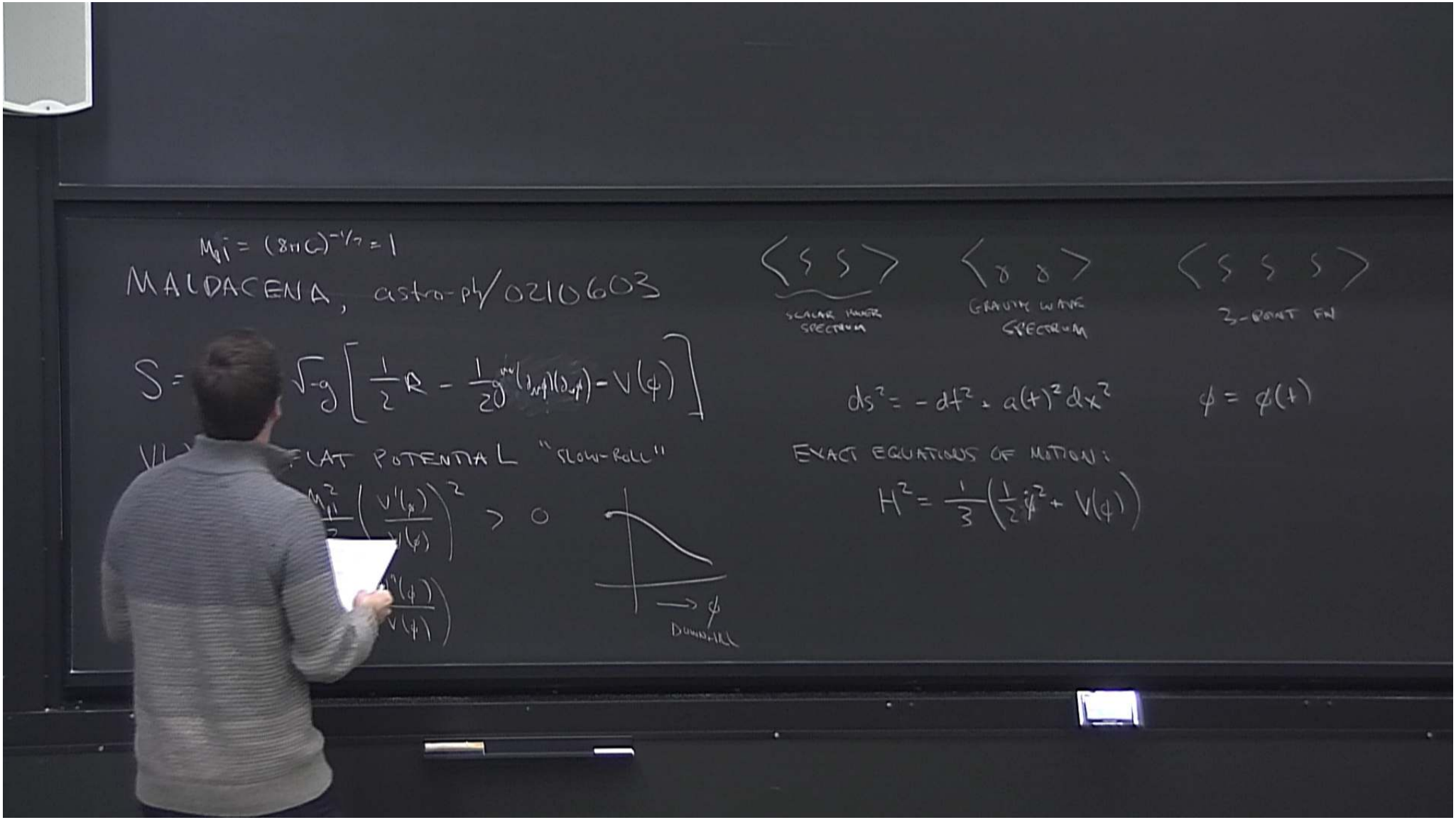
Title: Explorations in Cosmology-6

Date: Apr 13, 2015 10:15 AM

URL: http://pirsa.org/15040032

Abstract:


$M_{Pl} = (8\pi G)^{-1/2} = 1$
MALDACENA, astro-ph/0210603
$\langle \zeta\,\zeta \rangle$
SCALAR POWER SPECTRUM
$\langle \gamma\,\gamma \rangle$
GRAVITY WAVE SPECTRUM
$\langle \zeta\,\zeta\,\zeta \rangle$
3-POINT FN
$S = \int \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right]$
$ds^2 = -dt^2 + a(t)^2 dx^2$
$\phi = \phi(t)$
FLAT POTENTIAL "SLOW-ROLL"
EXACT EQUATIONS OF MOTION:
$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$
ϕ
RUNNING

$M_{Pl} = (8\pi G)^{-1/2} = 1$ $\rho = \log a$ $\dot{\rho} = H$

MALDACENA, astro-ph/0210603

$\langle \zeta\,\zeta \rangle$ SCALAR POWER SPECTRUM

$\langle \gamma\,\gamma \rangle$ GRAVITY WAVE SPECTRUM

$\langle \zeta\,\zeta\,\zeta \rangle$ 3-POINT FN

SLO...

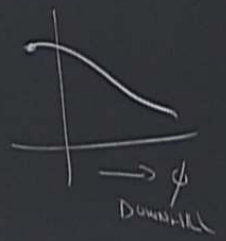
$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right]$$

$ds^2 = -dt^2 + a(t)^2 dx^2$ $\quad \phi = \phi(t)$

$V(\phi)$ IS A FLAT POTENTIAL "SLOW-ROLL"

$$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 > 0$$

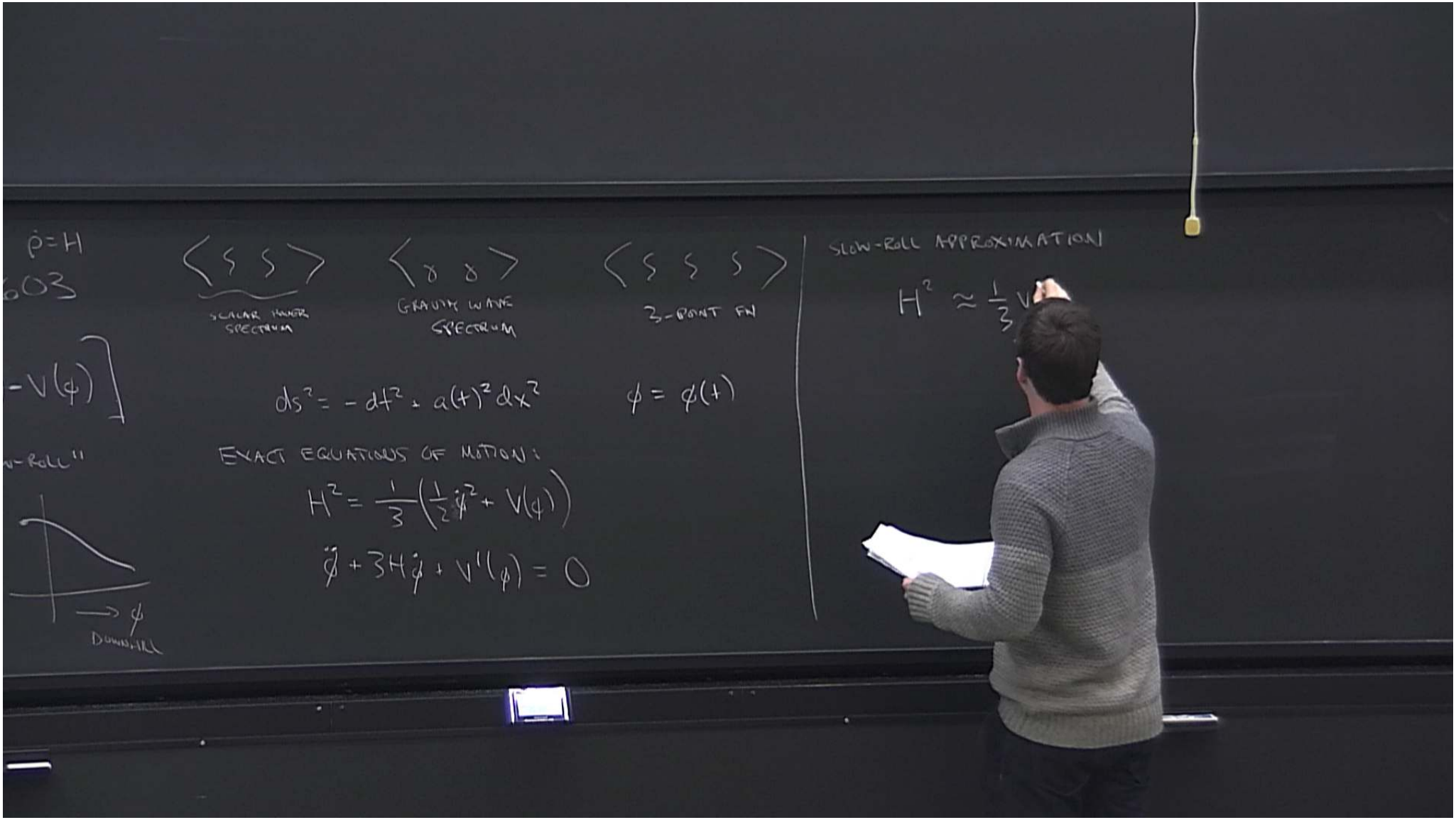
$$\eta = M_{Pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right)$$



EXACT EQUATIONS OF MOTION:

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$


$\dot{\rho}=H$
$\langle \zeta\,\zeta \rangle$
SCALAR POWER SPECTRUM
$\langle \gamma\,\gamma \rangle$
GRAVITY WAVE SPECTRUM
$\langle \zeta\,\zeta\,\zeta \rangle$
3-POINT FN
SLOW-ROLL APPROXIMATION
$H^2 \approx \frac{1}{3}V$
$ds^2 = -dt^2 + a(t)^2 dx^2$
$\phi = \phi(t)$
EXACT EQUATIONS OF MOTION:
$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
ϕ
DOWNHILL

$\dot{\rho} = H$

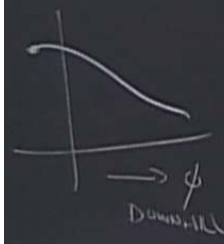
$\langle \zeta \zeta \rangle$ SCALAR POWER SPECTRUM

$\langle \gamma \gamma \rangle$ GRAVITY WAVE SPECTRUM

$\langle \zeta \zeta \zeta \rangle$ 3-POINT FN

$-V(\phi)]$

"SLOW-ROLL"



$ds^2 = -dt^2 + a(t)^2 dx^2$ $\quad \phi = \phi(t)$

EXACT EQUATIONS OF MOTION:

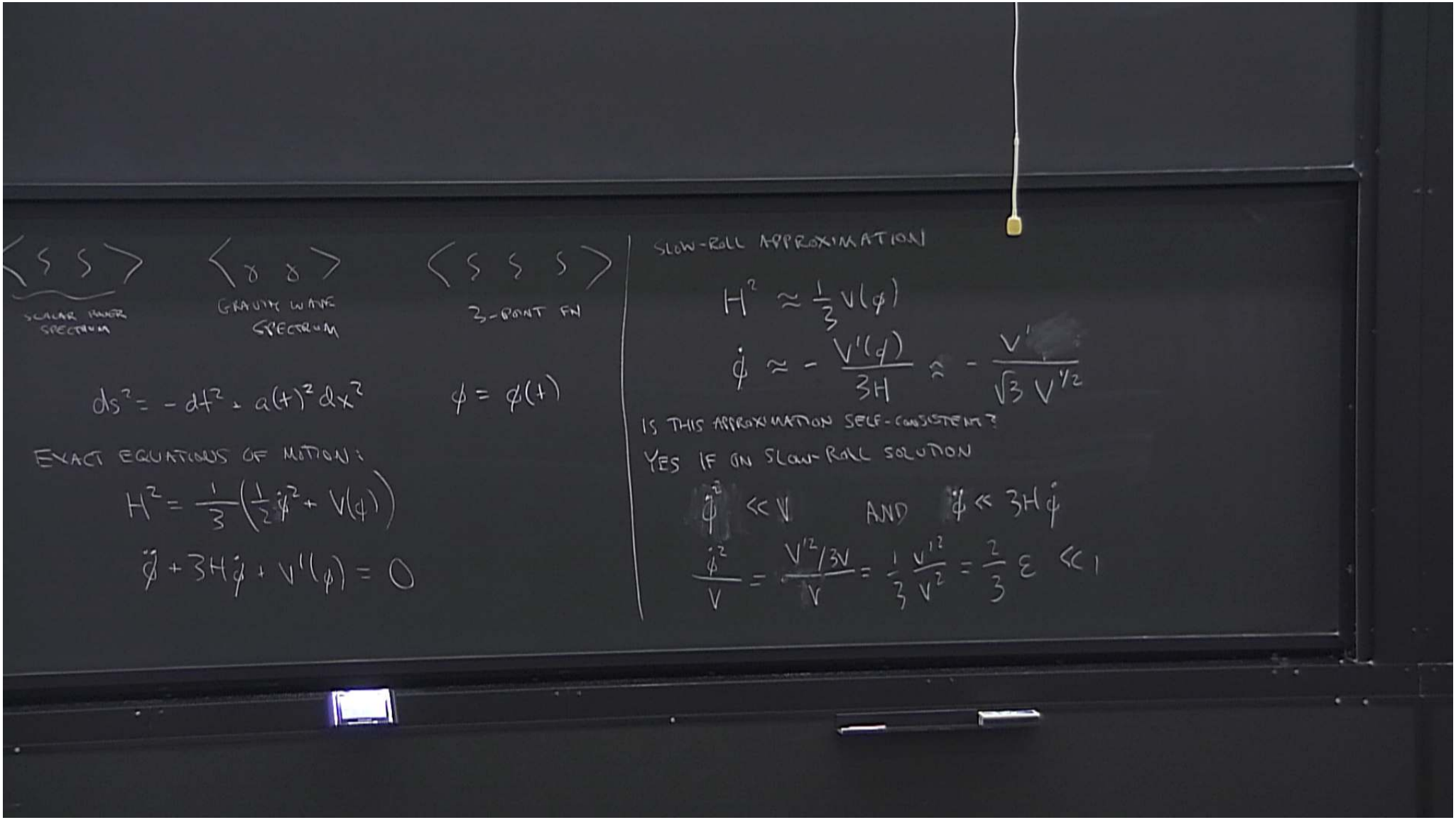
$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

