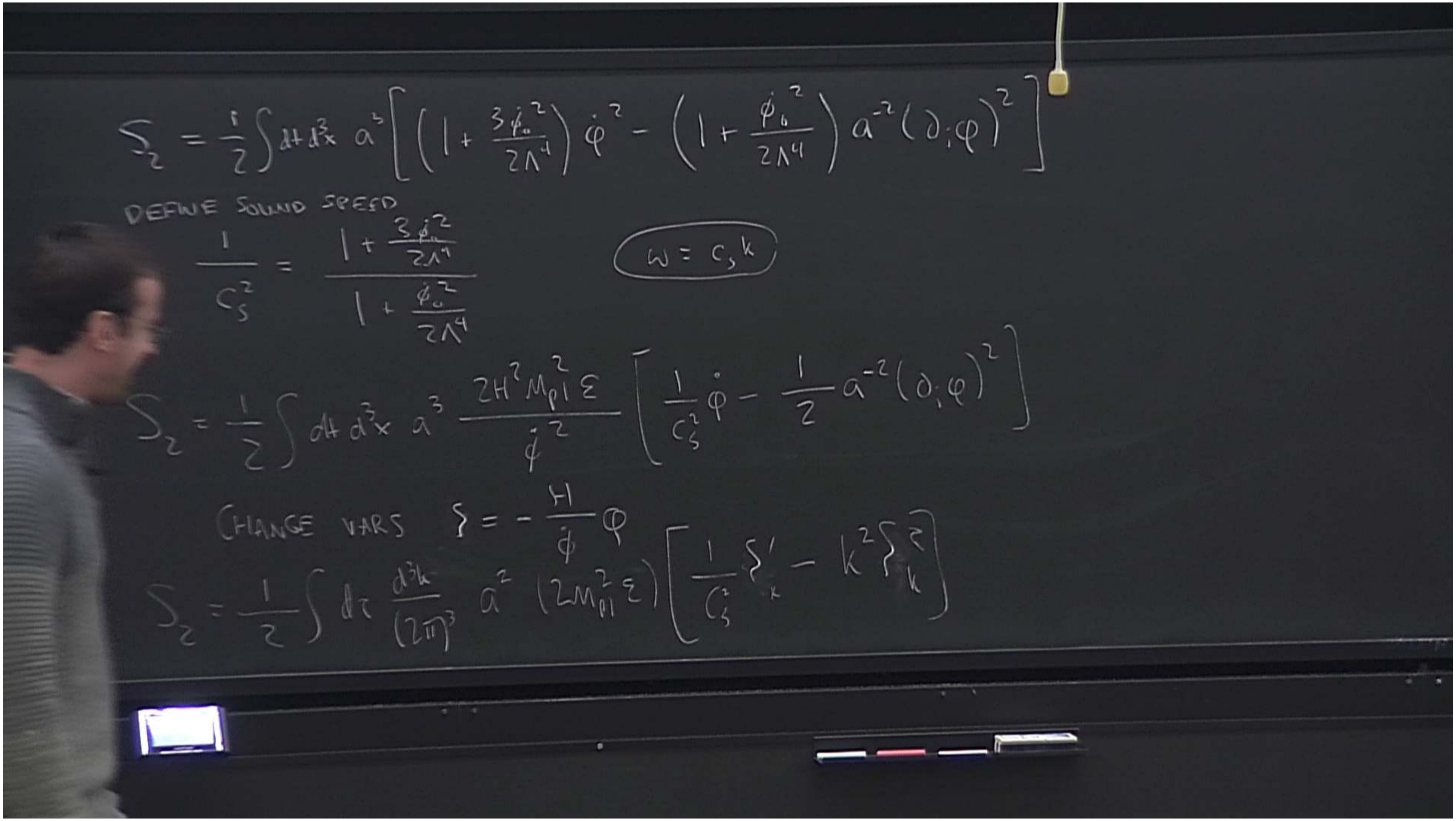
$$\frac{1}{c_s^2} = \frac{1+\frac{3\dot\phi_0^2}{2\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$$

$$\omega = c_s k$$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3 \frac{2H^2 M_{pl}^2 \varepsilon}{\dot\phi^2}\left[\frac{1}{c_s^2}\dot\varphi - \frac{1}{2}a^{-2}(\partial_i\varphi)^2\right]$$

CHANGE VARS $\zeta = -\frac{H}{\dot\phi}\varphi$

$$S_2 = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\, a^2\, (2M_{pl}^2\varepsilon)\left[\frac{1}{c_s^2}\varphi_k' - k^2\varphi_k\right]$$



$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3 \left[ \left(1 + \frac{3\dot{\phi}_0^2}{2\Lambda^4}\right)\dot{\varphi}^2 - \left(1 + \frac{\dot{\phi}_0^2}{2\Lambda^4}\right) a^{-2} (\partial_i \varphi)^2 \right]$
DEFINE SOUND SPEED
$\frac{1}{c_s^2} = \frac{1 + \frac{3\dot{\phi}_0^2}{2\Lambda^4}}{1 + \frac{\dot{\phi}_0^2}{2\Lambda^4}}$
$\omega = c_s k$
$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3 \frac{2H^2 M_{pl}^2 \varepsilon}{\dot{\phi}^2} \left[ \frac{1}{c_s^2}\dot{\varphi} - \frac{1}{2} a^{-2} (\partial_i \varphi)^2 \right]$
CHANGE VARS $\zeta = -\frac{H}{\dot{\phi}}\varphi$
$S_2 = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} a^2 (2M_{pl}^2 \varepsilon) \left[ \frac{1}{c_s^2}\zeta'_k - k^2 \zeta_k^2 \right]$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\left[\left(1+\frac{3\dot\phi_0^2}{2\Lambda^4}\right)\dot\varphi^2 - \left(1+\frac{\dot\phi_0^2}{2\Lambda^4}\right)a^{-2}(\partial_i\varphi)^2\right]$$

