Title: Explorations in Cosmology-10

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Abstract:

$$\zeta = \zeta_c + \frac{1}{2}\frac{\dot\phi}{\dot H}\zeta_c^2 + \frac{1}{8}\frac{\dot\phi^2}{H^2}\zeta_c^2 + \frac{1}{4}\frac{\phi^2}{H^2}\partial^{-2}(\zeta_c\partial^2\zeta_c)$$

$$= \zeta_c + \left(\frac{3\varepsilon}{4} - \frac{3}{2}\right)\zeta_c^2 + \frac{\varepsilon}{2}\partial^{-2}(\zeta_c\partial^2\zeta_c)$$

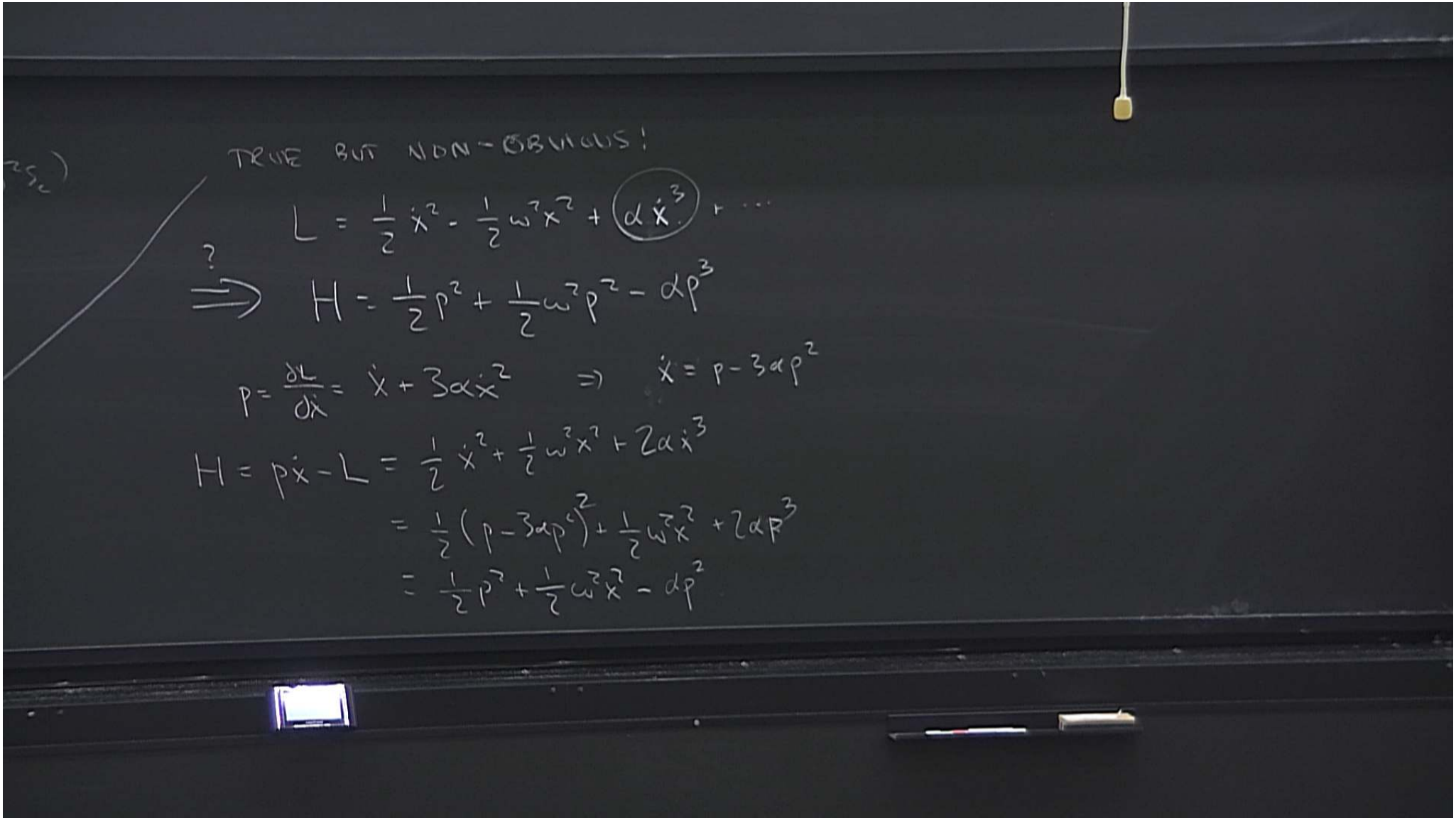
$$S_3 = \int dt\, d^3x\, \frac{\phi^4}{H^3}\, a^5\, \dot\zeta^2(\partial^{-2}\dot\zeta)$$

$$= \int d\tau\, d^3x\, 4\varepsilon^2 H\, a^3 (\zeta')^2(\partial^{-2}\zeta')$$

$$H_I = -\int d\tau\, d^3x\, 4\varepsilon^2 H\, a^3 (\zeta')^2(\partial^{-2}\zeta')$$

