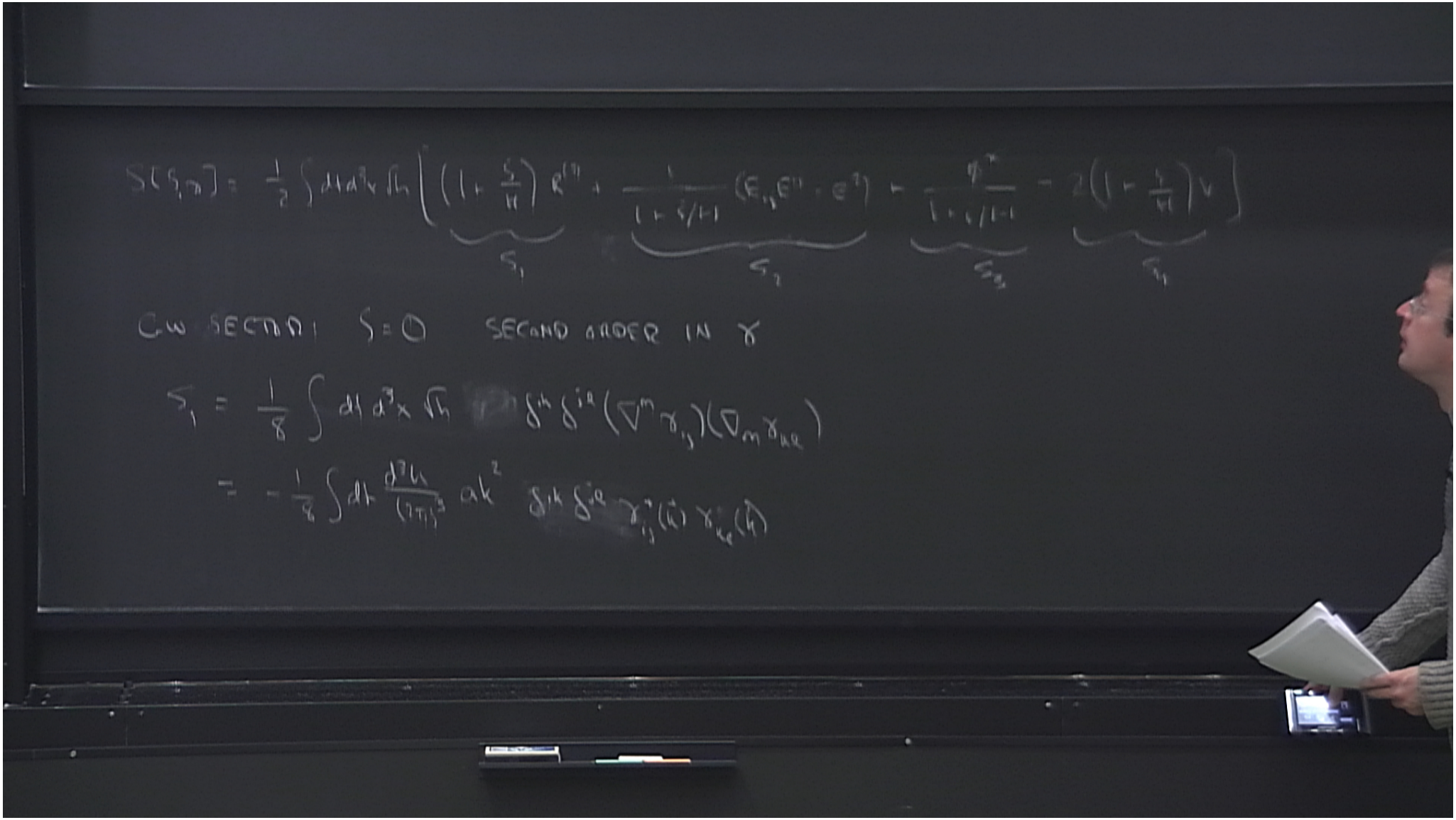
Title: Explorations in Cosmology-8

Date: Apr 15, 2015 10:15 AM

URL: http://pirsa.org/15040034

Abstract:


S[g,π] = ∫ dt d³x √h [(1 + 1/H) R^(3) + 1/(1 + 1/H) (E_ij E^ij − E²) − η²/(1 + 1/H) − 2(1 − 1/H) V]
S₁
S₂
S₃
S₄
GW SECTOR: S = 0 SECOND ORDER IN γ
S₁ = 1/8 ∫ dt d³x √h g^ik g^jℓ (∇^m γ_ij)(∇_m γ_kℓ)
= −1/8 ∫ dt d³k/(2π)³ ak² g_ik g_jℓ γ*_ij(k) γ^ij_kℓ(k)

$$S[\zeta,\gamma] = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\underbrace{\left(1+\frac{\dot\zeta}{H}\right)R^{(3)}}_{S_1} + \underbrace{\frac{1}{1+\dot\zeta/H}\left(E_{ij}E^{ij}-E^2\right)}_{S_2} + \underbrace{\frac{\dot\phi^2}{1+\dot\zeta/H}}_{S_3} - \underbrace{2\left(1+\frac{\dot\zeta}{H}\right)V}_{S_4}\right]$$

GW SECTOR: $\zeta = 0$ SECOND ORDER IN γ

$$S_1 = \frac{1}{8}\int dt\, d^3x\, \sqrt{h}\; g^{ik}g^{jl}\left(\nabla^m\gamma_{ij}\right)\left(\nabla_m\gamma_{kl}\right)$$

$$= -\frac{1}{8}\int dt\, \frac{d^3k}{(2\pi)^3}\, a k^2\, g^{ik}g^{jl}\,\gamma^*_{ij}(\vec{k})\,\gamma_{kl}(\vec{k})$$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, \Gamma_h \left(E_{ij} E^{ij} - E^2 \right)$$

$$E_{ij} = \frac{1}{2}\dot{h}_{ij}$$

$$h_{ij} = a^2 \left(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl} \right)$$

$$h^{ij} = a^{-2}\left(\delta^{ij} \right.$$

$$= H h_{ij} + \underbrace{\left(\frac{a^2}{2}\dot{\gamma}_{ij} + \frac{a^2}{4}\delta^{kl}\dot{\gamma}_{ik}\gamma_{jl} + \frac{a^2}{4}\delta^{kl}\gamma_{ik}\dot{\gamma}_{jl} \right)}_{E^{(1)}_{ij}}$$

$h^{ij} E^{(1)}_{ij} = 0$ TO SECOND ORDER IN γ_{ij}

$$S_2 = \frac{1}{2}\int dt\, d^3x\, \Gamma_h \left(E_{ij} E^{ij} - E^2 \right)$$

$$E_{ij} = \frac{1}{2}\dot{h}_{ij} \qquad h_{ij} = a^2\left(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl} \right) \qquad h^{ij} = a^{-2}\left(\delta^{ij} - \gamma^{ij} + \gamma^{ik}\gamma_{kj} \right)$$

$$= H h_{ij} + \underbrace{\left(\frac{a^2}{2}\dot{\gamma}_{ij} + \frac{a^2}{4}\delta^{kl}\dot{\gamma}_{ik}\gamma_{jl} + \frac{a^2}{4}\delta^{kl}\gamma_{ik}\dot{\gamma}_{jl} \right)}_{E^{(1)}_{ij}}$$

$$h^{ij} E^{(1)}_{ij} = 0 \quad \text{TO SECOND ORDER IN } \gamma \qquad \delta^{ij}\dot{\gamma}_{ij} = 0 \qquad \Rightarrow S_2 = \frac{1}{8}\int$$

$$\Rightarrow E = h^{ij} E_{ij} = 3H$$

$$E^{ij} E_{ij} = 3H^2 + \frac{1}{4}\delta^{ik}\delta^{jl}\dot{\gamma}_{ij}\dot{\gamma}_{kl}$$

$$h^{ij} = a^{-2}\left(\delta^{ij} - \delta^{ik}\delta^{jl}\gamma_{kl}\right)$$

$$\Rightarrow S_2 = \frac{1}{8}\int dt\, d^3x\, \sqrt{h}\; \delta^{ik}\delta^{jl}\,\dot{\gamma}_{ij}\,\dot{\gamma}_{kl}$$

$$\delta^{ij}\dot{\gamma}_{ij} = 0$$

$$S = \frac{1}{8}\int dt\, d^3x\, \sqrt{h}\, \delta^{ik}\delta^{jl}\left(\dot{\gamma}_{ij}\dot{\gamma}_{kl} - \nabla^m \gamma_{ij}\,\nabla_m \gamma_{ij}\right)$$

FOR EACH FOURIER MODE $\vec{k}$ WRITE $\gamma_{ij}(\vec{k},t) = \sum_{s=1}^{2} \gamma_{(s)}(\vec{k},t)\,\varepsilon_{ij}^{(s)}$

WHERE $\varepsilon_{ij}^{(s)}$ SATISFIES $\delta^{ij}\varepsilon_{ij}^{(s)} = 0$, $k^i \varepsilon_{ij}^{(s)} = 0$, AND $\delta^{ik}\delta^{jl}\varepsilon_{ij}^{(s)}\varepsilon_{kl}^{(s')} = 2\delta^{ss'}$

$\uparrow \vec{k} = k\hat{z}$

$$\varepsilon_{ij}^{(1)} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \qquad \varepsilon_{ij}^{(2)} = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}$$

$$S = \frac{1}{4}\int dt \frac{d^3k}{(2\pi)^3} \sum_s \left[a^3 \dot{\gamma}^{*}_{(s)}(\vec{k}) \dot{\gamma}_{(s)}(\vec{k}) - ak^2 \vec{\delta}^{*}_{(s)}(k) \vec{\delta}_{(s)}(k) \right]$$

$\Rightarrow$ EACH $\gamma_s(\vec{k},t)$ EVOLVES INDEPENDENTLY, ACTION IS

$$S = \frac{1}{4}\int dt \left[a^3 \dot{\gamma}^{*}_{k} \dot{\gamma}_k - ak^2 \gamma^{*}_k \gamma_k \right]$$

$$= \frac{1}{4}\int dt\, a^2 \left[\gamma'^{*}_k \gamma'_k - k^2 \gamma^{*}_k \gamma_k \right]$$

CONFORMAL TIME $dt = a\,d\tau$

$$\dot{X} = \frac{dx}{dt} \qquad X' = \frac{dx}{d\tau}$$

$$\frac{1}{(dt)^2} \to a^2$$

CANONICALLY NORMALIZE $\sigma = \frac{a}{\sqrt{2}}\gamma$

FREE QFT HAS

$$S = \frac{1}{2}\int d\tau \left[\sigma_k'^* \sigma_k' - \omega(k,\tau)^2 \sigma_k^* \sigma_k\right]$$

WHERE $\omega(k,\tau)^2 = k^2 - 2a^2H^2 - a^2\dot{H}$

$= k^2 - \frac{a''}{a}$ [MUKHANOV-SAKAI FORM]

FREE QFT HAS SOLUTION:

$$\hat{\sigma}_k^{H} = v_k(\tau)\, a_k + v_k^{*}(\tau)\, a_{-k}^{\dagger}$$

$$\hat{\gamma}_k^{H} = u_k(\tau)\, a_k + u_k^{*}(\tau)\, a_{-k}^{\dagger}$$

$$v_k''(\tau) + \omega(k\tau)^2 v_k(\tau) = 0$$

$$v_k(\tau) \to \frac{e^{-ik\tau}}{(2k)^{1/2}} \quad \text{AT EARLY TIMES}$$

$$u_k(\tau) = \frac{\sqrt{2}}{a} v_k(\tau)$$

$$u_k'' + 2aH u_k' + k^2 u_k = 0$$

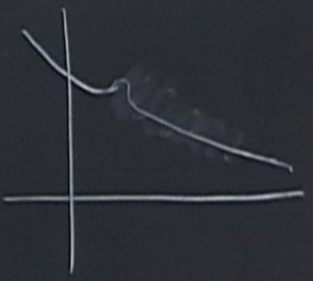
SOLUTION:

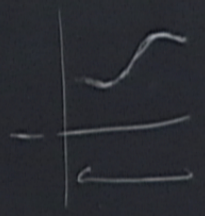
$$v_k''(\tau) + \omega(k,\tau)^2 v_k(\tau) = 0$$

$$v_k(\tau) \to \frac{e^{-ik\tau}}{(2k)^{1/2}} \quad \text{AT EARLY TIMES}$$

$$u_k(\tau) = \frac{\sqrt{2}}{a} v_k(\tau)$$

$$u_k'' + 2aH u_k' + k^2 u_k = 0$$





CANONICALLY NORMALIZE $\sigma = \frac{a}{\sqrt{2}}\gamma$

$$S = \frac{1}{2}\int d\tau \left[\sigma_k'^{*}\sigma_k' - \omega(k,\tau)^2 \sigma_k^{*}\sigma_k \right]$$

WHERE $\omega(k,\tau)^2 = k^2 - 2a^2H^2 - a^2\dot{H}$

$$= k^2 - \frac{a''}{a}$$

[MUKHANOV-SAKAI FORM]

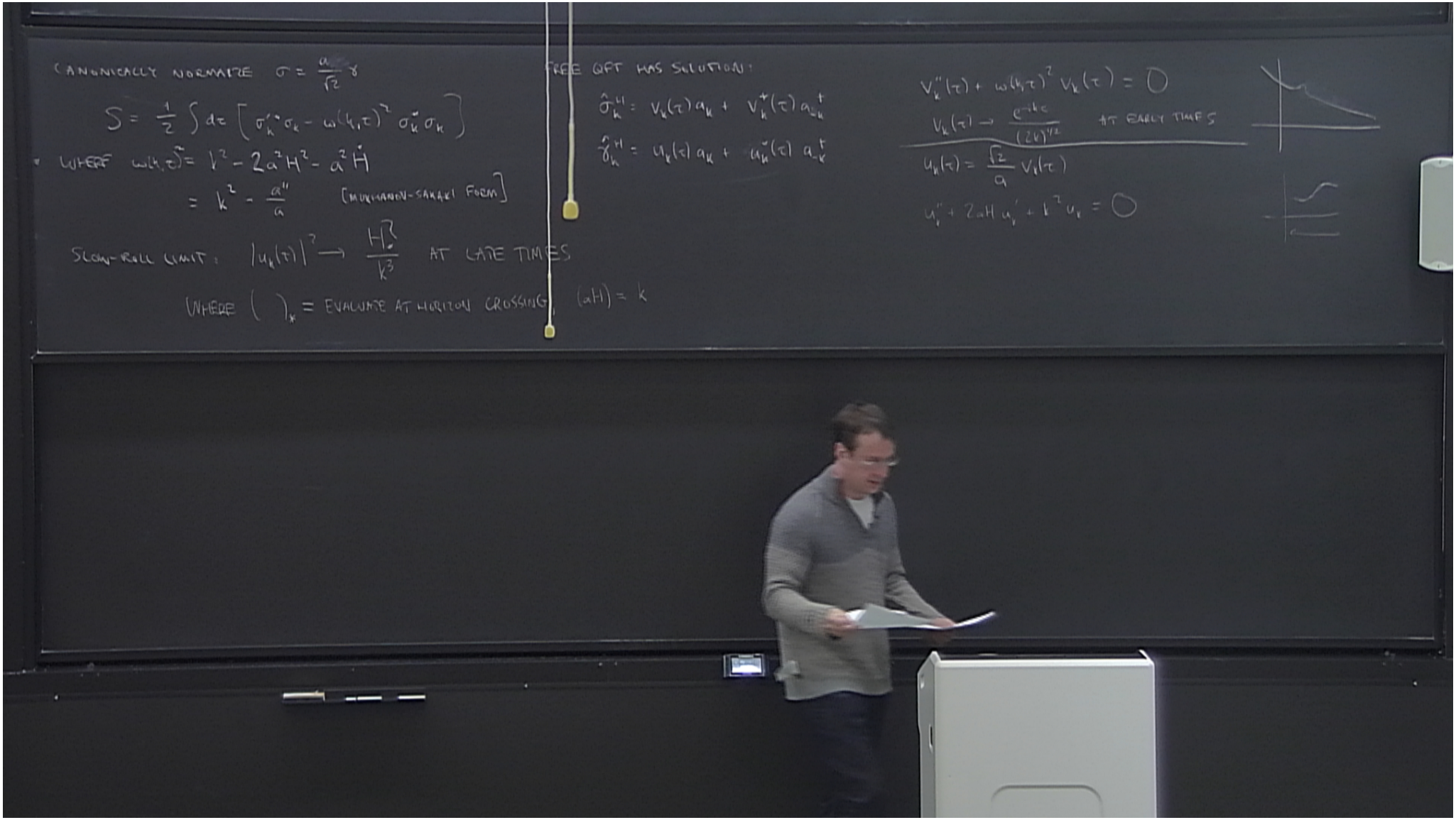
SLOW-ROLL LIMIT: $|u_k(\tau)|^2 \longrightarrow \frac{H_*}{k^3}$ AT LATE TIMES

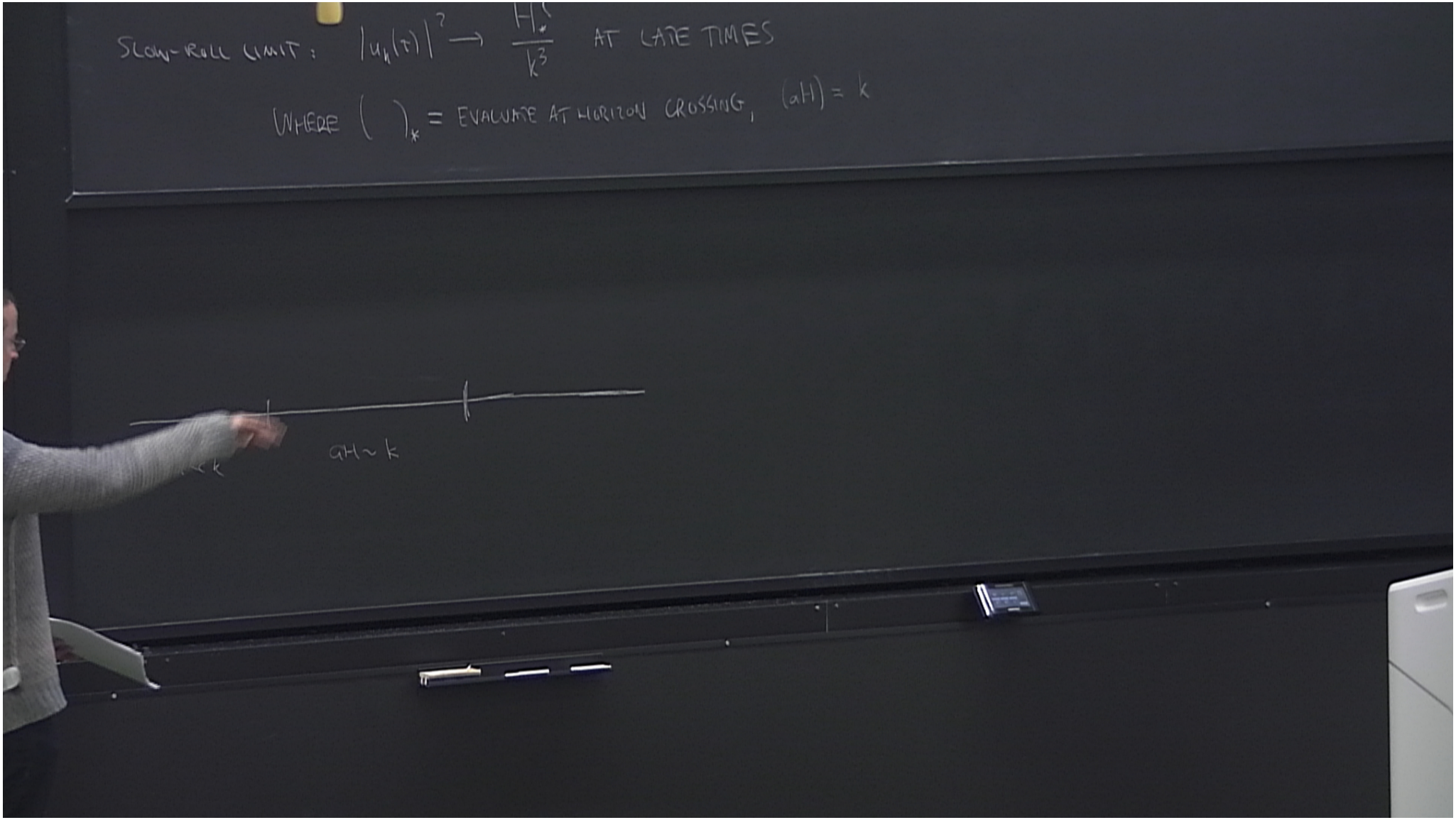
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, $(aH) = k$

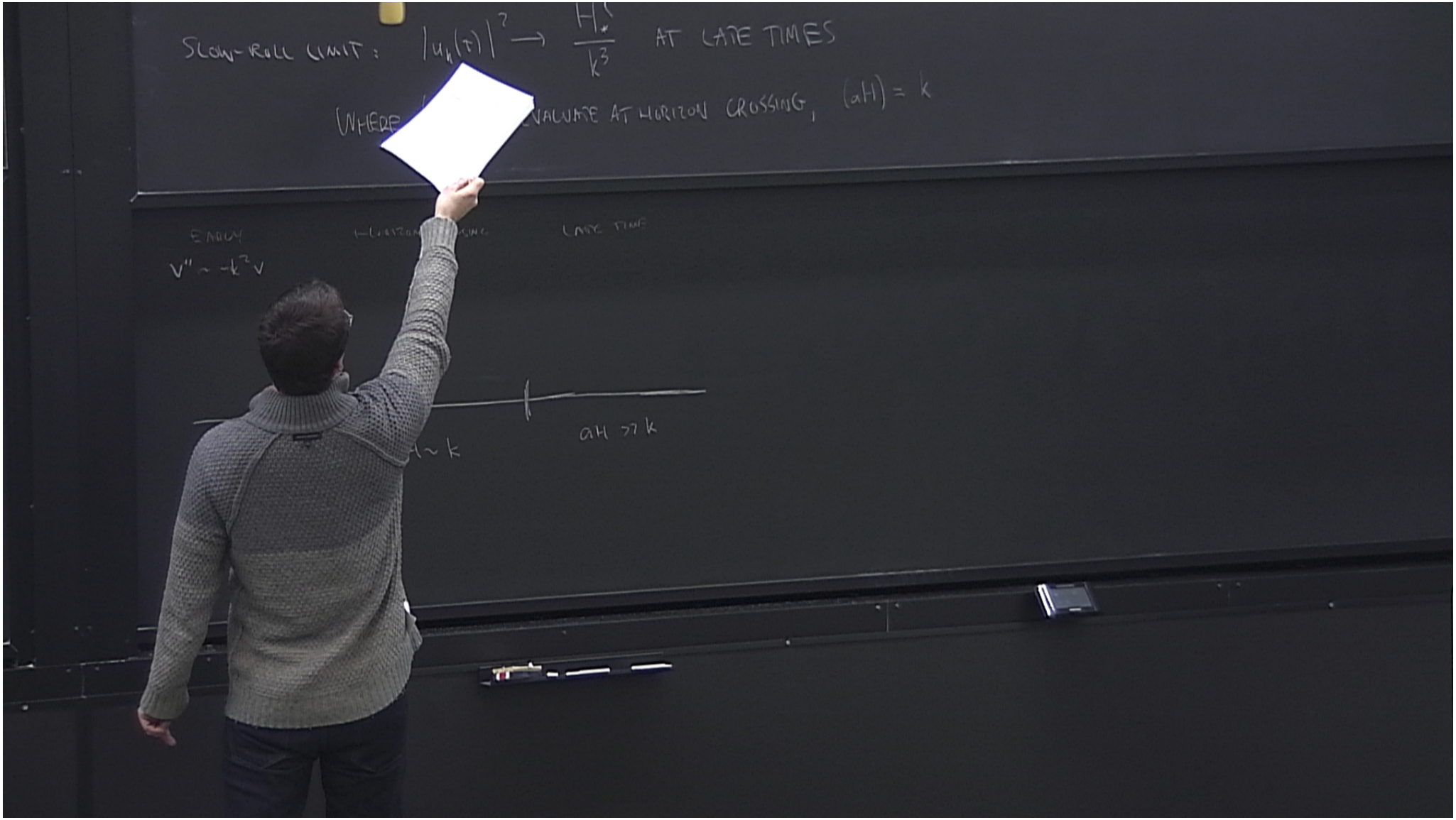
FREE QFT HAS SOLUTION:

