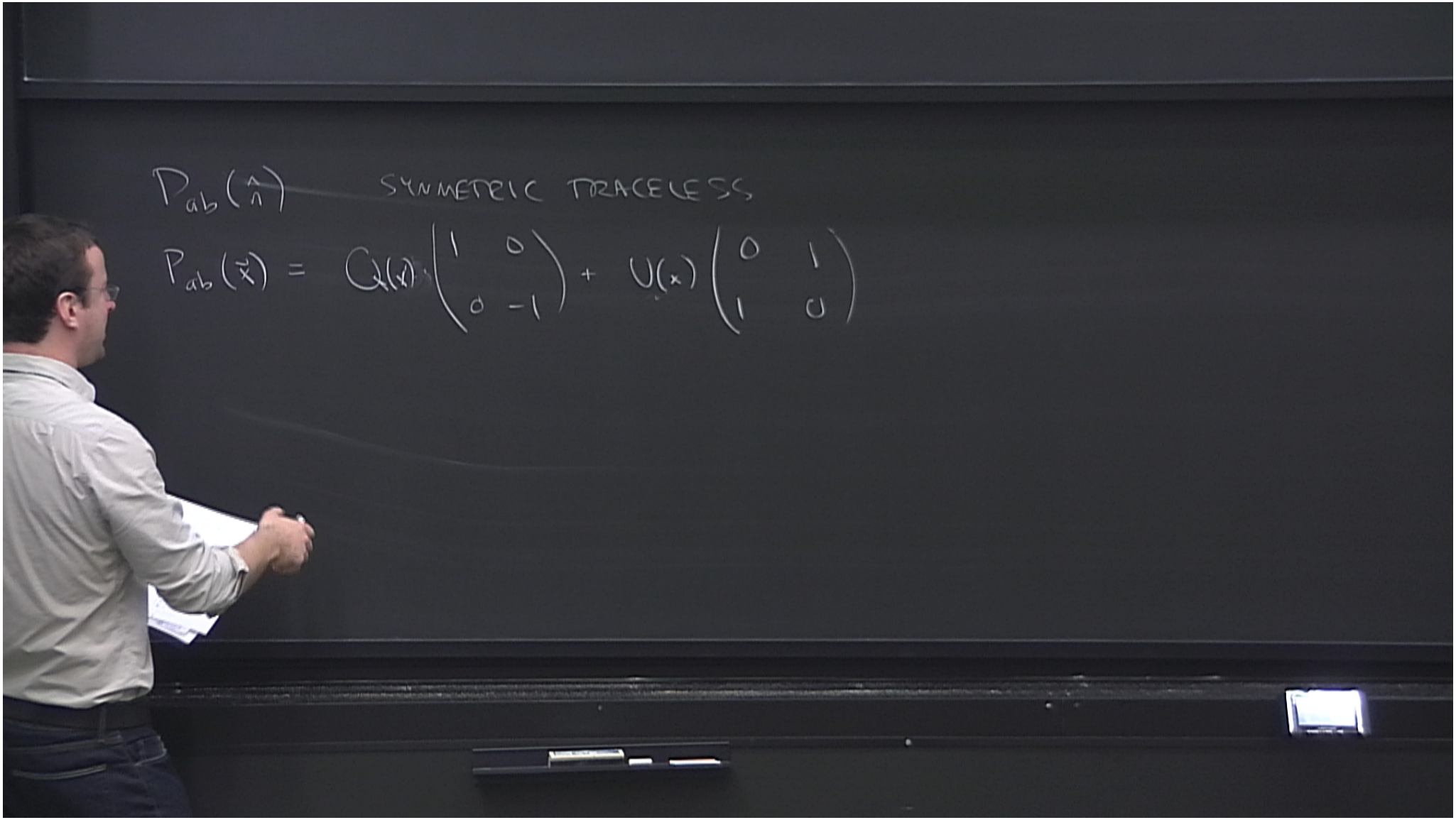
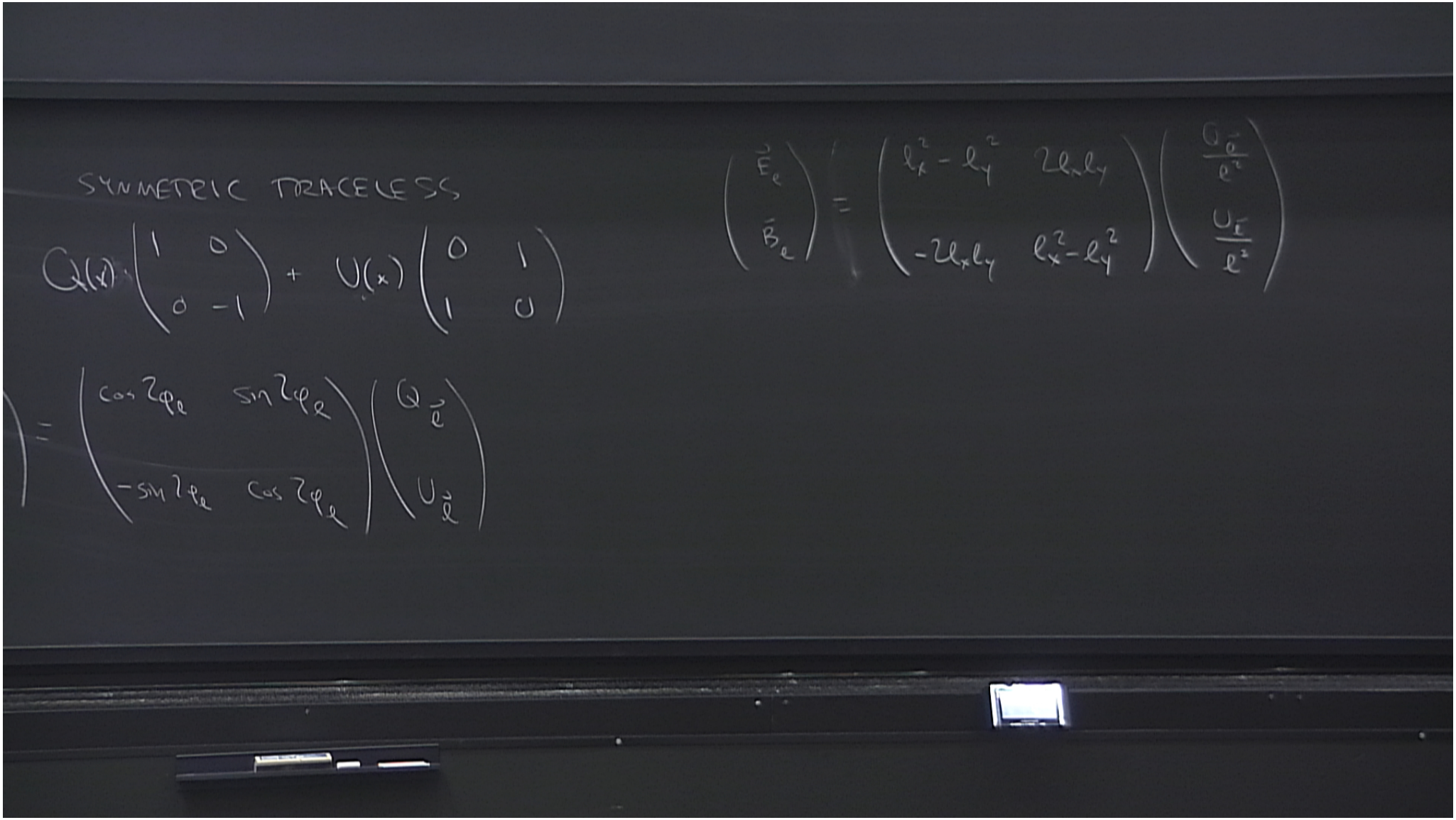
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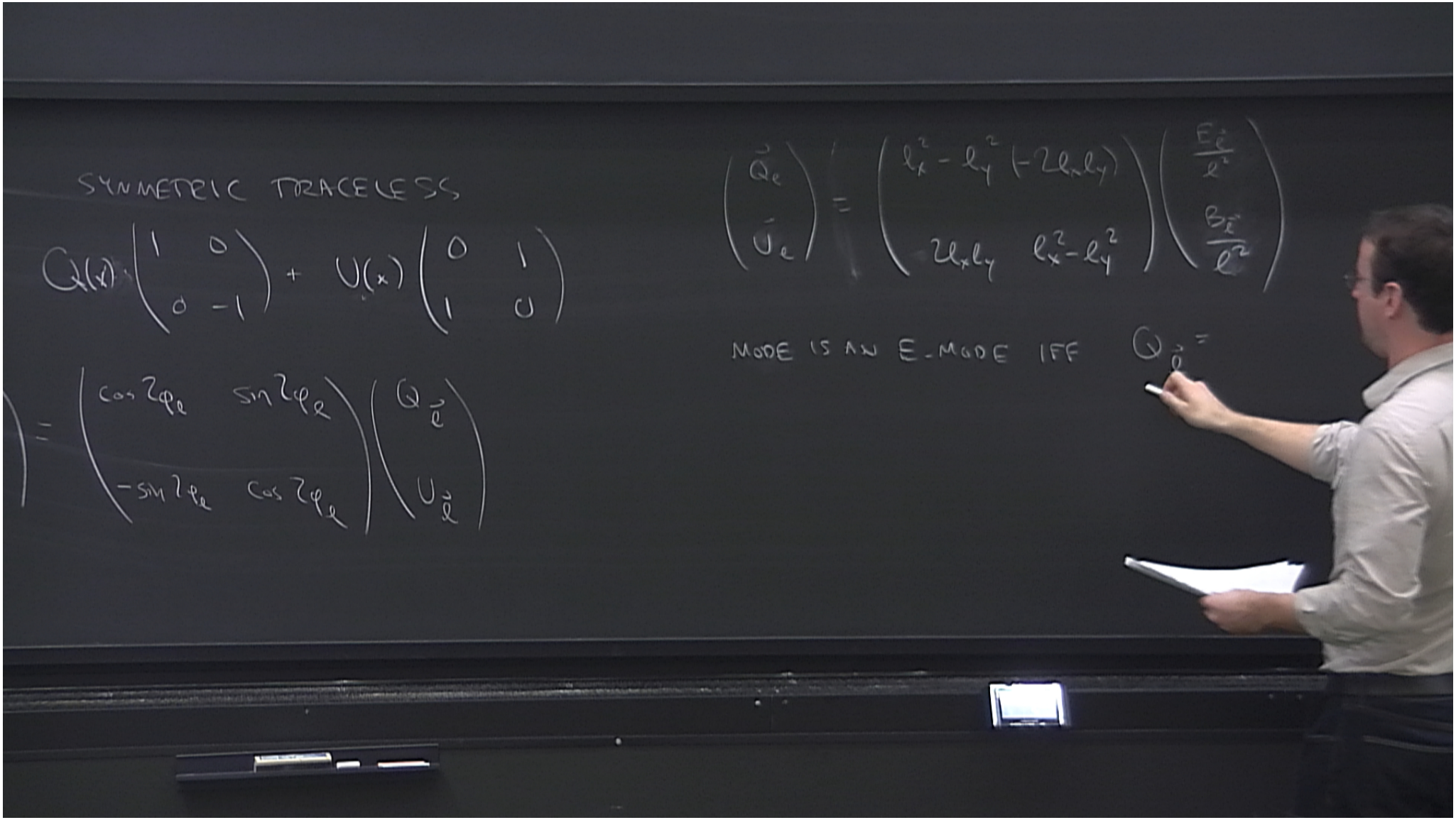
