Title: Explorations in Cosmology-9

Date: Apr 16, 2015 10:15 AM

URL: http://pirsa.org/15040035

Abstract:

QUADRATIC ACTION

$$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} \frac{\phi^2}{H^2} a^2 \left(\zeta_k'^2 - k^2 \zeta_k^2\right)$$

CANONICALLY NORMALIZE $\sigma_k = \frac{a\dot{\phi}}{H}\zeta_k$

$$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\sigma_k'^2 - \omega(k,\tau)^2 \sigma_k^2\right]$$

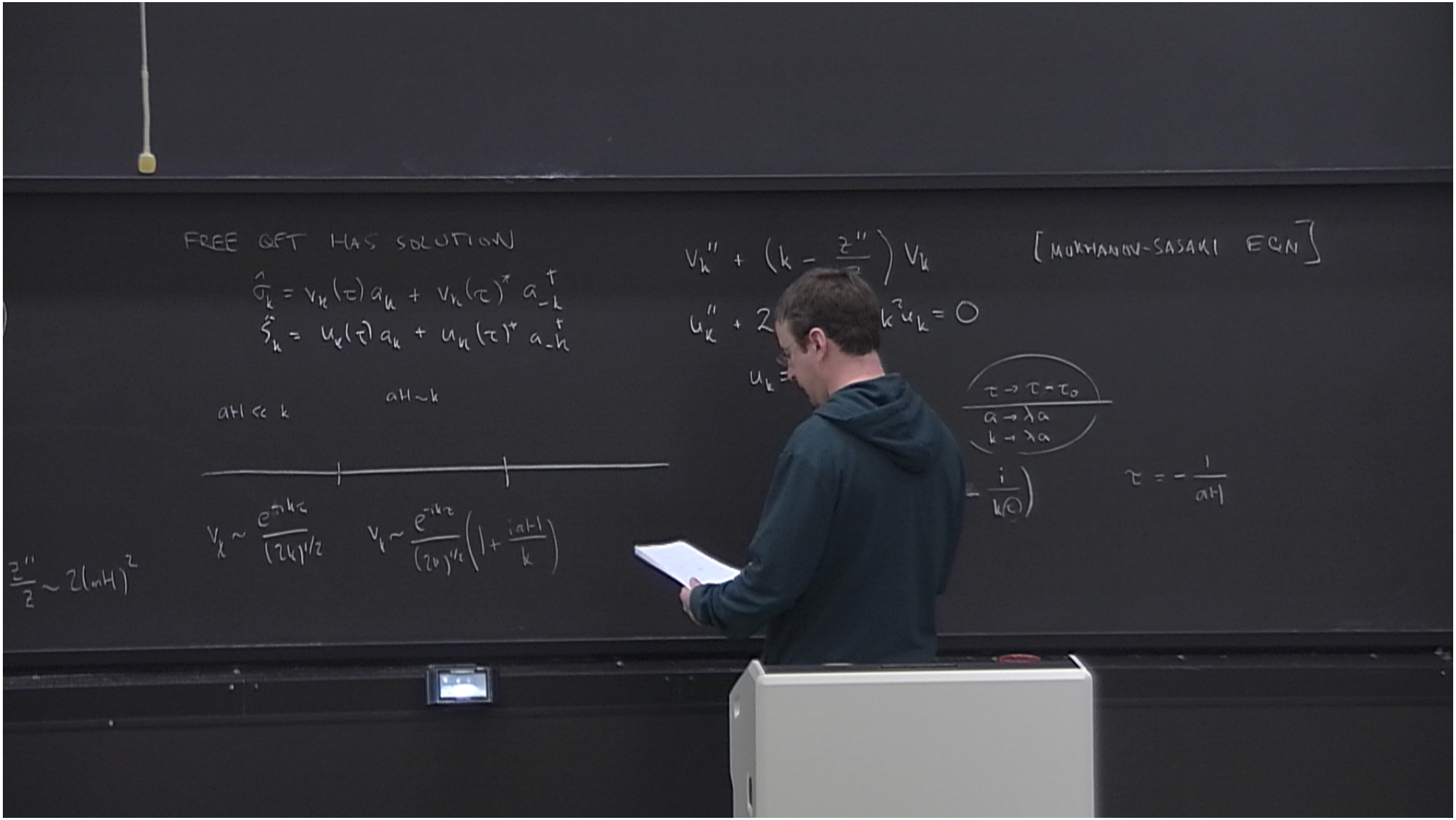
$$\omega(k,\tau)^2 = k^2 - \frac{z''}{z} \qquad z \overset{\text{def}}{=} \frac{a\dot{\phi}}{H}$$

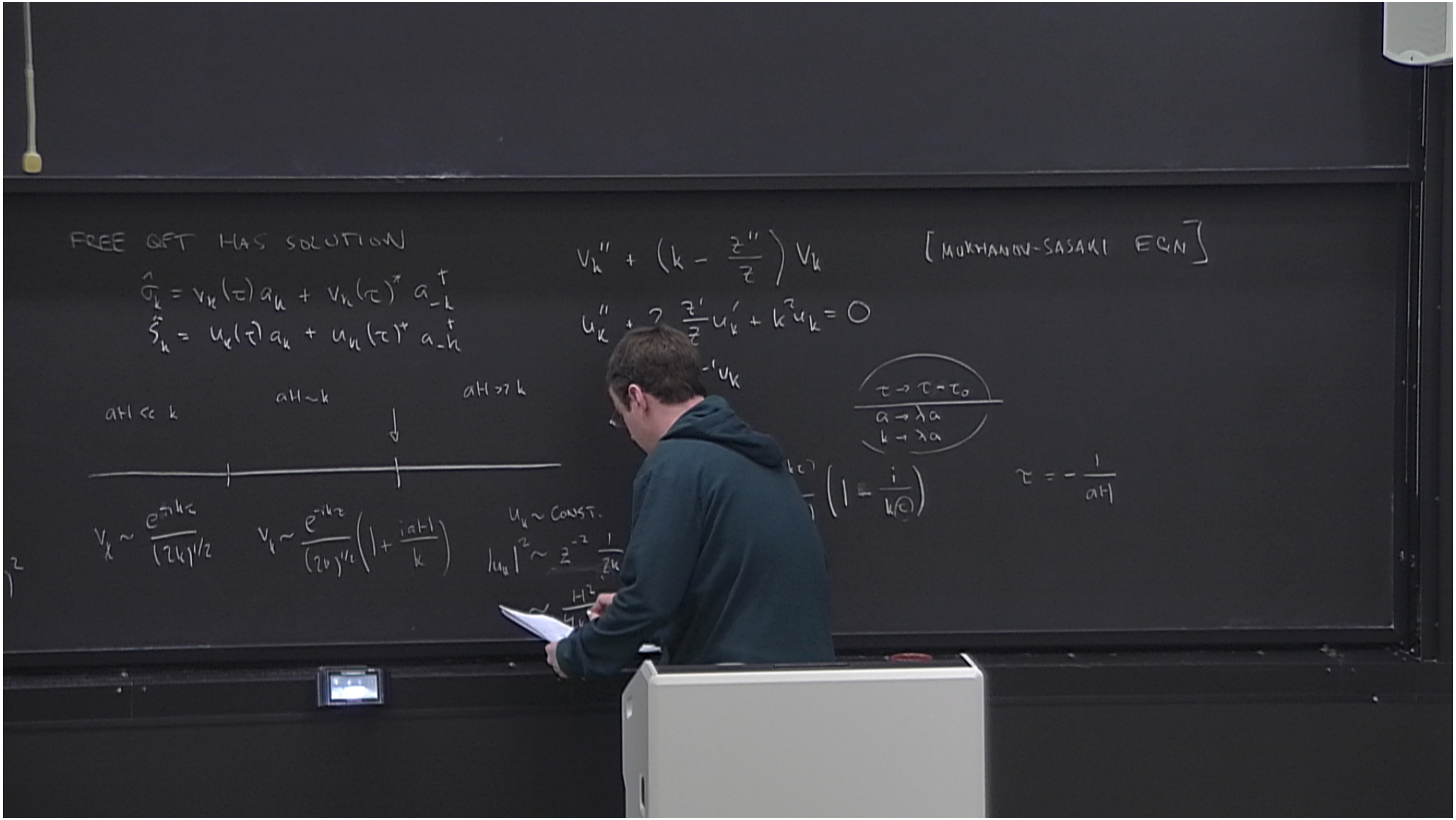
[TO LOWEST ORDER IN SR, $z \sim \sqrt{2\epsilon}\, a$ $\quad \frac{z'}{z} \sim aH$ $\quad \frac{z''}{z} \sim 2(aH)^2$