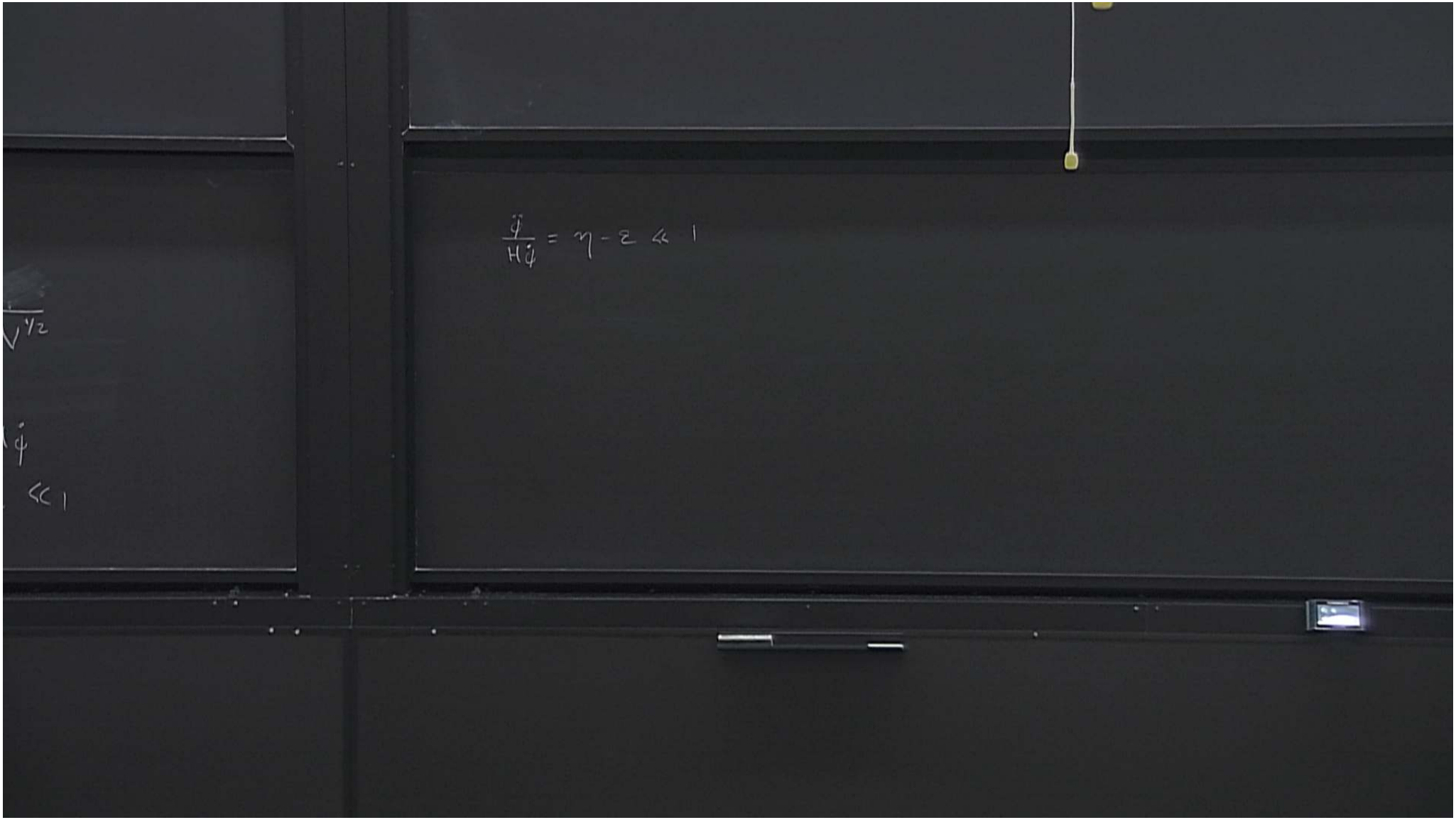
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

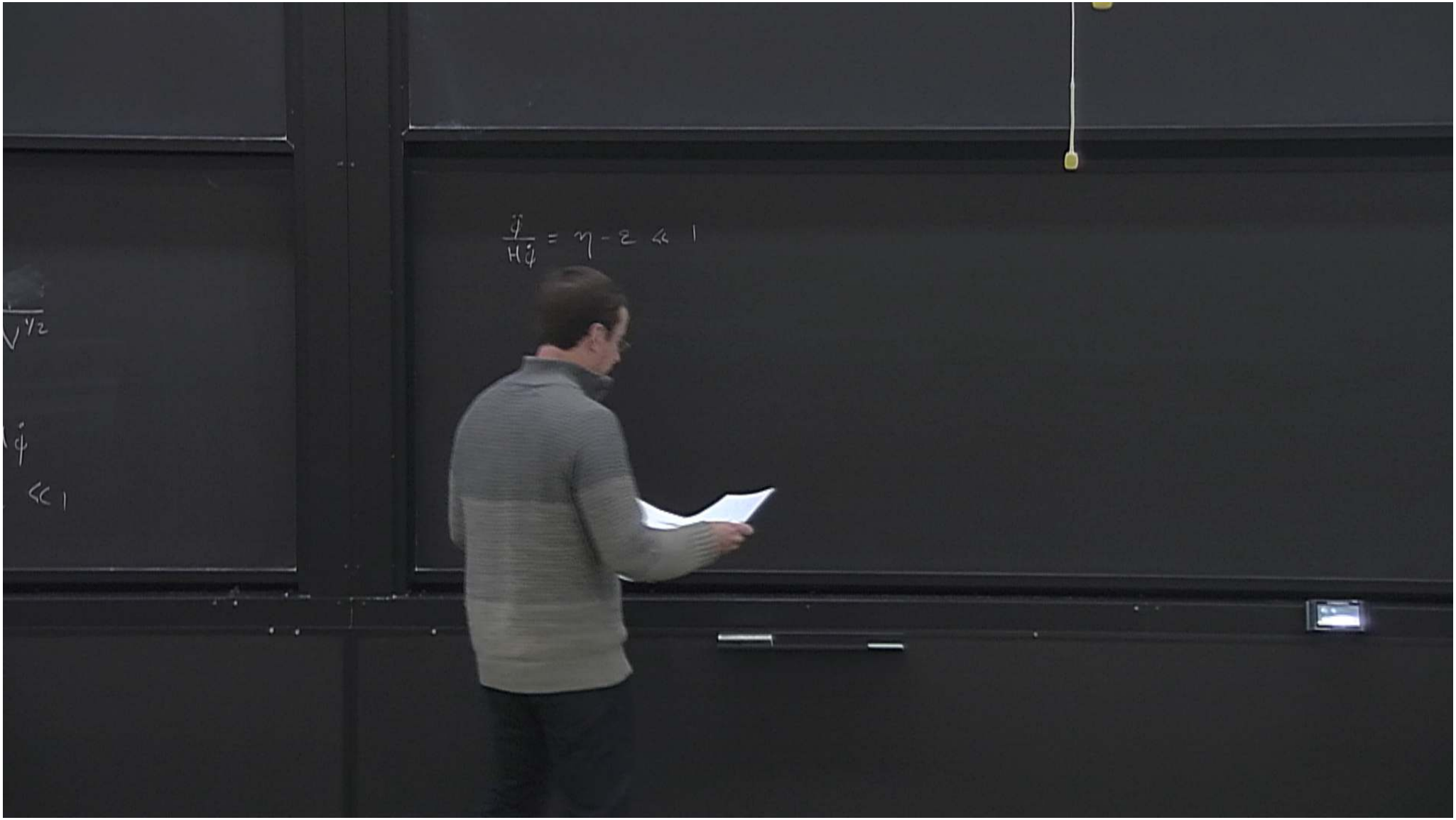
SLOW-ROLL APPROXIMATION

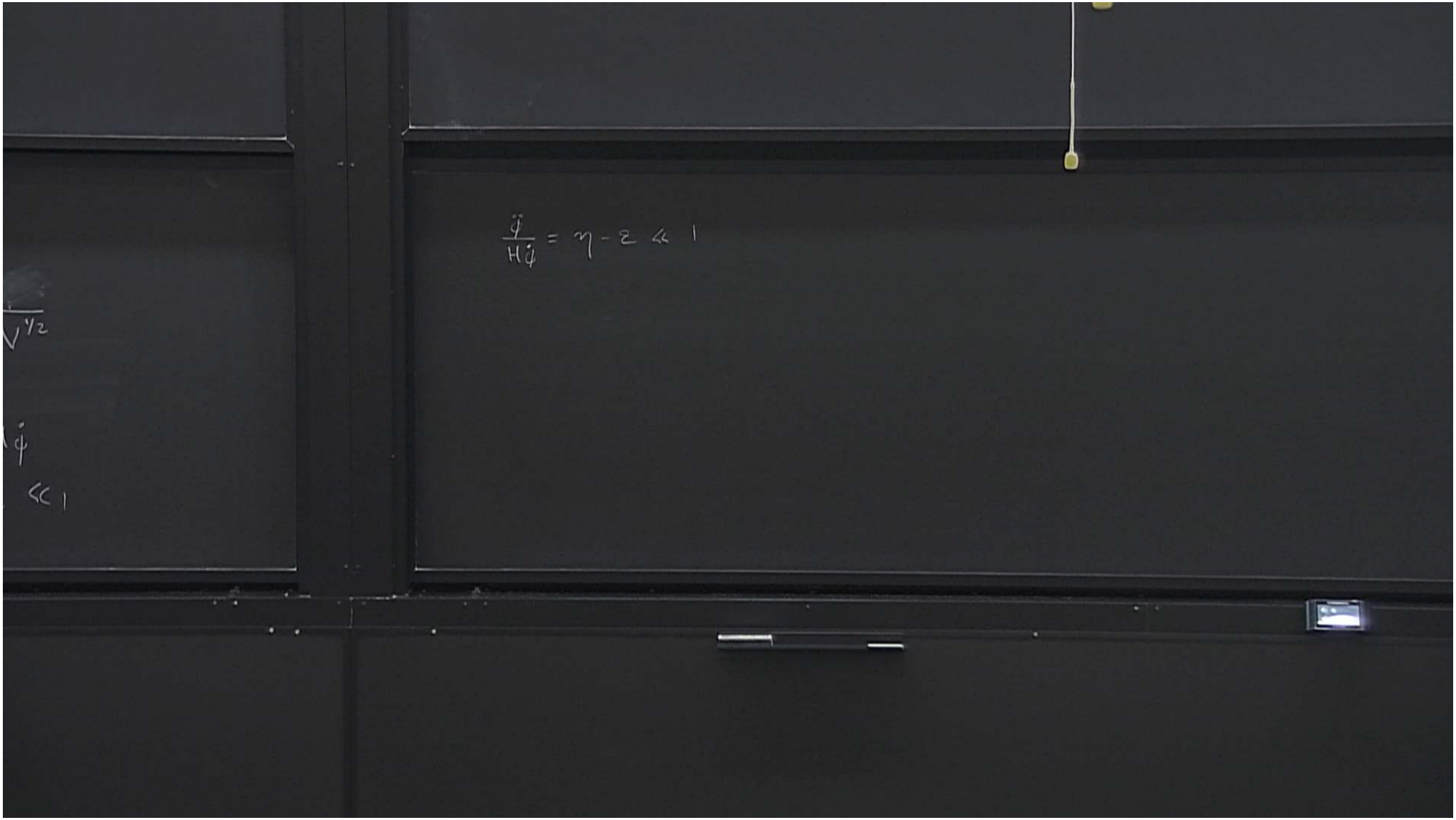
$$H^2 \approx \frac{1}{3}V(\phi)$$

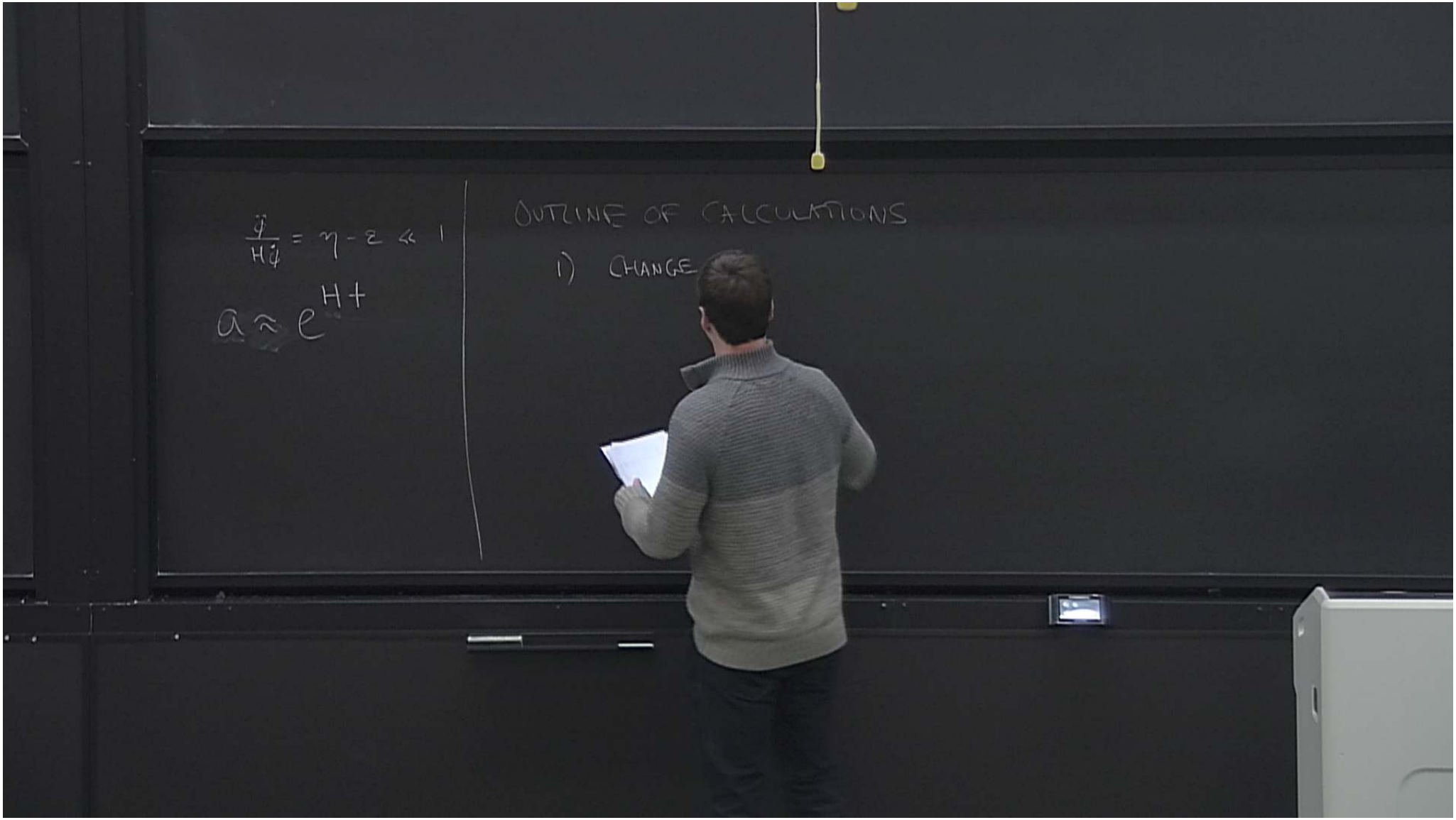
$$\dot{\phi} \approx -\frac{V'(\phi)}{3H} \approx -\frac{V'(\phi)}{}$$