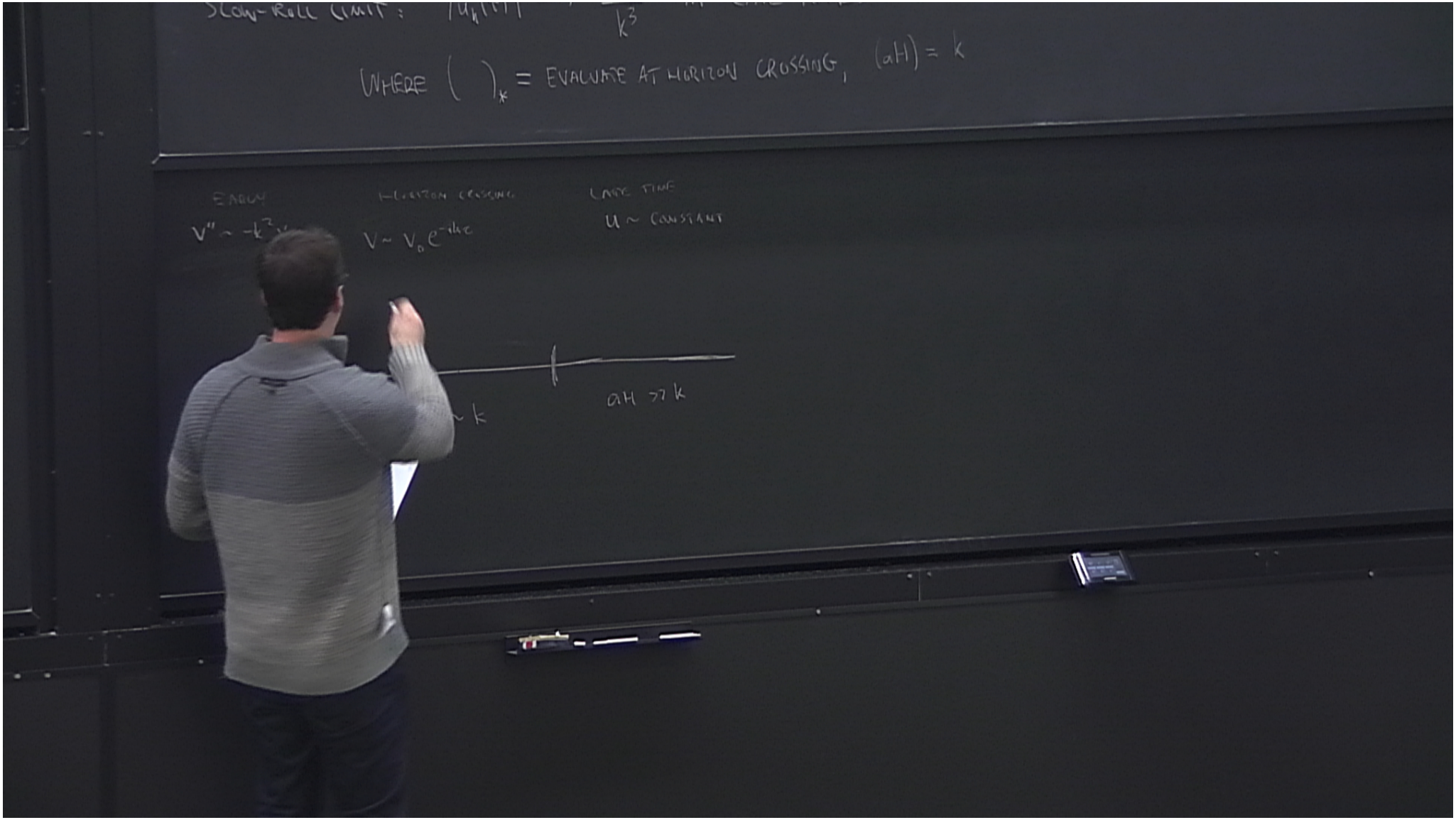
$\hat{\sigma}_k^{H} = V_k(\tau)\, a_k + \ldots$

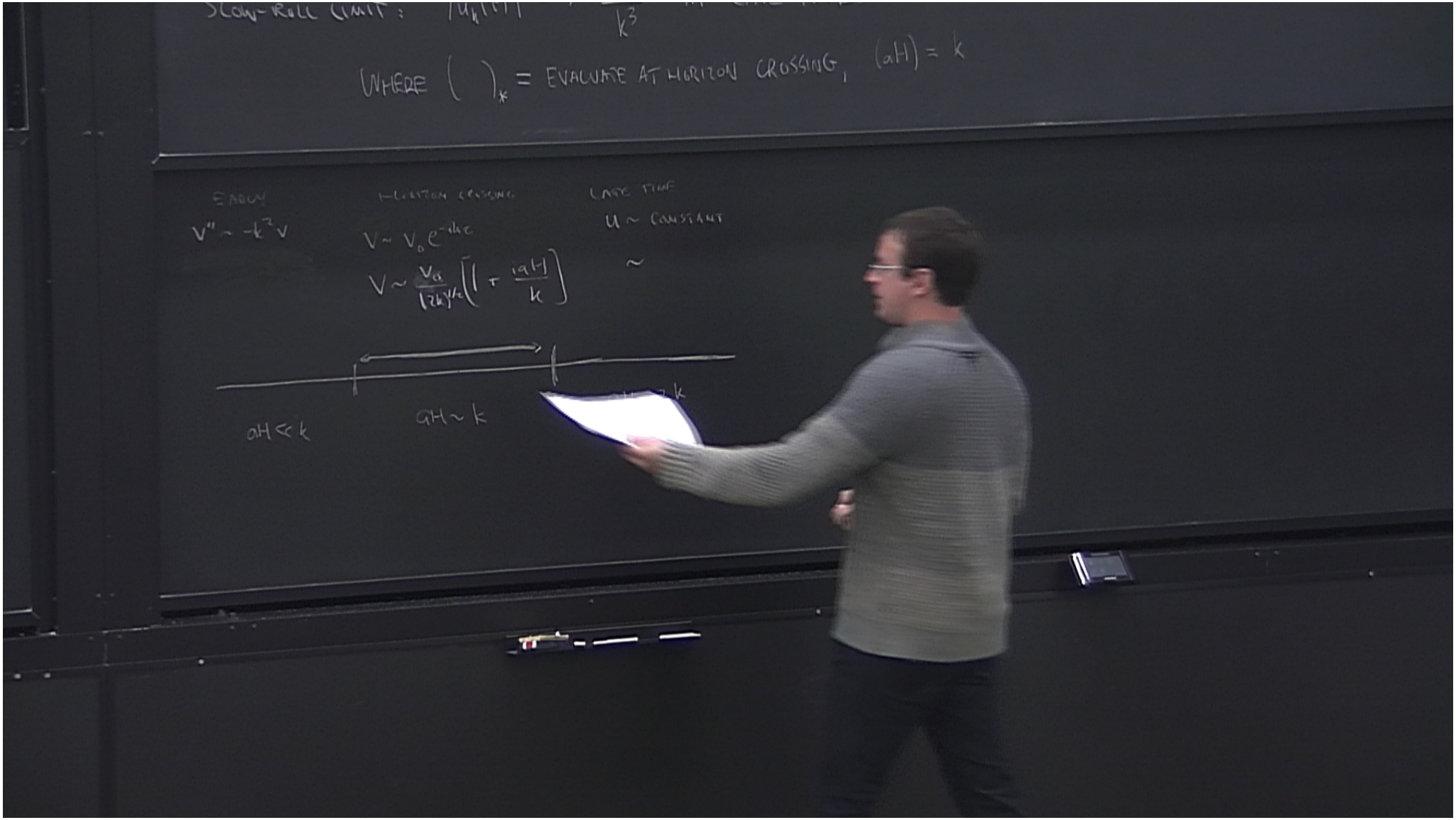
$\hat{\gamma}_k^{H} = u_k(\tau)\, a_k + \ldots$

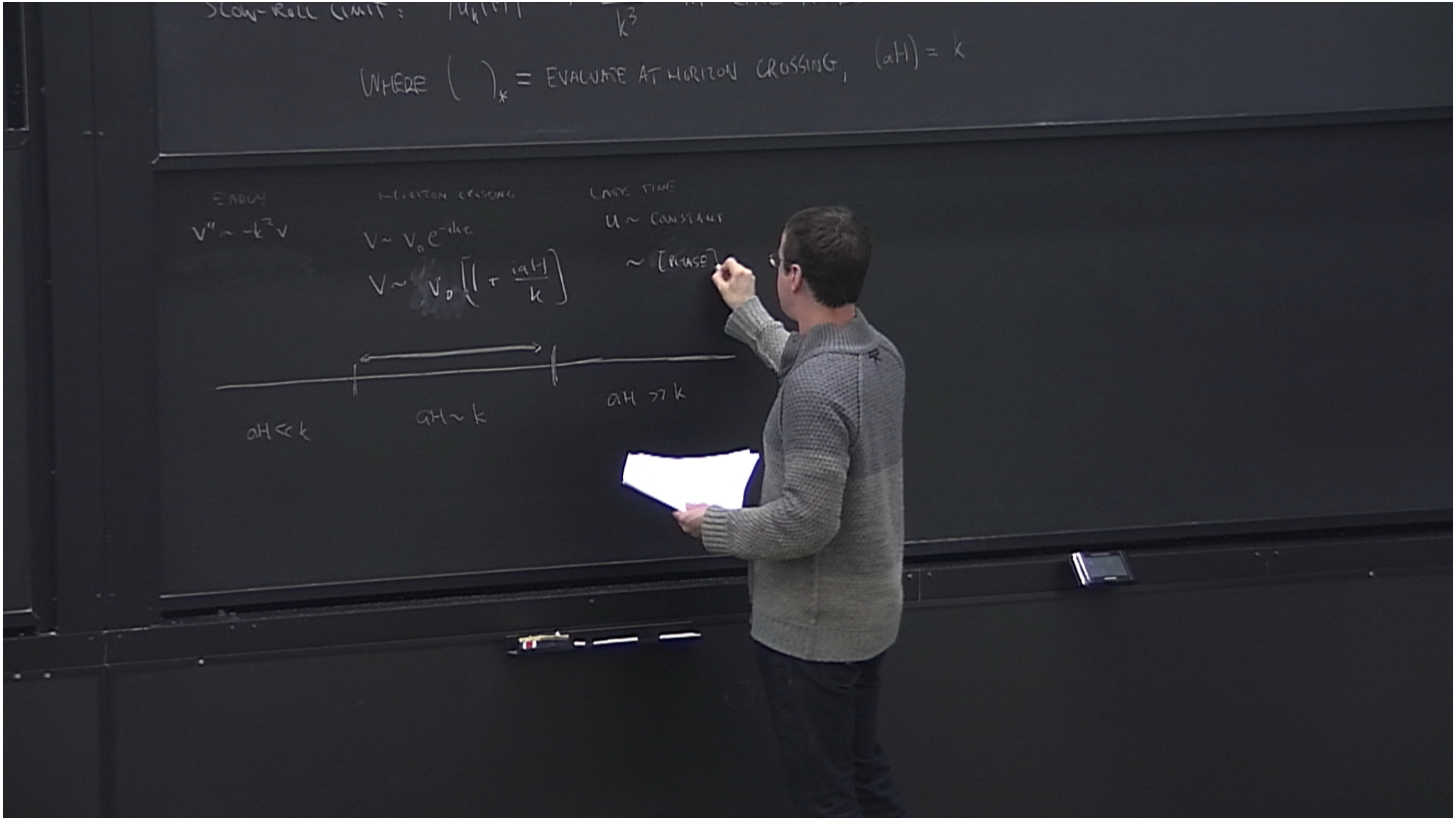

CANONICALLY NORMALIZE $\sigma = \frac{a}{\sqrt{2}}\gamma$
$S = \frac{1}{2}\int d\tau \left[\sigma_k'^* \sigma_k' - \omega(k,\tau)^2 \sigma_k^* \sigma_k\right]$
WHERE $\omega(k,\tau)^2 = k^2 - 2a^2H^2 - a^2\dot{H}$
$= k^2 - \frac{a''}{a}$ [MUKHANOV-SAKAKI FORM]
SLOW-ROLL LIMIT: $|u_k(\tau)|^2 \longrightarrow \frac{H_*^2}{k^3}$ AT LATE TIMES
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, $(aH) = k$
FREE QFT HAS SOLUTION:
$\hat{\sigma}_k^H = v_k(\tau) a_k + v_k^*(\tau) a_{-k}^\dagger$
$\hat{\gamma}_k^H = u_k(\tau) a_k + u_k^*(\tau) a_{-k}^\dagger$
$v_k''(\tau) + \omega(k,\tau)^2 v_k(\tau) = 0$
$v_k(\tau) \to \frac{e^{-ik\tau}}{(2k)^{1/2}}$ AT EARLY TIMES
$u_k(\tau) = \frac{\sqrt{2}}{a} v_k(\tau)$
$u_k'' + 2aH u_k' + k^2 u_k = 0$

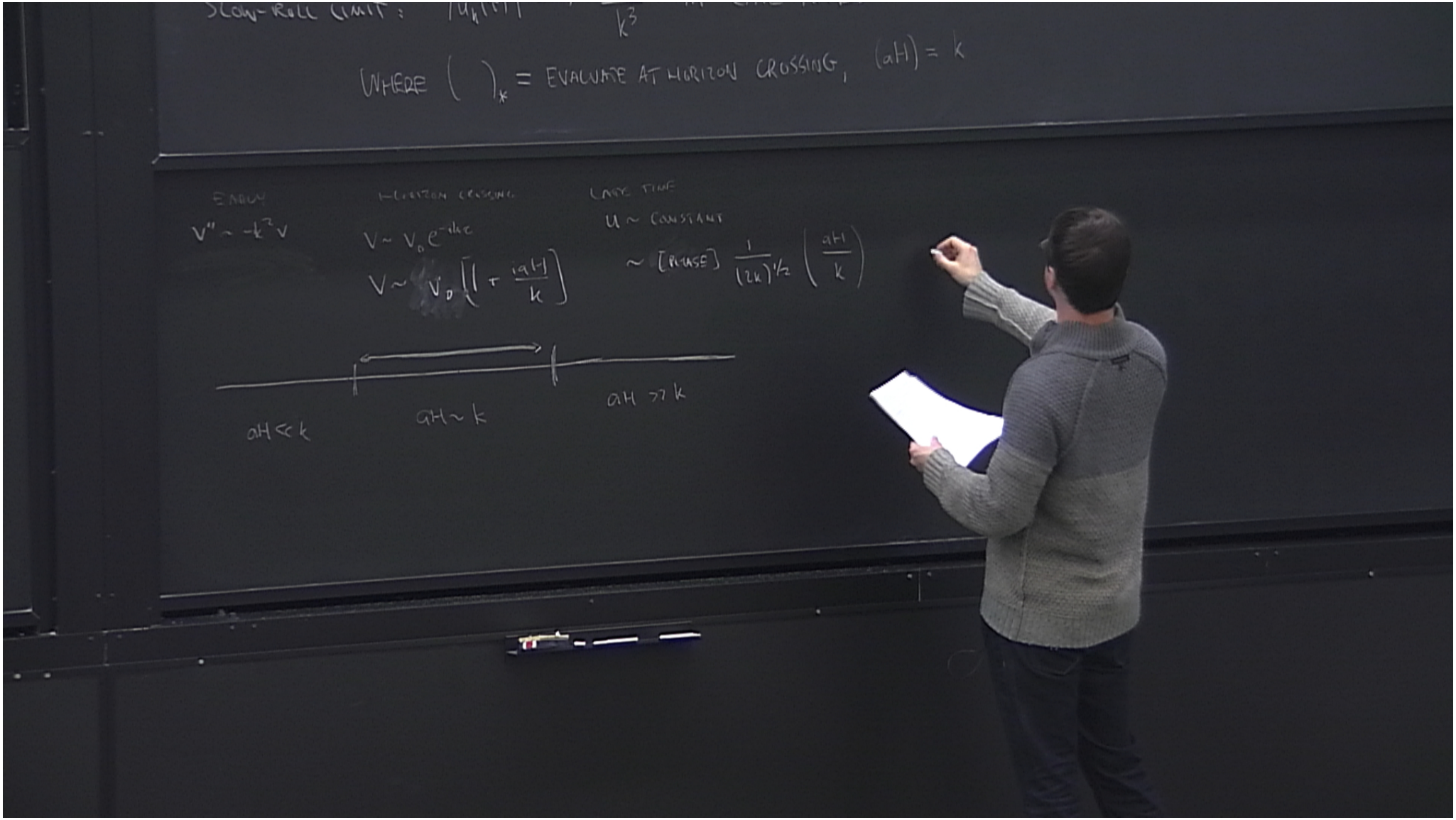

SLOW-ROLL LIMIT : $|u_k(\tau)|^2 \longrightarrow \frac{H_*^2}{k^3}$ AT LATE TIMES
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, $(aH) = k$
$aH \sim k$

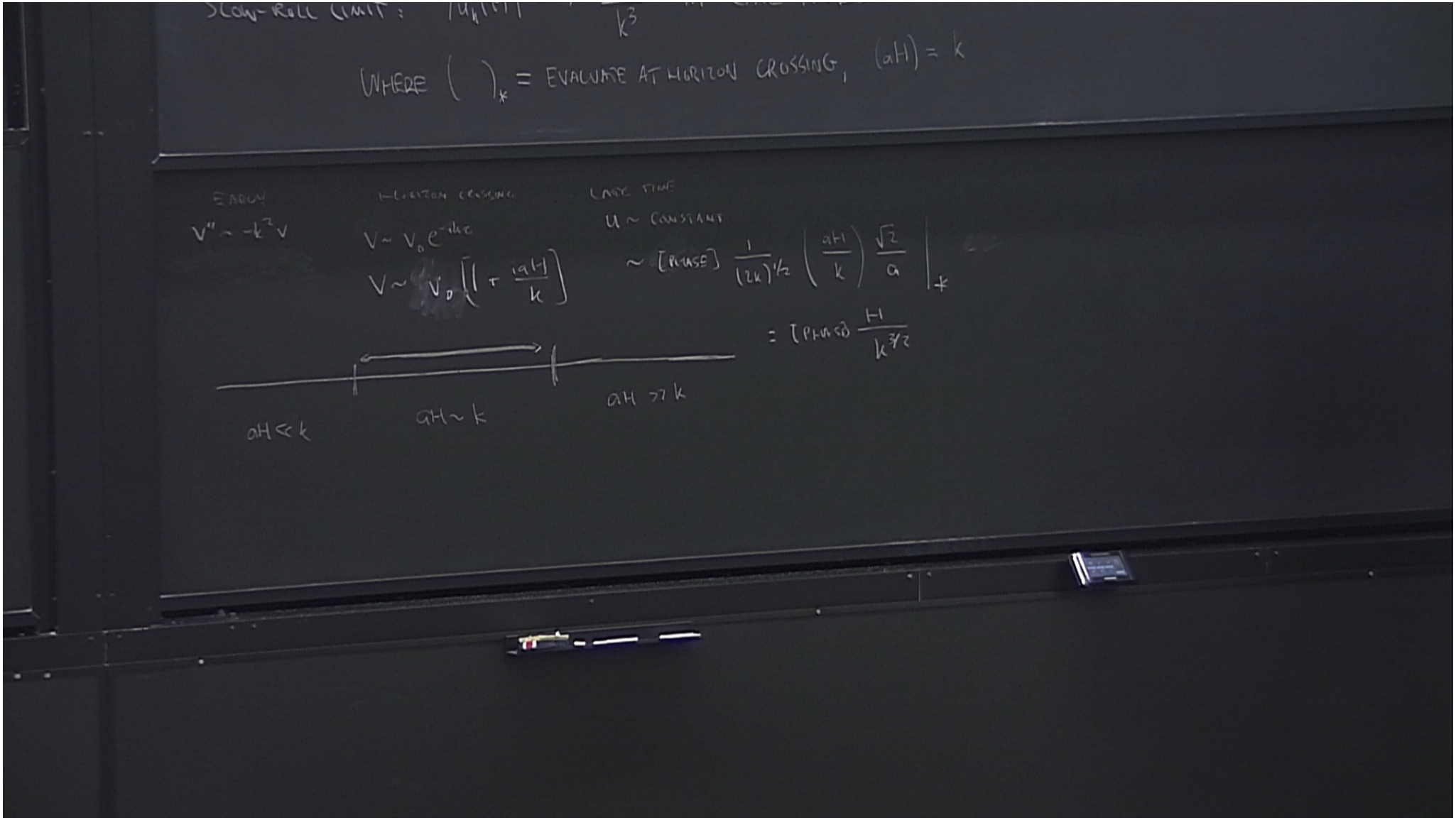

SLOW-ROLL LIMIT : $|u_k(\tau)|^2 \rightarrow \frac{H_*^2}{k^3}$ AT LATE TIMES
WHERE EVALUATE AT HORIZON CROSSING, $(aH) = k$
EARLY
LATE TIME
$v'' \sim -k^2 v$
$aH \gg k$

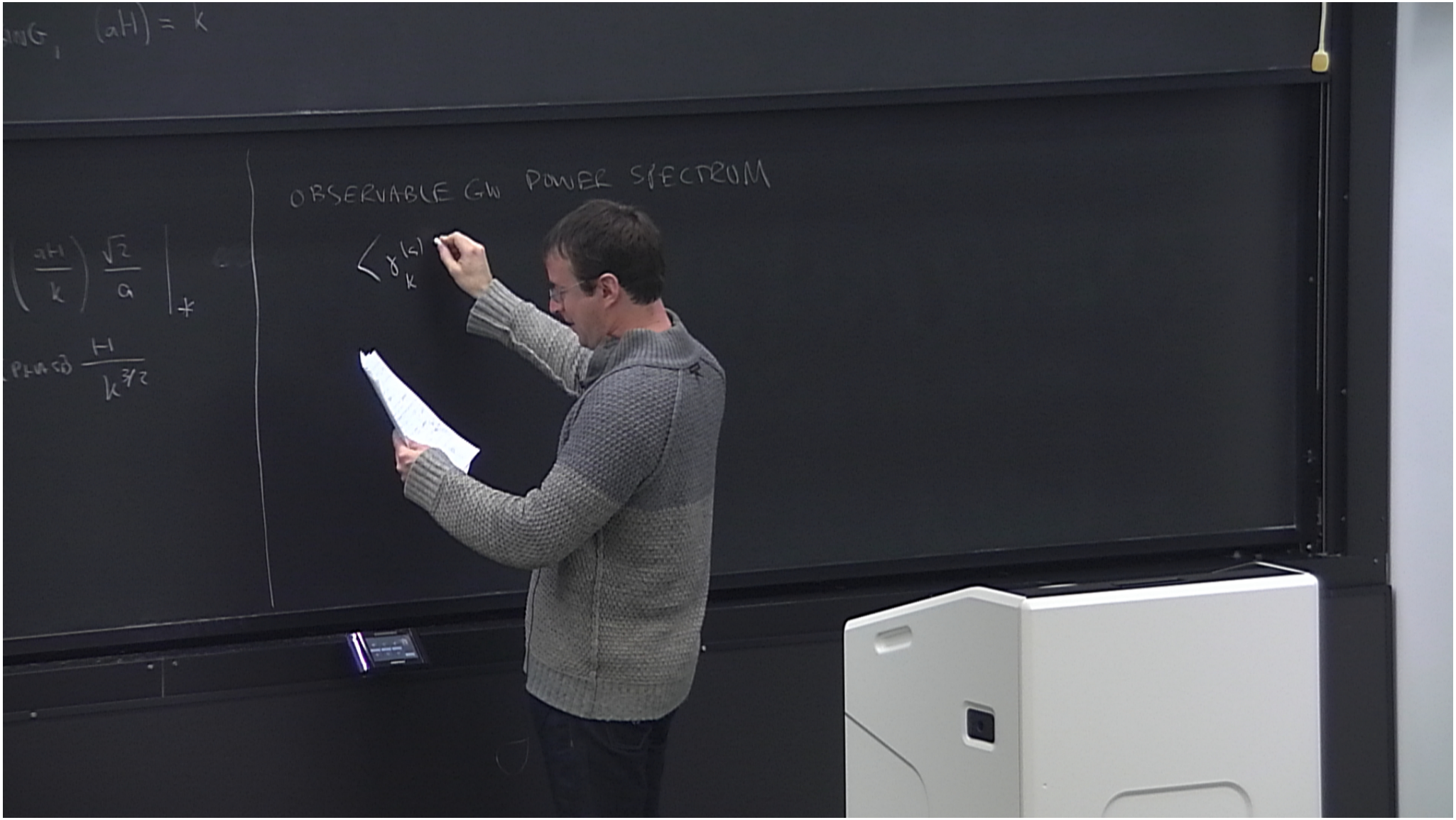

WHERE ( )* = EVALUATE AT HORIZON CROSSING, (aH) = k
EARLY
HORIZON CROSSING
LATE TIME
V ~ V0 e^-kc
U ~ CONSTANT
aH >> k

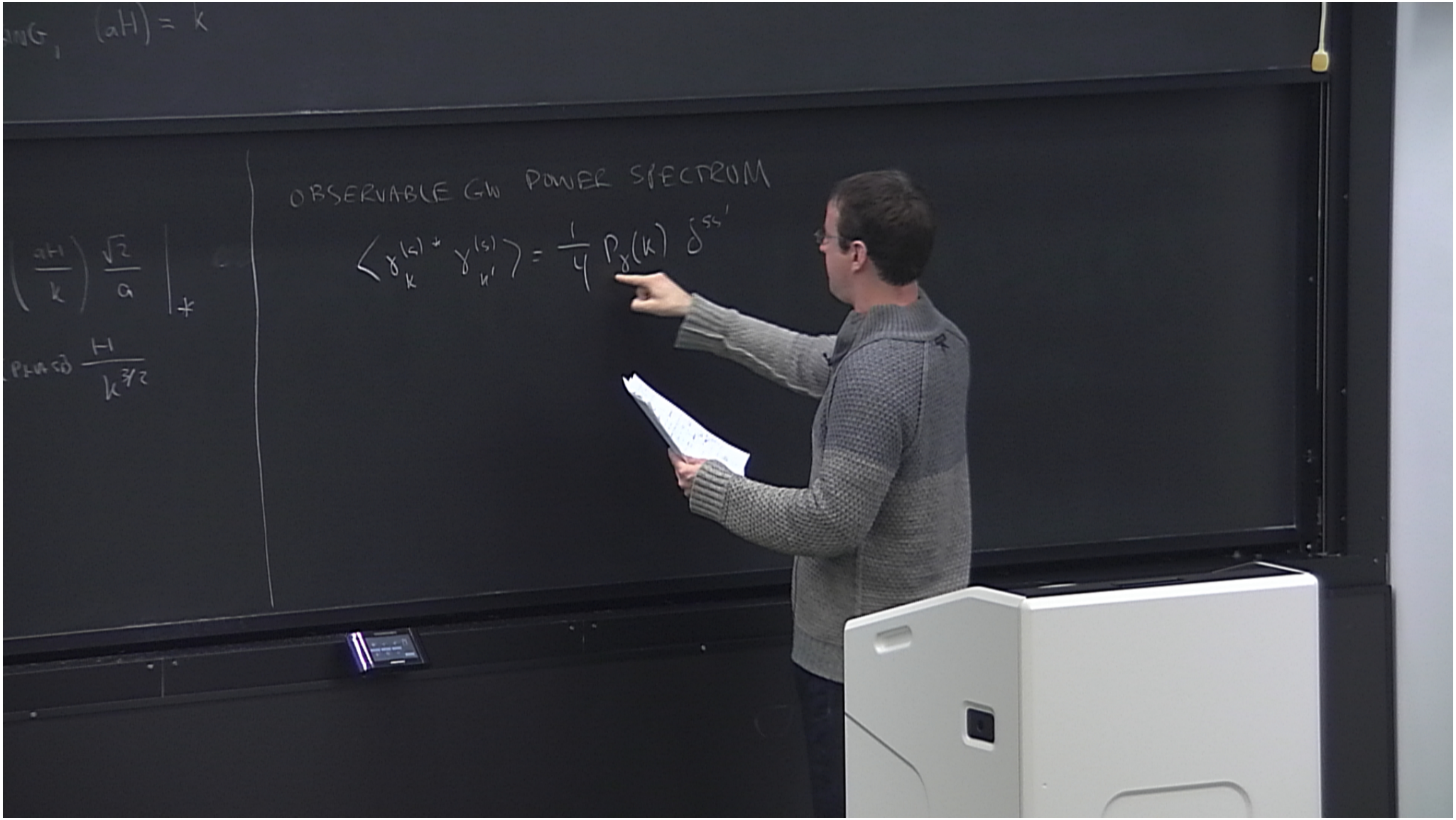

WHERE ( )* = EVALUATE AT HORIZON CROSSING, (aH) = k
EARLY
V'' ~ -k²v
HORIZON CROSSING
V ~ V₀e^{-ikτ}
V ~ V₀/(2k)^{1/2} [1 + |aH|/k]
LATE TIME
u ~ CONSTANT
~
aH ≪ k
aH ~ k