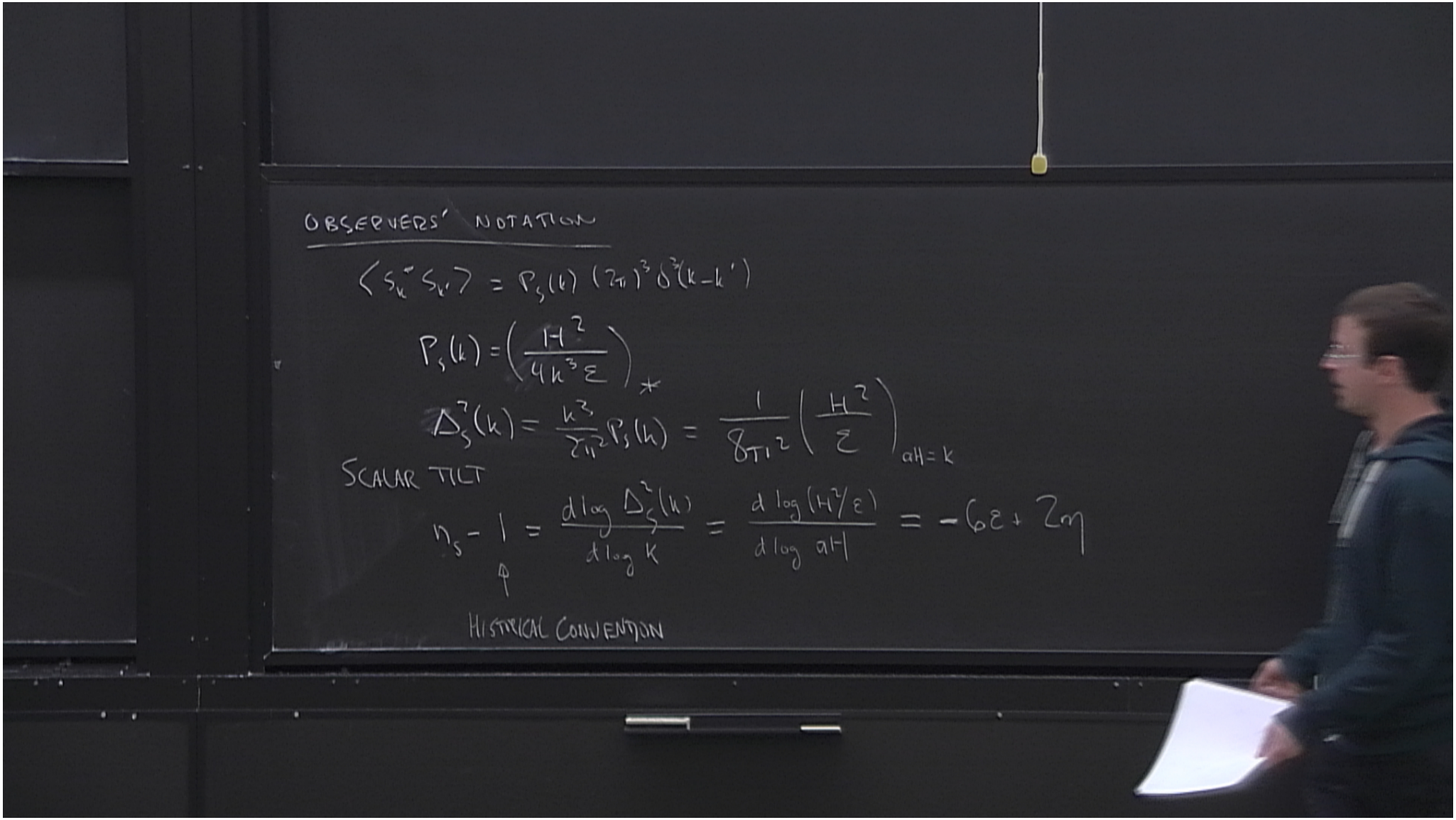
FREE QFT HAS SOLUTION

$$\hat{\sigma}_k = v_k(\tau)\, a_k + v_k(\tau)^*\, a_{-k}^\dagger$$
$$\hat{\zeta}_k = u_k(\tau)\, a_k + u_k(\tau)^*\, a_{-k}^\dagger$$

v_k''


FREE QFT HAS SOLUTION
$\hat{\sigma}_k = v_k(\tau)\, a_k + v_k(\tau)^* a_{-k}^\dagger$
$\hat{\zeta}_k = u_k(\tau)\, a_k + u_k(\tau)^* a_{-k}^\dagger$
$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k$
[MUKHANOV-SASAKI EQN]
$u_k'' + 2$ $k^2 u_k = 0$
$u_k =$
$\tau \to \tau - \tau_0$
$a \to \lambda a$
$k \to \lambda a$
$\tau = -\frac{1}{aH}$
$aH \ll k$
$aH \sim k$
$v_k \sim \frac{e^{-ik\tau}}{(2k)^{1/2}}$
$v_k \sim \frac{e^{-ik\tau}}{(2k)^{1/2}}\left(1 + \frac{iaH}{k}\right)$
$\frac{z''}{z} \sim 2(aH)^2$


FREE QFT HAS SOLUTION
$\hat{\sigma}_k = v_k(\tau)\, a_k + v_k(\tau)^* a_{-k}^\dagger$
$\hat{\zeta}_k = u_k(\tau)\, a_k + u_k(\tau)^* a_{-k}^\dagger$
$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k$
[MUKHANOV-SASAKI EQN]
$u_k'' + 2\frac{z'}{z} u_k' + k^2 u_k = 0$
$\tau \to \tau - \tau_0$
$a \to \lambda a$
$k \to \lambda a$
$aH \ll k$
$aH \sim k$
$aH \gg k$