TRUE BUT N

$$L = \frac{1}{2}\dot x^2$$

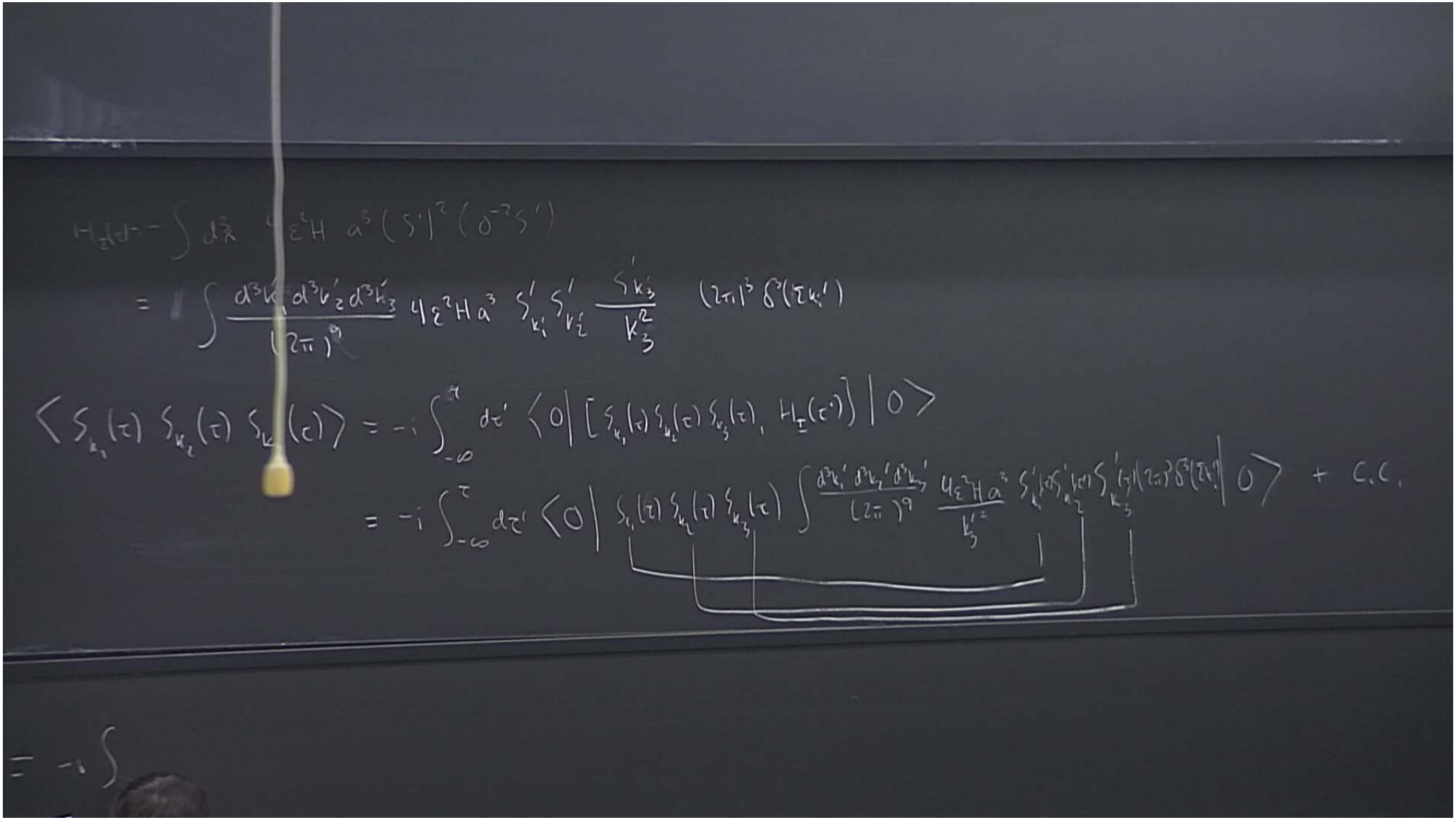

TRUE BUT NON-OBVIOUS!
$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 + \alpha\dot{x}^3 + \cdots$
$\Rightarrow H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 p^2 - \alpha p^3$
$p = \frac{\partial L}{\partial \dot{x}} = \dot{x} + 3\alpha\dot{x}^2 \Rightarrow \dot{x} = p - 3\alpha p^2$
$H = p\dot{x} - L = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega^2 x^2 + 2\alpha\dot{x}^3$
$= \frac{1}{2}(p - 3\alpha p^2)^2 + \frac{1}{2}\omega^2 x^2 + 2\alpha p^3$
$= \frac{1}{2}p^2 + \frac{1}{2}\omega^2 x^2 - \alpha p^2$

$$H_I(\tau) = -\int d^3x \; 4\varepsilon^2 H a^3 (\zeta')^2 (\partial^{-2}\zeta')$$

$$= \int \frac{d^3k_1' d^3k_2' d^3k_3'}{(2\pi)^3} 4\varepsilon^2 H a^3 \zeta'_{k_1}\zeta'_{k_2} \frac{\zeta'_{k_3}}{k_3^2} (2\pi)^3 \delta^3(\Sigma k_i')$$

$$\langle \zeta_{k_1}(\tau)\,\zeta_{k_2}(\tau)\,\zeta_{k_3}(\tau)\rangle = -i\int_{-\infty}^{\tau} d\tau' \langle 0|\left[\zeta_{k_1}(\tau)\zeta_{k_2}(\tau)\zeta_{k_3}(\tau),\, H_I(\tau')\right]|0\rangle$$

$$= -i\int_{-\infty}^{\tau} d\tau' \langle 0| \zeta_{k_1}(\tau)\zeta_{k_2}(\tau)\zeta_{k_3}(\tau)$$


$H_I(\tau) = -\int d^3x \; 4\epsilon^2 H a^3 (\zeta')^2 (\partial^{-2}\zeta')$
$= \int \frac{d^3k_1' d^3k_2' d^3k_3'}{(2\pi)^9} 4\epsilon^2 H a^3 \zeta'_{k_1'} \zeta'_{k_2'} \frac{\zeta'_{k_3'}}{k_3'^2} (2\pi)^3 \delta^3(\Sigma k_i')$
$\langle \zeta_{k_1}(\tau) \zeta_{k_2}(\tau) \zeta_{k_3}(\tau) \rangle = -i \int_{-\infty}^{\tau} d\tau' \langle 0 | [\zeta_{k_1}(\tau) \zeta_{k_2}(\tau) \zeta_{k_3}(\tau), H_I(\tau')] | 0 \rangle$
$= -i \int_{-\infty}^{\tau} d\tau' \langle 0 | \zeta_{k_1}(\tau) \zeta_{k_2}(\tau) \zeta_{k_3}(\tau) \int \frac{d^3k_1' d^3k_2' d^3k_3'}{(2\pi)^9} \frac{4\epsilon^2 H a^3}{k_3'^2} \zeta'_{k_1'} \zeta'_{k_2'} \zeta'_{k_3'} (2\pi)^3 \delta^3(\Sigma k_i') | 0 \rangle + c.c.$
$= -i \int$