DEFINE SOUND SPEED

$$\frac{1}{c_s^2} = \frac{1+\frac{3\dot\phi_0^2}{2\Lambda^4}}{1+\frac{\dot\phi_0^2}{2\Lambda^4}}$$

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$$S_2 = \frac{1}{2}\int dt\, d^3x\, a^3\,\frac{2H^2 M_{pl}^2\varepsilon}{\dot\phi^2}\left[\frac{1}{c_s^2}\dot\varphi - \frac{1}{2}a^{-2}(\partial_i\varphi)^2\right]$$

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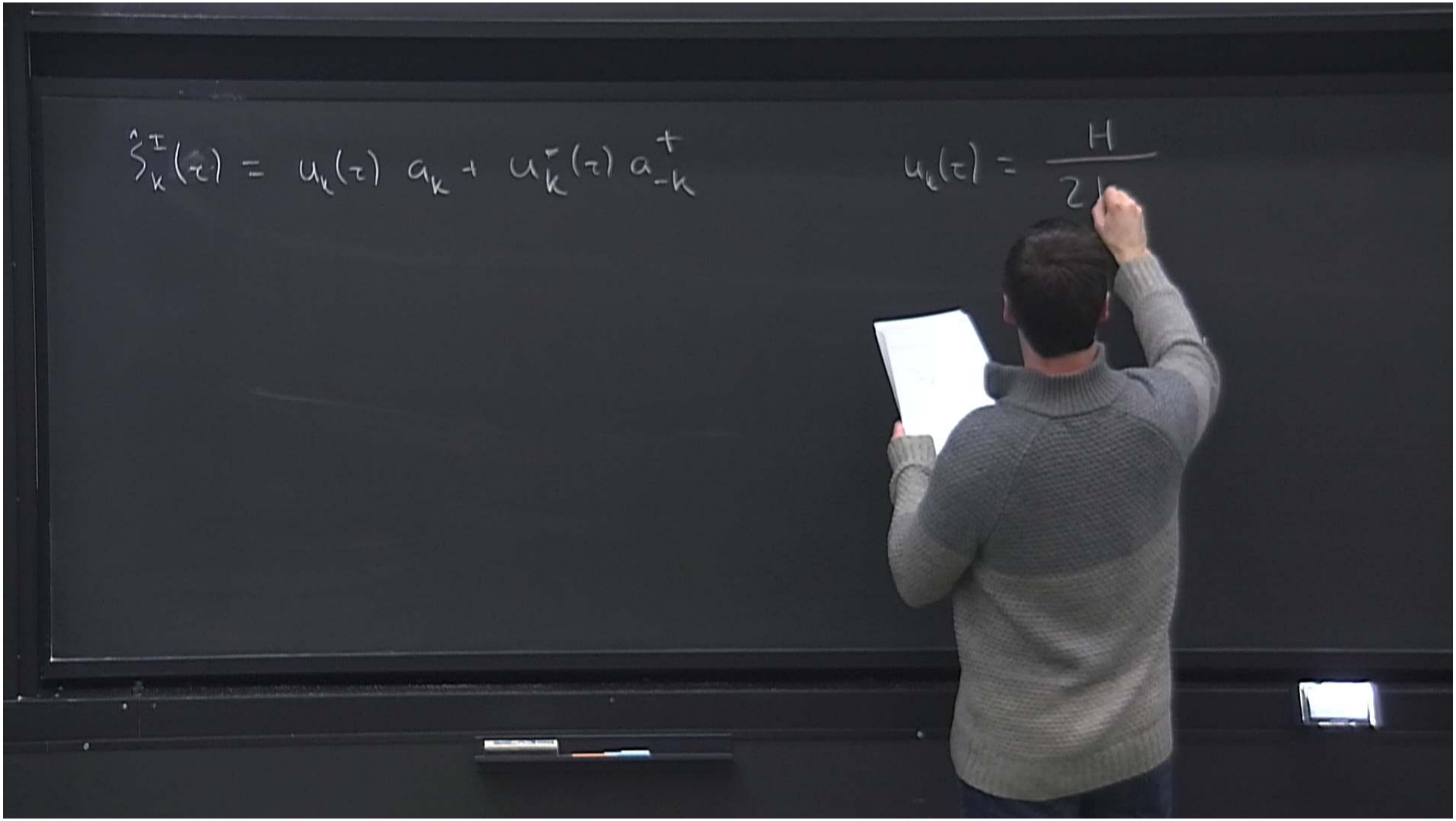
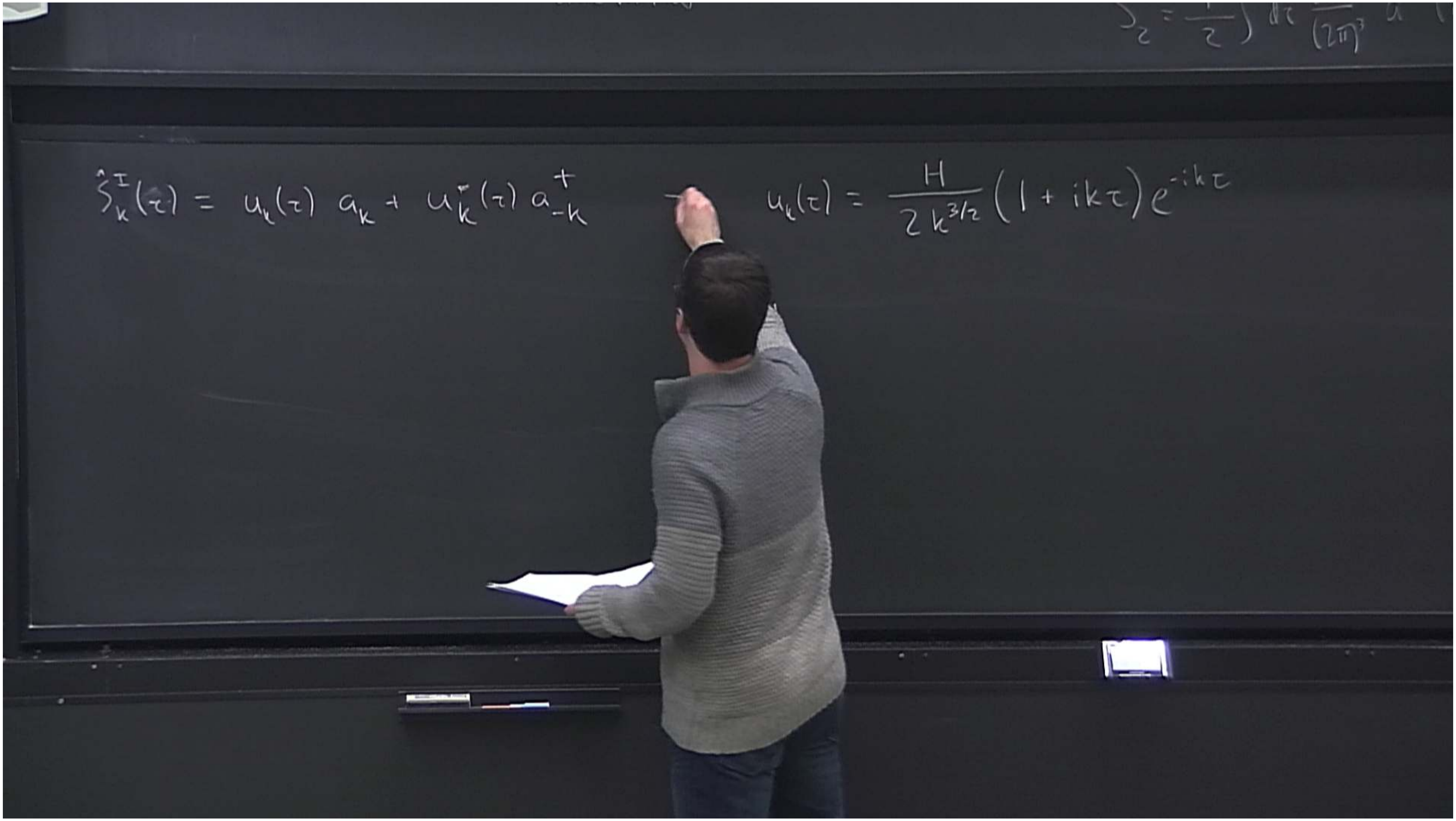
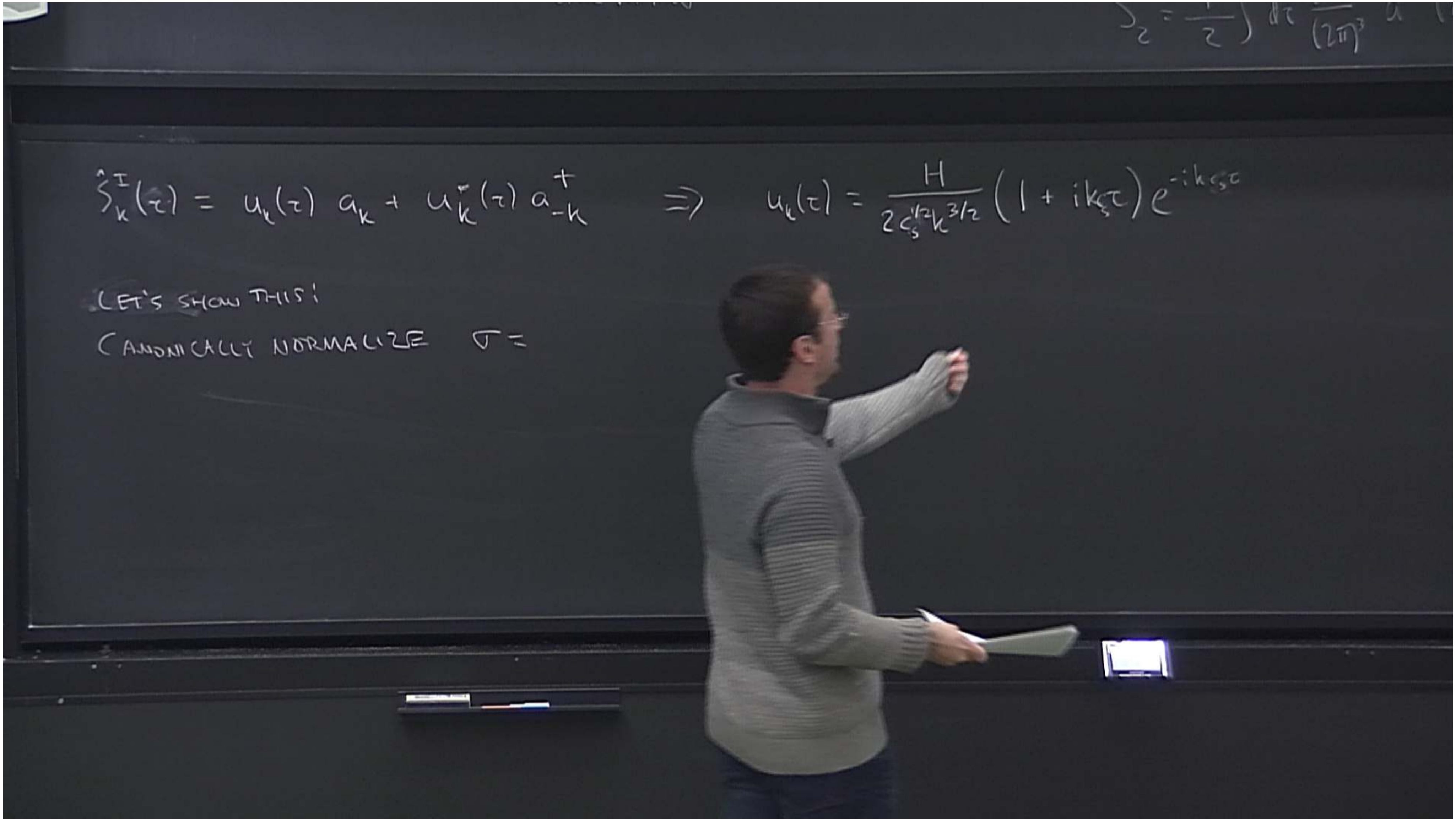
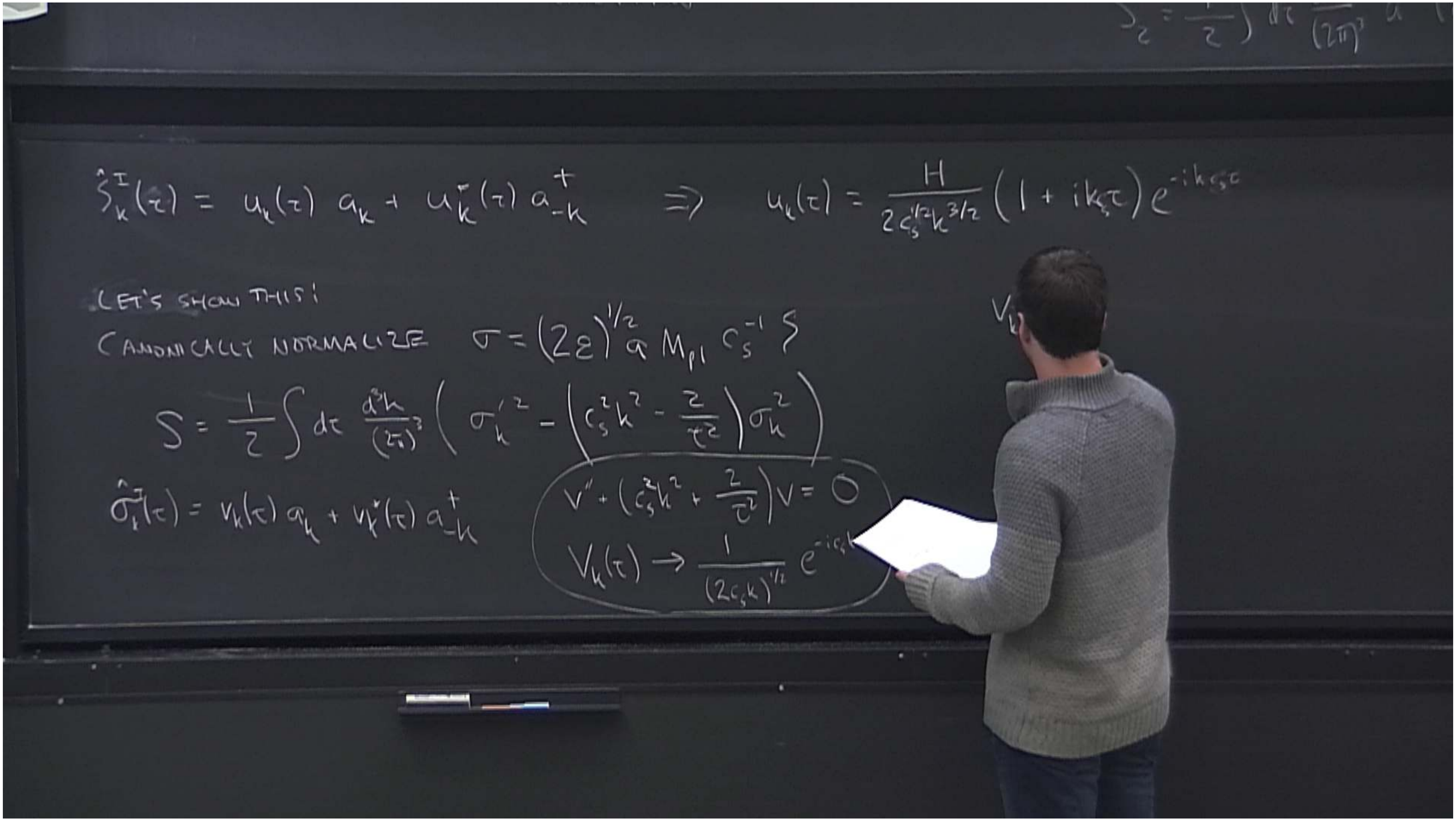