$v_k \sim \frac{e^{-ik\tau}}{(2k)^{1/2}}$
$v_k \sim \frac{e^{-ik\tau}}{(2k)^{1/2}}\left(1 + \frac{iaH}{k}\right)$
$u_k \sim$ CONST.
$\tau = -\frac{1}{aH}$


OBSERVERS' NOTATION
$\langle \zeta_k^* \zeta_{k'} \rangle = P_\zeta(k)\,(2\pi)^3 \delta^3(k-k')$
$P_\zeta(k) = \left(\frac{H^2}{4k^3\varepsilon}\right)_*$
$\Delta_\zeta^2(k) = \frac{k^3}{2\pi^2} P_\zeta(k) = \frac{1}{8\pi^2}\left(\frac{H^2}{\varepsilon}\right)_{aH=k}$
SCALAR TILT
$n_s - 1 = \frac{d\log \Delta_\zeta^2(k)}{d\log k} = \frac{d\log(H^2/\varepsilon)}{d\log aH} = -6\varepsilon + 2\eta$
HISTORICAL CONVENTION

NOTATION

$\mathcal{P} = P_s(k)\,(2\pi)^3\,\delta^3(k-k')$

$P_s(k) = \left(\frac{H^2}{4k^3\varepsilon}\right)_*$

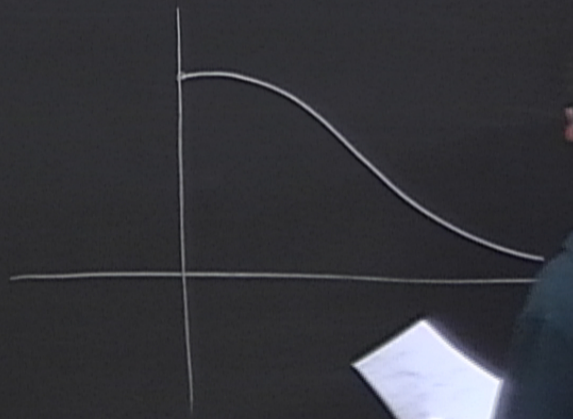
$\Delta_s^2(k) = \frac{k^3}{2\pi^2}P_s(k) = \frac{1}{8\pi^2}\left(\frac{H^2}{\varepsilon}\right)_{aH=k} + \cdots$

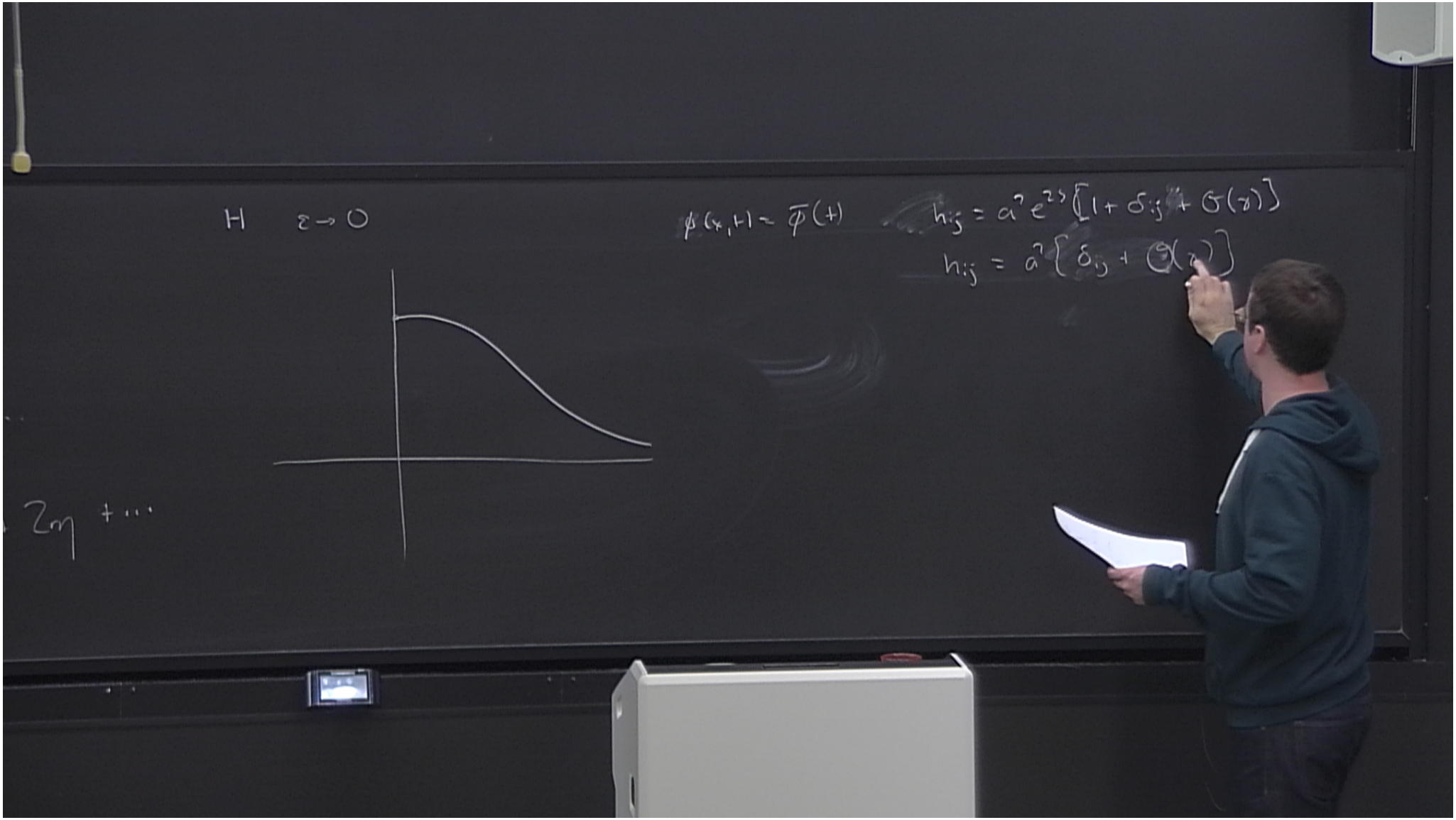
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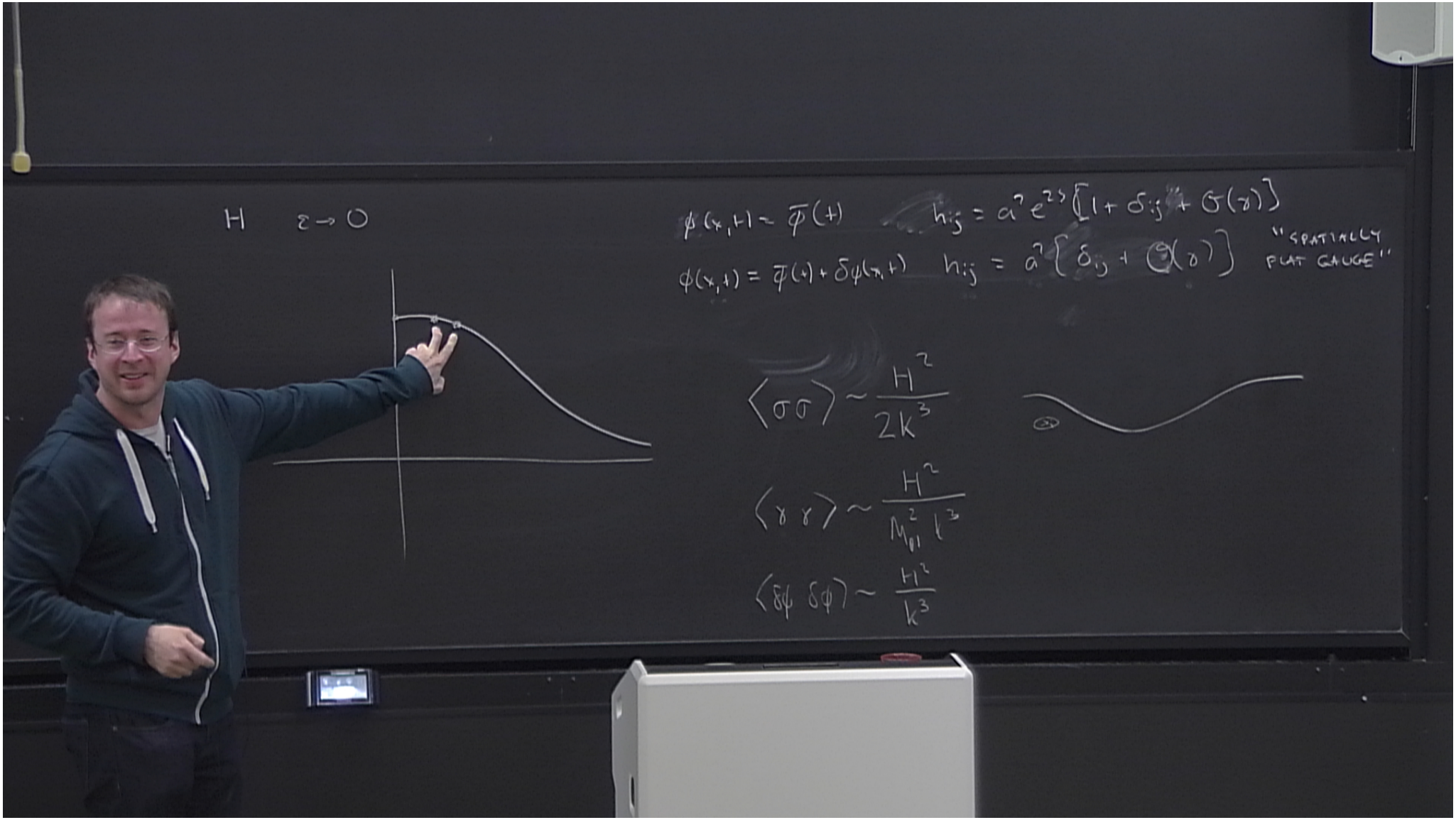
$n_s - 1 = \frac{d\log\Delta_s^2(k)}{d\log k} = \frac{d\log(H^2/\varepsilon)}{d\log aH}\Big| = -6\varepsilon + 2\eta + \cdots$