$$= -i \int_{-\infty}^{\tau} d\tau' \, (4\varepsilon^2 H^4 \alpha^3) \, u_{k_1}(\tau) u_{k_2}(\tau) u_{k_3}(\tau) \, u'^{\dagger}_{k_1}(\tau') u'^{\dagger}_{k_2}(\tau') u'^{\dagger}_{k_3}(\tau')$$

$$\times \left(\frac{2}{k_3^2} + \frac{2}{k_1^2} + \frac{2}{k_2^2} \right) (2\pi)^3 \delta^3(\Sigma k_i) + \text{c.c.}$$

$$\tau \to 0 \quad = -i \int_{-\infty}^{0} d\tau' \, 4\varepsilon^2 H^4 \, \frac{-1}{H^3 \tau'^3} \, \frac{H^6}{64 \varepsilon^3 k_1^2 k_2^2 k_3^2} \, \tau'^3 e^{i \Sigma k_i \tau'}$$

$$\times \left(\frac{2}{k_1^2} + \frac{2}{k_2^2} + \frac{2}{k_3^2} \right) (2\pi)^3 \delta^3(\Sigma k_i) + \text{c.c.}$$

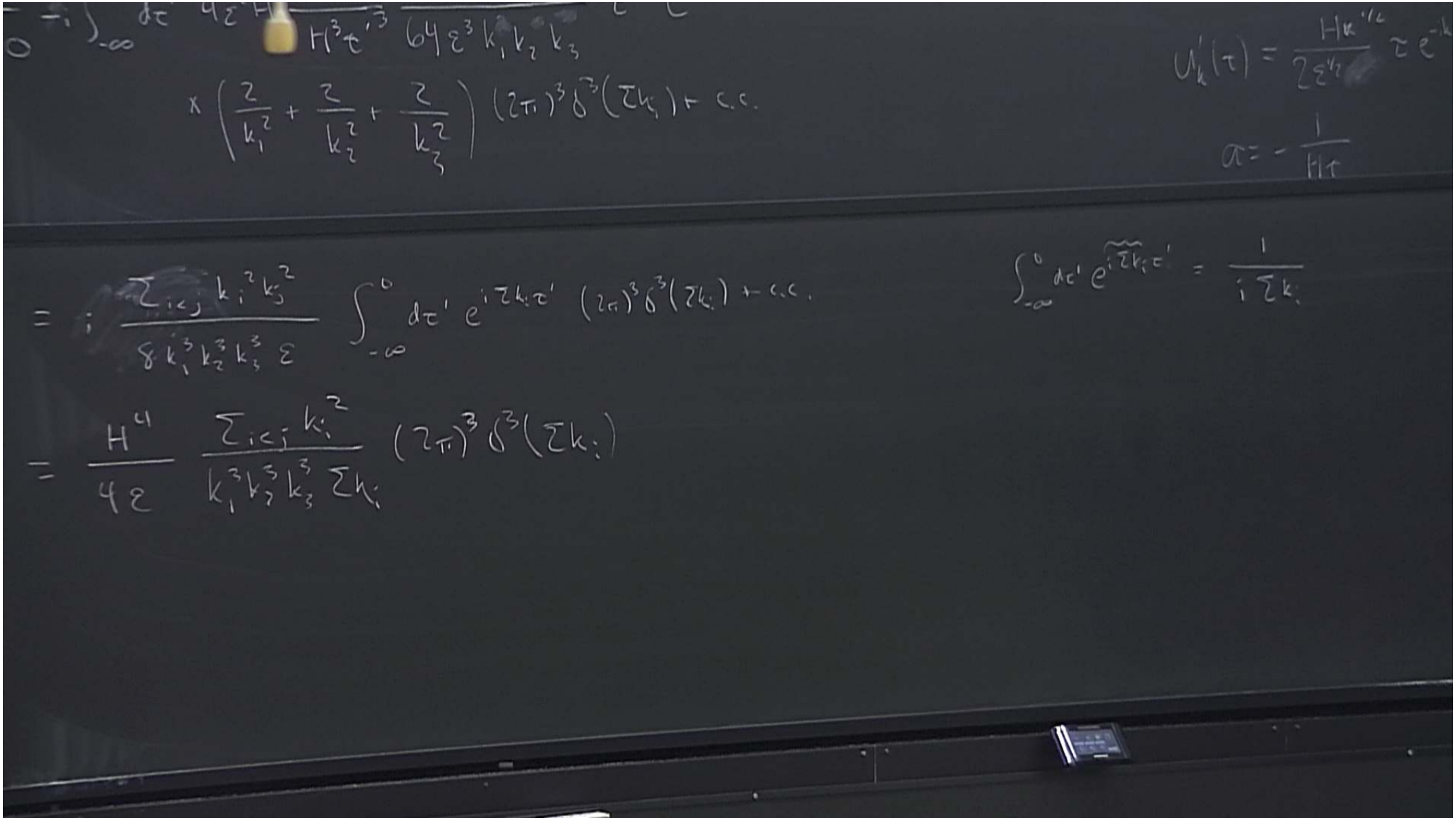
$$\hat{\zeta}^I_k = u_k(\tau) a_k + u_k^*(\tau) a^\dagger_{-k}$$