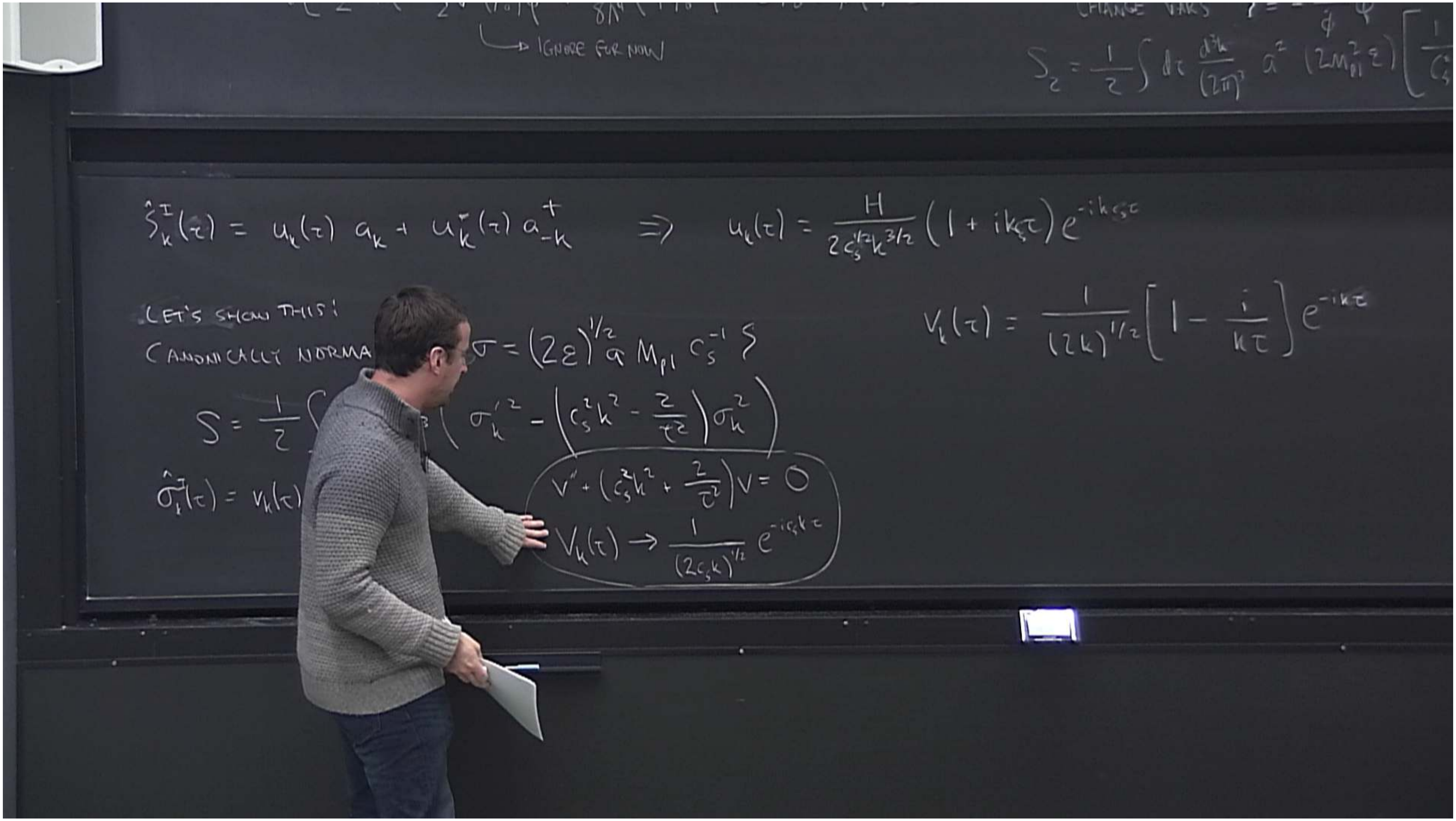
$$S_2 = \frac{1}{2}\int d\tau\,\frac{d^3k}{(2\pi)^3}\,a^2\,(2M_{pl}^2\varepsilon)\left[\frac{1}{c_s^2}\zeta_k'^2 - k^2\zeta_k^2\right]$$