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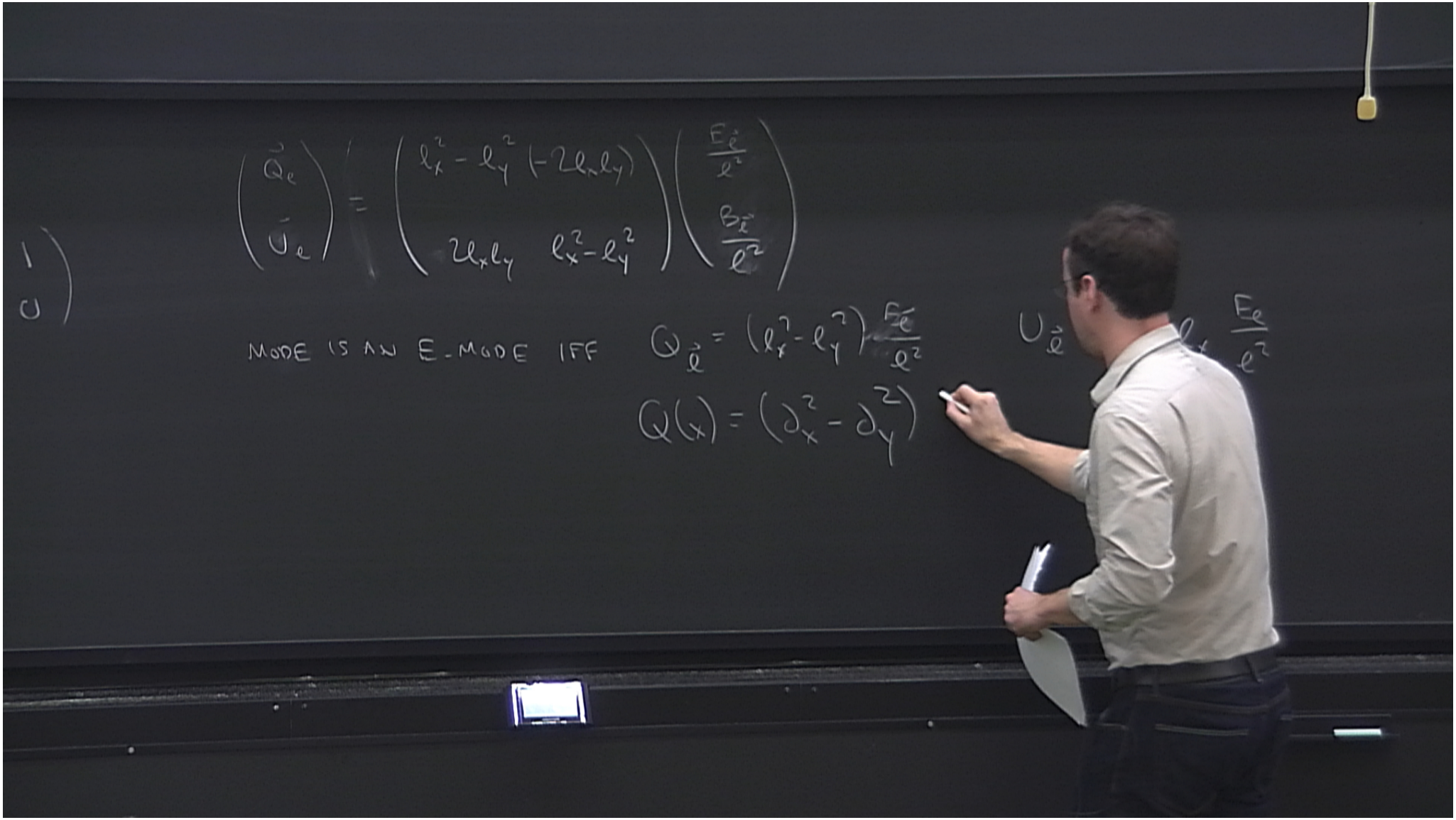
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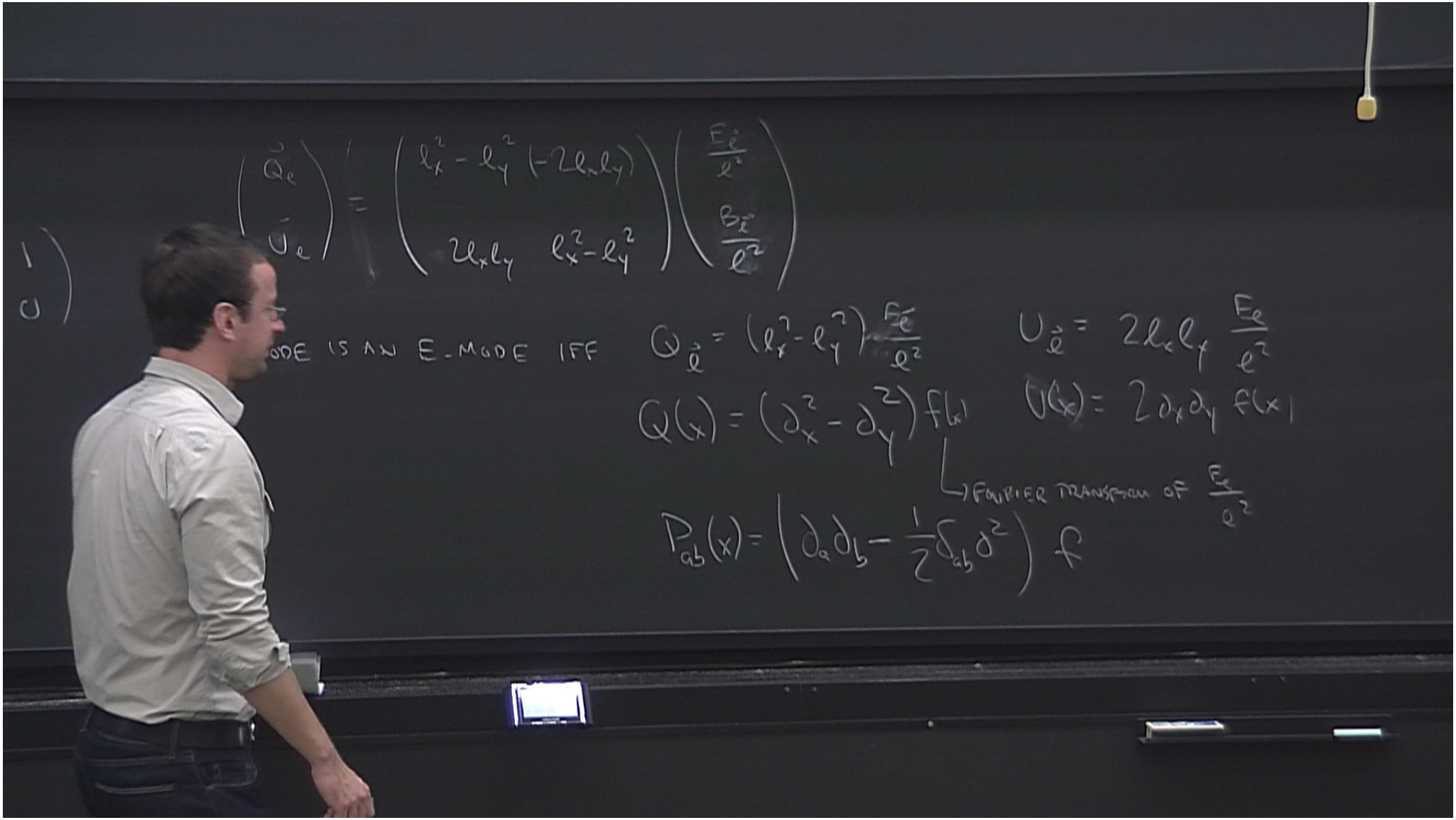
Abstract:

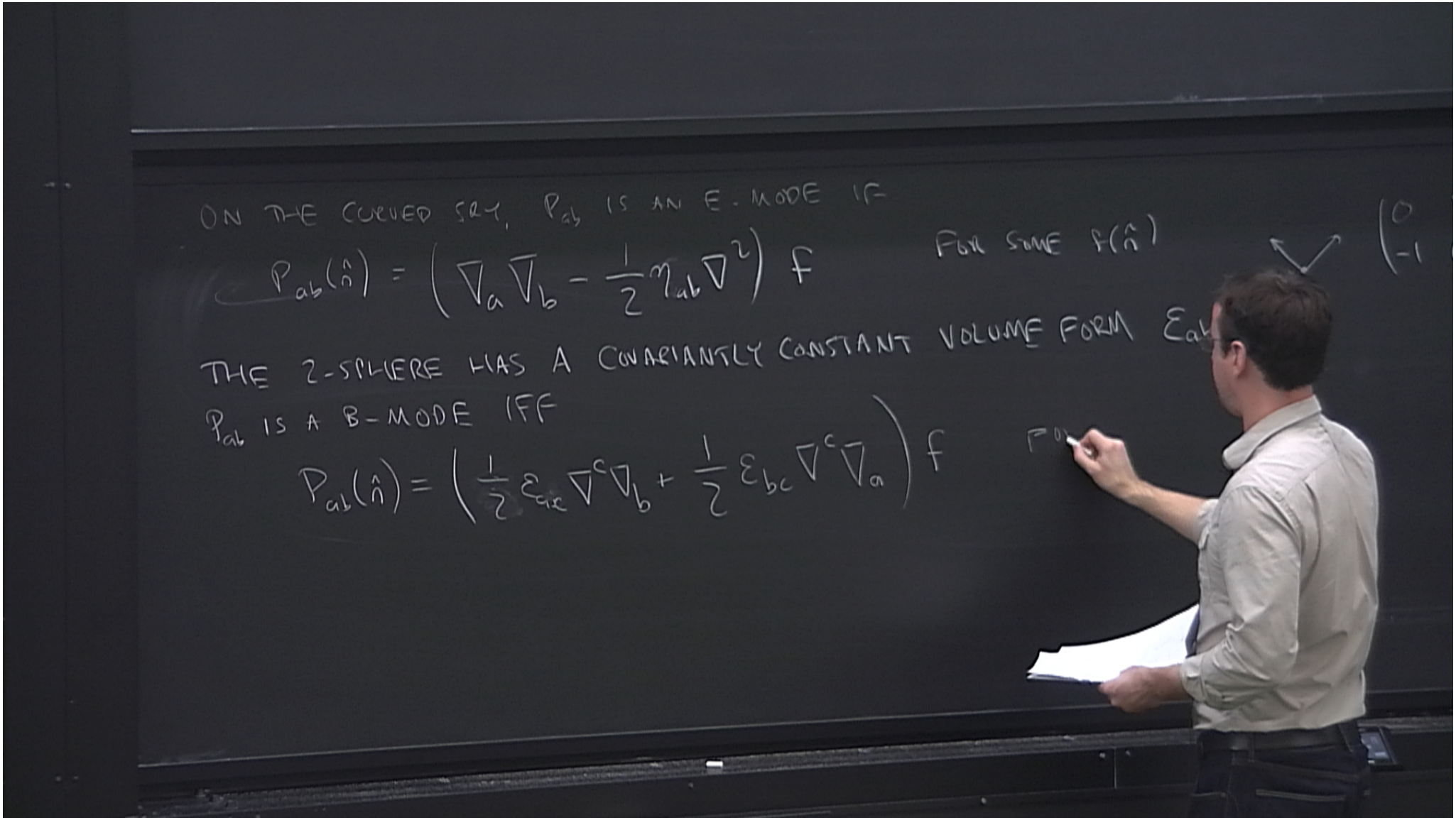

$P_{ab}(\hat{n})$ SYMMETRIC TRACELESS
$P_{ab}(\vec{x}) = Q(x)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + U(x)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$


SYMMETRIC TRACELESS
$Q(x)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + U(x)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
$= \begin{pmatrix}\cos 2\varphi_{\vec{e}} & \sin 2\varphi_{\vec{e}}\\ -\sin 2\varphi_{\vec{e}} & \cos 2\varphi_{\vec{e}}\end{pmatrix}\begin{pmatrix}Q_{\vec{e}}\\ U_{\vec{e}}\end{pmatrix}$
$\begin{pmatrix}\vec{E}_{\vec{e}}\\ \vec{B}_{\vec{e}}\end{pmatrix} = \begin{pmatrix}l_x^2 - l_y^2 & 2l_x l_y\\ -2l_x l_y & l_x^2 - l_y^2\end{pmatrix}\begin{pmatrix}\frac{Q_{\vec{e}}}{\ell^2}\\ \frac{U_{\vec{e}}}{\ell^2}\end{pmatrix}$


SYMMETRIC TRACELESS
$Q(\hat{x})\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + U(\hat{x})\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
$\begin{pmatrix}\vec{Q}_\ell\\ \vec{U}_\ell\end{pmatrix} = \begin{pmatrix}l_x^2 - l_y^2 & (-2l_x l_y)\\ 2l_x l_y & l_x^2 - l_y^2\end{pmatrix}\begin{pmatrix}\frac{E_{\vec{\ell}}}{\ell^2}\\ \frac{B_{\vec{\ell}}}{\ell^2}\end{pmatrix}$
$= \begin{pmatrix}\cos 2\varphi_\ell & \sin 2\varphi_\ell\\ -\sin 2\varphi_\ell & \cos 2\varphi_\ell\end{pmatrix}\begin{pmatrix}Q_{\vec{\ell}}\\ U_{\vec{\ell}}\end{pmatrix}$
MODE IS AN E-MODE IFF $Q_{\vec{\ell}} =$


MODE IS AN E-MODE IFF


$\begin{pmatrix} Q_{\vec{\ell}} \\ U_{\vec{\ell}} \end{pmatrix} = \begin{pmatrix} \ell_x^2 - \ell_y^2 & (-2\ell_x\ell_y) \\ 2\ell_x\ell_y & \ell_x^2 - \ell_y^2 \end{pmatrix} \begin{pmatrix} \frac{E_{\vec{\ell}}}{\ell^2} \\ \frac{B_{\vec{\ell}}}{\ell^2} \end{pmatrix}$
ODE IS AN E-MODE IFF
$Q_{\vec{\ell}} = (\ell_x^2 - \ell_y^2)\frac{E_{\vec{\ell}}}{\ell^2}$
$U_{\vec{\ell}} = 2\ell_x\ell_y \frac{E_{\vec{\ell}}}{\ell^2}$
$Q(x) = (\partial_x^2 - \partial_y^2) f(x)$
$U(x) = 2\partial_x\partial_y f(x)$
FOURIER TRANSFORM OF $\frac{E_{\vec{\ell}}}{\ell^2}$
$P_{ab}(x) = \left(\partial_a\partial_b - \frac{1}{2}\delta_{ab}\partial^2\right) f$


ON THE CURVED SKY, P_{ab} IS AN E-MODE IF
$P_{ab}(\hat{n}) = \left(\nabla_a \nabla_b - \frac{1}{2}\eta_{ab}\nabla^2\right) f$
FOR SOME $f(\hat{n})$
THE 2-SPHERE HAS A COVARIANTLY CONSTANT VOLUME FORM ε_{ab}
P_{ab} IS A B-MODE IFF
$P_{ab}(\hat{n}) = \left(\frac{1}{2}\varepsilon_{ac}\nabla^c\nabla_b + \frac{1}{2}\varepsilon_{bc}\nabla^c\nabla_a\right) f$

ON THE CURVED SKY, P_{ab} IS AN E-MODE IF

$$P_{ab}(\hat{n}) = \left(\nabla_a \nabla_b - \frac{1}{2}\eta_{ab}\nabla^2\right) f$$

FOR SOME $f(\hat{n})$

THE 2-SPHERE HAS A COVARIANTLY CONSTANT VOLUME FORM ε_{ab}

P_{ab} IS A B-MODE IFF

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FOR SOME $f(\hat{n})$

ON THE CURVED SKY, P_{ab} IS AN E-MODE IF

$$P_{ab}(\hat{n}) = \left(\nabla_a \nabla_b - \frac{1}{2}\eta_{ab}\nabla^2\right) f \qquad \text{FOR SOME } f(\hat{n})$$

THE 2-SPHERE HAS A COVARIANTLY CONSTANT VOLUME FORM ε_{ab}

P_{ab} IS A B-MODE IFF

$$P_{ab}(\hat{n}) = \left(\frac{1}{2}\varepsilon_{ac}\nabla^c\nabla_b + \frac{1}{2}\varepsilon_{bc}\nabla^c\nabla_a\right) f \qquad \text{FOR SOME } f(\hat{n})$$