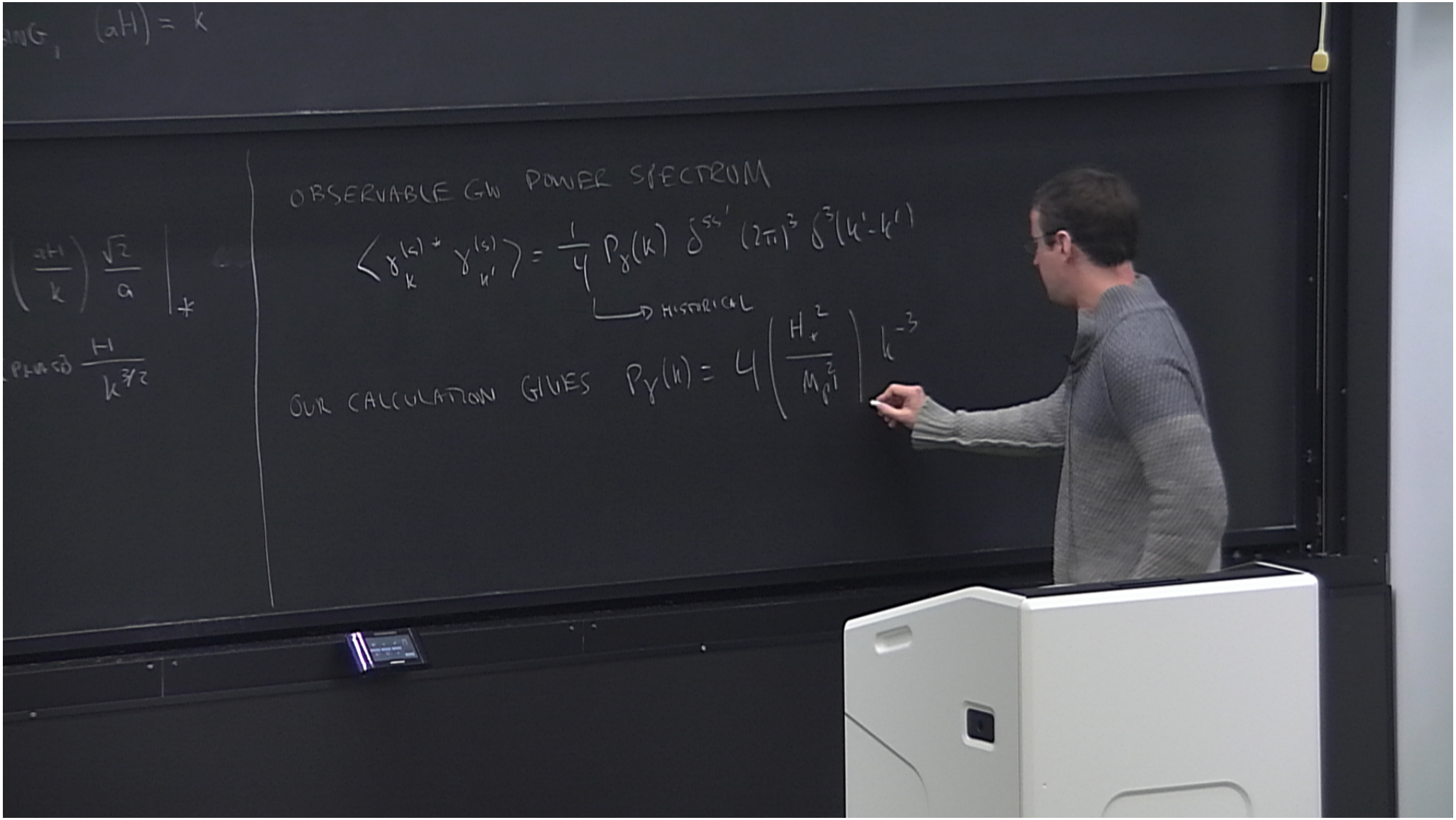

SLOW-ROLL LIMIT:
k^3
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, $(aH) = k$
EARLY
$v'' \sim -k^2 v$
HORIZON CROSSING
$v \sim v_0 e^{-ikc}$
$v \sim v_0 \left[(1 + \frac{iaH}{k}) \right]$
LATE TIME
$u \sim$ CONSTANT
$\sim$ [PHASE]
$aH \ll k$
$aH \sim k$
$aH \gg k$

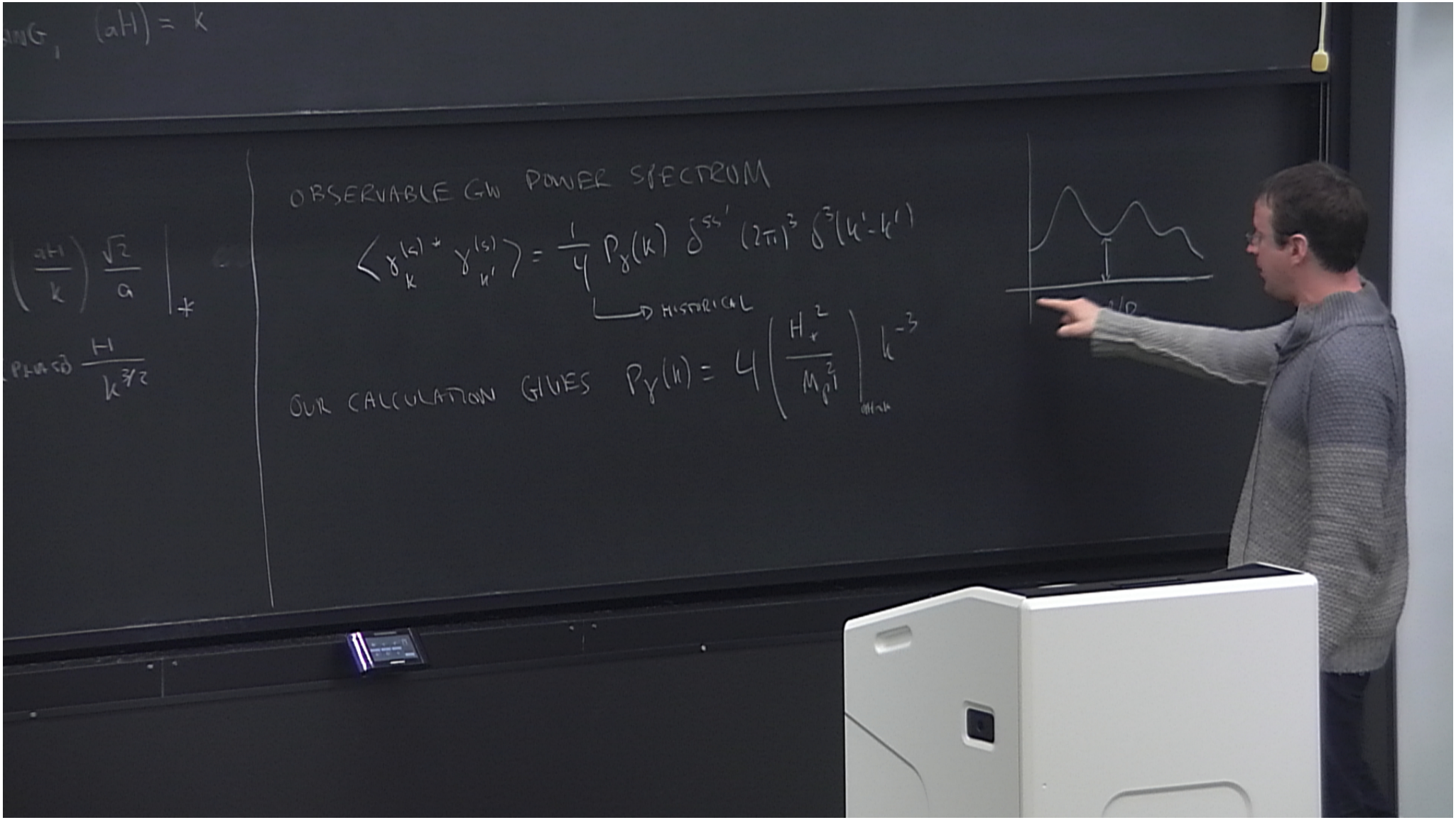

SLOW-ROLL LIMIT :
k^3
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, $(aH) = k$
EARLY
$v'' \sim -k^2 v$
HORIZON CROSSING
$v \sim v_0 e^{-ikt}$
$v \sim v_0 \left[1 + \frac{i(aH)}{k}\right]$
LATE TIME
$u \sim$ CONSTANT
$\sim$ [PHASE] $\frac{1}{(2k)^{1/2}} \left(\frac{aH}{k}\right)$
$aH \ll k$
$aH \sim k$
$aH \gg k$

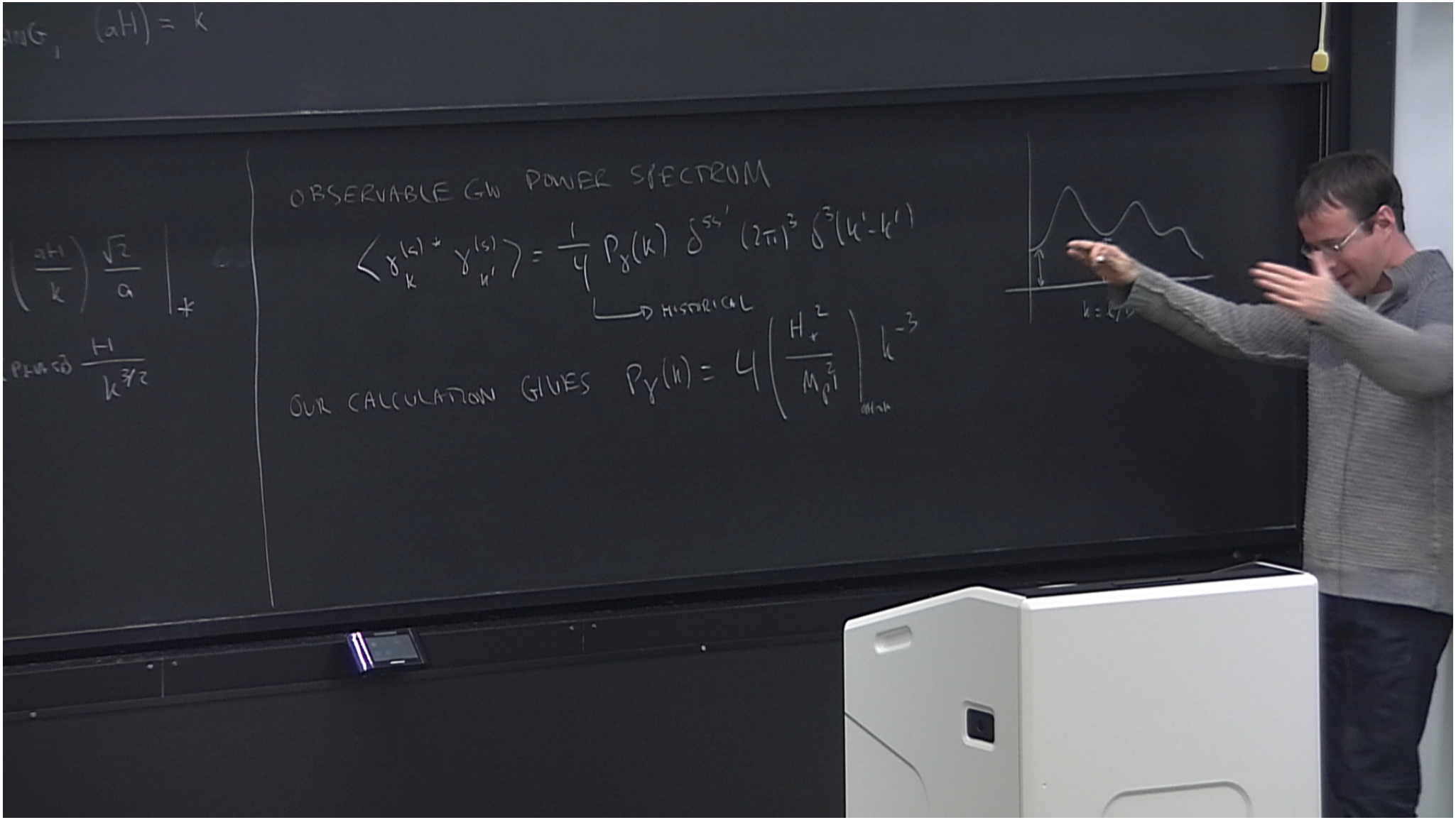

SLOW-ROLL LIMIT:
k^3
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, $(aH) = k$
EARLY
HORIZON CROSSING
LATE TIME
$v'' \sim -k^2 v$
$v \sim v_0 e^{-ikc}$
$v \sim v_0 \left[1 + \frac{iaH}{k}\right]$
$u \sim$ CONSTANT
$\sim$ [PHASE] $\frac{1}{(2k)^{1/2}} \left(\frac{aH}{k}\right) \frac{\sqrt{2}}{a} \Big|_*$
$= [\text{PHASE}] \frac{H}{k^{3/2}}$
$aH \ll k$
$aH \sim k$
$aH \gg k$

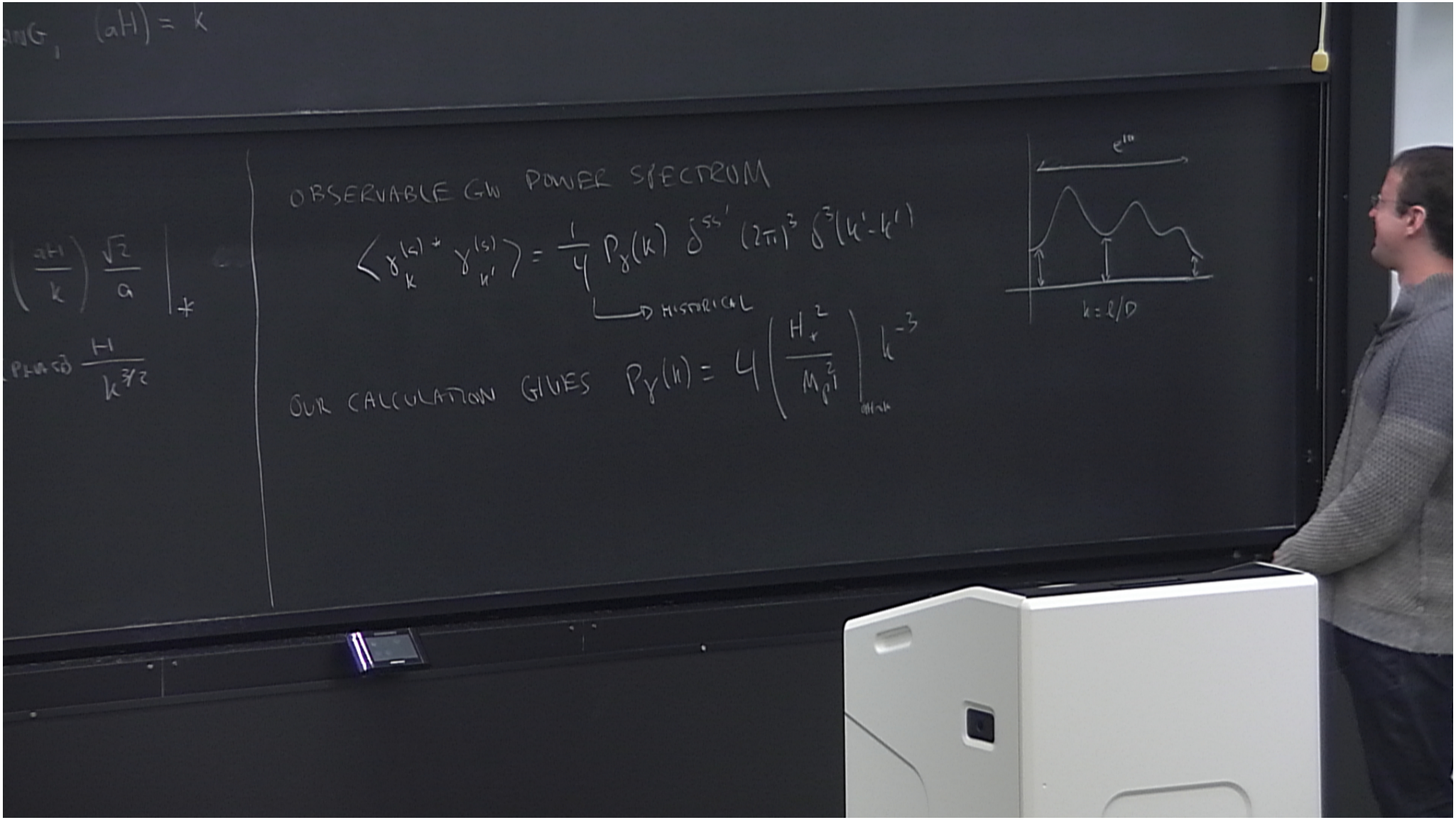

OBSERVABLE GW POWER SPECTRUM

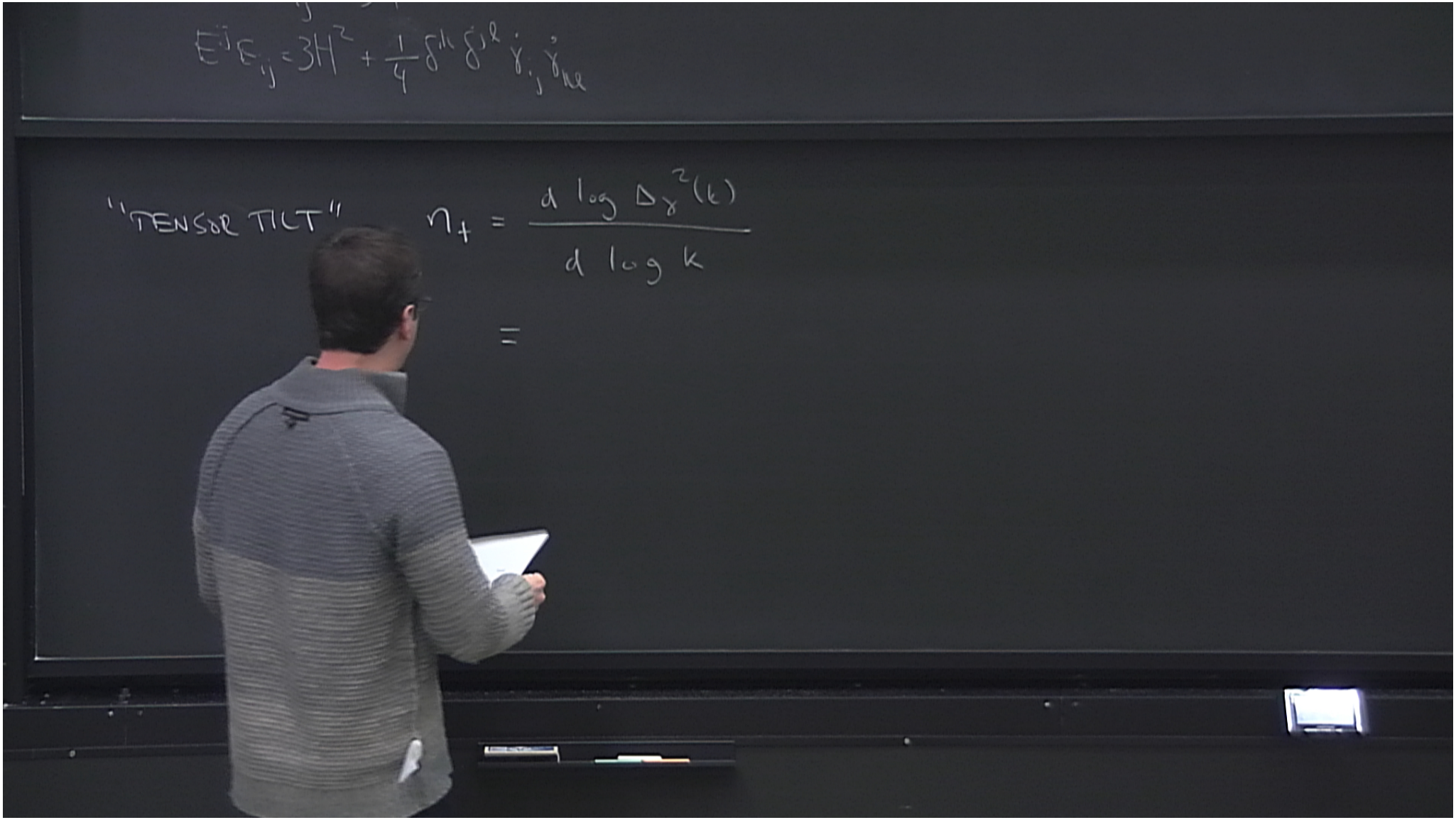

OBSERVABLE GW POWER SPECTRUM

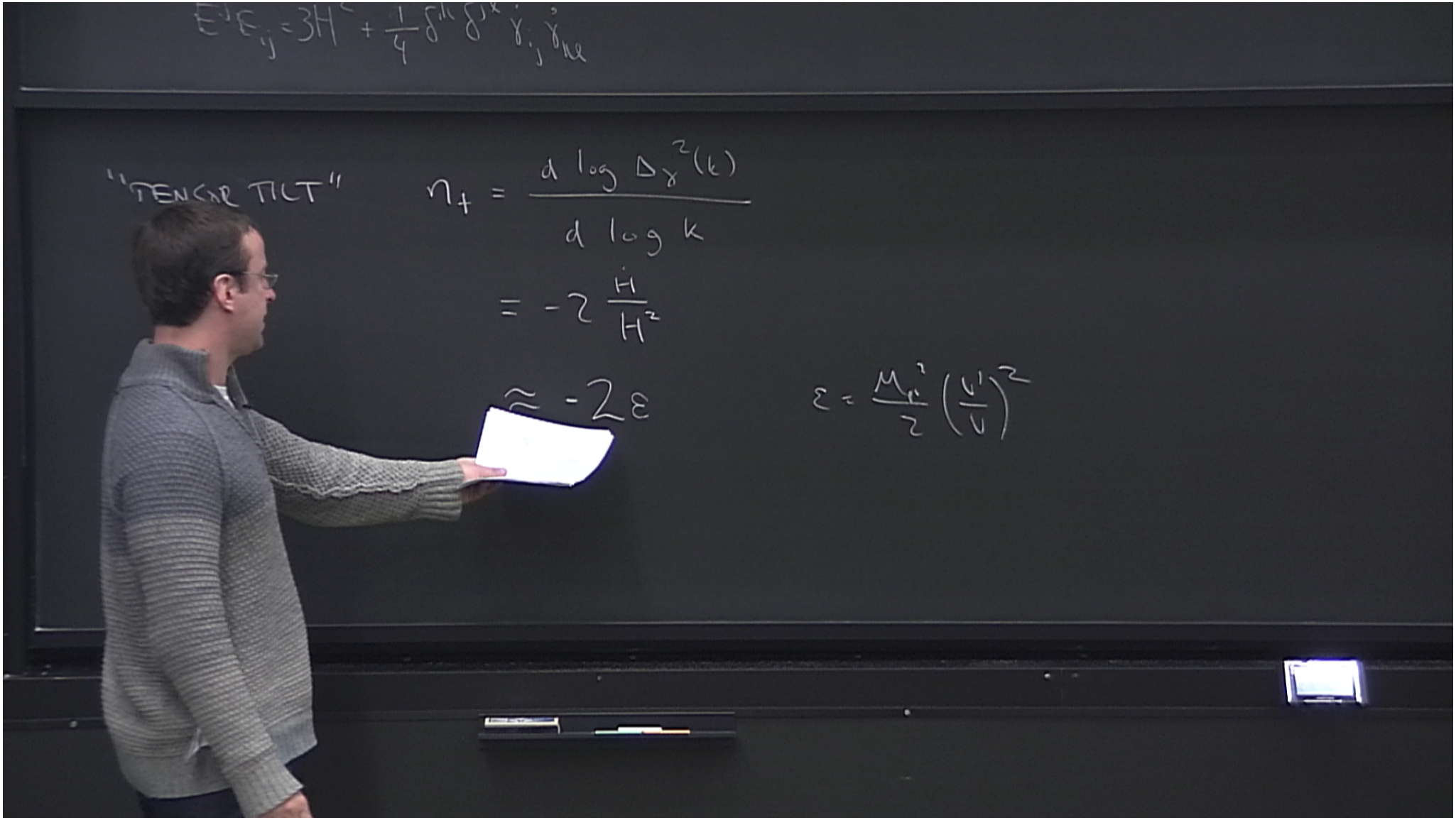

OBSERVABLE GW POWER SPECTRUM
$\langle \gamma^{(s)*}_{k} \gamma^{(s')}_{k'} \rangle = \frac{1}{4} P_\gamma(k) \delta^{ss'} (2\pi)^3 \delta^3(k'-k')$
HISTORICAL
OUR CALCULATION GIVES $P_\gamma(k) = 4 \left(\frac{H_*^2}{M_{pl}^2} \right) k^{-3}$