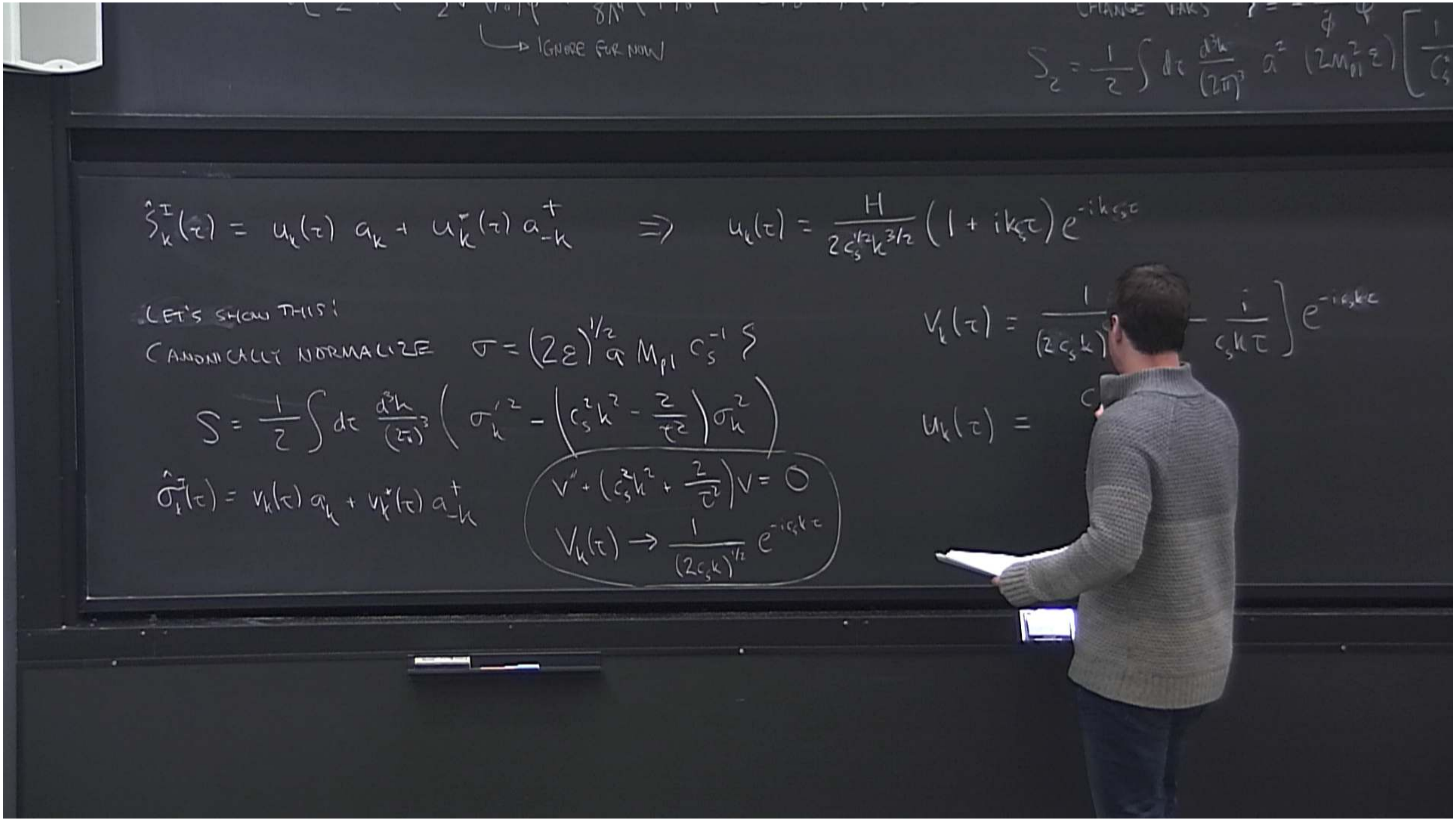

$\hat{\zeta}^I_k(\tau) = u_k(\tau)\, a_k + u^*_k(\tau)\, a^\dagger_{-k}$
$u_k(\tau) = \frac{H}{2}$