THEOREM: EVERY $P_{ab}(\hat{n})$ CAN BE UNIQUELY DECOMPOSED AS $P_{ab}(\hat{n}) = \left(\nabla_a\nabla_b - \frac{1}{2}\eta_{ab}\nabla^2\right) f_E + \left(\frac{1}{2}\varepsilon_{ac}\nabla$


THEOREM :

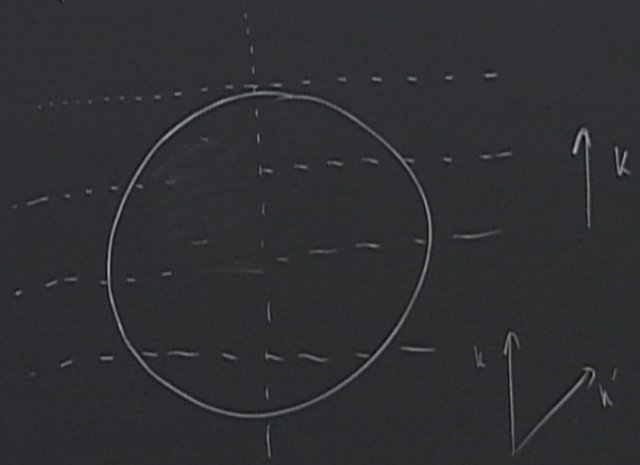
THEOREM: IN LINEAR P.T. FLUCTUATIONS IN S [OR ANY SCALAR] ONLY GENERATE E-MODES [PHYSICS ROT. INV., TRANS. INV. PARITY INV.]



$P_{ab}(\hat{n}) \rightarrow f_E = f_E(\theta) \rightarrow f_E($

$f_B = f_B(\theta)$

THEOREM: IN LINEAR P.T. FLUCTUATIONS IN S [OR ANY SCALAR] ONLY GENERATE E-MODES [PHYSICS ROT. INV., TRANS. INV. PARITY INV.]



$$P_{ab}(\hat{n}) \rightarrow f_E = f_E(\theta) \rightarrow f_E(\theta)$$
$$f_B = f_B(\theta) \rightarrow -f_B(\theta) = 0$$

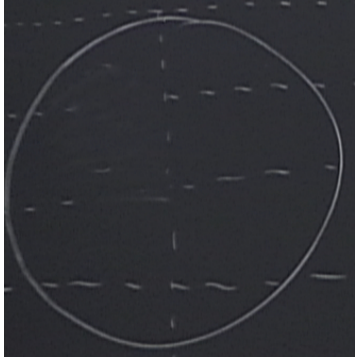
R P.T. FLUCTUATIONS IN S [OR ANY SCALAR]

GENERATE E-MODES [PHYSICS ROT. INV., TRANS. INV. PARITY INV.]

SOURCES OF B-MODES:

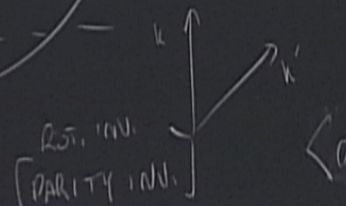
- SECOND ORDER [GRAVITA

- NON-SCALAR SOURCES



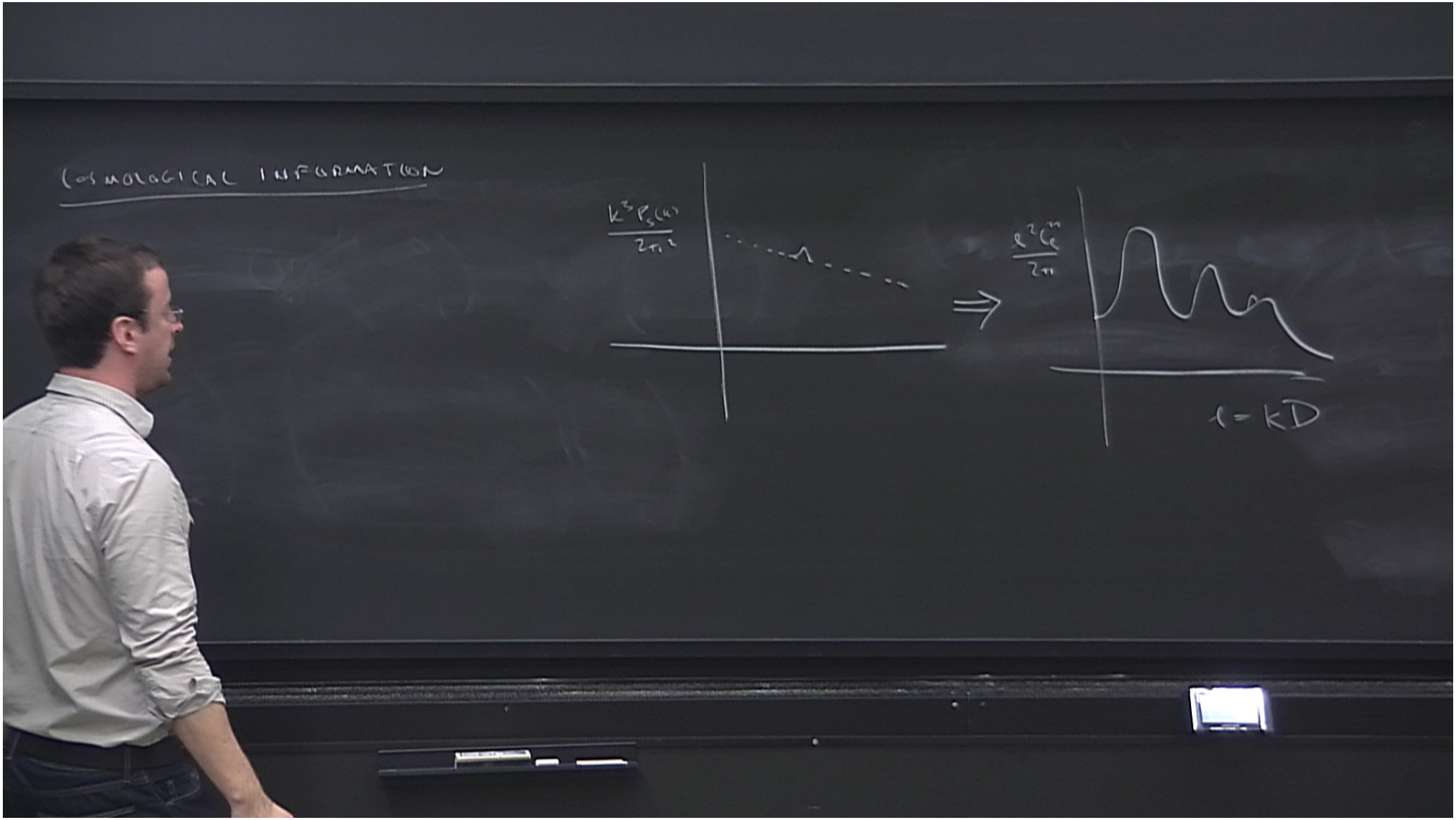
$P_{ab}(\hat{n}) \rightarrow f_E = f_E(\theta) \rightarrow f_E(\theta)$

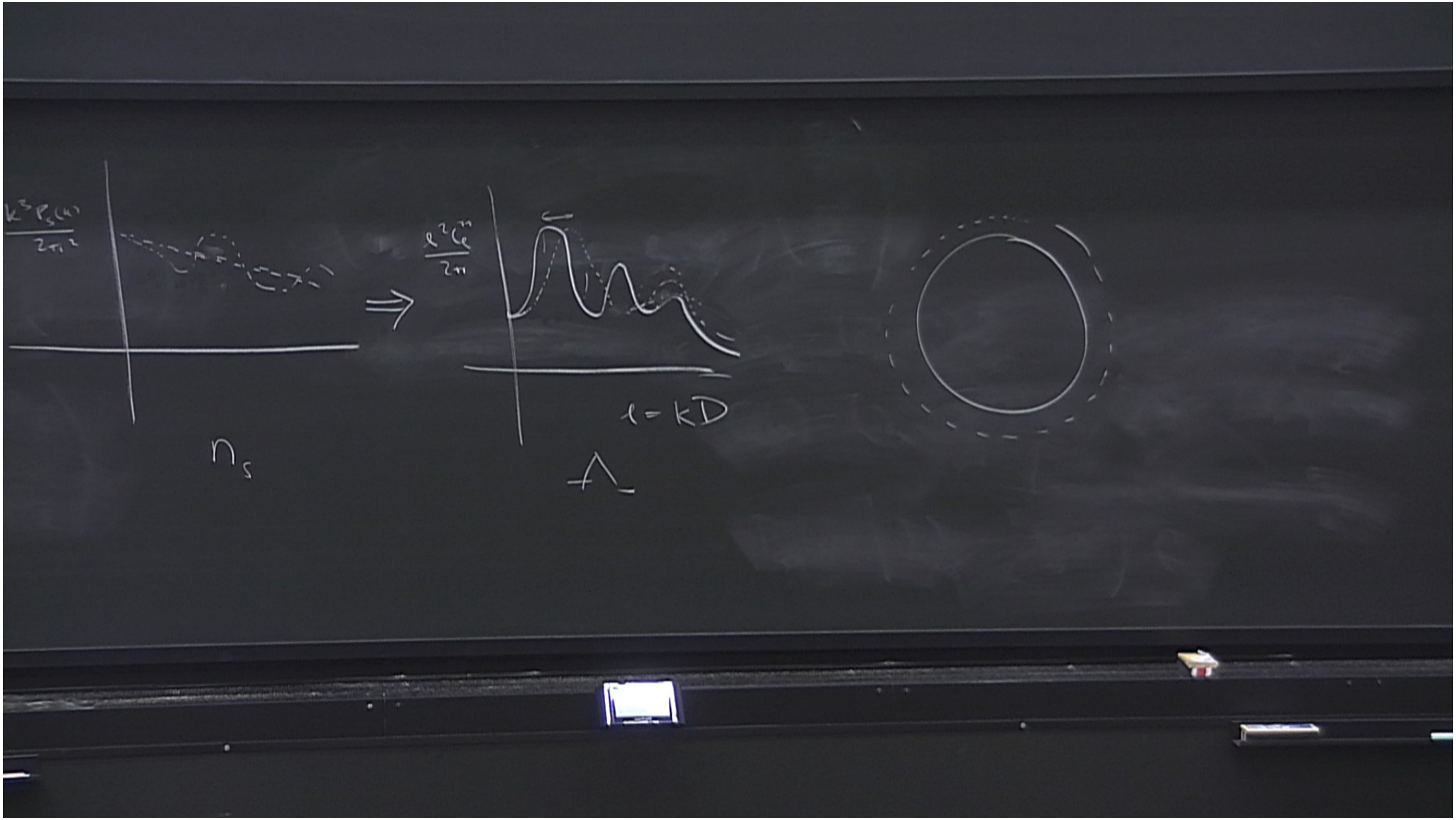
$f_B = f_B(\theta) \rightarrow -f_B(\theta) = 0$