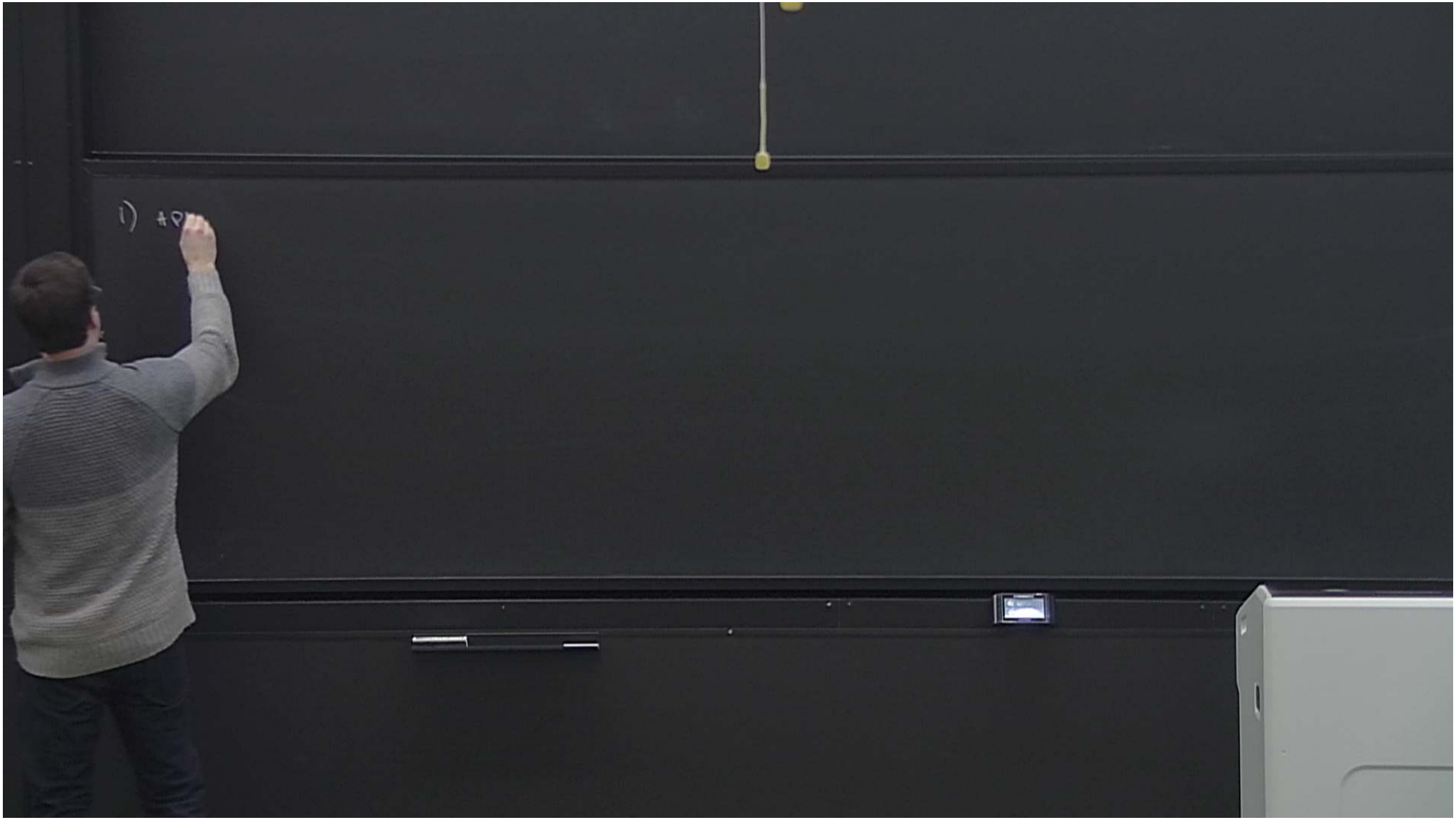

$\langle \zeta\,\zeta \rangle$
SCALAR POWER SPECTRUM
$\langle \gamma\,\gamma \rangle$
GRAVITY WAVE SPECTRUM
$\langle \zeta\,\zeta\,\zeta \rangle$
3-POINT FN
$ds^2 = -dt^2 + a(t)^2 dx^2$
$\phi = \phi(t)$
EXACT EQUATIONS OF MOTION:
$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
SLOW-ROLL APPROXIMATION
$H^2 \approx \frac{1}{3}V(\phi)$
$\dot{\phi} \approx -\frac{V'(\phi)}{3H} \approx -\frac{V'}{\sqrt{3}V^{1/2}}$
IS THIS APPROXIMATION SELF-CONSISTENT?
YES IF ON SLOW-ROLL SOLUTION
$\dot{\phi}^2 \ll V$ AND $\ddot{\phi} \ll 3H\dot{\phi}$
$\frac{\dot{\phi}^2}{V} = \frac{V'^2/3V}{V} = \frac{1}{3}\frac{V'^2}{V^2} = \frac{2}{3}\varepsilon \ll 1$

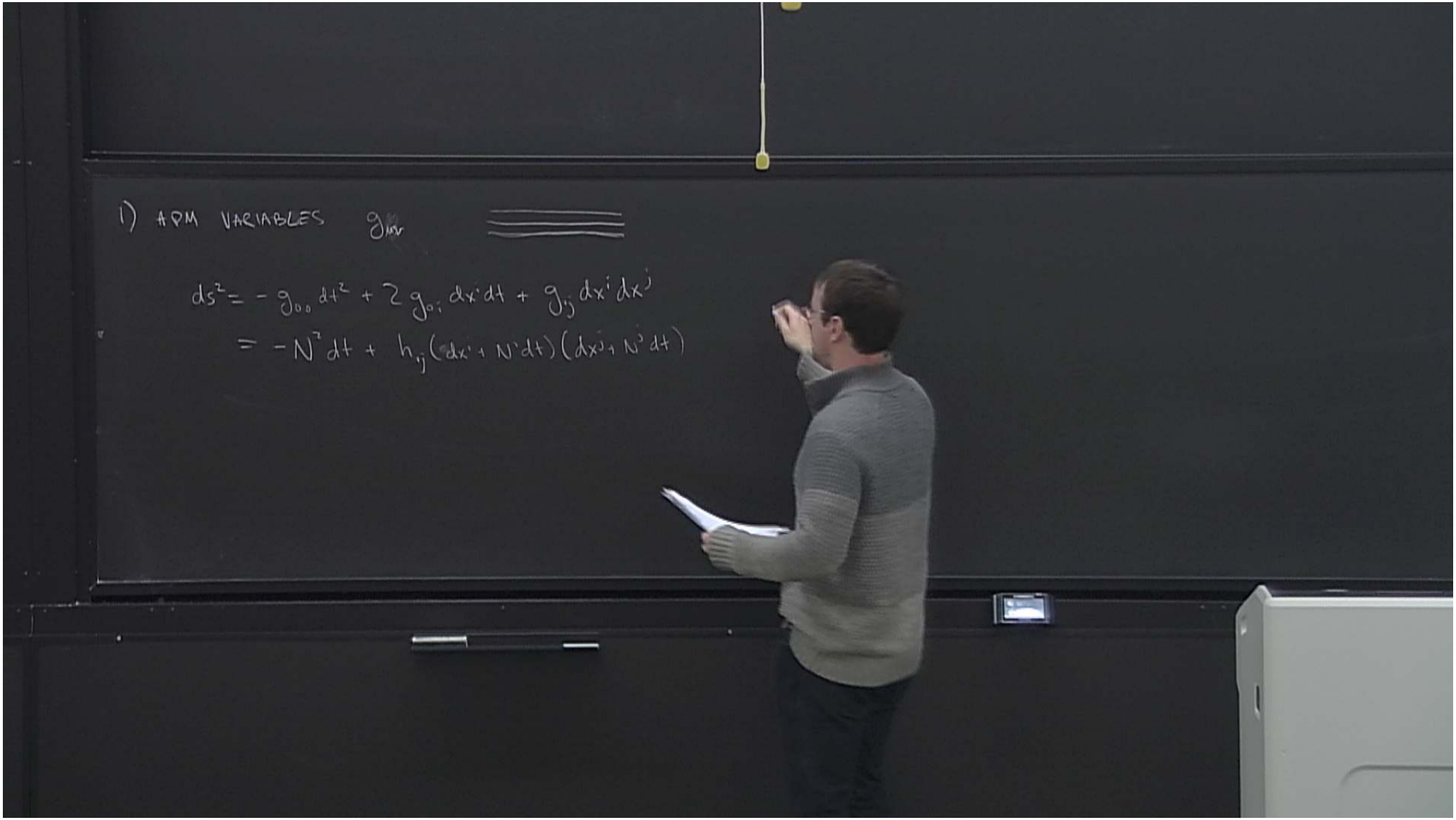

$\frac{\ddot{\varphi}}{H\dot{\varphi}} = \eta - \varepsilon \ll 1$

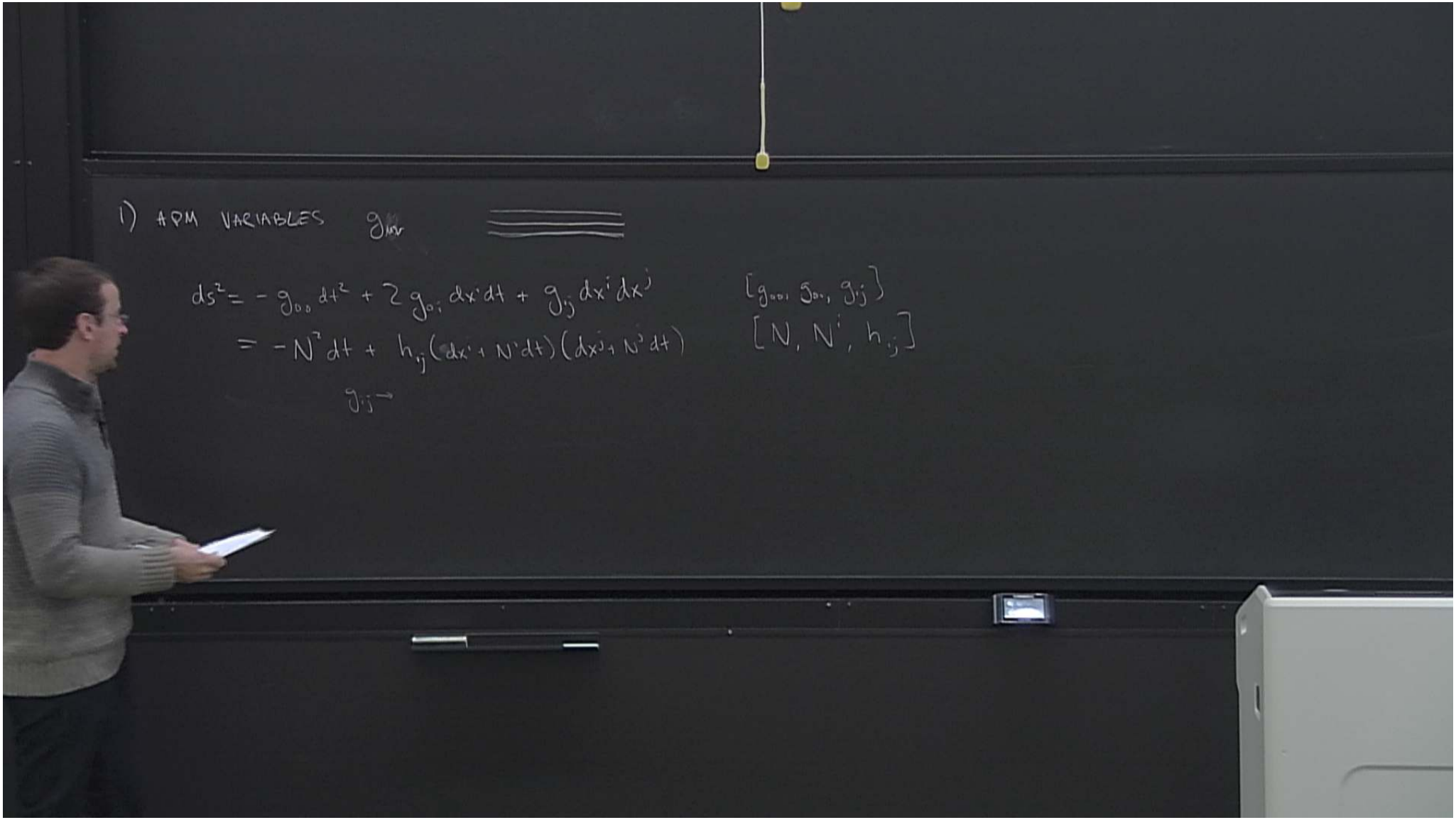

$\frac{\dot{\varphi}}{H\dot{\varphi}} = \eta - 2 \ll 1$
$\ll 1$

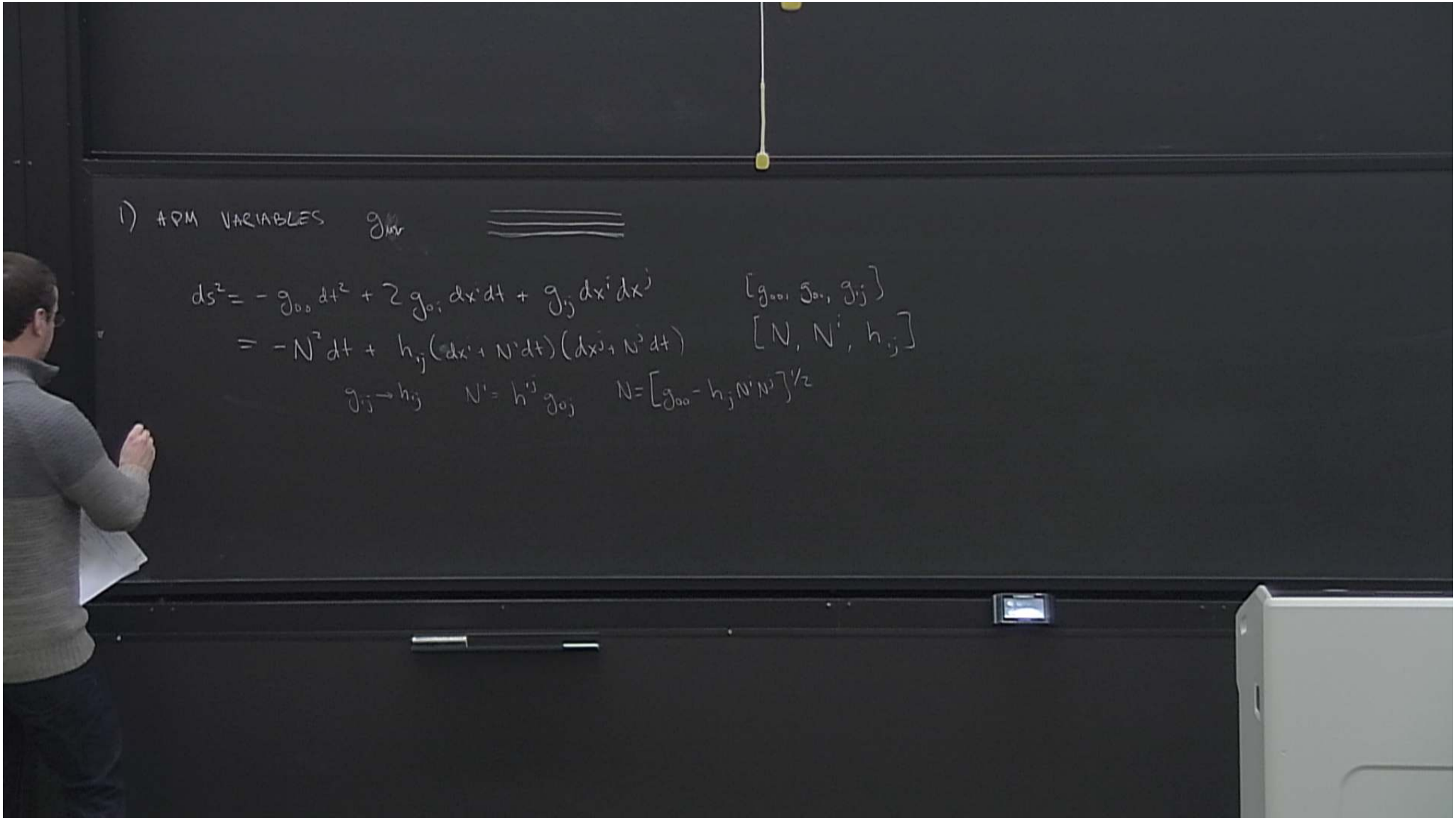

$\frac{\ddot{\varphi}}{H\dot{\varphi}} = \eta - 2 \ll 1$
$\sqrt{}^{1/2}$
$\dot{\varphi}$
$\ll 1$