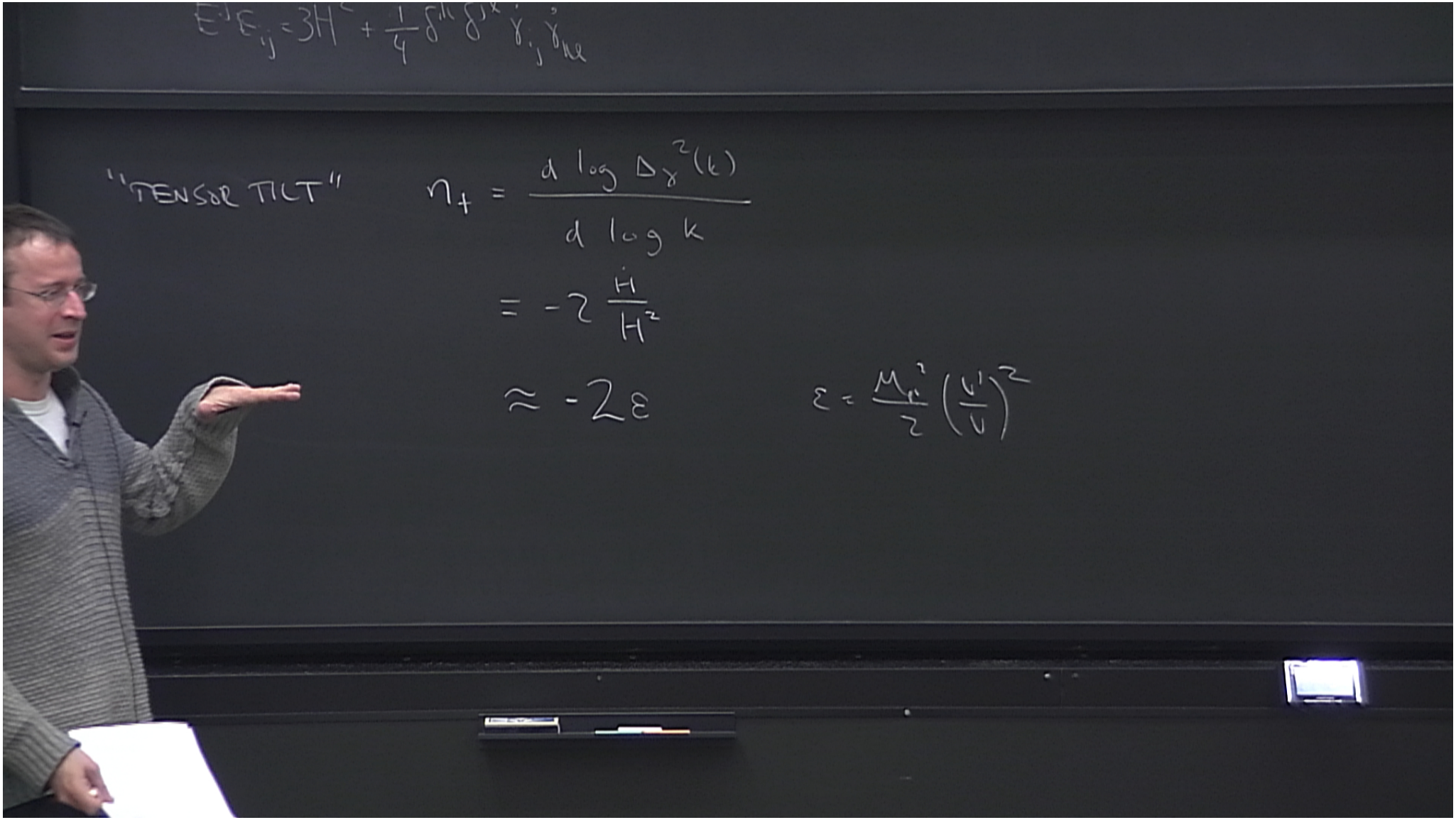

OBSERVABLE GW POWER SPECTRUM
HISTORICAL
OUR CALCULATION GIVES


OBSERVABLE GW POWER SPECTRUM
$\langle \gamma^{(s)*}_{k} \gamma^{(s')}_{k'} \rangle = \frac{1}{4} P_\gamma(k) \delta^{ss'} (2\pi)^3 \delta^3(k-k')$
HISTORICAL
OUR CALCULATION GIVES $P_\gamma(k) = 4 \left(\frac{H_*^2}{M_{pl}^2} \right) k^{-3}$


OBSERVABLE GW POWER SPECTRUM
HISTORICAL
OUR CALCULATION GIVES
k=l/D


$E^i E_{ij} = 3H^2 + \frac{1}{4} g^{kl} g^{rs} \dot{\gamma}_{ij} \dot{\gamma}_{kl}$
"TENSOR TILT"
$n_t = \frac{d \log \Delta_\gamma^2(k)}{d \log k}$
=


"TENSOR TILT"
$n_t = \frac{d \log \Delta_\gamma^2(k)}{d \log k}$
$= -2 \frac{\dot{H}}{H^2}$
$\approx -2\varepsilon$
$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$

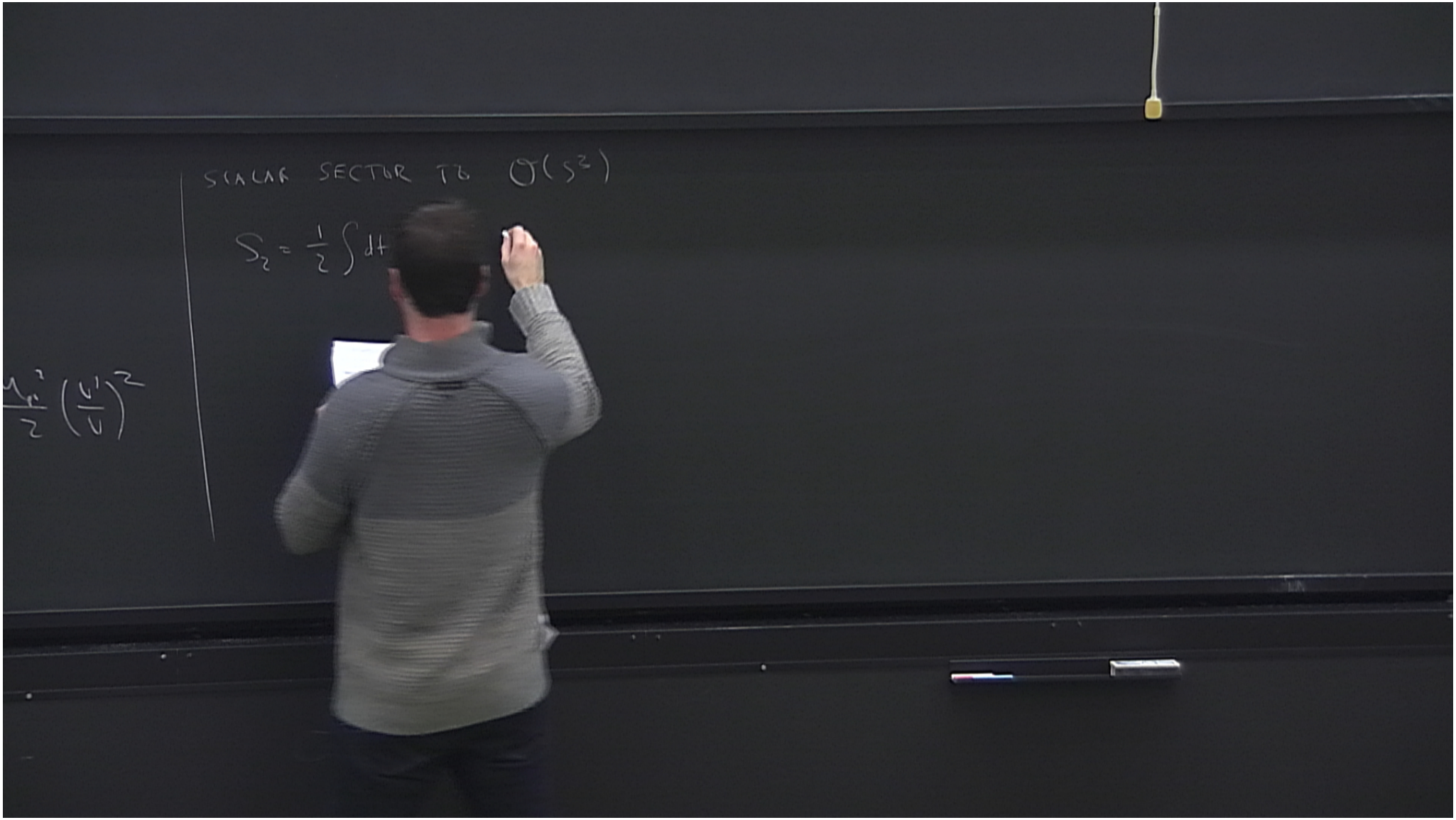

"TENSOR TILT"
$n_t = \frac{d \log \Delta_\gamma^2(k)}{d \log k}$
$= -2 \frac{\dot{H}}{H^2}$
$\approx -2\varepsilon$
$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$

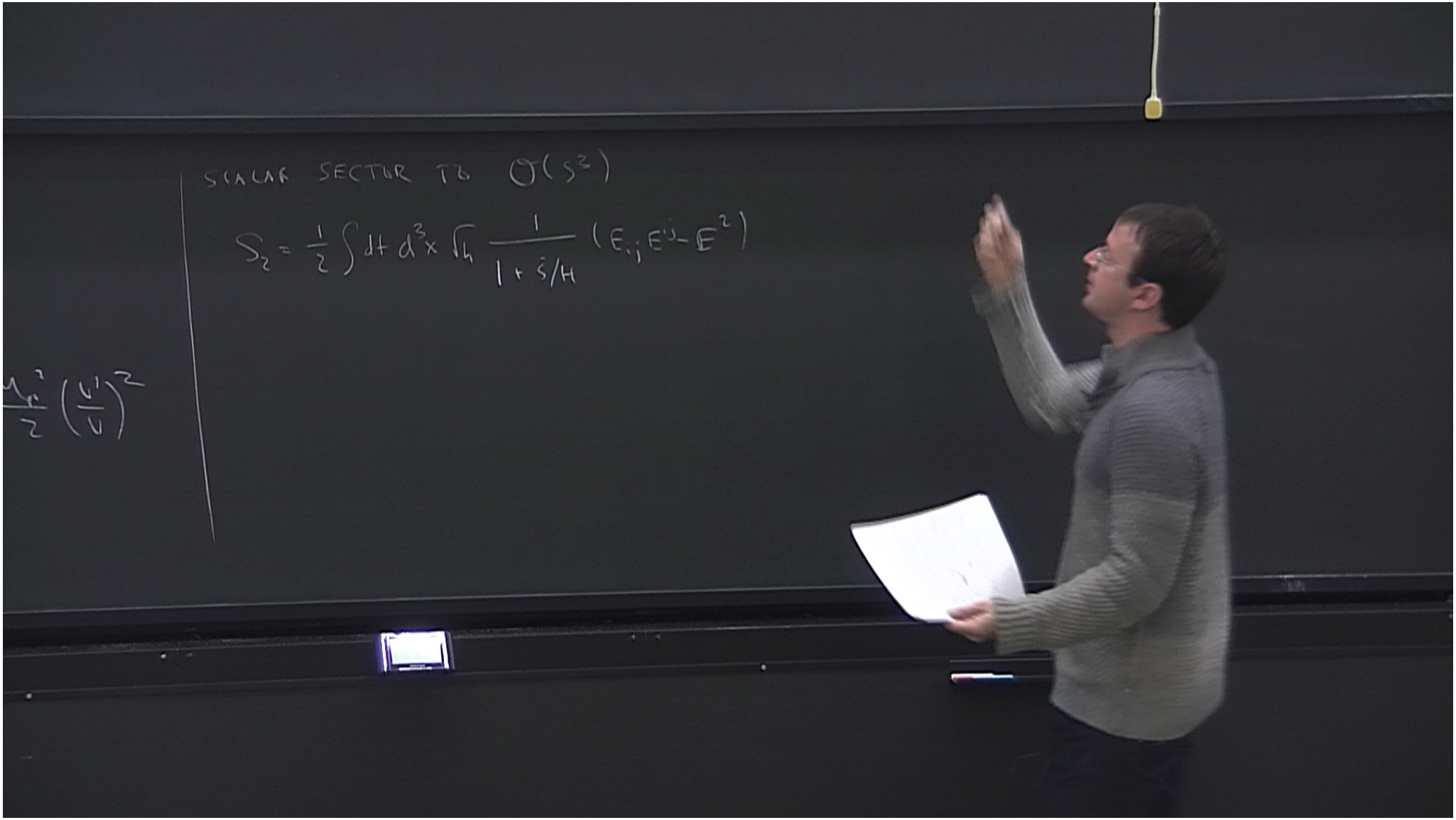
$E^i E_{ij} = 3H^2 + \frac{1}{4} g^{\mu\nu} g^{\alpha\beta} \gamma_{ij} \gamma_{\mu\nu}$

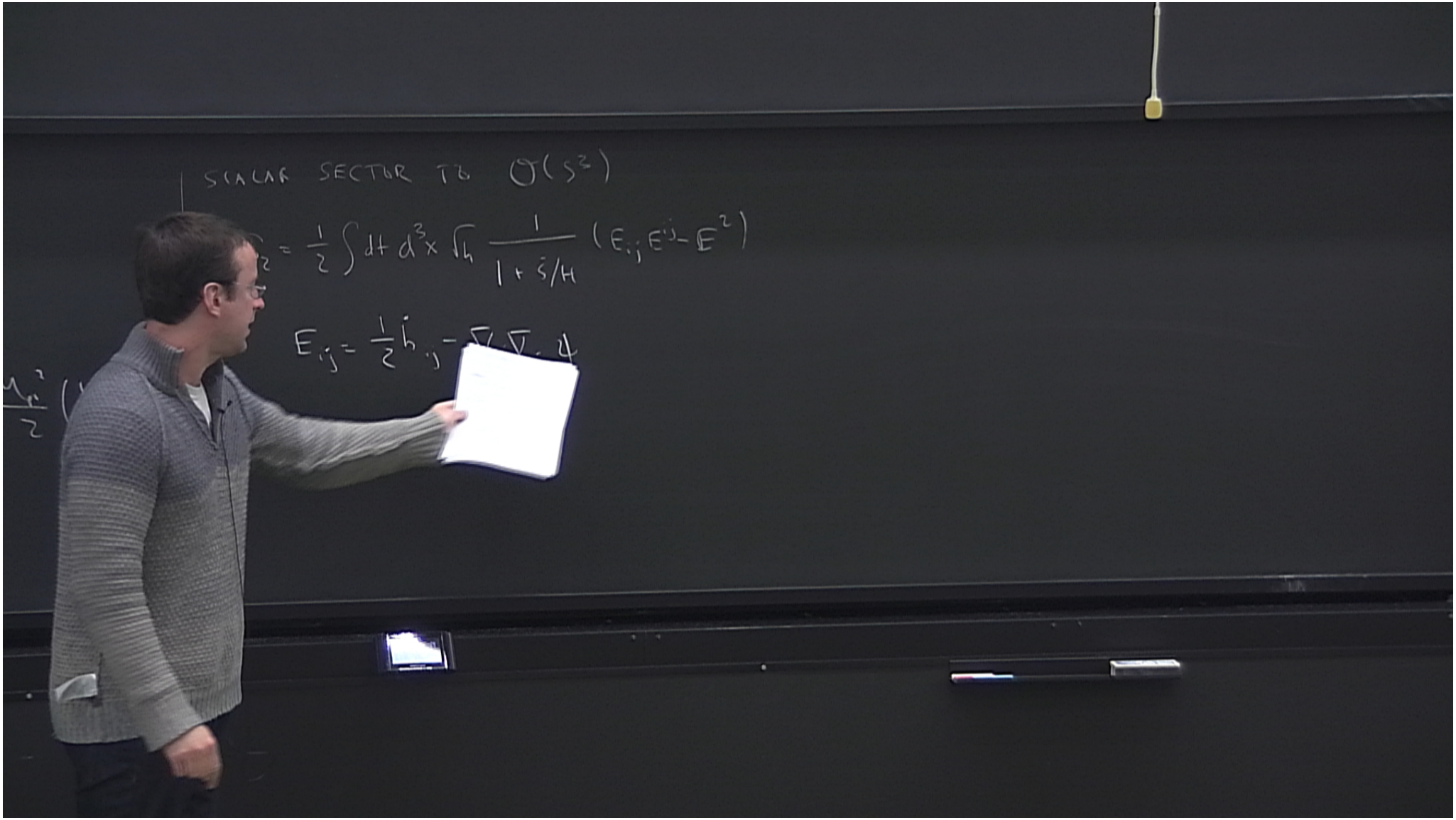
"TENSOR TILT" $\quad n_t = \dfrac{d \log \Delta_\gamma^2(k)}{d \log k}$

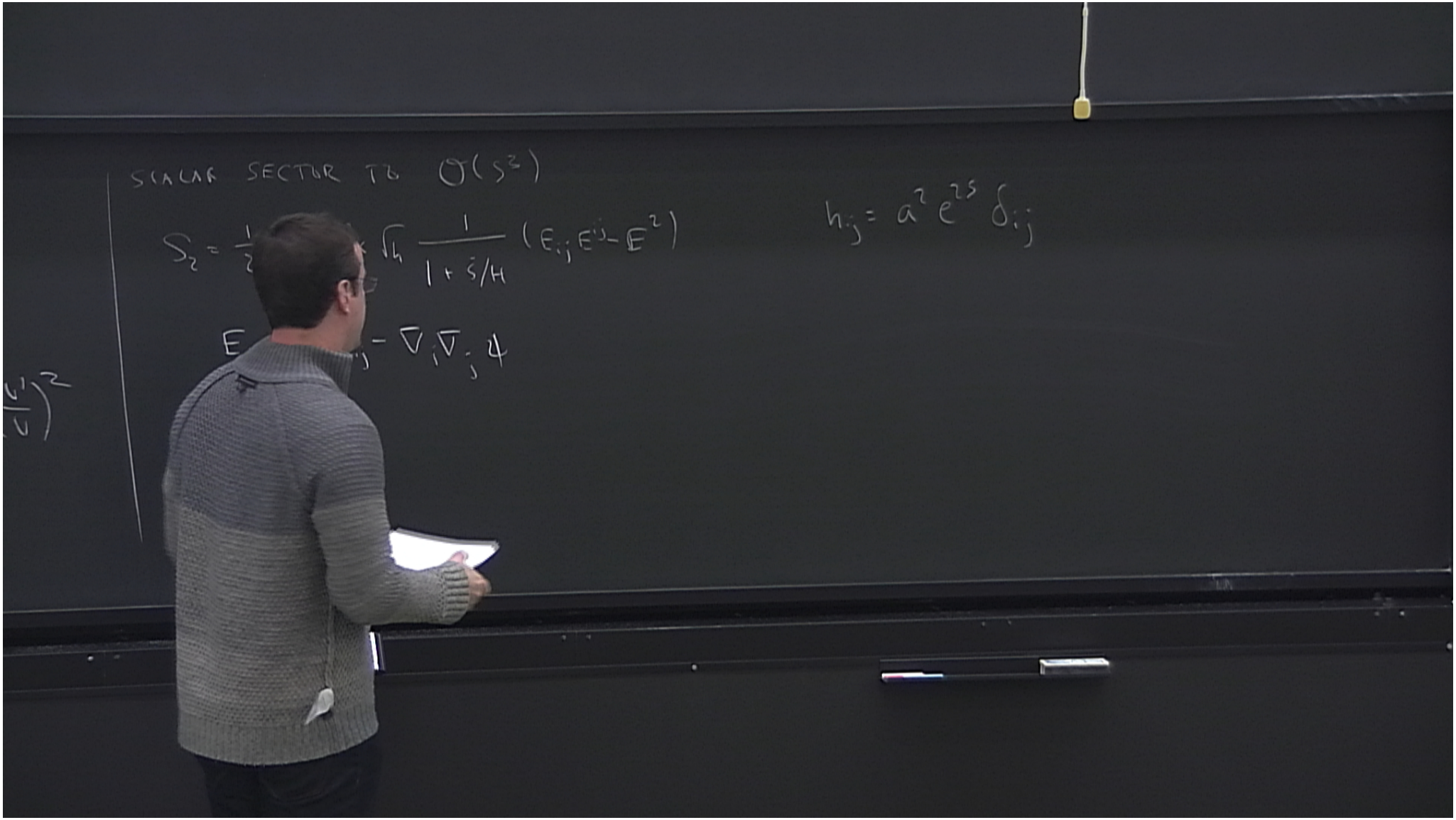
$= -2 \dfrac{\dot{H}}{H^2}$

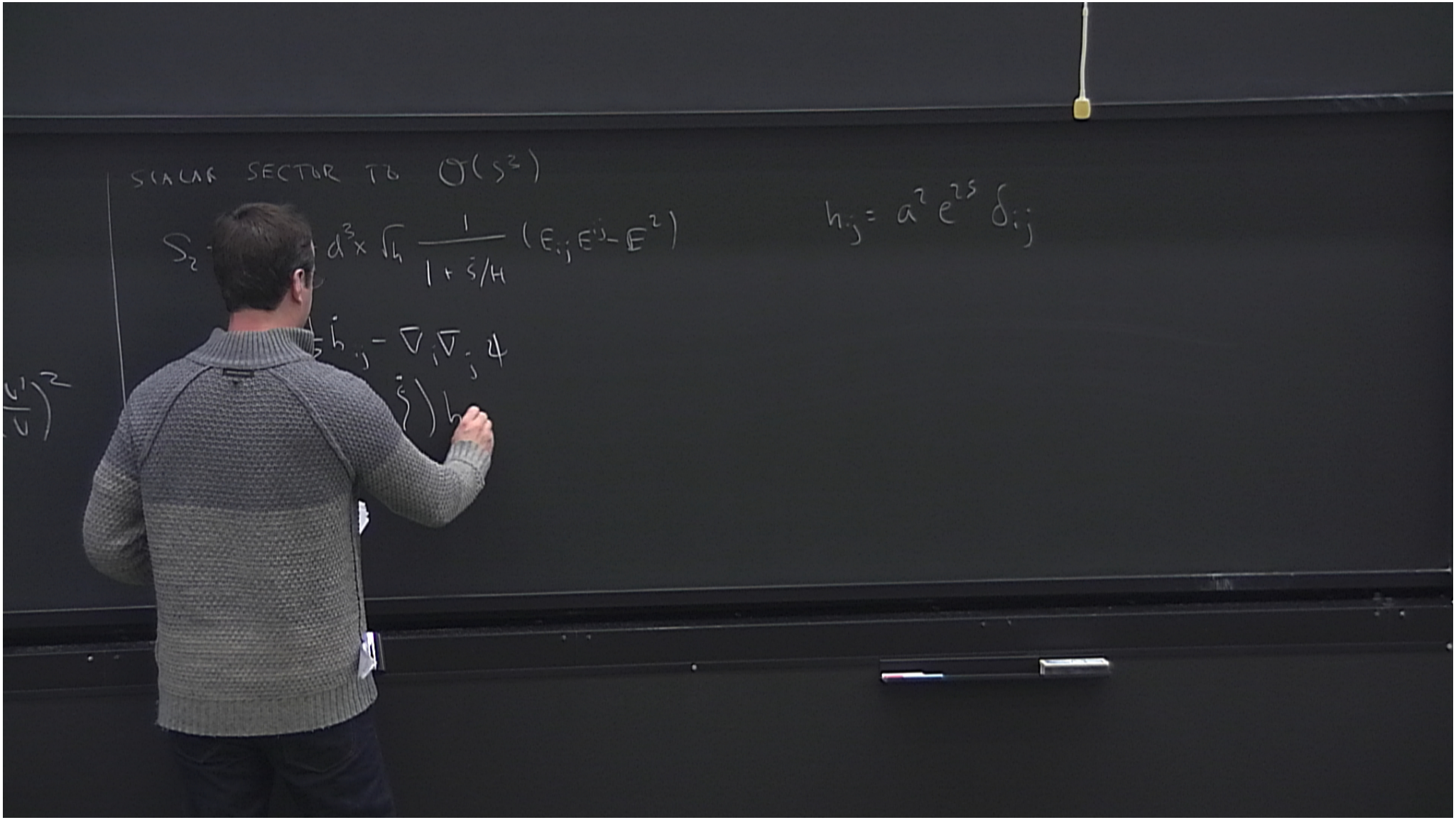
$\approx -2\varepsilon \qquad \varepsilon = \dfrac{M_{pl}^2}{2} \left(\dfrac{V'}{V}\right)^2$