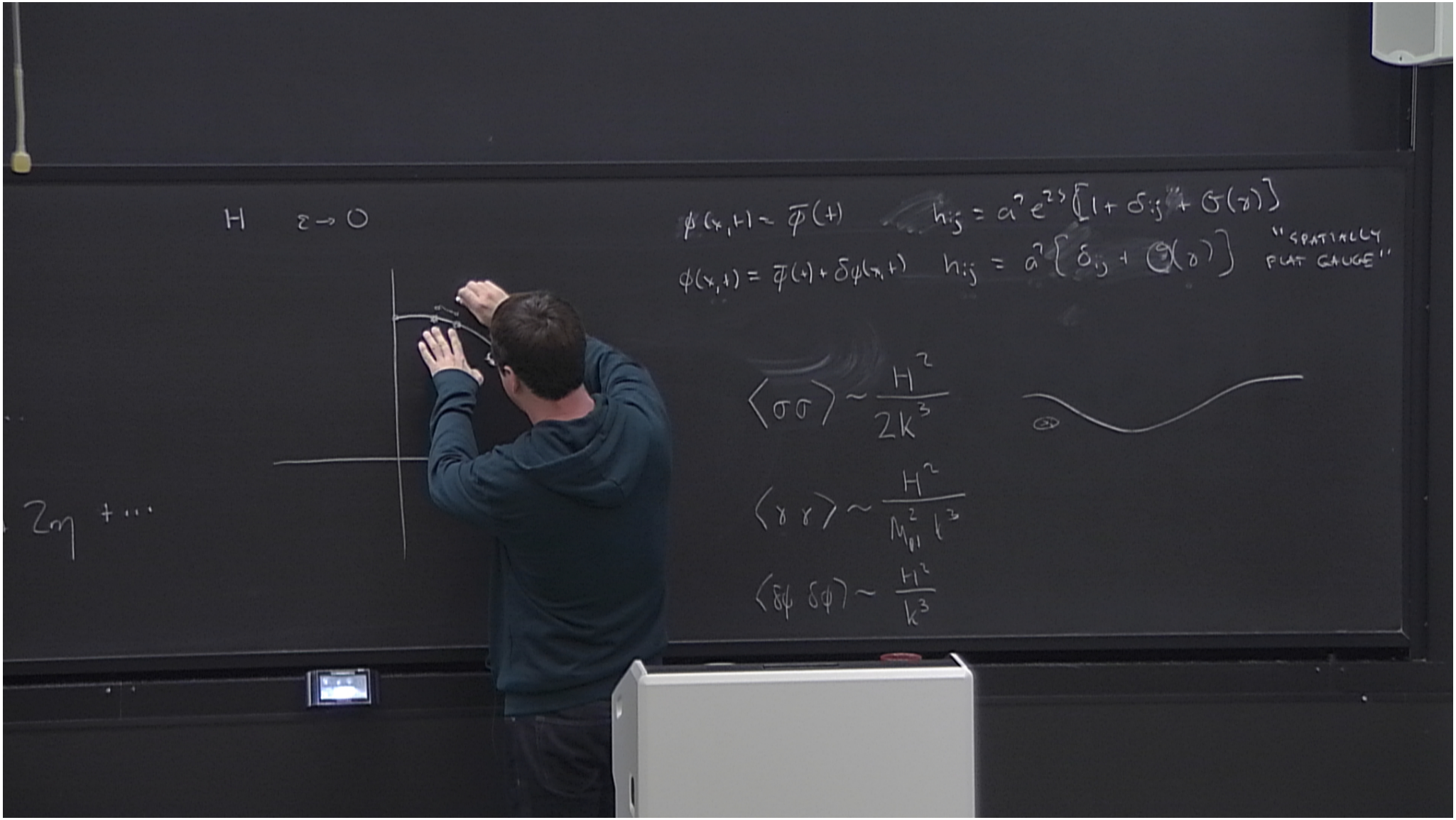
HISTORICAL CONVENTION

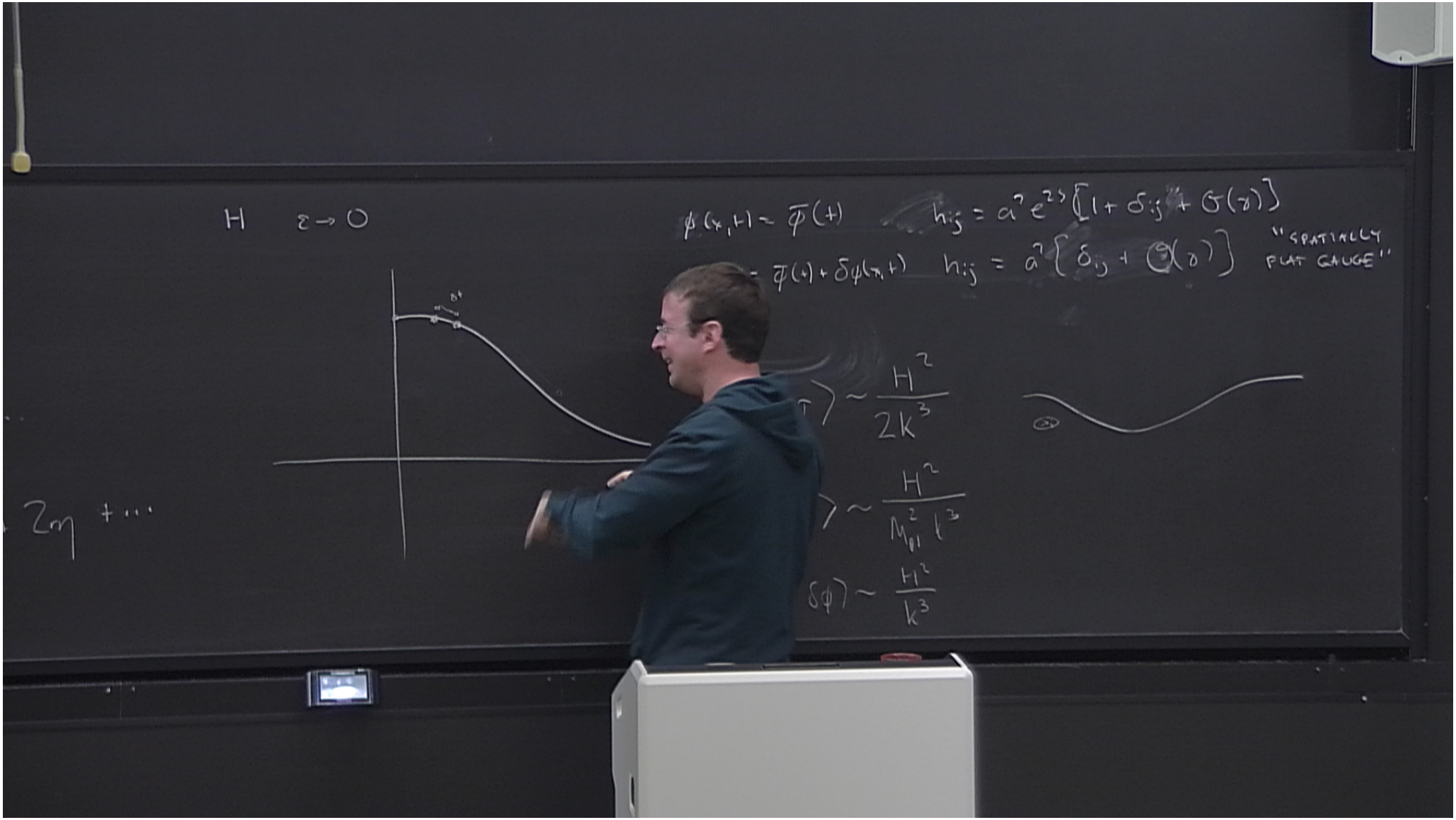
H $\quad \varepsilon \to 0$

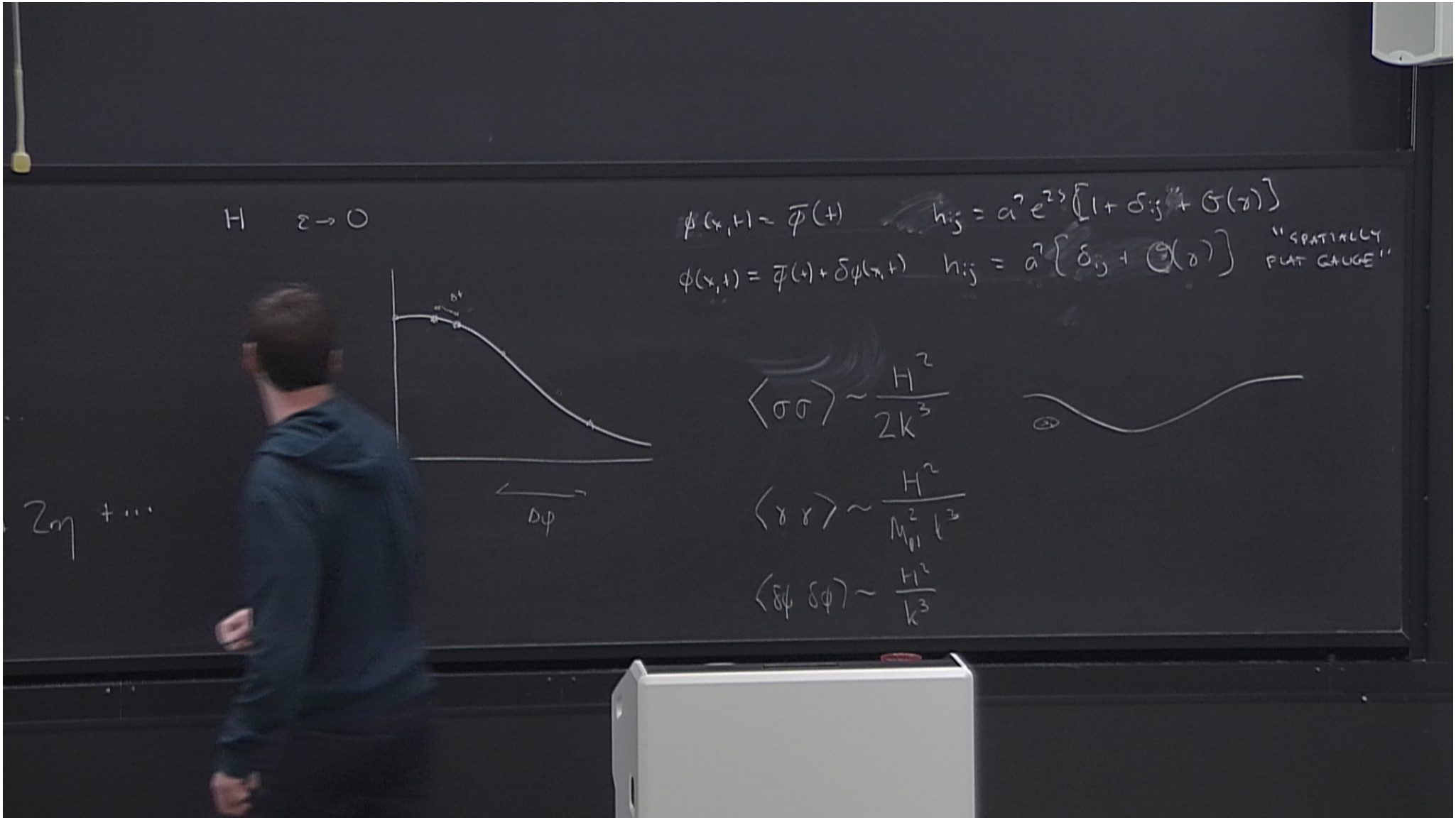


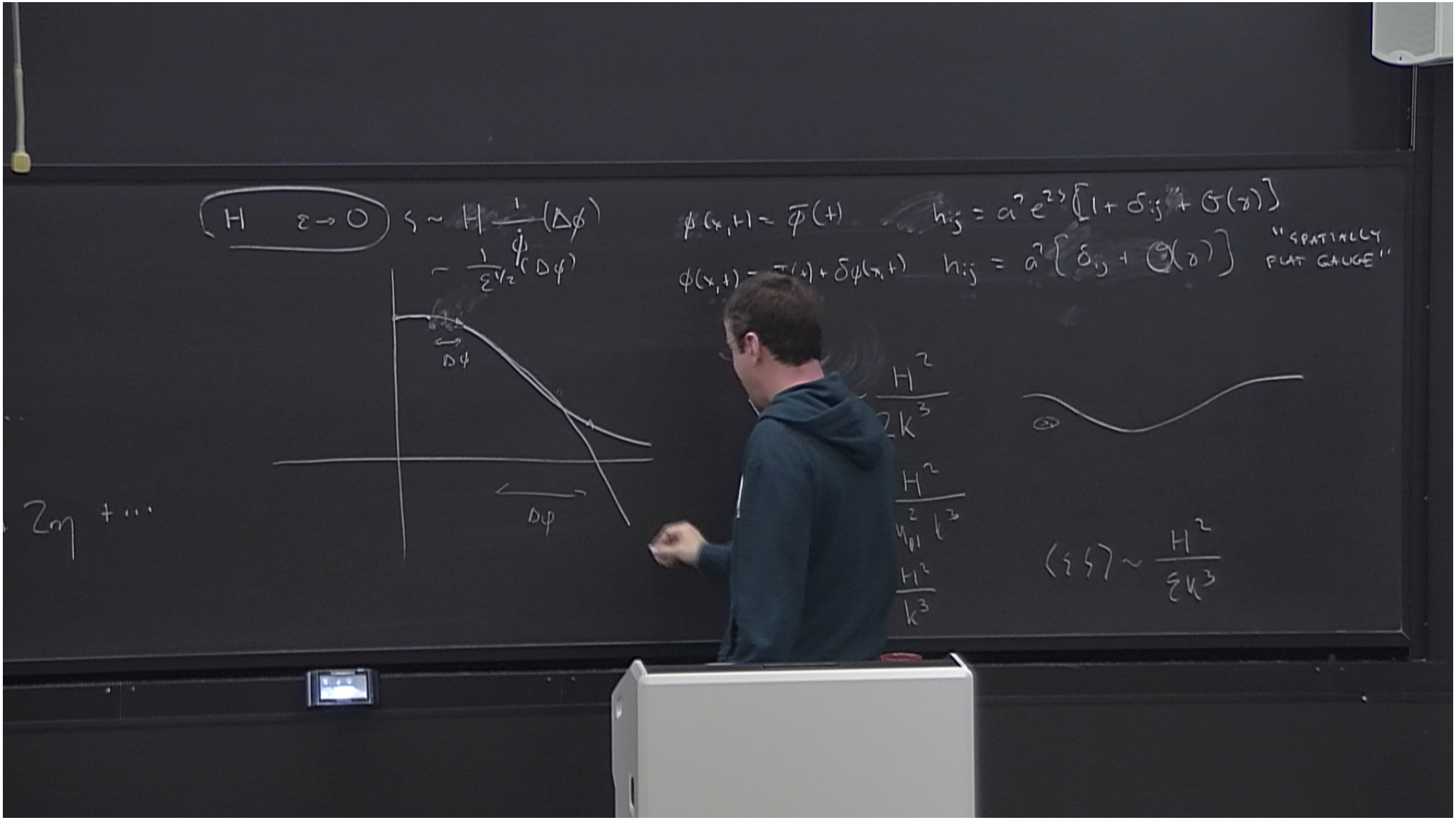

H ε → 0
φ(x,t) = φ̄(t)
h_ij = a² e^{2δ} [1 + δ_ij + σ(x)]
h_ij = a² [δ_ij + σ(x)]
2η + ...