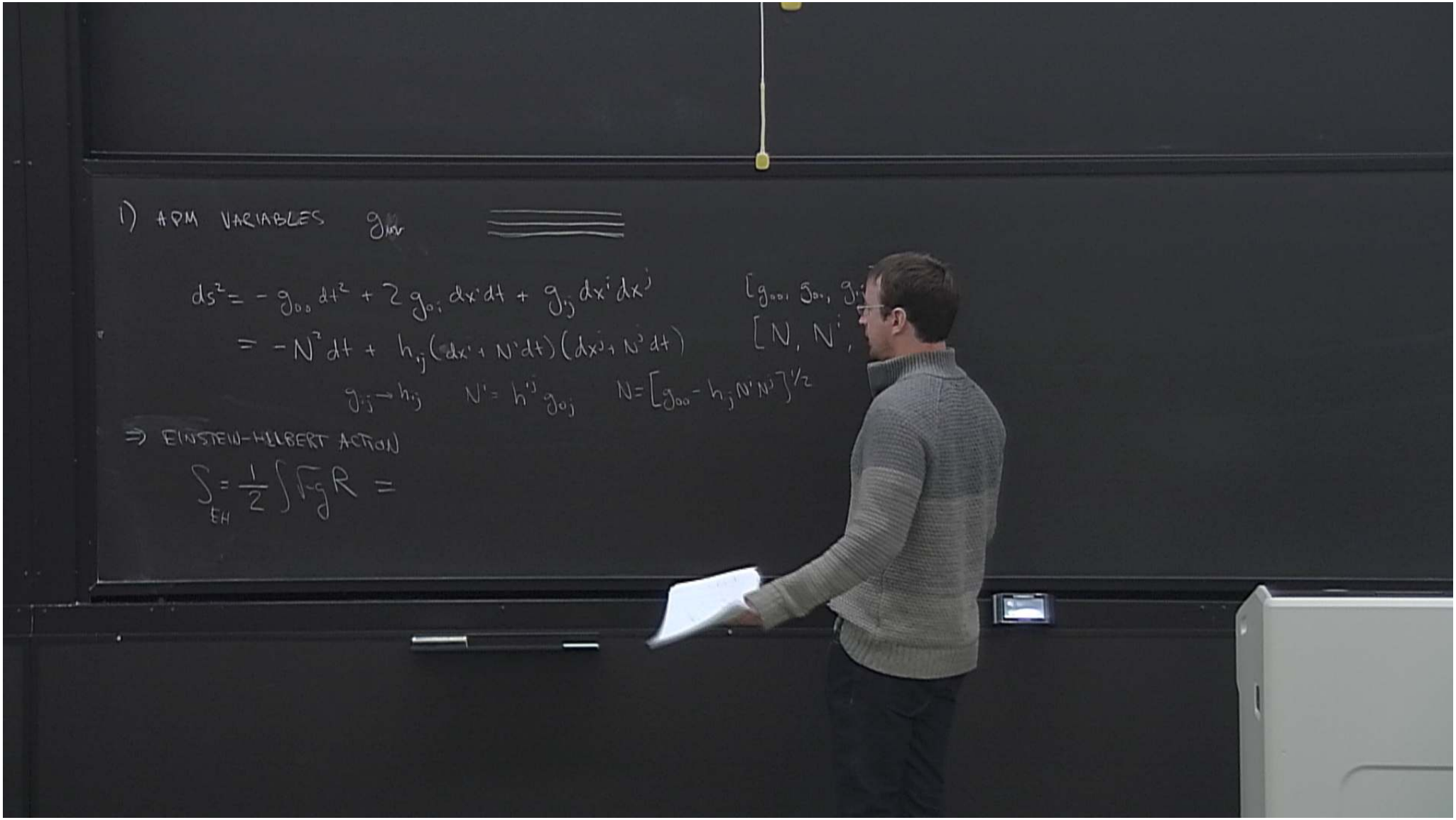

$\frac{\ddot{\varphi}}{H\dot{\varphi}} = \eta - \varepsilon \ll 1$
$a \approx e^{Ht}$
OUTLINE OF CALCULATIONS
1) CHANGE

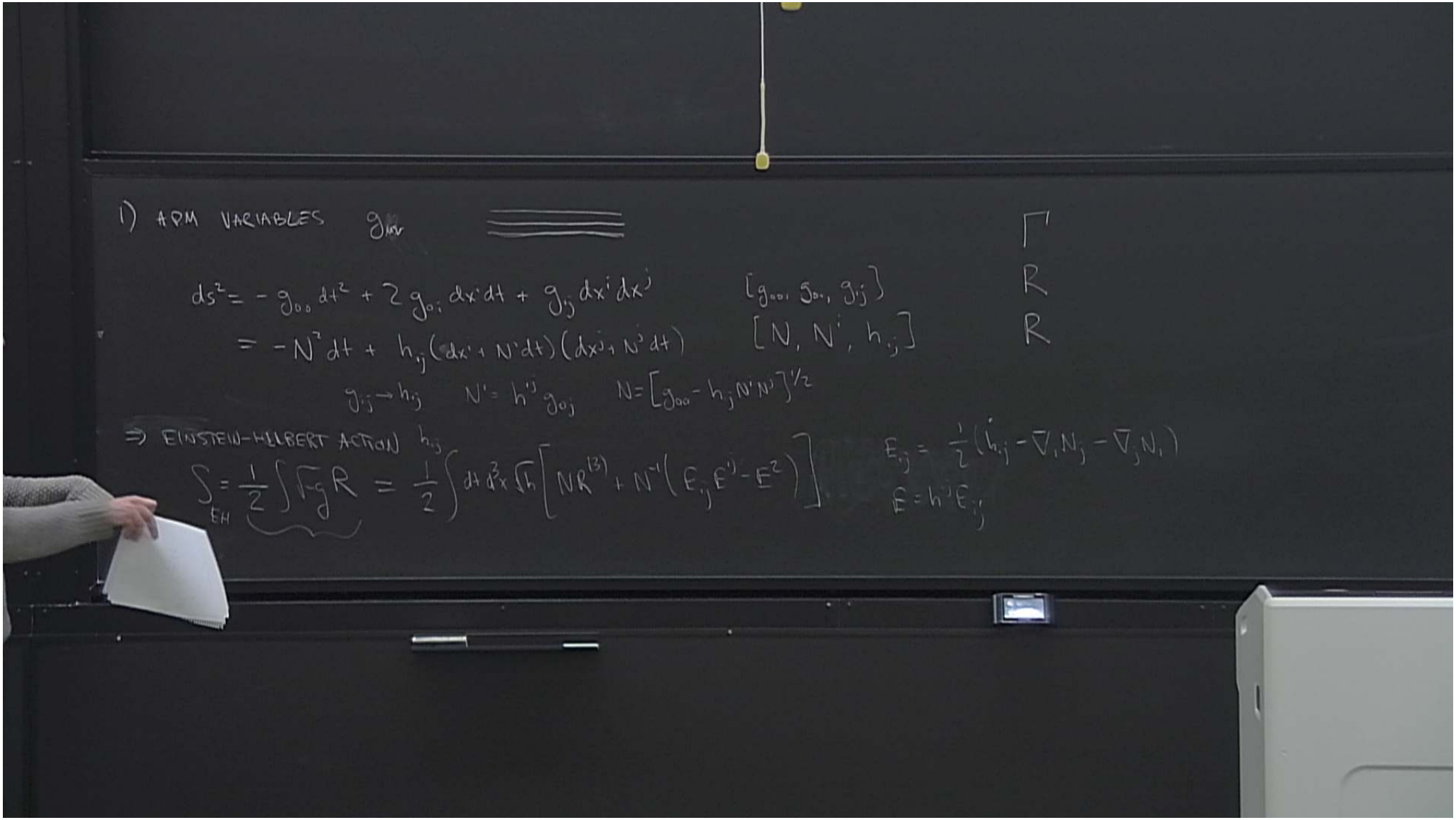

1)

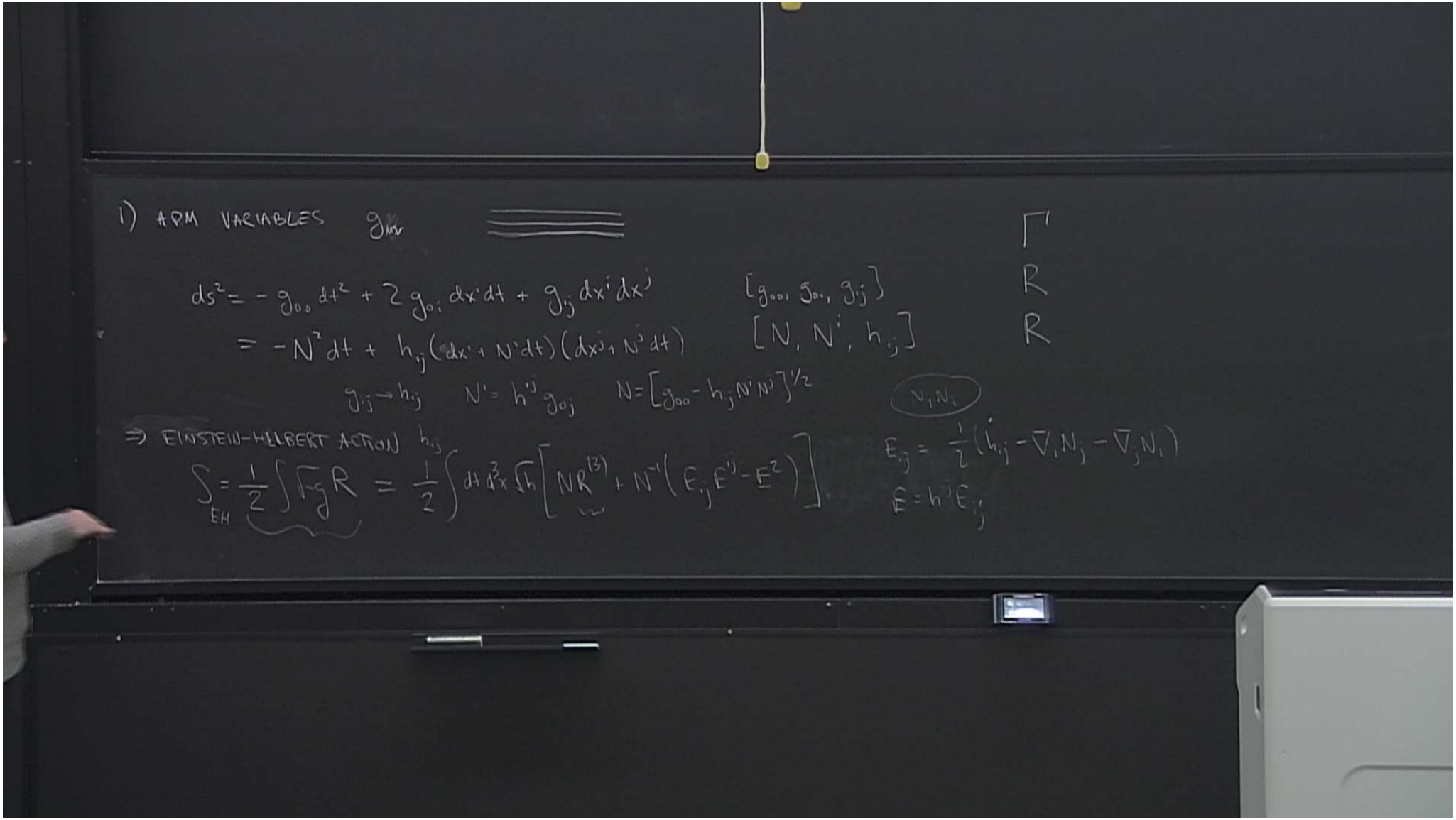

1) ADM VARIABLES $g_{\mu\nu}$
$ds^2 = -g_{00}\,dt^2 + 2\,g_{0i}\,dx^i\,dt + g_{ij}\,dx^i\,dx^j$
$= -N^2\,dt + h_{ij}(dx^i + N^i\,dt)(dx^j + N^j\,dt)$

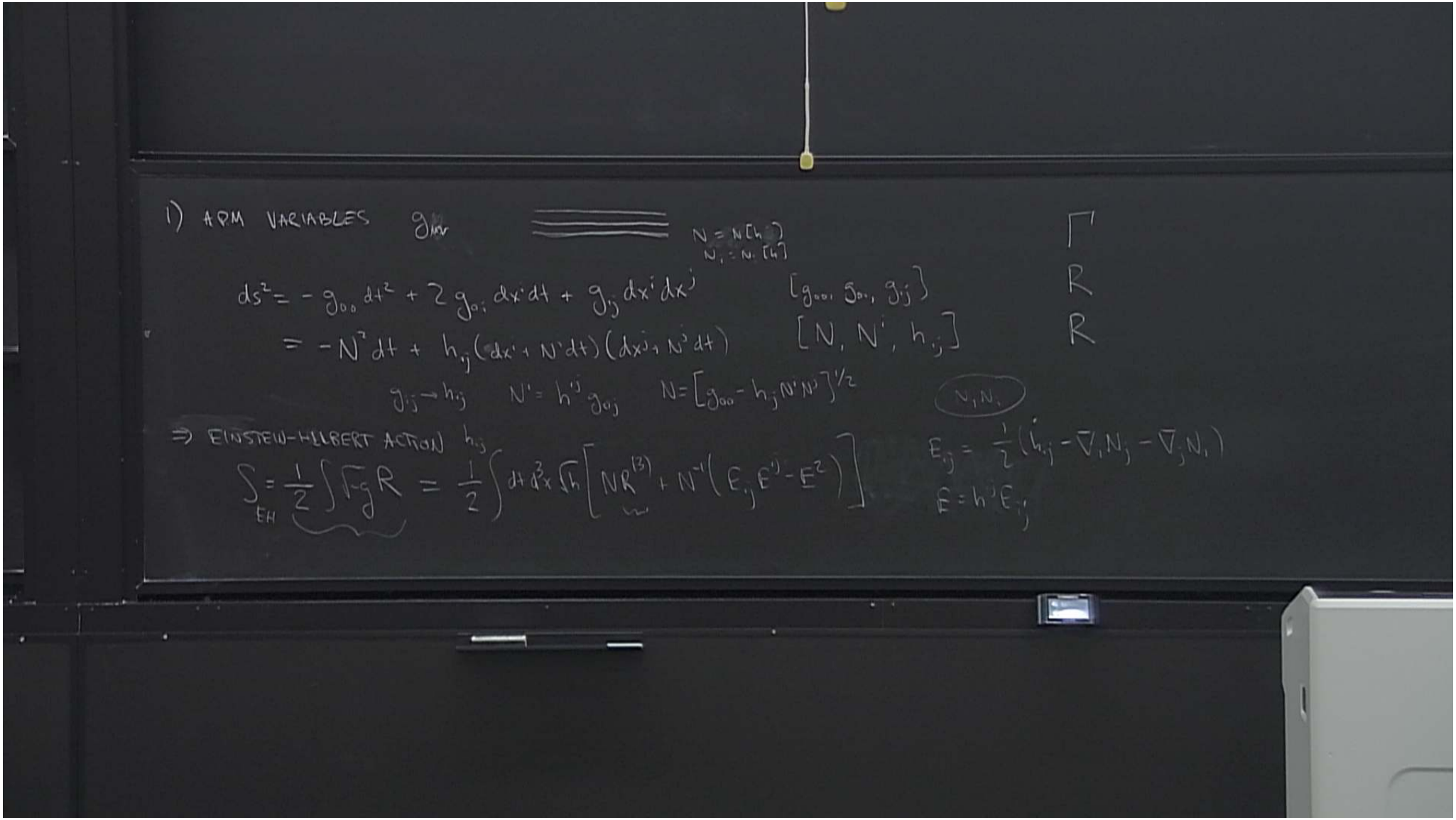

1) ADM VARIABLES $g_{\mu\nu}$
$ds^2 = -g_{00}\,dt^2 + 2g_{0i}\,dx^i dt + g_{ij}\,dx^i dx^j$ $[g_{00}, g_{0i}, g_{ij}]$
$= -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$ $[N, N^i, h_{ij}]$
$g_{ij} \rightarrow$