AT LOWEST ORDER IN SLOW ROLL

$$u_k(\tau) = \frac{H}{2\varepsilon^{1/2} k^{3/2}} (1 + ik\tau) e^{-ik\tau}$$

$$u'_k(\tau) = \frac{H}{2\varepsilon^{1/2} k^{1/2}} \tau e^{-ik\tau}$$

$$a = -\frac{1}{H\tau}$$


$\times \left(\frac{2}{k_1^2}+\frac{2}{k_2^2}+\frac{2}{k_3^2}\right)(2\pi)^3\delta^3(\Sigma k_i)+\text{c.c.}$
$u_k'(\tau)=\frac{Hk^{1/2}}{2\varepsilon^{1/2}}\tau e^{-k}$
$a=-\frac{1}{H\tau}$
$=i\frac{\Sigma_{i<j}k_i^2k_j^2}{8k_1^3k_2^3k_3^3\varepsilon}\int_{-\infty}^{0}d\tau' e^{i\Sigma k_i\tau'}(2\pi)^3\delta^3(\Sigma k_i)+\text{c.c.}$
$\int_{-\infty}^{0}d\tau' e^{i\Sigma k_i\tau'}=\frac{1}{i\Sigma k_i}$
$=\frac{H^4}{4\varepsilon}\frac{\Sigma_{i<j}k_i^2}{k_1^3k_2^3k_3^3\Sigma k_i}(2\pi)^3\delta^3(\Sigma k_i)$

$$\times \left(\frac{2}{k_1^2}+\frac{2}{k_2^2}+\frac{2}{k_3^2}\right)(2\pi)^3\delta^3(\Sigma k_i) + \text{c.c.}$$

$$u_k'(\tau) = \frac{Hk^{1/2}}{2\varepsilon^{1/2}}\tau e^{-}$$

$$a = -\frac{1}{H\tau}$$

$$= i\,\frac{\Sigma_{i<j} k_i^2 k_j^2}{8 k_1^3 k_2^3 k_3^3 \varepsilon}\int_{-\infty}^{0} d\tau'\, e^{i\Sigma k_i \tau'}\,(2\pi)^3\delta^3(\Sigma k_i) + \text{c.c.}$$

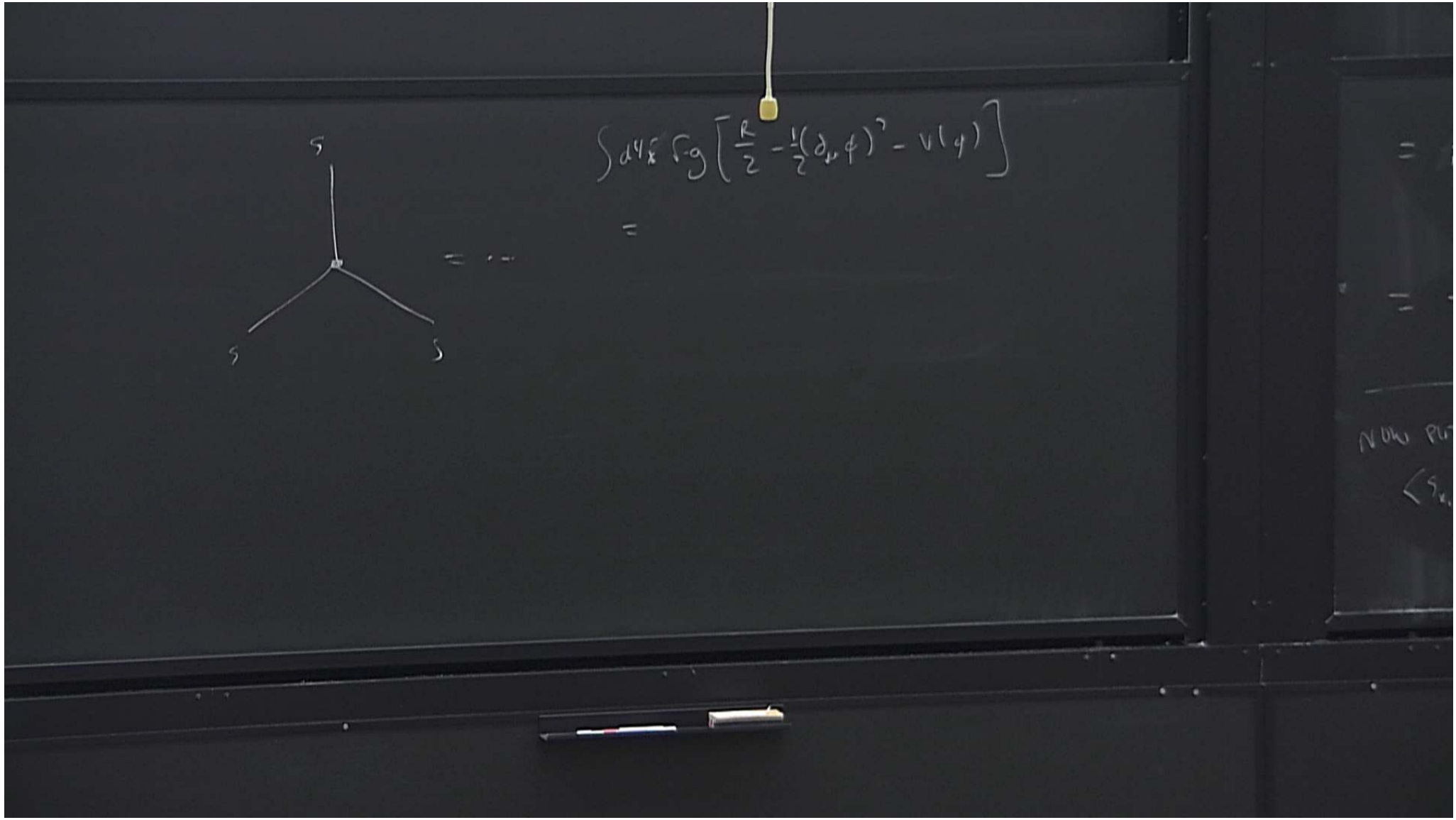
$$\int_{-\infty}^{0} d\tau'\, e^{i\widetilde{\Sigma k_i}\tau'} = \frac{1}{i\Sigma k_i}$$

$$= \frac{H^4}{4\varepsilon}\,\frac{\Sigma_{i<j} k_i^2}{k_1^3 k_2^3 k_3^3 \Sigma k_i}(2\pi)^3\delta^3(\Sigma k_i) \qquad \left[= \langle \zeta^c_{k_1}\zeta^c_{k_2}\zeta^c_{k_3}\right]$$