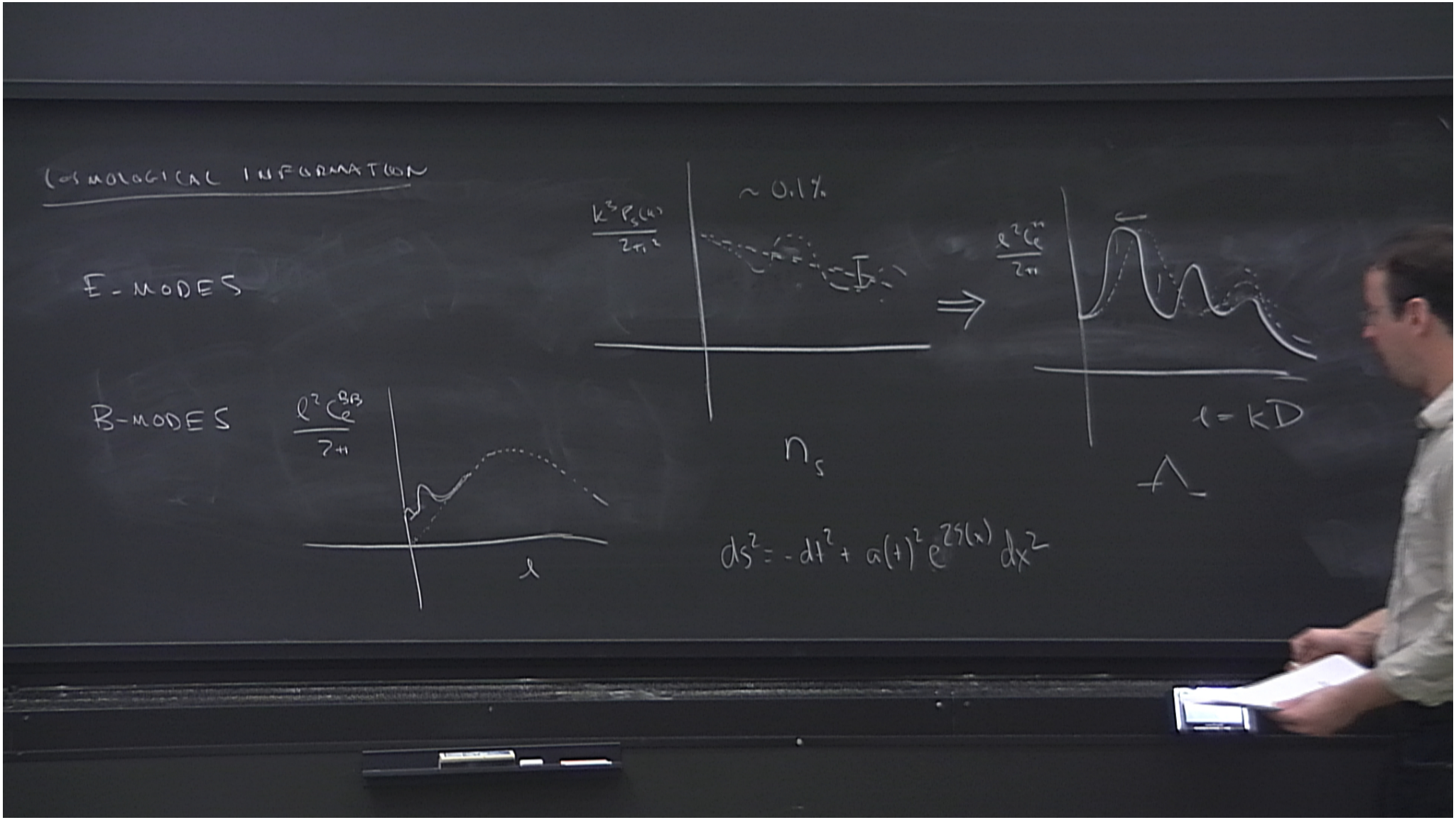


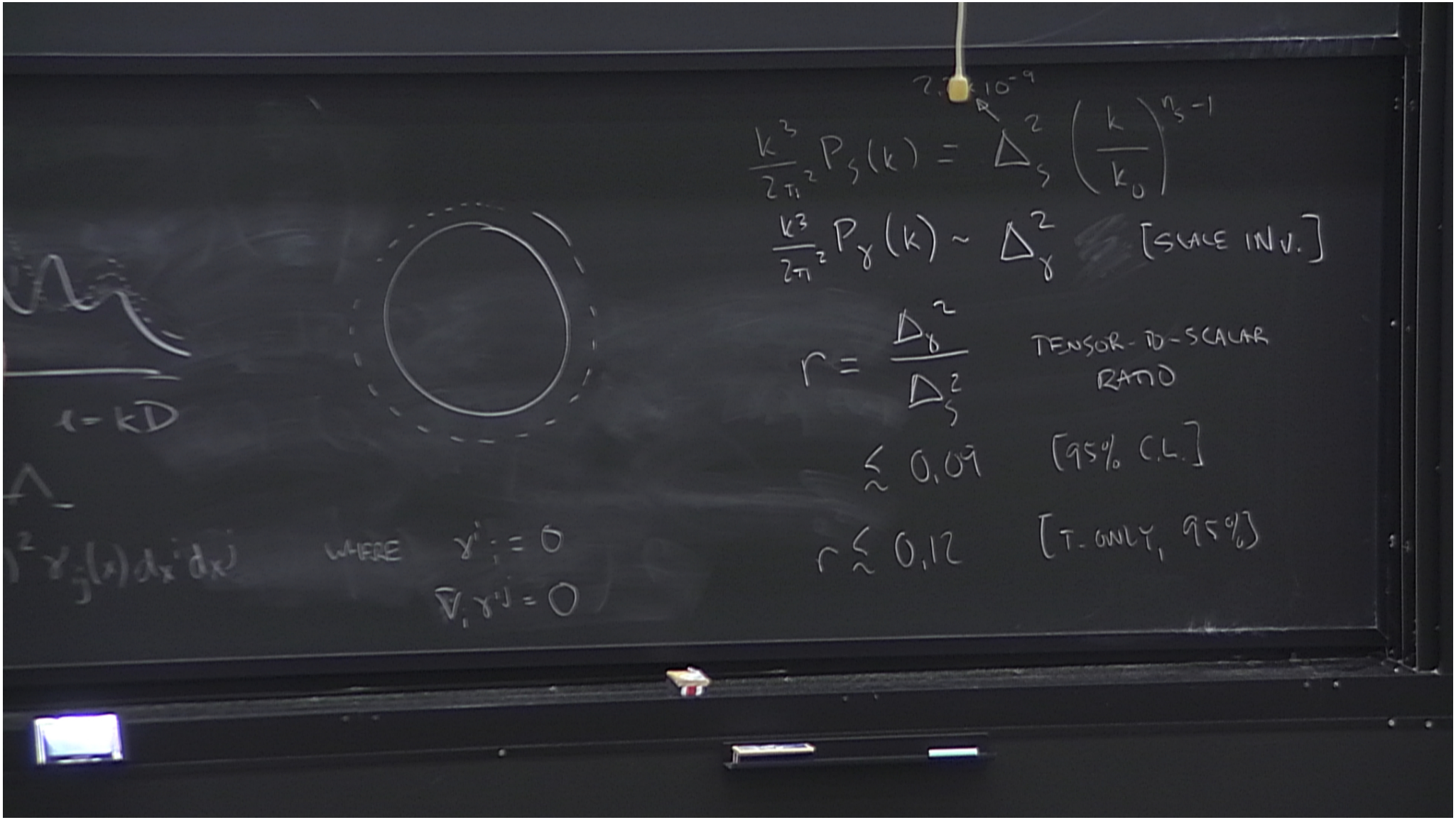
$C_\ell^{EB} = 0$

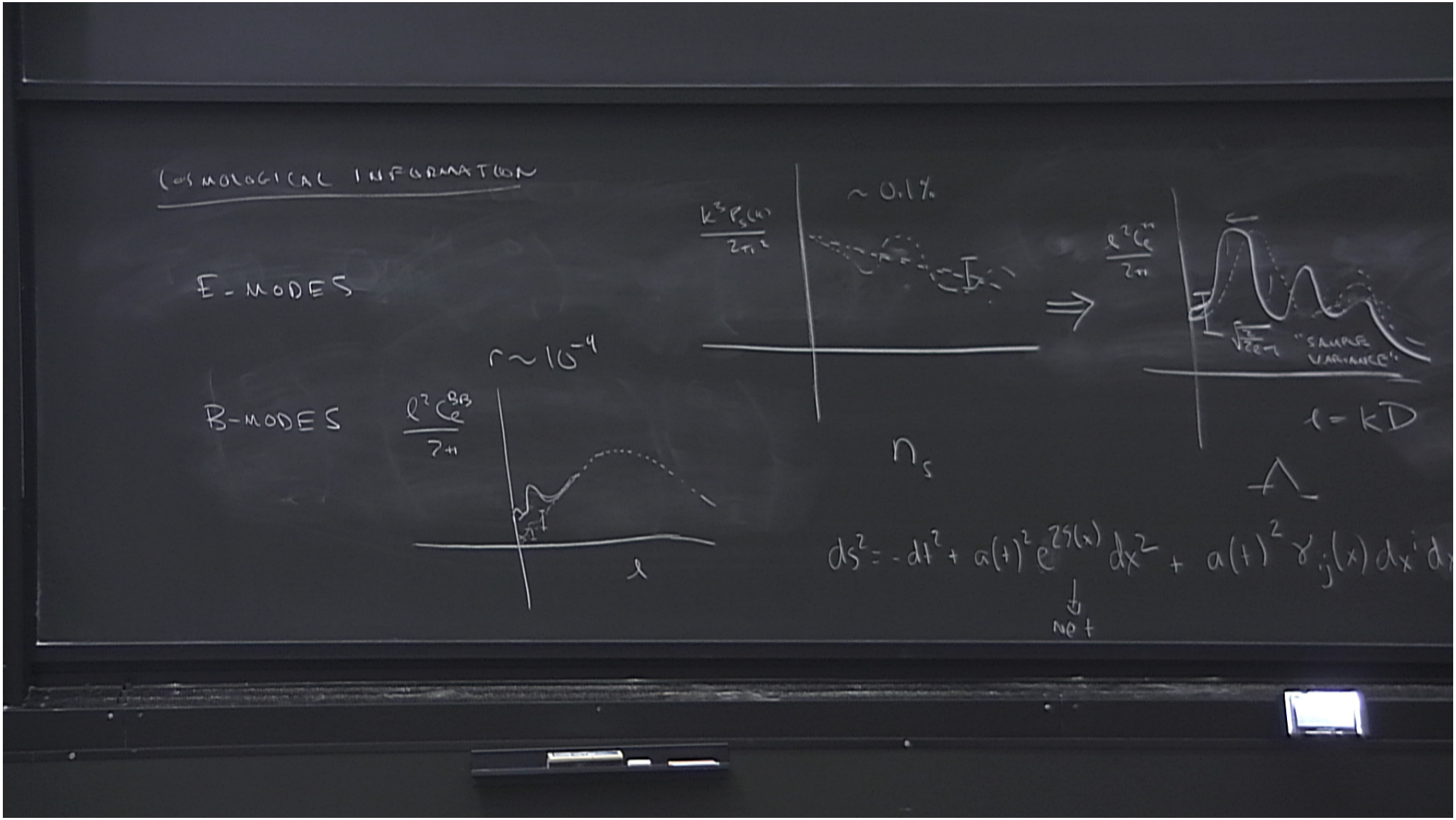
$\langle a^T_{\ell m} a^{B*}_{\ell' m'} \rangle = C_\ell^{TB} \delta_{\ell\ell'} \delta_{mm'} \Rightarrow C_\ell^{TB} = 0$


COSMOLOGICAL INFORMATION
$\frac{k^3 P_s(k)}{2\pi^2}$
$\frac{\ell^2 C_\ell}{2\pi}$
$\ell = kD$