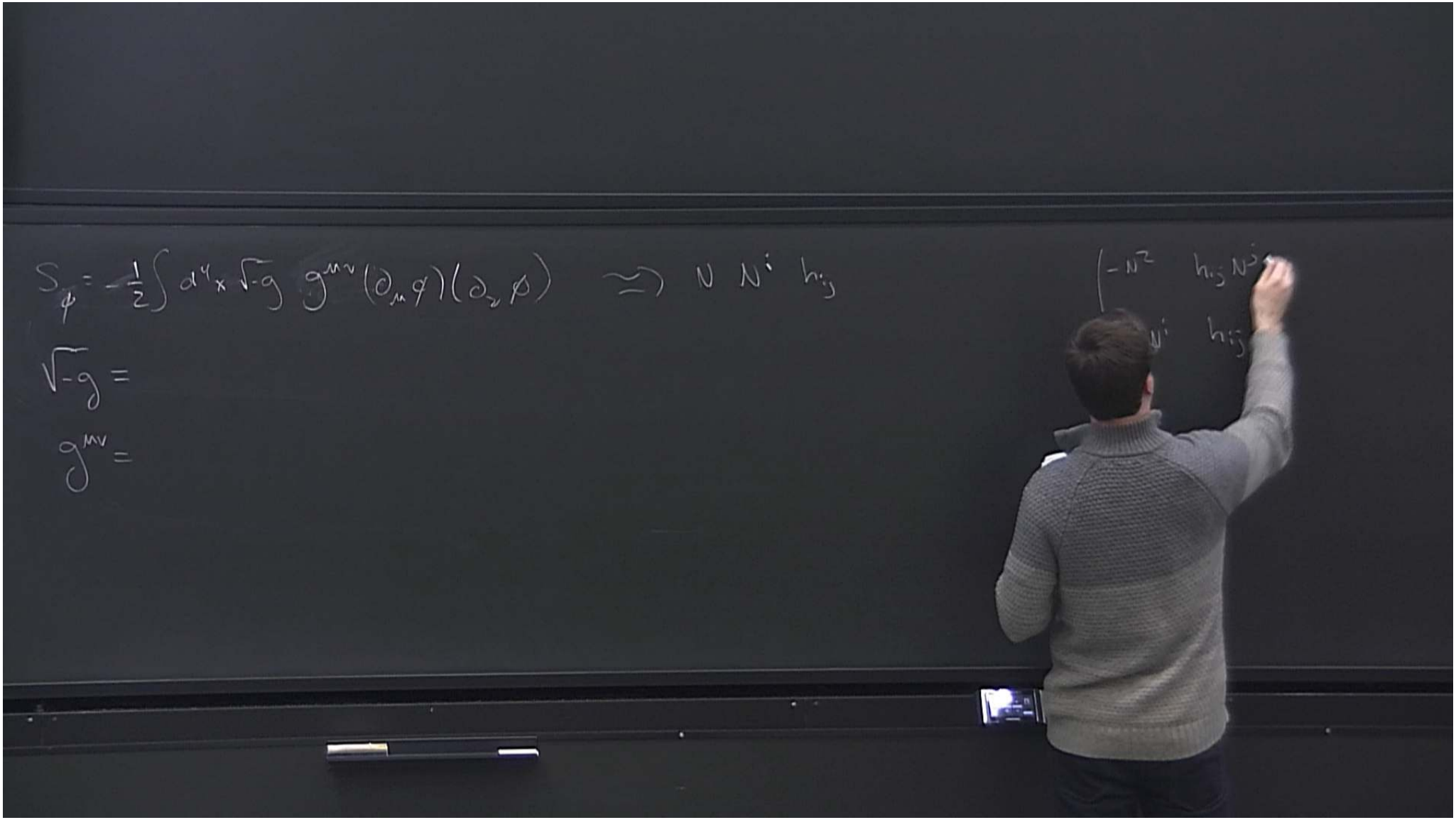

1) ADM VARIABLES $g_{\mu\nu}$
$ds^2 = -g_{00}\,dt^2 + 2\,g_{0i}\,dx^i dt + g_{ij}\,dx^i dx^j$ $[g_{00}, g_{0i}, g_{ij}]$
$= -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$ $[N, N^i, h_{ij}]$
$g_{ij} \to h_{ij}$ $N^i = h^{ij} g_{0j}$ $N = [g_{00} - h_{ij} N^i N^j]^{1/2}$


1) ADM VARIABLES $g_{\mu\nu}$
$ds^2 = -g_{00}\,dt^2 + 2g_{0i}\,dx^i dt + g_{ij}\,dx^i dx^j$
$= -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$g_{ij} \to h_{ij}$ $N^i = h^{ij} g_{0j}$ $N = [g_{00} - h_{ij} N^i N^j]^{1/2}$
⇒ EINSTEIN-HILBERT ACTION
$S_{EH} = \frac{1}{2}\int \sqrt{-g} R =$


1) ADM VARIABLES $g_{\mu\nu}$
$ds^2 = -g_{00}\,dt^2 + 2g_{0i}\,dx^i dt + g_{ij}\,dx^i dx^j$ $[g_{00}, g_{0i}, g_{ij}]$ Γ
$= -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$ $[N, N^i, h_{ij}]$ R
$g_{ij} \to h_{ij}$ $N^i = h^{ij} g_{0j}$ $N = [g_{00} - h_{ij} N^i N^j]^{1/2}$ R
⇒ EINSTEIN-HILBERT ACTION h_{ij}
$S_{EH} = \frac{1}{2}\int \sqrt{-g} R = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2)\right]$
$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
$E = h^{ij} E_{ij}$


1) ADM VARIABLES $g_{\mu\nu}$
$ds^2 = -g_{00}\,dt^2 + 2g_{0i}\,dx^i dt + g_{ij}\,dx^i dx^j$
$[g_{00}, g_{0i}, g_{ij}]$
$= -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$[N, N^i, h_{ij}]$
$g_{ij} \to h_{ij}$ $N^i = h^{ij} g_{0j}$ $N = [g_{00} - h_{ij} N^i N^j]^{1/2}$
N, N_i
⇒ EINSTEIN-HILBERT ACTION h_{ij}
$S_{EH} = \frac{1}{2}\int \sqrt{-g} R = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2)\right]$
$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
$E = h^{ij} E_{ij}$
Γ
R
R


1) ADM VARIABLES
$N = N[h_{ij}]$
$N_i = N_i[h_{ij}]$
$ds^2 = -g_{00}dt^2 + 2g_{0i}dx^i dt + g_{ij}dx^i dx^j$
$[g_{00}, g_{0i}, g_{ij}]$
$= -N^2 dt + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$[N, N^i, h_{ij}]$
$g_{ij} \to h_{ij}$ $N^i = h^{ij} g_{0j}$ $N = [g_{00} - h_{ij} N^i N^j]^{1/2}$
N, N_i
Γ
R
R
h_{ij}
$\Rightarrow$ EINSTEIN-HILBERT ACTION
$S_{EH} = \frac{1}{2}\int \sqrt{-g} R = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2)\right]$
$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
$E = h^{ij} E_{ij}$

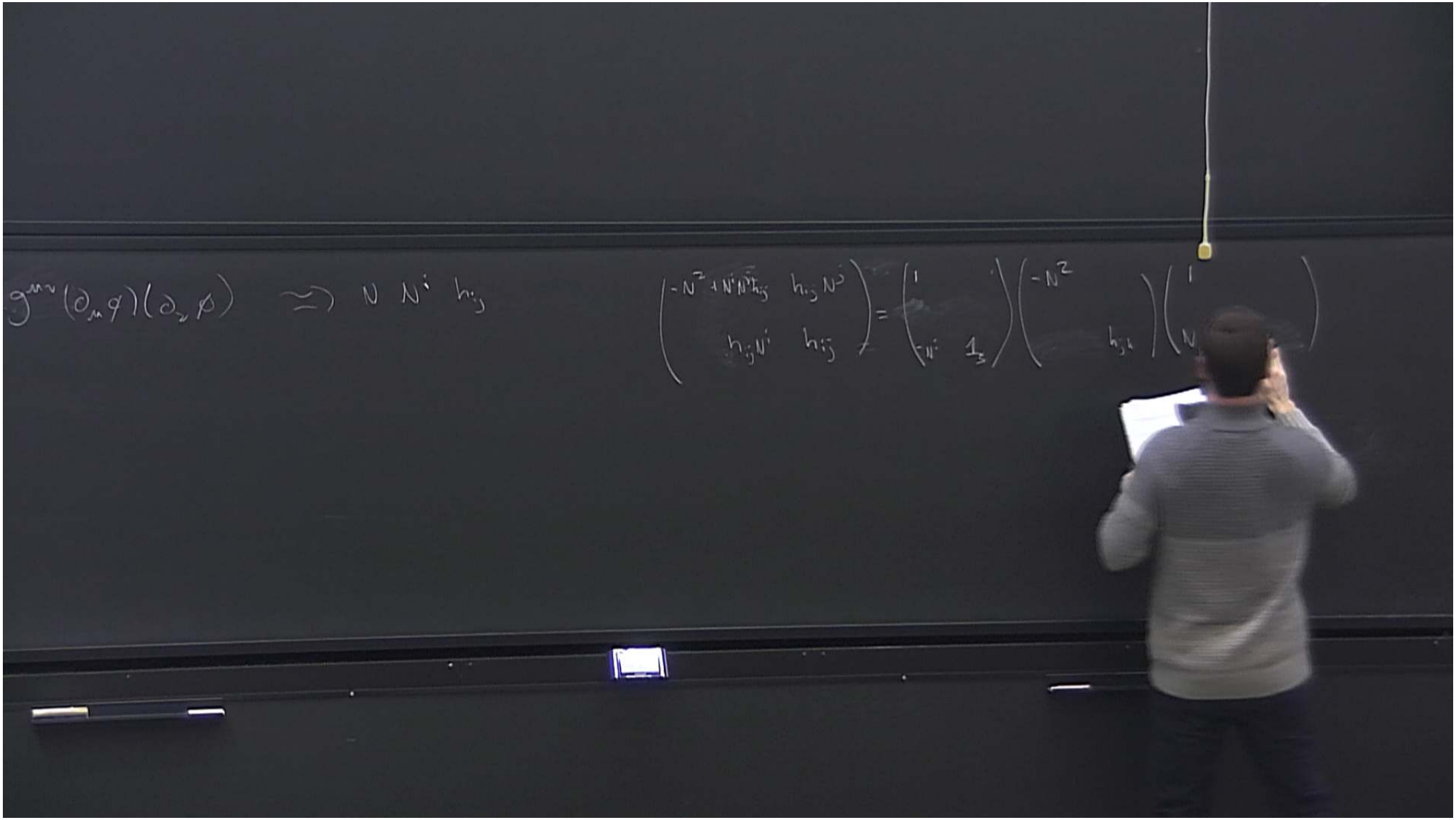

$S_\phi = -\frac{1}{2}\int d^4x \sqrt{-g}\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) \simeq N\, N^i\, h_{ij}$
$\sqrt{-g} =$
$g^{\mu\nu} =$
$(-N^2 \quad h_{ij}N^j$
$N^i \quad h_{ij}$

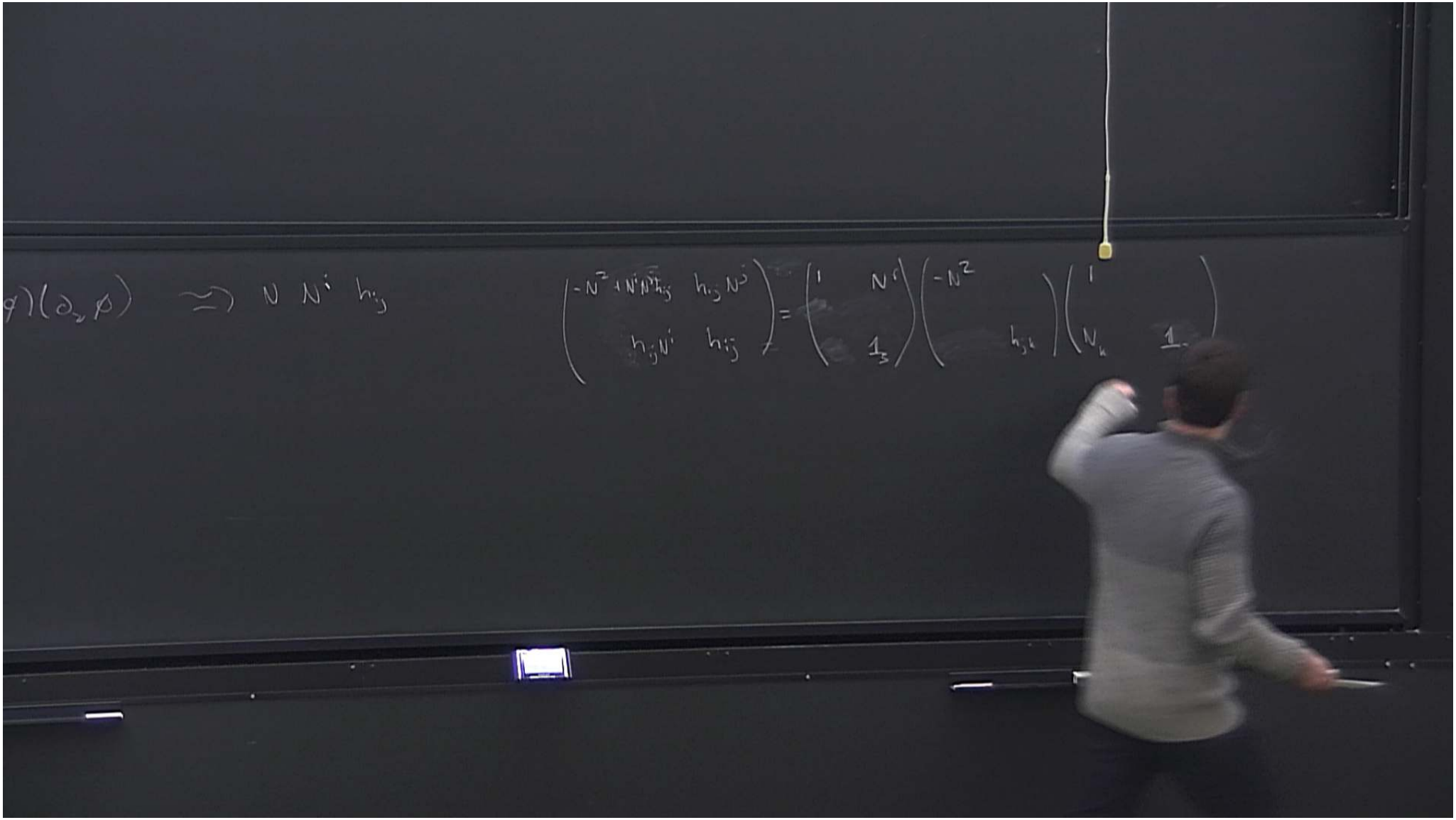
$$S_\phi = -\frac{1}{2}\int d^4x \sqrt{-g}\, g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) \quad \simeq \quad N\, N^i\, h_{ij}$$