NOW PUT EVERYTHING TOGETHER

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = \left[\frac{H^4}{4\varepsilon}\,\frac{\Sigma_{i<j} k_i^2 k_j^2}{k_1^3 k_2^3 k_3^3 \Sigma k_i} + \left(\frac{3\varepsilon}{4}-\frac{\eta}{2}\right)\frac{H^4\,\Sigma k_i^3}{8\varepsilon^2 k_1^3 k_2^3 k_3^3} + \frac{\varepsilon}{2}\,\frac{H^4\,\Sigma_{i\neq j} k_i^3 k_j^2}{16\varepsilon^2 k_1^3 k_2^3 k_3^3}\right]$$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \frac{H^4}{32\epsilon^2 k_1^3 k_2^3 k_3^3} \left[8\epsilon \frac{\sum_{i<j} k_i^2 k_j^2}{\sum k_i} + (3\epsilon - 2\eta) \sum k_i^3 + \epsilon \sum_{i \neq j} k_i k_j^2 \right]$$


$\int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \right]$
=
= ...
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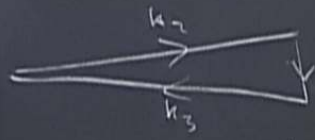
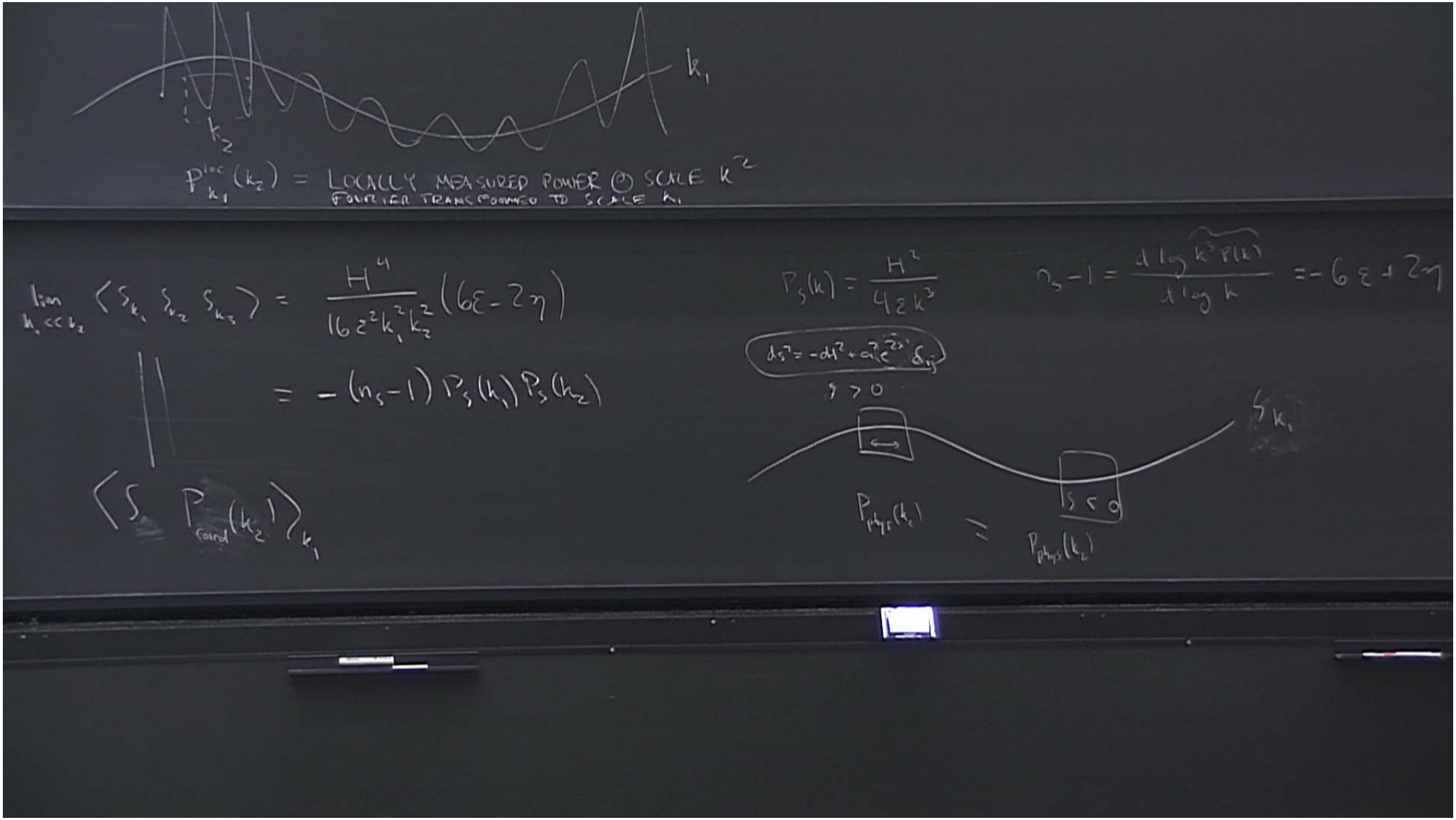
$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = \frac{H^4}{32\,\varepsilon^2 k_1^3 k_2^3 k_3^3} \left[8\varepsilon \frac{\sum_{i<j} k_i^2 k_j^2}{\sum k_i} + (3\varepsilon - 2\eta) \sum k_i^3 + \varepsilon \sum_{i \neq j} k_i k_j^2 \right] (2\pi)^3 \delta^3(\sum k_i)$$



