Title: Explorations in Cosmology-7

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Abstract:

$$S[N, N^i, h_{ij}, \phi] = \frac{1}{2}\int dt\, d^3x \sqrt{h}\Big[N R^{(3)} + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + \underbrace{N^{-1}\left(\dot\phi - N^i \partial_i \phi\right)^2}_{N^{-1}\dot\phi^2} - \underbrace{N\left(\nabla^i\phi\right)\left(\nabla_i\phi\right)}_{=0} - 2N V(\phi)\Big]$$

$$E_{ij} = \frac{1}{2}\left(\dot h_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

$$E = h^{ij} E_{ij}$$

MALDACENA GAUGE: $\phi(x,t) = \phi_0(t)$

$$h_{ij} = a(t)^2 e^{2\zeta(x,t)}\left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{ik}\delta^{jl}\gamma_{ij}\gamma_{kl}\right]$$

$$\delta^{ij}\gamma_{ij} = 0$$

$$\delta^{jk}\partial_k \gamma_{ij} = 0$$

$$S[N, N^i, h_{ij}, \phi] = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N R^{(3)} + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + \underbrace{N^{-1}\left(\dot\phi - N^i \partial_i \phi\right)^2}_{N^{-1}\dot\phi^2} - \underbrace{N\left(\nabla^i\phi\right)\left(\nabla_i\phi\right)}_{=0} - 2N\,V(\phi)\right]$$

$$E_{ij} = \frac{1}{2}\left(\dot h_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

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MALDACENA GAUGE: $\phi(x,t) = \phi_0(t)$

$$h_{ij} = a(t)^2 e^{2\zeta(x,t)}\left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{ik}\delta^{jl}\gamma_{ij}\gamma_{kl}\right]$$

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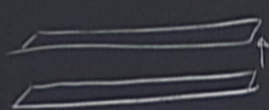

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$$\delta\int d^3x\,(\partial_i\phi)(\partial^i\phi) = -2\,\partial_i\partial^i\phi\,(\delta\phi)$$

$$\delta\int d^3x\sqrt{h}\,(\nabla_i\phi)(\nabla^i\phi) = -2\nabla^i\nabla_i\phi\,(\delta\phi)$$

$$2N\,V(\phi)\Big]$$

$$-2\partial_i\partial^i\phi\ (\delta\phi)$$

$$(\nabla^i\phi)$$

$$-2\nabla^i\nabla_i\phi\ (\delta\phi)$$

REMOVE NONDYNAMICAL DEGREES OF FREEDOM

EQS OF MOTION FOR N, N^i

$$R^{(3)} - N^{-2}\left(E_{ij}E^{ij} - E^2\right) - N^{-2}\dot{\phi}^2 - 2V = 0$$

$$\nabla_j\left[N^{-1}\left(E^{ij} - h^{ij}E\right)\right] = 0$$

SIMPLIFICATION: SUFFICES TO DETERMINE N, N^i TO FIRST ORDER IN ζ, γ

$$\delta S[N, N^i] = \left(\frac{\delta S}{\delta N}\right)[\delta N] + \left(\frac{\delta S}{\delta N^i}\right)[\delta N^i]$$

$$\begin{pmatrix} N \\ N^i \end{pmatrix} = \begin{pmatrix} \cdot & 0 \\ \cdot & 0 \end{pmatrix} \begin{pmatrix} \zeta \\ \gamma \end{pmatrix}$$

$$\delta N = (\#)\zeta + \nabla^i \nabla^j \gamma_{ij}(\vec{k})$$

SIMPLIFICATION #2: AT LINEAR LEVEL

1) CAN ASSUME $\gamma = 0$

2) CAN ASSUME $N^i = \nabla^i \psi$

$$h_{ij} = a^2(1+2\zeta)\delta_{ij} \qquad N = 1+\delta N \qquad N^i = \nabla^i \psi$$

$$\nabla^2\psi = \nabla^i\nabla_i\psi$$

$$(\nabla\psi)^2 = h^{ij}(\nabla_i\psi)(\nabla_j\psi)$$

$$E_{ij} = \frac{1}{2}\left(\dot h_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

$$= \frac{1}{2}\left(2H h_{ij} + 2\dot\zeta h_{ij} - 2\nabla_i\nabla_j\psi\right)$$