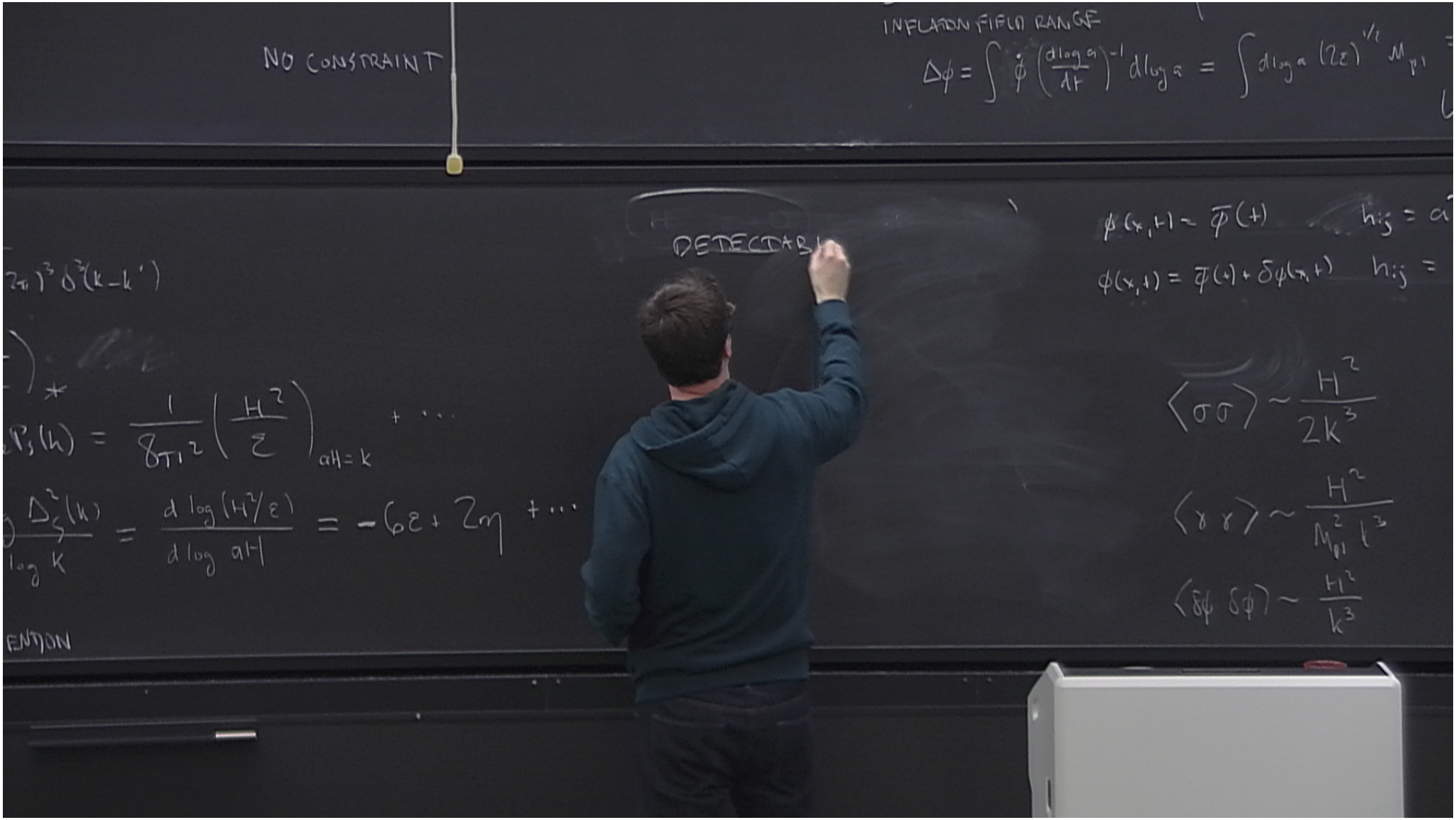

H ε → 0
φ(x,t) = φ̄(t)
φ(x,t) = φ̄(t) + δφ(x,t)
h_ij = a² e^{2ζ} [1 + δ_ij + σ(γ)]
h_ij = a² [δ_ij + σ(γ)]
"SPATIALLY FLAT GAUGE"
⟨σσ⟩ ~ H²/2k³
⟨γγ⟩ ~ H²/M_Pl² k³
⟨δφ δφ⟩ ~ H²/k³

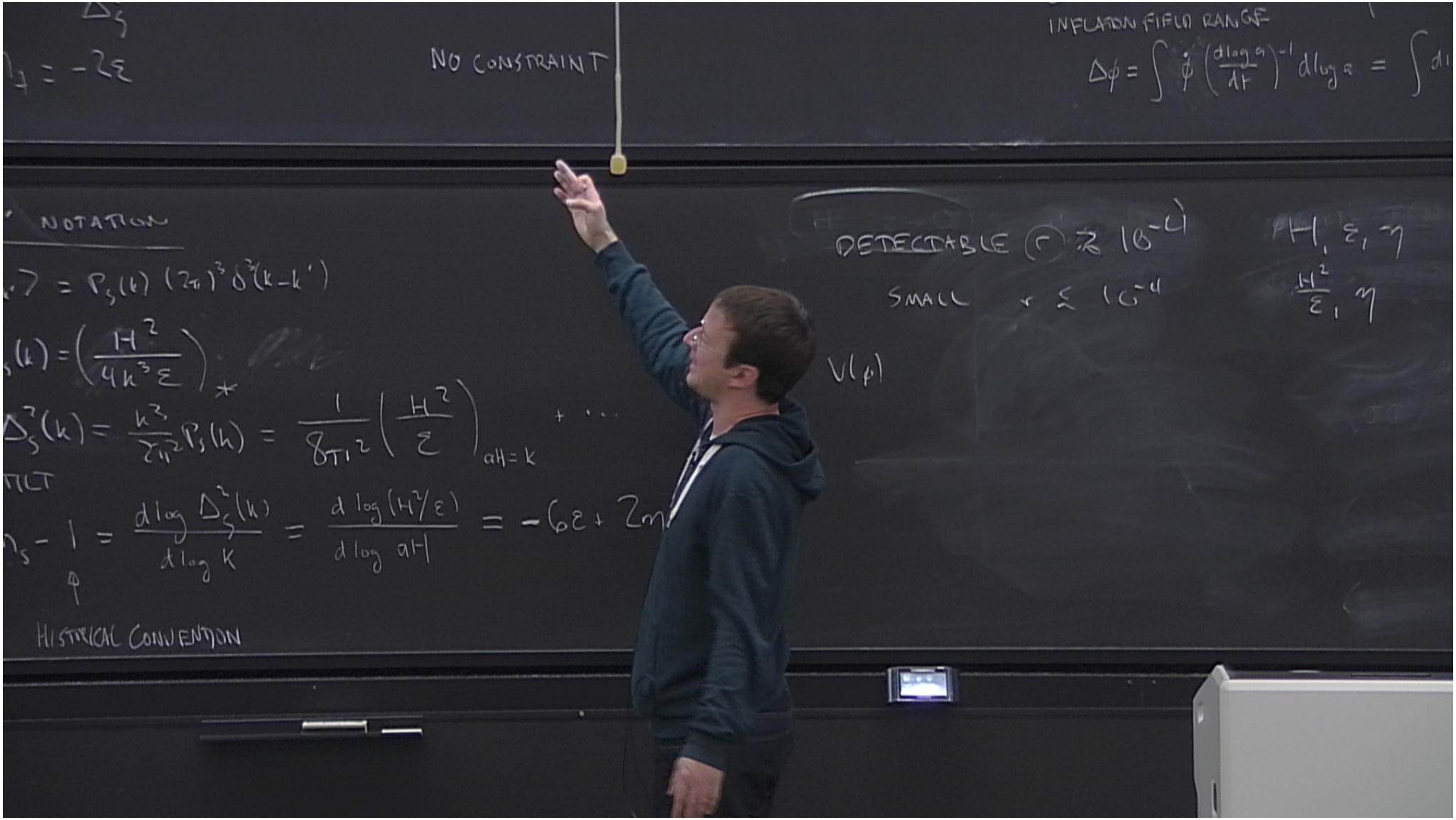

H ε → 0
φ(x,t) ≈ φ̄(t)
h_ij = a² e^{2ζ} [1 + δ_ij + σ(γ)]
φ(x,t) = φ̄(t) + δφ(x,t)
h_ij = a² [δ_ij + σ(γ)]
"SPATIALLY FLAT GAUGE"
⟨σσ⟩ ~ H²/2k³
⟨γγ⟩ ~ H²/M_Pl² k³
⟨δφ δφ⟩ ~ H²/k³
2η + ...

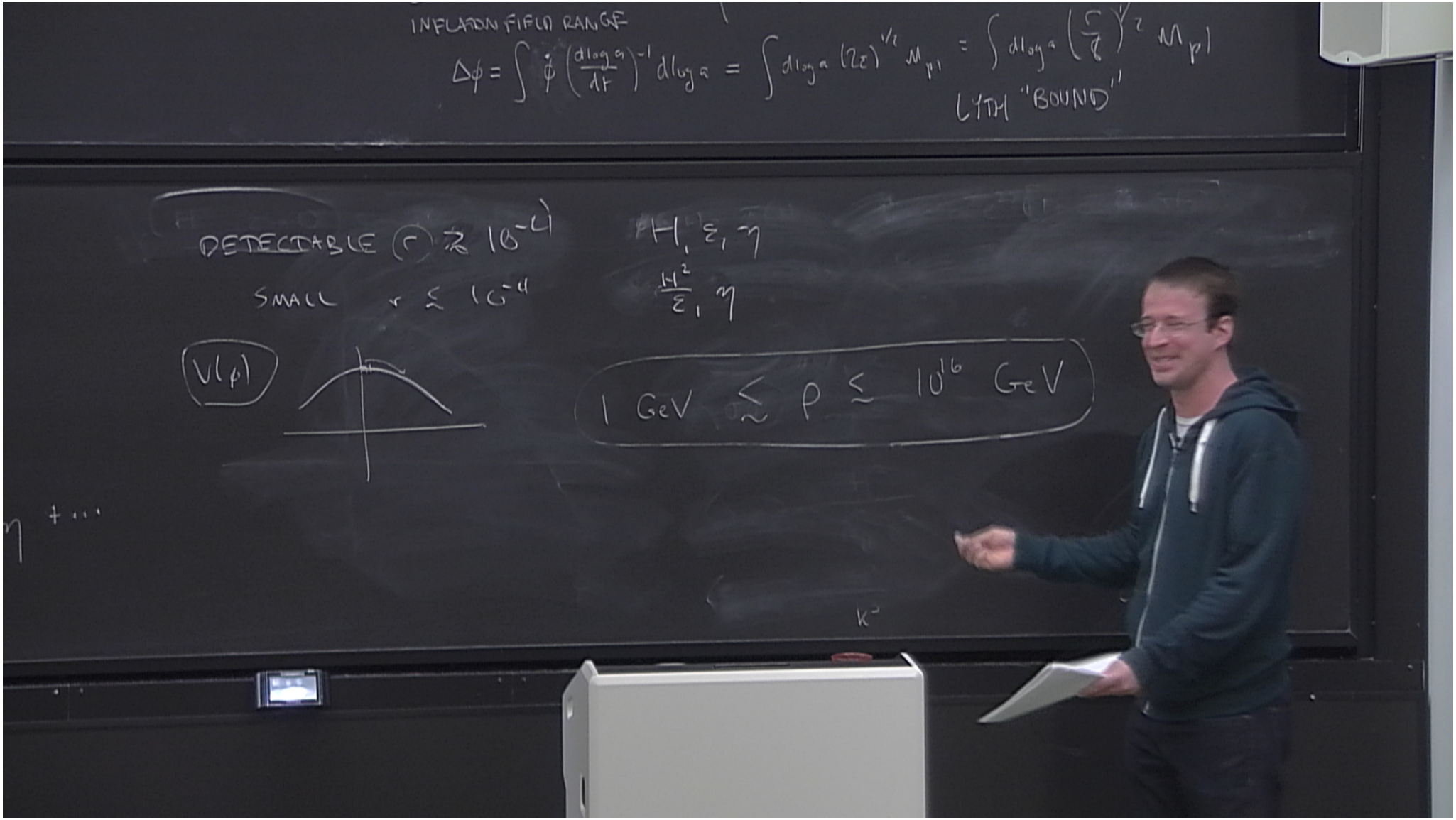

H ε → 0
φ(x,t) = φ̄(t)
h_ij = a² e^{2δ} [1 + δ_ij + G(δ)]
= φ̄(t) + δφ(x,t)
h_ij = a² [δ_ij + G(δ)]
"SPATIALLY FLAT GAUGE"
H²/2k³
H²/M_Pl² k³
H²/k³
2m + ...