SQUEEZED LIMIT $k_1 \ll k_2, k_3$


k_1
k_2
$P^{loc}_{k_1}(k_2)$ = LOCALLY MEASURED POWER @ SCALE k^{-2}
FOURIER TRANSFORMED TO SCALE k_1
$\lim_{k_1 \ll k_2} \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = \frac{H^4}{16\epsilon^2 k_1^2 k_2^2}(6\epsilon - 2\eta)$
$= -(n_s - 1) P_\zeta(k_1) P_\zeta(k_2)$
$\langle \zeta \, P_{found}(k_2) \rangle_{k_1}$
$P_\zeta(k) = \frac{H^2}{4\epsilon k^3}$
$n_s - 1 = \frac{d \log k^3 P(k)}{d \log k} = -6\epsilon + 2\eta$
$ds^2 = -dt^2 + a^2 e^{2\zeta} \delta_{ij}$
$\zeta > 0$
ζ_{k_1}
$\zeta < 0$
$P_{phys}(k_2)$ ≈ $P_{phys}(k_2)$

$$\times \left(\frac{2}{k_1^2} + \frac{2}{k_2^2} + \frac{2}{k_3^2} \right) (2\pi)^3 \delta^3\left(\textstyle\sum k_i\right) + \text{c.c.}$$

$P_\zeta(k)$

$$\zeta_k = \int d^3x\, \zeta(x) e^{ik\cdot x}$$

$$\langle \zeta_k \zeta_{k'} \rangle = P(k) (2\pi)^3 \delta^3(k - k')$$

$$n_s - 1 = \frac{d \log k^3}{d \log}$$

$$L' = (1+\varepsilon) L$$

$$k' = (1-\varepsilon) k$$

$$P'(k') = (1+3\varepsilon) P\left((1+\varepsilon) k'\right) = (1+3\varepsilon) P + k\varepsilon \frac{dP}{dK}$$

$\to 0 \quad \int_{-\infty} \quad H^3 \tau^{\ldots} \; 64 \varepsilon^3 k_1 k_2 k_3$

$$\times \left(\frac{2}{k_1^2} + \frac{2}{k_2^2} + \frac{2}{k_3^2} \right) (2\pi)^3 \delta^3(\Sigma k_i) + c.c.$$

$$u_k'(\tau) = \frac{H k}{2\varepsilon^{1/2}} \tau e^{-k\tau}$$

$$a = -\frac{1}{H\tau}$$

$P_s(k)$

$$S_k = \int d^3x \; S(x) e^{ik\cdot x}$$

$$\langle S_k S_{k'} \rangle = P(k) (2\pi)^3 \delta^3(k-k')$$

$$n_s - 1 = \frac{d \log k^3 P}{d \log k} = 3 + \frac{k}{P} \frac{dP}{dk}$$

$$L' = (1+\varepsilon) L$$

$$k' = (1-\varepsilon) k$$

$$P'(k') = (1+3\varepsilon) P\big((1+\varepsilon) k'\big) = (1+3\varepsilon) P + k\varepsilon \frac{dP}{dk}$$

$$= (1+\cancel{3\varepsilon}) P + k\varepsilon \frac{P}{k} \left(-\cancel{3} + n_s - 1 \right)$$

$$= (1 + (n_s - 1)\varepsilon) P$$

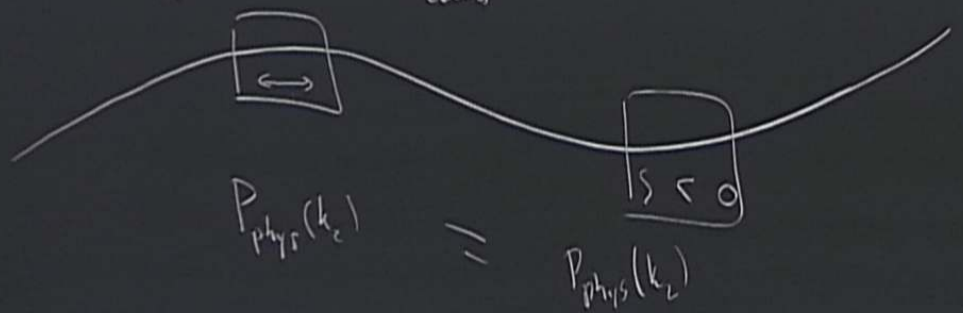
$= k^2$

$$P_s(k) = \frac{H^2}{4\epsilon k^3} \qquad n_s - 1 = \frac{d \log k^3 P(k)}{d \log k} = -6\epsilon + 2\eta$$

$$P_{coord}(k_z) = \left(1 - \xi (n_s - 1)\right) \bar{P}(k_z)$$

$ds^2 = -dt^2 + a^2 e^{2\xi} \delta_{ij}$

$\xi > 0$ $\quad L_{coord} = (1 - \xi) L_{phys}$



$P_{phys}(k_z) \equiv P_{phys}(k_z)$

k_2

$P^{loc}_{k_1}(k_2)$ = LOCALLY MEASURED POWER @ SCALE k^2
FOURIER TRANSFORMED TO SCALE k_1

$$\lim_{k_1 \ll k_2} \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = \frac{H^4}{16 \epsilon^2 k_1^2 k_2^2} (6\epsilon - 2\eta)$$

$$= -(n_s - 1) P_\zeta(k_1) P_\zeta(k_2)$$

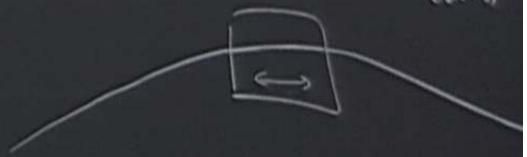
$$\langle \zeta \, P_{Coord}(k_2) \rangle_{k_1} = -(n_s - 1) P_\zeta(k_1) P_\zeta(k_2)$$

"CONSISTENCY RELATION" =

$$P_\zeta(k) = \frac{H^2}{4\epsilon k^3}$$

$ds^2 = -dt^2 + a^2 e^{2\zeta} \delta_{ij}$

$\zeta > 0$ L_{Coord} =

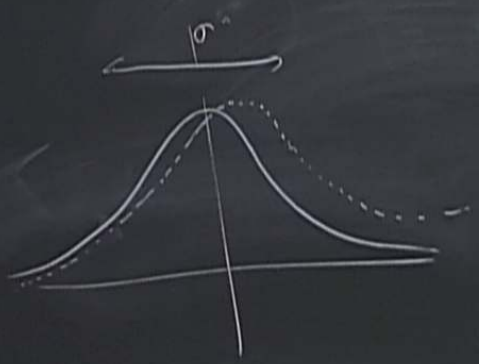


$P_{\phi\phi_T}(k_2)$ =

$\tau \to 0$ $\quad -\infty$

$$\times \left(\frac{2}{k_1^2} + \frac{2}{k_2^2} + \frac{2}{k_3^2} \right) (2\pi)^3 \delta^3(\vec{k}_i) + \text{c.c.}$$