SCALAR SECTOR TO $O(s^3)$
$S_2 = \frac{1}{2}\int dt$
$\frac{\mu_r^2}{2}\left(\frac{v'}{v}\right)^2$


SCALAR SECTOR TO $O(s^3)$
$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, \frac{1}{1+\dot{s}/H}\,(E_{ij}E^{ij} - E^2)$
$\frac{M_{Pl}^2}{2}\left(\frac{\dot{v}'}{v}\right)^2$


SCALAR SECTOR TO $O(\varepsilon^3)$
$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, \frac{1}{1+\dot{\varsigma}/H}\,(E_{ij}E^{ij} - E^2)$
$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_{(i}\nabla_{j)}\psi$


SCALAR SECTOR TO $O(\zeta^3)$
$h_{ij} = a^2 e^{2\zeta} \delta_{ij}$


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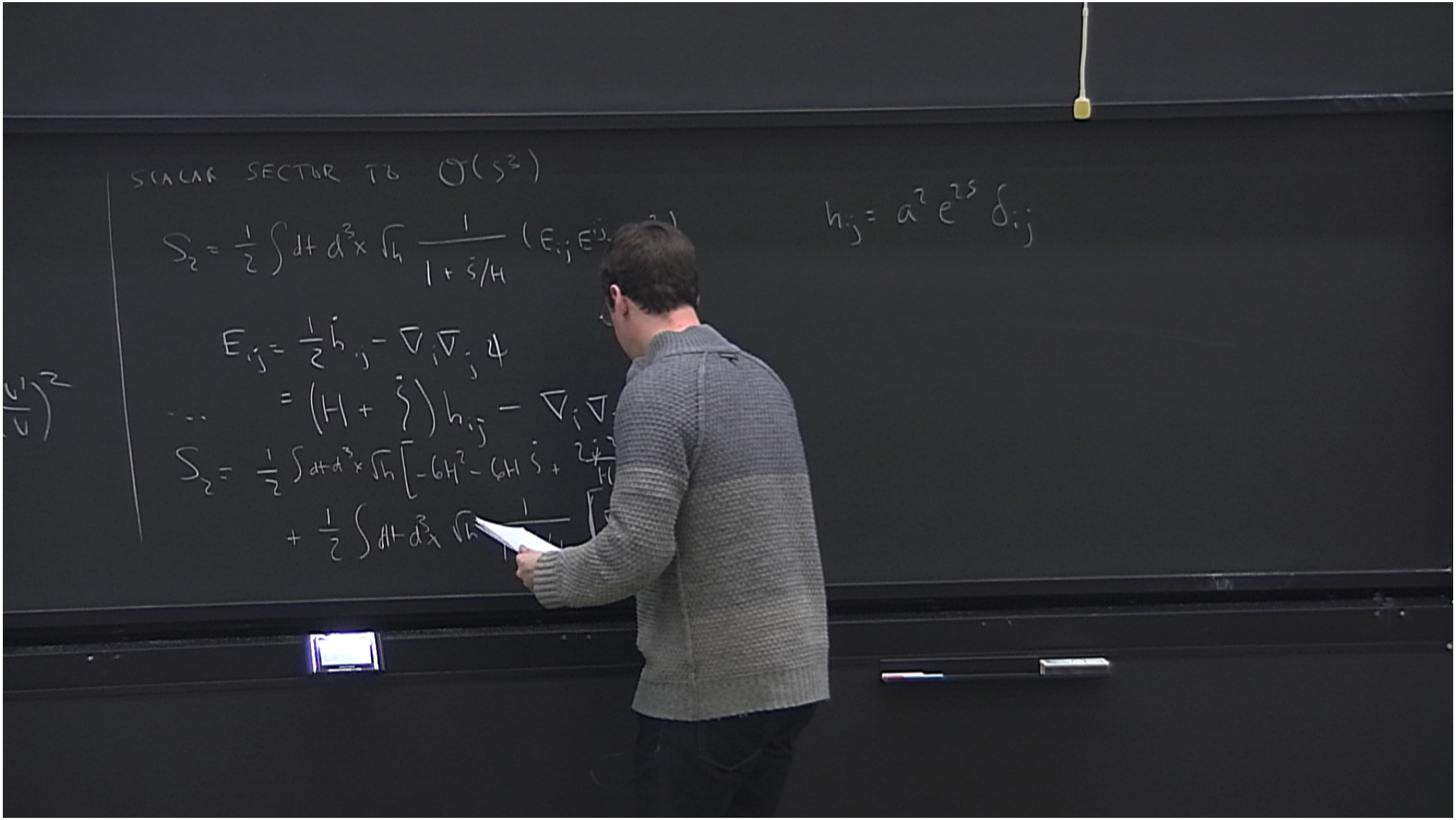
SCALAR SECTOR TO $O(\zeta^3)$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, \frac{1}{1+\dot{\zeta}/H}\left(E_{ij}E^{ij} - E^2\right)$$

$$h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

$$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_i\nabla_j\psi$$

$$= \left(H + \dot{\zeta}\right)h_{ij} - \nabla_i\nabla_j\psi$$


SCALAR SECTOR TO $O(s^3)$
$h_{ij} = a^2 e^{2s} \delta_{ij}$
$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_i \nabla_j \psi$
$= \left(H + \dot{s}\right) h_{ij} - \nabla_i \nabla$
$S_2 = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[-6H^2 - 6H\dot{s} +\right.$
$+ \frac{1}{2}\int dt\, d^3x \sqrt{h}$

SCALAR SECTOR TO $O(\zeta^3)$

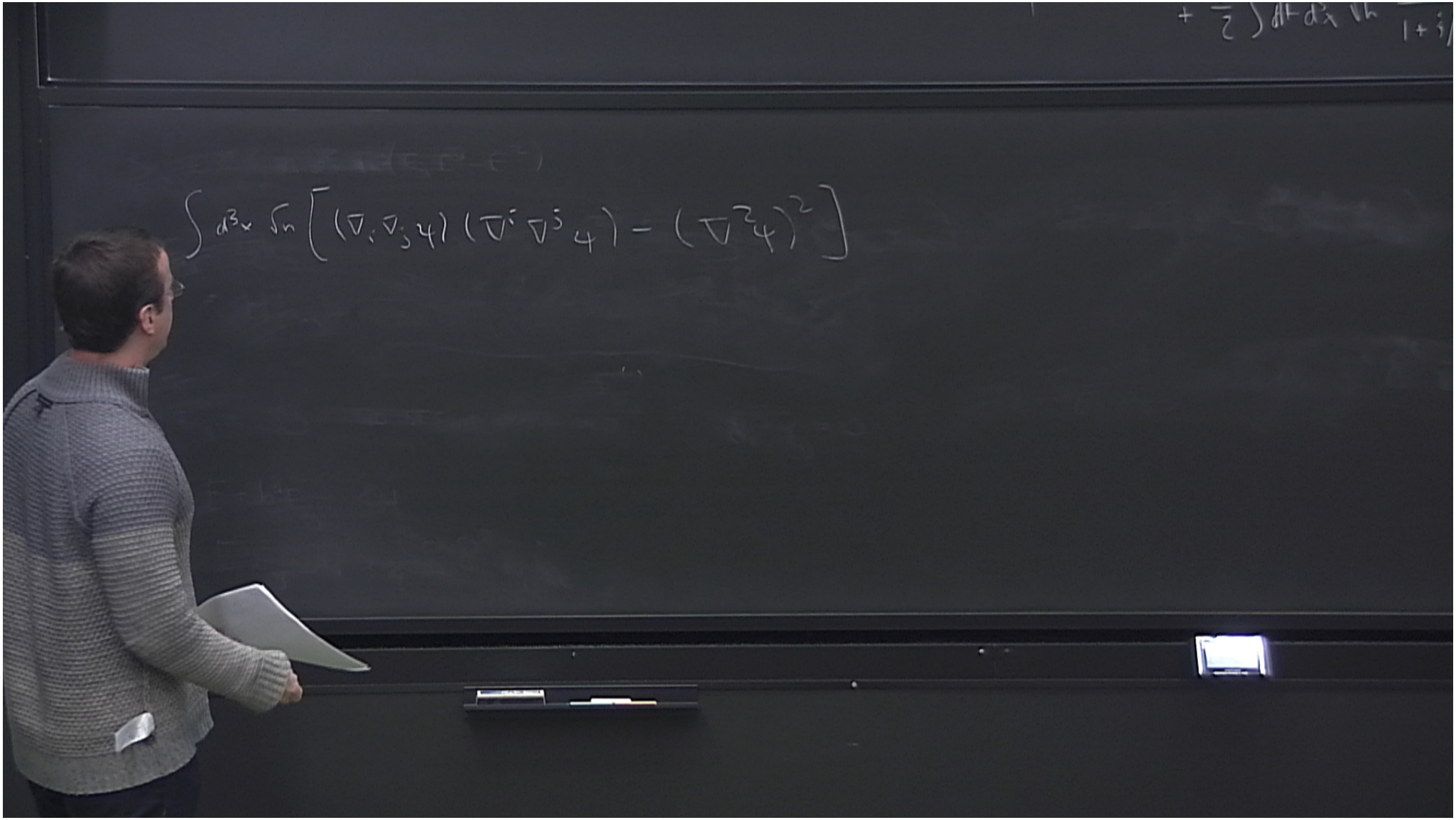
$$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, \frac{1}{1+\dot{\zeta}/H}\left(E_{ij}E^{ij} - E^2\right) \qquad h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

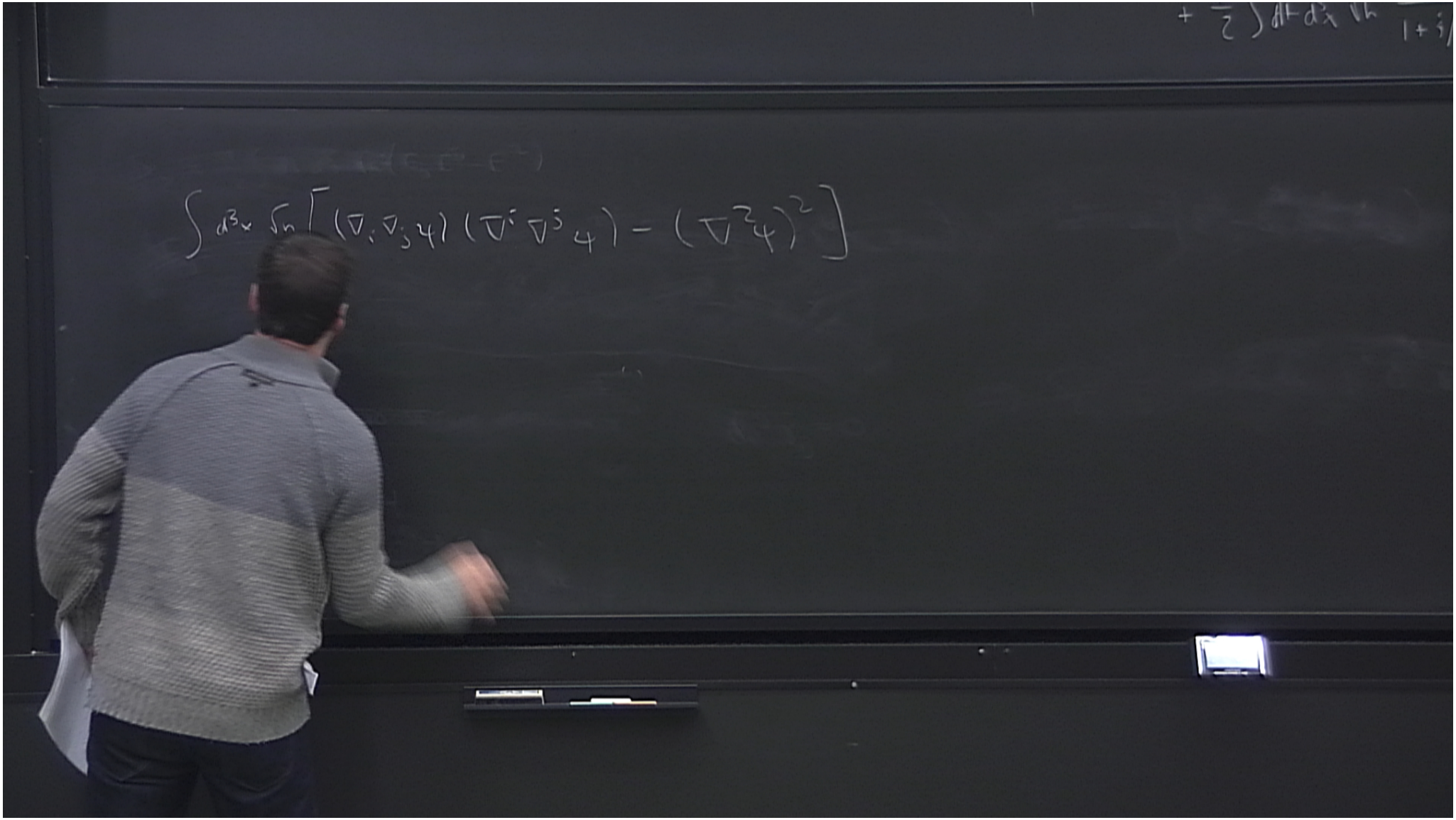
$$E_{ij} = \frac{1}{2}\dot{h}_{ij} - \nabla_i\nabla_j\psi$$

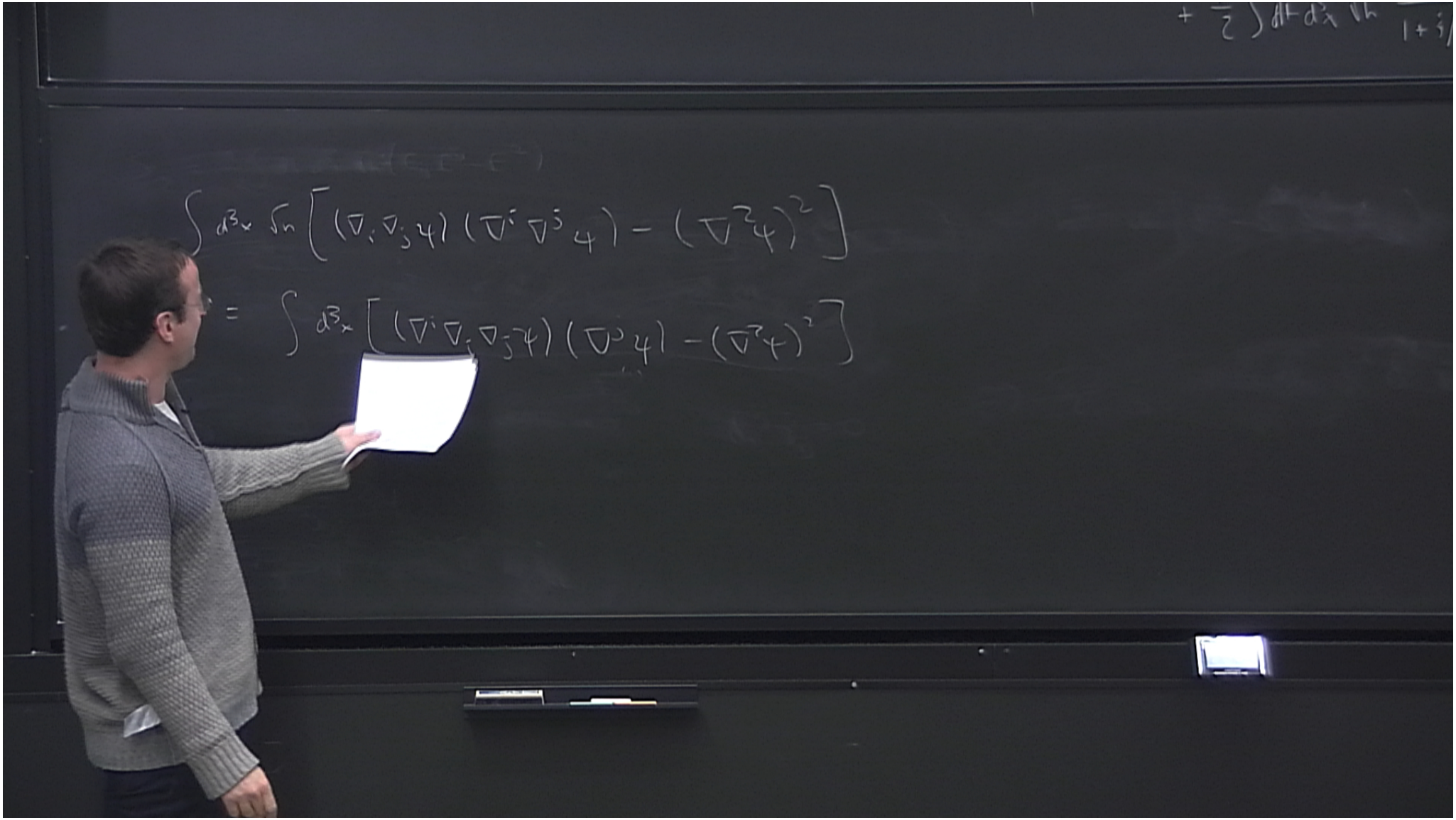
$$\ldots = \left(H + \dot{\zeta}\right)h_{ij} - \nabla_i\nabla_j\psi$$

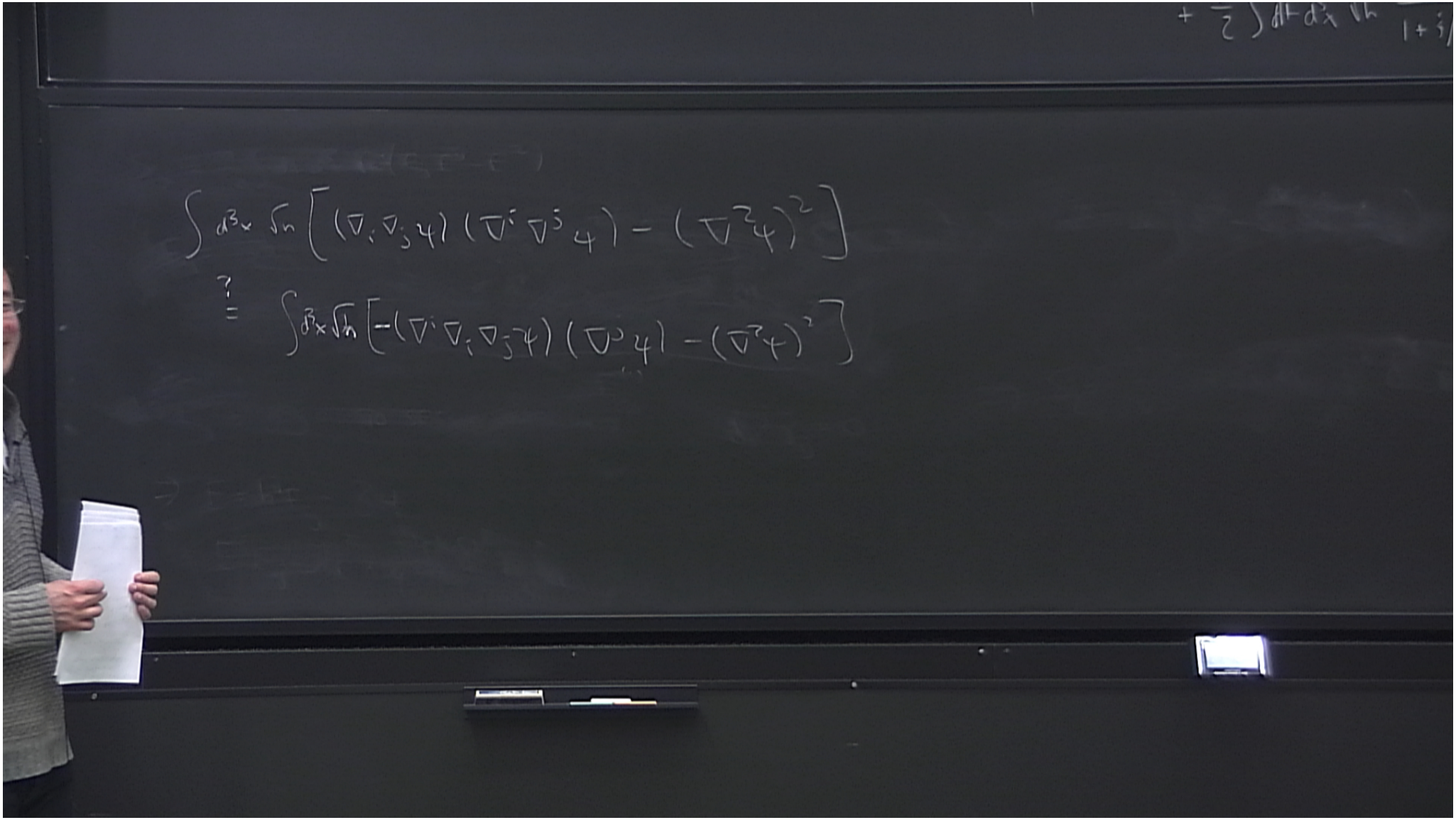
$$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-6H^2 - 6H\dot{\zeta} + \frac{2\dot{\psi}^2}{H}\dot{\zeta}\right]$$

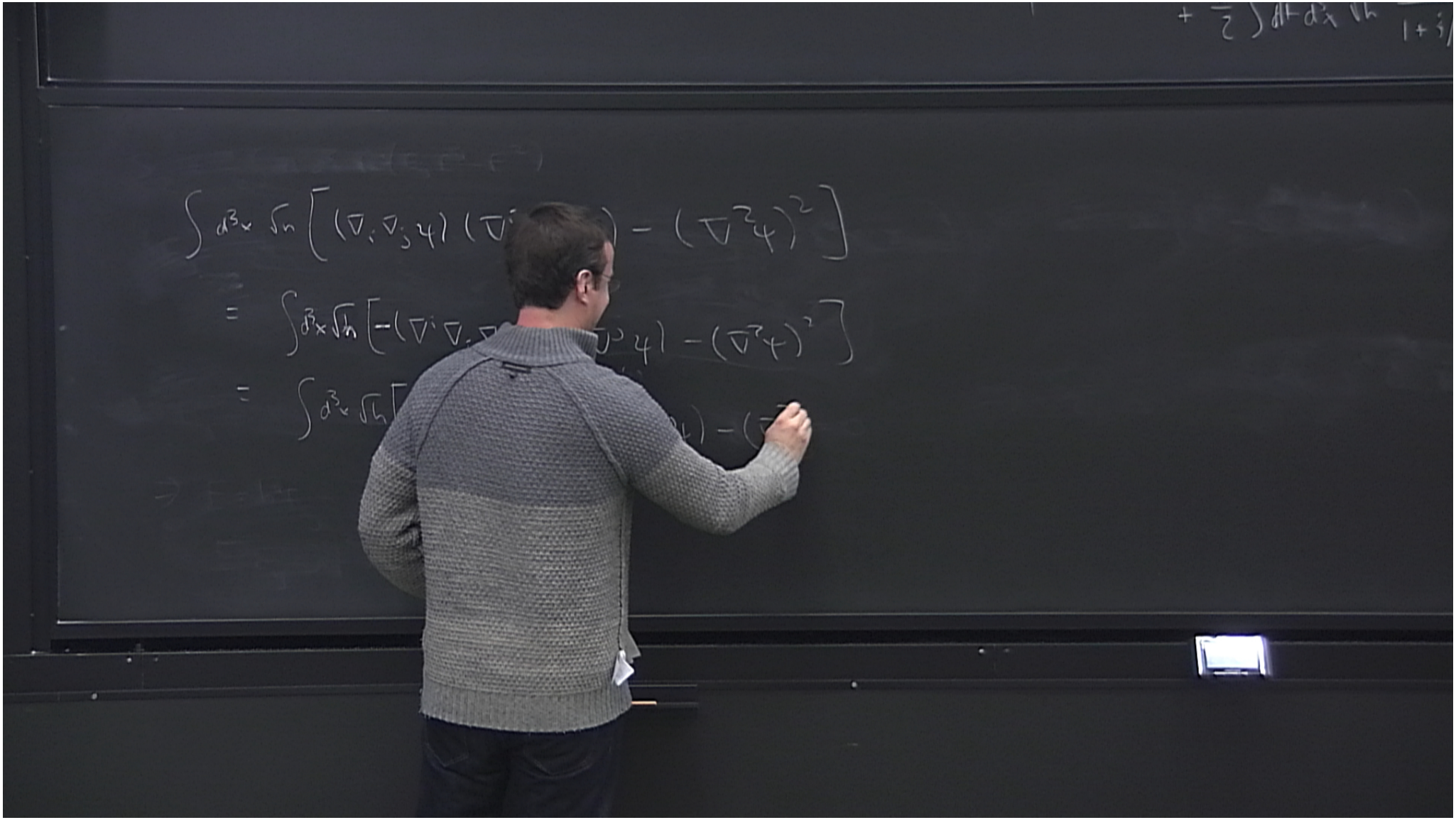
$$+ \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, \frac{1}{1+\dot{\zeta}/H}\left[\left(\nabla_i\nabla_j\psi\right)\left(\nabla^i\nabla^j\psi\right) - \left(\nabla^2\psi\right)^2\right]$$