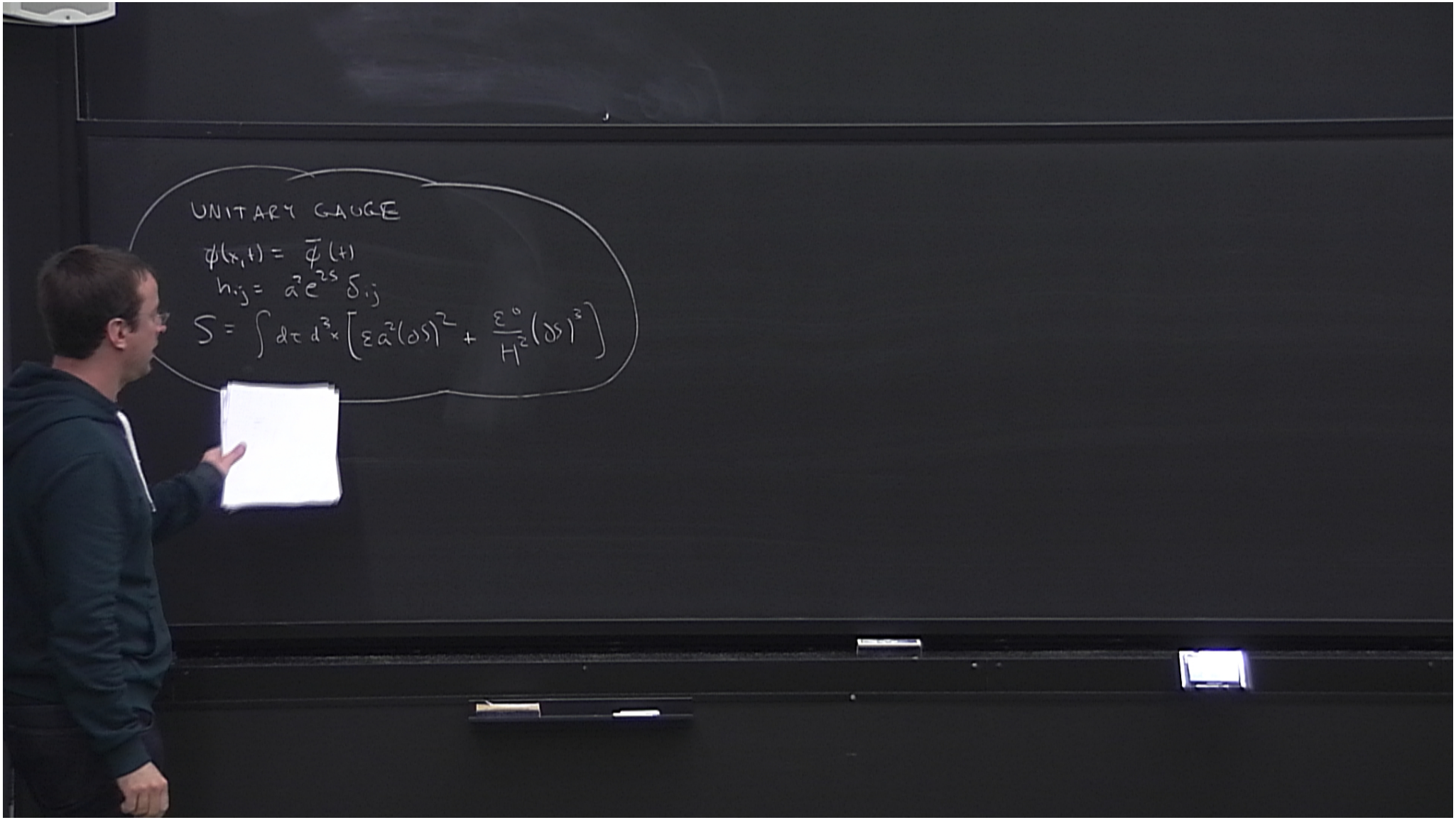

H ε → 0
φ(x,t) = φ̄(t)
h_ij = a² e^{2ζ} [1 + δ_ij + σ(γ)]
φ(x,t) = φ̄(t) + δφ(x,t)
h_ij = a² [δ_ij + σ(γ)]
"SPATIALLY FLAT GAUGE"
⟨σσ⟩ ~ H²/2k³
⟨γγ⟩ ~ H²/M_Pl² k³
⟨δφ δφ⟩ ~ H²/k³
Δφ
2η + ...

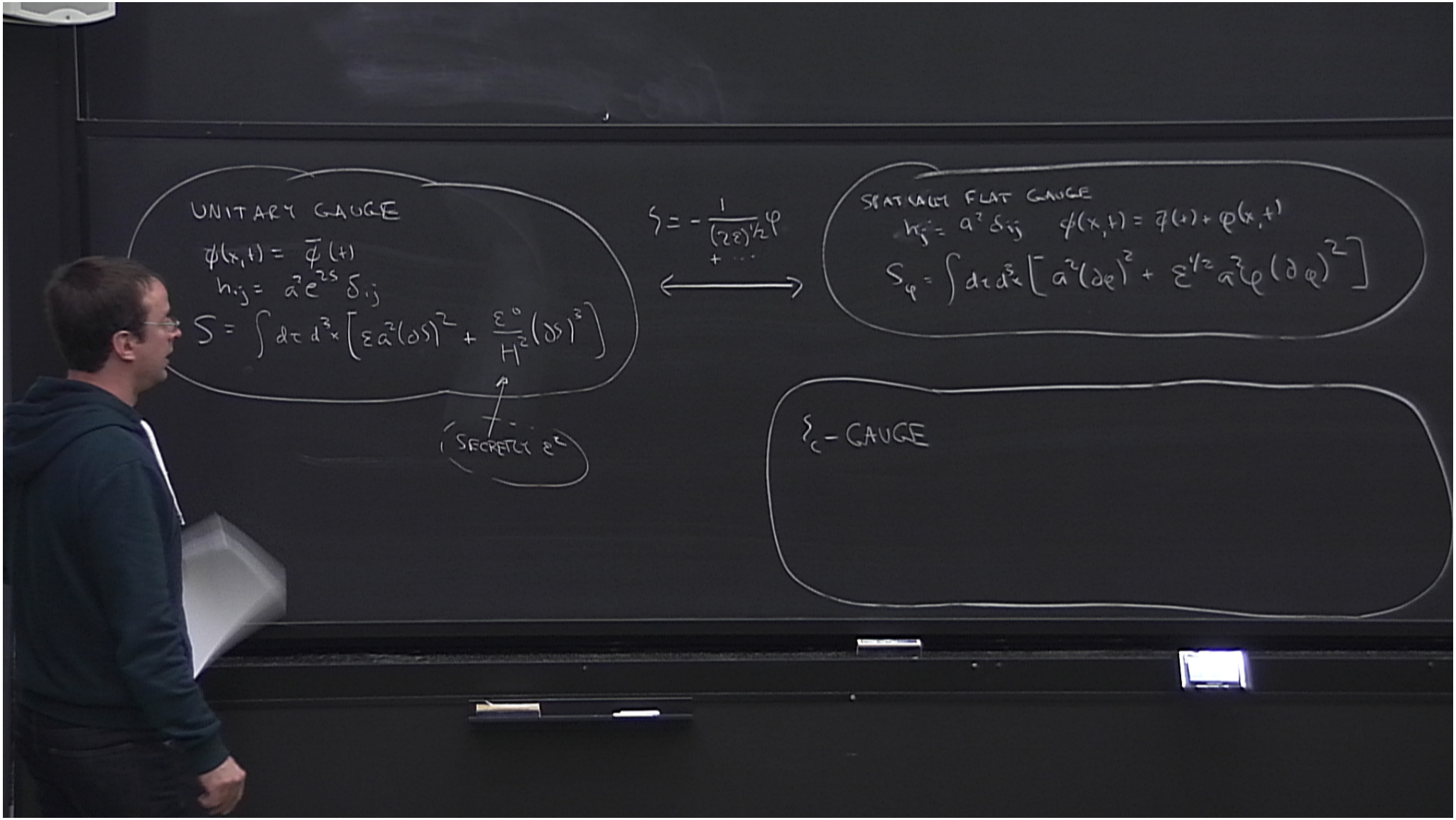

H ε → 0
ζ ~ H (1/φ̇) (Δφ)
~ (1/ε^{1/2}) (Δφ)
Δφ
Δφ
2m + ...
φ(x,t) = φ̄(t)
h_ij = a² e^{2ζ} [1 + δ_ij + σ(x)]
φ(x,t) = φ̄(t) + δφ(x,t)
h_ij = a² [δ_ij + σ(x)]
"SPATIALLY FLAT GAUGE"
H²/2k³
H²/ε M_Pl² k³
H²/k³
⟨ζζ⟩ ~ H²/εk³