DETECTABILITY



$\langle xx \rangle = \sigma^2$

$\langle x^3 \rangle = K \qquad K \ll \sigma^3$

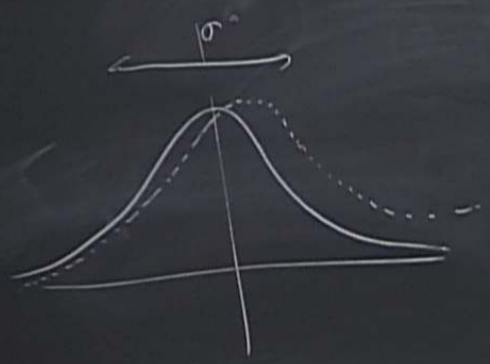
HOW MANY RANDOM SAMPLES (N) ARE NEEDED TO DETECT $K \neq 0$ AT 1σ?

$$\times \left(\frac{2}{k_1^2} + \frac{2}{k_2^2} + \frac{2}{k_3^2} \right) (2\pi)^3 \delta^3(\Sigma k_i) + c.c.$$

$$u_k'(\tau) = \frac{}{2\varepsilon}$$

$$a = -\frac{1}{H}$$

DETECTABILITY



$$\langle x x \rangle = \sigma^2$$

$$\langle x^3 \rangle = K \qquad K \ll \sigma^3$$

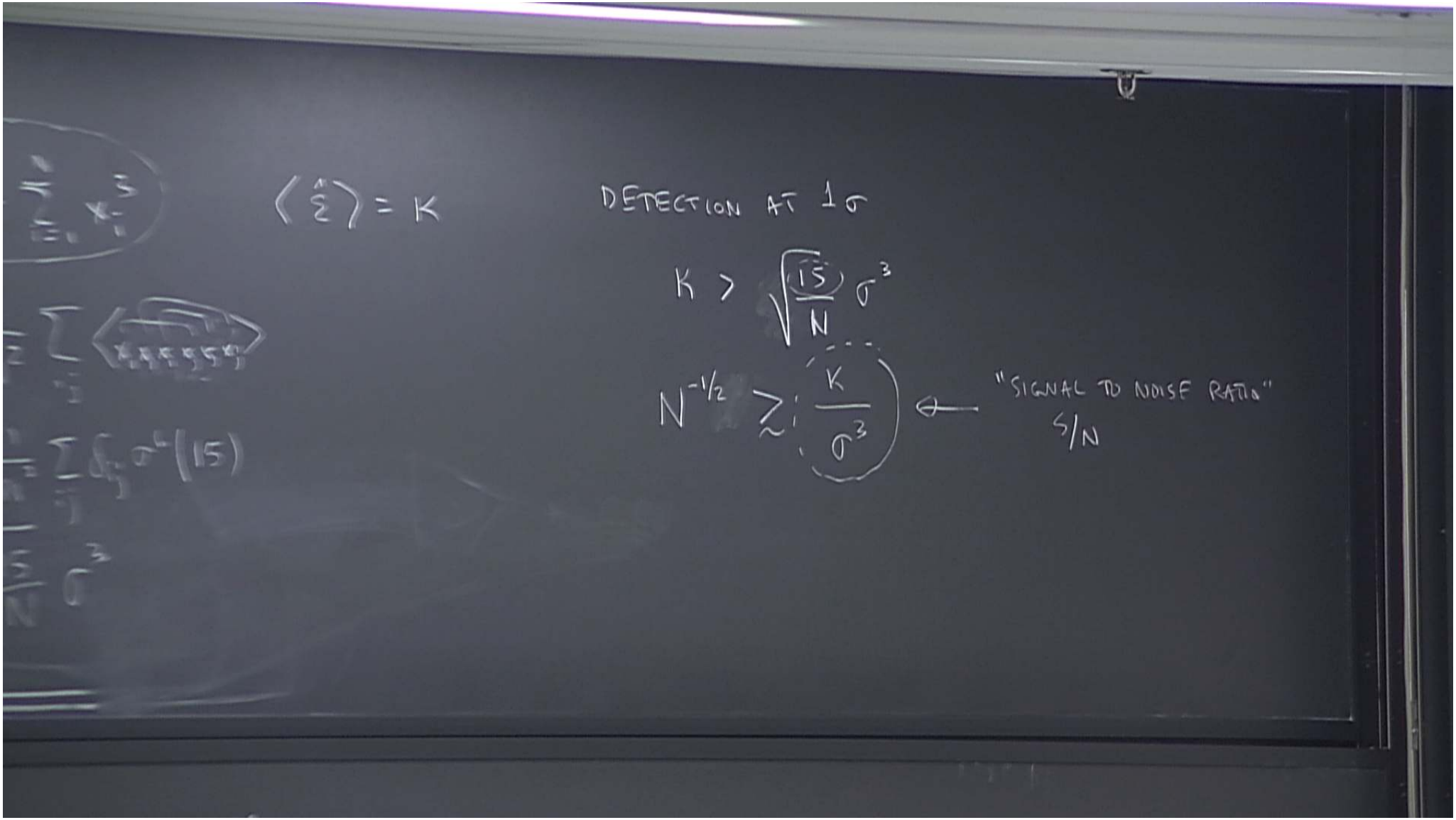
HOW MANY RANDOM SAMPLES N ARE NEEDED TO DETECT $K \neq 0$ AT 1σ?