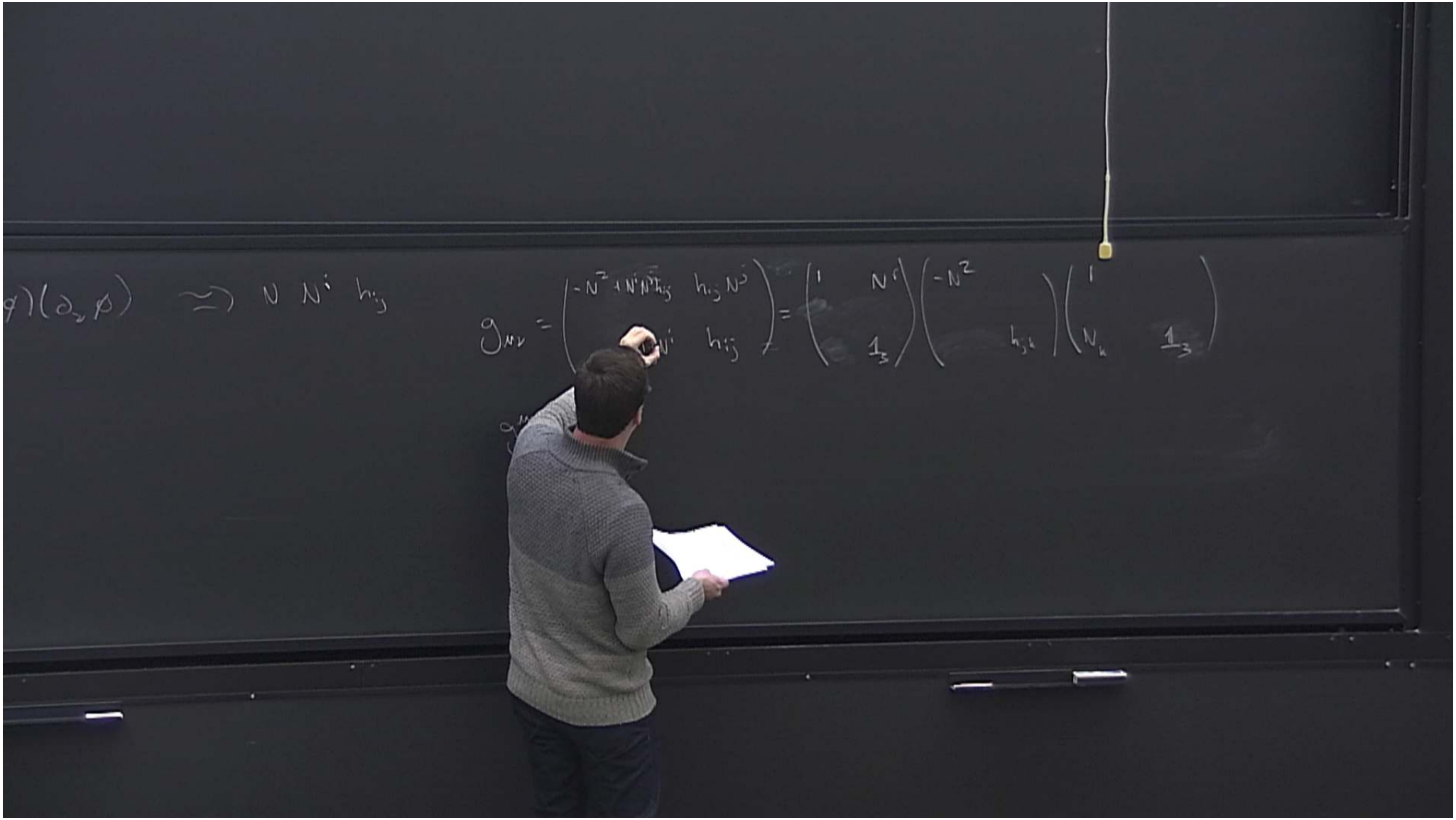
$$\begin{pmatrix} -N^2 + N^i N^j h_{ij} & h_{ij}N^j \\ h_{ij}N^i & h_{ij} \end{pmatrix}^{-1} = \; ?$$

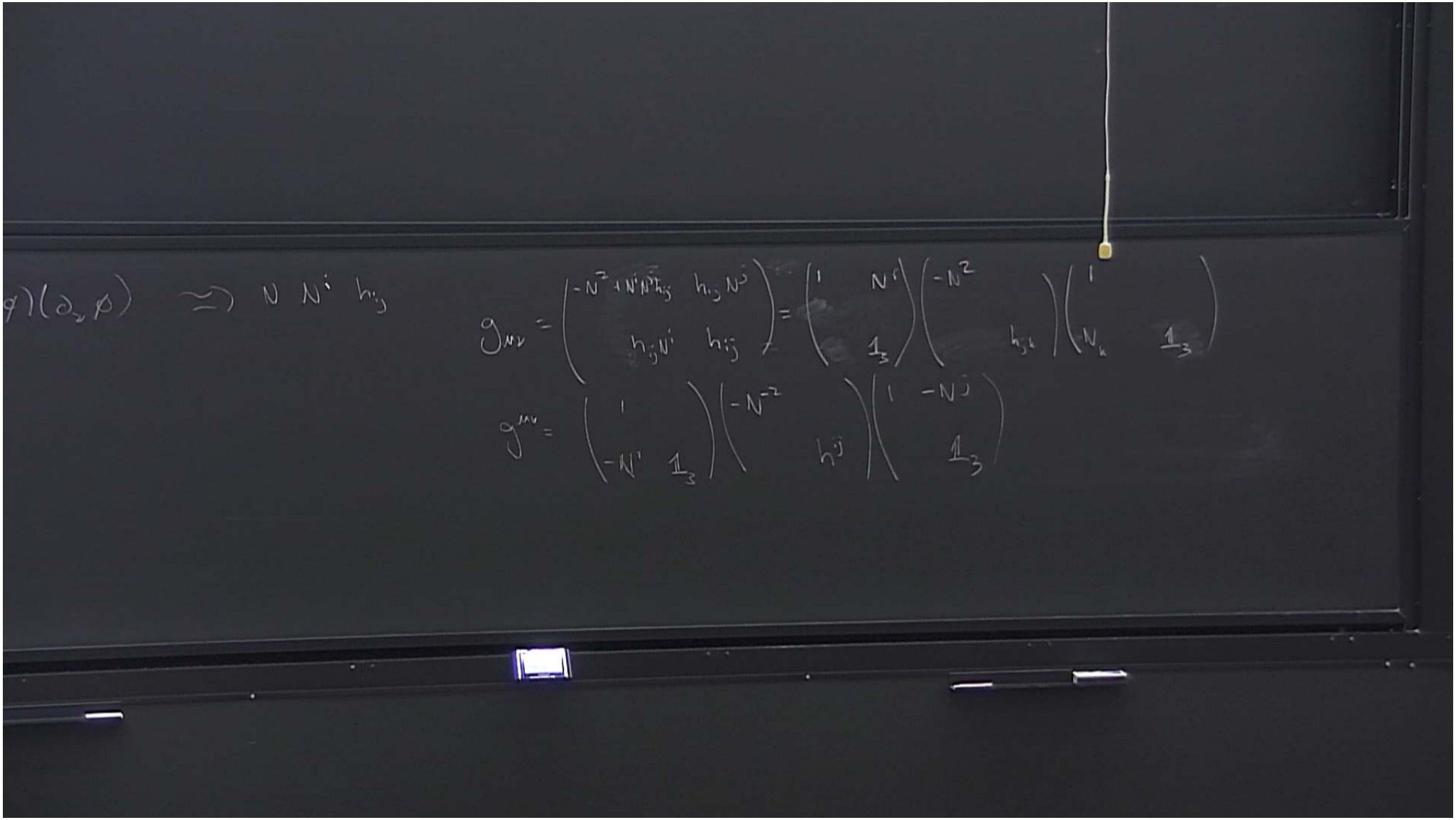
$\sqrt{-g} =$

$g^{\mu\nu} =$


$g^{\mu\nu}(\partial_\mu q)(\partial_\nu p) \supset N\, N^i\, h_{ij}$
$\begin{pmatrix} -N^2 + N^i N^j h_{ij} & h_{ij} N^j \\ h_{ij} N^i & h_{ij} \end{pmatrix} = \begin{pmatrix} 1 & \\ N^i & 1 \end{pmatrix} \begin{pmatrix} -N^2 & \\ & h_{ij} \end{pmatrix} \begin{pmatrix} 1 & \\ N^j & 1 \end{pmatrix}$


$\phi)(\partial_2 \phi) \simeq N N^i h_{ij}$
$\begin{pmatrix} -N^2 + N^i N^j h_{ij} & h_{ij} N^j \\ h_{ij} N^i & h_{ij} \end{pmatrix} = \begin{pmatrix} 1 & N^i \\ & 1_3 \end{pmatrix} \begin{pmatrix} -N^2 & \\ & h_{jk} \end{pmatrix} \begin{pmatrix} 1 & \\ N_k & 1 \end{pmatrix}$

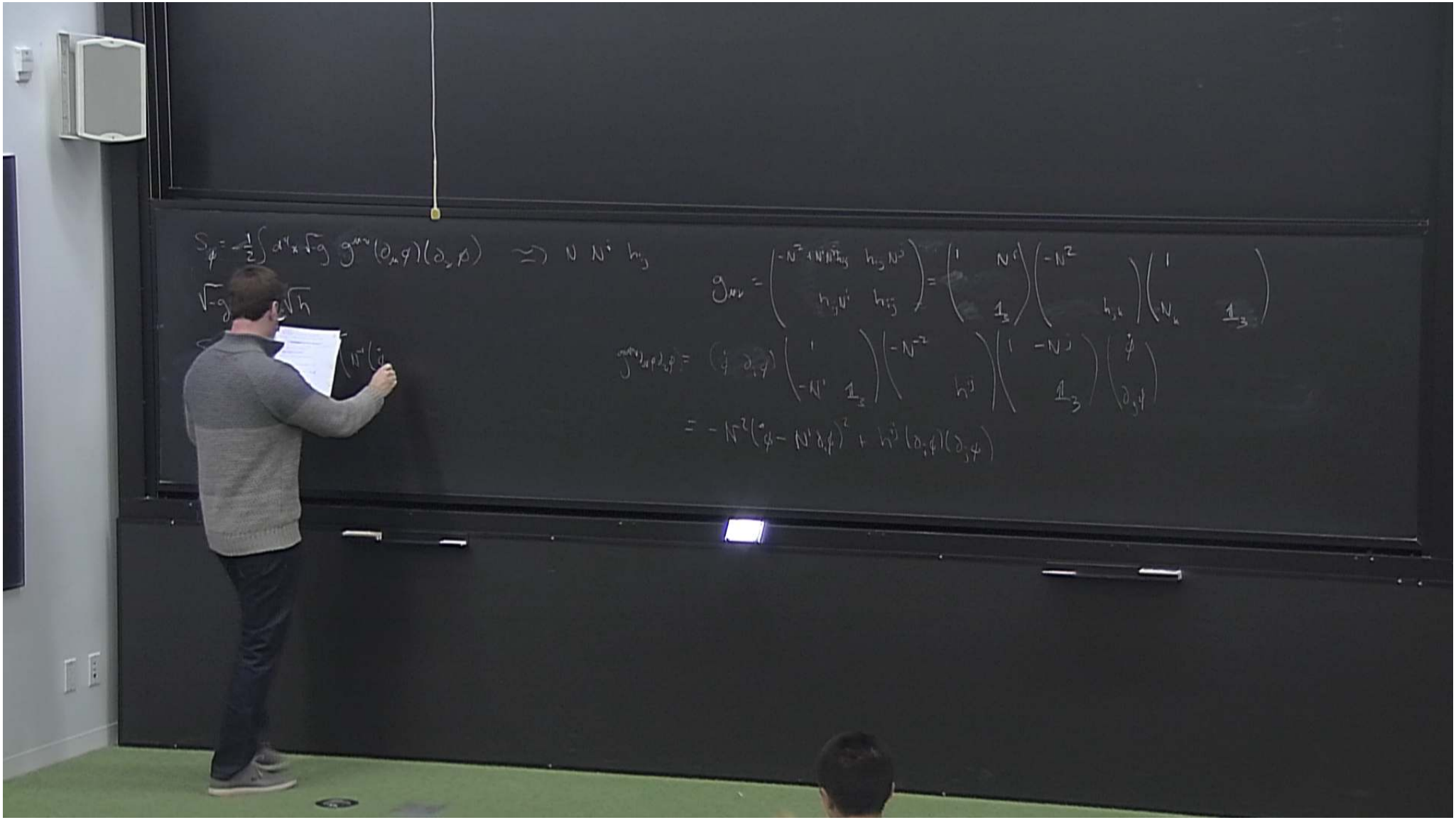

$\phi)(\partial_\nu \phi) \simeq N N^i h_{ij}$
$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j h_{ij} & h_{ij} N^j \\ N^i & h_{ij} \end{pmatrix} = \begin{pmatrix} 1 & N^i \\ & 1_3 \end{pmatrix} \begin{pmatrix} -N^2 & \\ & h_{jk} \end{pmatrix} \begin{pmatrix} 1 & \\ N_k & 1_3 \end{pmatrix}$