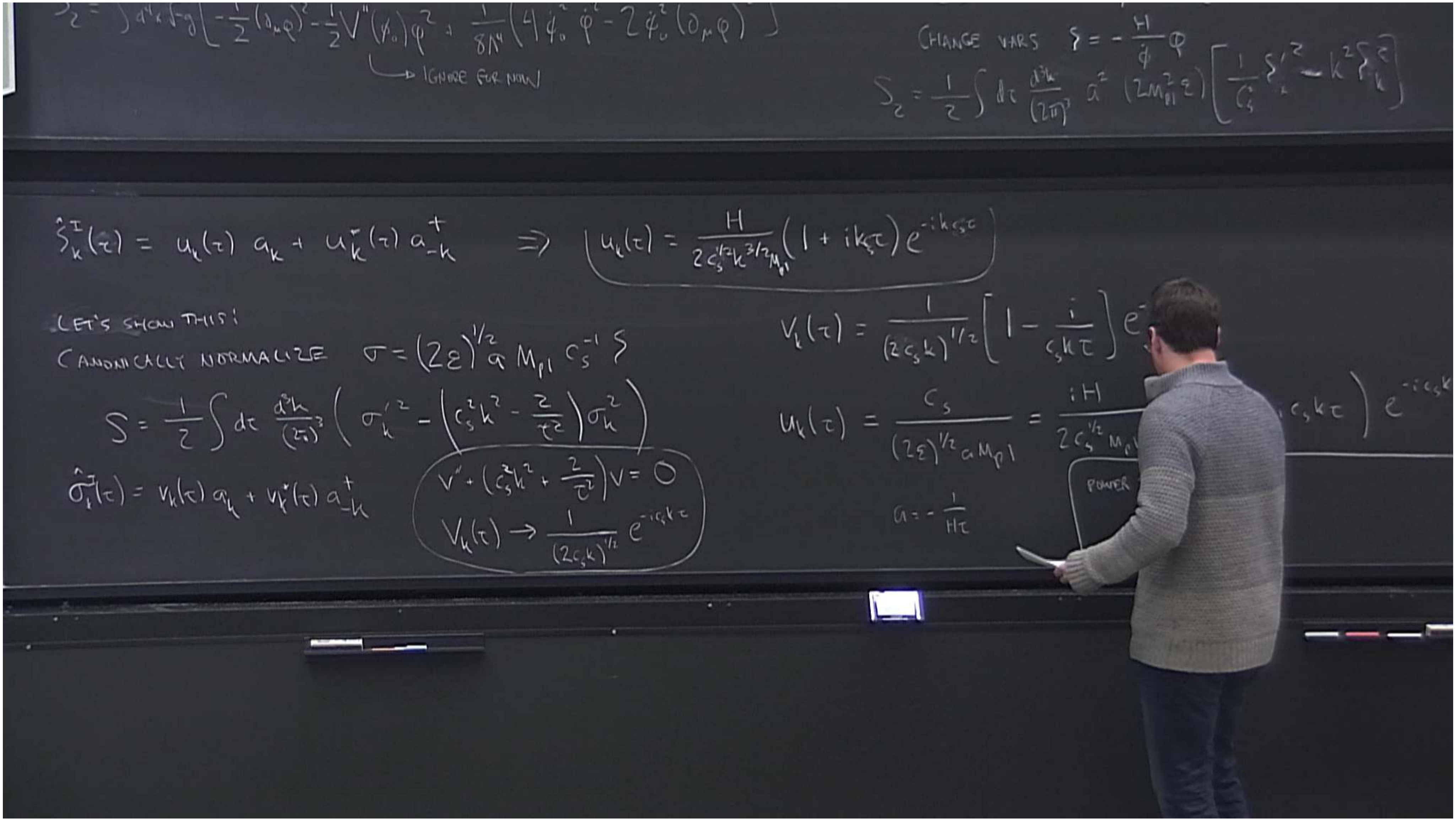

ζ̂^I_k(τ) = u_k(τ) a_k + u*_k(τ) a†_{-k}
u_k(τ) = H / (2 k^{3/2}) (1 + ikτ) e^{-ikτ}

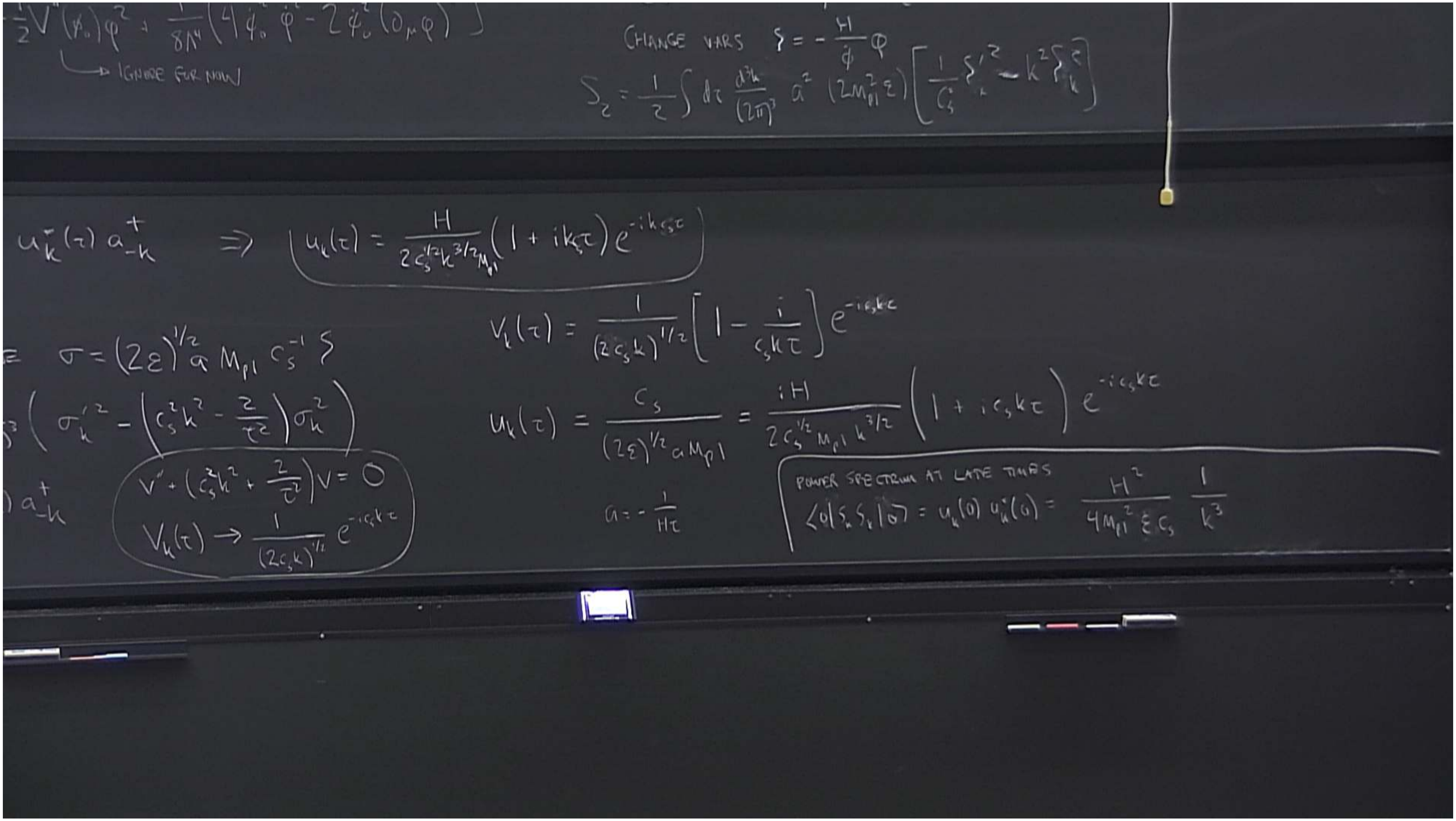

$\hat{\zeta}^I_k(\tau) = u_k(\tau)\, a_k + u^*_k(\tau)\, a^\dagger_{-k} \Rightarrow u_k(\tau) = \frac{H}{2 c_s^{1/2} k^{3/2}} (1 + i k c_s \tau) e^{-i k c_s \tau}$
LET'S SHOW THIS!
CANONICALLY NORMALIZE $\sigma =$