$$\hat{\varepsilon} = \frac{1}{N} \sum_{i=1}^{N} x_i^3 \qquad \langle \hat{\varepsilon} \rangle = K$$

$$\langle \hat{\varepsilon}^2 \rangle = \frac{1}{N^2} \sum_{ij} \langle x_i x_i x_i x_j x_j x_j \rangle$$

$$= \frac{1}{N^2} \sum_{ij} \delta_{ij} \sigma^6 (15)$$

$$\langle \hat{\varepsilon}^2 \rangle^{1/2} = \sqrt{\frac{15}{N}} \sigma^3$$

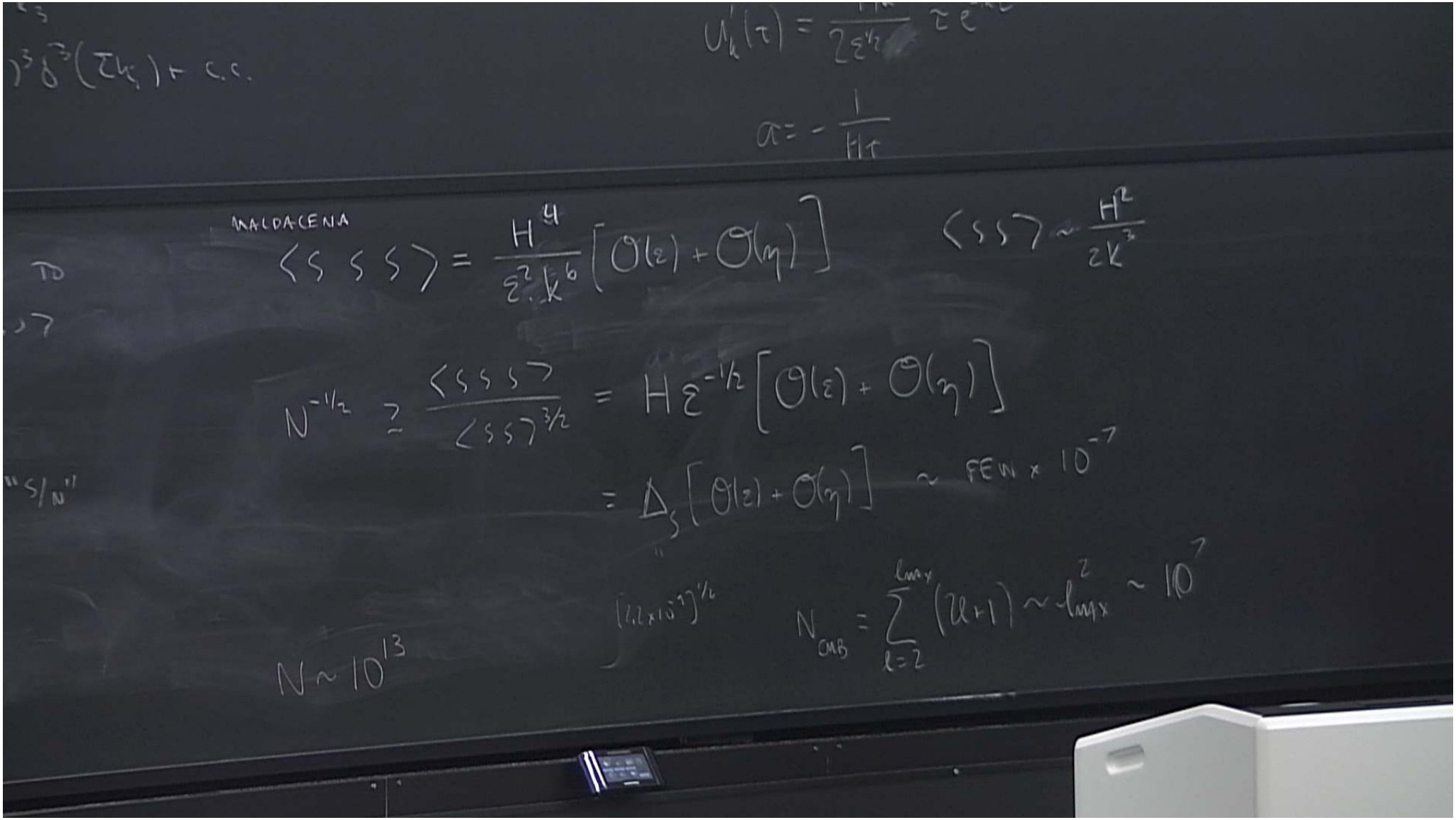

$\langle \hat{\Sigma} \rangle = K$
DETECTION AT 1σ
$K > \sqrt{\frac{15}{N}}\,\sigma^3$
$N^{-1/2} \gtrsim \frac{K}{\sigma^3}$
"SIGNAL TO NOISE RATIO"
S/N

$\tau \to 0$ $\quad -\infty$

$$\times \left(\frac{2}{k_1^2} + \frac{2}{k_2^2} + \frac{2}{k_3^2} \right) (2\pi)^3 \delta^3(\vec{k}_i) + \text{c.c.}$$

HOW MANY MODES N OF ζ ARE NEEDED TO DETECT A 3-PT FUNCTION $\langle \zeta\zeta\zeta \rangle$ AT 1σ?

$$N^{-1/2} = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} \quad \leftarrow \text{"S/N"}$$


MALDACENA
$\langle \zeta\zeta\zeta \rangle = \frac{H^4}{\varepsilon^2 k^6}\left[\mathcal{O}(\varepsilon) + \mathcal{O}(\eta)\right]$
$\langle \zeta\zeta \rangle \sim \frac{H^2}{2k^3}$
$N^{-1/2} \geq \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} = H\varepsilon^{-1/2}\left[\mathcal{O}(\varepsilon) + \mathcal{O}(\eta)\right]$
$= \Delta_\zeta\left[\mathcal{O}(\varepsilon) + \mathcal{O}(\eta)\right] \sim \text{FEW} \times 10^{-7}$
$N \sim 10^{13}$
$N_{CMB} = \sum_{\ell=2}^{\ell_{max}} (2\ell+1) \sim \ell_{max}^2 \sim 10^7$