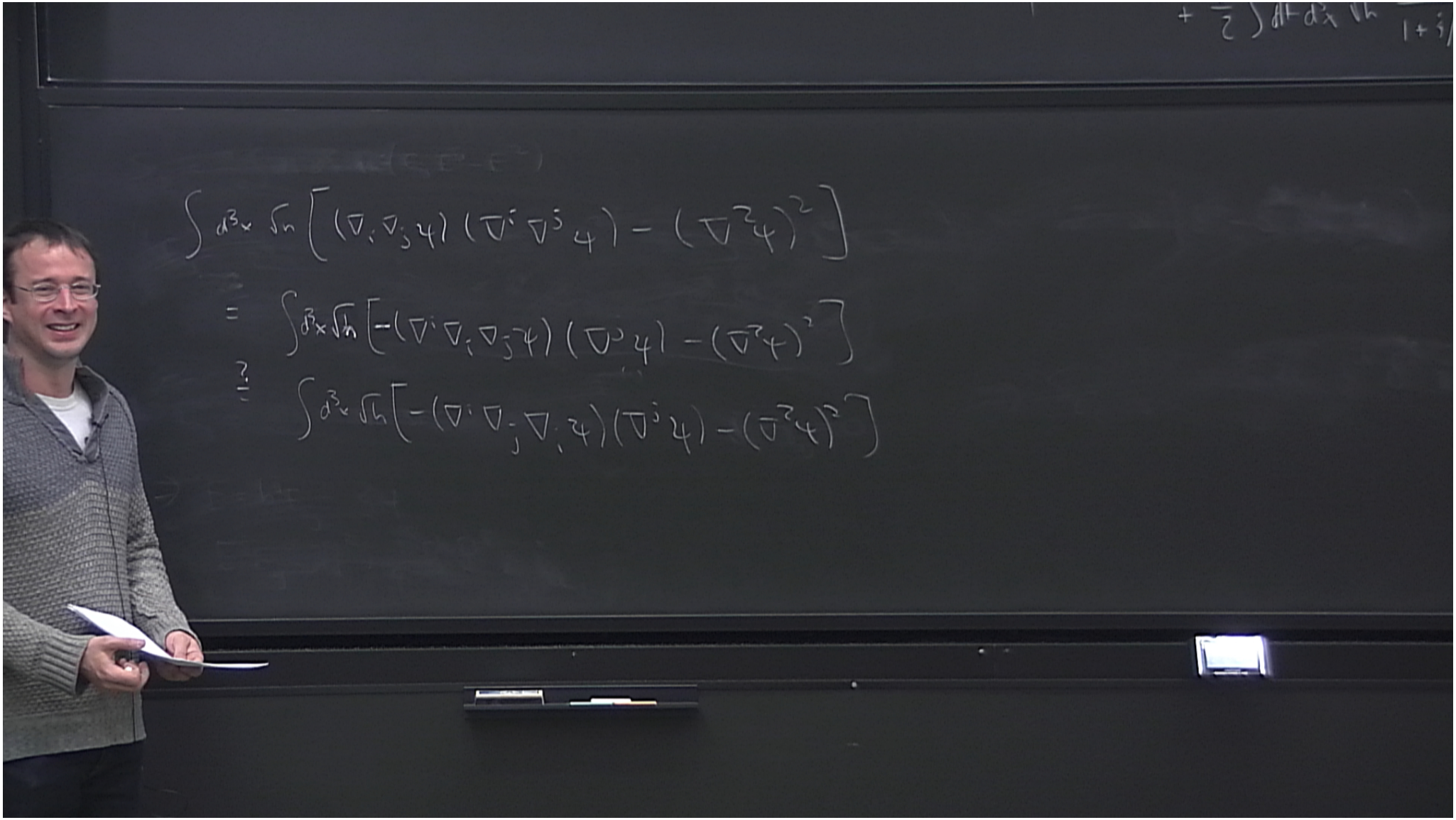

$\int d^3x \sqrt{h} \left[(\nabla_i \nabla_j \psi)(\nabla^i \nabla^j \psi) - (\nabla^2 \psi)^2 \right]$

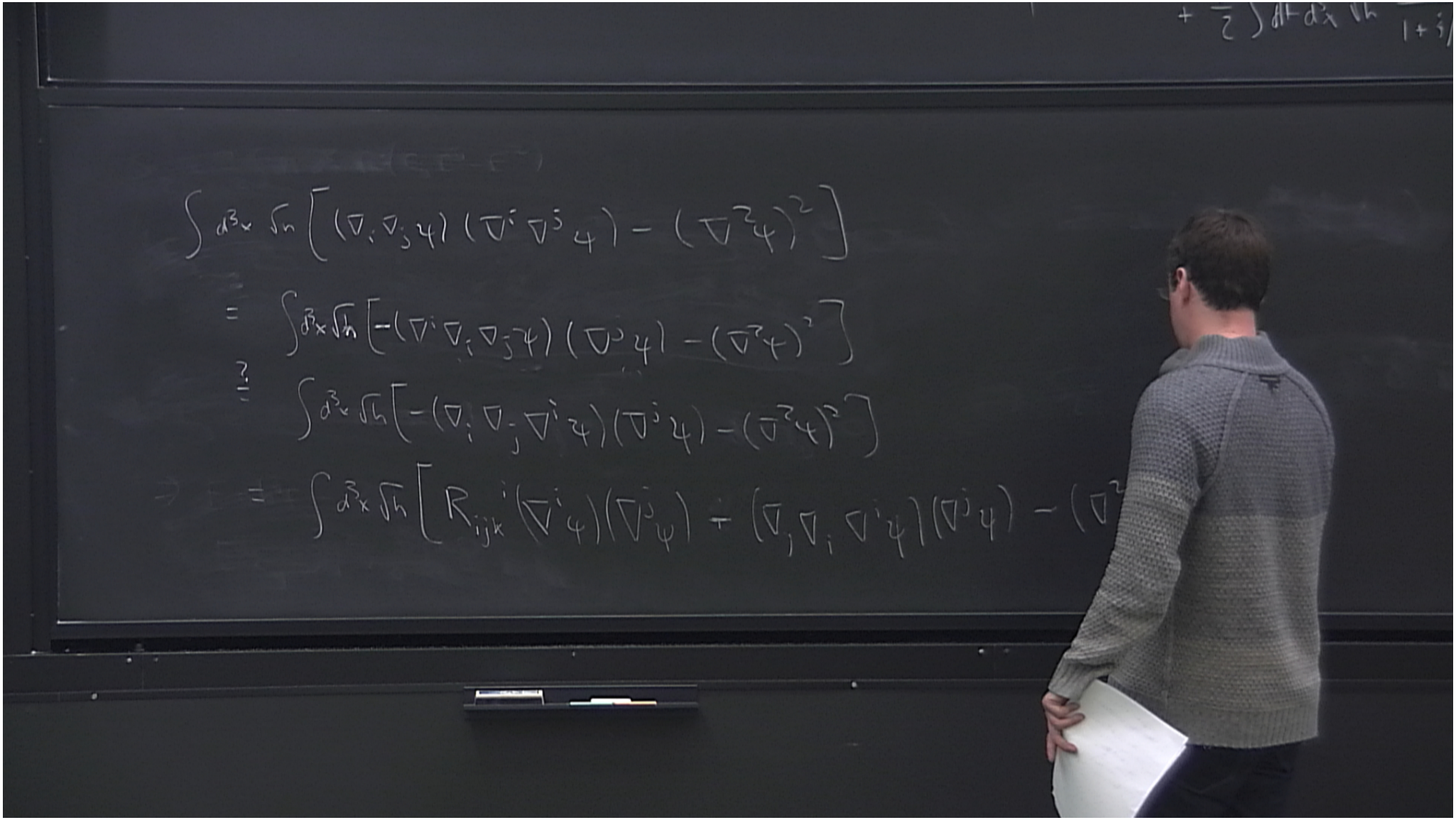

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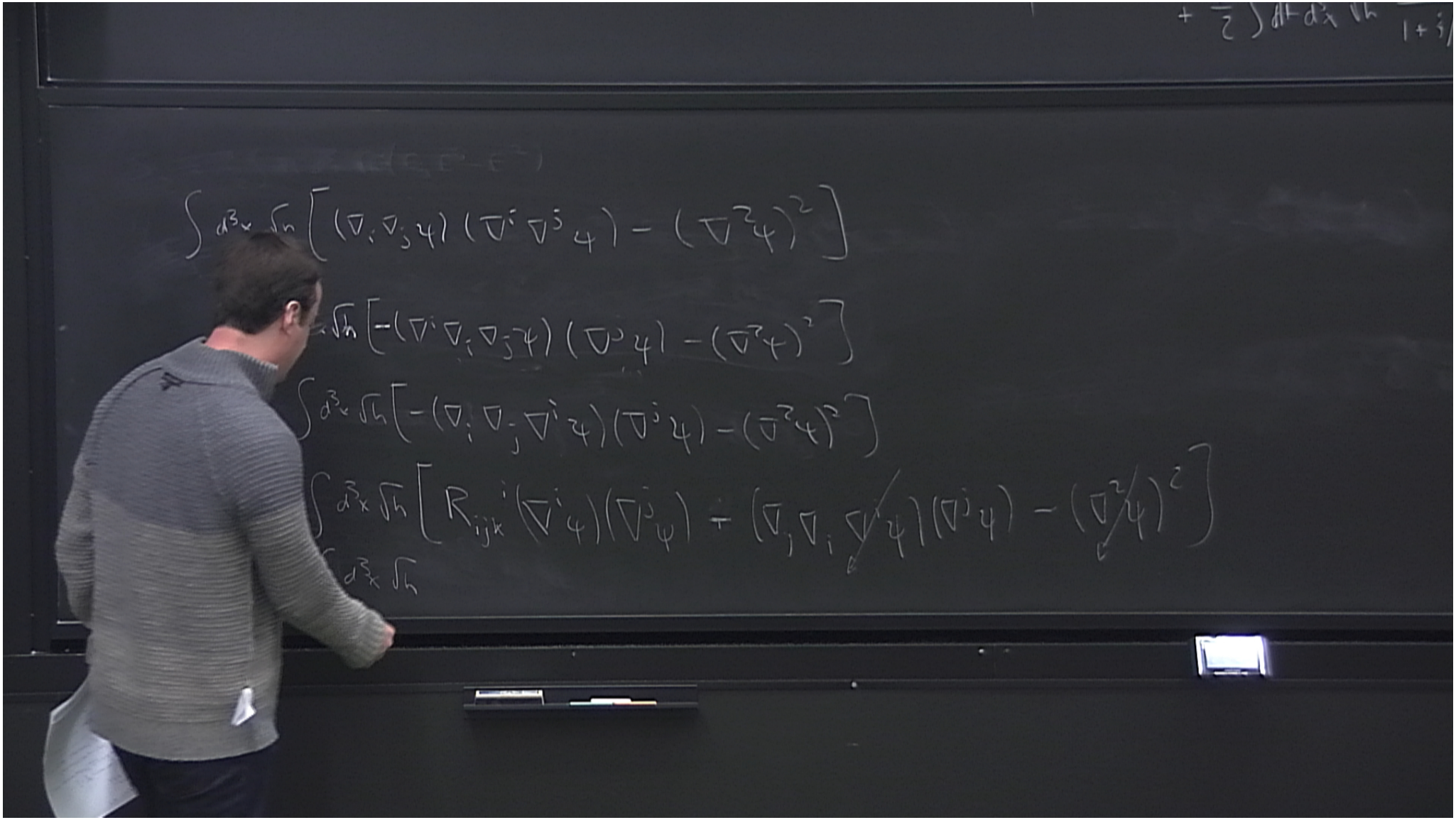

$\int d^3x \sqrt{h} \left[(\nabla_i \nabla_j \psi)(\nabla^i \nabla^j \psi) - (\nabla^2 \psi)^2 \right]$
$= \int d^3x \left[(\nabla^i \nabla_i \nabla_j \psi)(\nabla^j \psi) - (\nabla^2 \psi)^2 \right]$

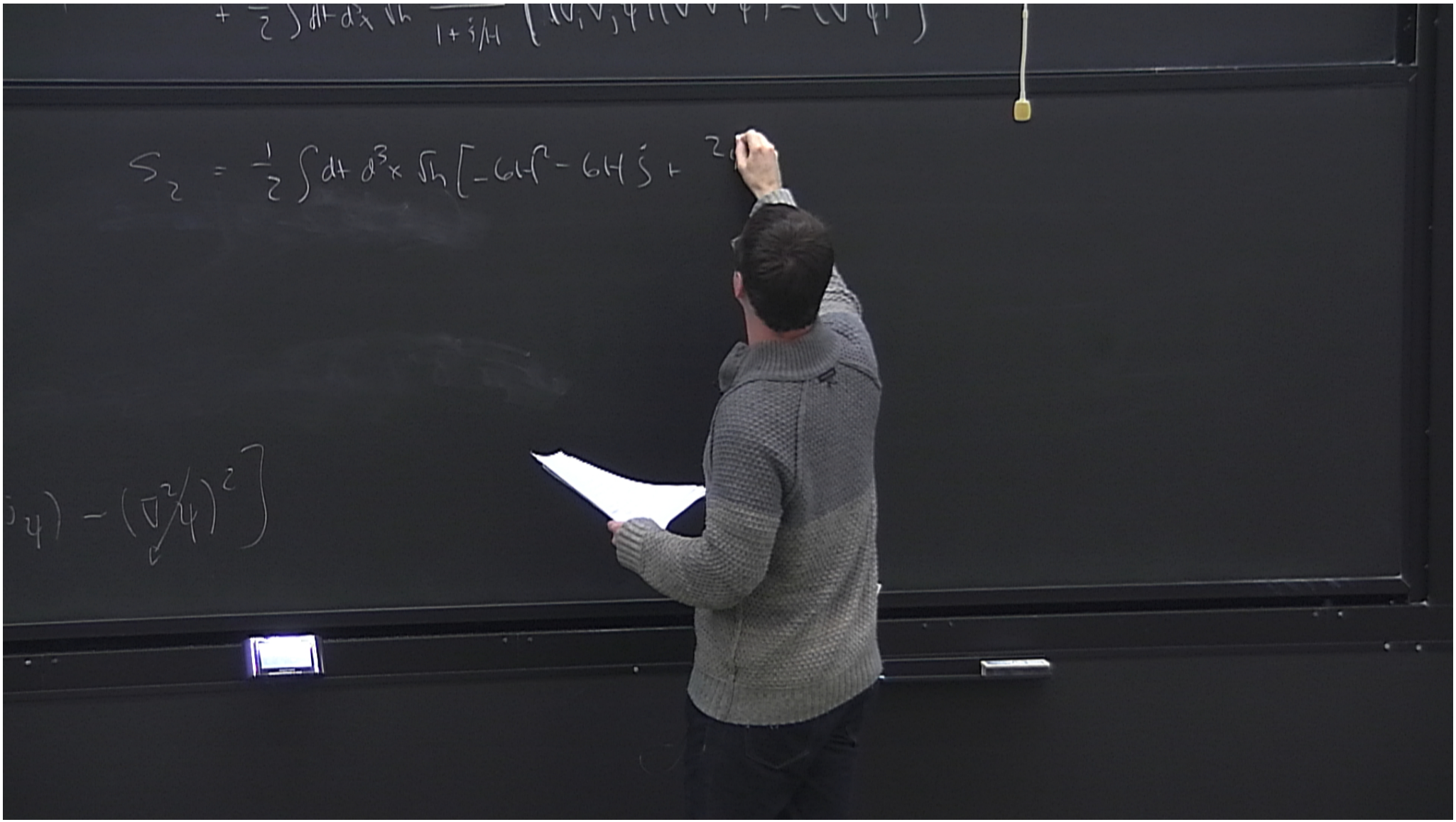

$\int d^3x \sqrt{h} \left[(\nabla_i \nabla_j \psi)(\nabla^i \nabla^j \psi) - (\nabla^2 \psi)^2 \right]$
$\stackrel{?}{=} \int d^3x \sqrt{h} \left[-(\nabla^i \nabla_i \nabla_j \psi)(\nabla^j \psi) - (\nabla^2 \psi)^2 \right]$

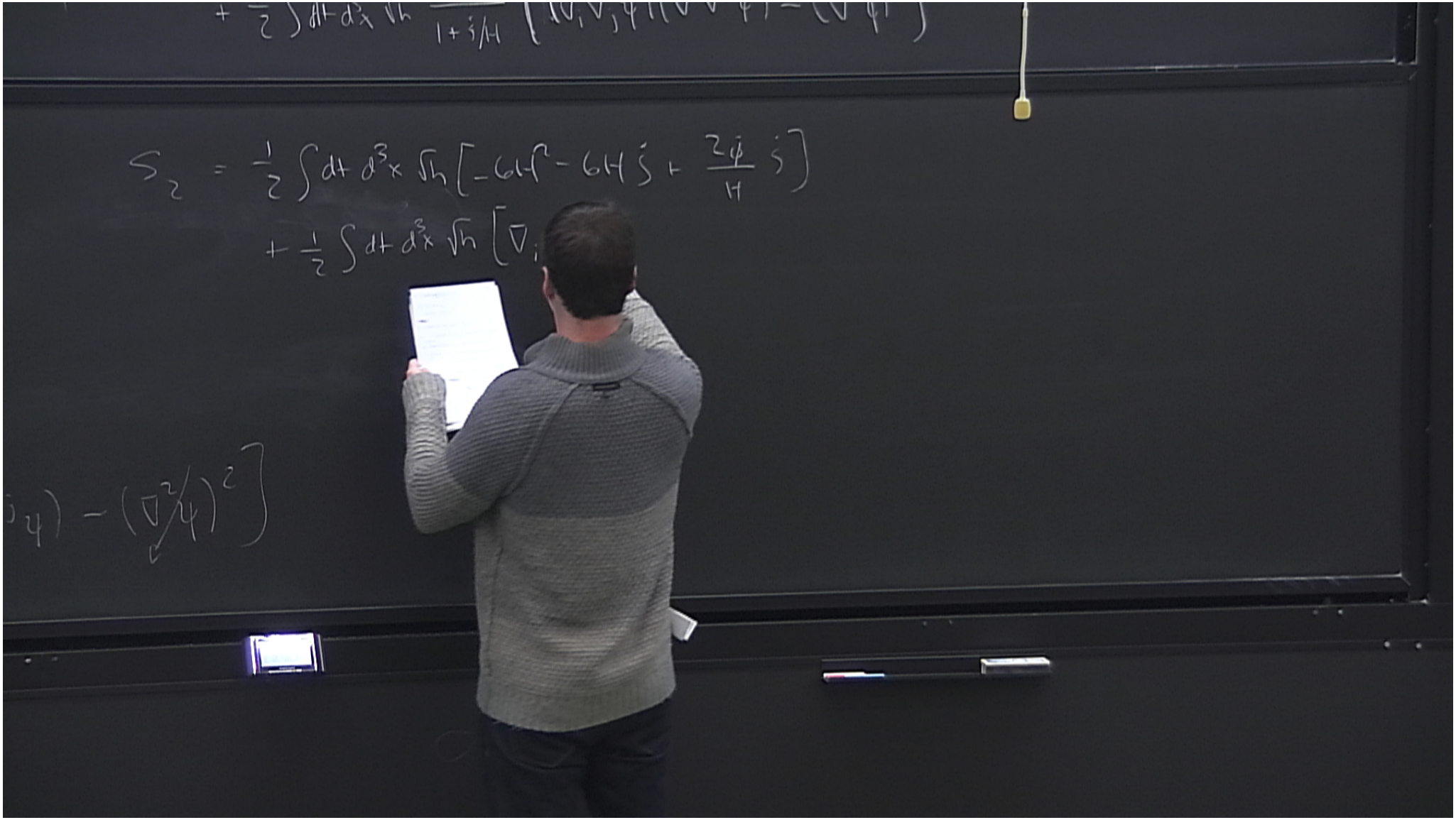


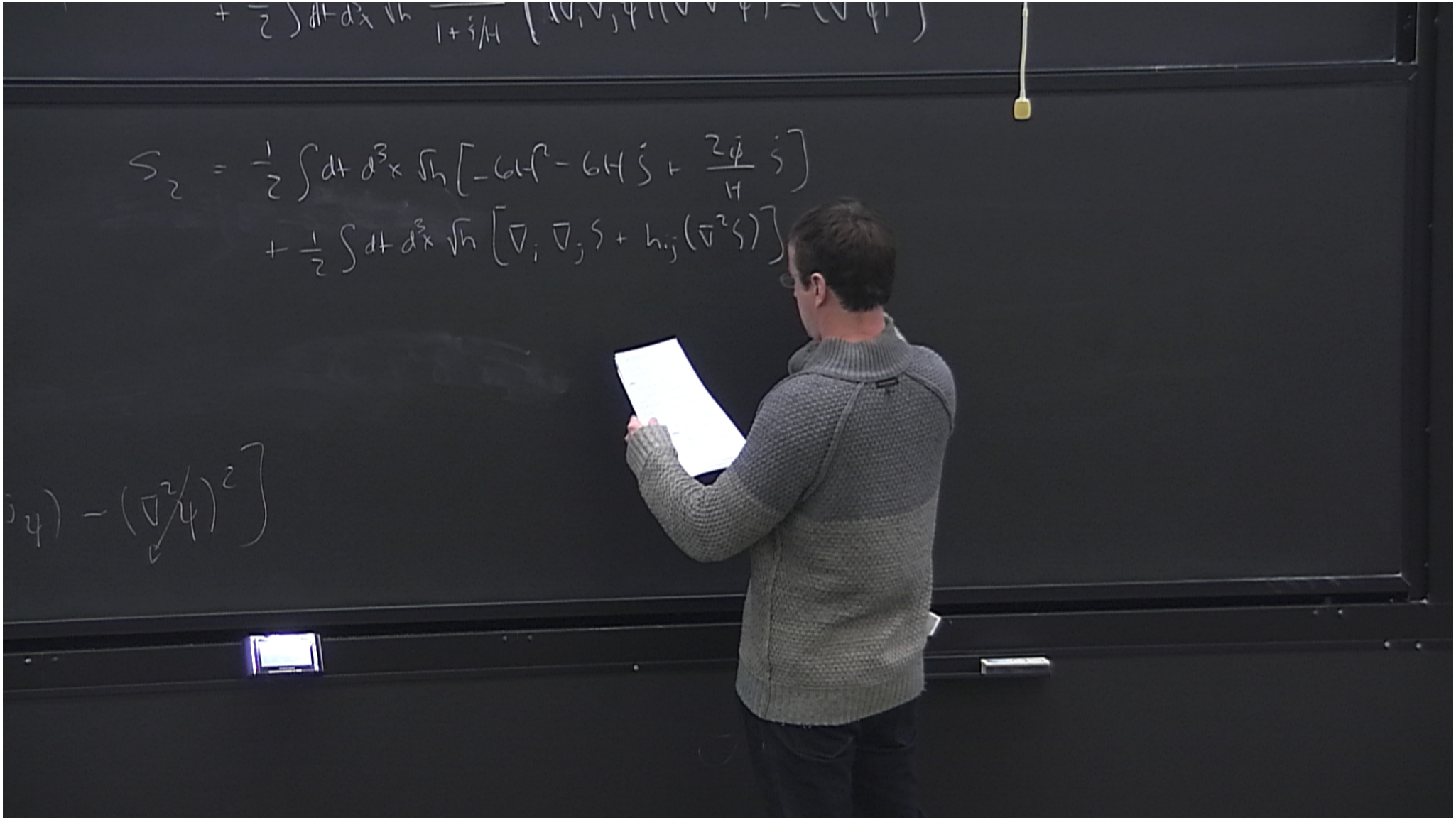

$\int d^3x \sqrt{h} \left[(\nabla_i \nabla_j \psi)(\nabla^i \nabla^j \psi) - (\nabla^2 \psi)^2 \right]$
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$\stackrel{?}{=} \int d^3x \sqrt{h} \left[-(\nabla_i \nabla_j \nabla^i \psi)(\nabla^j \psi) - (\nabla^2 \psi)^2 \right]$
$= \int d^3x \sqrt{h} \left[R_{ijk}{}^i (\nabla^i \psi)(\nabla^j \psi) + (\nabla_j \nabla_i \nabla^i \psi)(\nabla^j \psi) - (\nabla^2$


$\int d^3x \sqrt{h} \left[(\nabla_i \nabla_j \psi)(\nabla^i \nabla^j \psi) - (\nabla^2 \psi)^2 \right]$
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$\int d^3x \sqrt{h}$


$S_2 = \frac{1}{2}\int dt\, d^3x \sqrt{h}\,[-6H^2 - 6H\dot{\zeta} +$
$\nabla^2\zeta)^2]$

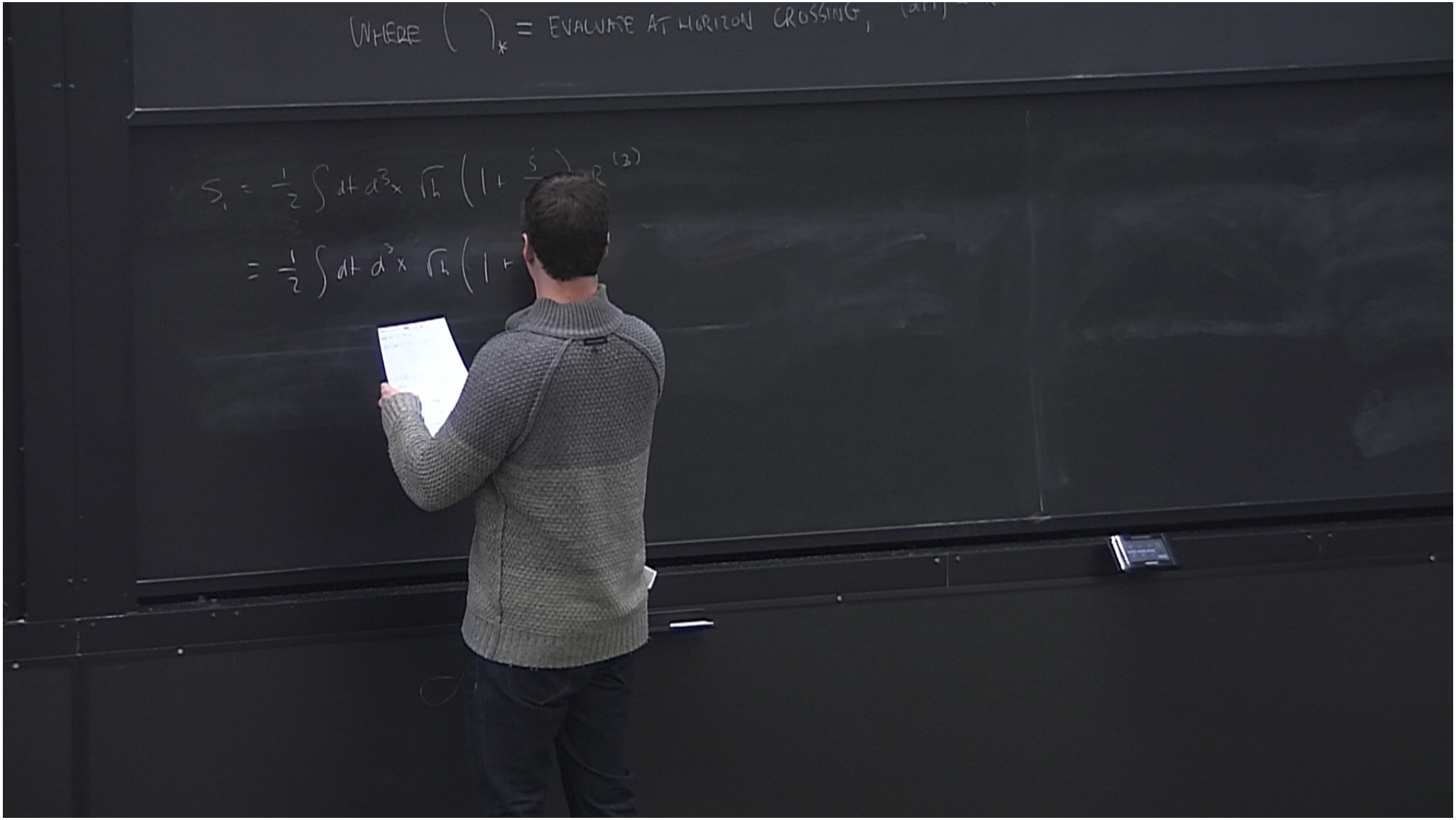

$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-6Hf^2 - 6H\dot{s} + \frac{2\dot{\phi}}{H}\dot{s}\right]$
$+ \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\nabla_i$
$\left.\left.\left.