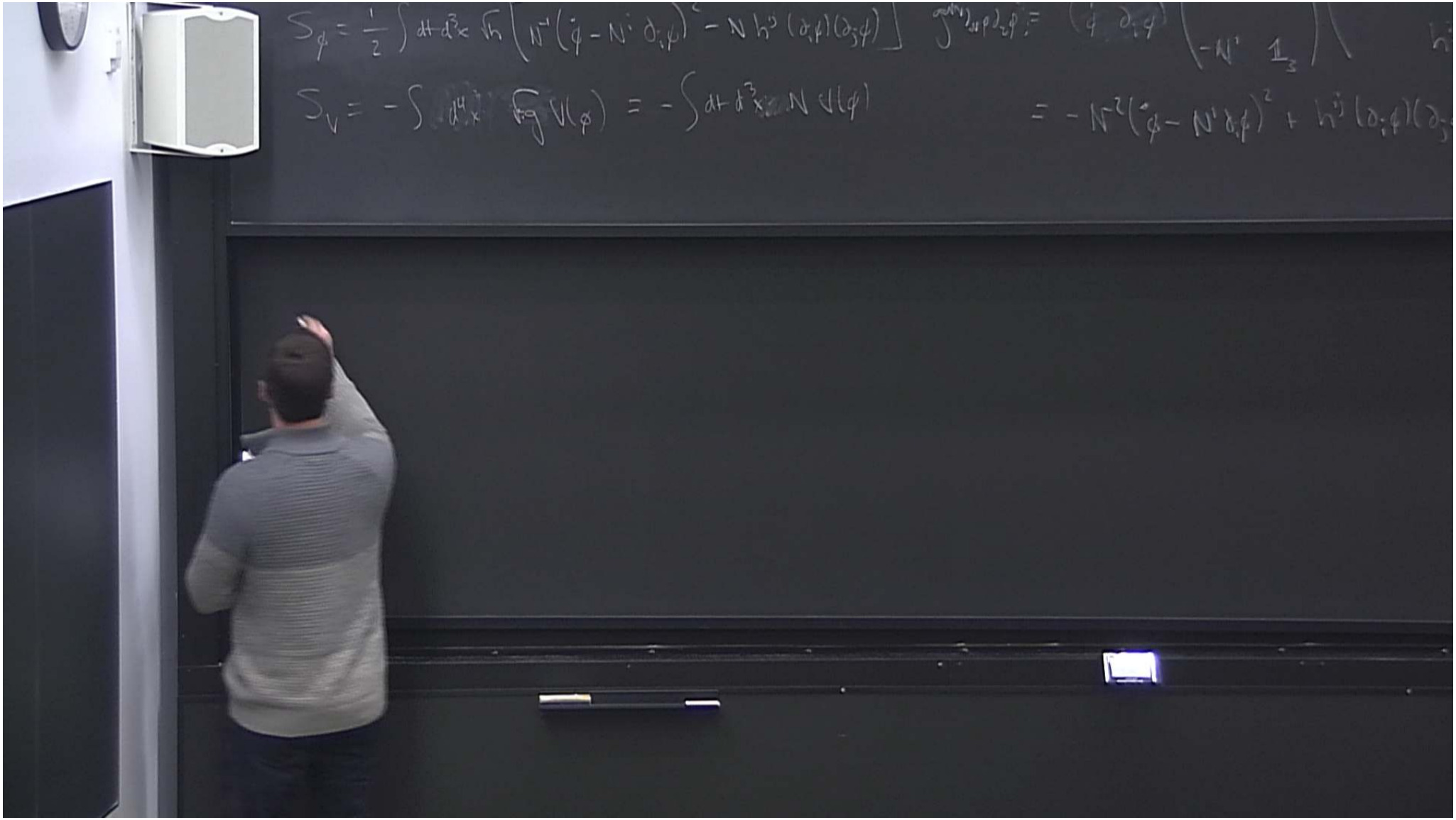


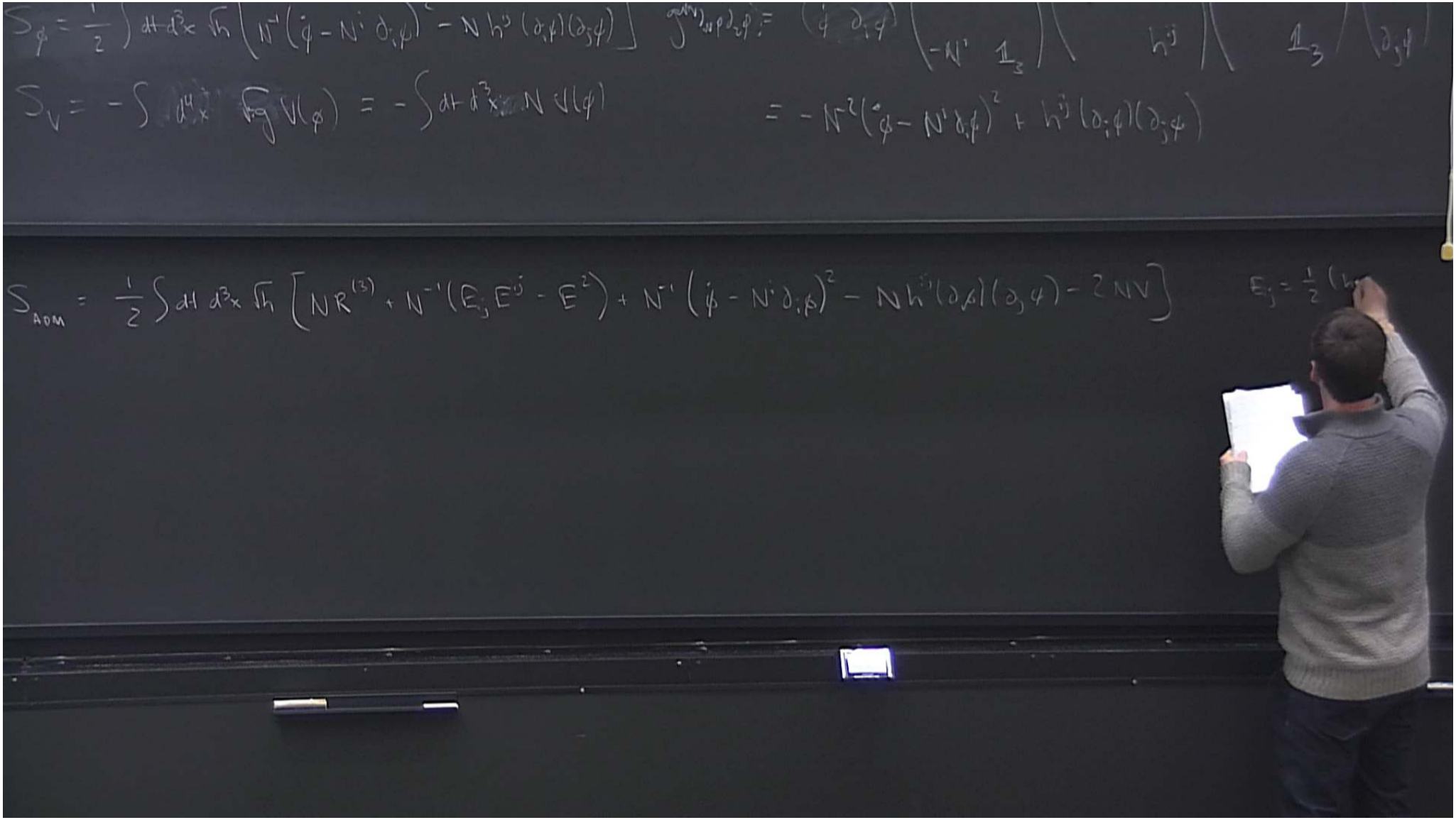
$$\phi)(\partial_\nu \phi) \rightsquigarrow N\, N^i\, h_{ij}$$

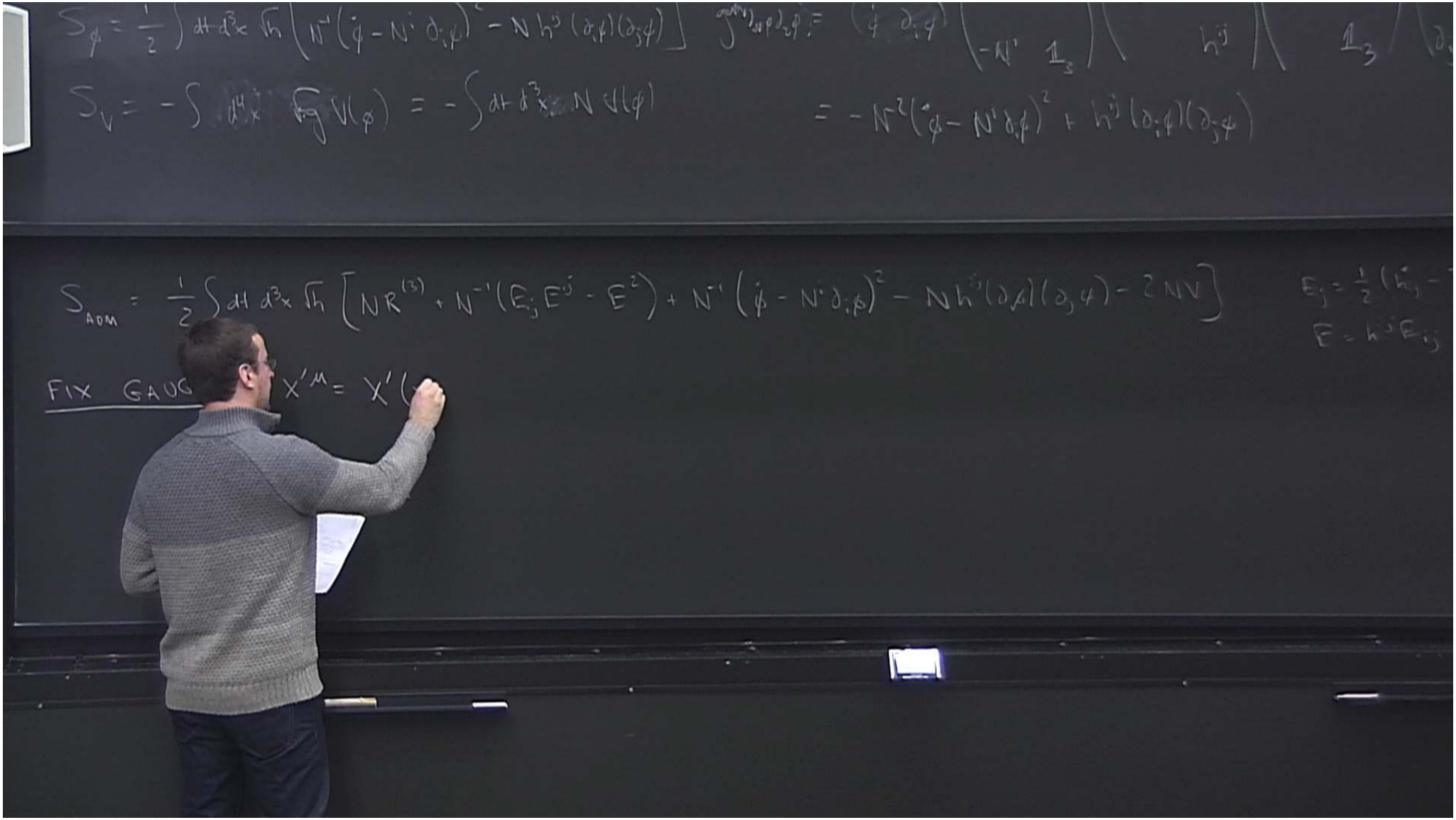
$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j h_{ij} & h_{ij} N^j \\ h_{ij} N^i & h_{ij} \end{pmatrix} = \begin{pmatrix} 1 & N^i \\ & \mathbb{1}_3 \end{pmatrix} \begin{pmatrix} -N^2 & \\ & h_{jk} \end{pmatrix} \begin{pmatrix} 1 & \\ N_k & \mathbb{1}_3 \end{pmatrix}$$

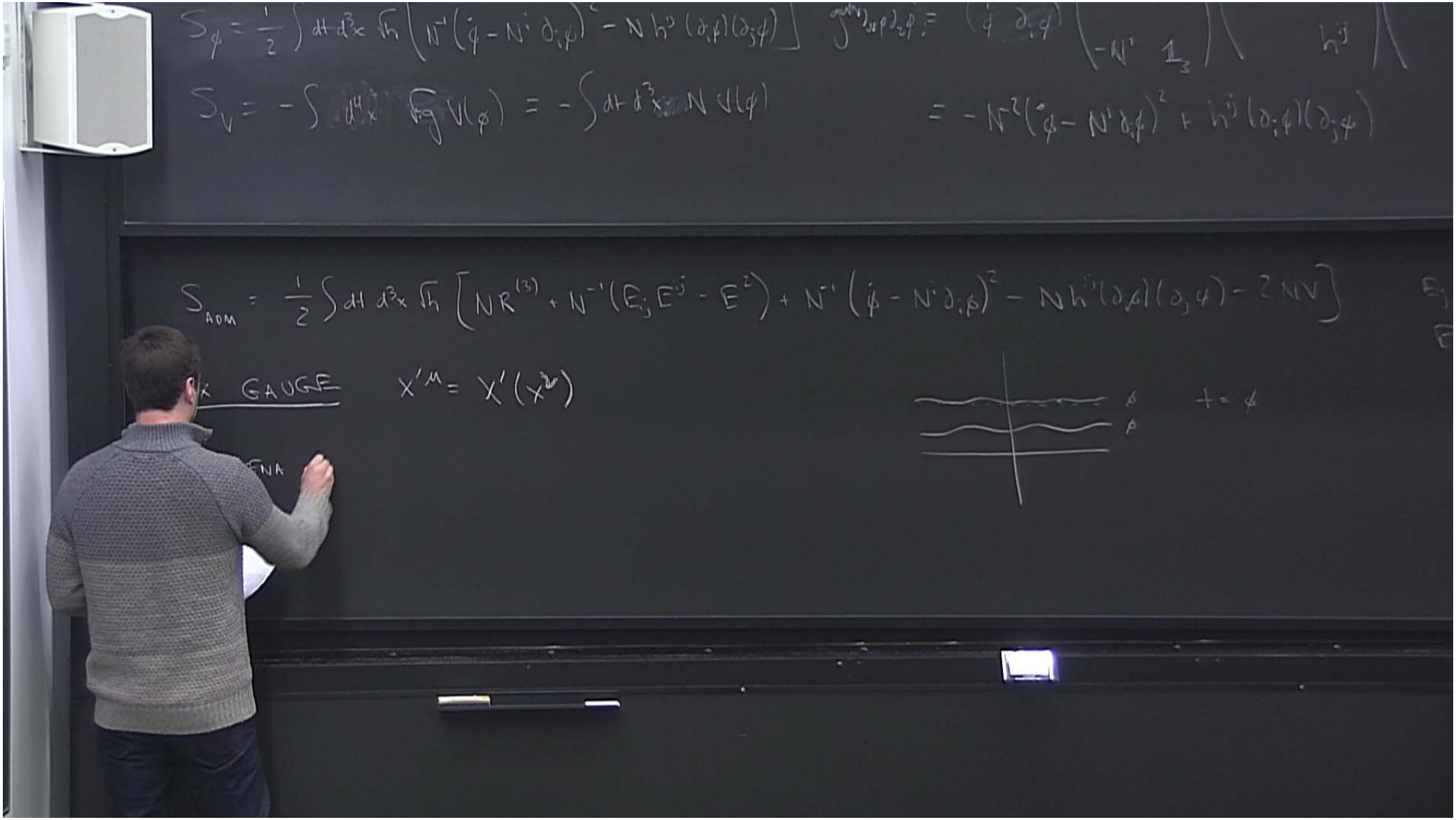
$$g^{\mu\nu} = \begin{pmatrix} 1 & \\ -N^i & \mathbb{1}_3 \end{pmatrix} \begin{pmatrix} -N^{-2} & \\ & h^{ij} \end{pmatrix} \begin{pmatrix} 1 & -N^j \\ & \mathbb{1}_3 \end{pmatrix}$$