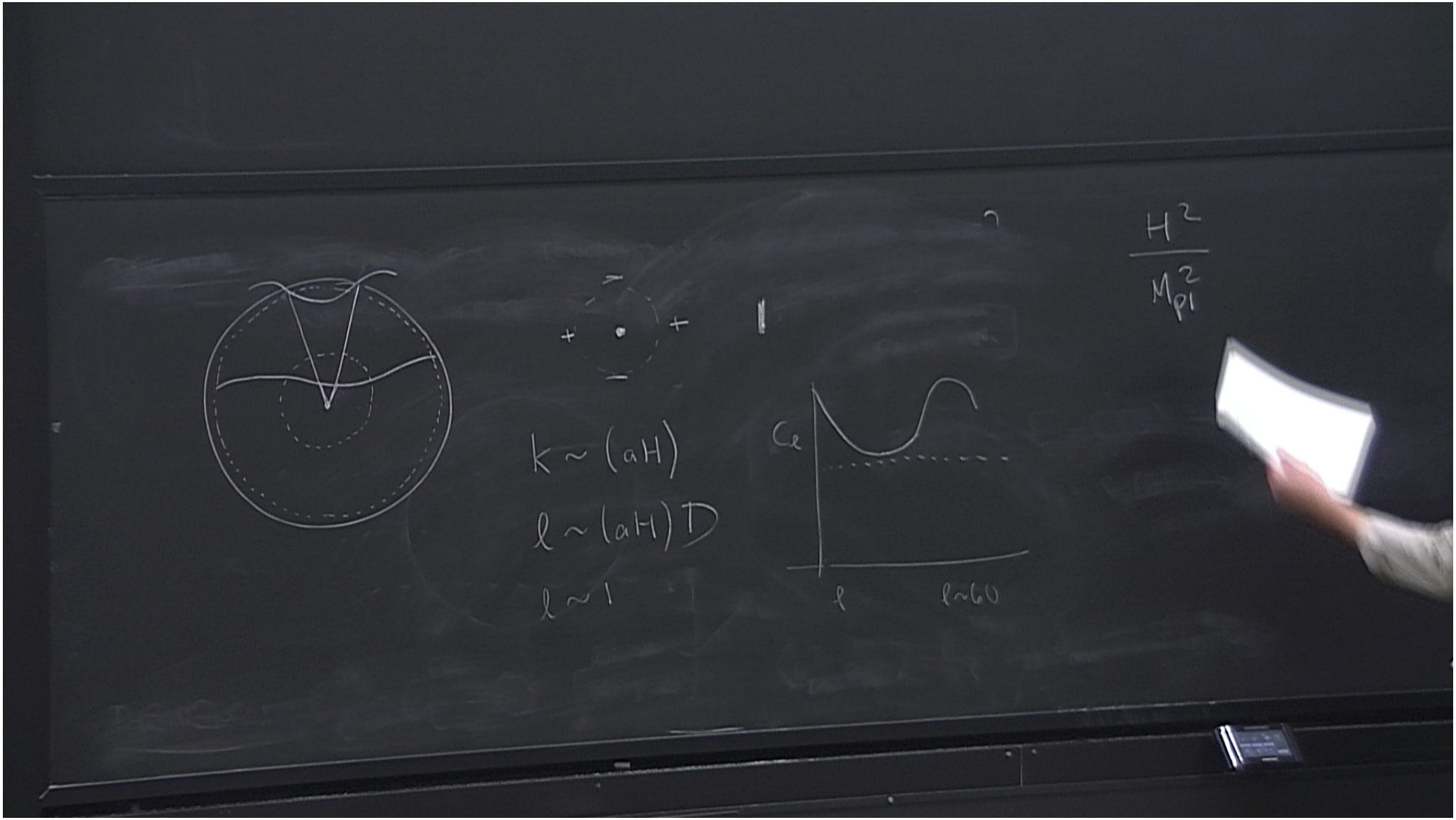

$\frac{k^3 P_s(k)}{2\pi^2}$
n_s
$\frac{\ell^2 C_\ell}{2\pi}$
$\ell = kD$
Δ_-

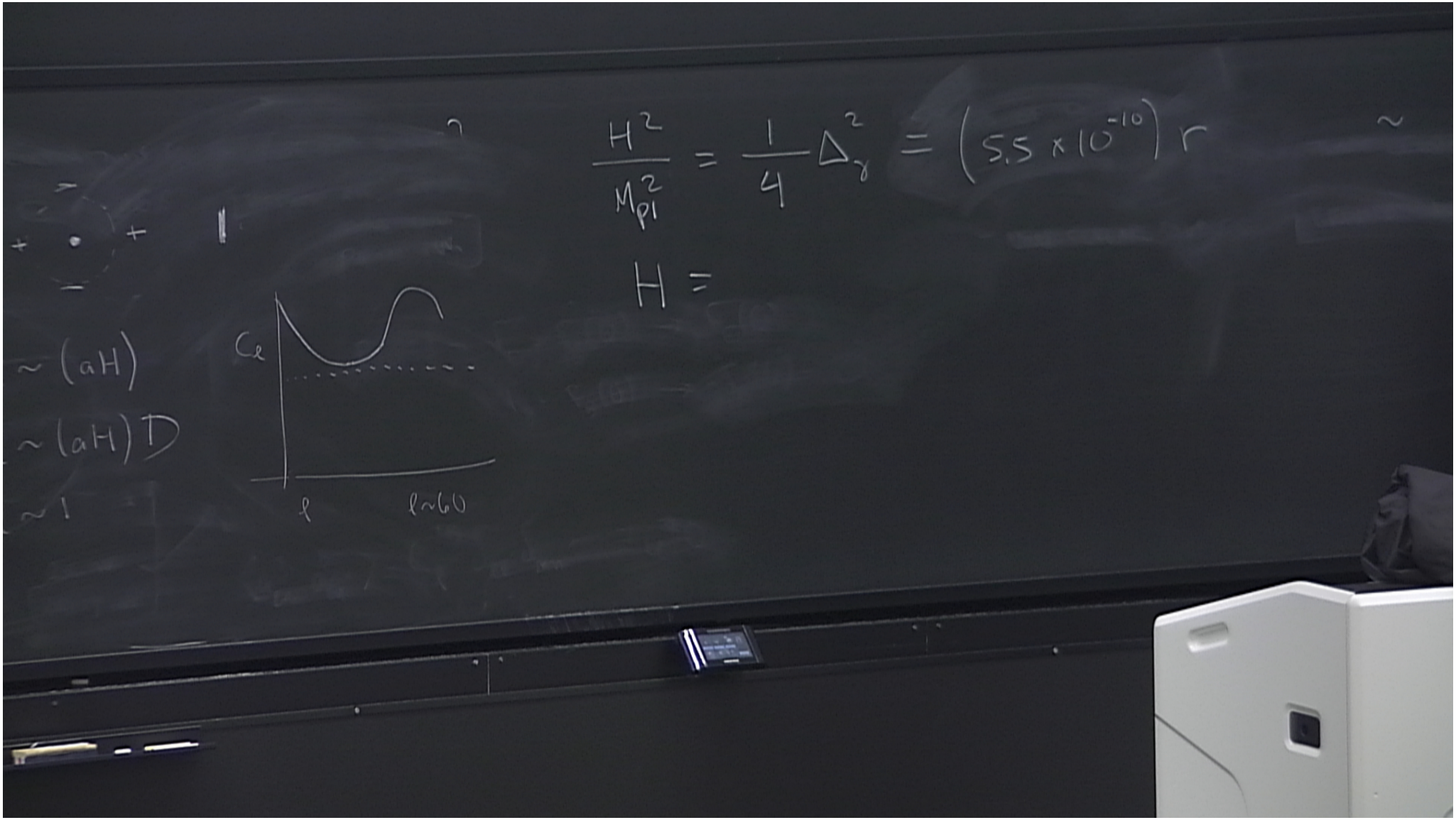

COSMOLOGICAL INFORMATION
E-MODES
B-MODES
~ 0.1%
n_s
$\ell = kD$
$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)} dx^2$

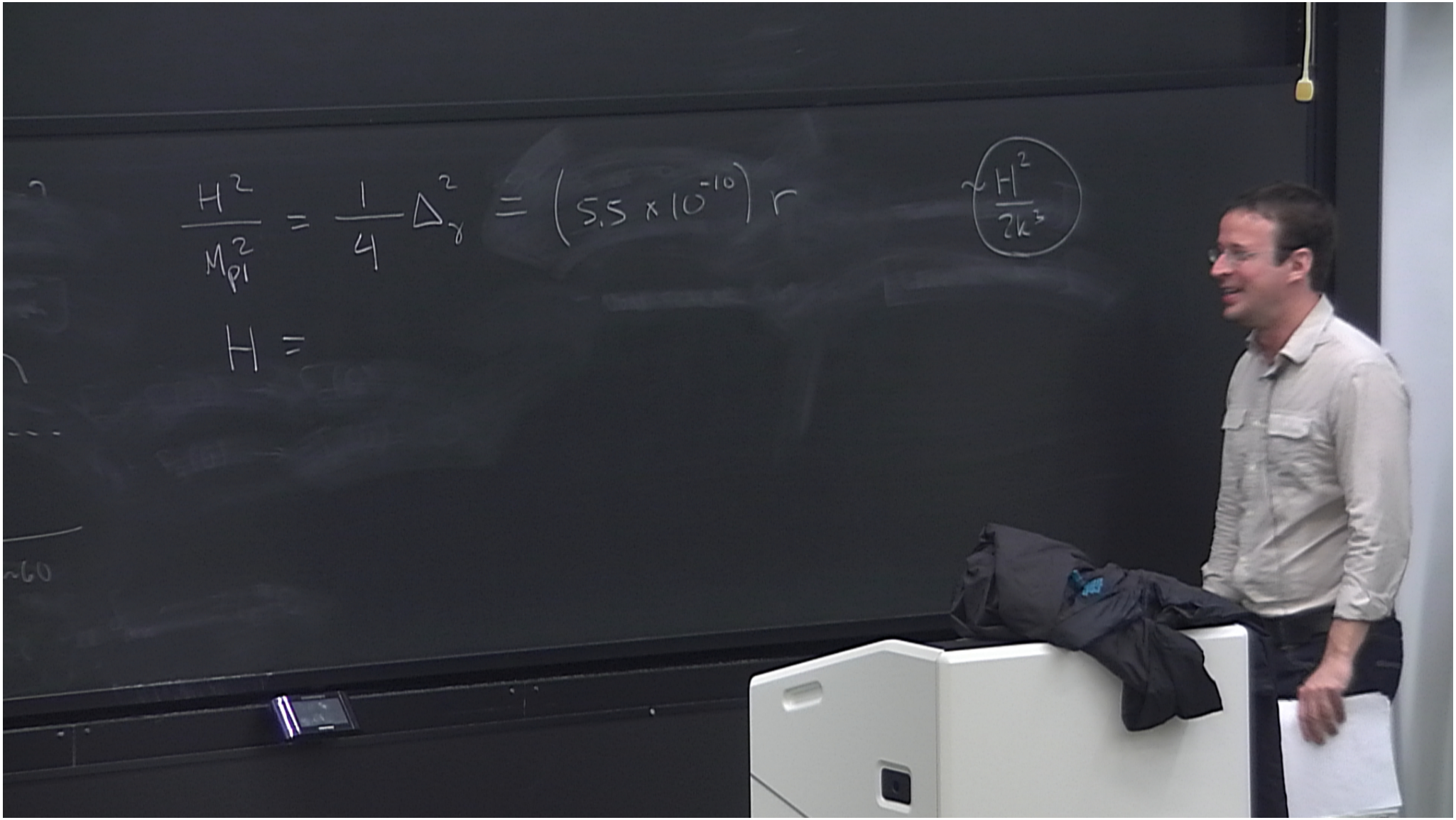

$\frac{k^3}{2\pi^2} P_S(k) = \Delta_S^2 \left(\frac{k}{k_0}\right)^{n_S - 1}$
$\frac{k^3}{2\pi^2} P_\gamma(k) \sim \Delta_\gamma^2$ [SCALE INV.]
$r = \frac{\Delta_\gamma^2}{\Delta_S^2}$ TENSOR-TO-SCALAR RATIO
$\lesssim 0.09$ [95% C.L.]
$r \lesssim 0.12$ [T. ONLY, 95%]
WHERE $\gamma^i{}_i = 0$
$\nabla_i \gamma^{ij} = 0$