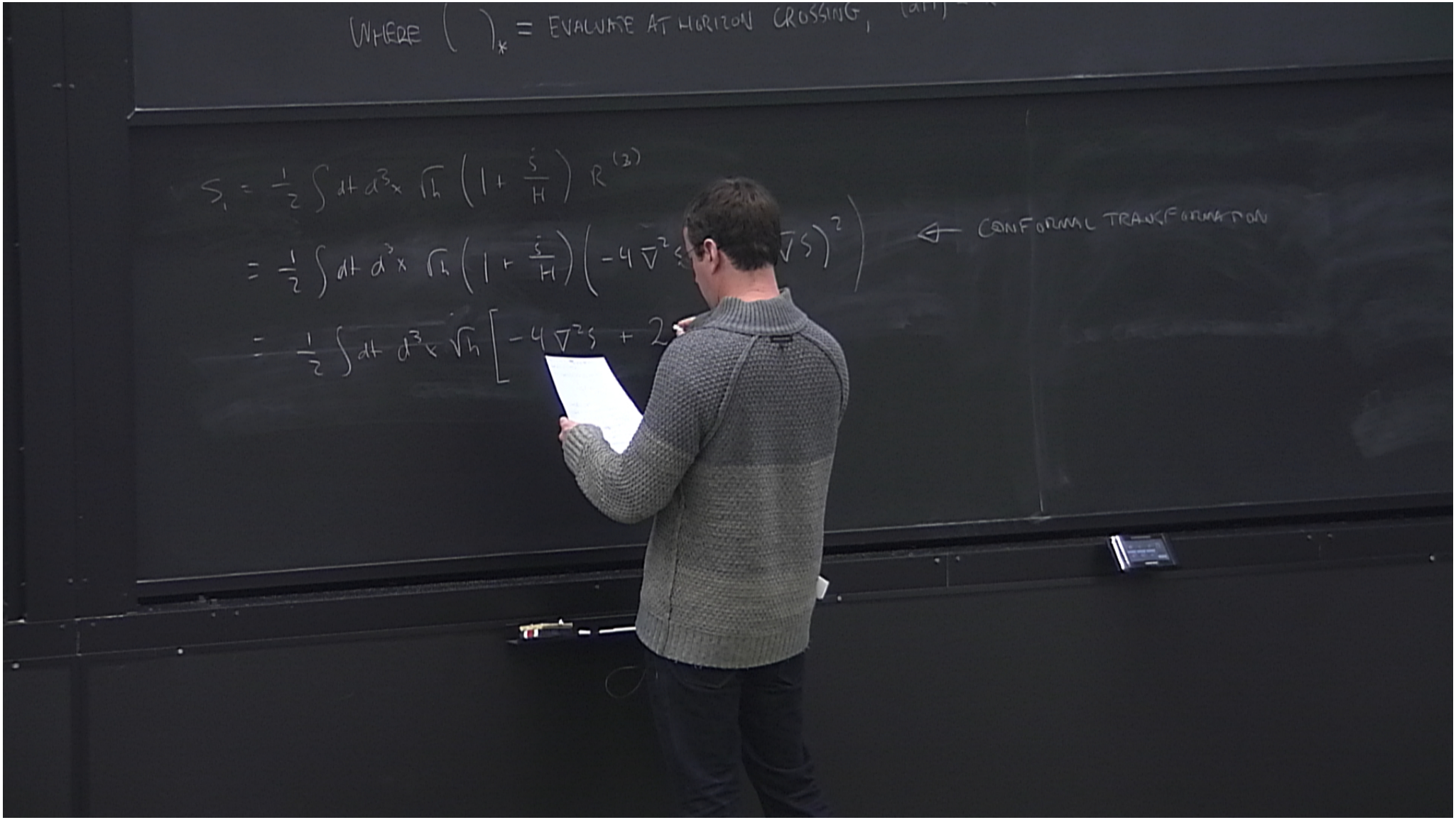

$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-6\dot{\zeta}^2 - 6H\zeta\dot{\zeta} + \frac{2\dot{\phi}}{H}\dot{\zeta}\right]$
$+ \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\nabla_i\nabla_j\zeta + h_{ij}(\nabla^2\zeta)\right]$

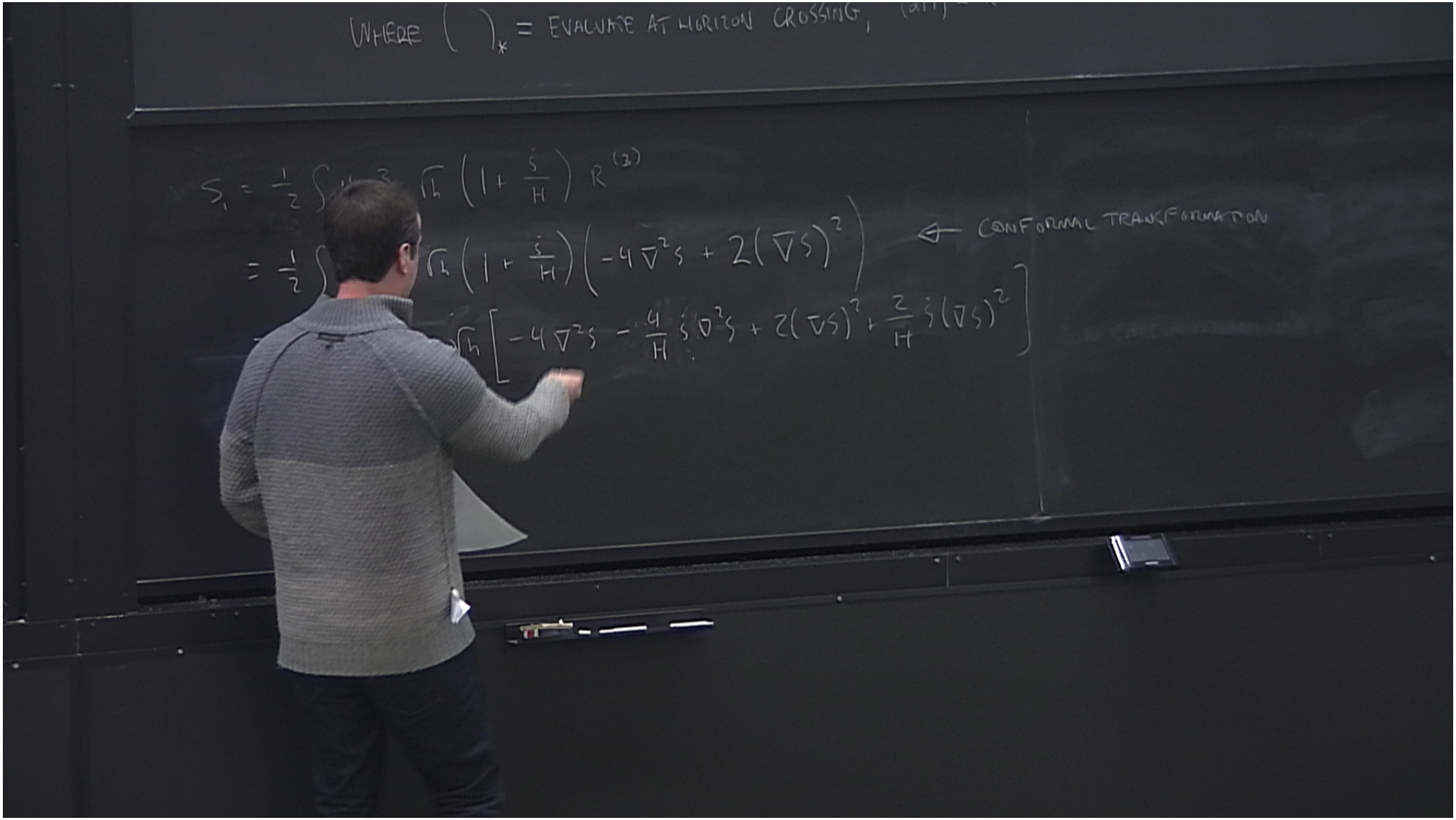
$$+\frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, \frac{}{1+\dot{\zeta}/H}\left[\nabla_i\nabla_j\psi(\nabla^i\psi)-(\nabla\psi)^2\right]$$

$$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-6H\dot{\zeta}^2 - 6H\zeta\dot{\zeta} + \frac{2\dot{\phi}}{H}\dot{\zeta}\right]$$
$$+\frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\nabla_i\nabla_j\zeta + h_{ij}(\nabla^2\zeta)\right](\nabla^i\psi)(\nabla^j\chi)$$
$$+\frac{1}{2}\int dt\, d^3x\, \sqrt{h}\,\frac{\nabla_i\dot{\zeta}}{H}\left[(\nabla^i\nabla^j\psi)(\nabla_j\psi)-(\nabla_i\psi)(\nabla^2\chi)\right]$$

$$\left.\vphantom{\frac{1}{2}}\right)-(\nabla^2\psi)^2\Big]$$


WHERE ( )* = EVALUATE AT HORIZON CROSSING,


WHERE ( )* = EVALUATE AT HORIZON CROSSING,
$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left(1+\frac{\dot{s}}{H}\right) R^{(3)}$
$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left(1+\frac{\dot{s}}{H}\right)\left(-4\nabla^2 s - (\nabla s)^2\right)$
$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-4\nabla^2 s + 2\right.$
← CONFORMAL TRANSFORMATION

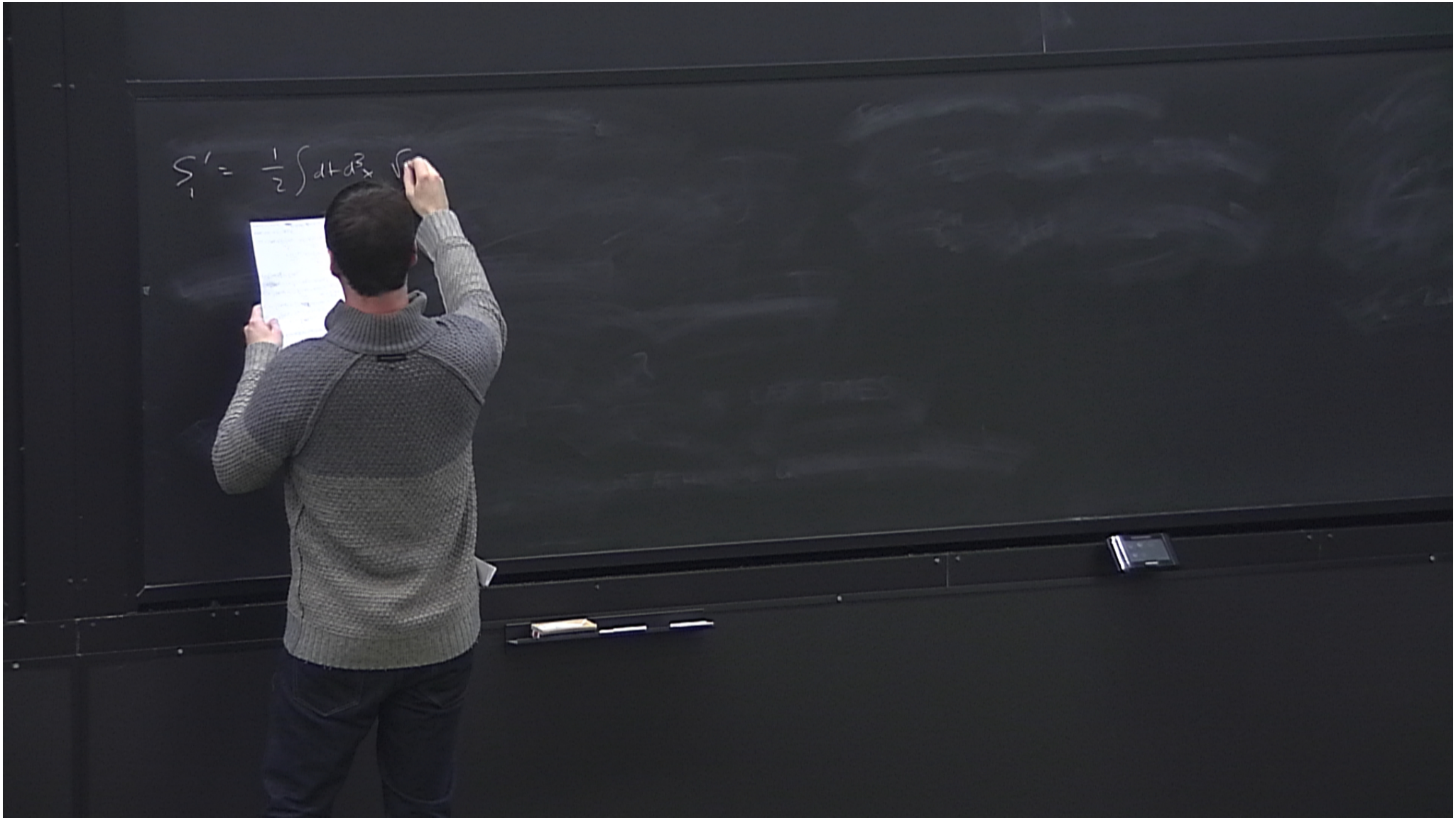

WHERE ( )* = EVALUATE AT HORIZON CROSSING,
$S_1 = \frac{1}{2}\int \sqrt{h}\left(1+\frac{\dot{s}}{H}\right)R^{(3)}$
$= \frac{1}{2}\int \sqrt{h}\left(1+\frac{\dot{s}}{H}\right)\left(-4\nabla^2 s + 2(\nabla s)^2\right)$
⇐ CONFORMAL TRANSFORMATION
$\sqrt{h}\left[-4\nabla^2 s - \frac{4}{H}\dot{s}\nabla^2 s + 2(\nabla s)^2 + \frac{2}{H}\dot{s}(\nabla s)^2\right]$

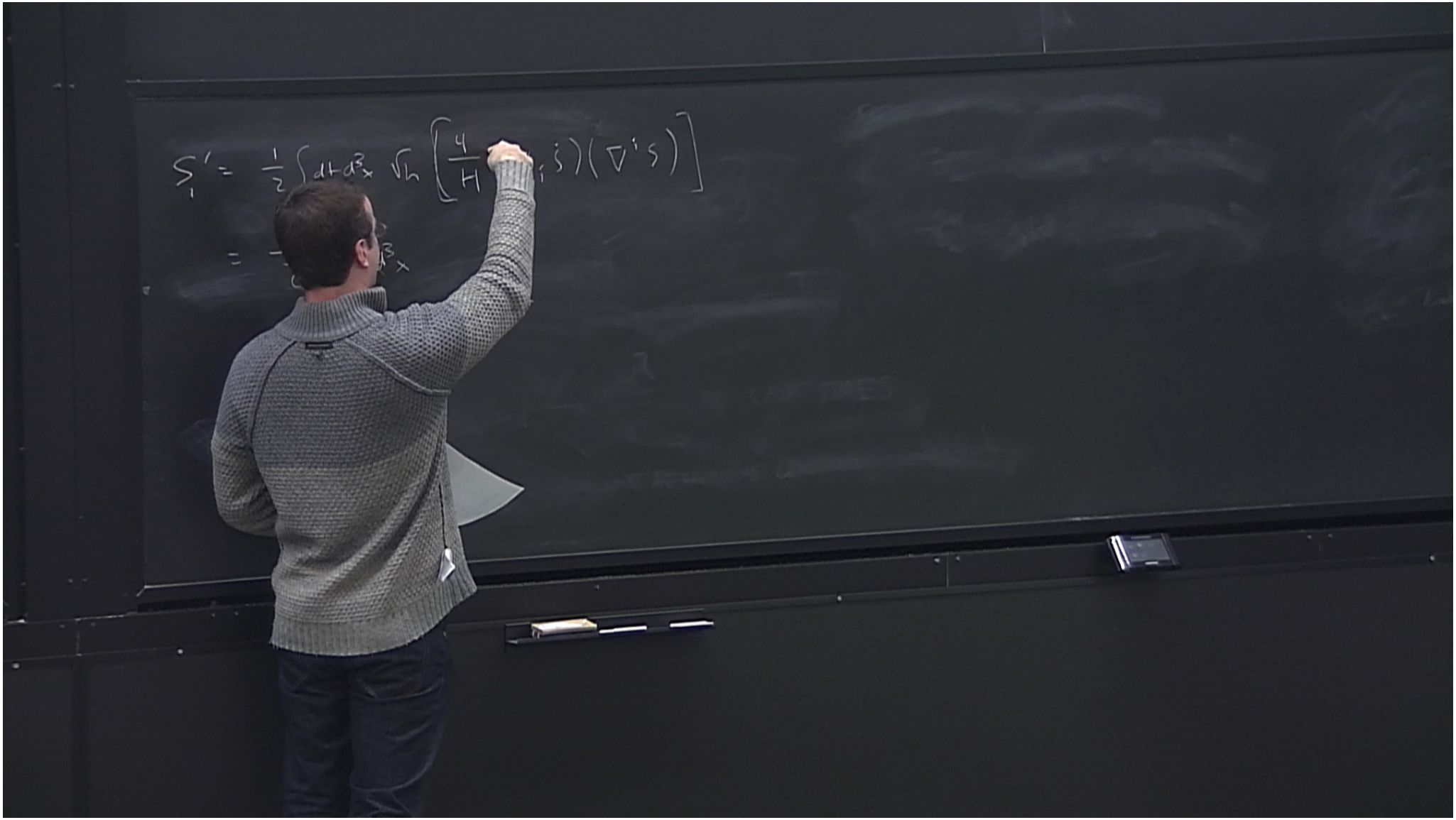
WHERE $(\ )_*$ = EVALUATE AT HORIZON CROSSING, (at)

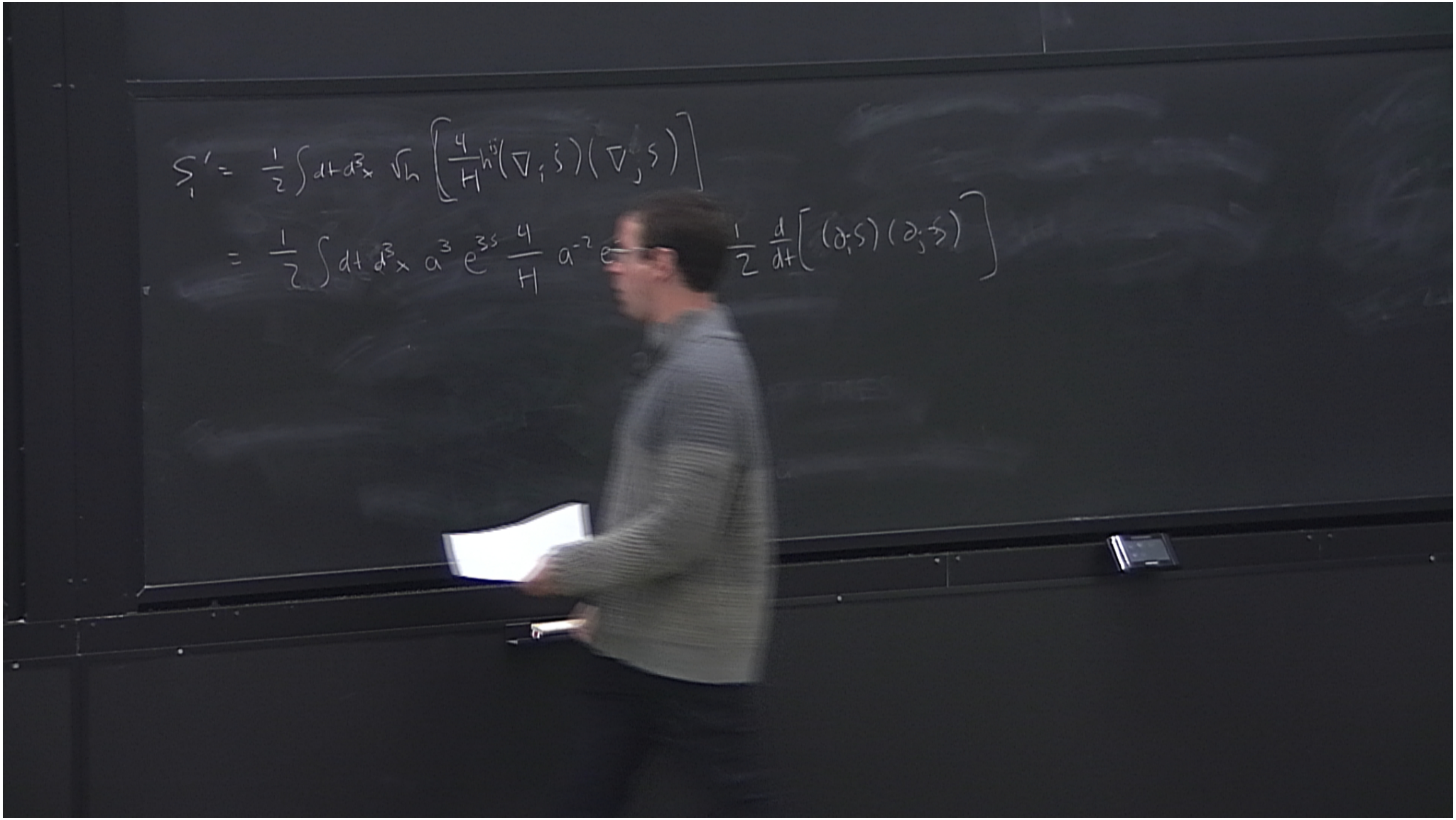
$$S_2 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left(1+\frac{\dot{s}}{H}\right) R^{(3)}$$