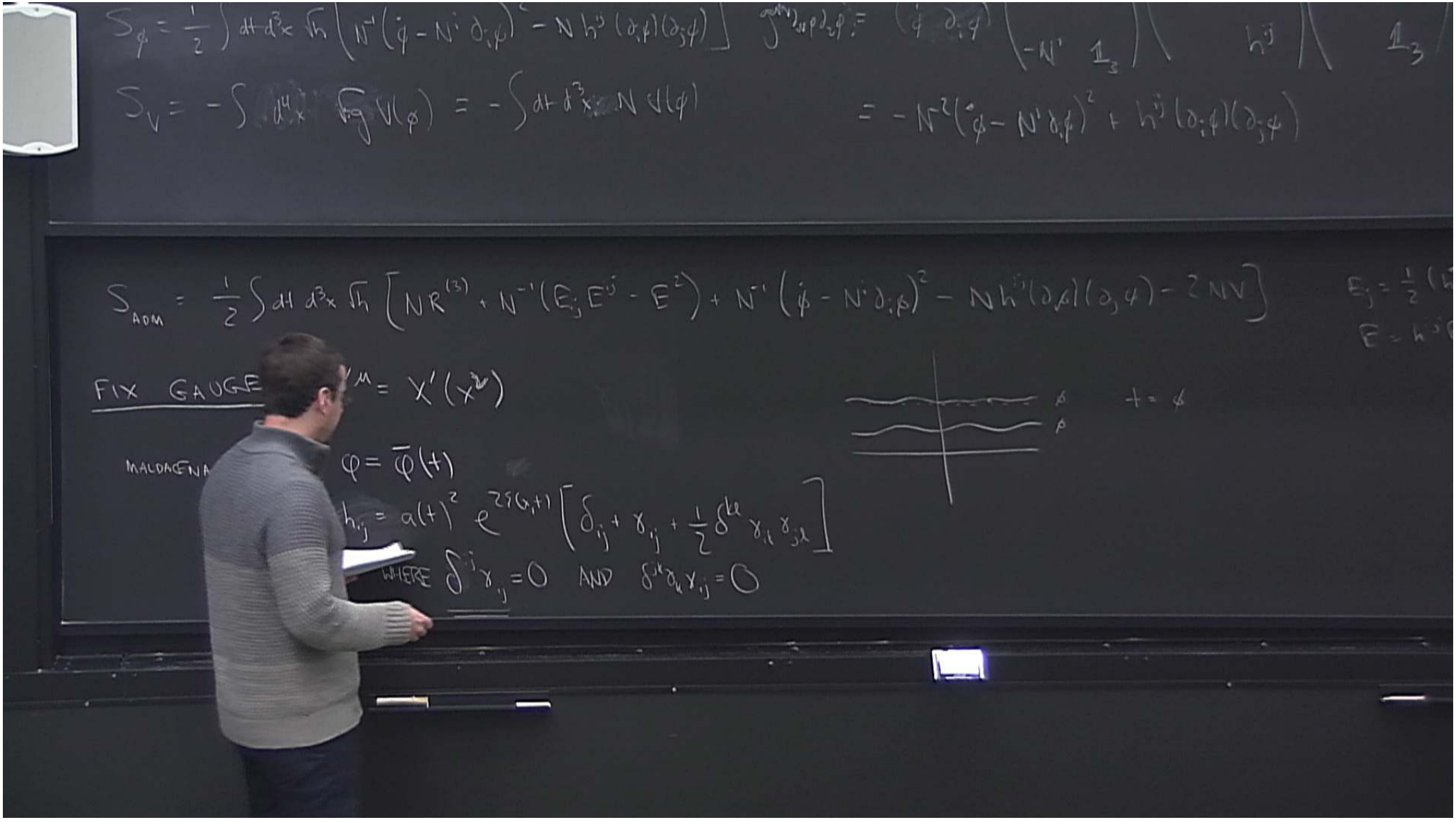

$S_\phi = -\frac{1}{2}\int d^4x \sqrt{-g}\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) \quad \supset \quad N\,N^i\,h_{ij}$
$\sqrt{-g} = N\sqrt{h}$
$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j h_{ij} & h_{ij}N^j \\ h_{ij}N^i & h_{ij} \end{pmatrix} = \begin{pmatrix} 1 & N^i \\ & \mathbb{1}_3 \end{pmatrix}\begin{pmatrix} -N^2 & \\ & h_{ij} \end{pmatrix}\begin{pmatrix} 1 & \\ N_k & \mathbb{1}_3 \end{pmatrix}$
$g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) = (\dot\phi \;\; \partial_i\phi)\begin{pmatrix} 1 & \\ -N^i & \mathbb{1}_3 \end{pmatrix}\begin{pmatrix} -N^{-2} & \\ & h^{ij} \end{pmatrix}\begin{pmatrix} 1 & -N^j \\ & \mathbb{1}_3 \end{pmatrix}\begin{pmatrix} \dot\phi \\ \partial_j\phi \end{pmatrix}$
$= -N^{-2}(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j\phi)$

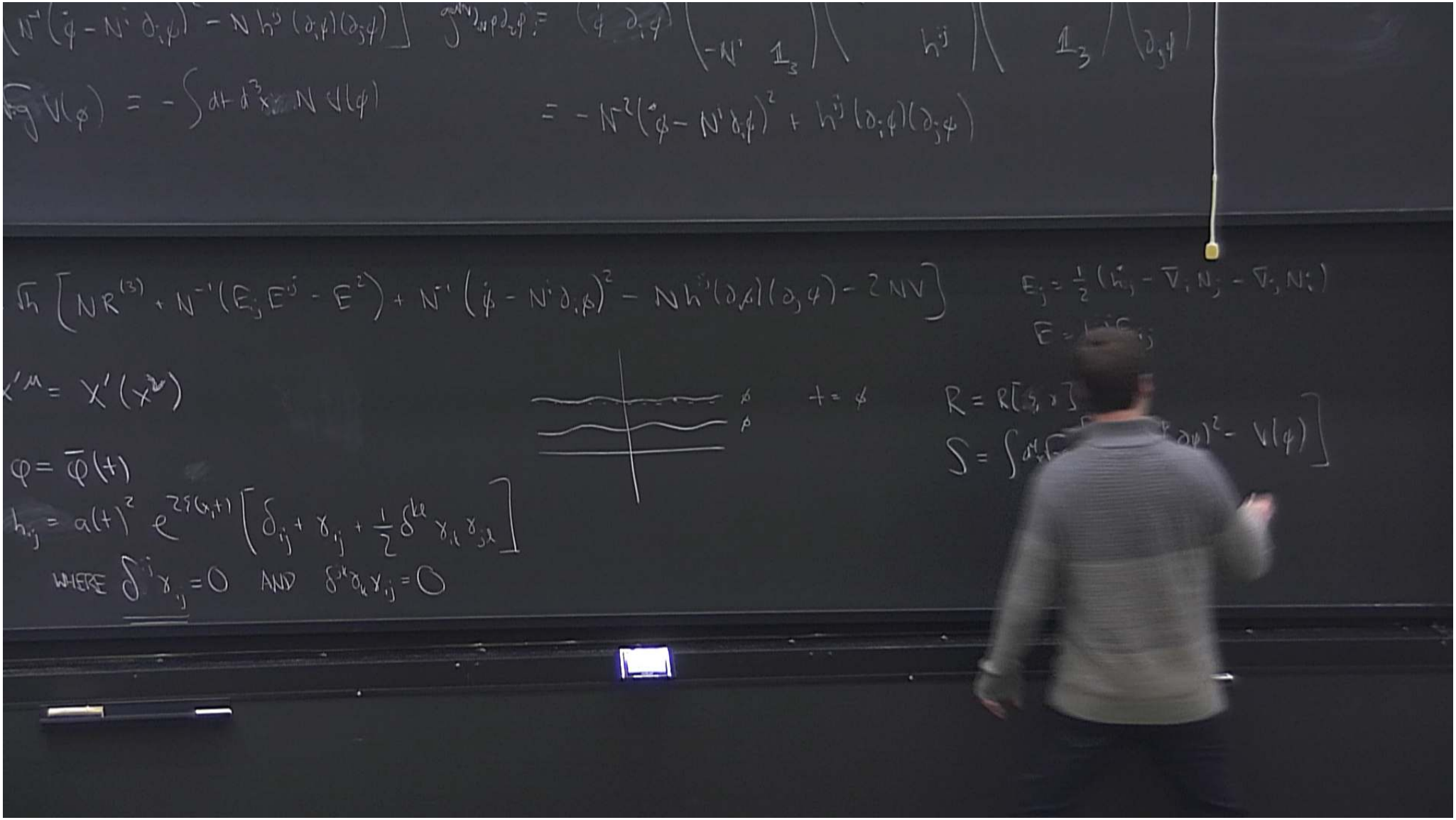

$S_\phi = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi)\right]$
$S_V = -\int d^4x \sqrt{-g}\, V(\phi) = -\int dt\, d^3x\, N V(\phi)$
$= -N^2(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j$

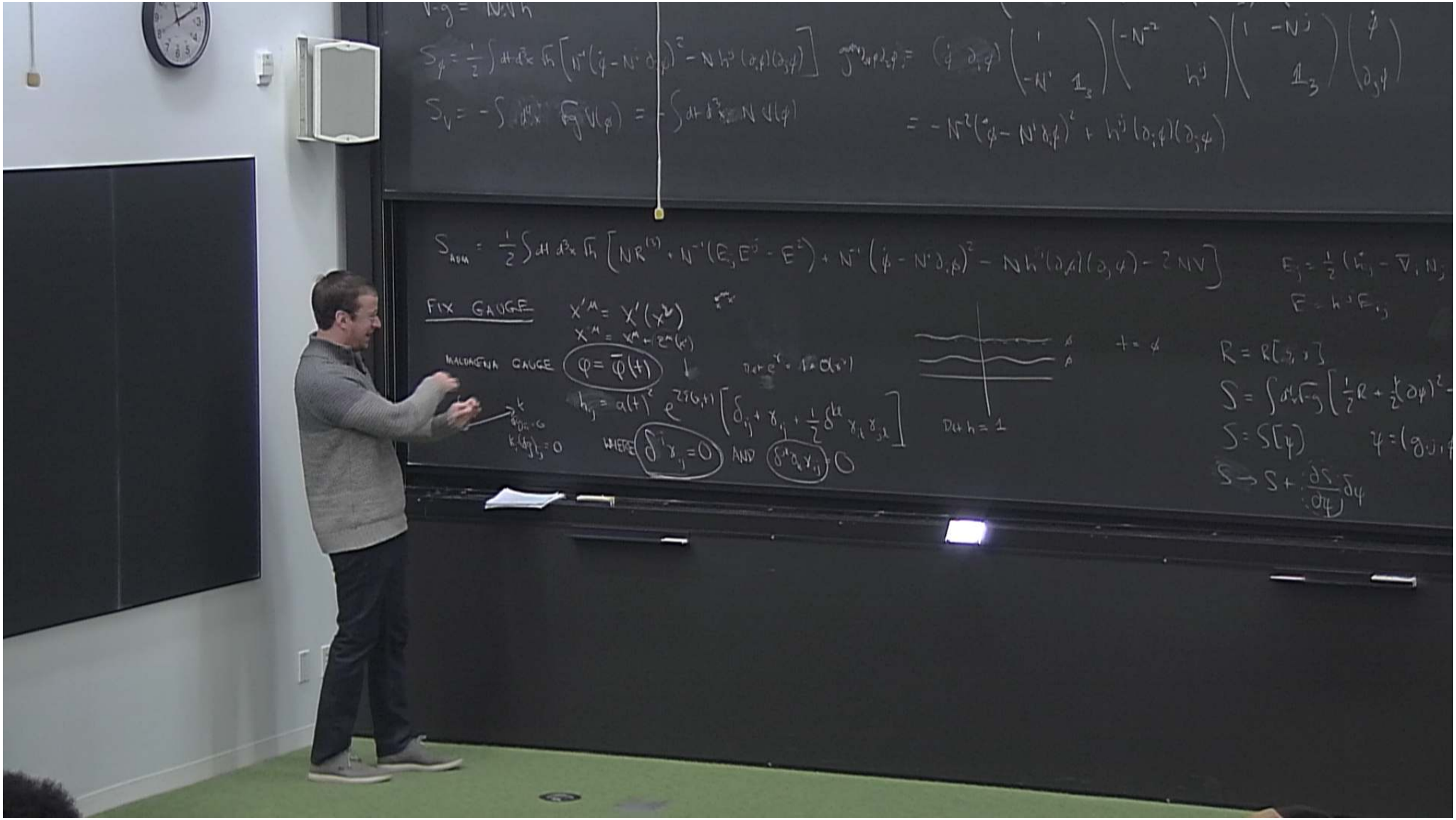

$S_\phi = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi)\right]$
$S_V = -\int d^4x\, \sqrt{-g}\, V(\phi) = -\int dt\, d^3x\, N V(\phi)$
$= -N^2(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j\phi)$
$S_{ADM} = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi) - 2NV\right]$
$E_{ij} = \frac{1}{2}(h$


$S_\phi = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi)\right]$
$S_V = -\int d^4x\, \sqrt{-g}\, V(\phi) = -\int dt\, d^3x\, N\, V(\phi)$
$= -N^2(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j\phi)$
$S_{ADM} = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi) - 2NV\right]$
$E_{ij} = \frac{1}{2}(\dot h_{ij} -$
$E = h^{ij}E_{ij}$
FIX GAUG
$x'^\mu = x'($


$S_\phi = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi)\right]$
$S_V = -\int d^4x \sqrt{-g}\, V(\phi) = -\int dt\, d^3x\, N V(\phi)$
$= -N^2(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j\phi)$
$S_{ADM} = \frac{1}{2}\int dt\, d^3x \sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi) - 2NV\right]$
GAUGE
$x'^\mu = x'(x^\mu)$
$t = \phi$


$S_\phi = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi)\right]$
$S_V = -\int d^4x\, \sqrt{-g}\, V(\phi) = -\int dt\, d^3x\, N\, V(\phi)$
$= -N^2(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j\phi)$
$S_{ADM} = \frac{1}{2}\int dt\, d^3x\, \sqrt{h}\left[N R^{(3)} + N^{-1}(E_i E^i - E^2) + N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi) - 2NV\right]$
FIX GAUGE
$= X'(x^\mu)$
$\varphi = \bar\varphi(t)$
$h_{ij} = a(t)^2 e^{2\zeta(x,t)}\left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl}\right]$
WHERE $\partial^i\gamma_{ij} = 0$ AND $\delta^{ij}\gamma_{ij} = 0$
$t = \phi$


$\sqrt{-g}\, V(\phi) = -\int dt\, d^3x\, N\, V(\phi)$
$= -N^{-2}(\dot\phi - N^i\partial_i\phi)^2 + h^{ij}(\partial_i\phi)(\partial_j\phi)$
$\sqrt{h}\left[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}(\partial_i\phi)(\partial_j\phi) - 2NV\right]$
$E_{ij} = \frac{1}{2}(\dot h_{ij} - \bar\nabla_i N_j - \bar\nabla_j N_i)$
$x'^\mu = x'(x^\nu)$
$t = \phi$
$R = R[\xi, \gamma]$
$\varphi = \bar\varphi(t)$
$h_{ij} = a(t)^2 e^{2\xi(x,t)}\left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl}\right]$
WHERE $\delta^{ij}\gamma_{ij} = 0$ AND $\delta^{jk}\gamma_k\gamma_{ij} = 0$


FIX GAUGE
MALDACENA GAUGE
WHERE
AND
Det h = 1

GAUGE $x'^\mu = x'(x^\mu)$

$x'^\mu = x^\mu + \xi^\mu(x)$

ENA GAUGE $\varphi = \bar{\varphi}(t)$

$h_{ij} = a(t)^2 e^{2\zeta(x,t)} \left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl} \right]$

WHERE $\delta^{ij}\gamma_{ij} = 0$ AND $\delta^{jk}\partial_k\gamma_{ij} = F_i = 0$

$\det e^{\gamma} = 1 + \alpha(\gamma^2)$

$\det h = 1$

$t = \phi$

$R = R[g, \gamma]$

$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}R + \frac{1}{2}(\partial\phi)^2 - V \right]$

$S = S[\psi]$ $\quad \psi = (g_{\mu\nu}, \phi)$

$S \to S + \frac{\partial S}{\partial \psi}\delta\psi$

$h_{ij} = e^{2\zeta(x,t)} \left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{jl} \right]$ $\quad \delta^{ij}\gamma_{ij} = 0$

$F_i = \delta^{jk}\partial_k\gamma_{ij} = 0$

$F_i \to F_i + \frac{1}{3}\partial_i\partial_k\xi^k + (\partial^2\xi^k)\delta_{ik} + O(\gamma\xi)$

$x'^i = x^i + \xi^i$

$\delta F_i(\vec{k}) = \left(-\frac{1}{3}k_i k_j - k^2\delta_{ij} \right)\xi^j(\vec{k})$

$\nabla^2\gamma_{ij} = 0$ $\quad \nabla^i h_{ij} = 0$