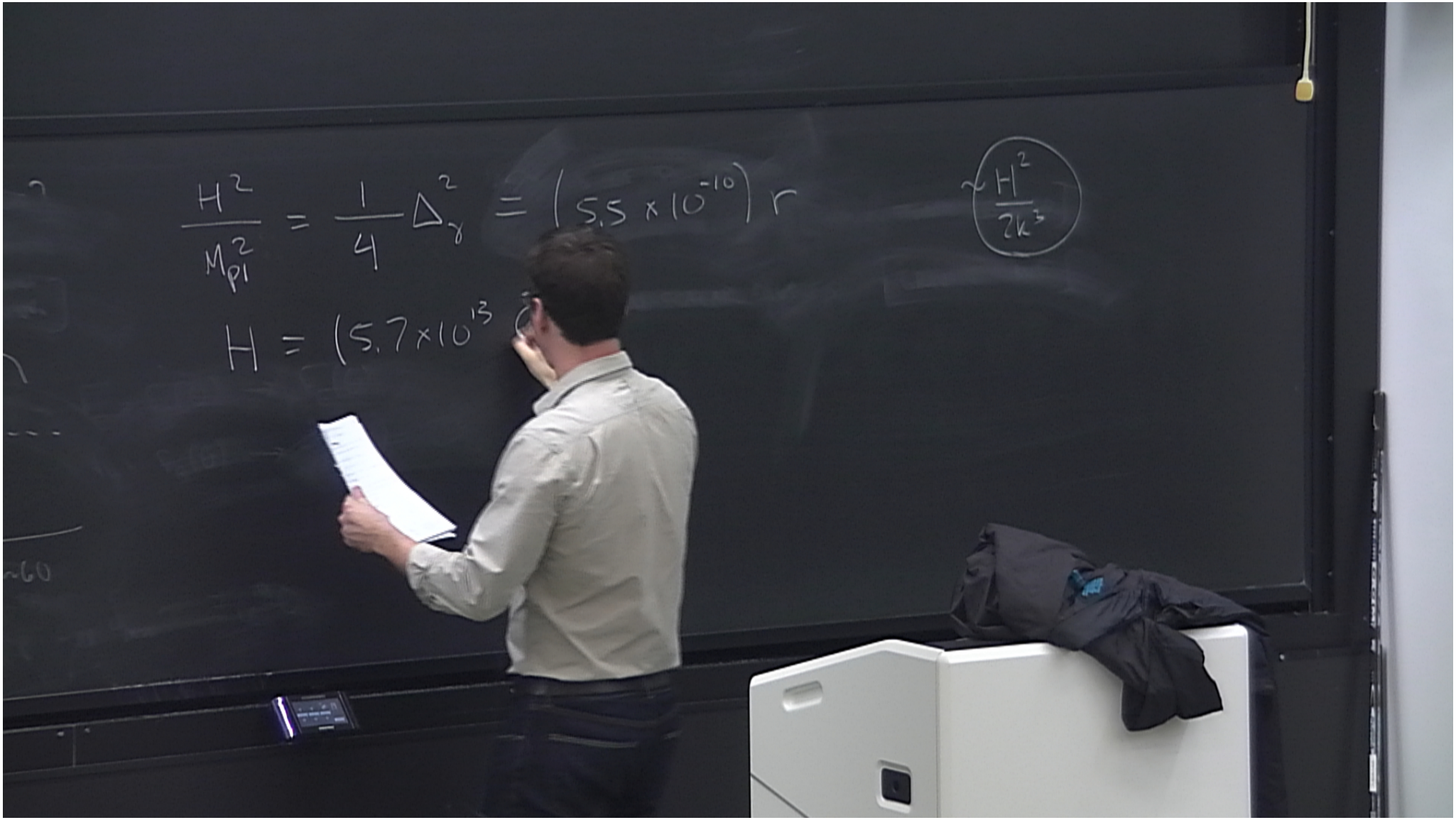

COSMOLOGICAL INFORMATION
E-MODES
B-MODES
r ~ 10^-4
~ 0.1%
"SAMPLE VARIANCE"
l = kD
n_s
ds² = -dt² + a(t)² e^{2ζ(x)} dx² + a(t)² γ_ij(x) dx^i dx
↓
NG t

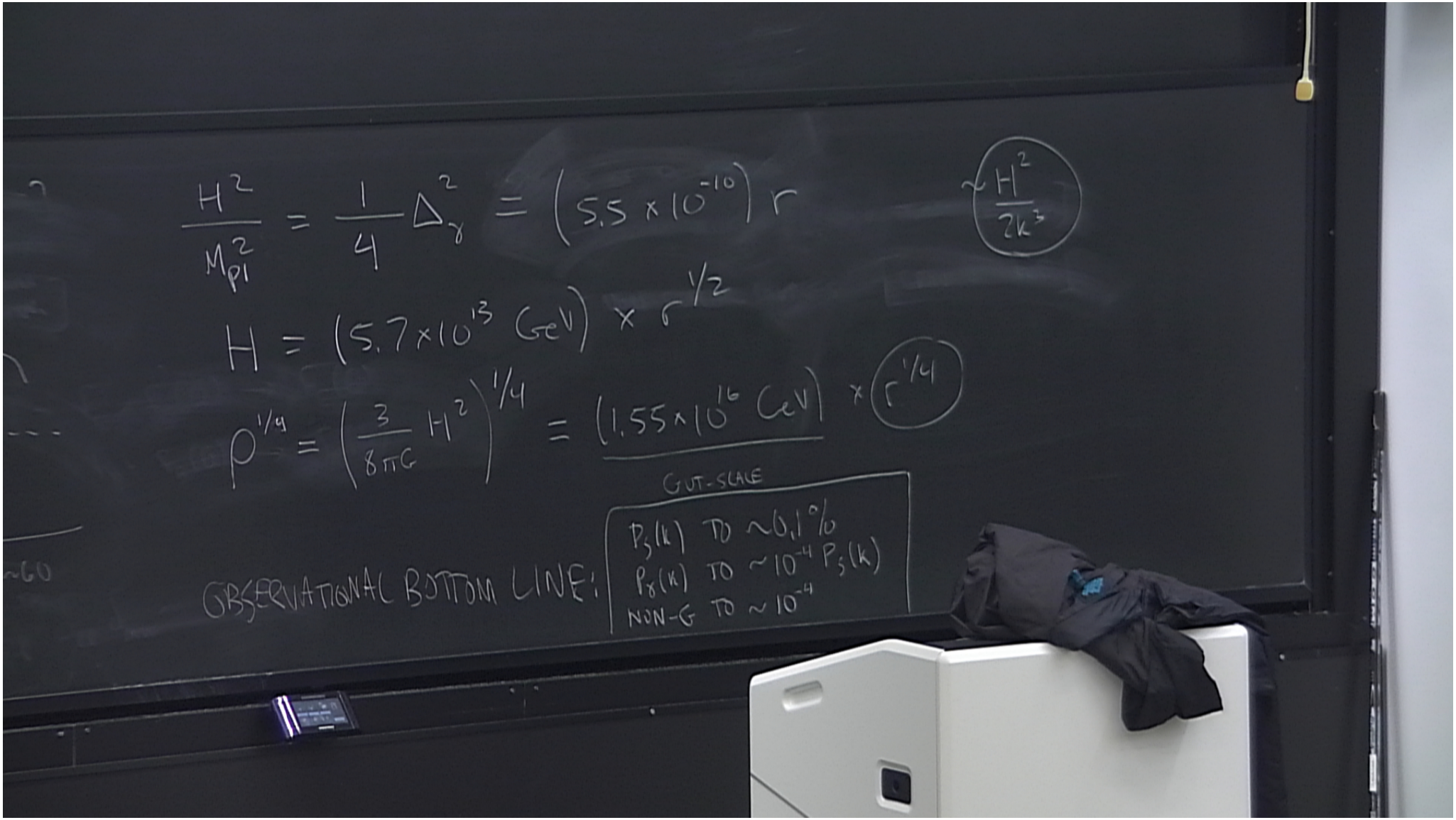



$\frac{H^2}{M_{Pl}^2}$
$k \sim (aH)$
$\ell \sim (aH)D$
$\ell \sim 1$
C_ℓ
ℓ
$\ell \sim 60$

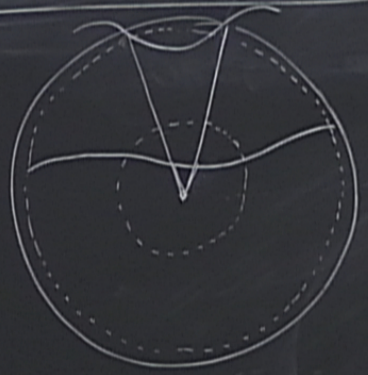

$\frac{H^2}{M_{pl}^2} = \frac{1}{4}\Delta_\gamma^2 = (5.5 \times 10^{-10})\, r$
$H =$
$\sim (aH)$
$\sim (aH)D$
~ 1
C_ℓ
ℓ
$\ell \sim 60$


$\frac{H^2}{M_{Pl}^2} = \frac{1}{4}\Delta_\gamma^2 = (5.5 \times 10^{-10})\, r$
$\sim \frac{H^2}{2k^3}$
$H =$


$\frac{H^2}{M_{Pl}^2} = \frac{1}{4}\Delta_\gamma^2 = (5.5 \times 10^{-10})\, r$
$\sim \frac{H^2}{2k^3}$
$H = (5.7 \times 10^{13}$


$\frac{H^2}{M_{Pl}^2} = \frac{1}{4}\Delta_\gamma^2 = (5.5\times10^{-10})\, r$
$\sim \frac{H^2}{2k^3}$
$H = (5.7\times10^{13}\ \text{GeV}) \times r^{1/2}$
$\rho^{1/4} = \left(\frac{3}{8\pi G}H^2\right)^{1/4} = (1.55\times10^{16}\ \text{GeV}) \times r^{1/4}$
GUT-SCALE
OBSERVATIONAL BOTTOM LINE:
$P_S(k)$ TO $\sim 0.1\%$
$P_T(k)$ TO $\sim 10^{-4}\, P_S(k)$
NON-G TO $\sim 10^{-4}$

MALDACENA ASTRO-PH/0210603

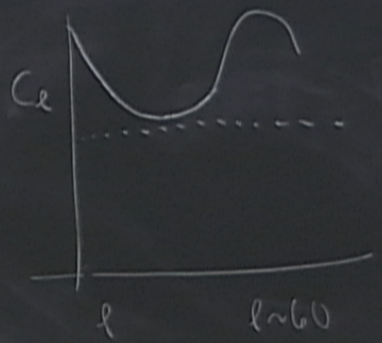




$k \sim (aH)$

$\ell \sim (aH) D$

$\ell \sim 1$



$$\frac{H^2}{M_{Pl}^2} = \frac{1}{4}\Delta$$

$H = (5.7\times$

$\rho^{1/4} = \left(\frac{3}{8\pi G}\right.$

OBSERVATIONAL B