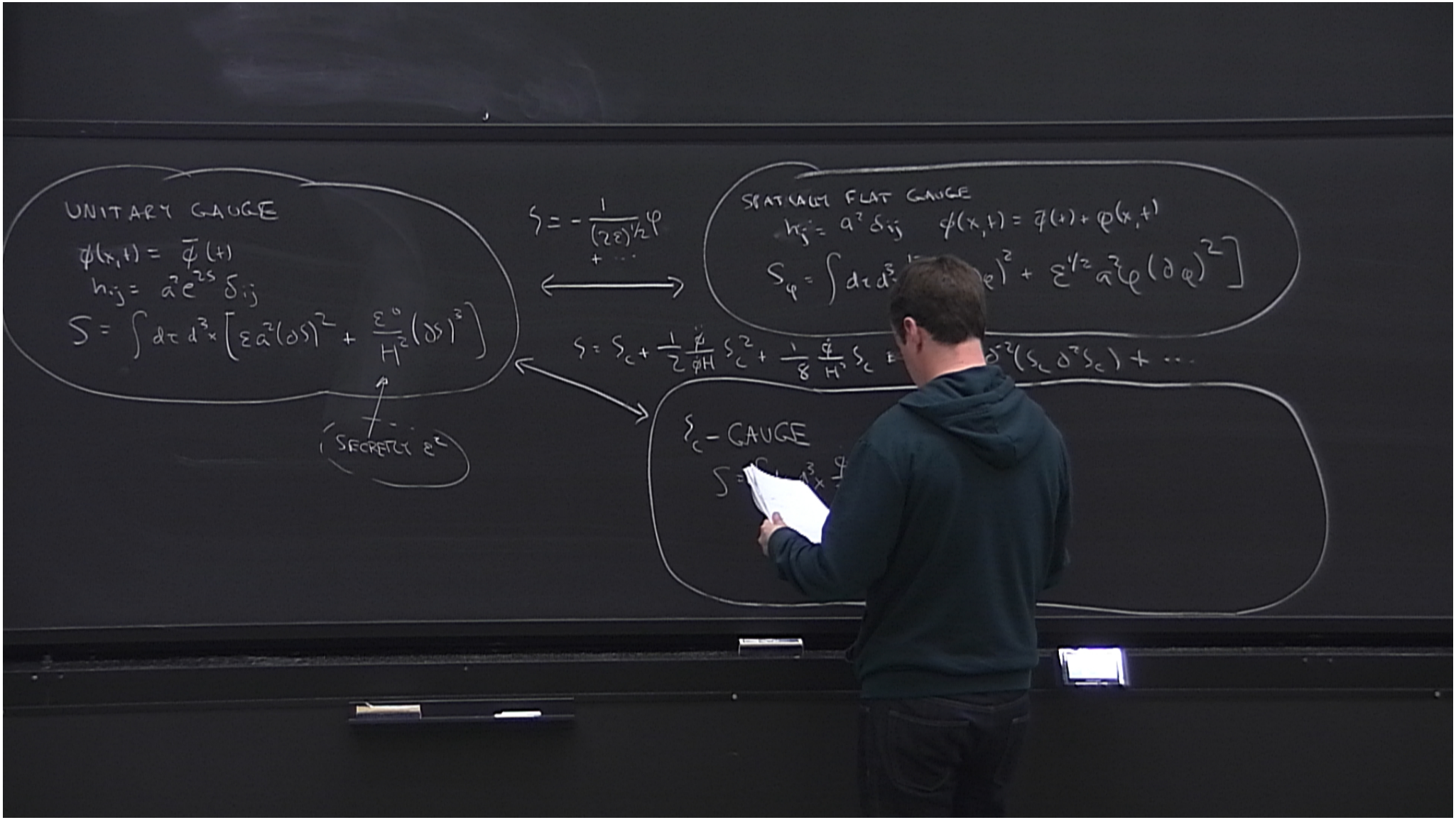

NO CONSTRAINT
INFLATON FIELD RANGE
$\Delta\phi = \int \dot{\phi} \left(\frac{d\log a}{dt}\right)^{-1} d\log a = \int d\log a\, (2\varepsilon)^{1/2} M_{pl}$
DETECTABL
$\phi(x,t) \sim \bar{\phi}(t)$
$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$
$\langle \sigma\sigma \rangle \sim \frac{H^2}{2k^3}$
$\langle \gamma\gamma \rangle \sim \frac{H^2}{M_{pl}^2 k^3}$
$\langle \delta\phi\, \delta\phi \rangle \sim \frac{H^2}{k^3}$
$\frac{d\log(H^2/\varepsilon)}{d\log aH} = -6\varepsilon + 2\eta + \cdots$

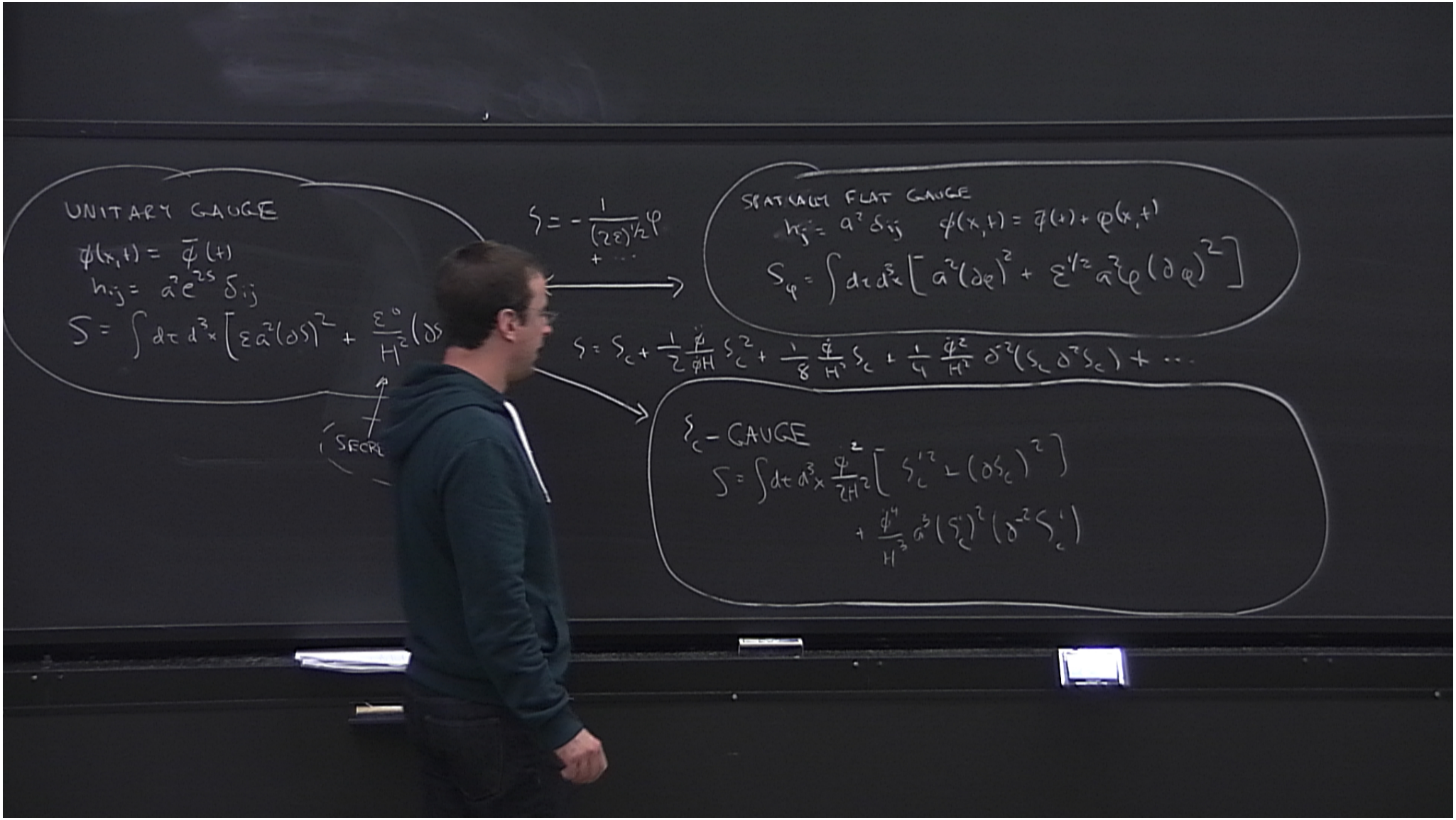

NO CONSTRAINT
INFLATION FIELD RANGE
NOTATION
DETECTABLE r ≳ 10^-1
SMALL r ≲ 10^-4
V(φ)
HISTORICAL CONVENTION


INFLATON FIELD RANGE
$\Delta\phi = \int \dot{\phi}\left(\frac{d\log a}{dt}\right)^{-1} d\log a = \int d\log a\,(2\epsilon)^{1/2} M_{pl} = \int d\log a \left(\frac{r}{8}\right)^{1/2} M_{pl}$
LYTH "BOUND"
DETECTABLE $r \gtrsim 10^{-4}$
SMALL $r \lesssim 10^{-4}$
H, ϵ, η
$\frac{H^2}{\epsilon}, \eta$
$V(\phi)$
$1\ \text{GeV} \lesssim \rho \lesssim 10^{16}\ \text{GeV}$
$\eta + \ldots$
k^3


UNITARY GAUGE
$\phi(x,t) = \bar{\phi}(t)$
$h_{ij} = a^2 e^{2\zeta} \delta_{ij}$
$S = \int d\tau\, d^3x \left[\varepsilon a^2 (\partial\zeta)^2 + \frac{\varepsilon^0}{H^2} (\partial\zeta)^3 \right]$


UNITARY GAUGE
$\phi(x,t) = \bar{\phi}(t)$
$h_{ij} = a^2 e^{2\zeta} \delta_{ij}$
$S = \int d\tau\, d^3x \left[\varepsilon a^2 (\dot{\zeta})^2 + \frac{\varepsilon^0}{H^2} (\partial\zeta)^3 \right]$
SECRETLY ε^2
$\zeta = -\frac{1}{(2\varepsilon)^{1/2}} \varphi + \dots$
SPATIALLY FLAT GAUGE
$h_{ij} = a^2 \delta_{ij}$ $\phi(x,t) = \bar{\phi}(t) + \varphi(x,t)$
$S_\varphi = \int d\tau\, d^3x \left[a^2 (\partial\varphi)^2 + \varepsilon^{1/2} a^3 \varphi (\partial\varphi)^2 \right]$
ζ_c - GAUGE


UNITARY GAUGE
$\phi(x,t) = \bar{\phi}(t)$
$h_{ij} = a^2 e^{2\zeta} \delta_{ij}$
$S = \int d\tau d^3x \left[\varepsilon a^2 (\partial\zeta)^2 + \frac{\varepsilon^0}{H^2} (\partial\zeta)^3 \right]$
SECRETLY ε^2
$\zeta = -\frac{1}{(2\varepsilon)^{1/2}} \varphi + \cdots$
SPATIALLY FLAT GAUGE
$h_{ij} = a^2 \delta_{ij}$ $\phi(x,t) = \bar{\phi}(t) + \varphi(x,t)$
ζ_c - GAUGE


UNITARY GAUGE
$\phi(x,t) = \bar{\phi}(t)$
$h_{ij} = a^2 e^{2\zeta} \delta_{ij}$
$S = \int dt\, d^3x \left[\varepsilon a^3 (\dot{\zeta})^2 + \frac{\varepsilon^0}{H^2} (\partial \zeta \right.$
$\zeta = -\frac{1}{(2\varepsilon)^{1/2}} \varphi + \cdots$
SPATIALLY FLAT GAUGE
$h_{ij} = a^2 \delta_{ij} \quad \phi(x,t) = \bar{\phi}(t) + \varphi(x,t)$
$S_\varphi = \int dt\, d^3x \left[a^3 (\dot{\varphi})^2 + \varepsilon^{1/2} a^3 \varphi (\partial \varphi)^2 \right]$
$\zeta = \zeta_c + \frac{1}{2} \frac{\dot{H}}{\dot{\phi} H} \zeta_c^2 + \frac{1}{8} \frac{\dot{\phi}}{H^3} \zeta_c + \frac{1}{4} \frac{\dot{\phi}^2}{H^2} \partial^{-2} (\zeta_c \partial^2 \zeta_c) + \cdots$
ζ_c - GAUGE
$S = \int dt\, d^3x \frac{\dot{\phi}^2}{2H^2} \left[\zeta_c'^2 - (\partial \zeta_c)^2 \right]$
$+ \frac{\dot{\phi}^4}{H^3} a^3 (\zeta_c')^2 (\partial^{-2} \zeta_c')$