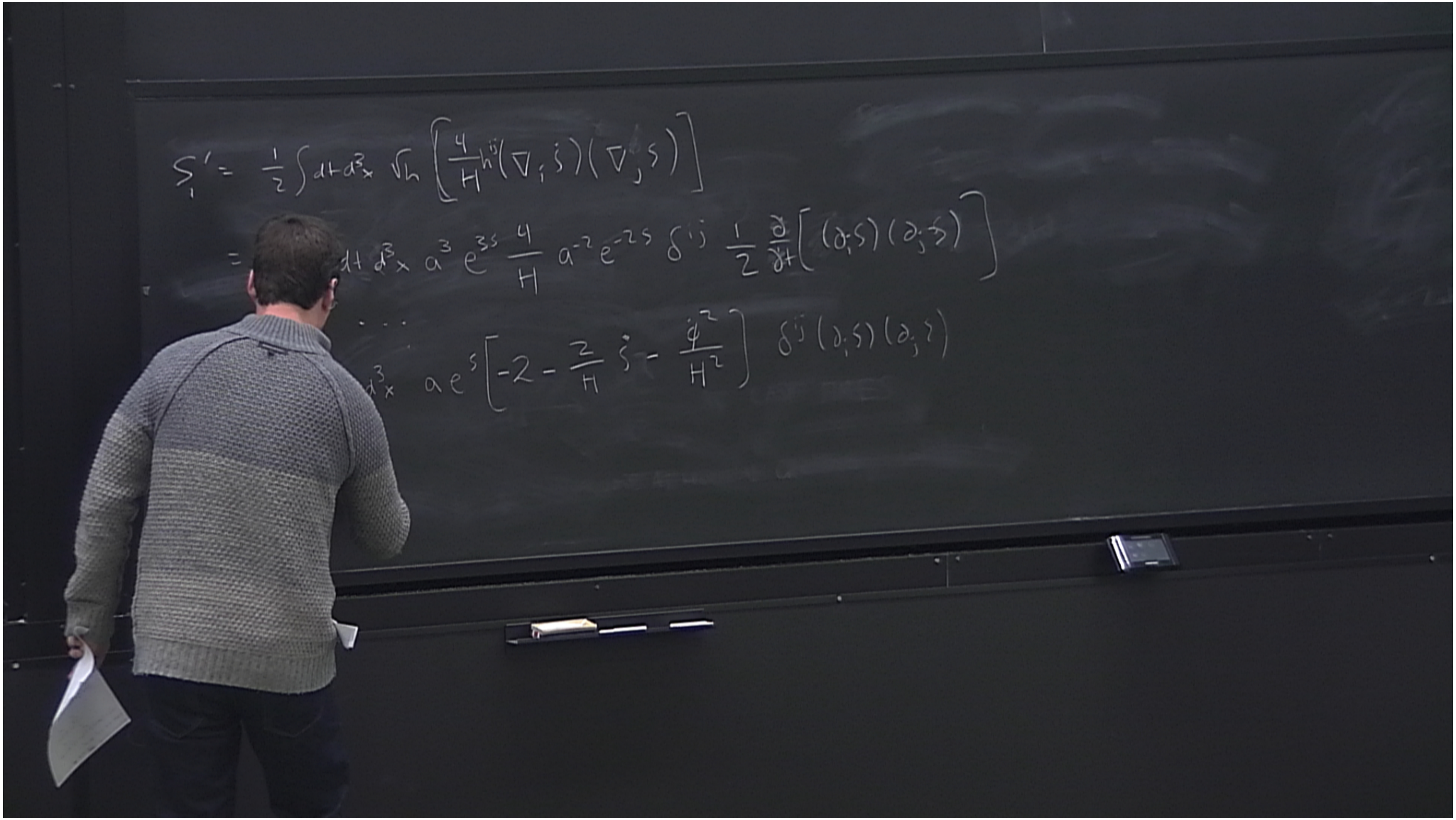
$$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left(1+\frac{\dot{s}}{H}\right)\left(-4\nabla^2 s + 2(\nabla s)^2\right)$$ ← CONFORMAL TRANSFORMATION

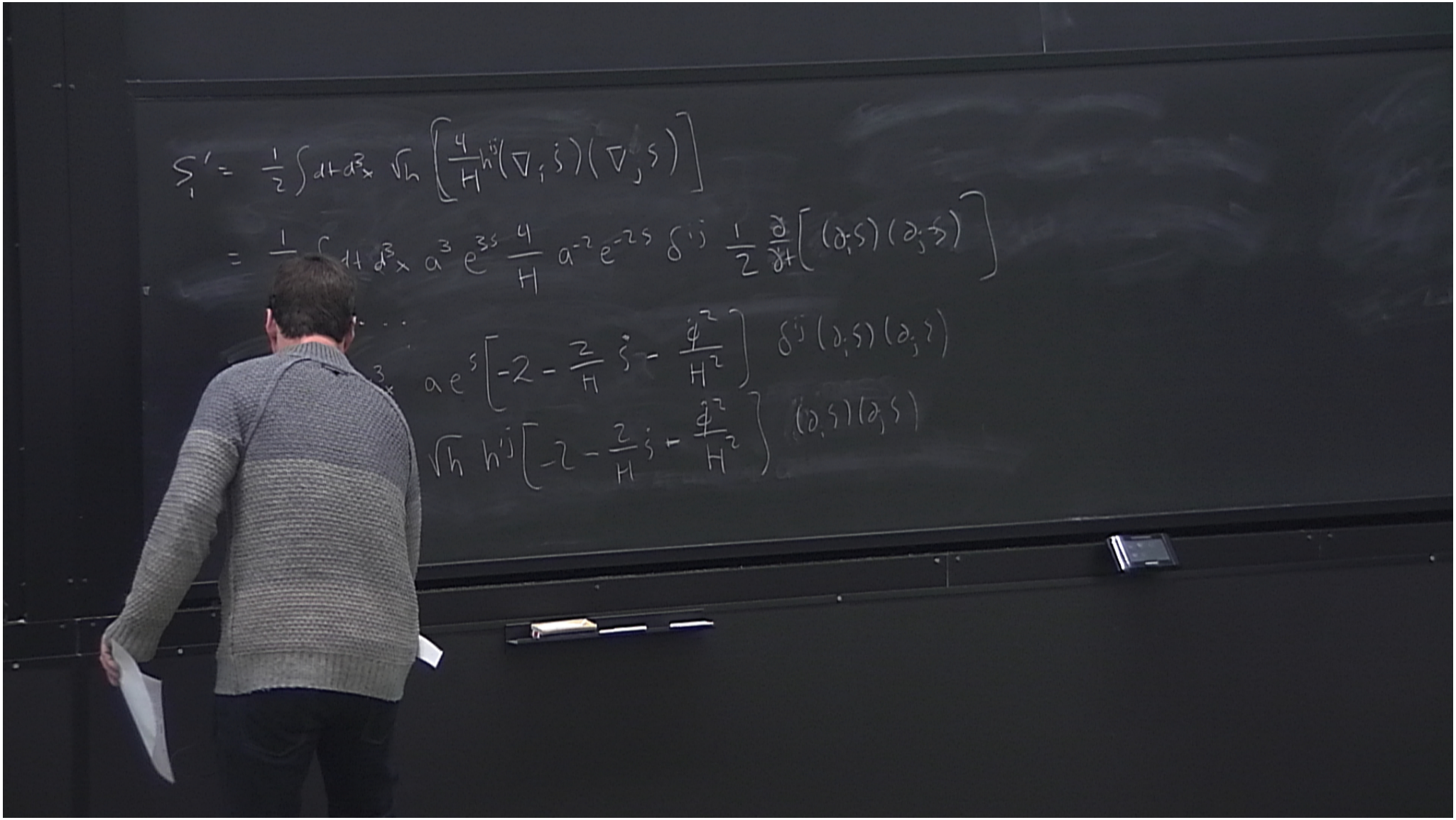
$$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-4\nabla^2 s - \frac{4}{H}\dot{s}\nabla^2 s + 2(\nabla s)^2 + \frac{2}{H}\dot{s}(\nabla s)^2\right]$$

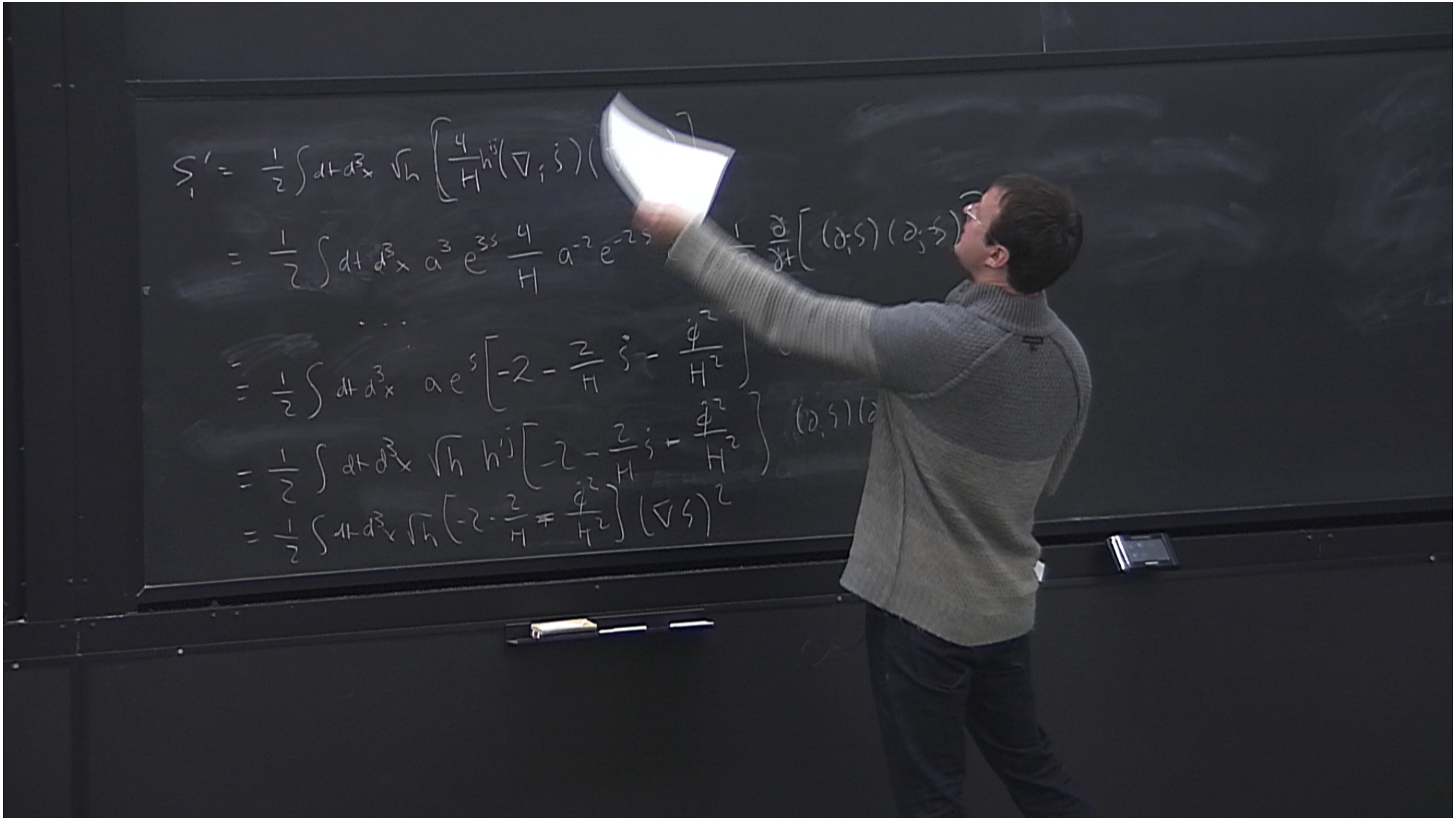

$S_1' = \frac{1}{2}\int dt\, d^3x \sqrt{}$

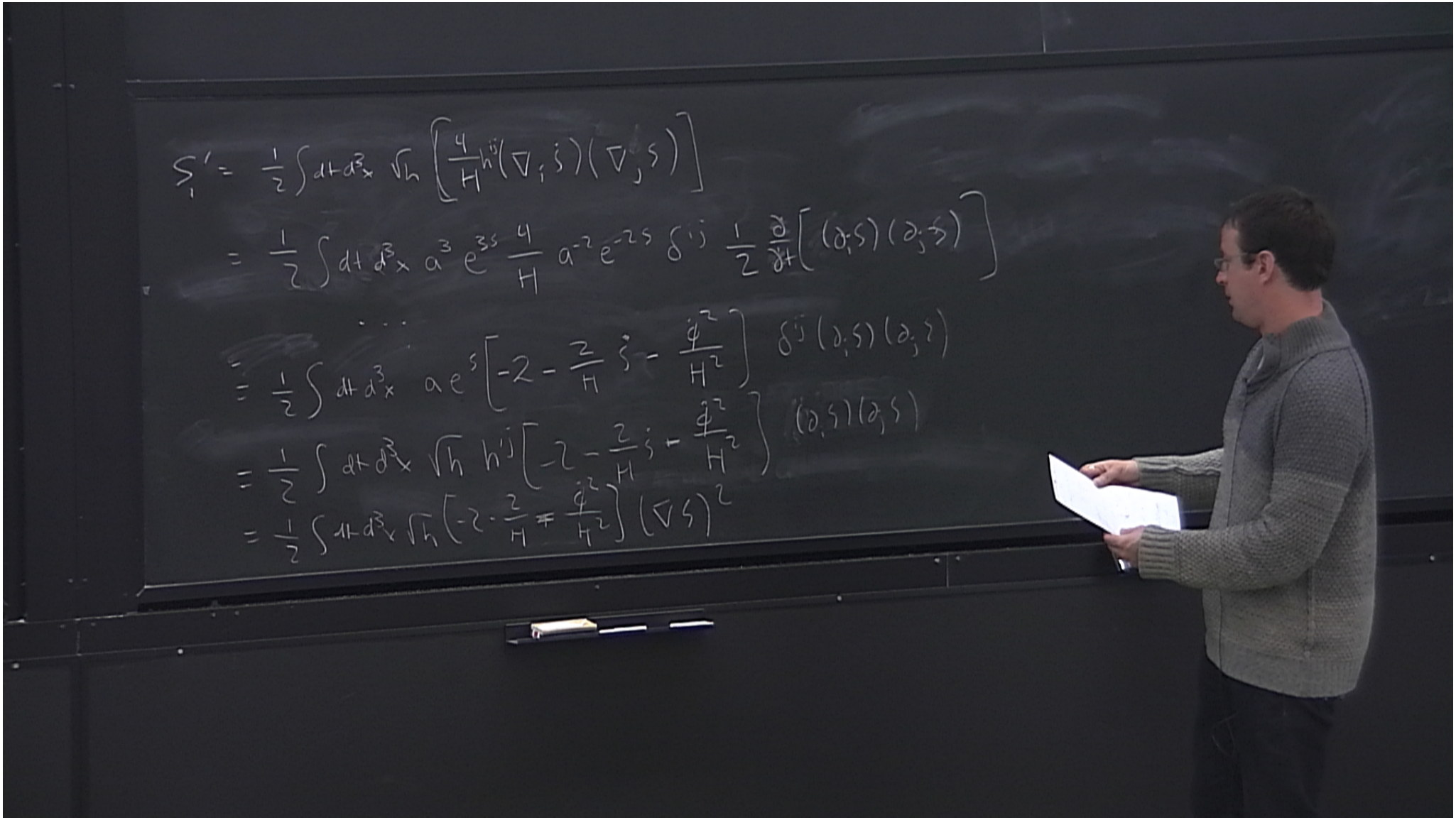

$S_1' = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\left(\frac{4}{H}\right.\right.$
$\left.\left.;i\right)(\nabla^i s)\right]$
$= -$
d^3x

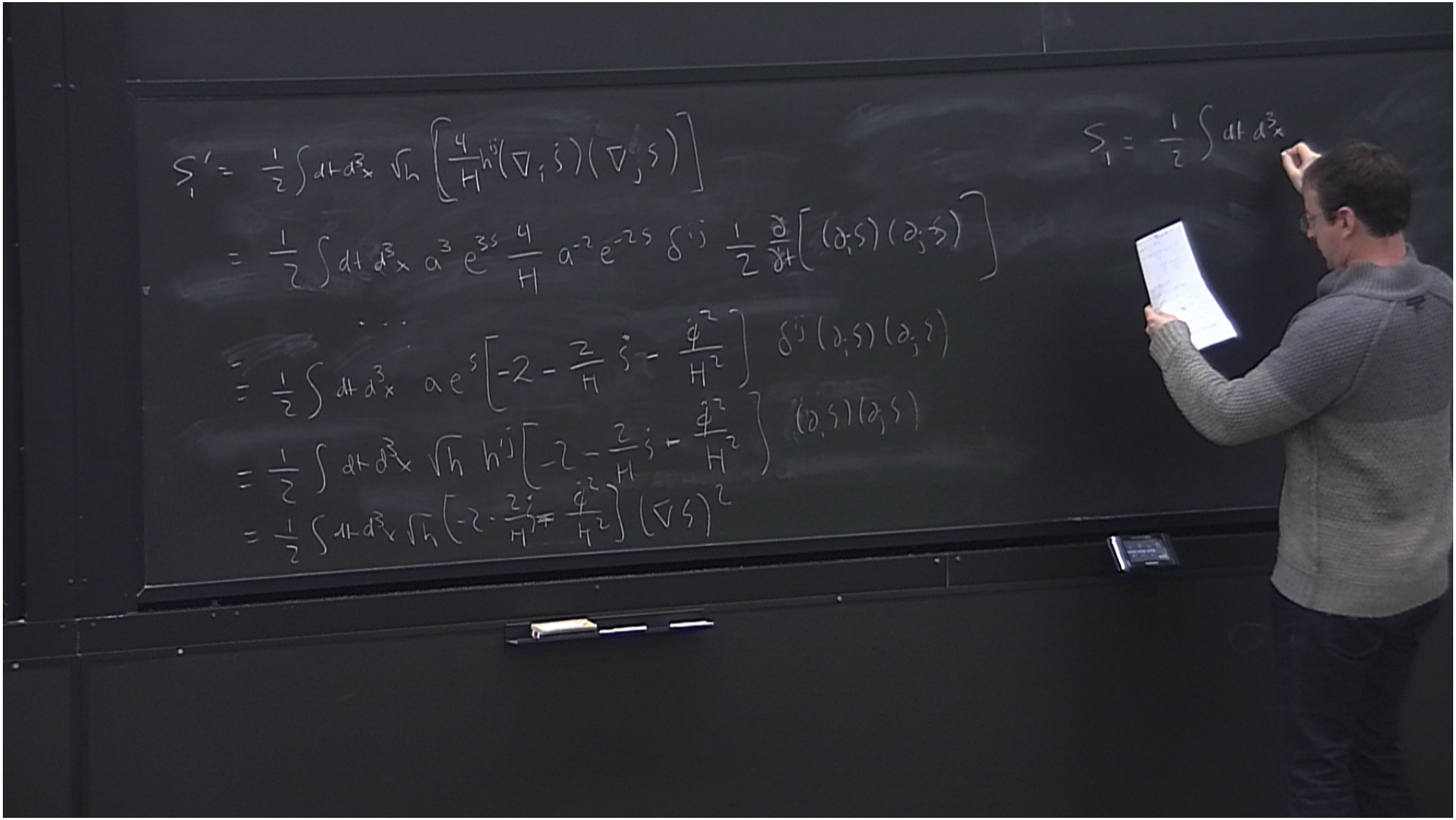

$S_1' = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\frac{4}{H}\dot{h}^{ij}(\nabla_i \zeta)(\nabla_j \zeta)\right]$
$= \frac{1}{2}\int dt\, d^3x\, a^3 e^{3\zeta}\frac{4}{H}a^{-2}e^{-2\zeta}\frac{1}{2}\frac{d}{dt}\left[(\partial_i \zeta)(\partial_j \zeta)\right]$

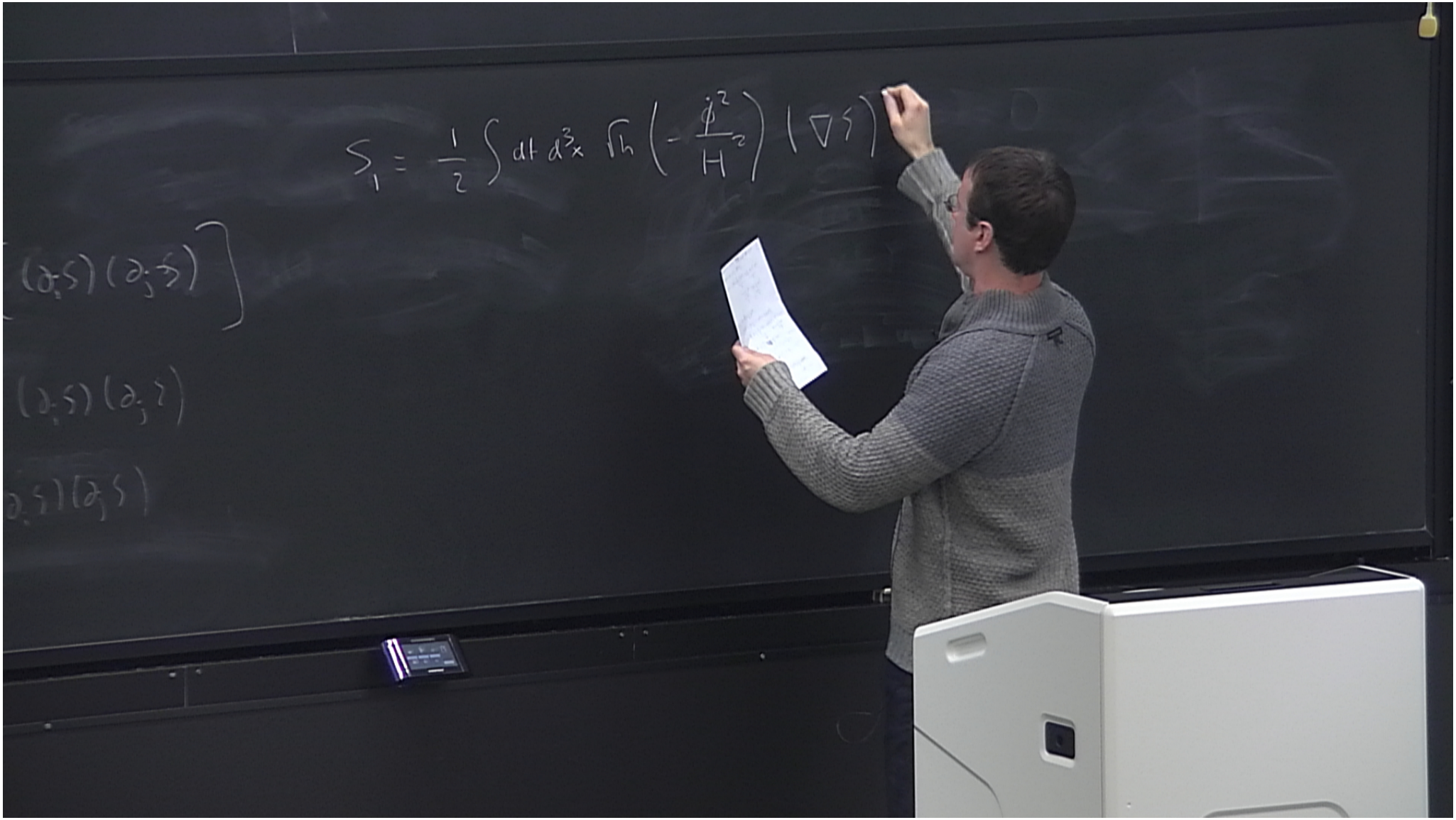

$S_1' = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\frac{4}{H}\dot{h}^{ij}(\nabla_i \zeta)(\nabla_j \zeta)\right]$
$= \int dt\, d^3x\, a^3 e^{3\zeta}\frac{4}{H} a^{-2} e^{-2\zeta}\delta^{ij}\frac{1}{2}\frac{\partial}{\partial t}\left[(\partial_i \zeta)(\partial_j \zeta)\right]$
$\int d^3x\, a e^{\zeta}\left[-2 - \frac{2}{H}\dot{\zeta} - \frac{\dot{\phi}^2}{H^2}\right]\delta^{ij}(\partial_i \zeta)(\partial_j \zeta)$


$S_1' = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\frac{4}{H}\dot{h}^{ij}(\nabla_i s)(\nabla_j s)\right]$
$= \frac{1}{2}\int dt\, d^3x\, a^3 e^{3s}\frac{4}{H}a^{-2}e^{-2s}\delta^{ij}\frac{1}{2}\frac{\partial}{\partial t}\left[(\partial_i s)(\partial_j s)\right]$
$ae^{s}\left[-2-\frac{2}{H}\dot{s}-\frac{\dot{\varphi}^2}{H^2}\right]\delta^{ij}(\partial_i s)(\partial_j s)$
$\sqrt{h}\,h^{ij}\left[-2-\frac{2}{H}\dot{s}-\frac{\dot{\varphi}^2}{H^2}\right](\partial_i s)(\partial_j s)$




$S_1' = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\frac{4}{H}h^{ij}(\nabla_i \dot{\zeta})(\nabla_j \zeta)\right]$
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$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left(-2\cdot\frac{2}{H}\dot{\zeta}-\frac{\dot{\phi}^2}{H^2}\right)(\nabla\zeta)^2$


$S_1' = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[\frac{4}{H}\dot{h}^{ij}(\nabla_i \zeta)(\nabla_j \zeta)\right]$
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$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\, h^{ij}\left[-2 - \frac{2}{H}\dot{\zeta} - \frac{\dot{\zeta}^2}{H^2}\right](\partial_i \zeta)(\partial_j \zeta)$
$= \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[-2 \cdot \frac{2}{H}\dot{\zeta} + \frac{\dot{\zeta}^2}{H^2}\right](\nabla \zeta)^2$
$S_1 = \frac{1}{2}\int dt\, d^3x$


$S_1 = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left(-\frac{\dot{\phi}^2}{H^2}\right)(\nabla\zeta)$
$(\partial_i\zeta)(\partial_j\zeta)\Big]$
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$\partial_i\zeta(\partial_j\zeta)$