$$= H h_{ij} + \dot\zeta h_{ij} - \nabla_i\nabla_j\psi$$

$$E = 3H + 3\dot\zeta - \nabla^2\psi$$

$$\bar{\eta}_{ab} = e^{2\omega(x)} \eta_{ab} \qquad D \text{ SPACETIME DIMS}$$

$$\bar{R} = e^{-2\omega}\left[R - 2(D-1)\eta^{ab}\nabla_a\nabla_b - (D-1)(D-2)\eta^{ab}\nabla_a\omega\, \nabla_b\omega \right]$$

$$= e^{-2\omega}R - 2(D-1)\bar{\eta}^{ab}\bar{\nabla}_a\bar{\nabla}_b + (D-1)(D-2)\bar{\eta}^{ab}(\bar{\nabla}_a\omega)(\bar{\nabla}_b\omega)$$

$$= Hh_{ij} + \dot{\zeta} h_{ij} - \nabla_i \nabla_j \psi$$

$$E = 3H + 3\dot{\zeta} - \nabla^2 \psi$$

SECOND EOM : . . .

$$2H(\nabla^i \delta N) h^{ij} - 2(\nabla^i \dot{\zeta}) h^{ij} = 0$$

$$\boxed{\delta N = \frac{\dot{\zeta}}{H}}$$

FIRST EOM

$$R^{(3)} = e^{-2\zeta}(0) - 4 h^{ij} \nabla_i \nabla_j \zeta + O(\zeta^2)$$

$$= -4 \nabla^2 \zeta$$

$$h_{ij} = a^2 \delta_{ij}$$

$$\bar{h}_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

$$0 = \left[-2V + 6H^2 - \dot{\phi}^2\right] - 4\nabla^2 \zeta - 4\nabla^2 \psi + 2\frac{\dot{\phi}^2}{H^2}\dot{\zeta}$$

$$H^2 = \frac{\rho}{3} = \frac{1}{3}\left(\frac{1}{2}\dot{\phi} + V\right)$$

$$\Rightarrow \psi = -\frac{\zeta}{H} + \frac{\dot{\phi}^2}{2H^2}\nabla^{-2}\dot{\zeta}$$

$$S[\zeta,\gamma] = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[\left(1+\frac{\dot{\zeta}}{H}\right) R^{(3)} + \frac{1}{1+\dot{\zeta}/H}\left(E_{ij}E^{ij} - E^2\right) + \frac{1}{1+\dot{\zeta}/H}\dot{\varphi}^2 - 2\left(1+\frac{\dot{\zeta}}{H}\right) V\right]$$

$$S[\zeta,\gamma] = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[\left(1+\frac{\dot\zeta}{H}\right)R^{(3)} + \frac{1}{1+\dot\zeta/H}\left(E_{ij}E^{ij} - E^2\right) + \frac{1}{1+\dot\zeta/H}\dot\phi^2 - 2\left(1+\frac{\dot\zeta}{H}\right)V\right]$$

$$h_{ij} = a^2 e^{2\zeta}\left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl}\right]$$

$$E_{ij} = \frac{1}{2}\dot h_{ij} - \nabla^2\psi$$

$$\psi = -\frac{\zeta}{H} + \frac{\dot\phi^2}{2H^2}\nabla^{-2}\dot\zeta$$

$$\delta S = [\zeta\zeta] + \cancel{[\zeta\gamma]} + [\gamma\gamma] + [\zeta\zeta\zeta]$$

AT QUADRATIC ORDER

$$S[\zeta,\gamma] = \frac{1}{2}\int dt\, \frac{d^3k}{(2\pi)^3}\left[Q_\zeta(k,t)\, \zeta(k)^*\, \zeta(k) + Q_\gamma(k,t)\, \delta^{ik}\delta^{jl}\, \gamma^*_{ij}(\vec{k})\, \gamma^*_{kl}(\vec{k}) \right]$$

CALCULATE S SETTING $\zeta = 0$ TO SECOND ORDER IN γ $\leftarrow$

SETTING $\gamma = 0$ TO THIRD ORDER IN ζ

$$= H h_{ij} + \zeta h_{ij} - \nabla_i \nabla_j \psi$$

$$E = 3H + \dot{\zeta} - \nabla^2 \psi$$

$$Q_e(k,t) = -\frac{a k^2}{4}$$

$$\frac{1}{2}\int dt\, d^3x \sqrt{h}\, R^{(3)} = \frac{1}{2}\int dt \frac{d^3k}{(2\pi)^3}\left(-\frac{a k^2}{4}\right)\left[\delta^{ik}\delta^{jl}\right]\gamma^*_{ij}(k)\gamma_{kl}(k)$$

$$= \frac{-1}{8}\int dt\, d^3x \sqrt{h}\,(\nabla_k \gamma_{ij})(\nabla^k \gamma^{ij})$$