$\hat{\zeta}^I_k(\tau) = u_k(\tau)\, a_k + u^*_k(\tau)\, a^\dagger_{-k} \Rightarrow u_k(\tau) = \frac{H}{2 c_s^{1/2} k^{3/2}}(1 + i k c_s \tau) e^{-i k c_s \tau}$
LET'S SHOW THIS!
CANONICALLY NORMALIZE $\sigma = (2\epsilon)^{1/2} a M_{pl} c_s^{-1} \zeta$
$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left(\sigma_k'^2 - \left(c_s^2 k^2 - \frac{2}{\tau^2}\right)\sigma_k^2\right)$
$\hat{\sigma}^I_k(\tau) = v_k(\tau)\, a_k + v^*_k(\tau)\, a^\dagger_{-k}$
$v'' + \left(c_s^2 k^2 + \frac{2}{\tau^2}\right) v = 0$
$V_k(\tau) \to \frac{1}{(2 c_s k)^{1/2}} e^{-i c_s k}$


IGNORE FOR NOW
CHANGE VARS
LET'S SHOW THIS:
CANONICALLY NORMA


IGNORE FOR NOW
CHANGE VARS
$S_2 = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} a^2 (2M_{pl}^2\varepsilon)$
$\hat{\zeta}^I_k(\tau) = u_k(\tau) a_k + u^*_k(\tau) a^\dagger_{-k} \Rightarrow u_k(\tau) = \frac{H}{2c_s^{1/2}k^{3/2}}(1 + ikc_s\tau)e^{-ikc_s\tau}$
LET'S SHOW THIS:
CANONICALLY NORMALIZE $\sigma = (2\varepsilon)^{1/2} a M_{pl} c_s^{-1} \zeta$
$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left(\sigma_k'^2 - \left(c_s^2k^2 - \frac{2}{\tau^2}\right)\sigma_k^2\right)$
$\hat{\sigma}^I_k(\tau) = v_k(\tau) a_k + v^*_k(\tau) a^\dagger_{-k}$
$v'' + \left(c_s^2k^2 + \frac{2}{\tau^2}\right)v = 0$
$V_k(\tau) \to \frac{1}{(2c_sk)^{1/2}} e^{-ic_sk\tau}$
$V_k(\tau) = \frac{1}{(2c_sk)}$
$\frac{i}{c_sk\tau}\Big] e^{-ic_sk\tau}$
$u_k(\tau) =$


IGNORE FOR NOW
CHANGE VARS $\zeta = -\frac{H}{\dot\phi}\varphi$
$S_2 = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} a^2 (2M_{pl}^2\varepsilon)\left[\frac{1}{c_s^2}\zeta_k'^2 - k^2\zeta_k^2\right]$
$\hat\zeta_k^I(\tau) = u_k(\tau)\, a_k + u_k^*(\tau)\, a_{-k}^\dagger \Rightarrow u_k(\tau) = \frac{H}{2c_s^{1/2}k^{3/2}M_{pl}}(1+ikc_s\tau)e^{-ikc_s\tau}$
LET'S SHOW THIS:
CANONICALLY NORMALIZE $\sigma = (2\varepsilon)^{1/2} a M_{pl} c_s^{-1}\zeta$
$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left(\sigma_k'^2 - \left(c_s^2k^2 - \frac{2}{\tau^2}\right)\sigma_k^2\right)$
$\hat\sigma_k^I(\tau) = v_k(\tau)\, a_k + v_k^*(\tau)\, a_{-k}^\dagger$
$v'' + \left(c_s^2k^2 + \frac{2}{\tau^2}\right)v = 0$
$v_k(\tau) \to \frac{1}{(2c_sk)^{1/2}} e^{-ic_sk\tau}$
$v_k(\tau) = \frac{1}{(2c_sk)^{1/2}}\left[1 - \frac{i}{c_sk\tau}\right]e$
$u_k(\tau) = \frac{c_s}{(2\varepsilon)^{1/2} a M_{pl}} = \frac{iH}{2c_s^{1/2}M_{pl}}$
$a = -\frac{1}{H\tau}$
POWER


IGNORE FOR NOW
CHANGE VARS $\zeta = -\frac{H}{\dot\phi}\varphi$
$S_2 = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} a^2 (2M_{pl}^2\varepsilon)\left[\frac{1}{c_s^2}\zeta'^2 - k^2\zeta_k^2\right]$
$u_k(\tau) = \frac{H}{2c_s^{1/2}k^{3/2}M_{pl}}(1 + ikc_s\tau)e^{-ikc_s\tau}$
$\sigma = (2\varepsilon)^{1/2} a M_{pl} c_s^{-1}\zeta$
$V_k(\tau) = \frac{1}{(2c_sk)^{1/2}}\left[1 - \frac{i}{c_sk\tau}\right]e^{-ic_sk\tau}$
$u_k(\tau) = \frac{c_s}{(2\varepsilon)^{1/2} a M_{pl}} = \frac{iH}{2c_s^{1/2}M_{pl}k^{3/2}}(1 + ic_sk\tau)e^{-ic_sk\tau}$
$v'' + \left(c_s^2k^2 + \frac{2}{\tau^2}\right)v = 0$
$V_k(\tau) \to \frac{1}{(2c_sk)^{1/2}}e^{-ic_sk\tau}$
$a = -\frac{1}{H\tau}$
POWER SPECTRUM AT LATE TIMES
$\langle 0|\zeta_k\zeta_k|0\rangle = u_k(0)\,u_k^*(0) = \frac{H^2}{4M_{pl}^2\varepsilon c_s}\frac{1}{k^3}$