Δ_ζ

$n_\zeta = -2\epsilon$ NO CONSTRAINT

INFLATON FI

$\Delta\phi = \int$

NONLINEAR OPERATIONS ⇒ NON-GAUSSIAN FIELDS
ON GAUSSIAN FIELDS

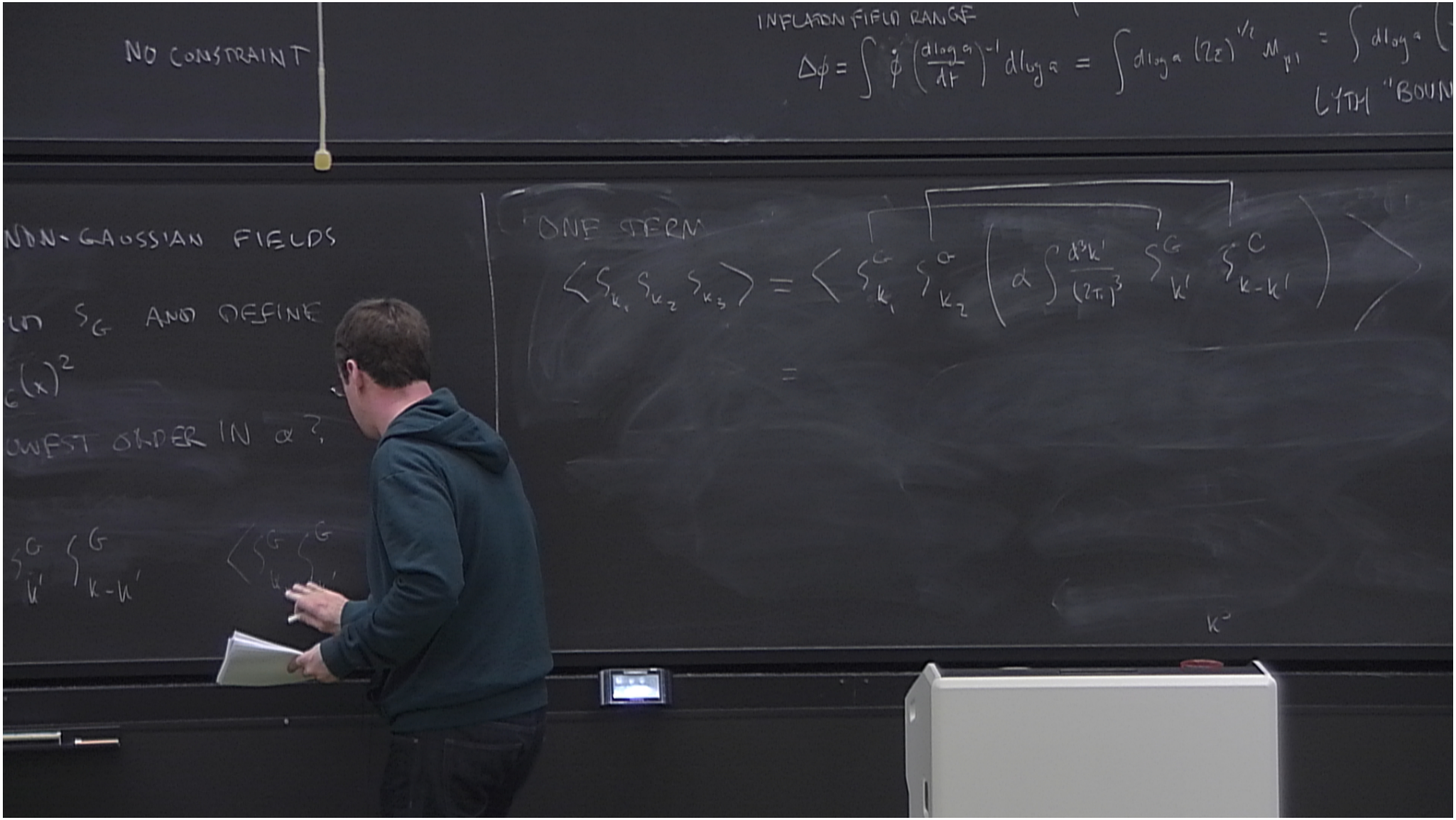
SUPPOSE I HAVE A GAUSSIAN FIELD ζ_G AND DEFINE

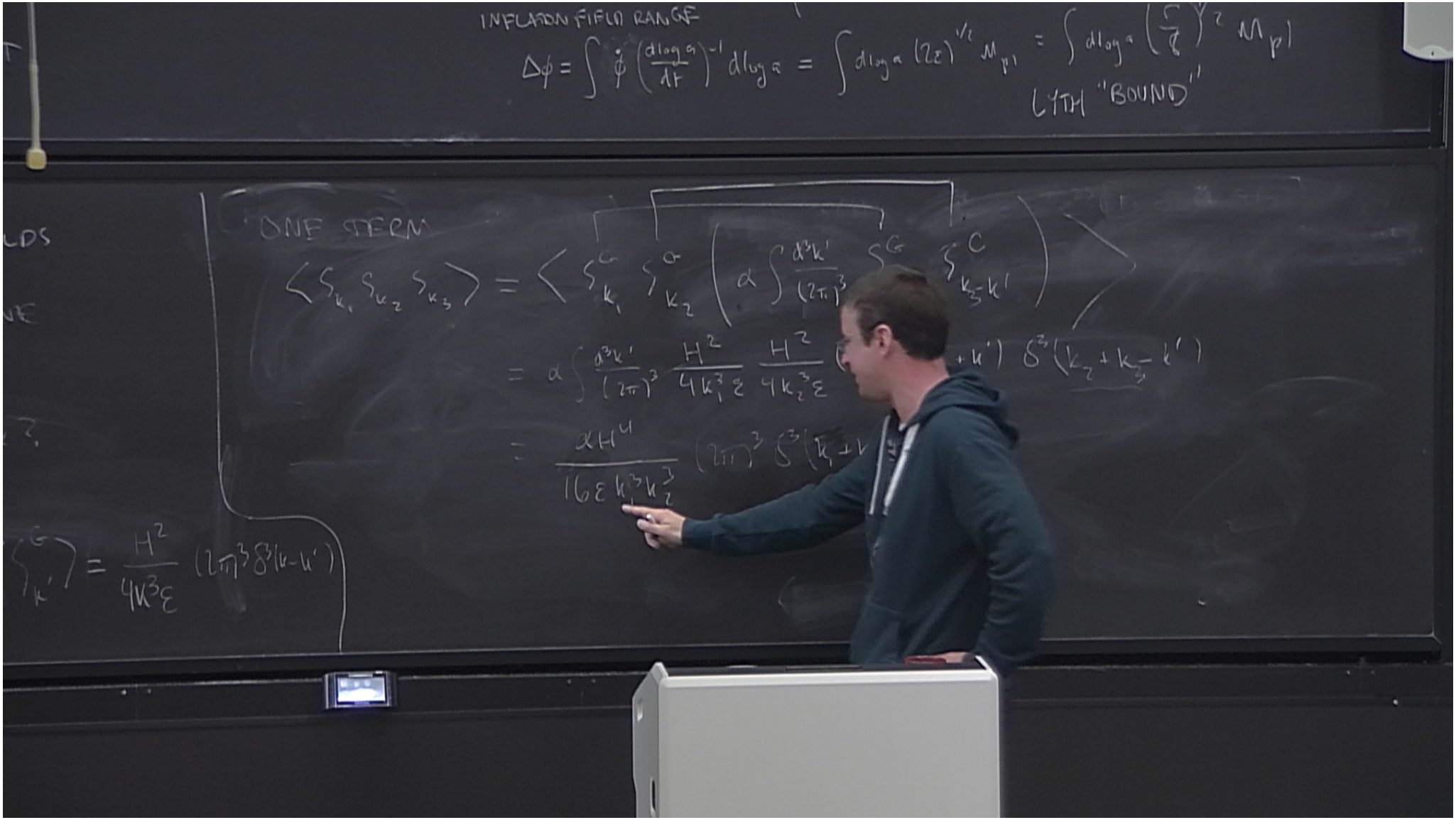
$$\zeta(x) = \zeta_G(x) + \alpha\,\zeta_G(x)^2$$

WHAT IS $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ TO LOWEST ORDER IN α?

FIRST WRITE

$$\zeta_k = \zeta^G_k + \alpha \int \frac{d^3k'}{(2\pi)^3}\, \zeta^G_{k'}\, \zeta^G_{k-k'}$$


NO CONSTRAINT
INFLATON FIELD RANGE
NON-GAUSSIAN FIELDS
ONE STERM


INFLATON FIELD RANGE
WITH "BOUND"
ONE TERM

STANDARD FLAT GAUGE

$h_{ij} = a^2 \delta_{ij} \quad \phi(x,t) = \bar{\phi}(t) + \varphi(x,t)$

$$S_\varphi = \int dt\, d^3x \left[a^3 (\partial\varphi)^2 + \varepsilon^{1/2} a^3 \varphi (\partial\varphi)^2 \right]$$

$\Rightarrow$

$$\frac{1}{2}\frac{\dot{\bar{\phi}}}{\phi H} \zeta_c^2 + \frac{1}{8}\frac{\dot{\bar{\phi}}^2}{H^2}\zeta_c + \frac{1}{4}\frac{\dot{\bar{\phi}}^2}{H^2}\partial^{-2}(\zeta_c \partial^2 \zeta_c) + \cdots$$

ζ_c - GAUGE

$$S = \int dt\, d^3x \frac{\dot{\bar{\phi}}^2}{2H^2}\left[\zeta_c'^2 - (\partial\zeta_c)^2 \right] + \frac{\dot{\bar{\phi}}^4}{H^3} a^3 (\zeta_c')^2 (\partial^{-2}\zeta_c')$$

SIMILARLY IF

$$\zeta = \zeta_G + \beta\, \partial^{-2}(\zeta_G \partial^2 \zeta_G)$$

$$\Rightarrow \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = \frac{\beta H^4}{16}$$